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THEORETICAL STUDY OF THE INTERACTION
BETWEEN RADIATIVE TRANSFER AND THE
TURBULENT EXCHANGE IN THE LOWER LAYERS
OF THE ATMOSPHERE

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16. Abstract

This paper treats the effect of radiation on turbulent exchanges in the lower atmosphere. It was performed within the more general framework of atmosphere-ocean interactions. The radiation transfer is first assumed constant. Then the temperature and humidity profiles are calculated to see whether the hypothesis is valid. A method of determining the radiation flux divergence is given. An analytical technique is developed to rapidly evaluate radiation transfer effects.

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NOTATION

Ox, Oy, Oz	Axes of the reference triad
u, v, w	Components of the velocity vector along the axes Ox, Oy, Oz
T	Absolute temperature
θ	Potential temperature
f	Specific humidity
τ	Turbulent momentum flux
s	Turbulent perceptible enthalpy flux
J	Turbulent water vapor flux
R	Long wavelength radiation flux
$\mathcal{L}$	Latent heat of vaporization of water($2,5 \times 10^6$ J.kg^{-1})
k	Karman constant (0.4)
g	Acceleration of gravity
ρ	Mass per volume of air
ρ_e	Mass per volume of atmospheric water vapor
C_p	Specific heat of air (
B	Bowen ratio
u_*	Friction velocity
θ_*	Reference temperature
F_*	Specific reference humidity
ν	Kinematic viscosity coefficient
D and k	Coefficients of molecular diffusivity for water vapor and heat

k_M, k_H and k_E	Coefficient of turbulent transfer for momentum, heat and water vapor
z_o	Dynamic roughness parameter
R_i	Richardson Number
L	Monin Oboukhov length
W	Optical thickness
ω	Modified optical thickness (defined in the text)
ε	Global emissivity

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THEORETICAL STUDY OF THE INTERACTION BETWEEN RADIATIVE TRANSFER AND TURBULENT EXCHANGE IN THE LOWER LAYERS OF THE ATMOSPHERE

Bernard Seguin

INTRODUCTION

Even though it only represents a small part of the earth's atmosphere, the surface layer is of particular interest. Most physical and biological processes which support all animal and vegetable life at the surface of the earth in effect depend directly on phenomena which characterize the lower layers of the atmosphere. Also, disciplines so different such as oceanography, agronomy, hydrology, atmospheric pollution studies, etc. are of fundamental interest in this region. In contrast with meteorology, which deals with the entire atmosphere, we can speak of micrometeorology for the surface layer.

This surface layer represents a particular subdivision of the planetary boundary layer. As is known, it is the region of the atmosphere which is affected by friction caused by the earth's surface. It is also the region in which the wind direction differs from the geostrophic wind. The surface layer corresponds to the first tens of meters of the atmosphere. This region depends directly on the adjacent surface below it. The wind direction is

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*Numbers in the margin indicate pagination in foreign text.

essentially uniform. It is useful to consider transforms of momentum, heat and mass as being constant along the vertical direction.

It is sometimes assumed that turbulent phenomena can be considered as being statistically stationary for a scale on the order of twenty minutes.

On the micrometeorology scale, the atmospheric motions are essentially turbulent and the turbulence has a great influence on all the phenomena. Thus the momentum, heat and mass fluxes, the intensities of which are determined at any time by the energy equilibrium established at the surface, are turbulent fluxes. The determination of the level of these fluxes is one of the main objectives of studies carried on in micrometeorology.

The direct measurement of these quantities from recording of fluctuations of various quantities has the advantage of not making use of any hypotheses. Nevertheless, it encounters experimental difficulties. Also, usually indirect measurements are preferred which use average values of the corresponding quantities. Semi-empirical theories are used and involve the notion of the turbulent transfer coefficient. The accuracy obtained essentially depends on the degree to which the various hypotheses used are valid. These are treated in a special study which make it possible to define the experimental conditions which must be maintained in order to minimize the errors which are introduced. /2

One of the hypotheses used in general is that the radiation heat flux remains approximately constant through the surface layer and consequently does not interfere with the turbulent heat flux which can then be calculated independently. Even though this hypothesis is explicitly or implicitly assumed in most work on the turbulent structure of the surface layer, it nevertheless

has been seriously placed/in doubt in recent studies.

The work which we will present here attempts to estimate the variations in radiation transfer through the surface layer of the atmosphere. We will estimate their effects on the distribution of turbulent heat fluxes in this region.

The present paper has the following plan:

- After recapitulating various simplifying hypotheses which lead to assuming that the vertical turbulent fluxes are conserved (Chapter I), the method of determining these fluxes by semi-empirical theories is treated (Chapter II). We will stress the hypothesis which involves the radiation transfer, and/the steps to control it are presented: the radiation transfer is initially assumed to be constant and then its vertical distribution is calculated from theoretical temperature and humidity profiles which result. This makes it possible to determine whether the initial hypothesis is valid or not (Chapter III). The theoretical temperature and humidity profiles are studied first of all (Chapter IV). Then the method of determining the divergence of the radiation flux from these profiles is presented (Chapter V). The details of the numerical calculations as well as their results are given (Chapter VI). An analytic method makes it possible to rapidly estimate the radiation transfer effects (Chapter VII). The general conclusions are finally given in (Chapter VIII).*

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*This paper was done within the more general framework of atmospheric-ocean interaction studies carried out at the I.M.S.T. Only the particular case of the ocean was considered in numerical applications. The results would be applicable to an arbitrary natural surface.

I. THE CLASSICAL THEORY AND NOTION OF "CONSTANT FLUX LAYER"

The theory of the turbulent surface layer of the atmosphere has been discussed in numerous works of which we would like to mention Sutton (1953), Priestley (1959), Lumley and Panofsky (1964), Monin and Yaglom (1966), Munn (1966), Zilitinkevitch (1970).

In the particular case of the ocean-atmosphere interface, the basic equations are presented and discussed in the article of Coantic (1969) which inspired the following paragraphs.

11. Simplified equations applicable to the atmosphere surface layer

The averaged equations applicable to the surface layer are established using the general statistical equations of atmosphere mechanics. The particular properties of the region under consideration will be taken into account:

- The variations and fluctuations of the specific mass are sufficiently small so that they do not influence either the definition of the macroscopic quantities (Favre 1965) nor the form of the averaged equations which follow.

- The effects of Coriolis forces can be assumed to be negligible. The interaction phenomena are then two dimensional on the average.

- These phenomena have a boundary layer character: the variation of the various variables is much more rapid along the vertical direction than along the direction of motion.

The following simplifications result:

$$\overline{V} = 0 \qquad (1)$$

$$\frac{\partial}{\partial y}(\overline{X}) = 0 \qquad (2)$$

$$\frac{\partial}{\partial z}(\overline{X}) \gg \frac{\partial}{\partial x}(\overline{X}) \qquad (3)$$

The equations which control the turbulent transfers of momentum, water vapor and enthalpy in the lower layers of the atmosphere are then written as:

$$\left\{ \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{w}\frac{\partial}{\partial z} \right\}(\rho\overline{u}) + \frac{\partial \overline{P}}{\partial x} = -\frac{\partial}{\partial z}\left\{ -\mu\frac{\partial \overline{u}}{\partial z} + \rho\,\overline{u'w'} \right\} \qquad (4)$$

$$\left\{ \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{w}\frac{\partial}{\partial z} \right\}(\rho\overline{F}) = -\frac{\partial}{\partial z}\left\{ -\rho D\frac{\partial \overline{F}}{\partial z} + \rho\,\overline{F'w'} \right\} \qquad (5)$$

$$\left\{ \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{w}\frac{\partial}{\partial z} \right\}(\rho C_p\overline{\theta} + \rho\mathscr{L}\overline{F}) = -\frac{\partial}{\partial z}\left\{ -k\frac{\partial \overline{\theta}}{\partial z} + \rho C_p\overline{\theta'w'} \right.$$
$$\left. -\rho\mathscr{L}D\frac{\partial \overline{F}}{\partial z} + \rho\mathscr{L}\overline{F'w'} + R \right\} \qquad (6)$$

and the equation of continuity takes on the simple form

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{w}}{\partial z} = 0 \qquad (7)$$

The simplifications introduced up to the present are generally valid. However, the system consisting of equations (4), (5), (6)

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and (7) is too complex to be easily solved. It can be simplified using additional hypotheses and of course there will be loss of generality.

12. Hypothesis of horizontal homogeneity

The processes studied are assumed to occur over horizontal ranges which are sufficiently large so that the phenomena can be considered to be "completely developed". In other words the averaged derivatives along the direction Ox can be completely ignored

$$\frac{\partial}{\partial x}(\bar{x}) = 0 \qquad (8)$$

In the case of the earth's surface, this hypothesis is sometimes open to question because of the existence of various heterogeneous features of various types. Numerous studies, theoretical as well as experimental, have been carried out recently on the subject. They make it possible to delimit the range of validity of the preceding hypothesis or the minimal horizontal distance in the wind direction from a heterogeneous region and for a given observation height (see for example Elliott 1958, Dyer 1963, Rider, Philip and Bradley 1963, Panofsky and Townsend 1964, Peterson 1969 etc...). In the particular case of the ocean, if we exclude coastal regions and contact regions of marine currents having different thermal properties, this hypothesis can be used much more extensively, because of the homogeneous nature of the ocean surface. /5

The advection terms in the first term of the preceding equations can then be ignored and the system is reduced to

$$\frac{\partial}{\partial t}\left(\rho \bar{u}\right) = -\frac{\partial}{\partial z}\left\{-\mu \frac{\partial \bar{u}}{\partial z} + \rho \overline{u'w'}\right\} \qquad (9)$$

$$\frac{\partial}{\partial t}\left(\rho \bar{F}\right) = -\frac{\partial}{\partial z}\left\{-\rho D \frac{\partial \bar{F}}{\partial z} + \rho \overline{F'w'}\right\} \qquad (10)$$

$$\frac{\partial}{\partial t}\left(\rho C_p \bar{\theta}\right) = -\frac{\partial}{\partial z}\left\{-\kappa \frac{\partial \bar{\theta}}{\partial z} + \rho C_p \overline{\theta'w'} + R\right\} \qquad (11)$$

13. Steady state hypothesis

Even though the question is still being discussed today, it
is generally assumed that the spectrum of dynamic turbulence in
the atmospheric surface layer is separated into two distinct parts
by a spectral gap which extends from a few tens of minutes to
several hours (see, for example, Lumley and Panofsky 1964, Panofsky
1969). These phenomena can consequently be considered as statis-
tically steady over a time interval which varies from five to
thirty minutes, depending on atmospheric conditions. Also,
except for transition periods (such as during the rising and
setting of the sun or when clouds pass by) the fundamental calori-
metric input produced by the sun's radiation varies little, and
the energy equilibrium can be considered as being established.

We can therefore write:

$$\frac{\partial}{\partial t}\left(\bar{x}\right) = 0 \qquad (12)$$

and the system of equations becomes

$$\frac{\partial}{\partial z}\left\{-\mu\frac{\partial \bar{u}}{\partial z} + \rho\overline{u'w'}\right\} = \frac{\partial \tau}{\partial z} = 0 \qquad (13)$$

$$\frac{\partial}{\partial z}\left\{-\rho D\frac{\partial \bar{F}}{\partial z} + \rho\overline{F'w'}\right\} = \frac{\partial J}{\partial z} = 0 \qquad (14)$$

$$\frac{\partial}{\partial z}\left\{-K\frac{\partial \bar{\theta}}{\partial z} + \rho C_p\overline{\theta'w'} + R\right\} = \frac{\partial}{\partial z}\left\{S + R\right\} = 0. \qquad (15)$$

The total momentum flux τ, the water vapor flux J and the enthalpy flux S + R , are then constant along the vertical direction.

Over almost all of the surface layer the transfer by molecular diffusion can be ignored with respect to the turbulent transport, and we can therefore write

$$\tau = \rho\overline{u'w'} = \tau_0 \qquad (16)$$

$$J = \rho\overline{F'w'} = J_0 \qquad (17)$$

$$S + R = S_0 + R_0 \quad \text{with} \quad S = \rho C_p\overline{\theta'w'} \qquad (18)$$

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14. Hypothesis regarding the conservation of radiation flux

The validity of the hypotheses which we have mentioned can be checked in each particular case, as we have seen. For example it is sufficient to verify that the selected space and time regions have been appropriately selected.

The system of equations (16), (17), (18) is then strictly valid. There are no turbulent quantities involved except for the radiation flux R, which has a different fundamental nature and cannot be determined using the same methods. In a general way, more or less implicitly, this radiation flux is assumed to be constant over the thickness of the surface layer. This hypothesis, which is justified because of the small atmospheric thickness (a few tens of meters) considered, as well as the high transparency of humid air, seems to be corroborated by the agreement between the theoretical positions and the experimental results which are most often observed.

It follows that the turbulent fluxes of momentum, water vapor, and enthalpy are in general considered as constants along the vertical direction

$$\tau = \tau_0 \tag{16}$$

$$J = J_0 \tag{17}$$

$$S = S_0 \tag{18}$$

This is why the surface layer is often called the "constant flux layer".

II. SEMI-EMPIRICAL RELATIONSHIPS BETWEEN THE PROFILES AND THE FLUXES

21. Methods of determining the turbulent fluxes

There are numerous such methods which use various techniques (see Readings, et al. 1969). Only two of them use the properties of turbulent flows. We will not mention the others here without evaluating their correctness.

In a general way, if χ is any one of the properties related to mass, the turbulent flux corresponding to it is of the form

$$\phi(\chi) = \rho \, \overline{w'\chi'} \tag{19}$$

Its value can thus be obtained by simultaneously recording instantaneous fluctuations of the property χ and the vertical component w at an arbitrary height z. The problem is essentially an experimental one connected with accuracy, response time and space definition of the transducers. The measurement of Reynolds stresses by this procedure is a classical one. The measurement of the heat flux is relatively well known (Businger et al. 1967). On the other hand, the determination of the water vapor flux (such as the carbon dioxide flux) is a delicate problem because of the difficulty of measuring humidity fluctuations (Dyer and Maher 1965, Miyake and Mac Bean 1970, Leducq 1970).

In spite of progress made in this direction, the required instruments are complex and expensive and are relatively delicate, which limits their use at the present time. This explains the

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development of other methods which require the introduction of additional hypotheses but which can be used in a relatively simple way. The fluxes are simply related to the vertical profiles of the average values of the corresponding properties by means of the turbulent transfer coefficients. These coefficients, defined in analogy to the molecule transfer coefficients, are determined in part by theoretical considerations and in part by empirical extensions which are the results of experimental data. This explains the name semi-empirical given to the method in question.

22. <u>Expression for the turbulent transfer coefficients</u>

The various coefficients are defined as

$$\tau = -\rho k_M \frac{\partial \bar{u}}{\partial z} \tag{20}$$

$$J = -\rho K_E \frac{\partial \bar{f}}{\partial z} \tag{21}$$

$$S = -\rho C_p K_H \frac{\partial \bar{\theta}}{\partial z} \tag{22}$$

The averaged values are obtained over conveniently selected time intervals (see paragraph 13).

k_M, k_H and k_E are turbulent transfer coefficients for the momentum, heat and water vapor, respectively, and we will briefly

recall the presently used expressions.

In the first approximation, we consider the case of adiabatic conditions. The gravity forces, or if we prefer, the effects of thermal stratification, are then negligible (in the surface layer, the adiabatic conditions are essentially equivalent to the thermal neutrality conditions defined by * 0).

The situation is analogous to the situation encountered in the laboratory in the internal part of the boundary layer over a rough plate. A logarithmic velocity distribution in the vertical direction results which is identical to the classical "wall law".

$$K_M = k\,u_*\,z \implies \frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k\,z}$$ and, by integration (23)

$$\bar{u}(z) = \frac{u_*}{k}\,\mathrm{Log}\,\frac{z}{z_0}$$ (24)

z_0 is an integration constant called the roughness parameter which translates the overall dynamic effects which are the result of the nature, height and spatial distribution of the surface uneveness.

Under adiabatic conditions, the various turbulent transfer processes depend only on the dynamic process: the various properties are transported in a passive way. In general it is assumed that the transfer coefficients are related by the simple relationships $\quad \frac{K_H}{K_M} = \alpha_H^0 \quad$ and $\quad \frac{K_E}{K_M} = \alpha_E^0$

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*Translator's note: Figures missing in foreign text.

where α_H^o and   are the inverses of the "turbulent Prandtl and Schmidt numbers".

In analogy with the friction velocity u_*, we can define a reference temperature θ_* and a reference humidity F_* such that

$$F_* = - \frac{J}{\rho k u_*}$$

(25)

$$\theta_* = - \frac{S}{\rho C_p k u_*}$$

(26)

The temperature and humidity profiles are then written in the form (Zilitinkevitch 1970, Monin 1970)

$$\theta(z) - \theta_o = \frac{\theta_*}{\alpha_H^o} \log \frac{z}{z_o} + \delta\theta_o$$

(27)

$$F(z) - F_o = \frac{F_*}{\alpha_E^o} \log \frac{z}{z_o} + \delta F_o$$

(28)

the quantities $\delta\theta_o$ and δF_o translate the deviation which exists between the mechanical energy transfer mechanisms, on the one hand, and those of other turbulent transfers, on the other hand, near the surface when it is very rough. Stated differently, the roughness parameter z_o, which is defined with a dynamic process, is not necessarily identical to those which could be defined for other

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transfer processes.

In effect, in the surface layer, the thermal neutrality conditions are rare. The temperature gradient is in general over-adiabatic during the day and there is inversion at night. Also, in the general case, the gravity force effects affect the structure of turbulent flow. The now classical works of Monin and Obukhov (again published by Monin and Yaglom 1966) made it possible to perform a strict analysis of the surface layer in the general case. Without going into details, we would like to state that these authors defined three external parameters in which any internal variable of the surface layer can be determined: these are the quantities u_*, z_o and L.

L is the Monin-Obukhov length. The expression for it is the following and is established by similitude laws

$$L = - \frac{u_*^3 \rho C_p \bar{\theta}}{k g S} \qquad (29)$$

The non-dimensional ratio $\frac{z}{L}$ is a variable which is <u>/10</u> characteristic for the state of stability of the atmosphere at a given level. It is substituted for the more classical Richardson number, Ri, to which it can be related.

We can write

$$\frac{k z}{u_*} \frac{\partial \bar{u}}{\partial z} = \phi_M \left(\frac{z}{L} \right) \qquad (30)$$

$$\frac{z}{\theta_*} \frac{\partial \bar{\theta}}{\partial z} = \phi_H \left(\frac{z}{L} \right) \qquad (31)$$

$$\frac{z}{F_*} \frac{\partial \bar{F}}{\partial z} = \phi_E \left(\frac{z}{L} \right) \qquad (32)$$

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This system is equivalent to the following system for the turbulent transfer coefficients

$$K_M = \frac{k u_* z}{\phi_M \left(\frac{z}{L}\right)}$$ (33)

$$K_H = \frac{k u_* z}{\phi_H \left(\frac{z}{L}\right)}$$ (34)

$$K_E = \frac{k u_* z}{\phi_E \left(\frac{z}{L}\right)}$$ (35)

In the particular case of adiabatic conditions, which we considered previously, $L \to \infty$ and $\frac{z}{L} \to 0$.

This makes it possible to determine the following conditions for the functions ϕ_M, ϕ_H and ϕ_E.

$$\phi_M (0) = 1$$ (36)

$$\phi_H (0) = \frac{1}{\alpha_H^0}$$ (37)

$$\phi_E (0) = \frac{1}{\alpha_E^0}$$ (38)

The functions ϕ_M, ϕ_H and ϕ_E could only be determined /11 theoretically up to the present.

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Their variations <u>as a function of the parameter</u> $\frac{z}{L}$ <u>can only be</u> established from experimental results.

By adopting a semi-empirical relationship for each of the functions ϕ_M, ϕ_H and ϕ_E, it becomes possible to directly determine the fluxes from the vertical profiles of the corresponding averaged quantities. This method is now used. Except for experimental problems, its accuracy depends essentially on the validity of the semi-empirical relationships used. Therefore it can be understood why the possible vertical variation of the radiation flux is of interest, not only during measurement taking but also when the micrometeorological measurements are interpreted.

<u>Note</u>: An extension to the definition of the Monin-Obukhov length must be made when the evaporation flux is large, in order to take into account the gravity effects which are the result of humidity gradients. The effects of stratification only appear as correction terms as we will see in the following. We did not take into account this generalization. /12

III. <u>PROBLEMS CAUSED BY THE VARIATION OF THE RADIATION FLUX AND THE METHOD PROPOSED FOR AVOIDING THEM.</u>

31. <u>General considerations on the radiation transfer in the atmospheric surface layer</u>

As we have seen, the hypothesis of conservation of the vertical radiation flux is a basic hypothesis of the classical theory of turbulent transfers in the vicinity of the surface. The lower layers of the atmosphere constitute an absorbing medium for long wavelengths, because of the presence of relatively large concentrations of water vapor and carbon dioxide. The presence of temperature gradients brings about a certain variation in the long

wavelength radiation flux. The determination of such effects is done in a classical way in meteorology (atmosphere thickness on the order of 500 m). A sufficient accuracy has been obtained by using diagrams (Elsasser, Kew, Yamamoto, etc...). It is more delicate in the region of the surface layer because of the small thicknesses considered. We were able to collect a certain number of papers on this question and we would like to mention the following: Brooks D. L. (1950), Deacon (1950), Robinson (1950), Yamamoto and Kondo (1959), Funk (1960), Funk (1961), Laikhimann et al (1961), Gaevskaya et al (1963), Elliott (1964), Godson (1965), Hamilton (1965), Kondratiev (1965), Adunkowski and Johnson (1965), Atwater (1966), Lieske and Stroschein (1966), Seo et al. (1968), Hinzpeter and Heinrich (1969), Faraponova (1969).

Based on these numerous papers, one could think that this problem had already been sufficiently studied. In reality, the large majority of the studies mentioned approached the question from a different angle, because an attempt is made to predict the variation of a non-steady thermal regime from certain experimental data.

In effect, within the surface layer assumed to be homogeneous in the horizontal direction, the equation which governs the turbulent enthalpy transfer can be written as

$$\frac{\partial}{\partial t}\left(\rho C_p \bar{\theta}\right) = -\frac{\partial}{\partial z}\left\{R+S\right\}$$

where
$$\rho C_p \frac{\partial \bar{\theta}}{\partial t} = -\left\{\frac{\partial R}{\partial z} + \frac{\partial S}{\partial z}\right\} \quad (39)$$

In the general case of non-steady conditions, the heating and cooling rate of the lower layers results in the combination of the divergences of the radiation and turbulent fluxes.

As far as the nocturnal cooling is concerned, the prediction of which is extremely important for economic reasons (prevention of ice, fog, etc.) and which is in general associated with small turbulent convections, $\frac{\partial S}{\partial z}$ can be considered negligible with respect to $\frac{\partial R}{\partial z}$. We then have

$$\rho C_p \frac{\partial \theta}{\partial t} \simeq - \frac{\partial R}{\partial z} \qquad (40)$$

The nocturnal cooling rate is therefore directly proportional to the radiation flux divergence and this is one of the reasons for the relatively large number of papers devoted to this question.

For the non-steady regime, the vertical variation of the turbulent flux can be derived experimentally from the equation (39), if $\frac{\partial R}{\partial z}$ is determined and $\frac{\partial \theta}{\partial t}$ is measured. Robinson (1950), Funk (1960), Elliott (1964) also compared the true cooling $\frac{\partial \theta}{\partial t}$ to the radiation cooling $\frac{1}{\rho C_p} \frac{\partial R}{\partial z}$. The difference corresponds to the term $\frac{1}{\rho C_p} \frac{\partial S}{\partial z}$.

We can therefore see that there are various ways of considering the problem. For our part, we proceed as follows: our goal is to verify the validity of the hypothesis made regarding the radiation transfer within the framework of the classical method of determining the turbulent fluxes. We are therefore led to assume that a steady regime exists. $\left(\frac{\partial \theta}{\partial t} = 0 \right)$ will then have the property that $\frac{\partial S}{\partial z}$

$$\frac{\partial S}{\partial z} = - \frac{\partial R}{\partial z} \qquad (41)$$

It is then clear that the method of determining the divergence of the radiation flux can be used in the non-steady regime. Consequently the numerical values obtained can be compared with

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those obtained by various authors which we mentioned.

32. Influence of the variation of the long wavelength radiation flux on the quantities defined in the constant flux surface layer

For a long time, in the study of turbulent transfer near the surface, it has been assumed that the divergence of the radiation flux could be considered negligible because of the small atmospheric thickness under consideration. This opinion was confirmed by the conclusions of Robinson (1950) who, according to experimental results, was led to believe that $\frac{\partial R}{\partial z}$ was negligible except possibly below 1 m. Nevertheless, we should note that the quantity considered in general was the relative variation of the radiation flux proper. This quantity is effectively very small. However, for the problem with which we are dealing, this is not the quantity we are concerned about. Instead we are concerned with the divergence of the radiation flux relative to the turbulent heat flux. /14

In effect equation (17) can be written as

$$S(z) = S_0 + R_0 - R(z) \quad (42)$$

where $\quad S(z) = S_0 \left[1 - \frac{R(z) - R_0}{S_0} \right]$ with $R(z) - R_0 = \int_{z_0}^{z} \left(\frac{\partial R}{\partial z} \right) dz$

It is therefore clear that the turbulent flux $S(z)$ cannot be considered as a constant and equal to S_0 except to the extent that $\int_{z_0}^{z} \left(\frac{\partial R}{\partial z} \right) dz$ is negligible with respect to S_0. A small relative variation in the radiation flux $\frac{R(z) - R_0}{R_0}$ can lead to a value of $\frac{R(z) - R_0}{S_0}$ which is not negligible with respect to 1 if S_0 is small with respect to R_0. It is therefore necessary to simultaneously know the quantities which characterize the radiation

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transfer on the one hand, and the turbulent transfer on the other hand. This justifies the interaction term which we have used in this section.

Under the conditions under discussion, the variation in the radiation flux R brings about a variation in the turbulent flux S, which then is assumed to be constant and equal to S_o. Thus an erroneous estimation of the flux F results when it is determined, if the temperature gradient and the coefficient k_H are known. Inversely, an erroneous estimation of k_H results when it is derived from measurements of the flux and of the gradient. In effect, strictly speaking k_H is defined

$$S(z) = - \rho C_p \, k_H \, \frac{\partial \bar{\theta}}{\partial z} \qquad (43)$$

If S is assumed to be constant, in reality we determine a global coefficient k_{Ho} such that

$$S_o = - \rho C_p \, k_{Ho} \, \frac{\partial \bar{\theta}}{\partial z} \qquad (44)$$

We can see that the relative error of k_H is equal to that of S.

$$\frac{k_H}{k_{Ho}} = \frac{S}{S_o} = 1 - \frac{\int_{z_o}^{z} \left(\frac{\partial R}{\partial z} \right) dz}{S_o}$$

The ratio $\frac{k_{Ho}}{k_H}$ is used jointly for determining the relationship $\phi_H\left(\frac{z}{L}\right)$, which introduces an error due to the influence of radiation transfer.

A certain number of authors have for a long time indicated
the possibility of radiation transfer (Priestley 1959, Mac Vehil
1964, Lumley and Panofsky 1964, Godson 1965, Webb 1970, Oke 1970).

Certain authors based their calculations on experimentally
determined profiles and detected this effect in certain particular
micrometeorological situations.

For the case which he studied, Elliott (1964) calculated a
variation of 14% in the turbulent flux, even though the wind
velocity was quite high (7 to 8 m/s). He also adds:

"However, much stronger inversions than the ones on which these
computations are based are frequently found. These stronger
inversions would lead to larger radiative cooling rates and
quite likely to eddy flux divergences which could not be ig-
nored in considering such low-level phenomena as fog-forma-
tion".

Also, Kraus, referenced by Munn (1966), studied three parti-
cular cases for which he found a variation of 2%, 7% and 50%,
respectively.

All these observations led Munn (1966) to write:

"The observed temperature structure in the boundary layer is
the integrated result of turbulent mixing and radiative trans-
fer. One of the unsolved problems of micrometeorology is the
determination of the relative importance of the two components".

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33. <u>Description of the method utilized for quantitatively</u>
<u>estimating the influence of radiation transfer</u>

The influence of radiation transfer seems to not have yet been studied in the most general case, where the various parameters are not determined numerically in advance and can take on arbitrary values corresponding to any micrometeorological situation. This global study has two advantages:

- on the one hand, the influence of the various parameters can be shown analytically, which makes it possible to better evaluate the relative importance of the various processes.

- on the other hand, generally valid equations could be then applied to each particular case. For each determination of the flux or of the turbulent transfer coefficient, an approximate evaluation of the error introduced by ignoring the radiation transfer can be obtained relatively easily. This evaluation seems to be necessary at the present time, as the studies on turbulent transfer mode recently by Webb (1970) and Oke (1970) have shown. These two authors deplore the absence or inaccuracy of methods which would make it possible to evaluate the influence of the radiation transfer during their experiments.

We are therefore forced to carry out a general study. It is based on the following scheme which was conceived by M. Coantic and which we personally developed while collaborating with him.

As generally assumed (and this is the point which we want to verify), the surface layer is first assumed to be a "constant flux layer" and the radiation flux variation is assumed to be negligible. According to hypothesis, the turbulent flux S is then constant.

It is then possible to establish theoretical temperature and
humidity profiles, if the fundamental parameters are given from
the beginning and if the usual semi-empirical relationships are
adopted. This method of calculation, which is a completely
classical one, is only valid if S is effectively constant. From
the temperature and humidity profiles it is possible to calculate
the divergence of the radiation flux. By integration its variation
over the surface can be calculated. It is assumed that steady
conditions prevail. The total enthalpy flux must then remain
effectively constant and the absolute value of the variation
obtained is equal to the variation of the turbulent flux S. Thus
S was assumed to be constant. If the variation given by the
calculation is small, i.e. on the order of magnitude of the errors
usually made in micrometeorology (on the order of 10%), the initial
hypothesis will remain approximately valid. On the other hand,
if the variation which results is higher, the hypothesis is in-
validated by the conclusions; the error introduced by removing
the influence of the radiation transfer cannot be considered as
negligible. The temperature and humidity profiles are different
from the ones which we used. The general solution of the problem /17
then requires recourse to a method of succesive approximations.

The study then consists of two main parts:

1. Establishment of theoretical temperature and humidity
    profiles in a constant flux layer

2. Calculation of the divergence and variation of the long
    wavelength radiation flux.

IV. DETERMINATION OF THE VERTICAL DISTRIBUTION OF TEMPERATURE AND HUMIDITY FOR THE HYPOTHESIS OF A CONSTANT FLUX LAYER

41. Parameters and relationships required for defining the profiles

The various transfer modes all depend directly on the dynamic process. The determination of the velocity profiles is indispensible for defining the vertical temperature and humidity profiles.

In the constant flux layer it is known that the dynamic process is defined by specifying three fundamental parameters u_*, z_o and L (Lumley and Panofsky 1964).

In the particular case of the ocean, and for the problem which we are treating here, it is generally assumed that the roughness of the interface is determined by the value of the friction velocity u_* (the statistical characteristics of the waves are assumed to be determined by the local wind velocity as the first approximation).

z_o is then a function of u_* which we will now examine. The number of fundamental parameters is reduced to two: u_* and L.

411. The method of calculating the velocity profile is then as follows:

From (3) we have

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k z} \phi_M \left(\frac{z}{L}\right) \tag{45}$$

by integration,

$$\bar{u}(z) = \frac{u_*}{k} \int \frac{\phi_M}{z} dz + C^{te} \tag{46}$$

At the interface, by definition of z_o $u(z) \longrightarrow 0$ when $z \longrightarrow z_o$.

Thus,
$$\bar{u}(z) = \frac{u_*}{k} \int_{z_o}^{z} \frac{\phi_M}{z} \, dz \qquad (47)$$

The velocity profile is thus completely determined by specifying the pair u_*, L if the functions $z_o(u_*)$ and $\phi_M\left(\frac{z}{L}\right)$ are known.

<u>412</u>. As far as the thermal process is concerned, equation <u>/19</u> (31) makes it possible to write

$$\frac{\partial \theta}{\partial z} = \frac{\theta_*}{z} \phi_H \left(\frac{z}{L}\right) \qquad (48)$$

or by integration
$$\theta(z) = \theta_* \int \frac{\phi_H\left(\frac{z}{L}\right)}{z} \, dz + C^{te}$$

The interface equation can be written as (see paragraph 22)

$$\theta(z) \longrightarrow \theta_o + \delta\theta_o \qquad \text{when} \qquad z \longrightarrow z_o$$

which leads to the expression for the temperature profile

$$\theta(z) - \theta_o = \delta\theta_o + \theta_* \int_{z_o}^{z} \frac{\phi_H}{z} \, dz \qquad (49)$$

It is therefore determined from the velocity profile by specifying $\theta_o, \delta\theta_o, \phi_H\left(\frac{z}{L}\right)$ and θ_* . In reality, the parameter θ_* is already known. In effect, θ_{**} is defined by the relationship (26) $\theta_* = -\frac{S}{\rho C_p k u_*}$. According to the definition of the

Monin-Obukhov length $\frac{S}{\rho C_p} = -\frac{u_*^3 \bar{\theta}}{kgL}$. Except for very pronounced

advection conditions, which are then eliminated according to hypothesis in our study, the natural exchange mechanism only results in moderate temperature gradients in the surface layer. $\bar{\theta}$ can be replaced by θ_o as an approximation to within 1%. We then have

$$\frac{S}{\rho c_p} = - \frac{u_*^3 \, \theta_o}{kg L}$$ (50)

By combining (26) and (48), we can then obtain θ_*:

$$\theta_* = \frac{u_*^2 \, \theta_o}{k^2 g L}$$ (51)

θ_* is then determined if u_*, L and θ_o are known.

Thus, for the thermal process, the velocity profile is assumed to be known and the determination of the temperature profile requires that the quantities θ_o, $\delta\theta_o$ and the function $\phi_H\left(\frac{z}{L}\right)$ be specified.

413. Finally, the evaportion process is defined in a analogous way. According to (32) in effect we have

$$\frac{\partial \bar{F}}{\partial z} = \frac{F_*}{z} \phi_E \left(\frac{z}{L}\right)$$ (52)

or

$$F(z) - F_o = \delta F_o + F_* \int_{z_o}^{z} \frac{\phi_E}{z} \, dz$$ (53)

with the condition at the interface $\bar{F}(z) \rightarrow F_o + \delta F_o$ when $z \longrightarrow z_o$.

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But F_* is simply related to Θ_* by the Bowen ratio

$$B = \frac{S}{\mathcal{L} J} \qquad (54)$$

In effect, by definition

$$F_* = -\frac{J}{\rho k u_*} \qquad (25)$$

$$\Theta_* = -\frac{S}{\rho C_p k u_*} \qquad (26)$$

or

$$\frac{F_*}{\Theta_*} = C_p \frac{J}{S} = \frac{\mathcal{C}_p}{\mathcal{L} B} \qquad (55)$$

which results in

$$F_* = \frac{C_p}{\mathcal{L} B} \frac{u_*^2 \Theta_o}{k^2 g L}$$

The humidity profile is therefore determined if the velocity and temperature profiles are known by specifying F_o, δF_o, $\phi_E\left(\frac{z}{L}\right)$ and the Bowen ratio.

In conclusion, the various profiles can be completely calculated in any case if the quantities $\bar{u}_*$, L, Θ_o, $\delta\Theta_o$, F_o, δF_o and B are specified and if the semi-empirical relationships $z_o(u_*)$, $\phi_H\left(\frac{z}{L}\right)$, $\phi_H\left(\frac{z}{L}\right)$ and $\phi_E\left(\frac{z}{L}\right)$ are known. We will now examine these various details.

42. <u>The roughness parameters on the surface of the ocean</u>

Because the dynamic process is predominant, it is necessary first of all to define the dynamic roughness parameter z_o. This

parameter, which is an integration constant which results from extrapolating the velocity profile, is a function of shape, height and spacing of obstacles which make up the roughness in the case where the surface is fixed. In the case of the ocean, the surface is deformable under the action of the wind, which leads to the development of waves. The waves have variable shapes and speeds in time and space. This brings about extremely complex dynamic effects which are not well known. In order to apply semi-empirical theories of atmospheric turbulence, it is generally assumed that these effects can be translated by means of the parameter z_0 which only depends on the local dynamic interaction, represented by the friction velocity u_*. All the information for this roughness parameter is of an experimental nature. The first experiments were carried out in the ocean, but it seems very difficult to interpret the results, as Figure 1 shows, taken from the book of Roll (1965).

The research carried out in the wind tunnel (Hidy and Plate 1967, Shemdin 1967, Karaki and Hsu 1968, Wu 1968 and 1969, etc...) made it possible to better understand this phenomenon. In the development of the various waves which influence the dynamic roughness of the ocean surface, there is not only the local turbulent stress caused by the wind and translated by the quantity u_*, but there is also the "fetch" X ("fetch" is a nautical term which designates the surface traversed by the wind. Thus in the case of the ocean, the distance to the shore or the origin of the perturbation). For the same wind velocity, the wind dimention increases with X (Figure 2). When X is small, u_* also varies as a function of X; because of the proximity of the "attack band" there is a complex variation of z_0 as a function of X (Figure 3).

In the present study, the flow is assumed to be completely developed and the role of "fetch" can be completely eliminated according to hypothesis. It will therefore be necessary, in order

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to extend the results obtained in the wind tunnel to the problem which we are considering, to isolate the only variation of z_o as a function of u_* for a fetch assumed to be infinite (in fact, a fetch on the order of 10 km seems to be sufficient).

For X given, two different flow regimes are observed corresponding to the wave dimensions which make up the roughness. For small u_*, the average heights of roughness are small and the flow regime corresponds to the "hydraulically smooth" regime. z_o is then connected with the thickness of the viscous sublayer and has the form $Z_o = a \frac{\nu}{u_*}$ (Monin 1970). For higher u_* values, the regime tends to become "hydraulically rough" after a transition region. Charnock established a theoretical relationship of the form $z_o = b \frac{u_*^2}{g}$. b depends on the value of the fetch according to the previous developments.

Overall this scheme has been confirmed by experimental results Hidy and Plate and Shemdin (Figures 4 and 5).

We should like to state that these authors obtained a limited "fetch" on the order of magnitude of the wind tunnel length.

The extension to a practically infinite "fetch" corresponding to the case of completely developed flow over the surface of the ocean was done by Wu (1969) (Figures 6 and 7). The results obtained are in agreement with the qualitative model which resulted from the wind tunnel studies and were the results of various observations.*

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*This agreement is different from the dispersion found by Roll. First of all the results collected by Roll are much older than those considered by Wu. Also the measurement accuracy has been considerably increased. The theories for the velocity profile shapes has advanced. In particular, the error associated with thermal stratification has been eliminated. (Continued on next page)

We therefore utilized the numerical values of z_0 from Figure 7.

For thermal and mass exchange processes the terms $\delta\theta_0'$ and $\delta F_0'$ translate the deviation which occurs close to the rough surface between the mechanical energy and the various other transfers. Various recent studies (Barry 1965, Chamberlain 1968, Cowan 1968, Thom 1968, Zilitinkevitch 1970) have shown that the terms could be correlated with the "roughness Reynolds number" $\frac{u_* z_0}{\nu}$.

$\delta\theta_0'$ and $\delta F_0'$ are increasing functions of the "macroviscosity" $u_* z_0$ defined by Sutton (1953). Consequently they must not be considered except for very rough surfaces, as Chamberlain (1968) showed in experiments.

The small values of the roughness parameter z_0 for the ocean surface then leads to negligible values of $\delta\theta_0'$ and $\delta F_0'$ and we therefore write

$$\delta\theta_0 = \delta F_0 = 0$$

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43. <u>Determination of the surface temperature and humidity and the Bowen ratio.</u>

It would first seem illogical to regroup the surface humidity and the Bowen ratio. These quantities have a common feature which is important for a study: strictly speaking, they cannot be considered as given quantities of the problem but instead

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*(Continued from preceding page) Finally, the role of the "fetch", which could be great for measurements near the coasts, has been considered as we showed. These various reasons lead us to believe that the measurement dispersions observed by Roll are due not to the phenomenon itself but to inaccuracies in the experimental determinations of z_0 and to the very different fetch effects.

must be considered as results.

As we stressed in the introduction, the distribution of energy between the various fluxes results in an equilibrium which is established at any time at the level of the surface because of the action of the heat input from the solar radiation.

Depending on the boundary conditions, in the air and also in the water, (wind velocity, air, temperature and humidity above the internal boundary layer, water temperature with depth) and the solar radiation absorption mechanism in the water, the system leads to a unique solution and the quantities u_*, L, Θ_{v_0}, f_0 and B as well as S and J are the results. The rigorous solution of the problem requires the complete solution of the system. We will consider the first part as being solved and will adopt plausible values of these parameters.

Among these, three are independently specified: u_*, L and Θ_0.

We will assume that F_0, the surface humidity, is the saturating humidity of sweet water at temperature Θ_0 and is therefore completely determined by specifying this parameter. F_0 also depends on the radius of curvature of the surface and the salinity of the water; also a surface film and molecular scale effect can have an influence. However, in the first approximation, the influence of these factors can be considered to be negligible.

The Bowen ratio B is more difficult to specify. It is not completely independent of the other parameters. Below the ocean its sign is generally determined by the sign of the heat flux S. The flux J is almost always directed upward (evaporation takes place). J is therefore positive except in an inversion situation. S is < 0 in an inversion situation. S is > 0 under

over-adiabatic conditions. Thus $B = \frac{S}{\mathcal{L}J}$ has the sign of S or the sign opposite to that of L.

We could then consider specifying various arbitrary values of B, which would be positive for $L < 0$ and negative for $L > 0$. B would then be an additional variable, comparable with u_*, L and θ_o. But the number of possible combinations between these various variables then becomes very large. We preferred utilizing the values obtained by Gordon and reported by Roll (1965) (page 254) where B was experimentally correlated with the temperature difference between the air and the surface. Even though there is no experimental justification for this correlation, we made use of it in this study in order to simplify our analysis.

44. Discussion of the semi-empirical relationships and ϕ_E, ϕ_H

This section alone could be the topic of a paper which would be as important as this entire paper. But this is not our purpose and we will give an outline of the problem and specify the expressions which we use.

First of all it should be stated that, by adapting a semi-empirical relationship of any kind, this does not fundamentally modify the significance of our results. As we will see in the following, to first order, the stratification effect is negligible and the adiabatic approximation $\phi_M = \phi_H = \phi_E = 1$ is sufficient. The form of the semi-empirical relationships does not enter except as correction factors.

The semi-empirical relationships ϕ_M, ϕ_H and ϕ_E translate the influence of the thermal stratification to mechanical energy transfers, heat transfer, and mass transfer, respectively. The use of similitude relationships makes use of the Monin-Obukhov

length L. It became possible to replace the empirical relation-
ships formally based on the Richardson number Ri [see for example
Deacon (1948)] by more satisfactory theoretical expressions.
Since the study of Monin and Obukhov date from 1954, these are
rather recent papers. It seems that a coherent theory is develop-
ing and we will now outline it.

441. We will start by studying the relationship $\phi_M\left(\frac{z}{L}\right)$ de-
fined by the equation (30)

$$\frac{k z}{u_*} \frac{\partial \bar{u}}{\partial z} = \phi_M\left(\frac{z}{L}\right)$$

For adiabatic conditions (see paragraph 22), i.e.
$\frac{z}{L} \rightarrow 0$ $\phi_M \rightarrow 1$. Monin and Obukhov considered the Taylor series
development of the function ϕ_M

$$\phi_M\left(\frac{z}{L}\right) = 1 + \alpha \frac{z}{L} + \alpha'\left(\frac{z}{L}\right)^2 + \dots$$

For small $\frac{z}{L}$ they limited themselves to the first term and
they wrote

$$\phi_M\left(\frac{z}{L}\right) = 1 + \alpha \frac{z}{L} \cdot \tag{56}$$

By integration we find the "log-linear law" which is well known

$$\bar{u}(z) = \frac{u_*}{k}\left[\text{Log} \frac{z}{z_0} + \alpha \frac{z}{L}\right] \tag{57}$$

The problem then reduces to determining the constant α. The /25
value 0.6 first proposed by Monin and Obukhov was found to be
small afterwards. Among the various determinations we can mention
those of Deacon (1962), Panofsky, Blackadar and Mac Vehil (1962),
who found 4 - 4.5 and 6 for unstable conditions, respectively.
For stable conditions, Mac Vehil proposes the value 7. Also,

Zilitinkevitch (1970) studied the various papers and gives an average value for α close to 1 for unstable conditions and 9 for stable conditions.

The dispersion in the obtained values can be understood because of the experimental uncertainties associated with the length L. In addition, the limitations in the expansion to the first term does not seem valid except for small values of $\frac{z}{L}$, which seriously limits the range of application of the log-linear law.

In this case, the second order term comes into play and obviously will prevent a correct determination of α.

This difficulty led a certain number of authors to find formulas which were valid over a more extended range. Among these, two are generally used.

The first formula carries the name KEYPS (formed from the initials of various authors which independently proposed it, Kazanski, Ellison, Yamamoto, Panofsky and Sellers). It can be derived from the turbulent energy balance using a few simplifications. It is written

$$\phi_M^4 - \beta \frac{z}{L} \phi_M^3 = 1 \qquad (58)$$

and integration leads to the following velocity profile (Klug 1967)

$$u(z) = \frac{u_*}{k} \left[\mathcal{f}(\gamma) - \mathcal{f}(\gamma_0) \right]$$

with

$$\mathcal{f}(\gamma) = \gamma - 2 \, \text{Arc tg} \, \gamma - 2 \, \text{Arc th} \, \gamma$$

$$\frac{\gamma^4 - 1}{\gamma^3} = \beta \frac{z}{L}$$

$$\frac{\gamma_0^4 - 1}{\gamma_0^3} = \beta \frac{z_0}{L}$$

β is determined experimentally: a value close to 18 was obtained.

For small $\frac{z}{L}$, the formula of KEYPS tends toward the expression of Monin and Obukhov with $\beta \simeq 4\alpha$, or $\alpha \simeq 4,5$.

Another formula was established by Swinbank (1964), who considered a quantity X (which is in a certain sense a generalization of z weighted by the thermal stratification effects) such that the relationship $\frac{kX}{u_*} \frac{\partial \bar{u}}{\partial X} = 1$ is always satisfied. The production of global kinetic energy including the effect of Archimedes forces is written as $\tau_0 \frac{\partial \bar{u}}{\partial z} + \frac{gS}{c_p\theta}$. Swinbank postulates that this quantity is equal to $\tau_0 \frac{\partial \bar{u}}{\partial x}$. This hypothesis makes it possible to determine ϕ_M by eliminating X between the two relationships. He obtains

$$\phi_M = \frac{z}{L} \left\{ 1 - \exp\left(-\frac{z}{L}\right) \right\}^{-1} \tag{59}$$

which results in the following "exponential" velocity profile

$$\bar{u}(z) = \frac{u_*}{k} \log \frac{L\left(\exp \frac{z}{L} - 1\right)}{z_0}$$

For small $\frac{z}{L}$, $\phi_M = 1 + \frac{1}{2}\frac{z}{L} + \frac{1}{4}\left(\frac{z}{L}\right)^2 + \cdots$

We again find the formula of Monin-Obukhov with $\alpha = 0,5.$

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These various forms can also be discussed theoretically. The formula of Monin-Obukhov, if it seems valid in the range $\left|\frac{z}{L}\right| < 0.1$, is not strictly applicable outside of this interval. The two other forms have a wider range of validity. However, the formula of Swinbank is also based on intuition as well as a physical fact. The formula of KEYPS uses a physical law which is better established, but the approximations used are rather coarse ones. Experimental verifications should be made to validate them. This comparison was carried out by Bernstein (1966) and his results are given in Figure 8. The formula of KEYPS seems to be best adapted as other authors have also stated. However, none of the formulas seems to have any advantage over any of the others.

In this study, the absolute value of $\left|\frac{z}{L}\right|$ does not exceed 1. Under these conditions the formula of Monin-Obukhov with $\alpha = 0.6$ seems to be just as valid as the formula of Swinbank. The formula of KEYPS seems to be slightly closer to the experimental results, but the resulting velocity profile would be more complex. We have adapted the simplest form, that of Monin-Obukhov.

<u>442.</u> For thermal processes $\phi_H\left(\frac{z}{L}\right)$ is defined by equation <u>/27</u> (34):

$$\phi_H\left(\frac{z}{L}\right) = \frac{k u_* z}{K_H},$$ (60)

According to (33) we have

$$\phi_M\left(\frac{z}{L}\right) = \frac{k u_* z}{K_M}$$ (61)

We then derive:

$$\phi_H\left(\frac{z}{L}\right) = \phi_M\left(\frac{z}{L}\right) \times \frac{K_M}{K_H}$$ (62)

ϕ_H is therefore derived from ϕ_M if the ratio $\frac{k_H}{k_M} = \alpha_H$ is known. According to equation (62) it is a function of   .

The ratio $\frac{k_H}{k_M}$ has been treated in many papers in addition to the investigations of the function ϕ_M. Various and sometimes contradictory results have been obtained. This can be explained by two factors:

1. The fact that the turbulent | fluxes τ and S are simul-
    taneously and strictly conserved in the vertical direc-
    tion should be verified by experiments. This would
    require experimental conditions which would be hard to
    find (see Chapter II). Also there is a problem of the
    radiation flux influence which is what is being treated
    in this paper.

2. The turbulent flux S must be measured directly and
    independently. Because of the development of a correla-
    tion measurement, which has been relatively recently
    developed and is not yet well known (Businger et al. 1967),
    certain results were derived from the surface energy
    balance. Often they have a considerable error.

In the unstable range $\frac{k_H}{k_M}$ increases with $\frac{z}{L}$ and can reach high values on the order of 3. The upper limit is 3.5 according to Monin (1970). On the other hand, in the stable range, $\frac{k_H}{k_M}$ decreases and reaches values on the order of 0.5 - 0.6.

The value of the ratio under adiabatic conditions is not determined exactly. For a long time it was assumed that $\chi_H^0 \approx 1$. Recent experiments (Zilitinkevitch 1970) give values on the order of 1.3 which corresponds to a turbulent Prandtl number close to 0.7. We prefer to use the classical value 1.

As far as the exact form of the function $\frac{K_H}{K_M}\left(\frac{z}{L}\right)$ is concerned, /28 only a few relationships have been established. For unstable conditions, Swinbank (1969) proposed an empirical equation $\frac{K_H}{K_M} = 2.7 \left|\frac{z}{L}\right|^{0.15}$ which is the result of experiments, the quality of which was not discussed, and which corresponds well with the results obtained by other authors (Businger 1966, Record and Cramer 1966, Charnock 1967, Cyer 1967, Swinbank and Dyer 1967, Deardorff 1968, Zilitinkevitch 1970). For stable conditions, only a few measurements have been carried out (Record and Cramer 1966). A wind tunnel study performed by Cermak and Arya (1970) resulted in the relationship $\frac{K_H}{K_M} = 0.5\left(\frac{z}{L}\right)^{-0.1}$ which corresponds well with observations of other authors.

Taking the continuity conditions into account for $\left|\frac{z}{L}\right| \to 0$, we therefore adopted the following relationships for $\quad$ (see Figure 9):

$$\frac{z}{L} < -0.02 \quad \frac{K_H}{K_M} = 2.7 \left|\frac{z}{L}\right|^{0.15} \tag{63}$$

$$-0.02 < \frac{z}{L} < 0.001 \quad \frac{K_H}{K_M} = 1 \tag{64}$$

$$\frac{z}{L} > 0.001 \quad \frac{K_H}{K_M} = 0.5\left(\frac{z}{L}\right)^{-0.1} \tag{65}$$

As far as the temperature profiles are concerned, we obtain the following equations by integration of equation (48)

$$\frac{z}{L} < -0{,}02 \qquad \theta(z) = \theta(0{,}02 L) + \frac{2{,}5 u_*^2}{g L}\left(\frac{1}{\sqrt{\left|\frac{z}{L}\right|}} - \frac{1}{\sqrt{0{,}02}}\right) \qquad (66)$$

$$-0{,}02 < \frac{z}{L} < 0{,}001 \qquad \theta(z) = \theta_0 + \theta_* \, \text{Log} \, \frac{z}{z_0} \qquad (67)$$

$$\frac{z}{L} > 0{,}001 \qquad \theta(z) = \theta(0{,}001 L) + 20\, \theta_*\left(\left|\frac{z}{L}\right|^{0{,}1} - 0{,}001^{0{,}1}\right) \qquad (68)$$

<u>443.</u> For the water vapor transfer $\phi_E\left(\frac{z}{L}\right)$ is related to $\phi_M\left(\frac{z}{L}\right)$ by means of the relationship

$$* \qquad (69)$$

The problem is identical to the one encountered in heat /29 transfer. The determination of the ratio $\frac{k_E}{k}$ is more delicate than determining the ratio $\frac{k_H}{k}$ because of the particular difficulties associated with the exact measurement of humidity and its fluctuations (Leducq 1970). It has since been assumed (Dyer 1967, Swinbank and Dyer 1967) that the heat and the water vapor are transported in a manner which is analogous, at least to the first approximation, in any stability range. Consequently we may assume the equation

$$k_H = k_E \qquad (70)$$

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*Translator's note: Equation (69) was not included in the foreign text.

Under these conditions, the temperature and humidity profiles are connected by a proportionality relationship and may be obtained

$$f(z) - f_0 = \frac{C_p}{\mathscr{L} B} \left(\theta(z) - \theta_0 \right)$$ (71)

45. Summary: Determination of the temperature and humidity profiles

In conclusion we will specify at the beginning arbitrary but plausible values of the fundamental variables u_*, L and θ_0.

The roughness parameter ' z_0 is derived from u_* according to the empirical relationship of Wu (1969) (Figure 7).

If u_*, L and θ_0 are known, it is possible to calculate θ_* and consequently $S = -\rho C_p k u_* \theta_*$. Then $\theta(z)$ can be determined by formulas (66), (67) and (68).

The Bowen ratio is then calculated from the temperature difference $\theta(z) - \theta_0$ using the empirical relationship reported by Roll (1965). The humidity profile is then obtained from (71) and J is given by $B = \frac{S}{\mathscr{L} J}$

Thus from the three parameters u_*, L and θ_0, the theoretical profiles of temperature and humidity are directly determined as well as the quantities z_0, θ_*, B, f_*, S and J.

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V. METHOD OF CALCULATION OF THE DIVERGENCE OF THE LONG WAVELENGTH RADIATION FLUX IN THE LOWER LAYERS OF THE ATMOSPHERE

51. Mechanism of the radiation flux divergence. Method of calculation

If there are no liquid or solid particles (fogs, clouds, etc.) the divergence of the longwave radiation flux is essentially caused by the presence of water vapor and carbon dioxide. These two gases have a spectrum which is characterized by considerable absorption in the infrared range. Depending on the H_2O and CO_2 content, each atmospheric layer absorbs a certain fraction of the radiation which it receives and radiates a certain energy, depending on its temperature. For thick layers, this mechanism can be assimilated into a conduction phenomenon according to certain authors (Brunt 1939). On a scale considered here (between the surface and 10 m), such a simplification cannot be used even as a first approximation.

The evaluation of the radiation flux is primarily carried out on the meteorological scale. Relatively simple methods which allow routine determinations can be used: these are the diagrams of Elsasser, Kew, Yamamoto, etc.. These methods are not adopted for our study, because they apply to layers which have a thickness of several hundred meters and are not satisfactory on the scale of the surface layer. It is necessary to use the basic radiation transfer equations, which were presented in a convenient way for this range by Bruinenberg (referenced by Kondratiev 1965). This work was extended by Brooks (1950) and by Funk (1961) who developed numerical methods for estimating the divergence of the radiation flux in the immediate vicinity of the ground. The method of Funk, more than the method of Brooks, has already been used by several authors (Elliott 1964, Hamilton, 1965, Lieske and

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Troschein 1967) and seems to be the more satisfactory one for the problems considered here. In order to correctly apply it, the mechanism which it represents must be well known. A certain number of precautions must be taken during the calculation. We analysed this in great detail. The calculations necessary to obtain results are long and complicated, and numerical calculation must be performed with a computer. This led Gaeuskaya et al. (1963) to reject the methods of Brooks and Funk because they were too complex. It seems that the phenomenon cannot be represented adequately by simple calculations.

52. Method of Funk (1961)

We will follow the development of Funk as he presented it. Let w be the optical thickness measured from the bottom (see paragraph 53). o, r and s are subscripts which correspond to values of the variables on the surface at the reference level z_r and at the top of the absorbing atmosphere, respectively.
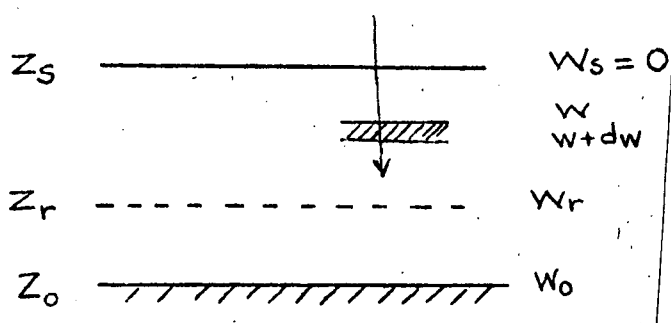
The radiation flux directed downwards of the layer having optical thickness w_r extends from the top of the atmosphere down to the level which is being studied z_r and which is received from this level and is given by (Moller 1957)

$$R\downarrow = \int_0^\infty dv \int_0^{w_r} \pi B_v(\theta) \frac{\partial \mathcal{T}_v(w_r - w)}{\partial w} dw \qquad (72)$$

where $B_v(\theta)$ is the emission intensity for the black body (Planck function) at the frequency v, corresponding to the temperature θ. $\tau_v(w_r - w)$ is the spectral transmission of the layer between w_r and w at this frequency.

τ_v is a function not only of the optical thickness but also of the temperature and pressure. H_2O is the principal absorber here (Brooks 1950, Funk 1961). The optical thickness of the water vapor will be substituted for the global optical thickness. The possible effects of CO_2 could be approximately taken into account with certain corrections.

For an isothermal layer, the emissivity ε is defined as the ratio of the total radiation flux emitted through one of its boundaries to that which would be emitted if it could be considered as a black body.

According to the Kirchoff law

$$\varepsilon(w_r - w)\sigma\theta^4 = \int_0^\infty \pi B_v(\theta)\left[1 - \tau_v(w_r - w)\right]dv \tag{73}$$

ε is a function of the same quantities as $*_v$.

In the micrometeorological range, the pressure is practically
 _/32
constant. Also it is possible to ignore the variation of ε with temperature because the deviations are so small, as Deacon (1950) had assumed and as Staley and Jurica (1970) have verified. ε then only depends on the water vapor concentration (except for possible correction for carbon dioxide).

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*Translator's note: Symbol was omitted in the foreign text.

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*Translator's note: Symbol was omitted in the foreign text.

R $\downarrow$| then becomes

$$R \downarrow = - \int_0^{W_r} \sigma \theta^4 \frac{\partial \varepsilon (w_r - w)}{\partial w} \, dw \Bigg| \qquad (74)$$

In the same way, if the adjacent layer underneath is treated as an isothermal atmosphere having infinite thickness, the ascending flux R $\uparrow$| at the level z_r is given by

$$R \uparrow = \int_{W_r}^\infty \sigma \theta^4 \frac{\partial \varepsilon (w_r - w)}{\partial w} \, dw = \sigma \theta_R^4 \left[1 - \varepsilon (w_o - w_r) \right] + \int_{W_r}^{W_o} \sigma \theta^4 \frac{\partial \varepsilon (w_r - w)}{\partial w} \, dw \Bigg| \qquad (75)$$

θ_R is the equivalent radiation temperature at the surface, i.e. the temperature of a black body which emits the same flux. At the level z_r, the divergence of the radiation flux is:

$$\left(\frac{\partial R}{\partial z} \right)_{z_r} = \left(\frac{\partial R \uparrow}{\partial z} - \frac{\partial R \downarrow}{\partial z} \right)_{z_r} = \left(\frac{\partial w}{\partial z} \right)_{z_r} \left(\frac{\partial R \uparrow}{\partial w} - \frac{\partial R \downarrow}{\partial w} \right) \Bigg| \qquad (76)$$

or

$$\left(\frac{\partial R}{\partial z} \right)_{z_r} = - \sigma \left(\frac{\partial w}{\partial z} \right)_{z_r} \left[- \int_0^{W_r} \theta^4 \frac{\partial^2 \varepsilon (w_r - w)}{\partial w^2} \, dw - \int_{W_r}^{W_o} \theta^4 \frac{\partial^2 \varepsilon (w - w_r)}{\partial w^2} \, dw \right.$$

$$\left. + \theta_R^4 \frac{\partial \varepsilon (w_o - w_r)}{\partial w_r} \right] \Bigg| \qquad (77)$$

As we will see in the following, the global emissivity is not known with great accuracy. Its second derivative is therefore also relatively poorly known. This is what led Brooks (1950) to integrate expression (77) by parts so that the first derivative $\frac{\partial \varepsilon}{\partial w}$| appears, which is better determined, and the term $\frac{\partial \theta}{\partial w}$| which is relatively well known from experiment.

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We then obtain

$$\left(\frac{\partial R}{\partial z}\right)_{Zr} = + \sigma \left(\frac{\partial w}{\partial z}\right)_{Zr} \left[\left(\theta_o^4 - \theta_R^4\right) \frac{\partial \mathcal{E}(w_o - w_r)}{\partial w_r} - \theta_s^4 \frac{\partial \mathcal{E}(w_r)}{\partial w_r} + \right. \tag{78}$$
$$\left. \int_0^{w_r} \frac{\partial \theta^4}{\partial w} \frac{\partial \mathcal{E}(w_r - w)}{\partial w} dw + \int_{w_r}^{w_o} \frac{\partial \theta^4}{\partial w} \frac{\partial \mathcal{E}(w - w_r)}{\partial w} dw \right]$$

In our particular problem, the temperature and humidity
profiles were known analytically. A second integration by parts
results in $\mathcal{E}(w)$ and $\frac{\partial^2 \theta^4}{\partial w^4}$, which was done by Funk and which
is of particular interest. This makes it possible to obtain the
following formula which will be used as the basis of our calcula-
tion.

$$\left(\frac{\partial R}{\partial z}\right)_{Zr} = \sigma \left(\frac{\partial w}{\partial z}\right)_{Zr} \left[\left(\theta_o^4 - \theta_R^4\right) \frac{\partial \mathcal{E}(w_o - w_r)}{\partial w_r} - \theta_s^4 \frac{\partial \mathcal{E}(w_r)}{\partial w_r} + \left(\frac{\partial \theta^4}{\partial w}\right)_{w_o} \mathcal{E}(w_r - w_o) \right.$$
$$\left. - \left(\frac{\partial \theta^4}{\partial w}\right)_o \mathcal{E}(w) - \int_0^{w_r} \frac{\partial^2 \theta^4}{\partial w^2} \mathcal{E}(w_r - w) dw - \int_{w_r}^{w_o} \frac{\partial^2 \theta^4}{\partial w^2} \mathcal{E}(w - w_r) dw \right] \tag{79}$$

53. Definition of the modified optical density ω

The optical density is defined by the relationship

$$dw = -\rho_e dz \implies w = -\int \rho_e dz + C^{te} \tag{80}$$

It is therefore assumed positive downwards and is zero at
the top of the absorbing atmosphere.

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This definition is difficult to use in our study in which we are interested in the layers close to the surface and where the variations in optical thickness which occur are extremely small in relative value.

w does not occur in the equations except in the differential expressions $\frac{\partial w}{\partial z}, \frac{\partial \theta^4}{\partial w}$ and $\frac{\partial^2 \theta^4}{\partial w^2}$, or in the form of finite differences $(w_r - w)$. It was advantageous to replace w by its fraction, which varies significantly near the surface. This quantity ω, which can be defined by the relationship $\omega = \int_0^z \rho_e dz$ was called the "modified optical thickness".

The global atmospheric optical density is $-\int_{z_s}^0 \rho_e dz$ and is therefore equal to $w + \omega$.

ω is therefore the complement of w. Their sum is equal to the global atmospheric thickness.

The various terms which occur in the formula of Funk are then modified in the following way

$$\frac{\partial w}{\partial z} = - \frac{\partial \omega}{\partial z} \qquad \text{since} \qquad \frac{\partial(w+\omega)}{\partial z} = 0 \tag{81}$$

$$\frac{\partial \theta^4}{\partial w} = - \frac{\partial \theta^4}{\partial \omega} \tag{82}$$

$$\frac{\partial^2 \theta^4}{\partial w^2} = + \frac{\partial^2 \theta^4}{\partial \omega^2} \tag{83}$$

$$\frac{\partial \varepsilon}{\partial w} = - \frac{\partial \varepsilon}{\partial \omega} \tag{84}$$

$$w_r - w = \omega - \omega_r \tag{85}$$

The equation of Funk is then written as

$$\left(\frac{\partial R}{\partial z}\right)_{z_r} = \sigma \left(\frac{\partial \omega}{\partial z}\right)_{z_r} \left[(\theta_0^4 - \theta_R^4) \frac{\partial \varepsilon (\omega_r - \omega_0)}{\partial \omega_r} - \theta_s^4 \frac{\partial \varepsilon (\omega_s - \omega_r)}{\partial \omega_r} + \left(\frac{\partial \theta^4}{\partial \omega}\right)_{\omega_0} \varepsilon (\omega_r - \omega_0) \right.$$
$$\left. - \left(\frac{\partial \theta^4}{\partial \omega}\right)_{\omega_s} \varepsilon (\omega_s - \omega_r) + \int_{\omega_0}^{\omega_r} \frac{\partial^2 \theta^4}{\partial \omega^2} \varepsilon (\omega_r - \omega) \, d\omega + \int_{\omega_r}^{\omega_s} \frac{\partial^2 \theta^4}{\partial \omega^2} \varepsilon (\omega_r - \omega) \, d\omega \right] \qquad (86)$$

54. <u>Calculation of the modified optical thickness</u>

According to our definition

$$\omega = \int_0^z \rho_e(z) \, dz$$

ρ_e is the specific mass of the water vapor at the level z under consideration. If ρ is the specific mass of humid air at the same level, the corresponding specific humidity F is defined by

$$F = \frac{\rho_e}{\rho} \quad ,$$

and consequently

$$\omega = \int_0^z \rho F(z) \, dz$$

In the region under consideration, the variations of ρ as a function of z are negligible to first order and

$$\omega = \rho \int_0^z F \, dz \qquad (87)$$

$\frac{\omega}{\rho}$ therefore represents a measure of the height weighted by the water vapor concentration in a certain sense.

By integrating equation (87) in parts, we obtain <u>/35</u>

$$\omega = \rho \left[z F(z) - \int z \frac{\partial F}{\partial z} \, dz \right] \qquad (88)$$

According to (32) $\quad Z \dfrac{\partial F}{\partial z} = F_* \phi_E \left(\dfrac{z}{L}\right)$

or $\qquad\qquad \omega = \rho \left[z\, F(z) - F_* \int \phi_E \left(\dfrac{z}{L}\right) dz \right]$ $\qquad\qquad$ (89)

Depending on the range of $\dfrac{z}{L}$ considered, ω is represented by different analytical expressions, corresponding to the expressions adapted for $\phi_E \left(\dfrac{z}{L}\right)$.

*For thermally neutral conditions, and for $- 0{,}02 \leqslant \dfrac{z}{L} \leqslant 0{,}001$

$$\phi_E \left(\dfrac{z}{L}\right) = 1 \implies \omega = \rho z \left[F(z) - F_* \right] \qquad\qquad (90)$$

*For over-adiabatic conditions $\dfrac{z}{L} < - 0{,}02$

$$\phi_E \left(\dfrac{z}{L}\right) \simeq \dfrac{1 + 0{,}6 \frac{z}{L}}{2{,}7 \left|\frac{z}{L}\right|^{0{,}25}} \implies \omega = \rho z \left[F(z) - \dfrac{F_*}{2{,}7} \left|\dfrac{z}{L}\right|^{0{,}75} \left(1{,}33 \left(\dfrac{z}{L}\right)^{-1} - 0{,}34 \right) \right] \qquad (91)$$

*For inversion conditions $\dfrac{z}{L} > 0{,}001$

$$\phi_E \left(\dfrac{z}{L}\right) \simeq \dfrac{1 + 0{,}6 \frac{z}{L}}{0{,}5 \left(\frac{z}{L}\right)^{-0{,}1}} \implies \omega = \rho z \left[F(z) - 2 F_* \left(\dfrac{z}{L}\right)^{0{,}1} \left(0{,}1 + 0{,}3 \dfrac{z}{L} \right) \right] \qquad (92)$$

55. Calculation of the expression $\dfrac{\partial \theta}{\partial \omega}$

We have $\dfrac{\partial \theta}{\partial \omega} = \left(\dfrac{\partial \theta}{\partial z}\right) \times \left(\dfrac{\partial z}{\partial \omega}\right)$

According to (87) $\dfrac{\partial \omega}{\partial z} = \rho F(z)$ and, according to (31)

$\dfrac{\partial \theta}{\partial z} = \dfrac{\theta_*}{z} \phi_H \left(\dfrac{z}{L}\right)$

Thus
$$\left(\frac{\partial \theta}{\partial \omega}\right) = \frac{\theta_*}{\rho z F(z)} \phi_H\left(\frac{z}{L}\right)$$
(93)

Thus,

*For thermally neutral conditions $\frac{\partial \theta}{\partial \omega} = \frac{\theta_*}{\rho z F(z)}$

for $-0,02 \leq \frac{z}{L} \leq 0,001$
(94)

*For over-adiabatic conditions $\frac{\partial \theta}{\partial \omega} = \frac{\theta_*}{\rho z F(z)} \frac{1+0,6\frac{z}{L}}{2,7\left|\frac{z}{L}\right|^{0,25}}$

for $\frac{z}{L} < -0,02$
(95)

*For inversion conditions $\frac{\partial \theta}{\partial \omega} = \frac{\theta_*}{\rho z F(z)} \frac{1+0,6\frac{z}{L}}{0,5\left(\frac{z}{L}\right)^{-0,1}}$

for $\frac{z}{L} > 0,001$
(96)

56. Calculation of the expression $\frac{\partial^2 \theta}{\partial \omega^2}$

We have

$$\frac{\partial^2 \theta}{\partial \omega^2} = \frac{\partial}{\partial \omega}\left(\frac{\partial \theta}{\partial \omega}\right) = \frac{\partial}{\partial z}\left(\frac{\partial \theta}{\partial \omega}\right) \times \frac{\partial z}{\partial \omega} = \frac{\theta_*}{\rho z F(z)} \frac{\partial}{\partial z}\left[\frac{\phi_H}{\rho z F(z)}\right]$$

$$= \frac{\theta_*}{\rho^2 F(z)}\left[\phi_H\left(\frac{-z\frac{\partial F}{\partial z} - F(z)}{[z F(z)]^2}\right) + \frac{1}{z F(z)} \frac{\partial \phi_H}{\partial z}\right] = \frac{\theta_*}{[\rho z F(z)]^2}\left[z\frac{\partial \phi_H}{\partial z} - \phi_H\left(1 + \frac{z\frac{\partial F}{\partial z}}{F(z)}\right)\right]$$

From (32) $z\frac{\partial F}{\partial z} = F_* \phi_E \simeq F_* \phi_H$ because $\phi_H \simeq \phi_E$

Thus $\frac{\partial^2 \theta}{\partial \omega^2} = -\frac{\theta_*}{[\rho z F(z)]^2}\left[\phi_H + \frac{F_*}{F(z)}\phi_H^2 - z\frac{\partial \phi_H}{\partial z}\right]$ which leads to the following

expressions

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E

*Thermally neutral conditions $\dfrac{\partial^2\theta}{\partial\omega^2} = -\dfrac{\theta_*}{[\rho z f(z)]^2}\left[1+\dfrac{F_*}{F(z)}\right]$

for $-0,02 \leqslant \dfrac{z}{L} \leqslant 0,001$ (98)

*For over-adiabatic conditions

$$\frac{\partial^2\theta}{\partial\omega^2} = -\frac{\theta_*}{[\rho z f(z)]^2}\left[\frac{1+0,6\frac{z}{L}}{2,7\left|\frac{z}{L}\right|^{0,25}}\left(1,25+\frac{F_*}{F}\left(\frac{1+0,6\frac{z}{L}}{2,7\left|\frac{z}{L}\right|^{0,25}}\right)\right)\right.$$
$$\left. -0,2\left|\frac{z}{L}\right|^{0,75}\right]$$ (99)

*For inversion conditions

$$\frac{\partial^2\theta}{\partial\omega^2} = -\frac{\theta_*}{(\rho z f(z))^2}\left[\frac{1+0,6\frac{z}{L}}{0,5\left|\frac{z}{L}\right|^{-0,1}}\left(0,9+\frac{F_*}{F}\left(\frac{1+0,6\frac{z}{L}}{0,5\left(\frac{z}{L}\right)^{-0,1}}\right)\right)-1,2\left(\frac{z}{L}\right)^{-1,1}\right]$$ (100)

57. Simplifications of the modified Funk equation (equation 86)

Funk (1961) utilized a development limited to first order for the quantities $\dfrac{\partial\theta^4}{\partial\omega}$ and $\dfrac{\partial^2\theta^4}{\partial\omega^2}$, taking into account the small temperature deviations which are observed in the surface layer.

We will now do the same, utilizing the expressions which were established in the preceding sections (55 and 56) in order to verify that the second order terms are indeed negligible.

In effect, $\dfrac{\partial\theta^4}{\partial\omega} = 4\theta^3\dfrac{\partial\theta}{\partial\omega} \simeq 4\theta_0^3\dfrac{\partial\theta}{\partial\omega}\left(1+3\dfrac{\theta-\theta_0}{\theta_0}+\ldots\right)$

The relative variation of θ is small (on the order of 1%) and we may write

$$\frac{\partial\theta^4}{\partial\omega} \approx 4\theta_0^3\frac{\partial\theta}{\partial\omega}$$ (101)

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Also $\dfrac{\partial^2 \theta^4}{\partial \omega^2} = 4\theta^3 \dfrac{\partial^2 \theta}{\partial \omega^2} + 12\,\theta^2 \left(\dfrac{\partial \theta}{\partial \omega}\right)^2 = 4\theta^3 \dfrac{\partial^2 \theta}{\partial \omega^2}\left[1 + \dfrac{3}{\theta}\dfrac{\left(\frac{\partial \theta}{\partial \omega}\right)^2}{\frac{\partial^2 \theta}{\partial \omega^2}}\right]$

According to the expressions which were established for $\dfrac{\partial \theta}{\partial \omega}$ and $\dfrac{\partial^2 \theta}{\partial \omega^2}$ for adiabatic conditions, and assuming that F_* is small compared with F, which we will justify in the following, we may write $\omega \simeq pz\,f(z)$ (see (90)) and we have:

$$\frac{\partial \theta}{\partial \omega} \simeq \frac{\theta_*}{\omega}$$

$$\frac{\partial^2 \theta}{\partial \omega^2} \simeq -\frac{\theta_*}{\omega^2}$$

Thus

$$\frac{\left(\frac{\partial \theta}{\partial \omega}\right)^2}{\frac{\partial^2 \theta}{\partial \omega^2}} \simeq \theta_* \quad \text{and} \quad 1 + \frac{3}{\theta}\frac{\left(\frac{\partial \theta}{\partial \omega}\right)^2}{\frac{\partial^2 \theta}{\partial \omega^2}} \simeq 1 + \frac{3\,\theta_*}{\theta}$$

Thus $3\,\dfrac{\theta_*}{\theta}$ always is less than 1% and we can therefore write:

$$\frac{\partial^2 \theta^4}{\partial \omega^2} \simeq + 4\,\theta_0^3\,\frac{\partial^2 \theta}{\partial \omega^2} \tag{102}$$

The expansion carried out by Funk therefore seems justified and the Funk formula can be written in the following simplified form:

form:

$$\left(\frac{\partial R}{\partial z}\right)_{z_r} = 4\,\sigma\,\theta_0^3 \left(\frac{\partial \omega}{\partial z}\right)_{z_r}\left[(\theta_0 - \theta_R)\frac{\partial \mathcal{E}(\omega_r - \omega_0)}{\partial \omega_r} - \frac{\theta_0}{4}\,\frac{\partial \mathcal{E}(\omega_s - \omega_r)}{\partial \omega_r} + \left(\frac{\partial \theta}{\partial \omega}\right)_{\omega_0}\mathcal{E}(\omega_r - \omega_0)\right.$$

$$\left. - \left(\frac{\partial \theta}{\partial \omega}\right)_{\omega_s}\mathcal{E}(\omega_s - \omega_r) + \int_{\omega_0}^{\omega_s}\frac{\partial^2 \theta}{\partial \omega^2}\,\mathcal{E}(|\omega - \omega_r|)\,d\omega \right] \tag{103}$$

Or, $\left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_0}\mathcal{E}(\omega_r - \omega_0) - \left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_s}\mathcal{E}(\omega_s - \omega_r) = \left[\left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_0} - \left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_s}\right]\mathcal{E}(\omega_r - \omega_0)$

$+\left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_s}\left[\mathcal{E}(\omega_r - \omega_0) - \mathcal{E}(\omega_s - \omega_r)\right] = \int_{\omega_s}^{\omega_0}\dfrac{\partial^2 \theta}{\partial \omega^2}\,\mathcal{E}(\omega_r - \omega_0)\,d\omega + \left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_s}\left[\mathcal{E}(\omega_r - \omega_0) - \mathcal{E}(\omega_s - \omega_r)\right]$

Formula (103) then finally becomes:

$$\left(\frac{\partial R}{\partial z}\right)_{zr} = 4\sigma\theta_o^3 \left(\frac{\partial \omega}{\partial z}\right)_{zr} \left[(\theta_o - \theta_R)\frac{\partial \varepsilon(\omega_r - \omega_o)}{\partial \omega_r} - \frac{\theta_o}{4}\frac{\partial \varepsilon(\omega_s - \omega_r)}{\partial \omega_r}\left(\frac{\partial \theta}{\partial \omega}\right)_{\omega_s}\right.$$

$$\left[\varepsilon(\omega_r - \omega_b) - \varepsilon(\omega_s - \omega_r)\right] + \int^{\omega_s}\frac{\partial^2\theta}{\partial \omega^2}\left[\varepsilon(\omega_r - \omega_b) - \varepsilon(|\omega - \omega_r|)\right]d\omega\Big]\Big| \qquad (104)$$

58. Discussion of the emissivity curve $\varepsilon(\omega)$

By definition, ε is the global emissivity obtained by integrating over the entire frequency spectrum corresponding to the optical thickness ω. It can be determined either by integrating the elementary emissivity. , corresponding to the various wavelengths of the spectrum of directly from absorption measurements of infrared radiation for various optical thicknesses.

In the surface layer, the variation of ε is a function of the pressure and temperature can be ignored (Deacon 1950) and we will utilize the values obtained for the atmospheric pressure on the ground and for a temperature close to 20° C.

As a comparison, we studied the values of ε given in the articles of Brooks (1950), Deacon (1950), Funk (1961), Huhn (1964), Zdunkowski and Johnson (1965), Atwater (1966), Stone and Manabe (1968), Staley and Jurica (1970) and the work of Fleagle and Businger (1963), Kondratiev (1965), Haltiner and Martin (1967). This revue of the literature made it possible to establish the following table where the emissivity values obtained by various authors are shown for the corresponding optical thicknesses (the emissivity values are given in %)

	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1
BROOKS (1950)	0,29	2,44	11,55	24,56	39,59	57,5
DEACON (1950)		2,8	11,8	25	48,6	
HOWARD et al. (1955)		2,6	8,9	19,8	36,5	52,5
cited by STONE and MANABE (1968)						
ELSASSER (1960)		5	14	28	41	54
MÖLLER and ZDUNKOWSKI (1962)	0,27	2,47	11,5	28,9	45,4	59,5
KUHN (1964)	0,58	4,0	12,8	24,9	38,1	54,3
STALEY and JURICA (1970)	1,96	5,65	14,3	27,3	41,8	60,0

*Commas represent decimal points.

Examination of Figure 10 and study of this table show that there is quite a large dispersion which is explained by the difficulty of experimentally determining the function $\varepsilon(\omega)$. Very few measurements have been made for $\omega \leqslant 10^{-4}$ cm H_2O, which is particularly unfavorable for a study where small optical thicknesses must be taken into consideration.

The consequences of these two remarks will be examined in the following.

It seems necessary to select a more valid emissivity curve. We agree with Kondratiev (1965) who, according to our comparative study adapted the curve proposed by Brooks D. L. (1950) (derived from experiments of Brooks, F. A. and Robinson). We agree with this because of the quality of the corresponding measurements and the relative intermediate position of these values. According to recent works of Kuhn (1964) and Staley and Jurica (1970), it seems that greater emissivities must be considered in the range $\omega \leqslant 10^{-3}$ cm H_2O. It seems therefore preferable to confirm this tendency by additional work.

We would like to note that the emissivities considered do
not correspond to absorption by water vapor and therefore do not
consider carbon dioxide. This question is quite controversial.
Brooks (1950) justified the omission of CO_2 by again using the idea
of Elsasser according to which the quasicomplete absorption within
the $\lceil \cdot \mu \rfloor$ band at the temperature under consideration leads to a
negligible role for CO_2 in the divergence of the radiation flux.
This point was contested by Cooley (referenced by Haltiner and
Martin (1967), page 256.) According to Deacon (1950), the statement
of Elsasser would be valid for high layers of the atmosphere,
but absorption by layers at heights less than 100 m would not be
sufficient to make the role of CO_2 effectively negligible. It
seems more correct to take into account the influence of CO_2 by con-
sidering an additional absorption which can be directly added to
the water vapor absorption, as Deacon did. This type of calculation
assumes that the absorption bands do not overlap except over a
very small wavelength interval, which seems to be valid for the
lower layers of the atmosphere because of the small values of
optical thickness (Deacon 1950). Deacon therefore established
emissivity curves based on values of Brooks for water vapor but
corrected by an additional emissivity due to CO_2, which is
assumed to be present for an average concentration of 0.03% in
volume. (This hypothesis is obviously an approximate one but is
acceptable because it only influences a correction term). The
values obtained by Deacon were utilized by Funk (1961 in the
application of his numerical method. We would like to state
that within the framework of our study, the possible effect of
CO_2 is of no great practical importance, because of the disper-
sion of the emissivity values corresponding to absorption by water
vapor only. The values of Deacon, which take into account the
CO_2 effect, even though they are obviously larger than those
of Brooks, are within the fluctuation range which is shown in
Figure 10. The two problems cannot be separated. The only
important point to know is whether the indeterminacy of the

emissivity curve affects the results in a significant way. The calculations will then be carried out with two different emissivity curves, the one of Brooks (1950) and the one of Deacon (1950). By comparing the obtained results we can evaluate the role of CO_2 and the influence of the different emissivity values for water vapor alone at the same time.

Another unfavorable point must be considered. This is the scarcity or absence of values of ε for $\omega \leqslant 10^{-4}$ cm H_2O . In our study, optical thicknesses as small as 10^{-8} cm H_2O must be taken into consideration. Just like Zdunkowski and Johnson (1965) and Atwater (1966), an extrapolation relationship must be used. Assuming that ε must be essentially proportional to ω (Fleagle and Businger 1963) for very small values of optical thickness, it seems that an equation having the form $\varepsilon = a \log (1 + b \omega)$ is more approapriate for $\omega \leqslant 10^{-4}$ cm H_2O (a and b are determined for each case by continuity conditions at the point $\omega = 10^{-4}$ cm H_2O of $\varepsilon(\omega)$ and $\frac{\partial \varepsilon}{\partial \omega}$).

VI. NUMERICAL CALCULATION OF THE RADIATION FLUX DIVERGENCE: /40 METHOD AND RESULTS

61. Conditions for carrying out the numerical calculation

If we know the analytical expressions for the temperature profile, humidity profile, modified optical thickness ω, and functions $\frac{\partial \theta}{\partial \omega}$ and $\frac{\partial^2 \theta}{\partial \omega^2}$, the calculation of the radiation flux divergence according to formula (104) can be done numerically for different values of u_*, L and θ_0, if θ_R and z_s are defined and if $\varepsilon(\omega)$ is known.

Continuing the discussion in paragraph (58) and after performing numerical studies on the computer, the following expressions were obtained for representing the emissivity curves.

According to

BROOKS $\quad \omega \geqslant 10^{-4}$ cm H_2O $\qquad \mathcal{E} = 0,5777 + 0,0847 \, Log \, \omega + 0,0027(Log \, \omega)^2$ (105)

$\qquad \omega \leqslant 10^{-4}$ cm H_2O $\qquad \mathcal{E} = 0,260 \, Log \, (1 + 1,04.10^3 \, \omega)$ (106)

According to

DEACON $\quad \omega \geqslant 10^{-4}$ cm H_2O $\qquad \mathcal{E} = 0,7875 + 0,1420 \, Log \, \omega + 0,0065(Log \, \omega)^2$ (107)

$\qquad \omega \leqslant 10^{-4}$ cm H_2O $\qquad \mathcal{E} = 0,055 \, Log \, (1 + 7,00.10^3 \, \omega)$ (108)

z_s was defined as the altitude at the top of the absorbing atmosphere. This introduces a problem, because we assume certain variation laws for the temperature and humidity which are obviously only valid in the surface layer. By continuing them and extra-polating them to higher altitudes, unreasonable results are obtained. As Funk (1961) showed, and as we were able to deter-mine in our calculations, the influence of the upper layers (above 100 m) is very small. The temperature and humidity gradients are primarily of importance in the very low layers. It is there-fore possible to limit oneself to a value of z_s on the order of 100 m, which amounts to assuming that the temperature and humidity gradients are negligible in the upper layers.

Also it is possible to ignore the terms $\dfrac{\theta_o}{4} \dfrac{\partial \mathcal{E}(\omega_s - \omega_r)}{\partial \omega_r}$ and $\left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_s} \left[\mathcal{E}(\omega_r - \omega_o) - \mathcal{E}(\omega_s - \omega_r)\right]$ in equation (104) as Funk (1961) showed. The validity of this approximation was verified numerically and can be justified directly. In effect, the first term is close to $\dfrac{\theta_o}{4} \dfrac{\partial \mathcal{E}(\omega_s)}{\partial \omega_r}$, where ω_R is small compared with ω_s. $\dfrac{\partial \mathcal{E}}{\partial \omega}$ is a rapidly decreasing function as we shall see later on. Conse-quently $\dfrac{\partial \mathcal{E}}{\partial \omega}(\omega_s)$ is small. Also the second term is close to $\left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_s} \mathcal{E}(\omega_s)$. Thus, $\mathcal{E}(\omega_s)$ is always smaller than 1 and $\left(\dfrac{\partial \theta}{\partial \omega}\right)_{\omega_s}$

is on the order of $\frac{\partial x}{\omega s}$, which is very small compared with the other terms. ω_s is large with respect to the values of ω within the surface layer.

The problem posed by the definition of θ_R is a more delicate one. θ_R intervenes through the term $\theta_o - \theta_R$, which represents the deviation between the real temperature and the equivalent radiation temperature at the surface. θ_R is an experimental quantity which is difficult to introduce into the calculation. Also, in the case of the ocean, whose surface emissivity is very close to 1, it seems that $\theta_o - \theta_R$ does not exceed 0.5° C (Lecomte and Deschamps 1970). Consequently, this term is much less important than above the ground where the deviation can reach 10° C. In order to avoid the introduction of additional variables for the numerical calculation, we will assume that $\theta_R = \theta_o$ and we will later on examine the consequences of this hypothesis.

Under these conditions, equation (104) finally becomes

$$\left(\frac{\partial R}{\partial z}\right)_{Z_r} = 4\sigma \theta_o^3 \left(\frac{\partial \omega}{\partial z}\right)_{Z_r} \left[\int_{\omega_o}^{\omega_s} \frac{\partial^2 \theta}{\partial \omega^2} \left[\mathcal{E}(\omega_r - \omega_o) - \mathcal{E}(|\omega - \omega_r|)\right] d\omega\right] \quad (109)$$

This relatively simple expression has advantages, because of the form of the integral in the brackets. In effect, if we again consider the initial expression (79) used by Funk (1961) Hamilton (1965), Lieske and Troschein (1967), we can see that the terms $\left(\frac{\partial \theta^4}{\partial \omega}\right)_{\omega_o} \mathcal{E}(\omega_r - \omega_o)$ and $\int_{\omega_o}^{\omega_r} \frac{\partial^2 \theta^4}{\partial \omega^2} \mathcal{E}(\omega_r - \omega) d\omega$ are essentially of the same order of magnitude and are also much more important than the other terms. The result therefore is essentially given by the difference of these two terms. Separate estimation of them can lead to a considerable error, as Lieske and Troschein (1966) remarked. The establishment of an analytical expression for this difference makes it possible to limit the errors resulting from a numerical calculation in a significant way.

This is important to us and as far as we know, it has not yet been done when the Funk formula was used.

The calculation of the integral (109) is too complex to be carried out analytically in the general case. Therefore we must carry out a numerical calculation. The calculations were first done by hand for particular cases. But the number of possible combinations of the data and the requirement for decomposing the integral into a sufficient number of intervals rapidly led us to using a computer. The numerical calculation was performed by M. R. Tomassone from the Biometry Department of the INRA (CNRZ - 78-Jouy-en-Josas) on an IBM 360-50. The program which was established in collaboration with him is given in the Appendix. It was built in order to determine the influence of the emissivity curve on the results by comparing the values obtained with the curves of Deacon and Brooks. It is also used to determine the effect of a simplification of the profiles when the logarithmic relationship is substituted for the complete expressions.

The calculation was carried for 50 different situations corresponding to five values of friction velocity u_*

$$u_* = 0,08 \text{ m/s} \quad (u_{10} \simeq 2,8 \text{ m/s}) \Longrightarrow z_0 \simeq 10^{-5} \text{ m}$$
$$u_* = 0,16 \text{ m/s} \quad (u_{10} \simeq 5 \text{ m/s}) \Longrightarrow z_0 \simeq 5.10^{-5} \text{ m}$$
$$u_* = 0,28 \text{ m/s} \quad (u_{10} \simeq 8 \text{ m/s}) \Longrightarrow z_0 \simeq 10^{-4} \text{ m}$$
$$u_* = 0,5 \text{ m/s} \quad (u_{10} \simeq 12 \text{ m/s}) \Longrightarrow z_0 \simeq 5.10^{-4} \text{ m}$$
$$u_* = 0,65 \text{ m/s} \quad (u_{10} \simeq 15 \text{ m/s}) \Longrightarrow z_0 \simeq 10^{-3} \text{ m}$$

and ten values of the Monin-Obukhov length.

$$L \text{ (m)} \quad -5, \quad -16, \quad -100, \quad -500, \quad -1000, \quad +1000, \quad +500, \quad +100, \quad +16, \quad +$$

The values of $\left.\dfrac{\partial R}{\partial z}\right|$ and $\left.R(z) - R_0\right|$ are obtained at the levels

$$z = 1 \text{ mm}, \quad 5 \text{ mm}, \quad 1 \text{ cm}, \quad 5 \text{ cm}, \quad 10 \text{ cm}, \quad 50 \text{ cm}, \quad 1 \text{ m}, \quad 5 \text{ m}, \quad \text{and} \quad 10 \text{ m}.$$

For each situation, the calculations were made using the following hypotheses:

Hypothesis 1 - emissivity of Deacon (for a concentration of 0.03% of CO_2)

Hypothesis 2 - emissivity of Brooks

Hypothesis 3 - emissivity of Brooks and logarithmic profile expression

62. Presentation and discussion of results obtained with Hypothesis 1

Hypothesis 1 corresponds to the general solution of the problem, because the complete expression for the profile is used and the role of CO_2 is taken into account.

The results obtained for $\left.\dfrac{\partial R}{\partial z}\right)$ under different situations are given in the tables of Appendix II. Examination of them leads to the following conclusions.

621. The form of the profiles $\left.\dfrac{\partial R}{\partial z}\right]$ always remains essentially similar to itself. Figures 11 and 12 show typical profiles corresponding to the stable and unstable ranges.

622. The profile shape is very close to the one used in previous studies, and is similar to both theoretical ones (Zdunkowski and Johnson (1965), Atwater (1966), Lieske and Troschein (1967) as well as experimental ones (Yamamoto and Kondo (1959), Hamilton (1965) and primarily those of Hinzpeter and Heinrich (1969) in the range 0-10 m.)

The divergence in the radiation flux, no mater what its sign, decreases rapidly with altitude and always maintains the same sign. However, the maximum does not correspond to the surface itself but is slightly shifted (about 1 mm to 10 cm, depending on the case). For the air layer immediately adjacent to the surface (on the order of 1 mm), there is an inversion of the phenomena which correspond to heating when there is cooling above it and vice versa. These results already obtained by Zdunkowski and Johnson (1965) in a similar study and also by Godson (1965) by a different method make it possible to explain the fact that the minimal nocturnal temperature often occurs not at the surface but slightly above it (1 mm - 10 cm, depending on the author). Numerous experimental observations of this phenomena have been collected by Oke (1970), who himself demonstrated this fact.

623. Variation of the ratio $\dfrac{R(z) - R_0}{S_q}$ as a function of z is shown in Figure 12 for various stability ranges. The influence of the radiation transfer appears as an increasing function with altitude. The divergence has the same sign. Thus, as Robinson (1950) predicted and as Figure 11 shows, if the term $\dfrac{\partial R}{\partial z}$ becomes small above 1 m in height, it is not possible to conclude that the influence of radiation transfer is negligible above this height and no statements regarding the conservation of turbulent flux can be made. This conclusion has often been reached.

624. As we indicated previously, the relative variation of this latter flux is equal to the ratio $\dfrac{R(z) - R_0}{S_\partial}$. In the following table we will give the calculated values at the level z = 10 meters.*

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*For large wind velocities, Monin-Obukhov lengths such as L = 16 m or 5 m lead to deviations $\theta_{10} - \theta_0$ which are much greater than those produced in natural mechanisms and can therefore not be considered plausible ones. Consequently, we did not give the corresponding results.

L	− 5 m	− 16 m	− 100 m	− 500 m	−1000 m	+1000 m	+ 500 m	+ 100 m	+ 16 m	+ 5 m
θ_0 = 0,08 m/s	− 3,7	− 1,15	− 0,20	− 0,04	− 0,02	+ 0,02	+ 0,03	+ 0,18	+ 1,20	+ 3,7
$\dfrac{(R_{10}) - R_0}{S_0}$	0,72	0,80	0,87	0,88	0,87	0,88	0,92	0,98	1,13	1,37
θ_0 = 0,16 m/s	− 13,5	− 4,2	− 0,74	− 0,13	− 0,07	+ 0,07	+ 0,12	+ 0,66	+ 4,4	+ 13,4
$\dfrac{(R_{10}) - R_0}{S_0}$	0,32	0,34	0,37	0,39	0,37	0,39	0,40	0,43	0,51	0,62
θ_0 = 0,28 m/s		− 11,5	− 2,0	− 0,34	− 0,17	+ 0,16	+ 0,34	+ 1,82	+ 12,0	
$\dfrac{(R_{10}) - R_0}{S_0}$		0,18	0,20	0,21	0,20	0,19	0,22	0,24	0,28	
θ_0 = 0,5 m/s			− 2,4	− 0,87	− 0,44	+ 0,44	+ 0,91	+ 4,84		
$\dfrac{(R_{10}) - R_0}{S_0}$			0,10	0,10	0,10	0,10	0,11	0,27		
θ_0 = 0,65 m/s			− 9,20	− 1,47	− 0,74	+ 0,75	+ 1,53	+ 8,20		
$\dfrac{(R_{10}) - R_0}{S_0}$			0,066	0,068	0,066	0,068	0,072	0,080		

The relative variation of the turbulent flux resulting from the calculation is sometimes very large and most often greater than 10 or 20%, which could be established as an arbitrary limit for assuming that the hypothesis of flux conservation between 0 and 10 m is valid. Nevertheless we should stress that these numbers do not necessarily correspond to the effective variation of S but is the result of our method: if S is assumed to be initially constant, the variations in S given in the table can be

derived from it. The effective variation of S will be smaller in general, but must be taken into account completely in most cases.*

625. According to the above table, it seems that the deter- /45 mining factor (for a given surface temperature and therefore given surface humidity) is the wind velocity. The smaller it is, the greater this effect will be. This is logical since the value of the turbulent flux relative to the value of the radiation flux is then reduced. For the same reason, the relative variation of the turbulent flux is greater at a given velocity in an inversion situation than it is in an over-adiabatic situation. This confirms the generally held opinion according to which the influence of the radiation transfer is primarily noticeable during the night (and even more if the wind is weak). However, even in an over-adiabatic situation or, in general, in daytime conditions, the effect can be noticed for weak winds, which was expressed in the work by Lumley and Panofsky (1964) and Munn (1966).

63. Influence of the emissivity

The results described in the preceding chapter were obtained for emissivity values given by Deacon.

We can ask whether a priori the important role played by small optical thicknesses in our study would not lead to considerable deviations in our result. In effect, if the values of ε adopted by Brooks and Deacon, for example, only differed by 10 to 20% for $\omega \geq 10^{-4}$ cm H_2O, a much larger difference would be introduced in the extrapolation formulas. The ratio $\dfrac{\varepsilon \text{ BROOKS}}{\varepsilon \text{ DEACON}}$ would go to 0.67 when ω goes to zero.

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*This point will be discussed in the conclusion.

62

The study of the ratios $\left|\dfrac{\left(\frac{\partial R}{\partial z}\right)_{BROOKS}}{\left(\frac{\partial R}{\partial z}\right)_{DEACON}}\right|$ and $\left|\dfrac{(R(z)-R_0)_{BROOKS}}{(R(z)-R_0)_{DEACON}}\right|$, especially for $z = 10$ m , makes it possible to study the deviation which could result by adopting any kind of emissivity curve. These ratios, calculated for a certain number of cases, are given in the following tables.

$\dfrac{\left(\frac{\partial R}{\partial z}\right)_{z=10} BROOKS}{\left(\frac{\partial R}{\partial z}\right)_{z=10} DEACON}$ $\qquad$ L $\qquad$ u_*	− 5 m	− 1 000 m	+ 1000 m	+ 5 m
0,08 m/s	0,75	0,80	0,80	0,75
0,28 m/s	0,80	0,78	0,80	0,80
0,65 m/s	0,79	0,80	0,81	0,79

$\dfrac{(R_{(10)}- R_0)_{BROOKS}}{(R_{(10)}-R_0)_{DEACON}}$ $\qquad$ L $\qquad$ u_*	− 5 m	− 1000 m	+ 1000 m	+ 5 m
0,08 m/s	0,90	0,91	0,90	0,96
0,28 m/s	0,91	0,90	0,88	0,97
0,65 m/s	0,92	0,90	0,90	0,97

The differences in the divergence are on the order of 20%. The differences in the flux variation are about 10%. These numbers can be considered to show the influence of CO_2 or the uncertainty connected with the emissivity value for water vapor only. It therefore seems that the high values of $\left|\dfrac{R_{(10)} - R_0}{50}\right|$ obtained in the preceding paragraph cannot be attributed to excessive values of ε, because the contribution of CO_2 does not

F

exceed 10%. On the other hand, the increased emissivity values
which were the result of recent studies by Kuhn and by Staley and
Jurica could not lead to a higher number for the relative variation
in turbulent flux.

In conclusion, the problem of exactly defining the emissivity
values does not seem to be the determining one, as Zdunkowski and
Johnson (1965) and Jurica (1966) found. Thus the latter concluded,
"Thus, we may consider that the Brooks method shows that there
is a moderate sensitivity to emissivity values when considered as
external parameters. It seems that any calculation based on a
reasonable collection of values, no matter from what source, will
lead to results which can be considered equivalent for all practi-
cal applications."

64. <u>Influence of stability. Logarithmic approximation of the
profiles.</u>

The influence of stability was demonstrated by the numerical
results given in paragraph 62. The appearance of the curves for
$\dfrac{R(z) - R_0}{S_0}$ and those shown in Figure (12) suggests the definition
of an empirical stability function $\varphi\dfrac{z}{L}$ such that

$$\frac{R(z) - R_0}{S_0} = 4\,\sigma\,\theta_0^3\,\theta_*\,\varphi_1\left(u_*, z_0, L\right)\,\varphi\left(\frac{z}{L}\right)$$

The dispersion of the various values of $\varphi\left(\dfrac{z}{L}\right)$ obtained is
relatively small, which justifies the method. The averaged
values are given in the following table   * /47

$\dfrac{z}{L}$	- 1	- 0,6	- 0,3	- 0,1	-0,05	-0,02	0	+0,02	+0,05	+ 0,1	+ 0,3	+ 0,6	+ 1
$\dfrac{\varphi\left(\frac{z}{L}\right)}{\varphi(0)}$	0,89	0,91	0,95	0,99	1	1	1	1,05	1,06	1,10	1,12	1,25	1,4

*Commas represent decimal points.

L = 10 m is a more modern example and shows that the influence of stability in the first 10 meters of the atmosphere will be less than 15% in the over-adiabatic regime and 40% in the inversion regime.

It seems that the simplified logarithmic form of the profiles will not bring about important errors in estimating the relative variation of turbulent flux. The second order effects can be taken into account by the following empirical relationships:

$$\varphi\left(\frac{z}{L}\right) = \varphi_{(0)}\left(1 + 0,15 \frac{z}{L}\right) \quad \text{for} \quad L < 0 \tag{110}$$

$$\varphi\left(\frac{z}{L}\right) = \varphi_{(0)}\left(1 + 0,4 \frac{z}{L}\right) \quad \text{for} \quad L > 0 \tag{111}$$

In order to confirm this opinion we carried out the calculation using hypothesis 3, the Brooks emissivity and the logarithmic expression for the profiles. Comparison with the results obtained for hypothesis 2 lead to the following table.[*]

$\frac{z}{L}$	− 1	− 0,6	− 0,3	−0,1	−0,05	−0,02	0	+0,02	+0,05	+ 0,1	+0,3	+0,6	1
$\dfrac{(R(z) - R(0))_{hyp2}}{(R(z) - R(0))_{hyp3}}$	0,90	0,93	0,96	0,98	1,02	1	1	1	1,03	1,04	1,10	1,27	1,52

*Commas represent decimal points.

Agreement with the preceding table is very satisfactory.

Nevertheless, it is obvious that the loss of accuracy corresponding to the simplified logarithmic relationships in the numerical calculation is not an advantage. The reduction in calculation time on the computer is not significant. On the other hand, it becomes difficult to find an analytical solution because of the complexity of the relationships used. However it is possible if we limit ourselves to logarithmic expressions, as we will see.

ANALYTICAL CALCULATION AND ESTABLISHMENT OF SIMPLIFIED <u>/48</u>
RELATIONSHIPS

71. Interest in an analytical calculation

Even though the calculation cannot be done except by using
a certain number of approximations, it is of interest for two
reasons:

--on the one hand, it makes it possible to have approximate
calculation methods for the divergence which are simple
and fast (the computer is replaced by the slide rule) which
are then of interest for the experimentor

--on the other hand, the relative influence of micrometeoro-
logical parameters can be shown.

72. Analytical expression of the functions θ and ω

The temperature and humidity profiles are reduced to the
logarithmic form

$$\theta(z) - \theta_o = \theta_* \, Log \, \frac{z}{z_o} \qquad (112)$$

$$F_{(z)} - F_o = F_* \, Log \, \frac{z}{z_o} \qquad (113)$$

According to paragraphs 54, 55, and 56, ω , $\frac{\partial \theta}{\partial \omega}$ et $\frac{\partial^2 \theta}{\partial \omega^2}$ are
written as:

$$\omega = \rho z F(z) \left[1 - \frac{F_*}{F(z)} \right] \qquad (114)$$

66

$$\frac{\partial \theta}{\partial \omega} = \frac{\theta_*}{\rho z F(z)}$$

$$\frac{\partial^2 \theta}{\partial \omega^2} = -\frac{\theta_*}{[\rho z F(z)]^2}\left[1 + \frac{F_*}{F(z)}\right]$$

(116)

According to the calculated humidity profiles, $\left.\frac{F_*}{F(z)}\right|$ always /49 remains less than 0.1. The relatively large values of this ratio correspond to relatively great heights. We can therefore ignore the term $\left.\frac{F_*}{F(z)}\right|$ in equations 114, 115, 116 which leads to the following equation:

$$\omega = \rho z F(z)$$

(117)

$$\frac{\partial \theta}{\partial \omega} = \frac{\theta_*}{\omega}$$

(118)

$$\frac{\partial^2 \theta}{\partial \omega^2} = -\frac{\theta_*}{\omega^2}$$

(119)

73. Analytical representation of the emissivity curve

Such a representation must approximate the experimental curve in a satisfactory way at least over the optical thickness range used in the calculation. Also it must have a simple form which will not cause too much complexity in the analytic calculation of the integral in equation (104).

Such expressions have already been used for numerical calculations. Thus, Hamilton (1965) proposes a rather simple decomposition of the curve $\varepsilon(\omega)$, into two parts of straight lines, which meet at the value $\omega = 8.10^{-4}$ cm H_2O. Atwater (1966) utilized the decomposition of Elliott and Stevens in which the curve is represented by line segments for the intervals limited by the values 10^{-5}, 10^{-4}, 10^{-3} and 10^{-2} cm H_2O. Finally, Jurica (1966) also utilized analytical expressions for $\frac{\partial \varepsilon}{\partial \omega}$ which enter the formula of Brooks, but he does not give them explicitly.

Even though the expression utilized by Atwater seems to have the required accuracy, the decomposition into several intervals is cumbersome for the analytical calculation. We also look for a relationship better suited for our study. Since we could not find a simpler relationship which extends over the entire variation range of ω, we had to consider two intervals.

For $\qquad \omega \geqslant 10^{-4}$ cm H_2O

the form of the emissivity curve of Brooks in the semi-logarithmic representation led to the use of a hyperbolic tangent function of the form:

$$\frac{\varepsilon}{\varepsilon_1} = 1 + A\,\mathrm{Th}\left(B\,\mathrm{Log}\frac{\omega}{\omega_1}\right) = 1 + A\frac{\left(\frac{\omega}{\omega_1}\right)^{2B} - 1}{\left(\frac{\omega}{\omega_1}\right)^{2B} + 1} \qquad (120)$$

ε_1, A and B are numerical constants.

Such a simple expression cannot cover the entire variation /50 range of ω with the desired accuracy. The introduction of higher order terms would lead to too much complexity.

The contribution of layers such as $\omega \geqslant 10^{-1}$ cm H_2O seems to be quite small. We determined ε_1, A and B such that expression (120) approximates the experimental curve over the interval $\sqrt{} 10^{-4}$ cm H_2O , 10^{-1} cm $H_2O \sqrt{}$ in the best way.

The adopted values are

$$\begin{cases} A = 1,125 \\ B = 0,25 \\ _1 = 0,218 \end{cases}$$

As Figure 13 shows, agreement with the experimental data is satisfactory over the interval $\sqrt{} 10^{-4}$ cm H_2O, 4×10^{-2} cm $H_2O \sqrt{}$. There is a slight divergence which appears above 4×10^{-2} cm H_2O . This influence is less than 1%. All attempts to make an improvement on the side of large optical thicknesses led to a divergence on the small optical thickness side, which was much more detrimental to the calculation accuracy.

The corresponding analytical representation of the first derivative $\frac{\partial \varepsilon}{\partial \omega}$ is shown in Figure 14. It also seems to be satisfactory considering the dispersion of the experimental values (the high values of $\frac{\partial \varepsilon}{\partial \omega}$ proposed by Fleagle and Businger (1963) and which were contested by Staley and Jurica (1970) were not included).

For $\omega \leqslant 10^{-4}$ cm H_2O, we used an extrapolation formula of the form $\varepsilon = a$ Log $(1 + b\omega)$ (Cf 58); a and b are determined from continuity at the point 10^{-4} cm H_2O.

We have $\quad \varepsilon = 0,260$ Log $(1 + 1,04 \times 10^3 \omega)$ (121)

This expression as well as the one corresponding to $\left.\frac{\partial \varepsilon}{\partial \omega}\right|$ are also shown in Figures 13 and 14.

74. Details of the calculation proper

This calculation is based on the equation (104) in which the terms $\frac{\theta_o}{4} \left.\frac{\partial \varepsilon(\omega_s - \omega_r)}{\partial \omega_r}\right|$ and $\left(\frac{\partial \theta}{\partial \omega}\right)_{\omega_i} \left[\varepsilon(\omega_i - \omega_o) - \varepsilon(\omega_s - \omega_r)\right]\bigg|$ can be neglected as we saw. We therefore find:

$$\left(\frac{\partial R}{\partial z}\right)_{z_r} = -4\sigma\theta_o^3 \left(\frac{\partial \omega}{\partial z}\right)_{z_r} \left[-(\theta_o - \theta_R)\frac{\partial \varepsilon(\omega_r - \omega_o)}{\partial \omega_r} + \left|\frac{\partial \theta}{\partial \omega}\varepsilon(|\omega_r - \omega|)\right|_{\omega_o}^{\omega_s} \right.$$
$$\left. - \int_{\omega_o}^{\omega_s} \frac{\partial^2\theta}{\partial \omega^2}\varepsilon(|\omega - \omega_r|)\, d\omega \right]\bigg| \tag{122}$$

Taking 117, 118 and 119 into account we obtain:

$$\left(\frac{\partial R}{\partial z}\right)_{z_r} = -4\sigma\theta_o^3\theta_* \rho F(z_r)\left[-\frac{(\theta_o - \theta_R)}{\theta_*}\frac{\partial \varepsilon(\omega_r - \omega_o)}{\partial \omega_r} + \left|\frac{1}{\omega}\varepsilon(|\omega - \omega_r|) + \int_{\omega_o}^{\omega_i\ \omega_s}\frac{\varepsilon(|\omega - \omega_r|)}{\omega^2}d\omega\right|\right]\bigg| \tag{123}$$

The calculation amounts to calculating the expression
$\times = \left|\frac{1}{\omega}\varepsilon(|\omega_r - \omega|)\right|_{\omega_b}^{\omega_i\ \omega_o} + \int_{\omega_o}^{\omega_s}\frac{\varepsilon(\omega_r - \omega)}{\omega^2}d\omega\bigg|$ which is done in different ways depending on the value of ω_r.

731. In the most complex case $\omega_r \gtrsim 1{,}1 \times 10^{-4}$ cm H_2O, 5 intervals must be considered.

I. $\left[\omega_o,\ 0{,}1\ \omega_r\right]\bigg|$

Let us set $\big|$ $|\omega_r - \omega| = \omega_r - \omega\big|$

and $\omega_r \geqslant \omega_r - \omega \geqslant 0,9\,\omega_r |\cdot|$

This latter condition leads to assuming that $\dfrac{\partial \mathcal{E}(\omega_r - \omega)}{\partial \omega}\Big|$ is approximately constant and equal to $\dfrac{\partial \mathcal{E}(\omega_r)}{\partial \omega_r}\Big|$.

By integrating by parts $X_1 = X(\omega_0, 0,1\,\omega_r) = \displaystyle\int_{\omega_0}^{0,1\,\omega_r} \dfrac{\partial \mathcal{E}(\omega_r - \omega)}{\partial \omega} \times \dfrac{1}{\omega} \times d\omega\Big|$

$\qquad = -\dfrac{\partial \mathcal{E}(\omega_r)}{\partial \omega} \left[\text{Log}\, \dfrac{0,1\,\omega_r}{\omega_0} \right]$

II. $\quad \left[0,1\omega_x,\ \omega_x - 10^{-4} \right]$

$\qquad |\omega_r - \omega| = \omega_r - \omega$

$\qquad \omega_r \cdot \omega \geqslant 10^{-4} \implies$ eq (120)

$X_2 = X(0,1\,\omega_r\ ,\ \omega_r - 10^{-4}) = -2A\,\mathcal{E}_1 \sqrt{\omega_i} \left[\dfrac{1}{\omega_0 + \omega_r} \left(\dfrac{1}{10^{-2} + \sqrt{\omega_i}} - \dfrac{1}{\sqrt{\omega_i} - \sqrt{0,9\omega_r}} \right) \right.$

$+ \dfrac{2\sqrt{\omega_i}}{(\omega_i + \omega_r)^2} \left(\text{Log}\, \dfrac{\sqrt{\omega_r - 10^{-4}}}{10^{-2} + \sqrt{\omega_i}} + \text{Log}\, \dfrac{\sqrt{0,9\omega_r} + \sqrt{\omega_i}}{\sqrt{0,1\,\omega_r}} \right) - \dfrac{\omega_0 - \omega_r}{(\omega_0 + \omega_r)^2} \dfrac{1}{\sqrt{\omega_r}} \left(\text{Arcth}\, \dfrac{10^{-2}}{\sqrt{\omega_r}} - \text{Arcth}\, \sqrt{0,9} \right) \left. \right]$ $\Big/ \underline{52}$

III. $\quad \left[\omega_r - 10^{-4}, \omega_r \right]$

$\qquad |\omega_r - \omega| = \omega_r - \omega$

$\qquad \omega_r - \omega \leqslant 10^{-4} \implies$ eq (121)

$X_3 = X(\omega_r - 10^{-4}\ ,\ \omega_r) = -\left(\dfrac{\partial \mathcal{E}}{\partial \omega}\right)_{\omega_r} \left[\mathcal{E}(10^{-4}) + \text{Log}\, \dfrac{\omega_r}{\omega_r - 10^{-4}} \right]$

IV. $\quad \left[\omega_r, \omega_r + 10^{-4} \right]$

$\qquad |\omega_r - \omega| = \omega - \omega_r$

$\qquad \omega_r - \omega \leqslant 10^{-4} \implies$ eq (121)

$X_4 = X(\omega_r, \omega_r + 10^{-4}) = -\left(\dfrac{\partial \mathcal{E}}{\partial \omega}\right)_{\omega_r} \left[-\mathcal{E}(10^{-4}) + \text{Log}\, \dfrac{\omega_r}{\omega_r + 10^{-4}} \right]$

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V. $\left[\omega_r + 10^{-4} \;,\; \omega_s\right]$

We are then referred to case II with $|\omega_1 - \omega| = \omega - \omega_r$, and we find:

$$X_5 = -2A\varepsilon_1 \sqrt{\omega_1} \left[\frac{1}{\omega_0 + \omega_r} \left(\frac{1}{\sqrt{\omega_s} + \sqrt{\omega_1}} + \frac{1}{\sqrt{\omega_1} - 10^{-2}} \right) + \frac{2\sqrt{\omega_0}}{(\omega_0 + \omega_r)^2} \left(Log \frac{10^{-2} + \sqrt{\omega_1}}{\sqrt{\omega_r + 10^{-4}}} \right. \right.$$

$$\left. \left. + Log \frac{\sqrt{\omega_s}}{\sqrt{\omega_s} + \sqrt{\omega_0}} \right) - \frac{\omega_0 - \omega_r}{(\omega_0 + \omega_r)^2} \frac{1}{\sqrt{\omega_r}} \left(Arc\,tg \frac{\sqrt{\omega_s}}{\sqrt{\omega_r}} - Arc\,tg \frac{10^{-2}}{\sqrt{\omega_r}} \right) \right]$$

Thus for the numerical calculation, we see that ω_s is a quantity which is not well defined.

For the analytical calculation, we may set $\omega_s = 100 \; \omega_r$ and ignore the contribution of the upper layers.

We finally find:

$$X = -\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega_r} Log \frac{\omega_r}{\omega_0} + \left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega_r} \left[Log\,10 - Log \frac{\omega_r}{\omega_1 \cdot 10^{-4}} + Log \frac{\omega_r}{\omega_r + 10^{-4}} \right] + X_2 + X_5$$

But X_2 and X_5 are defined numerical functions which always depend on ω_r.

We can therefore set

$$X = -\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega_r} \left[Log \frac{\omega_r}{\omega_0} + R_1(\omega_r) \right] \tag{124}$$

$\underline{R_1(\omega_r)}$ $\underline{\text{is a function which can be expressed in analytical form}}$ $\underline{\text{and which can be calculated once and for all.}}$

732. In the case $10^{-4} \leq \omega r \leq 1,1 \cdot 10^{-4}$

$$0,1 \, \omega_r \geq \omega_r - 10^{-4}$$

The intervals II and III are regrouped into the single interval $[0,1\omega_r, \omega_r]$. The intervals I, IV and V remain unchanged. We then obtain an expression having the form (124), $R_1(\omega_r)$ is this time defined by a different analytical expression.

733. In the case $\omega_o \leq \omega_r \leq 10^{-4}$ cm H_2O the intervals I, II, and III are regrouped into the interval $[\omega_b, \omega_r]$ and the intervals IV and V remain unchanged. Using the same calculation method, we again find expression (124) and even here we find an analytical expression for $R_1(\omega_r)$.

734. In conclusion, in the general case and for any ω_r we can write

$$X = -\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega_r} \left[\text{Log} \frac{\omega_r}{\omega_o} + R_1(\omega_r) \right]$$

$R_1(\omega_r)$ is a defined function and its analytical expression differs, depending upon the interval under consideration. But obviously it remains continuous over the entire variation interval of ω_r. It was numerically calculated and is shown in Figure 15 (it should be noted that $R_1(\omega_r)$ is a negative quantity for

$\omega_r \leq 3.10^{-3}$ cm H_2O) .

735. According to equation (123) and the expression for X, we finally find:

$$\left(\frac{\partial R}{\partial z}\right)_{z_r} = 4\sigma\theta_o^3 \, \theta_* \, \rho F(z_r) \left[\frac{\theta_o - \theta_R}{\theta_*} \frac{\partial \varepsilon(\omega_r - \omega_o)}{\partial \omega_r} + \left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega_r} \text{Log} \frac{\omega_r}{\omega_o} + R_1(\omega_r) \right]$$

As soon as $z_r/z_o \gg 10$, which occurs most frequently, $\frac{\omega_r}{\omega_o} > 10$, and we find:

$$\left(\frac{\partial R}{\partial z}\right)_{z_r} = 4\,\sigma\theta_o^3\,\theta_* \rho\,F(z_r)\left[\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega r}\left(\log\frac{\omega_r}{\omega_o} + \frac{\theta_o - \theta_R}{\theta_*}\right) + R_1(\omega_r)\right] \quad (125)$$

With this expression we can directly calculate $\left(\frac{\partial R}{\partial z}\right)$ from values of $\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega r}$ and $R_1(\omega_r)$. This is done using Figures 15 and 16. The calculation is simple, because it is sufficient to calculate $\omega_o = \rho\,F_o\,z_o$ and $\omega_r = \rho\,F(z_r)\,z_r$.

It would also be necessary to know $\theta_o - \theta_R$, which we ignored in the numerical calculation. We can determine its influence by means of an analytical calculation. Let us consider that:

$$\log\frac{\omega_r}{\omega_o} = \log\frac{z_r F(z_r)}{z_o f_o} \simeq \log\frac{z_r}{z_o} \simeq \frac{\theta(z_r) - \theta_o}{\theta_*}$$

We can see that in equation (124) $\theta_o - \theta_R$ plays the role which is similar to the role of $\theta(z_r) - \theta_o$. Its influence is negligible if $\left|\frac{\theta_o - \theta_R}{\theta(z_r) - \theta_o}\right| \ll 1$. If this is not true and because $\theta_o - \theta_R$ is still positive, this term tends to increase the value of $\frac{\partial R}{\partial z}$ for $\theta(z_r) - \theta_o \geq 0$, i.e., in an inversion situation or, on the other hand, to be reduced for $\theta(z_r) - \theta_o \leq 0$, i.e., in an over-adiabatic condition. This was confirmed by the numerical results obtained by Zdunkowski and Johnson (1965) and Zdunkowski, Henderson and Hales (1966). In any case, the small value of this term in the ocean makes it possible to ignore it, except if the temperature difference between the air and the water is small. In this case it would have to be determined by experiment. The correct determination of the surface temperature of the ocean θ_o can never be done except by means of a radiometric method, which will then result in a value

corresponding to Θ_R. This is inconvenient, because the sum of the terms in parentheses in (124) is equal to $\dfrac{\Theta_{(z_r)} \cdot \Theta_R}{\Theta_*}$ and therefore Θ_o is not involved.

736. The analytical expression which was obtained for $\left.\dfrac{\partial R}{\partial z}\right|$ /55 makes it possible to establish the expression for the radiation flux variation $R_{(z)} - R_o$ by means of an integration:

$$R(z) - \tilde{R}_o = \int_{z_o}^{z} \left(\frac{\partial R}{\partial z}\right)_{z_r} dz_r = 4\sigma\Theta_o^3 \Theta_* \int_{z_o}^{z} \left[\left(\frac{\partial \omega}{\partial z}\right)\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{\omega r}\left[\log \frac{\omega r}{\omega_o} + \frac{\Theta_o - \Theta_R}{\Theta_*}\right] + R_1(\omega r)\right] dz_r$$

$$= 4\sigma\Theta_o^3 \Theta_* \int_{\omega_o}^{\omega} \left[\left(\frac{\partial \varepsilon}{\partial \omega}\right)\left[\log \frac{\omega}{\omega_o} + \frac{\Theta_o - \Theta_R}{\Theta_*}\right] + R_1(\omega)\right] d\omega$$

or $R(z) - R_o = 4\sigma\Theta_o^3 \Theta_* \left[\left[\varepsilon(\omega)\log\frac{\omega}{\omega_o}\right]_{\omega_o}^{\omega} + \int_{\omega_o}^{\omega}\left[R_1(\omega r) - \frac{\varepsilon(\omega r)}{\omega r}\right] d\omega r\right]$

$\varepsilon(\omega r)$ is a well defined function of ω_r. This is also true for $R_1(\omega) - \frac{\varepsilon(\omega r)}{\omega r}$ and, consequently, for the integral $\int_{\omega_o}^{\omega}\left[R_1(\omega r) - \frac{\varepsilon(\omega r)}{\omega r}\right] d\omega r$ which can be called $R_2(\omega_r)$. It was calculated numerically and is shown in Figure 16.

We then finally obtain:

$$\boxed{R(z_r) - R_o = 4\sigma\Theta_o^3 \Theta_* \left[\varepsilon(\omega r)\left[\log\frac{\omega r}{\omega_o} + \frac{\Theta_R - \Theta_o}{\Theta_*}\right] + R_2(\omega r) - R_2(\omega)\right]} \quad (126)$$

75. Discussion of the simplified formulas /56

The analytical calculation leads to simplified formulas (125) and (126) which make it possible to rapidly calculate the values

of $\frac{\partial R}{\partial z}\big|$ and $R_{(z)} - R_o\big|$ at a level z without a computer, if u_*, θ_o, L are known in our problem or if $u_*, \theta_o, L, z_o, \theta_R\big|$ and B are known in the general case (for example, above the ground). These formulas assume that the form of the various profiles can be considered logarithmic. We have seen that the errors introduced by a strong stratification can reach 40% in the inversion range and 20% in the over-adiabatic range.

Considering the large number of approximations which we have introduced, we must now determine the error of the calculation. The results of the analytical calculation and the numerical calculation (both carried out for hypothesis 3: Brooks emissivity and logarithmic profiles) were compared for different situations. In all the cases studied, this comparison leads to conclusions which are similar to those which can be derived from the typical cases shown in Figures 17 and 18. According to Figure 17 which shows the radiation flux divergence, the agreement seems satisfactory overall, even though the numerical calculation introduces a certain dispersion in the points, which can probably be attributed to calculation of the integral which occurs in the expression for $\frac{\partial R}{\partial z}\big|$. The ordinates are plotted on a logarithmic scale and the deviation between the two methods of calculation between 1 and 10 m, which do not look very great according to Figure 17, leads to an appreciable difference for the integrated term $R_{(z)} - R_o\big|$, as Figure 18 shows. We are now faced with a problem of determining which of the two methods is more reliable. Considering the approximations made for the two cases and the details of the two methods, we believe that the numerical calculation is probably afflicted with the greatest amount of errors.

This point of view confirms the difficulty in estimating the radiation flux divergence accurately using the numerical method of Funk from a limited number of temperature and humidity

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measurements. This also explains the general disagreement observed (Funk 1961, Hamilton 1965, Lieske and Troschein 1966) between the direct determinations of this divergence and the calculated predictions.

76. Examination of the influence of various micrometeorological parameters

Assuming that $\theta_0 = \theta_R$, equation (126) leads to:

$$\left|\frac{R(z)-R}{S_0}\right| = \left|\frac{4 \cdot \sigma \theta_0^3 \; \theta_* \; \left[\varepsilon(\omega) Log \frac{w}{w_0} + R_2(\omega) - R_2(\omega_0)\right]}{\rho C_p \, k \, \theta_* \, u_*}\right| \qquad \underline{/57}$$

or

$$\left|\frac{S-S_0}{S_0}\right| = \frac{4\sigma}{\rho C_p \, k} \; \frac{\theta_0^3}{u_*} \left[\varepsilon(\omega) Log \frac{z \, F(z)}{z_0 F_0} + R_2(\omega) - R_2(\omega_0)\right] \qquad (127)$$

It follows that the relative variation of the turbulent flux resulting from the effect of radiation transfer:

- is proportional to θ_0^3

- is inversely proportional to u_*

- increases with $F_{(z)}$, i.e. with average air humidity

- decreases slightly with z_0

For the ocean, the variation range of θ_0 is quite small and therefore the term θ_0^3 will vary slightly. Also, z_0 depends on u_*. Thus, for the ocean, $\left|\frac{S-S_0}{S_0}\right|$ is practically inversely proportional to u_* and increases with the average air humidity. This confirms the results of the numerical calculation. Also, we are now in a position to draw another conclusion by estimating the

relative effects of stability. This made it possible to draw
Figure 19 which clearly shows the influence of the various para-
meters. In particular, we can see that the air increases very
rapidly when the friction velocity drops below 10 cm/second.

VIII. Conclusion

A rigorous study of heat transfer in the surface layer of the
atmosphere requires knowledge of the interaction mechanisms between
the radiation transfer and the turbulent exchanges. Up to now,
this topic has only been studied in a limited number of works. Its
influence was placed in perspective in certain theoretical and
experimental studies. The classical theories on turbulent heat
transfer in the surface layers do not take into account the possible
effects of radiation transfer. Consequently errors can be made
either during the establishment of semi-empirical predictions or
when experimental results are interpreted.

We attempted to determine these errors by using absurd reason-
ing: the turbulent flux is initially assumed to be constant, and
we then calculate the variation of the same flux which is a logical
consequence of this hypothesis. The numerical results show that
in certain situations, the relative variations are much larger
than the limit of 10 to 20% which could be established, considering
the tolerable error levels in micrometeorology. It therefore seems
that in the general case and a priori it is not allowable to assume
that the influence of radiation transfer is negligible.

The dominating factor is the wind velocity. Small values
of it lead to large variations. Thermal stratification only enters
in on the second order level. This point allows one to find an
approximate analytical solution to the problem using temperature
and humidity profiles which obey the classical logarithmic re-
lationships. Simplified formulas were established with which it

is possible to rapidly evaluate the effects of the radiation flux. In addition, this method has the advantage of clearly demonstrating the role of various parameters. We finish with a quantitative determination of the unfavorable effects, such as small wind velocities, increased humidity values and nocturnal inversion regimes. In addition, appreciable errors can be made if we deliberately ignore the influence of radiation transfer during the day and during periods of low wind velocity and large humidity.

The calculations were carried out assuming clear skies. The <u>/59</u> fact that there was no fog could only increase the influence of radiation transfer (Zdunkowski et al. (1966) concluded that there was a smaller fog effect than the one we predicted). A large $\theta_0 - \theta_R$ difference would lead to results which would be quite different from the ones we obtained. The surface emissivity must therefore be taken into account.

Except for these points, the uncertainty in the emissivity values of water vapor for small optical thicknesses (less than 10^{-4} cm H_2O) and the uncertainty in the carbon dioxide effect, as well as deviations observed between the result obtained numerically and analytically, contributed to an appreciable uncertainty in our results, which could be reduced by additional work. Even though this uncertainty limits the accuracy of the preceding results, it is, nevertheless not large enough to invalidate their significance.

Considered from a certain point of view, the preceding work can be considered to have a negative quality. This is because we are only able to define conditions under which the radiation transfer can be considered negligible. We are not able to determine the corrections which must be made for taking it into account. In order to do this, a method of successive approximations would have to be developed. We assumed that the flux was constant, and

we ended with a variation of x_1 % . We would have to perform a new calculation based on this variation of x_1 %, which would then lead to a resulting variation of x_2 % . Assuming that the calculation converges (very close tests make it possible to believe that this will be true), we should very rapidly arrive at a rigorous solution of the problem. Unfortunately, this method, although promising cannot be developed because of the present state of the theory. In fact the turbulent flux S is not constant, and the Monin-Obukhov length L must be considered as a function of altitude and therefore cannot play the role of a length scale which would be developed over the entire surface layer. An extension of the present theories which consider the turbulent fluxes of momentum and water vapor as being constant but use variable turbulent heat fluxes, would be necessary.

Assuming that this problem is solved, we can assume that the method under consideration would converge rather quickly. In effect, the divergence of the radiation flux is primarily determined by the shape of the humidity and temperature profiles very close to the surface. The latter would not be greatly modified by variations of the turbulent flux, which affect primarily the upper layers of the surface layer. Considering the uncertainty which affects the calculations of the radiation flux divergence, we can assume that the margin of error on this latter quantity would very quickly rise above the significant margin for iteration processes. For temperature and humidity profiles, the difference would not be greater than the one which results for velocity profiles when a constant friction term is adopted (flat plate case) or when a variable friction is adopted (tube case).   /60
On the other hand, the effect of radiation transfer will certainly be important during a calculation of the values of turbulent heat transfer coefficients from experimental profiles in the surface layer. This explains in part the dispersion of the results for the ratio $\dfrac{k_H}{k_M}$. In order to evaluate it precisely, it would be

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necessary either to remove the influence of radiation transfer (which is possible in the wind tunnel) or to at least reduce it by avoiding determinations when the wind is too slow and the air humidity is too great. Also the fluxes would have to be measured as close as possible to the surface.

This study does not pretend to entirely resolve the problem posed. Instead we give a first approach which should lead the way to further work. This work should take into account certain aspects which we have separated out due to the definition of our problem, in particular the effects of advection and the effects of non-steady conditions, which must be considered in the general case.

APPENDIX I

```
  DIMENSION U(5),ZOM(5),OBUKO(10)
  DIMENSION Z(15),X(15),TETA(15),EHUM(15),W(15),T2W2(15),V(15),Y(15)
  DIMENSION WR(10),ZR(10),DIV(10),FI(10),CUMUL(10),T3W3(15),EMIS(15)
1 FORMAT(////,3X,'VITESSE DU VENT A 10 M., U10= ',F 9.1,'....RUGOSITE,
 1..............ZO= ',E 9.3,'..L.MONIN-OBUKHOV,..........L= ',F9.0)
2 FORMAT(/    ,1H ,'.USTAR=   ',F10.4,',FLUX    QH=',F10.4,',EVAPO=
 1',F10.5,',BOWEN= ',F10.4,'     ,A=     ,F10.5)
3 FORMAT(1H ,100('*'))
4 FORMAT(1H   ,        HAUTEUR    *    TEMPERATURE      *       HUMIDITE    *
 1          W    *            D2T/DW2        *')
5 FORMAT(1H ,5(E15.8,4X,1H*))
6 FORMAT(1H0,'NP-R *          WR   *                ZR   *    INTEGRALE  *
 2DIVERGENCE    *      CUMUL   *')
7 FORMAT(1H, I4,1X,'*',5(E13.7,1X,'*'))


          INITIALISATION DES VALEURS CONSTANTES DE L'ANALYSE

 N=14
 AK=0,4
 TETO=293.
 U(1)=2.8
 U(2)=5.
 U(3)=8.
 U(4)=12.
 U(5)=15.
 ZOM(1)=0.00001
 ZOM(2)=0.00005
 ZOM(3)=0.0001
 ZOM(4)=0.0005
 ZOM(5)=0.001
 OBUKO(1)=-1000.
 OBUKO(2)=-500.
 OBUKO(3)=-100.
 OBUKO(4)=-16
 OBUKO(5)=-5
 DO 100 I=6,10
 J=11-I
 OBUKO(I)=-OBUKO(J)
 EO=23.27
 A1=13000


      MX EST LE NUMERO DE L'UNITE DE SORTIE

 MX=3
```

```
                    VALEUR STANDARD DES HAUTEURS.
  X(1)=0.
  X(2)=0.00001
  X(3)=0.00005    .
  X(4)=0.0001
  X(5)=0.0005
  X(6)=0.001
  X(7)=0.005
  X(8)=0.01
  X(9)=0.05
  X(10)=0.1
  X(11)=0.5
  X(12)=1.
  X(13)=5.
  X(14)=10.
  X(15)=100.


             VALEURS DES PARAMETRES DE LA COURBE D'EMISSIVITE

  B0=0.7875
  B1=0.1420
  B2=0.0065


  J'INDIQUE ICI LE NOMBRE DE COUPLES(U10,ZO)ETUDIES,A L'EXPLOITATION,NTEST=5
  DO 130 I=1,5
  U10=U(I)
  ZO=ZOM(I)


             ON DOIT D'ABORD CALCULER LE NOMBRE DE POINTS DU PROFIL ET LES
             VALEURS POSSIBLES DES HAUTEURS,JAMAIS ZERO...
  NP=N+1-I
  DO 120 J=1,NP
  K=I+J
20 Z(J)=X(K)


             ON EFFECTUE LES CALCULS POUR DES VALEURS FIXEES DE LA LONGUEUR
             DE MONIN-OBUKHOV

  DO 130 J=1,10
  IH=1
  DO 130 IJ=1,2
  AL=OBUKO(J)
  WRITE(3,1)U10,ZO,AL
  UETOI=USTAR(AK,U10,AL,ZO,INDIC)

             CALCUL DU PROFIL DE TEMPERATURE

  CALL PROTP(AL,AK,UETOI,ZO,TETO,NP,Z,TETA,IX,IJ)
  TET10=TETA(NP)

             CALCUL DU RAPPORT DE BOWEN

  B=BOWEN(TET10,TETO)
```

```
              CALCUL DU FLUX DE CHALEUR ET DE L'EVAPORATION
              CALCUL DU PROFIL D'HUMIDITE

    CALL FLUEV(UETOI,AL,B,,QHO,EVAPO)
    CALL PROHU(EO,B,NP,TETA,TETO,EHUM)

              CALCUL DES ELEMENTS INTERVENANT DANS LA DIVERGENCE DU FLUX
              RADIATIF

    A2=TETO*UETOI*UETOI/(AK*AK*AL*9.81)

              CALCUL DES DEUX TABLEAUX CONTENANT W ET LA DERIVEE SECONDE
              DE TETA PAR RAPPORT A W.

    CALL EOPTI(A1,A2,B,NP,EHUM,Z,W,T2W2,AL,IJ)
    WRITE(MX,2)UETOI,QHO,EVAPO,B,A2
    WRITE(MX,3)
    WRITE(MX,4)
    WRITE(MX,3)
    DO 135 K=1,NP
135 WRITE(MX,5)Z(K),TETA(K),EHUM(K),W(K),T2W2(K)

              CALCUL DES VALEURS ZR ET WR

    DO 140 K=1,10
    L=NP-K+1
    M=11-K
    WR(M)=W(L)
140 ZR(M)=Z(L)

              CALCUL DES EMISSIVITES

08 DO 150 M=1,10
    DO 163 K=1,NP
    V(K)=W(K)-WR(M)
    IF(V(K))164,162,164
162 EMIS(K)=0.
    GO TO 163
164 IF (ABS(V(K))-1.E-4) 165,165,161
161 IF(IH.NE.3) GO TO 300
    IX=2
    EMIS(K)=0,218*(1.+1.125*(SORT(ABS(V(K)))-7.9E-2)/(SORT(ABS(V(K)))+17.9E-2))
     GO TO 163
165 IF(IH.NE.3) GO TO 301
    IX=2
    EMIS(K)=0.260*ALOG(1.+1.043E3*(ABS(V(K))))
    GO TO 163
301 EMIS(K)=0.055*ALOG(1.+7.E3*ABS(V(K)))
163 T3W3(K)=T2W2(K)*(EMIS(1)-EMIS(K))

              INTEGRATION DE LA FONCTION T3W3 TABULEE DANS L'INTERVALLE V(1) A V(NP)
              L'INTEGRALE VAUT FI.SSP PAGE 289
84 CALL QTFG(V,T3W3,Y,NP)
    FI(M)=Y(NP)
```

```
              CALCUL DE LA DIVERGENCE AU NIVEAU ZR

    L=NP-M+1
  0 DIV(M)=-22.4E-9*TETO**3*EHUM(L)*FI(M)/A1

              CALCUL DES AIRES CUMULEES DE DIV EN FONCTION DE Z

    CUMUL(1)=1.E-3*DIV(1)
    DO 170 M=2,10
  0 CUMUL(M)=CUMUL(M-1)+(ZR(M)-ZR(M-1))*(DIV(M)+DIV(M-1))*.5

    WRITE(MX,6)
    DO 180 M=1,10
  0 WRITE(MX,7)M,WR(M),ZR(M),FI(M),DIV(M),CUMUL(M)
    GO TO (210,130),IX
  0 IH=3
    GO TO 208
  C CONTINUE
    CALL EXIT
    END
```

SOUS-PROGRAMMES

```
    FUNCTION USTAR(AK,U10,AL,ZO,INDIC)

    CALCUL DE LA VITESSE DE FROTTEMENT USTAR(U ETOILE)
    ...AK...CONSTANTE DE KARMAN
    ...U10..VITESSE DU VENT A 10 METRES
    ...AL...LONGUEUR DE MONIN-OBUKHOV
    ...ZO...PARAMETRE DE RUGOSITE
    ...INDIC..O SI LE CALCUL EST NUMERIQUEMENT POSSIBLE
          ..1 SI LE TERME INTERVENANT DANS LE LOGARITHME EST NUL ON POURRA AMELIORER
             CE POINT EN IMPOSANT AU TERME D'ETRE SUPERIEUR A EPS

    INDIC=0
    EPS=0.
    X=AL*(EXP(10./AL)-1.)/ZO
    IF(X-EPS)100,100,110
  0 INDIC=1
    USTAR=0.
    RETURN
  0 USTAR=AK*U10/ALOG(X)
    RETURN
    END
```

```
SUBROUTINE PROTP (AL,AK,USTAR,ZO,TETO,N,Z,TETA,IX,IJ)
DIMENSION Z(1),TETA(1)

      CALCUL D'UN PROFIL DE TEMPERATURE TETA(Z) EN FONCTION DE Z
      TABLEAU CONTENANT LES DIFFERENTES VALEURS DE LA HAUTEUR DES OBSERVATIONS,
      DIMENSION N
      ...AL....LONGUEUR DE MONIN-OBUKHOV.
      ...AK....CONSTANTE DE KARMAN.
      ...USTAR.VITESSE DE FROTTEMENT
      ...ZO....PARAMETRE DE RUGOSITE
      ...N.....NOMBRE DE VALEURS DIFFERENTES DE LA HAUTEUR
      ...TETA..TABLEAU CONTENANT LES RESULTATS DE DIMENSION N
      ...TETO..VALEUR FIXE DU PROFIL POUR ZO
   GO TO (201,202),IJ
201 T=ABS(AL*EXP(-0.02)-1.)
   IX=1
   G=9.81
   T=TETO*(1.+USTAR*USTAR*ALOG(T/ZO)/AK/AK/G/AL)
   DO 90 I=1,N
   IF(AL)100,100,121
100 BL=-.02*AL
   IF(ABS(Z(I)-BL).EL.0.000001)Z(I)=BL
   IF(Z(I)-BL)110,110,120
110 CL=ABS(BL)
   TETA(I)=T+2.8*USTAR*USTAR*(1./Z(I)**.5-1./CL**.5)/G/ABS(AL)**.5
   GO TO 90
120 S=ABS(AL*(EXP(Z(I)/AL)-1.))
   IF (ABS(S).LE.0.00001)TETA(I)=TETO
   IF(TETA(I).EQ.TETO)GO TO 90
   TETA(I)=TETO*(1.+USTAR*USTAR*ALOG(S/ZO)/AK/AK/G/AL)
   GO TO 90
121 DL=0.001*AL
   IF(ABS(Z(I)-DL).LE.0.000001)Z(I)=DL
   IF(Z(I)-DL) 110,110,122
122 TPRIM=ABS(AL*(EXP(0.001)-1))
   TPROM=TETO*(1.+USTAR*USTAR*ALOG(TPRIM/ZO)/AK/AK/G/AL)
   TETA(I)=TPROMº20.*USTAR*USTAR*TETO*((Z(I)/AL)**0.1-0.001**0.1)/AK/AK/G/AL
90 CONTINUE
90 RETURN
202 TETA(I)=TETO*(1.+USTAR**2*ALOG(Z(I)/ZO)/AK/AL/AK/G)
   IX=2
   RETURN
   END
```

```
FUNCTION BOWEN(TET10,TETO)
      CALCUL DU RAPPORT DE BOWEN

   DELTA=TET10-TETO
   IF(DELTA)100,100,110
100 BOWEN=.1-.09*DELTA
   MX=3
   RETURN
110 BOWEN=-.8*DELTA
   RETURN
   END
```

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```
SUBROUTINE FLUEV(USTAR,AL,BOWEN,QHO,EVAPO)

        CETTE SUBROUTINE CALCULE
         ..QHO....FLUX DE CHALEUR SENSIBLE
         ..EVAPO.EVAPORATION
        CONNAISSANT
         ..USTAR.VITESSE DE FROTTEMENT
         ..AL....LONGUEUR DE MONIN-OBUKHOV,NON NULLE
         ..BOWEN.RAPPORT DE BOWEN,NON NUL

QHO=-.9E+04*USTAR*USTAR*USTAR/AL
EVAPO=1.5E-02*QHO/BOWEN
MX=3
RETURN
END



SUBROUTINE PROHU(EO,BOWEN,N,TETA,TETO,EHUM)
DIMENSION TETA(1),EHUM(1)

        CALCUL DU PROFIL D'HUMIDITE CONNAISSANT CELUI DE TEMPERATURE
        ...EO.....VALEUR DE L'HUMIDITE POUR Z=ZO PARAMETRE DE RUGOSITE
        ...BOWEN..RAPPORT DE BOWEN
        ...N......NOMBRE DE VALEURS DIFFERENTES DE LA HAUTEUR
        ...TETA...TABLEAU DE DIMENSION N CONTENANT LES VALEURS DU PROFIL DE TEMPERATURE
        ...TETO...CONSTANTE CORRESPONDANT A LA VALEUR DE ZO
        ...EHUM...TABLEAU DE DIMENSION N CONTENANT LES VALEURS DU PROFIL D'HUMIDITE

 DO 100 I=1,N
0 EHUM(I)=EO+.66*(TETA(I)-TETO)/BOWEN
 MX=3
 RETURN
 END
```

```
SUBROUTINE EOPTI(A1,A2,B,NP,EHUM,Z,W,T2W2,AL,IJ)
DIMENSION EHUM(1),Z(1),W(1),T2W2(1),X(15)

    EN   ENTREE    A1   EST UNE CONSTANTE.
                   A2   EST UN  PARAMETRE CONSTANT LORS D'UN APPEL DU SOUS-PROGRAMME
                   B    EST UN PARAMETRE CONSTANT EGAL AU RAPPORT DE BOWEN
                   NP   EST LE NOMBRE DE POINTS DU PROFIL
                 EHUM DIMENSION NP CONTIENT PROFIL D'HUMIDITE
                   Z    ............... ............. DE HAUTEUR
    EN SORTIE      W    ....................OMEGA (CF.TEXTE)
                   T2W2.....................LA DERIVEE SECONDE DE
                                         TETA PAR RAPPORT A OMEGA)
R=0.66*A2/B
DO 300 I=1,NP
IF(IJ.EQ.2)GO TO 110
IF(AL)110,110,120
CL=-0.02*AL
IF(ABS(Z(I)-CL).LE.0.000001)Z(I)=CL
IF (IJ.EQ.2)GO TO 111
IF(Z(I)-CL) 111,111,112
W(I)=Z(I)*(EHUM(I)-R)/A1
T2W2(I)=-A2*(1.+R/EHUM(I))*(A1/Z(I)/EHUM(I))**2
GO TO 300
```

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```
2  W(I)=R*Z(I)**.75*(1.33*AL/Z(I)-.34)/(2.7*(ABS(AL))**.75)
   W(I)=EHUM(I)-W(I)
   W(I)=W(I)*Z(I)/A1
   X(I)=R*(1.+.6*Z(I)/AL)/(EHUM(I)*2.7*(Z(I)/ABS(AL))**.25)+.75
   X(I)=.2*(Z(I)/ABS(AL))**.75+(1.+.6*Z(I)/AL*X(I))/(2.7*(Z(I)/ABS(AL)**.25)
   T2W2(I)= X(I)*(-A2*(A1/Z(I)/EHUM(I))**2)
   GO TO 300
0  DL=0.001*AL
   IF(ABS(Z(I)-DL).LE.0.000001)Z(I)=DL
   IF(Z(I)-DL)111,111,122
2  W(I)=R*2*((Z(I)/AL)**.1)*(.9+.3*Z(I)/AL)
   W(I)=EHUM(I)-W(I)
   W(I)=Z(I)*W(I)/A1
   X(I)=R*2*(1.+.6*Z(I)/AL)*(Z(I)**.1)/EHUM(I)*(AL**.1)+1.1
   X(I)=1.2*(Z(I)/AL)**1.1+(1.+6*Z(I)/AL)*2*((Z(I)/AL)**.1)*X(I)
   T2W2(I)= X(I)*(-A2*(A1/Z(I)/EHUM(I))**2)
0  CONTINUE
   RETURN
   END
```

APPENDIX II

TABLE OF NUMERICAL VALUES OF $\dfrac{\partial R}{\partial z}$ (mW/cm^2/m)

$$u_{10} = 15 \text{ m/s}$$

L (m)	-5	-16	-100	-500	-1000	+1000	+500	+100	+16	+5
$\vartheta_{10} - \vartheta_0$			$-9{,}17$	$-1{,}46$	$-0{,}73$	$+0{,}75$	$+1{,}53$	$+8{,}20$		
$S_0 (\text{mW/cm}^2)$			$+25{,}2872$	$+4{,}9924$	$+2{,}49$	$-2{,}4841$	$-4{,}9600$	$-24{,}4769$		
$Z(m)$ $10{-}3$			$+1{,}09$	$+2{,}35.10^{-1}$	$+1{,}51.10^{-1}$	$-1{,}77.10^{-1}$	$-3{,}61.10^{-1}$	$-1{,}88$		
10^{-2}			$+1{,}68$	$+1{,}03.10^{-}$	$+4{,}55.10^{-3}$	$-6{,}56.10^{-3}$	$-2{,}01.10^{-2}$	$-1{,}91.10^{-1}$		
10^{-1}			$-6{,}62.10^{-1}$	$-1{,}39.10^{-1}$	$-7{,}19.10^{-2}$	$+7{,}30.10^{-2}$	$+1{,}40.10^{-1}$	$+6{,}32.10^{-1}$		
1			$-2{,}66.10^{-1}$	$-5{,}91.10^{-2}$	$-2{,}96.10^{-2}$	$+2{,}79.10^{-2}$	$+6{,}01.10^{-2}$	$+3{,}20.10^{-1}$		
5			$-9{,}02.10^{-2}$	$-1{,}92.10^{-2}$	$-9{,}35.10^{-3}$	$+9{,}37.10^{-3}$	$+1{,}97.10^{-2}$	$+1{,}09.10^{-1}$		
10			$-1{,}14.10^{-1}$	$-2{,}28.10^{-2}$	$-1{,}12.10^{-2}$	$+1{,}38.10^{-2}$	$+2{,}96.10^{-2}$	$+1{,}79.10^{-1}$		

$$u_{10} = 12 \text{ m/s}$$

L(m)	-5	-16	-100	-500	-1000	$+1000$	$+500$	$+100$	$+16$	$+5$
$\theta_{10} - \theta_0$			$-5,41$	$-0,87$	$-0,44$	$+0,44$	$+0,91$	$+4,84$		
$S_0(\text{mW/cm}^2)$			$+10,4026$	$+2,0556$	$+1,0263$	$-1,0232$	$-2,0432$	$-10,0522$		
Z(m) 10^{-3}			$+5,54.10^{-1}$	$+1,28.10^{-1}$	$+6,82.10^{-2}$	$-7,62.10^{-2}$	$-1,56.10^{-1}$	$-8,27.10^{-1}$		
10^{-2}			$-1,43.10^{-1}$	$-3,18.10^{-2}$	$-1,69.10^{-2}$	$+1,64.10^{-2}$	$+2,90.10^{-2}$	$+9,32.10^{-2}$		
10^{-1}			$-4,78.10^{-1}$	$-9,83.10^{-2}$	$-5,04.10^{-2}$	$+5,02.10^{-2}$	$+9,76.10^{-2}$	$+4,48.10^{-1}$		
1			$-1,78.10^{-1}$	$-3,68.10^{-2}$	$-1,84.10^{-2}$	$+1,74.10^{-2}$	$+3,72.10^{-2}$	$+1,97.10^{-1}$		
5			$-6,03.10^{-2}$	$-1,12.10^{-2}$	$-5,54.10^{-3}$	$+5,73.10^{-3}$	$+1,20.10^{-2}$	$+6,58.10^{-2}$		
10			$-6,28.10^{-2}$	$-1,32.10^{-2}$	$-6,50.10^{-3}$	$+7,97.10^{-3}$	$+1,70.10^{-2}$	$+1,02.10^{-1}$		

L (m)	− 5	− 16	− 100	− 500	− 1000	+ 1000	+ 500	+ 100	+ 16	+ 5
$\Theta_{10} - \Theta_0$		− 11,43	− 2,01	− 0,34	− 0,17	+ 0,17	+ 0,34	+ 1,82	+ 12,04	
S_0 (mW/cm²)		+ 13,0612	+ 1,9578	+ 0,3875	+ 0,1935	− 0,1930	− 0,3855	− 1,9074	− 11,100	
Z(m) $10-3$		+ $6,17.10^{-1}$	+ $1,27.10^{-1}$	+ $2,85.10^{-2}$	+ $1,47.10^{-2}$	− $1,55.10^{-2}$	− $3,23.10^{-2}$	− $1,78.10^{-1}$	− 1,52	
10^{-2}		− $7,81.10^{-1}$	− $1,20.10^{-1}$	− $2,79.10^{-2}$	− $1,45.10^{-2}$	+ $1,40.10^{-2}$	+ $2,69.10^{-2}$	+ $1,17.10^{-1}$	+ $2,93.10^{-1}$	
10^{-1}		− 1,05	− $2,00.10^{-1}$	− $4,17.10^{-2}$	− $2,12.10^{-2}$	+ $2,08.10^{-2}$	+ $4,07.10^{-2}$	+ $1,90.10^{-1}$	+ 1,14	
1		− $4,49.10^{-1}$	− $6,84.10^{-2}$	− $1,38.10^{-2}$	− $6,92.10^{-3}$	+ $6,60.10^{-3}$	+ $1,39.10^{-2}$	+ $7,36.10^{-2}$	+ $4,54.10^{-1}$	
5		− $1,26.10^{-1}$	− $2,13.10^{-2}$	− $4,09.10^{-3}$	− $2,02.10^{-3}$	+ $2,12.10^{-3}$	+ $4,42.10^{-3}$	+ $2,40.10^{-2}$	+ $1,70.10^{-1}$	
10		− $1,07.10^{-1}$	− $1,99.10^{-2}$	− $4,47.10^{-3}$	− $2,22.10^{-3}$	+ $2,73.10^{-3}$	+ $5,82.10^{-3}$	+ $3,49.10^{-2}$	+ $3,37.10^{-1}$	

$$u_{10} = 5 \text{ m/s}$$

L (m)	-5	-16	-100	-500	-1000	$+1000$	$+500$	$+100$	$+16$	$+5$
$\theta_{10} - \theta_0$	$-13,51$	$-4,19$	$-0,73$	$-0,13$	$-0,07$	$+0,06$	$+0,12$	$+0,66$	$+4,40$	$+13,38$
S_0 (mW/cm²)	$+9,8031$	$+2,6638$	$+0,4008$	$+0,0794$	$+0,0396$	$-0,0395$	$-0,0790$	$-0,3911$	$-2,2849$	$-6,0285$
Z(m) 10^{-3}	$+2,77.10^{-1}$	$+1,00.10^{-1}$	$+2,36.10^{-2}$	$+5,35.10^{-3}$	$+2,71.10^{-3}$	$-2,90.10^{-3}$	$-6,25.10^{-3}$	$-3,70.10^{-2}$	$-3,90.10^{-1}$	$-1,80$
10^{-2}	$-1,39$	$-4,06.10^{-1}$	$-6,79.10^{-2}$	$-1,48.10^{-2}$	$-7,50.10^{-3}$	$+7,18.10^{-3}$	$-1,39.10^{-2}$	$+6,33.10^{-2}$	$+2,37.10^{-1}$	$+1,88.10^{-1}$
10^{-1}	$-1,55$	$-4,50.10^{-1}$	$-8,35.10^{-2}$	$-1,71.10^{-2}$	$-8,59.10^{-3}$	$+8,37.10^{-3}$	$-1,64.10^{-2}$	$+7,75.10^{-2}$	$+4,66.10^{-1}$	$+1,05$
1	$-5,53.10^{-1}$	$-1,71.10^{-1}$	$-2,67.10^{-2}$	$-5,26.10^{-3}$	$-2,64.10^{-3}$	$+2,53.10^{-3}$	$-5,31.10^{-3}$	$+2,78.10^{-2}$	$+1,71.10^{-1}$	$+3,87.10^{-1}$
5	$-1,57.10^{-1}$	$-4,74.10^{-2}$	$-7,72.10^{-3}$	$-1,51.10^{-3}$	$-7,68.10^{-4}$	$+8,02.10^{-4}$	$-1,66.10^{-3}$	$+8,91.10^{-3}$	$+6,35.10^{-2}$	$+2,12.10^{-1}$
10	$-1,24.10^{-1}$	$-3,87.10^{-2}$	$-7,00.10^{-3}$	$-1,61.10^{-3}$	$-7,99.10^{-4}$	$+9,81.10^{-4}$	$-2,10.10^{-3}$	$+1,25.10^{-2}$	$+1,19.10^{-1}$	$+6,45.10^{-1}$

$$u_{10} = 2,8 \text{ m/s}$$

| L (m)	- 5	- 16	- 100	- 500	- 1000	+ 1000	+ 500	+ 100	+ 16	+ 5
$\Theta_{10} - \Theta_0$	- 3,70	- 1,15	- 0,20	- 0,04	- 0,02	+ 0,02	+ 0,03	+ 0,18	+ 1,20	+ 3,67
S_0 (mW/cm²)	1,1572	0,3198	0,0485	0,0096	0,0048	- 0,0048	- 0,0096	- 0,0474	- 0,2793	- 0,7525
Z (m) 10^{-3}	$- 6,82.10^{-2}$	$- 1,75.10^{-2}$	$- 1,95.10^{-3}$	$- 3,37.10^{-4}$	$- 1,72.10^{-4}$	$+ 1,00.10^{-4}$	$+ 9,12.10^{-5}$	$- 9,7.10^{-4}$	$- 4,51.10^{-2}$	$- 3,12.10^{-1}$
10^{-2}	$- 4,89.10^{-1}$	$- 1,52.10^{-1}$	$- 2,55.10^{-2}$	$- 5,22.10^{-3}$	$- 2,62.10^{-3}$	$+ 2,49.10^{-3}$	$+ 4,88.10^{-3}$	$+ 2,3.10^{-2}$	$+ 1,02.10^{-1}$	$+ 1,73.10^{-1}$
10^{-1}	$- 4,72.10^{-1}$	$- 1,45.10^{-1}$	$- 2,50.10^{-2}$	$- 4,99.10^{-3}$	$- 2,48.10^{-3}$	$+ 2,39.10^{-3}$	$+ 4,73.10^{-3}$	$+ 2,24.10^{-2}$	$+ 1,36.10^{-1}$	$+ 3,25.10^{-1}$
1	$- 1,47.10^{-1}$	$- 4,58.10^{-2}$	$- 7,39.10^{-3}$	$- 1,45.10^{-3}$	$- 7,18.10^{-4}$	$+ 6,89.10^{-4}$	$+ 1,45.10^{-3}$	$+ 7,55.10^{-3}$	$+ 4,68.10^{-2}$	$+ 1,08.10^{-1}$
5	$- 4,17.10^{-2}$	$- 1,24.10^{-2}$	$- 2,04.10^{-3}$	$- 4,12.10^{-4}$	$- 2,06.10^{-4}$	$+ 2,16.10^{-4}$	$+ 4,44.10^{-4}$	$+ 2,36.10^{-3}$	$+ 1,67.10^{-2}$	$+ 5,60.10^{-2}$
10	$- 3,11.10^{-2}$	$- 9,8.10^{-3}$	$- 1,76.10^{-3}$	$- 4,23.10^{-4}$	$- 2,08.10^{-4}$	$+ 2,53.10^{-4}$	$+ 5,34.10^{-4}$	$+ 3,17.10^{-3}$	$+ 2,99.10^{-2}$	$+ 1,63.10^{-1}$
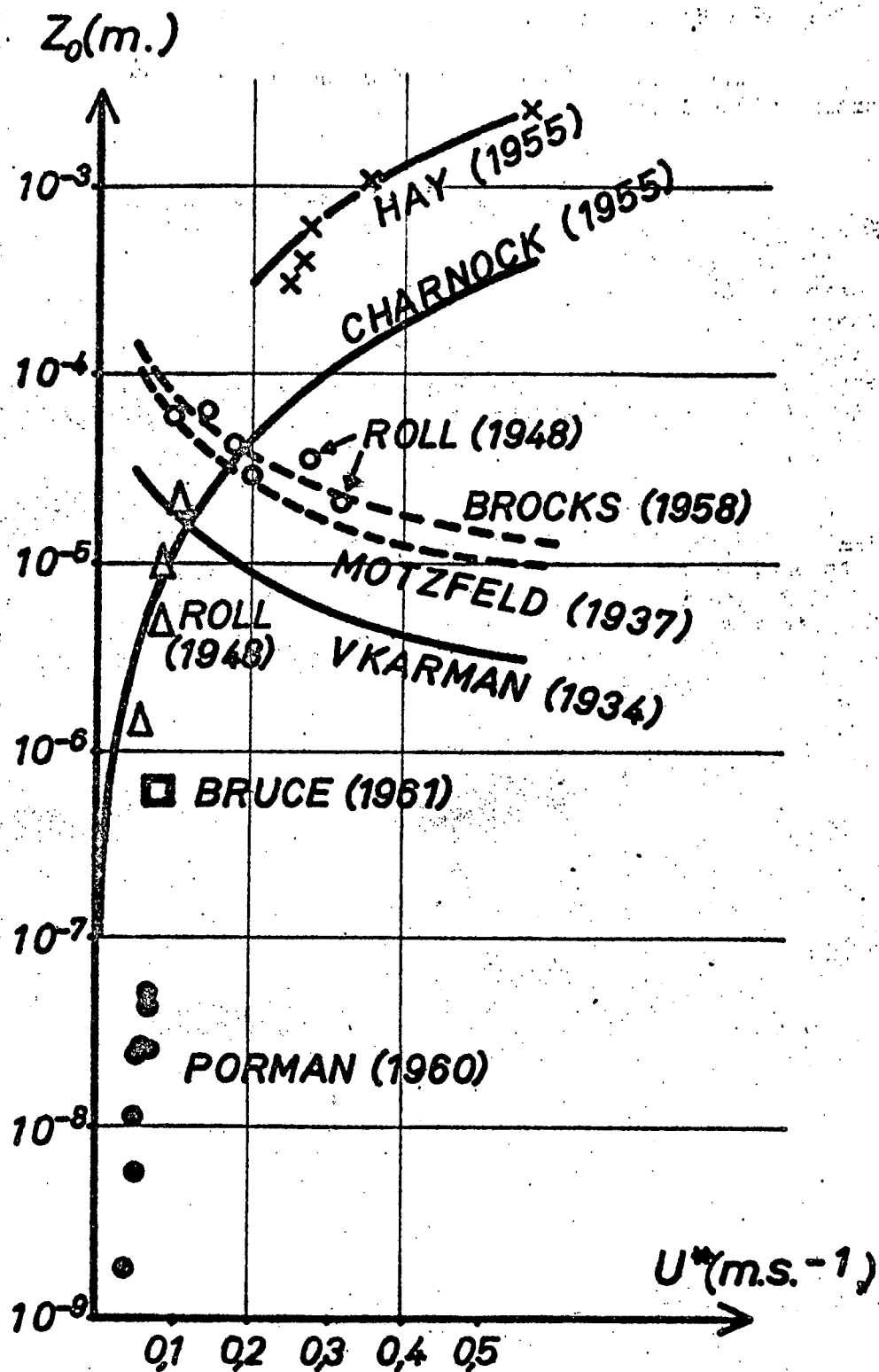
Figure 1. Summary of observations of the variation of z_o as a function of U_* (according to Roll 1965).
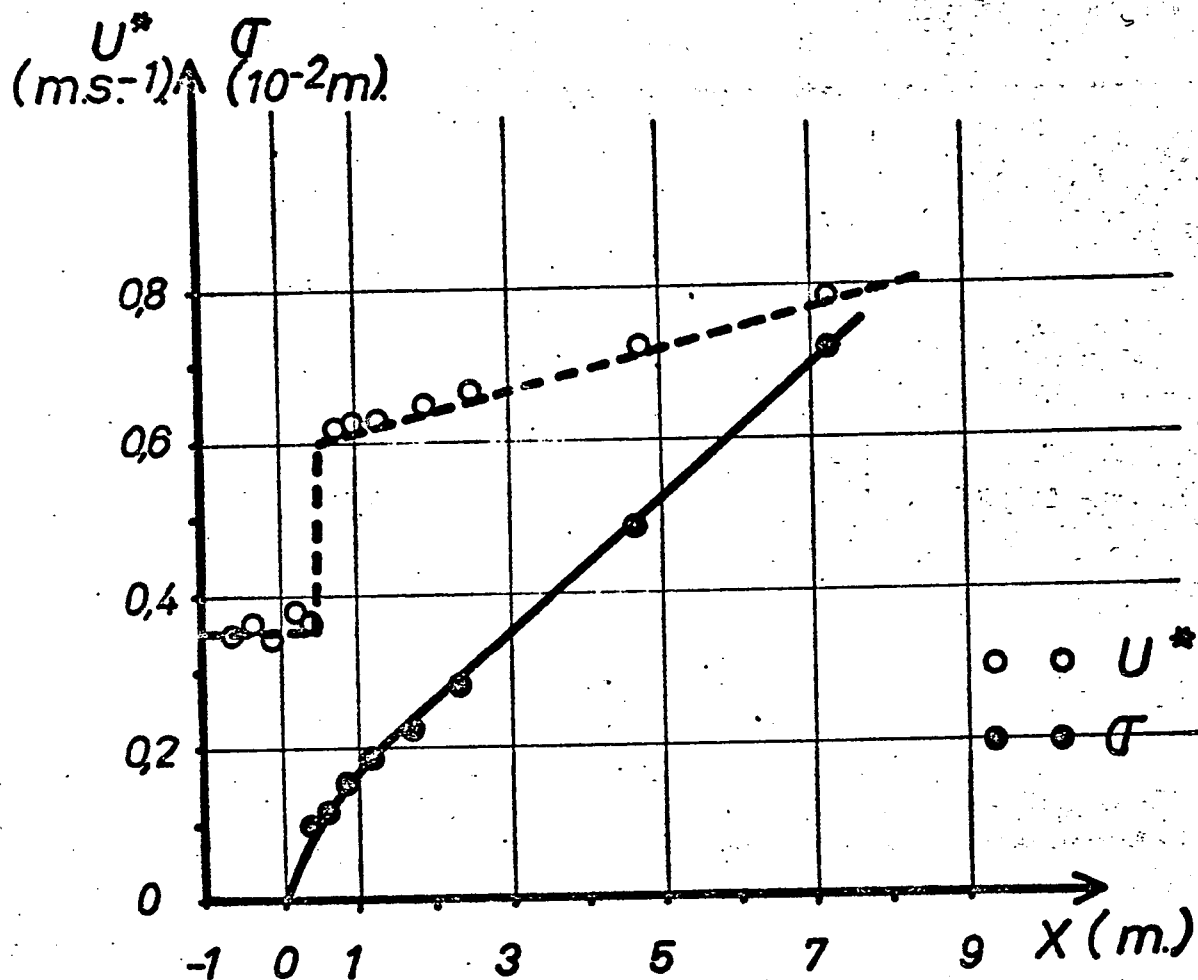
Figure 2. Variation of U[*] as a function of X and σ (according to Hidy and Plate 1967).
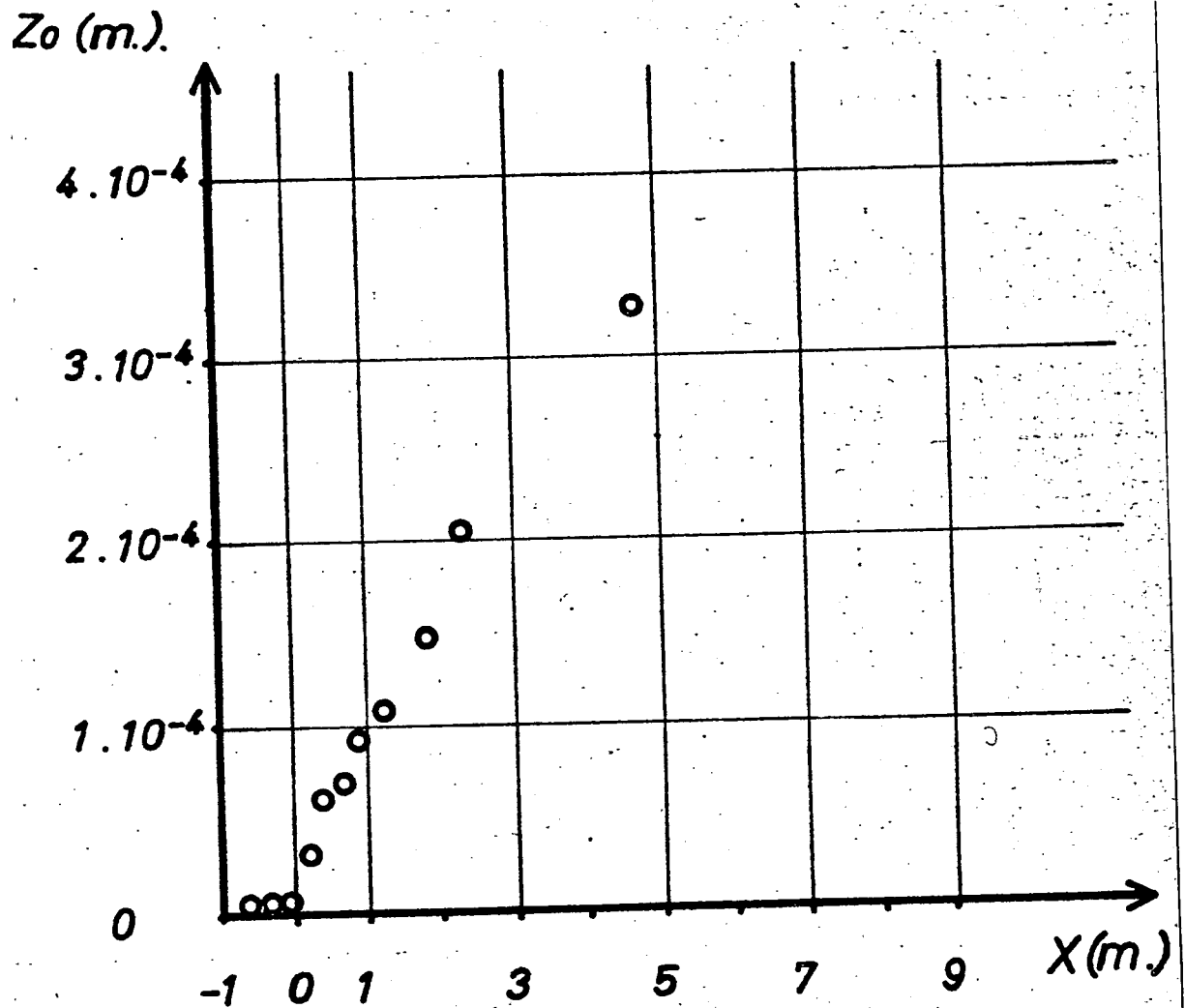
Figure 3. Variation of z_o as a function of X (according to Hidy and Plate 1967).
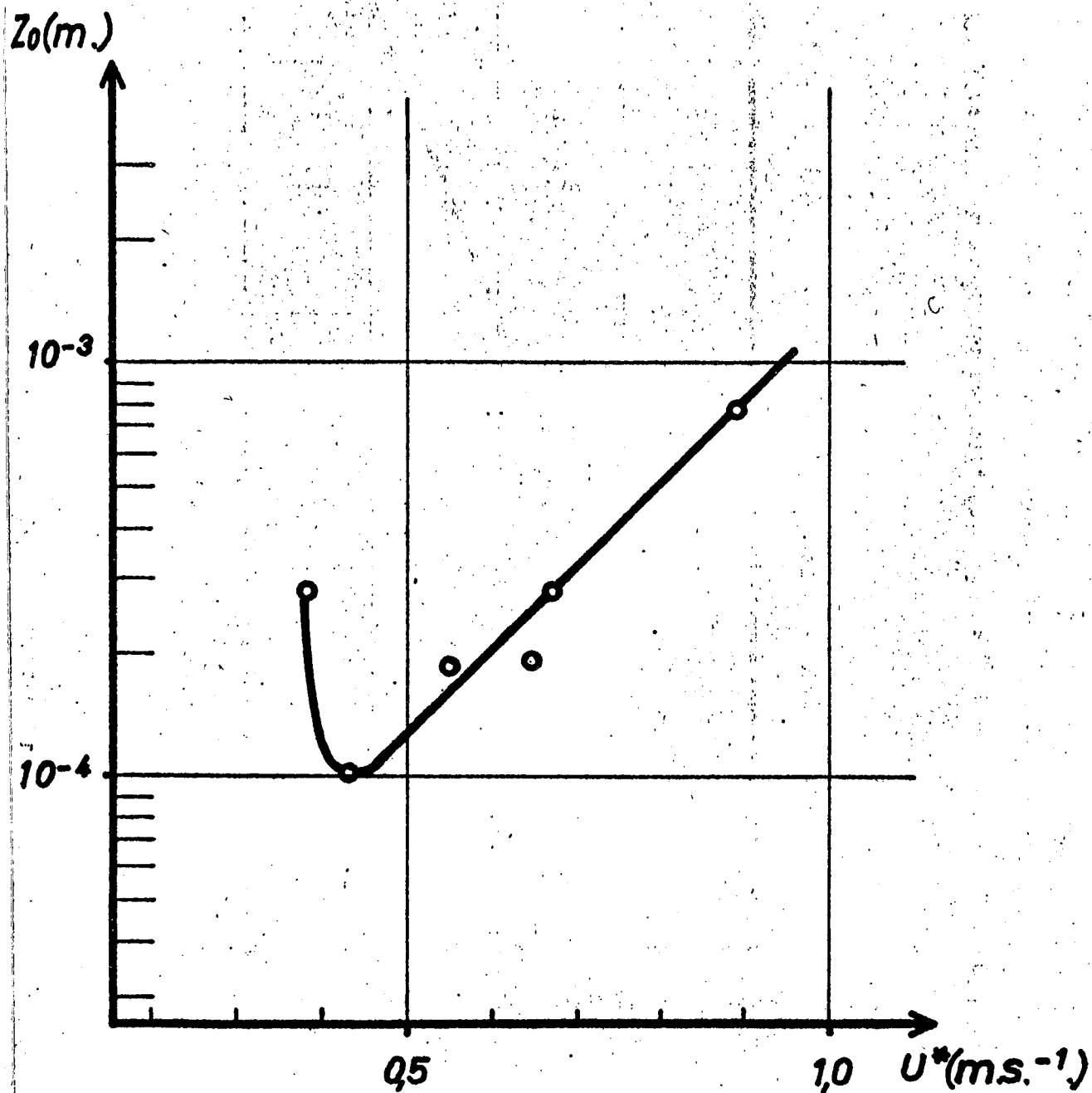
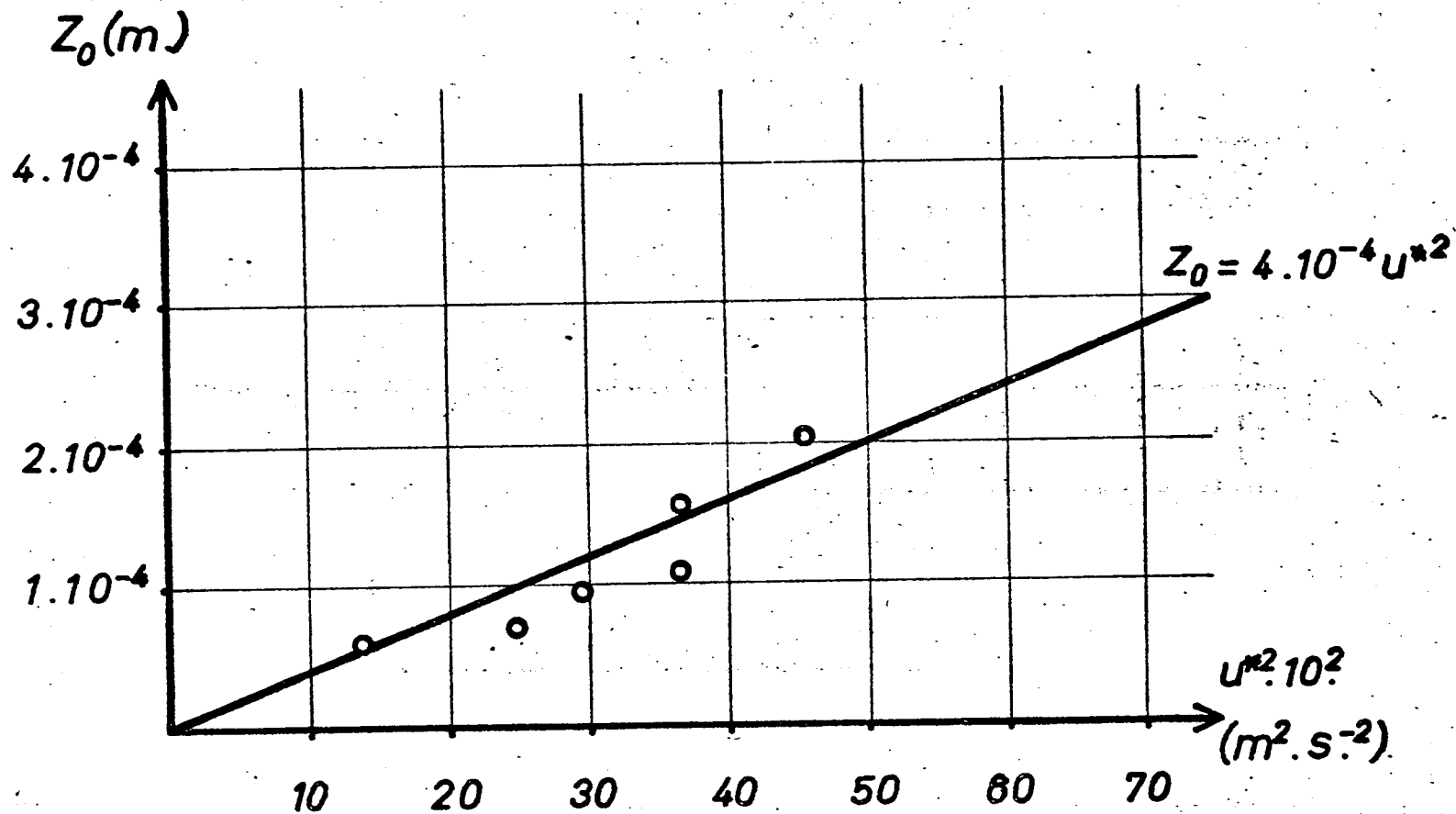Figure 4. Variation of z_o as a function of U_* (according to Shemdin 1967).

Figure 5. Relationship between z_o and U^{*2} (according to the results of Hidy and Plate 1967).

Type of wind	Breeze	Moderate wind										Strong wind									
Velocity U_{10m}		1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29																				
State of the ocean	0	1	2	3	4			5		6	7		8	9
Number on Beaufort scale	0	1	2	3	4	5	6	7	8	9	10		11	12
Aerodynamic flow conditions	Smooth	Transi- tion 1		Rough		Tr. 2	z_0 constant		

Figure 6. Table summarizing the roughness characteristics of the ocean surface (according to Wu 1969).
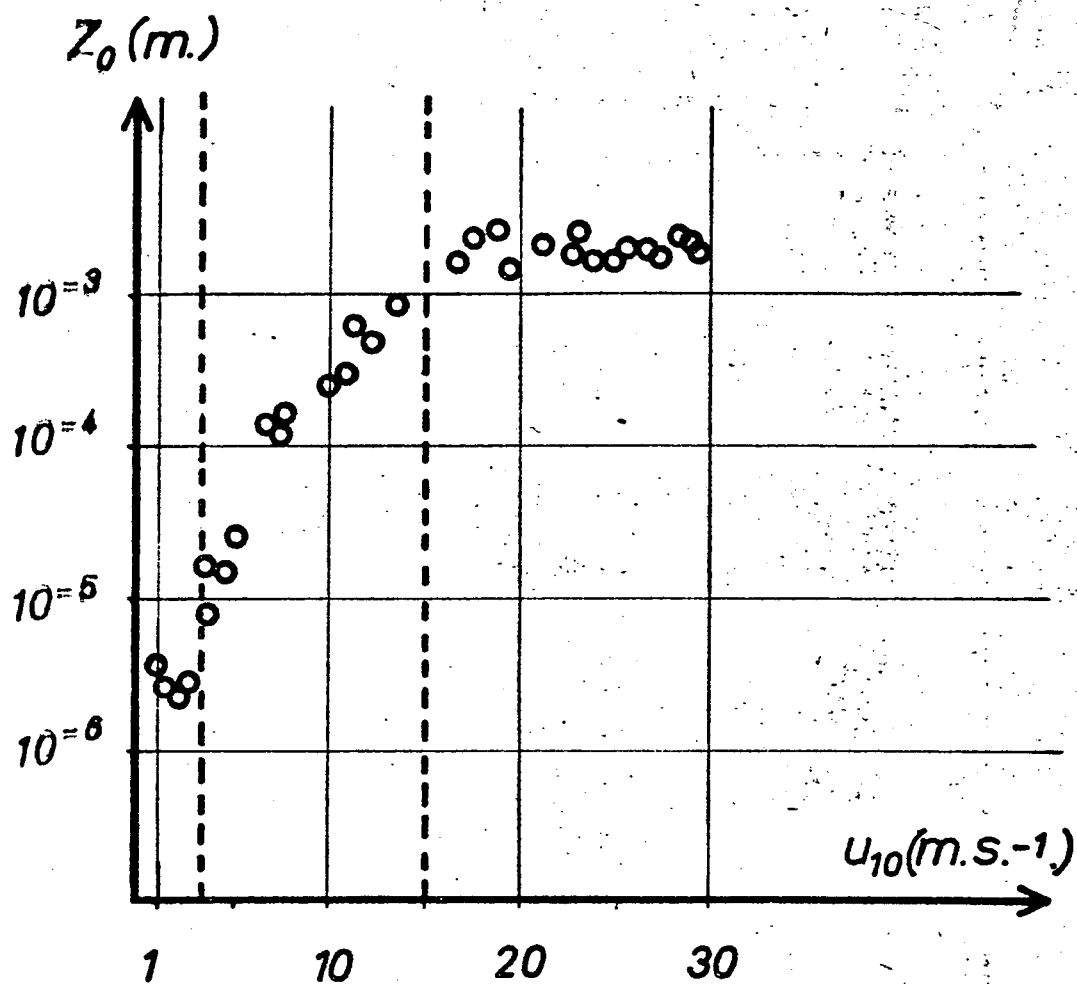
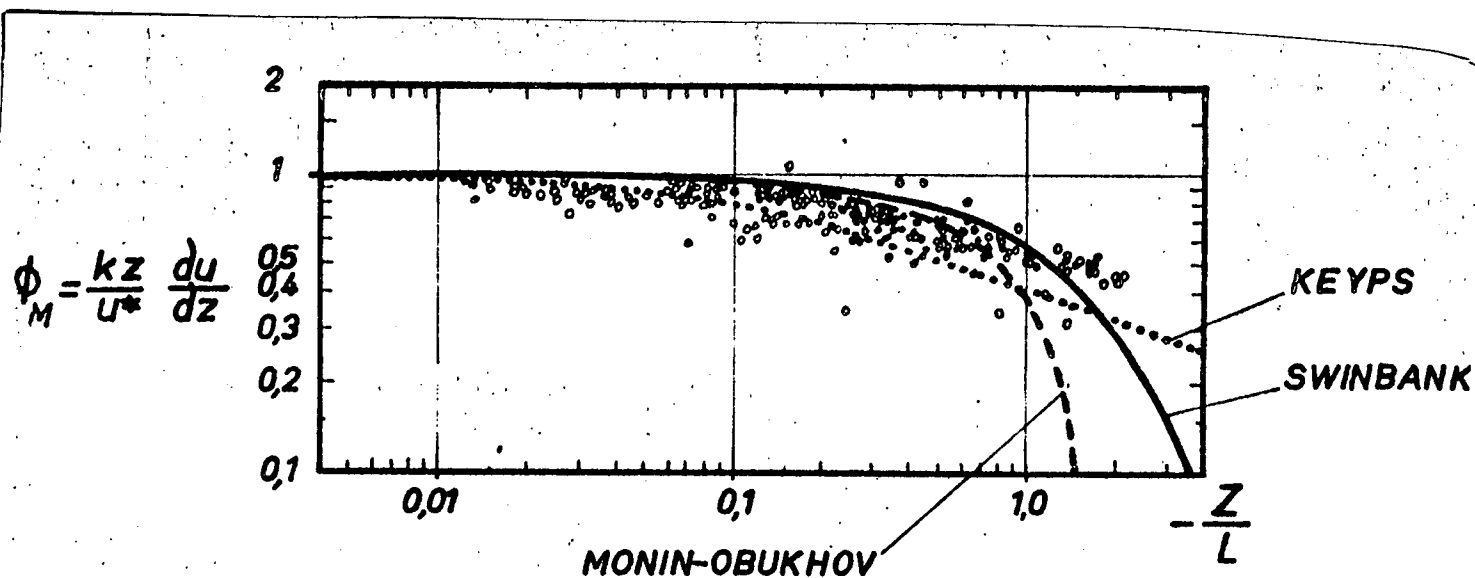Figure 7. Variation of z_o as a function of U_{10} in the ocean (according to Wu 1969).

Figure 8.   Comparison of theoretical expressions of Monin-Obukhov
Swinbank and KEYPS with experimental data, in the case of instability
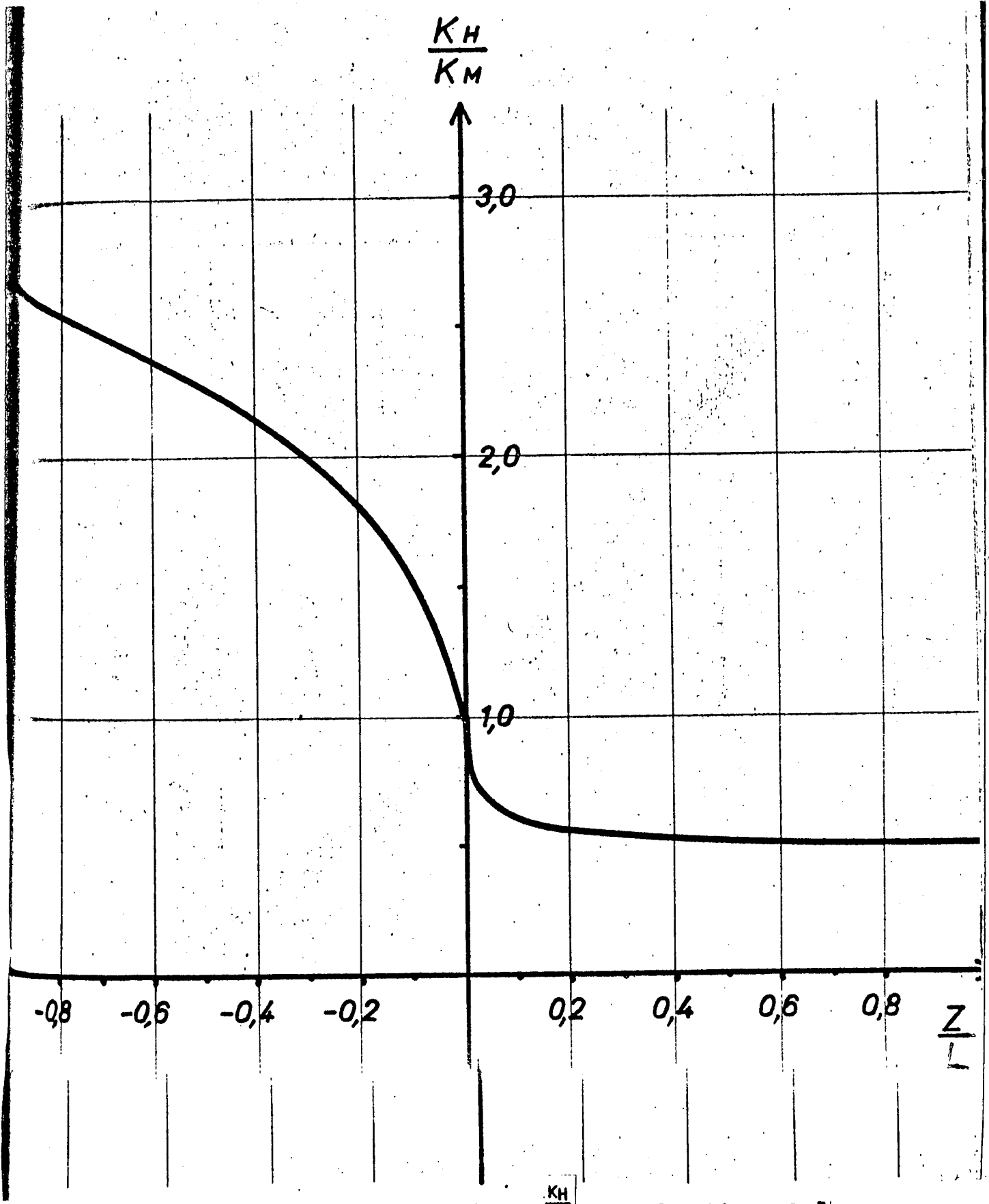(according to Bernstein 1966).

Figure 9. Variation of the ratio $\dfrac{K_H}{K_M}$ as a function of $\dfrac{z}{L}$ using relationships from the present study.
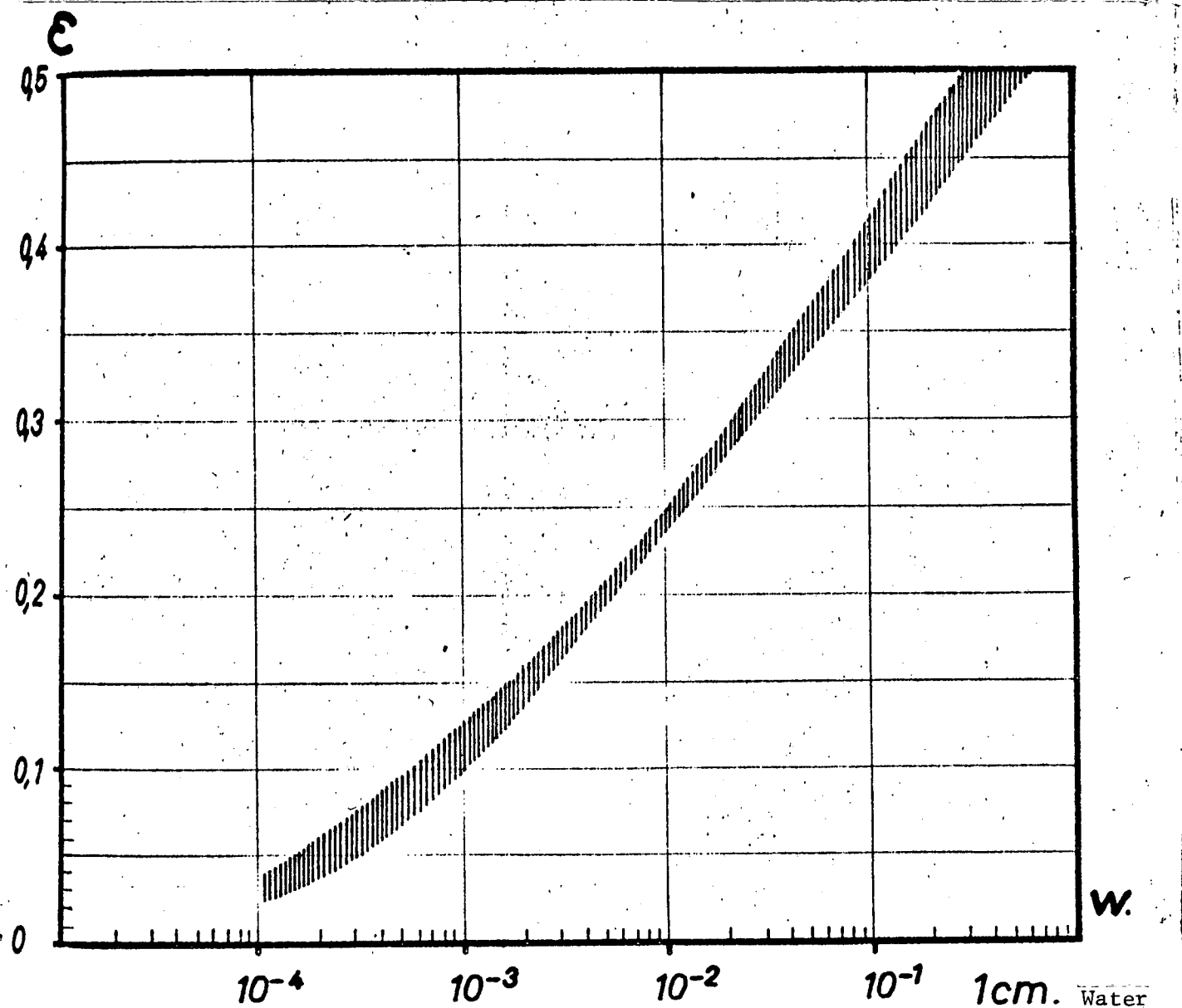
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B

Figure 10. Region of variation of the experimental determinations of the emissivity function (work of Brooks, Robinson, Bruinenberg, Elsasser and Kuhn).
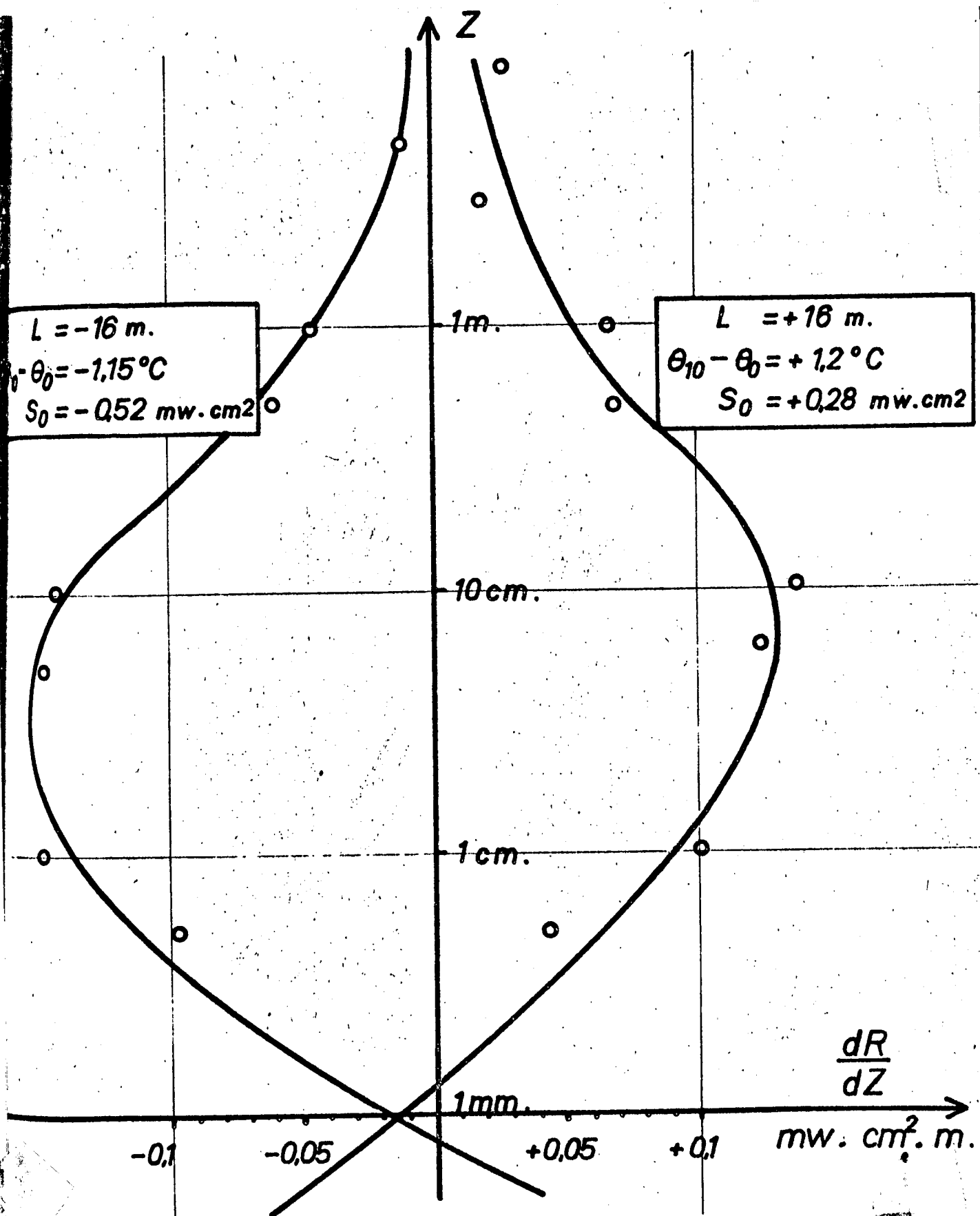
Figure 11. Examples of numerical calculation results for two situations : inversion and over-adiabatic conditions
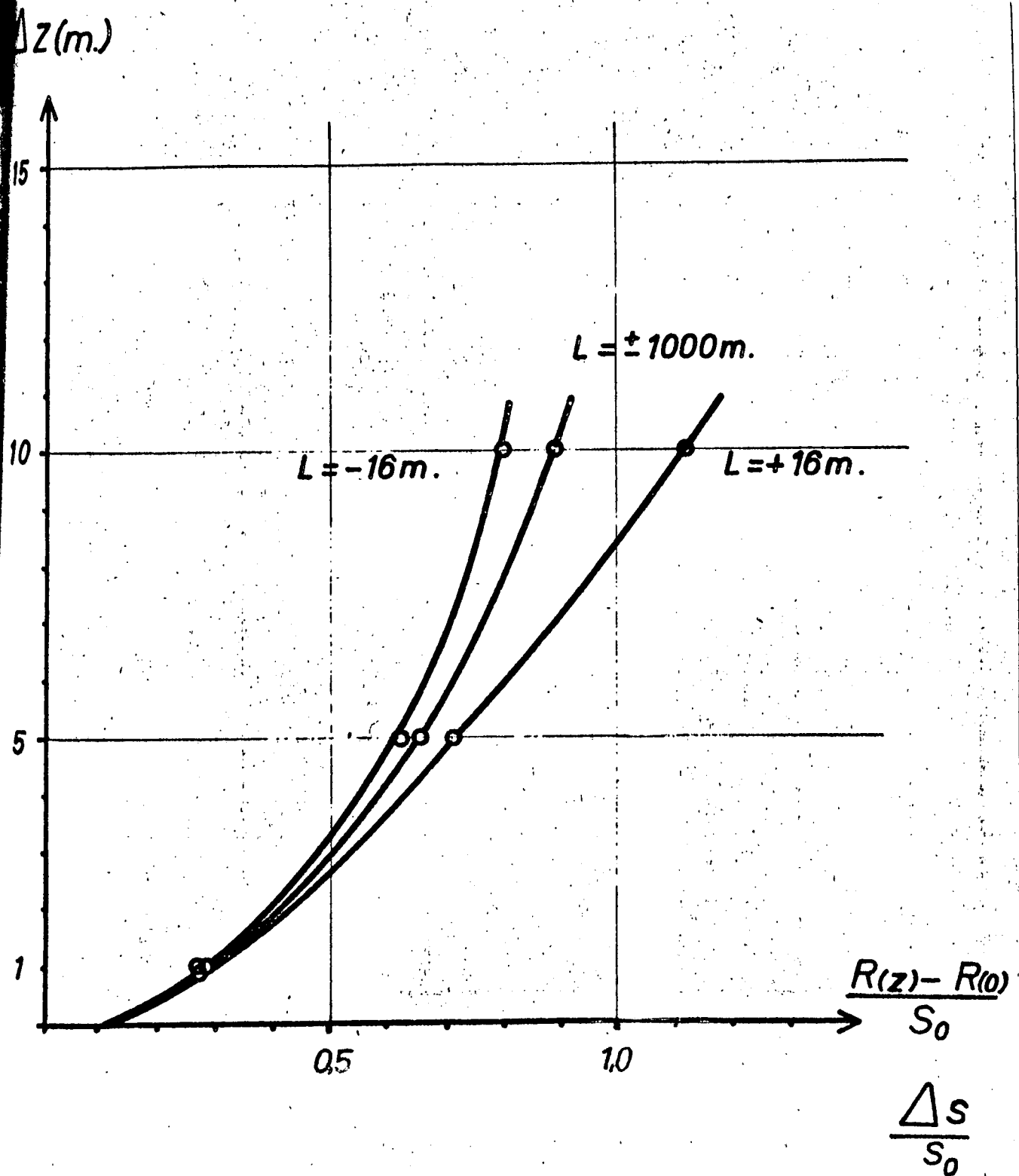
$(u_{10} = 2.8 \text{ m.s}, \Theta_0 = 20\text{° C})$

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Figure 12. Characteristic results of the numerical calculations obtained for $\Theta_o = 20^\circ$ C – $u_{10} = 2,8$ m.s and different stability regions.
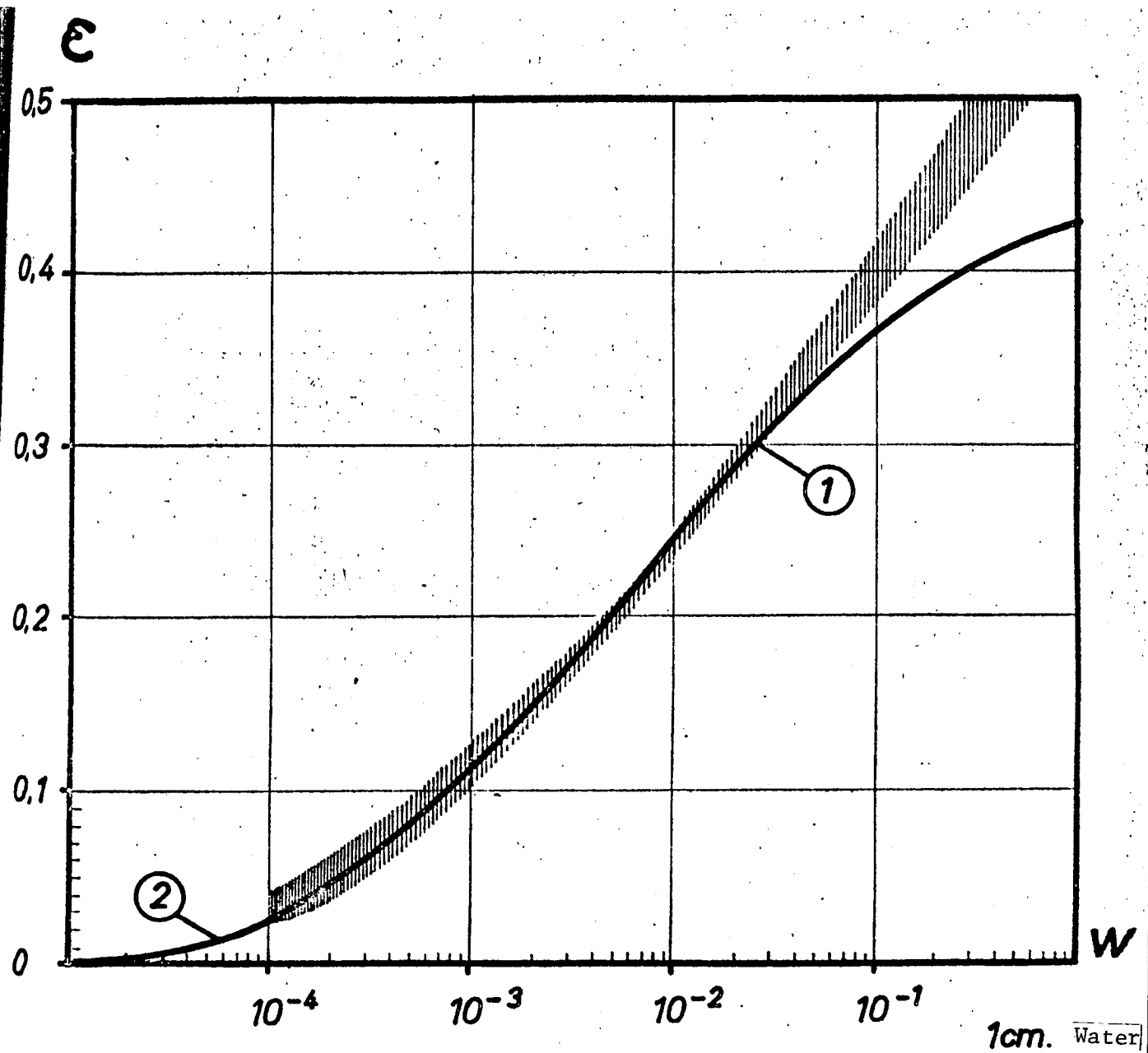
Figure 13. Analytical representation of the emissivity
curve $\varepsilon_1(w)$

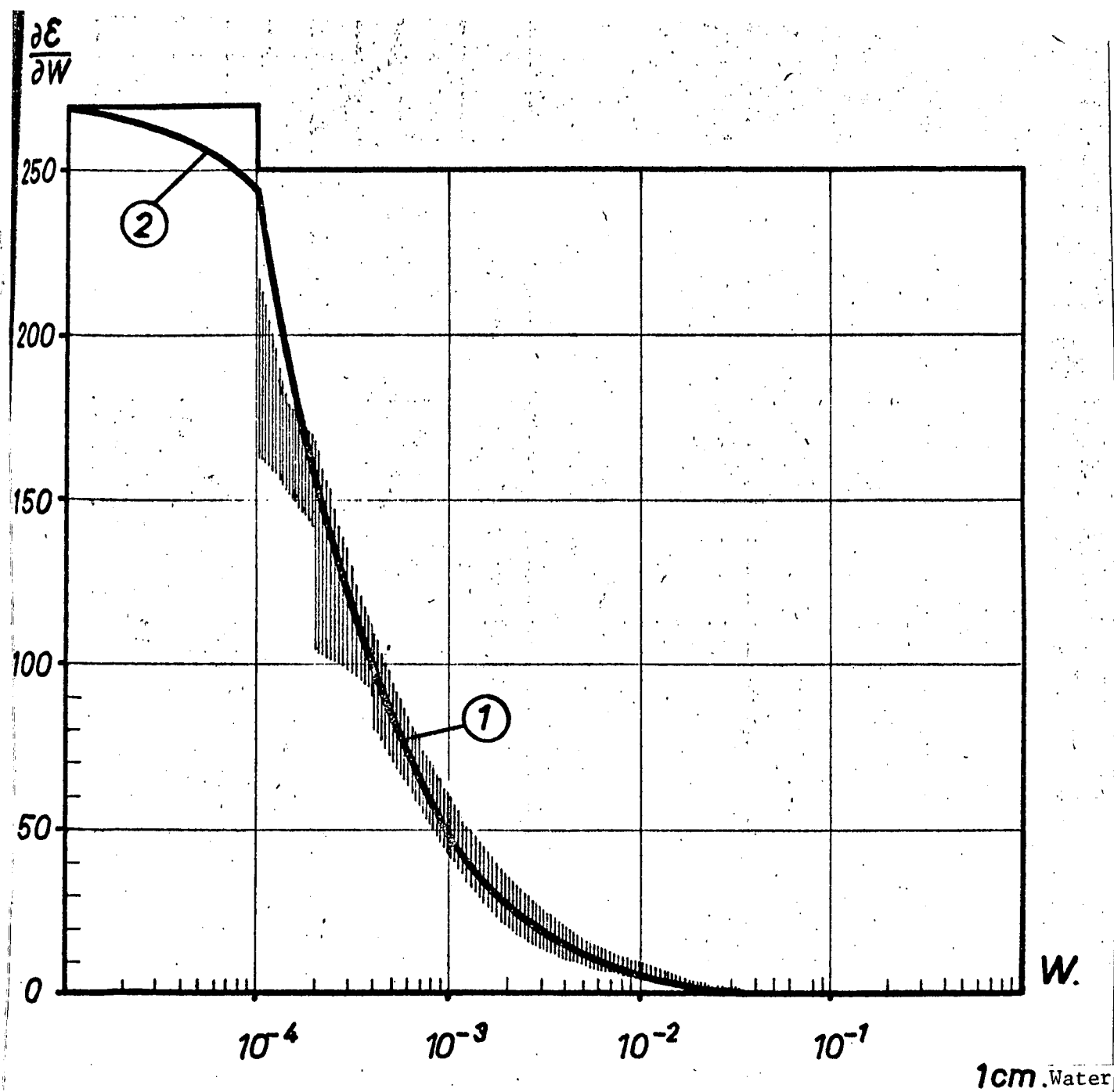Figure 14.   Analytical representation of the first
derivative $\dfrac{\partial \mathcal{E}}{\partial w}\Big|$
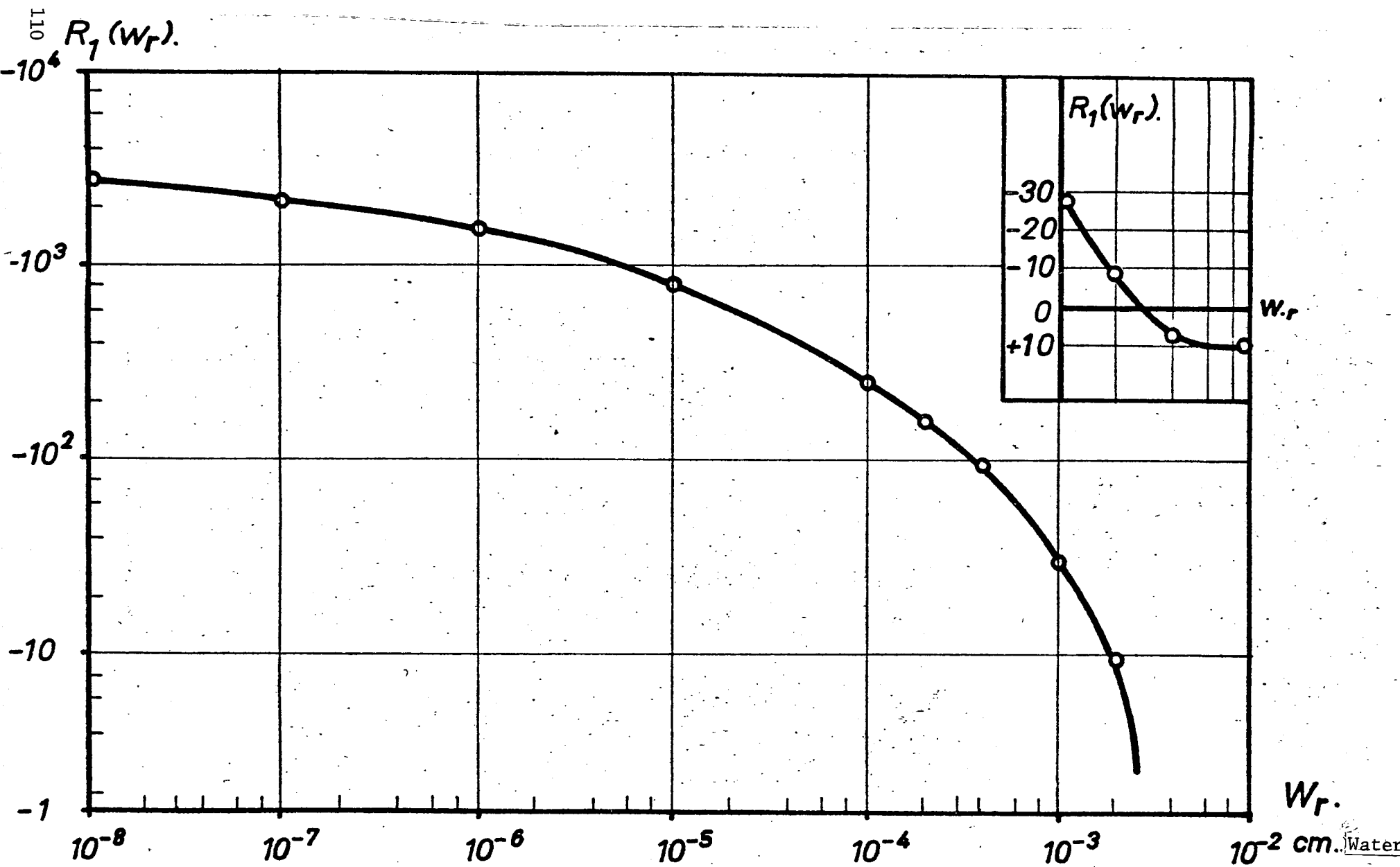
Figure 15.   Numerical representation of the function $R_1(w_{\frac{1}{2}})$
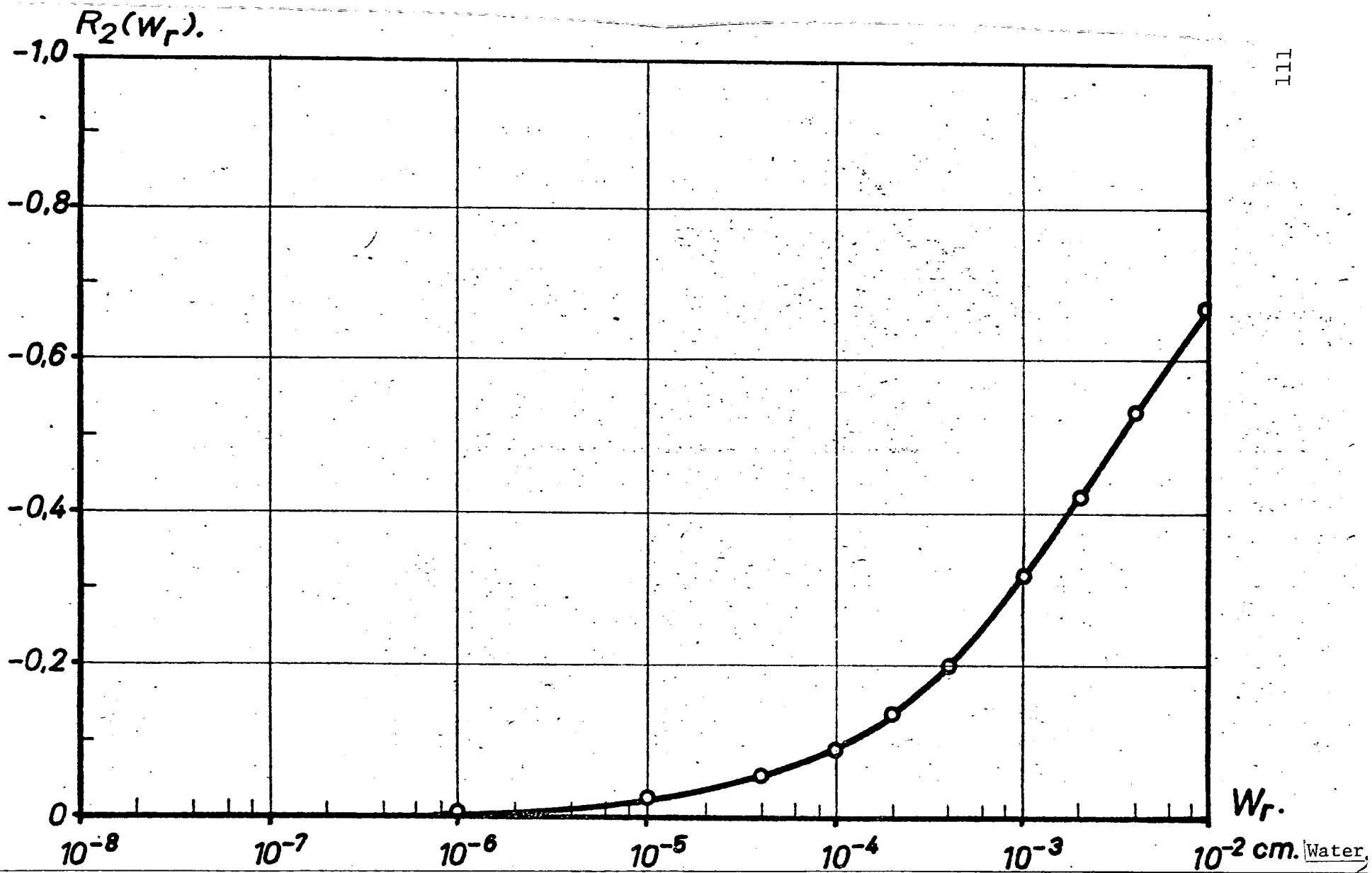
Figure 16. Numerical representation of the function $R_2(w_r)$
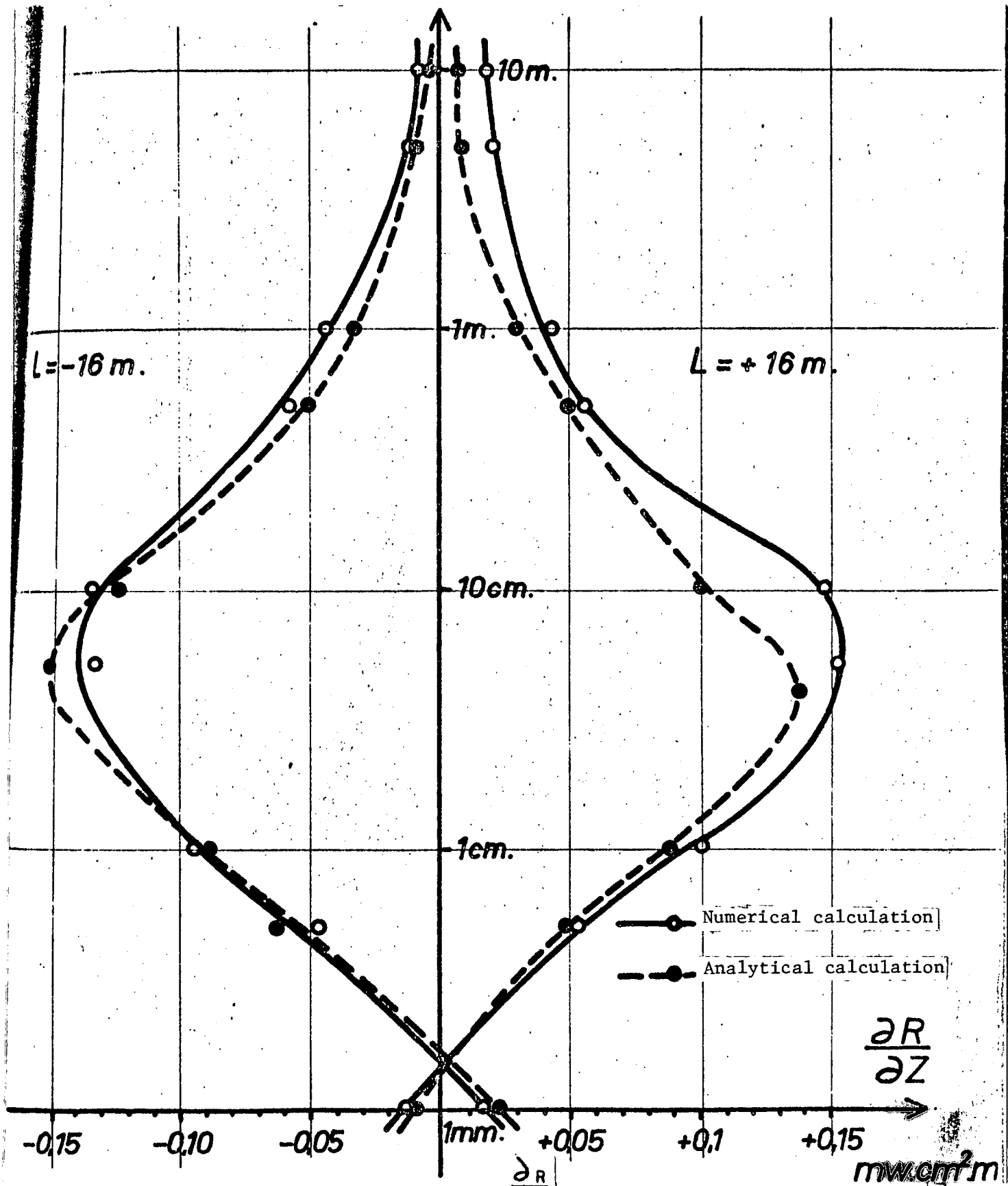
Figure 17. Results obtained for $\dfrac{\partial R}{\partial z}$ using the simplified logarithmic profile expression. Comparison between numerical calculation and analytical development ($\Theta_o = 20^\circ$ C – $u_{10} = 2.8$ m.s)

Figure 18. Results obtained for $\dfrac{R(z)-R(o)}{S_o}$ using the simplified logarithmic profile expression. Comparison between numerical calculation and analytical development ($\Theta_o = 20° C - u_{10} = 2,8$ m.s)

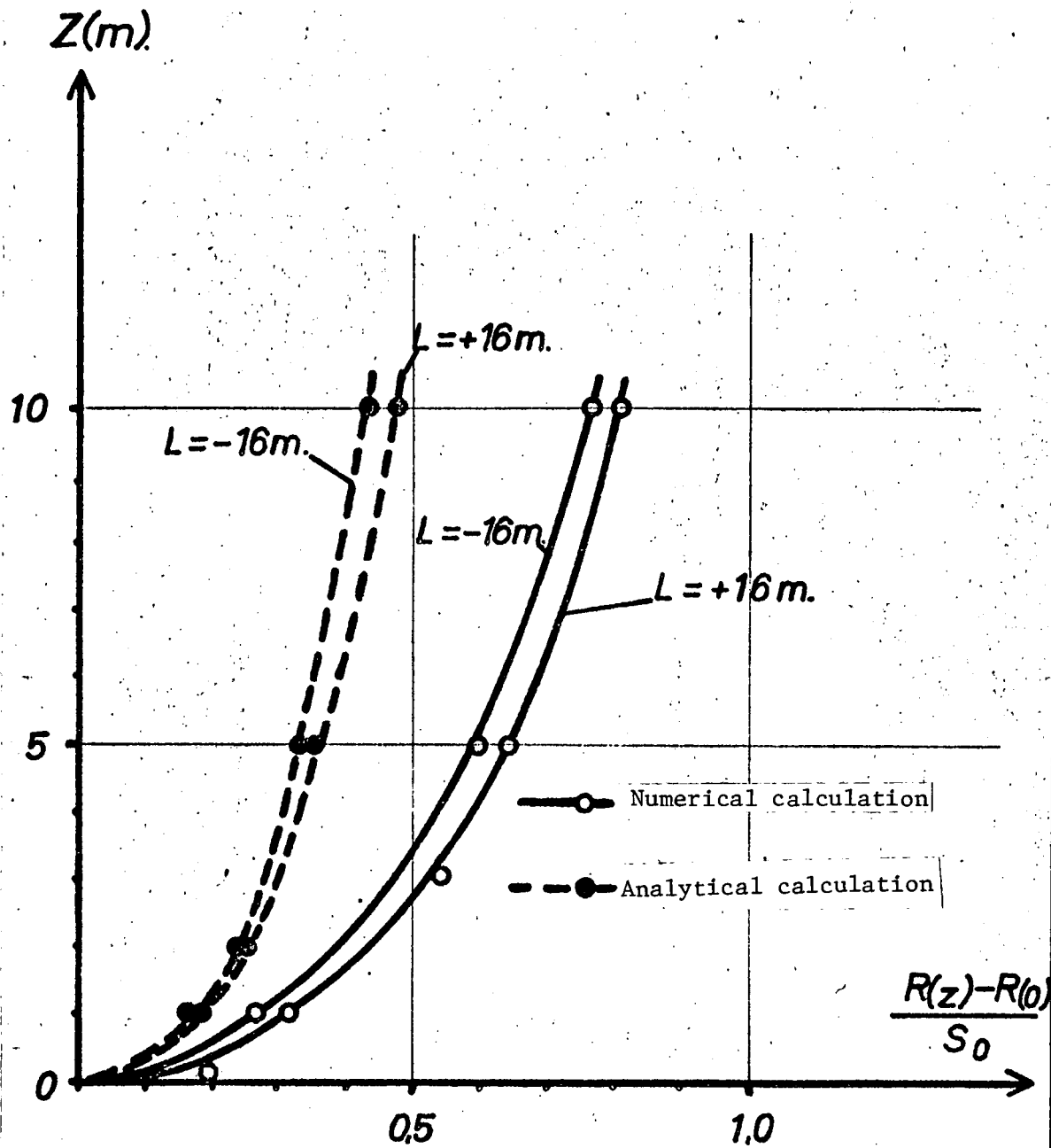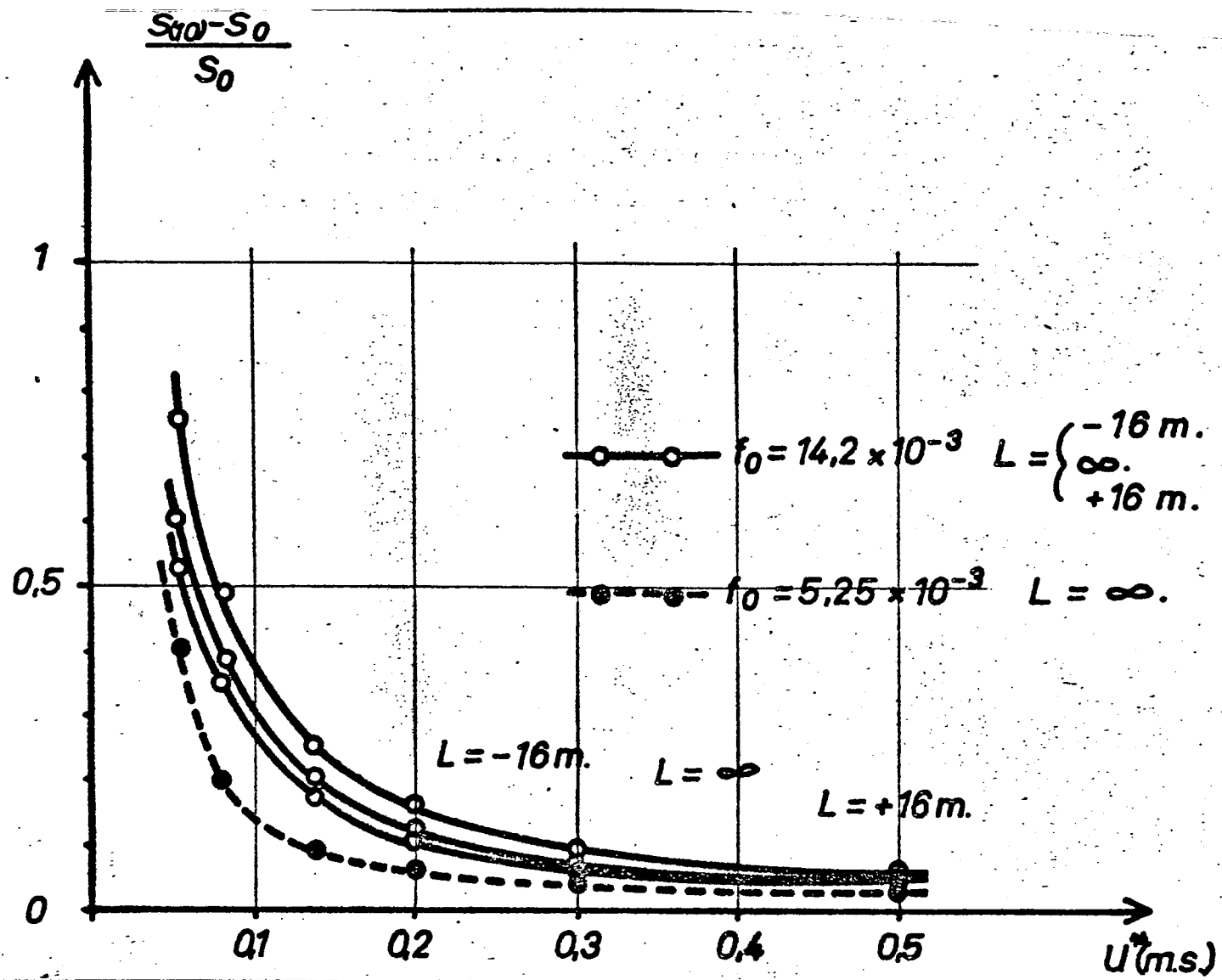Figure 19. Calculated variations of the turbulent flux S between 0 and 10 m as a function of the friction velocity of u* (for various humidities and stability regions and the emissivities of Brooks)

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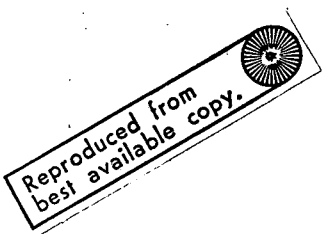
APPENDIX I

NUMERICAL CALCULATION PROGRAM (FORTRAN IV)

```
DIMENSION U(5),ZOM(5),OBUKO(10)
DIMENSION Z(15),X(15),TETA(15),EHUM(15),W(15),T2W2(15),V(15),Y(15)
DIMENSION WR(10),ZR(10),DIV(10),FI(10),CUMUL(10),T3W3(15),EMIS(15)
FORMAT(////,3X,'WIND VELOCITY AT 10 M., U10= ',F 9.1,'....ROUGHNESS,
1...........ZO= ',E 9.3,'..L.MONIN-OBUKHOV,.........L= ',F9.0)
FORMAT(/   ,1H ,'.USTAR=   ',F10.4,',FLUX   QH=',F10.4,',EVAPO=
1',F10.5,',BOWEN=  ',F10.4,'    ,A=    ,F10.5)
FORMAT(1H ,100('*'))
4 FORMAT(1H ,     HEIGHT    *   TEMPERATURE    *   HUMIDITY    *
1     W      *     D2T/DW2     *')
FORMAT(1H ,5(E15.8,4X,1H*))
FORMAT(1H0,'NP-R *          WR    *          ZR   *   INTEGRAL    *
2DIVERGENCE    *      CUMUL   *')
FORMAT(1H, I4,1X,'*',5(E13.7,1X,'*'))
```

INITIALIZATION OF CONSTANT ANALYSIS VALUES

```
N=14
AK=0,4
TETO=293.
U(1)=2.8
U(2)=5.
U(3)=8.
U(4)=12.
U(5)=15.
ZOM(1)=0.00001
ZOM(2)=0.00005
ZOM(3)=0.0001
ZOM(4)=0.0005
ZOM(5)=0.001
OBUKO(1)=-1000.
OBUKO(2)=-500.
OBUKO(3)=-100.
OBUKO(4)=-16
OBUKO(5)=-5
DO 100 I=6,10
J=11-I
OBUKO(I)=-OBUKO(J)
ZO=23.27
Z1=13000
```

MX IS THE NUMBER OF THE OUTPUT UNIT

```
          STANDARD HEIGHT VALUES
  X(1)=0.
  X(2)=0.00001
  X(3)=0.00005
  X(4)=0.0001
  X(5)=0.0005
  X(6)=0.001
  X(7)=0.005
  X(8)=0.01
  X(9)=0.05
  X(10)=0.1
  X(11)=0.5
  X(12)=1.
  X(13)=5.
  X(14)=10.
  X(15)=100.


          VALUES OF THE EMISSIVITY CURVE PARAMETERS

  BO=0.7875
  B1=0.1420
  B2=0.0065

UP TO HERE THE NUMBER OF PAIRS (U10,ZO) STUDIED, USED FOR EVALUATION, NTEST=5
  DO 130 I=1,5
  U10=U(I)
  ZO=ZOM(I)


          WE CAN SEE THAT THE NUMBER OF PROFILE POINTS AND THE
          POSSIBLE HEIGHT VALUES, NEVER ZERO
  NP=N+1-I
  DO 120 J=1,NP
  K=I+J
20 Z(J)=X(K)


          CALCULATIONS FOR FIXED VALUES OF THE MONIN-OBUKHOV
          LENGTHS ARE BEING CARRIED OUT

  DO 130 J=1,10
  IH=1
  DO 130 IJ=1,2
  AL=OBUKO(J)
  WRITE(3,1)U10,ZO,AL
  UETOI=USTAR(AK,U10,AL,ZO,INDIC)

          CALCULATION OF THE TEMPERATURE PROFILE

  CALL PROTP(AL,AK,UETOI,ZO,TETO,NP,Z,TETA,IX,IJ)
  TET10=TETA(NP)

          CALCULATION OF THE BOWEN RATIO

  B=BOWEN(TET10,TETO)
```

```
      CALCULATION OF THE HEAT FLUX AND EVAPORATION
      CALCULATION OF THE HUMIDITY PROFILE

CALL FLUEV(UETOI,AL,B,QHO,EVAPO)
CALL PROHU(EO,B,NP,TETA,TETO,EHUM)

      CALCULATION OF ELEMENTS INVOLVED IN THE RADIATION FLUX
      DIVERGENCE

A2=TETO*UETOI*UETOI/(AK*AK*AL*9.81)

      CALCULATION OF THE TWO TABLES CONTAINING W AND THE SECOND
      DERIVATIVE OF THETA WITH RESPECT TO W

CALL EOPTI(A1,A2,B,NP,EHUM,Z,W,T2W2,AL,IJ)
WRITE(MX,2)UETOI,QHO,EVAPO,B,A2
WRITE(MX,3)
WRITE(MX,4)
WRITE(MX,3)
DO 135 K=1,NP
35 WRITE(MX,5)Z(K),TETA(K),EHUM(K),W(K),T2W2(K)

      CALCULATION OF THE VALUES ZR AND WR

DO 140 K=1,10
L=NP-K+1
M=11-K
WR(M)=W(L)
40 ZR(M)=Z(L)

      CALCULATION OF EMISSIVITIES

08 DO 150 M=1,10
DO 163 K=1,NP
V(K)=W(K)-WR(M)
IF(V(K))164,162,164
62 EMIS(K)=0.
GO TO 163
64 IF (ABS(V(K))-1.E-4) 165,165,161
61 IF(IH.NE.3) GO TO 300
IX=2
EMIS(K)=0,218*(1.+1.125*(SORT(ABS(V(K)))-7.9E-2)/(SORT(ABS(V(K)))+17.9E-2))
GO TO 163
65 IF(IH.NE.3) GO TO 301
IX=2
EMIS(K)=0.260*ALOG(1.+1.043E3*(ABS(V(K))))
GO TO 163
01 EMIS(K)=0.055*ALOG(1.+7.E3*ABS(V(K)))
63 T3W3(K)=T2W2(K)*(EMIS(1)-EMIS(K))

      INTEGRATION OF THE FUNCTION T3W3 TABULATED IN THE INTERVAL V(1) TO V(NP)
      THE INTEGRAL EQUALS FI.SSP PAGE 289
CALL QTFG(V,T3W3,Y,NP)
FI(M)=Y(NP)
```

/25

CALCULATION OF THE DIVERGENCE AT LEVEL ZR

```
L=NP-M+1
DIV(M)=-22.4E-9*TETO**3*EHUM(L)*FI(M)/A1
```

CALCULATION OF CUMULATIVE AREAS OF DIV OF THE FUNCTION OF Z

```
CUMUL(1)=1.E-3*DIV(1)
DO 170 M=2,10
CUMUL(M)=CUMUL(M-1)+(ZR(M)-ZR(M-1))*(DIV(M)+DIV(M-1))*.5

WRITE(MX,6)
DO 180 M=1,10
WRITE(MX,7)M,WR(M),ZR(M),FI(M),DIV(M),CUMUL(M)
GO TO (210,130),IX
IH=3
GO TO 208
CONTINUE
CALL EXIT
END
```

SUB-PROGRAMS

```
FUNCTION USTAR(AK,U10,AL,ZO,INDIC)
```

CALCULATION OF THE FRICTION VELOCITY USTAR(U STAR)
...AK...KARMAN CONSTANT
...U10..WIND VELOCITY AT 10 METERS
...AL...MONIN-OBUKHOV LENGTH
...ZO...ROUGHNESS PARAMETER
...INDIC..0 IF THE CALCULATION IS NUMERICALLY POSSIBLE
        ..1 IF THE TERM IN THE LOGARITHM IS ZERO, THIS POINT COULD BE
            IMPROVED BY SPECIFYING THAT THE TERM HAVE AN ORDER HIGHER
            THAN THAT OF EPS

```
INDIC=0
EPS=0.
X=AL*(EXP(10./AL)-1.)/ZO
IF(X-EPS)100,100,110
INDIC=1
USTAR=0.
RETURN
USTAR=AK*U10/ALOG(X)
RETURN
END
```

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```fortran
SUBROUTINE PROTP (AL,AK,USTAR,ZO,TETO,N,Z,TETA,IX,IJ)
DIMENSION Z(1),TETA(1)

      CALCULATION OF A TEMPERATURE PROFILE THETA(Z) IS A FUNCTION OF Z
      TABLE CONTAINING THE VARIOUS VALUES OF THE OBSERVATION HEIGHTS,
      DIMENSION N
      ...AL....MONIN-OBUKHOV LENGTH
      ...AK....KARMAN CONSTANT
      ...USTAR.FRICTION VELOCITY
      ...ZO....ROUGHNESS PARAMETER
      ...N.....NUMBER OF DIFFERENT HEIGHT VALUES
      ...THETA.TABLE CONTAINING THE RESULTS OF DIMENSION N
      ...TETO..FIXED PROFILE VALUE FOR ZO
```

```fortran
   GO TO (201,202),IJ
11 T=ABS(AL*EXP(-0.02)-1.)
   IX=1
   G=9.81
   T=TETO*(1.+USTAR*USTAR*ALOG(T/ZO)/AK/AK/G/AL)
   DO 90 I=1,N
   IF(AL)100,100,121
10 BL=-.02*AL
   IF(ABS(Z(I)-BL).EL.0.000001)Z(I)=BL
   IF(Z(I)-BL)110,110,120
0  CL=ABS(BL)
   TETA(I)=T+2.8*USTAR*USTAR*(1./Z(I)**.5-1./CL**.5)/G/ABS(AL)**.5
   GO TO 90
0  S=ABS(AL*(EXP(Z(I)/AL)-1.))
   IF (ABS(S).LE.0.00001)TETA(I)=TETO
   IF(TETA(I).EQ.TETO)GO TO 90
   TETA(I)=TETO*(1.+USTAR*USTAR*ALOG(S/ZO)/AK/AK/G/AL)
   GO TO 90
1  DL=0.001*AL
   IF(ABS(Z(I)-DL).LE.0.000001)Z(I)=DL
   IF(Z(I)-DL) 110,110,122
2  TPRIM=ABS(AL*(EXP(0.001)-1))
   TPROM=TETO*(1.+USTAR*USTAR*ALOG(TPRIM/ZO)/AK/AK/G/AL)
   TETA(I)=TPROM=20.*USTAR*USTAR*TETO*((Z(I)/AL)**0.1-0.001**0.1)/AK/AK/G/AL
0  CONTINUE
0  RETURN
2   TETA(I)=TETO*(1.+USTAR**2*ALOG(Z(I)/ZO)/AK/AL/AK/G)
   IX=2
   RETURN
   END


   FUNCTION BOWEN(TET10,TETO)
         CALCULATION OF THE BOWEN RATIO

   DELTA=TET10-TETO
   IF(DELTA)100,100,110
0  BOWEN=.1-.09*DELTA
   MX=3
   RETURN
0  BOWEN=.8*DELTA
```

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```
SUBROUTINE FLUEV(USTAR,AL,BOWEN,QHO,EVAPO)

      THIS SUBROUTINE CALCULATES
        ..QHO....FLUX OF SENSIBLE HEAT
        ..EVAPO.EVAPORATION
      WHEN THE FOLLOWING ARE KNOWN
        ..USTAR.FRICTION VELOCITY
        ..AL....NON ZERO MONIN-OBUKHOV LENGTH
        ..BOWEN.NON ZERO BOWEN RATIO

QHO=-.9E+04*USTAR*USTAR*USTAR/AL
EVAPO=1.5E-02*QHO/BOWEN
MX=3
RETURN
END




SUBROUTINE PROHU(EO,BOWEN,N,TETA,TETO,EHUM)
DIMENSION TETA(1),EHUM(1)

      CALCULATION OF THE HUMIDITY PROFILE IF THE TEMPERATURE PROFILE IS KNOWN
        ...EO.....VALUE OF THE HUMIDITY FOR Z=ZO ROUGHNESS PARAMETER
        ...BOWEN...BOWEN RATIO
        ...N......NUMBER OF DIFFERENT HEIGHT VALUES
        ...TETA...N DIMENSION TABLE CONTAINING THE TEMPERATURE PROFILE VALUES
        ...TETO...CONSTANT CORRESPONDING TO THE VALUE OF ZO
        ...EHUM...N DIMENSIONAL TABLE CONTAINING THE HUMIDITY PROFILE VALUES

DO 100 I=1,N
EHUM(I)=EO+.66*(TETA(I)-TETO)/BOWEN
MX=3
RETURN
END




SUBROUTINE EOPTI(A1,A2,B,NP,EHUM,Z,W,T2W2,AL,IJ)
DIMENSION EHUM(1),Z(1),W(1),T2W2(1),X(15)

   AS INPUT    A1   IS A CONSTANT.
               A2   IS A CONSTANT PARAMETER WHEN THE SUB-PROGRAM IS CALLED
               B    IS A CONSTANT PARAMETER EQUAL TO THE BOWEN RATIO
               NP   IS THE NUMBER OF PROFILE POINTS
               EHUM DIMENSION NP CONTAINS THE HUMIDITY PROFILE
               Z                              HEIGHT
   AS OUTPUT   W                     OMEGA (SEE TEXT)
               T2W2                  SECOND DERIVATIVE OF
                                     TETA WITH RESPECT TO OMEGA)

A=0.66*A2/B
DO 300 I=1,NP
IF(IJ.EQ.2)GO TO 110
IF(AL)110,110,120
CL=-0.02*AL
IF(ABS(Z(I)-CL).LE.0.000001)Z(I)=CL
IF (IJ.EQ.2)GO TO 111
IF(Z(I)-CL) 111,111,112
```

```
2 W(I)=R*Z(I)**.75*(1.33*AL/Z(I)-.34)/(2.7*(ABS(AL))**.75)
  W(I)=EHUM(I)-W(I)
  W(I)=W(I)*Z(I)/A1
  X(I)=R*(1.+.6*Z(I)/AL)/(EHUM(I)*2.7*(Z(I)/ABS(AL))**.25)+.75
  X(I)=.2*(Z(I)/ABS(AL))**.75+(1.+.6*Z(I)/AL*X(I))/(2.7*(Z(I)/ABS(AL))**.25)
  T2W2(I)= X(I)*(-A2*(A1/Z(I)/EHUM(I))**2)
  GO TO 300
  DL=0.001*AL
  IF(ABS(Z(I)-DL).LE.0.000001)Z(I)=DL
  IF(Z(I)-DL)111,111,122
  W(I)=R*2*((Z(I)/AL)**.1)*(.9+.3*Z(I)/AL)
  W(I)=EHUM(I)-W(I)
  W(I)=Z(I)*W(I)/A1
  X(I)=R*2*(1.+.6*Z(I)/AL)*(Z(I)**.1)/EHUM(I)*(AL**.1)+1.1
  X(I)=1.2*(Z(I)/AL)**1.1+(1.+6*Z(I)/AL)*2*((Z(I)/AL)**.1)*X(I)
  T2W2(I)= X(I)*(-A2*(A1/Z(I)/EHUM(I))**2)
  CONTINUE
  RETURN
  END
```

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