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Guidance Strategies and Analysis for Low-Thrust Navigation

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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
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National Aeronautics and Space Administration

PREFACE

The work described in this report was performed by the Mission Analysis Division of the Jet Propulsion Laboratory.

CONTENTS

ABSTRACT

A low-thrust guidance algorithm suitable for operational use has been formulated. A constrained linear feedback control law has been obtained using a minimum terminal miss criterion and restricting control corrections to constant changes for specified time periods. Both fixed- and variable-time-of-arrival guidance were considered. The performance of the guidance law was evaluated by applying it to the approach phase of the 1980 rendezvous mission with the comet Encke.

I. INTRODUCTION

Guiding a low-thrust spacecraft requires techniques differing from those which have been employed for ballistic spacecraft. The low-thrust engine, which provides a continuous, controllable accelerating force acting on the vehicle, not only increases its targeting ability but also increases the demands imposed upon its navigation system. Considerable effort has been expended in the development of a variety of low-thrust guidance schemes (Refs. 1-18) which range from the extremely simple to the mathematically elegant.

However, none of these methods offers a realistic solution to the problem of operational guidance when actual mission constraints must be considered. In general, all of them generate control programs or control program corrections which are unacceptable from an operations viewpoint because they require control accelerations that may be beyond the capability of the spacecraft (the problem of controllability), or that are continuously varying in a way that imposes excessive hardware requirements for implementation. In addition, some guidance laws delay corrections until an optimum time (with respect to some preselected performance criterion such as minimum fuel); and although this policy at first may appear to be desirable, it can lead to controllability problems if additional errors or disturbances occur between the guidance update time and the selected correction time.

The purpose of this memorandum is to present a practical algorithm which may be used for both analysis and operational guidance. The development of the steering law is based on the use of linear perturbation theory to correct a nominal trajectory in a way that minimizes violations of desired terminal mission constraints. The corrections are required to occur prior to a prespecified time, thus causing an immediate attempt to improve the trajectory before additional errors occur and possibly give rise to problems of controllability. In addition, the implementation deficiencies of earlier schemes are avoided by restricting the correction policy to control variable changes which are constant for specified time intervals and are of limited magnitude. Section II contains a complete derivation of the deterministic (i.e., operational) form of the guidance equations, and Section III presents the statistical formulation of those same equations as required for a statistical guidance analysis.

In order to evaluate the effectiveness of the basic algorithm, it was employed in a preliminary analysis of the approach phase of a rendezvous with the comet Encke. Section IV describes that analysis and discusses the performance of the guidance scheme.

II. THE LOW-THRUST GUIDANCE ALGORITHM

In general, a nominal mission trajectory is designed to satisfy terminal constraints

$$\psi\left[x^*(t_f^*),\ t_f^*\right] = 0 \tag{1}$$

where

ψ = m vector set of constraint equations

$x^*(t)$ = nominal trajectory state, a 6 vector

t_f^* = nominal mission final time

For guidance purposes, it is assumed that Eq. (1) has the form

$$\psi\left[x^*(t_f^*) - y(t_f^*)\right] = 0 \tag{2}$$

where

$y(t)$ = state of the target set, a 6 vector

Since the actual mission trajectory generally deviates from the nominal, the terminal constraint equations may not be satisfied at the end of the actual trajectory. That is,

$$\psi\left[x(t_f) - y(t_f)\right] = e_f \neq 0 \tag{3}$$

where

$x(t)$ = actual trajectory state, a 6 vector

t_f = actual mission final time

e_f = constraint violations, an m vector

Equation (3) can be approximated by the linear terms in a Taylor's Series expansion about the nominal trajectory:

$$\psi_x\left[x^*(t_f^*) - y(t_f^*)\right]\left[\delta x_f^* + (\dot{x}_f^* - \dot{y}_f^*)\delta t_f\right] = e_f \tag{4}$$

where

$\delta x_f^* = x(t_f^*) - x^*(t_f^*)$ = state deviation at t_f^*

$\delta t_f = t_f - t_f^*$ = final time variation

$\dot{x}_f^* = \dot{x}^*(t_f^*)$ = rate of change of $x^*(t)$ at t_f^*

$\dot{y}_f^* = \dot{y}(t_f^*)$ = rate of change of $y(t)$ at t_f^*

ψ_x = m x 6 matrix of partial derivatives of ψ with respect to x evaluated at t_f^*

In order for the actual trajectory to satisfy Eq. (2), e_f must be zero. A natural guidance law, therefore, would be one that generates a control program defining a trajectory which nulls out e_f. However, because of the inherent controllability problems associated with low-thrust flight, it may not be possible to eliminate the terminal constraint violations with the available control effort. Consequently, a guidance law seeking to null e_f is unrealistic. An alternative and more practical approach is to develop a strategy which minimizes

$$J = e_f^T S e_f \tag{5}$$

where

S = diagonal weighting matrix

$$S_{ii} > 0, \qquad i = 1 \text{ to } m$$

A guidance procedure of this type yields the smallest end constraint violations attainable with the control available and thus avoids the problem of controllability.

By employing the expanded form (Eq. 4) of e_f, minimization of J becomes a problem of determining the minimizing final state and time deviations δx_f^* and δt_f. Expansion of the actual trajectory about the nominal gives δx_f^* as a function of initial state deviations $\delta x_0 = x(t_0) - x^*(t_0)$, and deviations from the nominal control program $\delta u(t) = u(t) - u^*(t)$;

$$\delta x_f^* = \Phi(t_f^*, t_0)\delta x_0 + \int_{t_0}^{t_f} \Phi(t_f^*, \tau) f_u(\tau, x^*, u^*)\delta u(\tau)\, d\tau \tag{6}$$

where

$\Phi(t, t_0)$ = state transition matrix of the nominal trajectory

$f_u(t, x^*, u^*)$ = partial derivative of the equations of motion with respect to control u evaluated on the nominal trajectory

$\delta u(t)$ = control deviations, a 3 vector

Replacement of δx_f^* by Eq. (6) transforms the problem into the minimization of J as a function of $\delta u(t)$ and δt_f.

In order to define a practical and useful guidance law, the control deviations are required to satisfy a set of auxiliary conditions. Let t_c be a specified time, $t_c \leq t_f^*$, and subdivide the interval (t_0, t_c) into M equal subdivisions (t_i, t_{i+1}), $i = 0$ to $M - 1$. Then for $\delta u(t)$ to be admissible, it must satisfy

(1) $\delta u(t) = \delta u_i = \text{constant}, \; t \epsilon (t_i, t_{i+1})$

(2) $\delta u(t) = 0, \; t > t_c$

(3) $-\alpha \leq \delta u_i \leq \alpha, \; \alpha = \text{specified 3 vector}$

Substitution of Eqs. (4) and (6) and the admissible control program into Eq. (5) reduces the problem to the minimization of the function

$$J = \left[\xi + \Gamma\delta\hat{u} + (\dot{x}_f^* - \dot{y}_f^*)\delta t_f\right]^T A \left[\xi + \Gamma\delta\hat{u} + (\dot{x}_f^* - \dot{y}_f^*)\delta t_f\right] \tag{7}$$

subject to

$$-\hat{\alpha} \leq \delta\hat{u} \leq \hat{\alpha} \tag{8}$$

where

$$A = \psi_x^T \left[x^*(t_f^*) - y(t_f^*)\right] S \psi_x \left[x^*(t_f^*) - y(t_f^*)\right]$$

$$\xi = \Phi(t_f^*, t_0)\delta x_0$$

$$\Gamma = \left[\hat{\Gamma}_0 \mid \hat{\Gamma}_1 \mid \cdots \mid \hat{\Gamma}_{M-1}\right], \qquad 6 \times 3M \text{ matrix}$$

$$\hat{\Gamma}_i = \int_{t_i}^{t_{i+1}} \Phi(t_f^*, \tau) f_u(\tau, x^*, u^*) d\tau, \qquad i = 0 \text{ to } M - 1$$

$$\delta\hat{u}^T = \left[\delta u_0^T \mid \delta u_1^T \mid \cdots \mid \delta u_{M-1}^T\right], \qquad 3M \text{ vector}$$

$$\hat{\alpha}^T = \left[\alpha^T \mid \alpha^T \mid \cdots \mid \alpha^T\right], \qquad 3M \text{ vector}$$

Performing the minimization of J with respect to δt_f yields

$$\delta t_f = -\left[(\dot{x}_f^* - \dot{y}_f^*)^T A (\dot{x}_f^* - \dot{y}_f^*)\right]^{-1} \cdot \left[(\dot{x}_f^* - \dot{y}_f^*)^T A \xi + (\dot{x}_f^* - \dot{y}_f^*)^T A \Gamma \delta\hat{u}\right] \tag{9}$$

By introduction of Eq. (9) into Eq. (7), J reduces to simply a function of $\delta\hat{u}$:

$$J = \left[\xi + \Gamma\delta\hat{u}\right]^T \tilde{A} \left[\xi + \Gamma\delta\hat{u}\right] \tag{10}$$

where

$$\tilde{A} = \left[I - \frac{(\dot{x}_f^* - \dot{y}_f^*)(\dot{x}_f^* - \dot{y}_f^*)^T A}{(\dot{x}_f^* - \dot{y}_f^*)^T A (\dot{x}_f^* - \dot{y}_f^*)}\right]^T A \left[I - \frac{(\dot{x}_f^* - \dot{y}_f^*)(\dot{x}_f^* - \dot{y}_f^*)^T A}{(\dot{x}_f^* - \dot{y}_f^*)^T A (\dot{x}_f^* - \dot{y}_f^*)}\right]$$

A qualitative discussion of the significance of the reduced form (Eq. 10) for J is presented in Appendix A.

Since J is now a positive semidefinite quadratic function of $\delta\hat{u}$, it may not have a unique or easily computed minimum. These difficulties are removed by augmenting J with an additional term to form a new function $\hat{J}$:

$$\hat{J} = J + \delta\hat{u}^T W \delta\hat{u} \qquad (11)$$

The matrix W is a positive semidefinite weighting matrix defined by the following procedure:

(1) Find the orthogonal transformation which diagonalizes the symmetric matrix $\Gamma^T \tilde{A} \Gamma$,

$$Q(\Gamma^T \tilde{A} \Gamma) Q^T = D$$

$$(\Gamma^T \tilde{A} \Gamma) = Q^T D Q \qquad (12)$$

where

D = diagonal matrix, Q = orthogonal matrix

(2) Define a diagonal matrix B,

$$B = \text{diagonal } (b_i) \qquad (13)$$

where

$b_i = 0$ for $D_{ii} \neq 0$

$b_i > 0$ for $D_{ii} = 0$

(3) Form W,

$$W = Q^T B Q \qquad (14)$$

Because of the term $\delta\hat{u}^T W \delta\hat{u}$, $\hat{J}$ is a positive definite quadratic function with a unique minimum. Moreover, as shown in Appendix B, the unconstrained minimum of $\hat{J}$ is also an unconstrained minimum of J, and by a suitable choice of the b_i, the constrained minimum of $\hat{J}$ closely approaches a constrained minimum of J.

Straightforward minimization of $\hat{J}$ subject to the constraints of Eq. (8) yields the feedback control law

$$\delta\hat{u} = -\left(\Gamma^T \tilde{A} \Gamma + W + \Lambda\right)^{-1} \Gamma^T \tilde{A} \Phi(t_f^*, t_0) \delta x_0 \tag{15}$$

where

Λ = diagonal (λ_i), $\lambda_i \geq 0$ are the Lagrange multipliers which enforce the constraints on $\delta\hat{u}$

Since the matrix $(\Gamma^T \tilde{A} \Gamma + W) = Q^T (D + B)Q$ is nonsingular, the inverse required in Eq. (15) always exists. Equation (15) can be substituted back into Eq. (9) to give δt_f in feedback form:

$$\delta t_f = -\left[\left(\dot{x}_f^* - \dot{y}_f^*\right)^T A \left(\dot{x}_f^* - \dot{y}_f^*\right)\right]^{-1} \left(\dot{x}_f^* - \dot{y}_f^*\right)^T A \cdot$$

$$\left[I - \Gamma\left(\Gamma^T \tilde{A} \Gamma + W + \Lambda\right)^{-1} \Gamma^T \tilde{A}\right] \Phi(t_f^*, t_0) \delta x_0 \tag{16}$$

The controls δt_f and $\delta\hat{u}$ are coupled only through the matrix Λ; consequently, in the guidance calculations $\delta\hat{u}$ can be found independently of δt_f, the multipliers can be determined, and then δt_f can be computed.

The basic variable-time-of-arrival guidance algorithm described in this section can also be used for fixed-time-of-arrival guidance by setting $(\dot{x}_f^* - \dot{y}_f^*) = 0$, $\tilde{A} = A$, and omitting δt_f computations. Equation (15) again defines the control law $\delta\hat{u}$, which for this fixed-time case generates a trajectory minimizing terminal constraint violations e_f at the nominal final time t_f^*.

III. STATISTICAL GUIDANCE ANALYSIS

Section II presented a set of deterministic guidance equations defining control corrections based on a given state deviation. In mission planning and performance studies, however, statistical analysis techniques are normally employed, and only a statistical description of the state deviations — the state covariance — is available. Consequently, for analysis purposes, it is necessary to have a statistical formulation of the guidance laws which utilizes only the state covariance. In a straightforward manner the following expressions can be derived for the covariance of the final time and control deviations and for the final state errors remaining after the application of the guidance strategy:

$$E\left[\delta t_f^2\right] = \sigma_{t_f}^2 = \left[(\dot{x}_f^* - \dot{y}_f^*)^T A(\dot{x}_f^* - \dot{y}_f^*)\right]^{-2} (\dot{x}_f^* - \dot{y}_f^*)^T A \cdot$$
$$\left[I - \Gamma\left(\Gamma^T\tilde{A}\Gamma + W + \Lambda\right)^{-1}\Gamma^T\tilde{A}\right]\hat{Z} \cdot$$
$$\left[I - \Gamma\left(\Gamma^T\tilde{A}\Gamma + W + \Lambda\right)^{-1}\Gamma^T\tilde{A}\right]^T A(\dot{x}_f^* - \dot{y}_f^*) \tag{17}$$

$$E\left[\delta\hat{u}\delta\hat{u}^T\right] = U = \left(\Gamma^T\tilde{A}\Gamma + W + \Lambda\right)^{-1}\Gamma^T\tilde{A}\hat{Z}\tilde{A}\Gamma\left(\Gamma^T\tilde{A}\Gamma + W + \Lambda\right)^{-1} \tag{18}$$

$$E\left[\delta x(t_f^*)\,\delta x^T(t_f^*)\right] = \overline{X}_f = \left[I - \Gamma\left(\Gamma^T\tilde{A}\Gamma + W + \Lambda\right)^{-1}\Gamma^T\tilde{A}\right]\hat{Z} \cdot$$
$$\left[I - \Gamma\left(\Gamma^T\tilde{A}\Gamma + W + \Lambda\right)^{-1}\Gamma^T\tilde{A}\right] \tag{19}$$

where

$$\hat{Z} = \Phi(t_f^*, t_0)\hat{X}_0\Phi^T(t_f^*, t_0)$$

$\hat{X}_0 = E\left[\delta x(t_0)\,\delta x^T(t_0)\right]$ = state estimate covariance at time t_0

The covariances (17 - 19) represent ensemble averages of time, control, and final state deviations based on a guidance policy which employs the feedback control gain matrix

$$G = -\left(\Gamma^T \widetilde{A} \Gamma + W + \Lambda\right)^{-1} \Gamma^T \widetilde{A} \tag{20}$$

In the deterministic case, the matrix, Λ, and therefore G, are selected to enforce constraints on the control variables. In a statistical analysis, however, only the covariance of the controls is computed, and a question arises as to the choice of Λ. Setting Λ to zero gives the unconstrained control gain matrix and represents an unrealistic choice since unrestricted control is unavailable on actual missions. A more reasonable approach is to define Λ such that the standard deviations of the control variables satisfy constraints in a manner similar to the deterministic controls, i.e.,

$$0 \leq \sigma_{\delta u} \leq \overline{\alpha} \tag{21}$$

where

$\sigma_{\delta u}$ = 3M vector of control standard deviations

$\overline{\alpha}$ = 3M vector of limiting values

The use of constraint (21) in selecting Λ permits practical statistical guidance analysis to be performed with a constrained control law. The procedure is, therefore, an improvement over previous methods which employed the weighting matrix technique to reduce control deviations to acceptable levels.

IV. GUIDANCE ALGORITHM EVALUATION — ENCKE RENDEZVOUS GUIDANCE

In order to evaluate its capabilities, the guidance algorithm was used for a statistical analysis of the approach phase of a 1980 rendezvous mission with the comet Encke. Orbit determination was accomplished by processing daily on-board optical and range observations with a Kalman Filter (Ref. 19). The standard deviations of the angle pointing error, target center finding error, and range measurement error were taken as 100 arc seconds, 10 km, and 1 km, respectively. The vehicle was assumed to be experiencing a random acceleration which was modeled as a first-order Gauss-Markov process. The acceleration vector components were assumed uncorrelated and spherically distributed with a standard deviation of 1.8% of the nominal thrust-acceleration level and a correlation time of 5 days. At the start of the approach phase, the standard deviations of the state errors were chosen to represent the relative comet-spacecraft state uncertainty due to the comet ephemeris uncertainty. The values selected were 30,000 km in position and 11.57 m/s (1000 km/day) in velocity.

The guidance algorithm was employed in four different modes of operation:

(1) Every day a single control correction δu_0 was computed and applied for a period of 1 day. The final time was held fixed.

(2) Every 2 days, two control corrections, δu_0 and δu_1, were computed and applied for successive periods of 1 day each. The final time was held fixed.

(3) Same as (1), but with the final time variable.

(4) Same as (2), but with the final time variable.

The control variables for the analysis were the components of the thrust-acceleration vector, and the limiting values on their standard deviations were set at 10% of the nominal thrust-acceleration level.

The guidance scheme performance for modes 1 and 2 is shown by Figs. 1 and 2, which give the final rms position and velocity errors as a function of the weighting matrix S and the time prior to nominal rendezvous at which guidance was initiated. For this rendezvous mission, S is a 6 x 6 diagonal matrix with the first 3 diagonal elements set to S_p (the position weight) and the second 3 set to S_v (the velocity weight). Figs. 3 and 4 give the final rms errors for modes 3 and 4 as a function of S for a 30-day guidance initiation time; the corresponding mode 1 and 2 results are repeated for comparison purposes. Figs. 5 and 6 show the mode 2 and 4 performance when the constraint limit on one of the controls is reduced to 5% of the nominal thrust-acceleration; the corresponding results for the original control constraints are also included for comparison.

B

The effectiveness of the guidance scheme can be judged simply by determining whether the final state errors can be reduced below some acceptable limiting values. For the Encke rendezvous, those limits were set at 1000 km and 4 m/s. It is clear from Figs. 1 and 2 that there exist some values of S for which successful rendezvous with modes 1 and 2 is possible, provided guidance is initiated 40 days and 35 days, respectively, prior to nominal encounter. Moreover, the trends indicated by Figs. 3 and 4 suggest that for a variable-final-time mode, those initiation times may be reduced almost 5 days with little loss in performance. Figs. 5 and 6, on the other hand, show that tighter control constraints degrade guidance effectiveness, and consequently earlier initiation will be required to achieve rendezvous. Again, permitting final time to vary improves performance.

General conclusions from the guidance scheme evaluation include the following:

(1) Because trajectory correction with a low-thrust vehicle requires a significant time period, early guidance initiation is required in order to obtain small final errors.

(2) The weighting matrix S provides a trade-off in final position and velocity errors; the effect of the trade-off is more pronounced for the later initiation times, which have the greater overall final errors.

(3) The two-correction policy is better than the single-correction one for rendezvous, since 6 rather than 3 control variables are being used to control the 6 final state deviations. But for a flyby ($S_p = 1$, $S_v = 0$), two corrections offer no improvement over one.

(4) The variable-final-time mode of operation significantly improves performance by providing an additional control variable.

V. CONCLUDING REMARKS

This memorandum described and evaluated a low-thrust guidance algorithm that has been designed to allow for actual mission constraints. It is a linear perturbation scheme with the following primary features:

(1) Violations in the terminal mission constraints are minimized.

(2) Control corrections are required to occur early in the trajectory.

(3) Control corrections are constant over specified time intervals.

(4) Control corrections are explicitly bounded.

By having these features, the algorithm avoids the controllability and implementation difficulties associated with other guidance laws.

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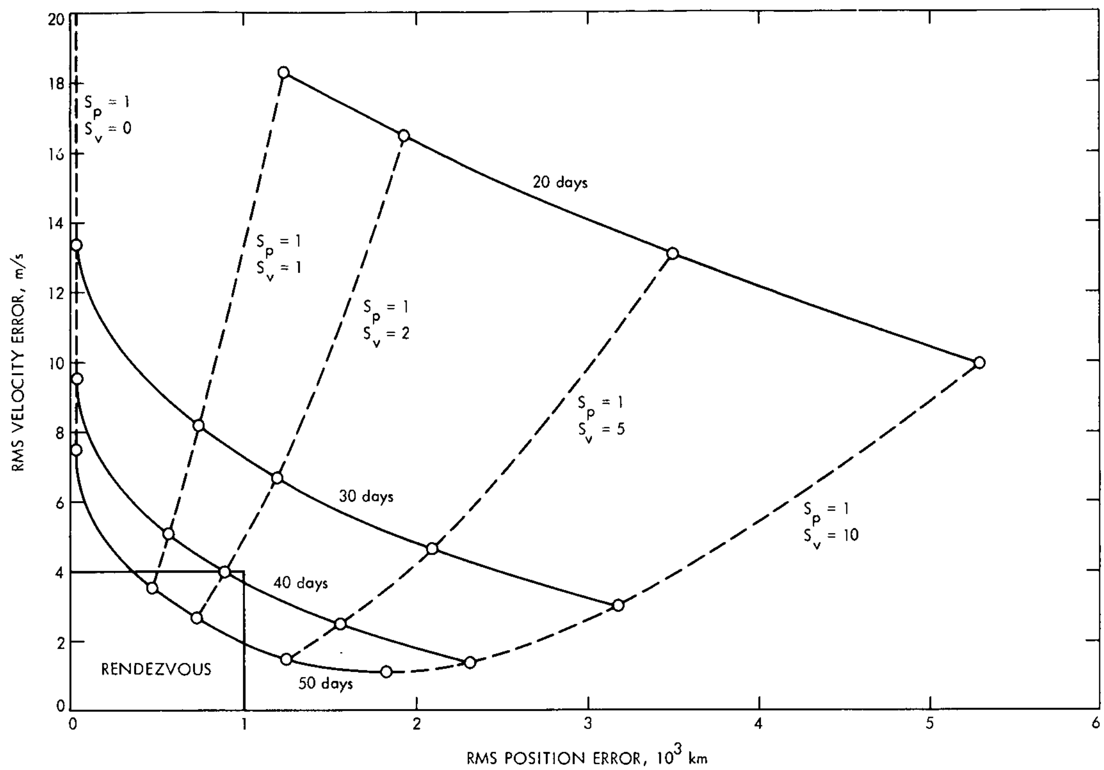

Fig. 1. RMS final state errors vs flight time and weighting parameters, mode 1 guidance

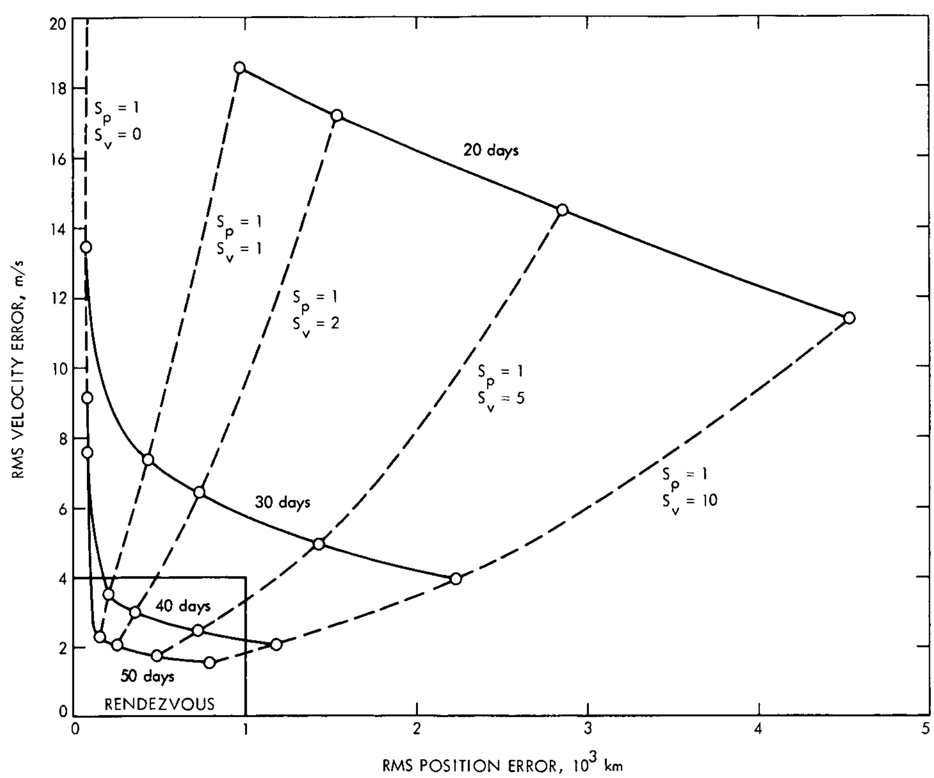

Fig. 2. RMS final state errors vs flight time and weighting parameters, mode 2 guidance

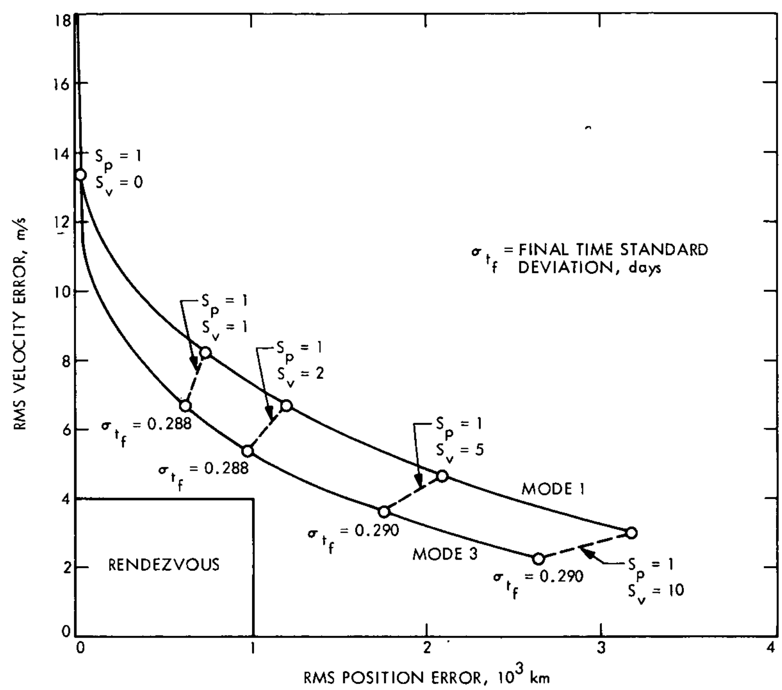

Fig. 3. RMS final state errors vs weighting parameters for 30-day rendezvous, mode 3 guidance

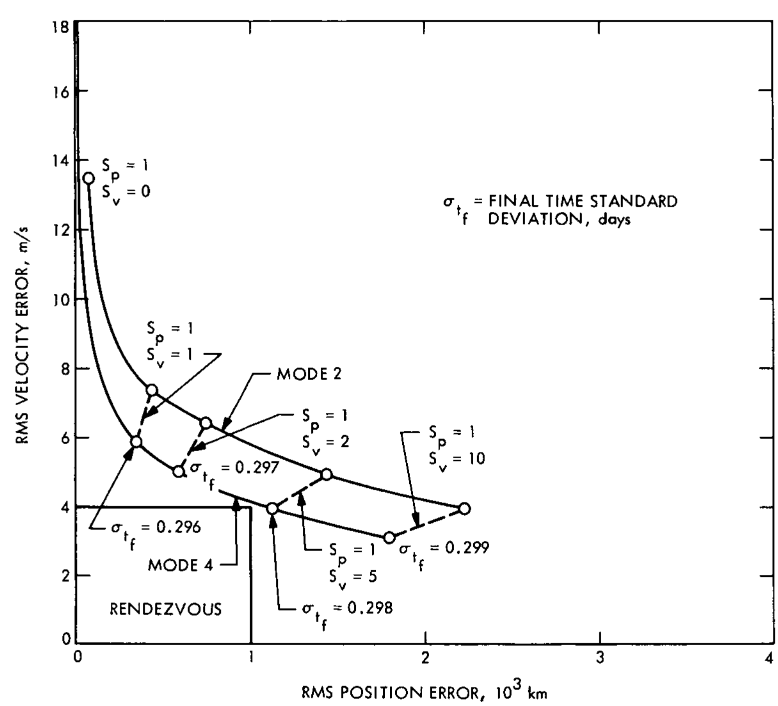

Fig. 4. RMS final state errors vs weighting parameters for 30-day rendezvous, mode 4 guidance

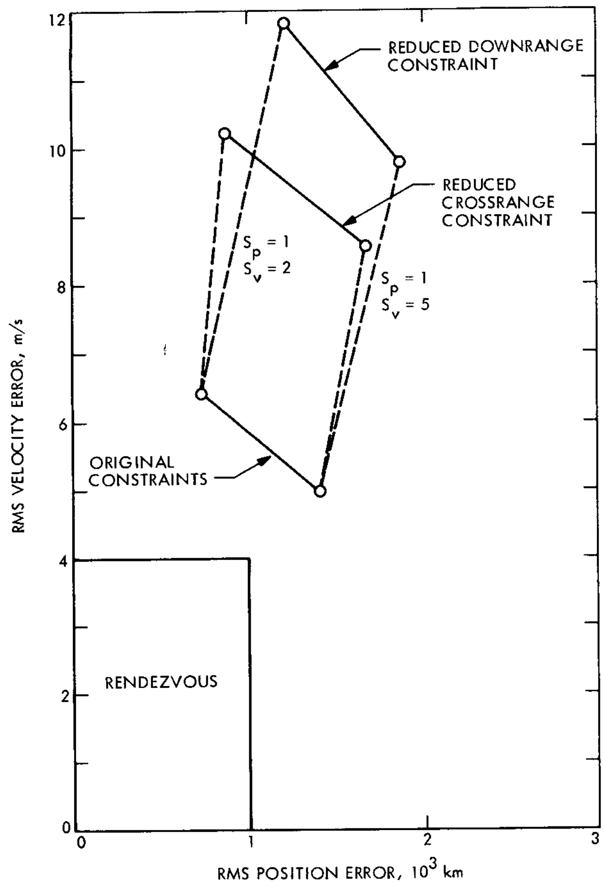

Fig. 5. RMS final state errors with reduced control constraints for 30-day rendezvous, mode 2 guidance

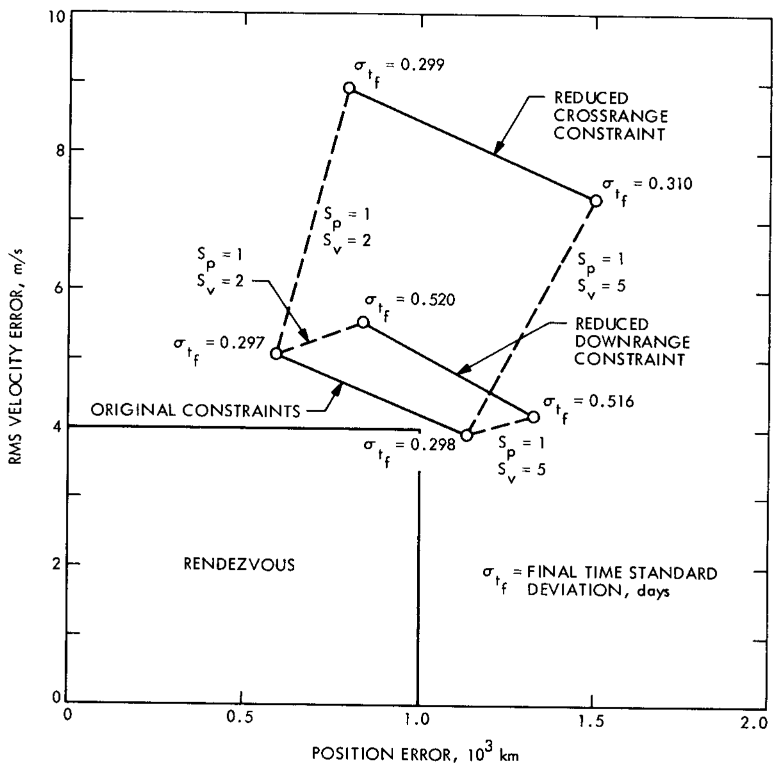

Fig. 6. RMS final state errors with reduced control constraints for 30-day rendezvous, mode 4 guidance

APPENDIX A
THE REDUCED J FUNCTION AND THE FINAL TIME VARIATION

The function J (Eq. 10) can be written in the form

$$J = (\delta x_f^*)^T \tilde{A} (\delta x_f^*)$$

where

$$\tilde{A} = E^T A E$$

$$E = I - \left[(\dot{x}_f^* - \dot{y}_f^*)^T A (\dot{x}_f^* - \dot{y}_f^*) \right]^{-1} (\dot{x}_f^* - \dot{y}_f^*)(\dot{x}_f^* - \dot{y}_f^*)^T A$$

Let the time derivative of the terminal constraints, evaluated at the end of the nominal trajectory, be denoted by the vector q,

$$q = \psi_x \left[x^*(t_f^*) - y(t_f^*) \right] (\dot{x}_f^* - \dot{y}_f^*)$$

and let the vector p be defined as

$$p = \psi_x \left[x^*(t_f^*) - y(t_f^*) \right] E \delta x_f^*$$

In a straightforward manner, it may be shown that when S is the identity, p is orthogonal to q:

$$q^T p = (\dot{x}_f^* - \dot{y}_f^*)^T \psi_x^T \left[x^*(t_f^*) - y(t_f^*) \right] \psi_x \left[x^*(t_f^*) - y(t_f^*) \right] E \delta x_f^*$$

$$= (\dot{x}_f^* - \dot{y}_f^*)^T A E \delta x_f^* = 0 \cdot \delta x_f^* = 0$$

The vector p represents the constraint violations at t_f^* which are orthogonal to the nominal time derivative of the constraints. For a suitable choice of units for e_f, the matrix S may always be selected as the identity; consequently, it is clear that minimization of the reduced form of J as a function of $\widehat{\delta u}$ simply yields the smallest possible p, i.e., $J = p^T p$. The final time variation δt_f is then used to null out remaining terminal errors along the direction q.

APPENDIX B
THE WEIGHTING MATRIX W

The primary functions of the weighting matrix defined by Eqs. (12-14) are

(1) To guarantee a unique minimizing control $\delta\hat{u}$.

(2) To permit the minimizing control to be computed in a straightforward and systematic manner.

(3) To guarantee that the control $\delta\hat{u}$ can always be expressed in feedback form, i.e., Eq. (15).

For the particular choice of W, it can be shown that in the unconstrained case, the control which minimizes the function $\hat{J}$ (Eq. 11) also minimizes the function J (Eq. 10). In order to verify this property, the following lemma is required.

LEMMA: Let the m x m orthogonal matrix Q be the diagonalizing transformation for the matrix $(\Gamma^T \tilde{A} \Gamma)$, where $\tilde{A}$ is an n x n positive semidefinite symmetric matrix and Γ is an arbitrary n x m matrix, i.e., $Q\Gamma^T \tilde{A} \Gamma Q^T = D$. The matrix D is an m x m diagonal. Let $\eta = Q\Gamma^T \tilde{A} \xi$ be an m vector where ξ is an arbitrary n vector. Then if element D_{ii} of matrix D is zero, component η_i of η is also zero.

Proof: Let the orthogonal matrix V diagonalize $\tilde{A}$, then $V\tilde{A}V^T = \overline{A}$, where $\overline{A}$ is diagonal. It follows that $(Q\Gamma^T V^T)\overline{A}(V\Gamma Q^T) = D$. Let p_{ij} be an element of matrix $Q\Gamma^T V^T$, then

$$\sum_{j=1}^{n} p_{ij}^2 \overline{A}_{jj} = D_{ii}$$

for each i.

Since $\tilde{A}$ is positive semidefinite, $\overline{A}_{jj} \geq 0$. Consequently, if $D_{ii} = 0$, then $p_{ij}^2 \overline{A}_{jj} = 0$ for all j, or $p_{ij} \overline{A}_{jj} = 0$ for all j. The vector η may be written as $\eta = (Q\Gamma^T V^T)\overline{A}(V\xi)$. The i'th component then is

$$\eta_i = \sum_{j=1}^{n} p_{ij} \overline{A}_{jj} r_j$$

where r_j is a component of the vector $r = V\xi$. If $D_{ii} = 0$, $p_{ij}\overline{A}_{jj} = 0$ for all j; consequently, $\eta_i = 0$ as required.

With the use of the above lemma, it is easy to establish the following property of the unconstrained control minimizing $\hat{J}$.

Property: The control $\widehat{\delta u}$ which minimizes the function

$$\hat{J} = \left[\xi + \Gamma\widehat{\delta u}\right]^T \widetilde{A} \left[\xi + \Gamma\widehat{\delta u}\right] + \widehat{\delta u}^T W \widehat{\delta u}$$

also minimizes

$$J = \left[\xi + \Gamma\widehat{\delta u}\right]^T \widetilde{A} \left[\xi + \Gamma\widehat{\delta u}\right]$$

Proof: Let the vector w be defined as $w = Q\widehat{\delta u} + (D + B)^{-1}\eta$, then $\widehat{\delta u} = Q^T\left[w - (D + B)^{-1}\eta\right]$. Substitution of this expression into the function J yields

$$J = \xi^T \widetilde{A}\xi + w^T Dw + 2w^T\left[I - D(D + B)^{-1}\right]\eta$$

$$-\eta^T (D + B)^{-1}\left[2(D + B) - D\right](D + B)^{-1}\eta$$

If $D_{ii} = 0$, it follows by definition that $B_{ii} > 0$, and by lemma that $\eta_i = 0$; also, if $D_{ii} > 0$, $B_{ii} = 0$. Consequently, J may be reduced to the form

$$J = w^T Dw + \xi^T \widetilde{A}\xi - \eta^T (D + B)^{-1} D(D + B)^{-1}\eta$$

A minimum of J clearly occurs at $w = 0$; that is, $Q\widehat{\delta u} + (D + B)^{-1}\eta = 0$, which implies $\widehat{\delta u} = -Q^T(D + B)^{-1}\eta$. But this control is the one obtained by minimizing $\hat{J}$; therefore, the control which minimizes $\hat{J}$ also minimizes J.

In the constrained case, $\widehat{\delta u}$ may not possess the above property. However, by selecting the elements of B sufficiently small when forming W, the minimum of J can be approached to any desired accuracy. Let $\widehat{\delta u}$ minimize $\hat{J}$ subject to the constraint $-\hat{\alpha} \le \widehat{\delta u} \le \hat{\alpha}$, then it is necessary that

$$(\Gamma^T \widetilde{A}\Gamma + W)\widehat{\delta u} + \Gamma^T \widetilde{A}\xi = -\Lambda\widehat{\delta u} \qquad \text{(B-1)}$$

where Λ is the diagonal matrix of non-negative Lagrange multipliers. For B sufficiently small, it is clear there exists a non-negative diagonal matrix $\hat{\Lambda}$ such that

$$\Lambda\widehat{\delta u} + W\widehat{\delta u} = \hat{\Lambda}\widehat{\delta u} + \epsilon \tag{B-2}$$

where ϵ is a small residual error vector. Therefore, by substitution of Eq. (B-2) into Eq. (B-1),

$$(\Gamma^T\tilde{A}\Gamma)\widehat{\delta u} + \Gamma^T\tilde{A}\xi \approx -\hat{\Lambda}\widehat{\delta u}$$

but this is simply the condition for the minimization of J. Consequently, $\widehat{\delta u}$ is an approximation to the constrained minimizing control for J. For computational purposes, the elements of B should be small with respect to those of D, but should be large enough to avoid numerical difficulties in the computation of the inverse matrix $(\Gamma^T\tilde{A}\Gamma + W)^{-1}$.

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