N73-18679

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EFFECTS OF BACKLASH AND DEAD BAND ON TEMPERATURE CONTROL OF THE PRIMARY LOOP OF A CONCEPTUAL NUCLEAR BRAYTON SPACE POWERPLANT

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1973

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM X-2754		

4. Title and Subtitle	5. Report Date
EFFECTS OF BACKLASH AND DEAD BAND ON TEMPERATURE CONTROL OF THE PRIMARY LOOP OF A CONCEPTUAL NUCLEAR BRAYTON SPACE POWERPLANT	March 1973
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Edward J. Petrik	E-7276
	10. Work Unit No.
9. Performing Organization Name and Address	503-25
Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135	11. Contract or Grant No.
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Memorandum
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

An analytical study was made of the stability of a closed-loop liquid-lithium temperature control of the primary loop of a conceptual nuclear Brayton space powerplant. The operating point was varied from 20 to 120 percent of design. A describing-function technique was used to evaluate the effects of temperature dead band and control coupling backlash. From the system investigation, it was predicted that a limit cycle will not exist with a temperature dead band, but a limit cycle will exist when backlash is present. The results compare favorably with a digital computer simulation.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Nuclear space power reactor Nonlinear control Temperature control Control system	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	22	$3.00

* For sale by the National Technical Information Service, Springfield, Virginia 22151

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SUMMARY

An analytical investigation was made of the stability of a closed-loop liquid-lithium temperature control of the primary loop of a conceptual nuclear Brayton space powerplant. In this conceptual system, the lithium-cooled primary loop is coupled directly to an inert-gas power conversion loop by a heat exchanger. The reactor control drum is driven by a stepping motor.

For this investigation, the operating point of the nuclear Brayton space powerplant was varied parametrically from 20 to 120 percent of design. Describing functions are used to represent temperature dead band and control coupling backlash. Transfer functions are used to represent the nuclear Brayton space powerplant.

From the investigation, it was determined that (1) the system is stable over a large operating range, (2) a limit cycle will not exist with a temperature dead-band control, (3) a limit cycle exists if either friction-controlled backlash or inertia controlled backlash is present in the control loop.

The effects of temperature dead band and backlash were also investigated by using a digital computer to simulate the nuclear Brayton space powerplant. The results compare favorably with the analytical results.

INTRODUCTION

In the coming years, the electric power requirements for the nation's projected space programs will continue to increase. To meet these projected requirements, the Lewis Research Center has been participating in a technology program aimed at the design of a high-power nuclear Brayton space powerplant. The heat source being considered is a compact, fast-spectrum nuclear reactor. The design thermal output of the reactor is 2.17 megawatts, and the design operating lifetime for the reactor and powerplant is set at 50 000 hours. For this study it is assumed that the reactor exit coolant temperature is maintained by rotation of control drums to effect changes in reactivity. A stepping motor is used to drive the reactor control drum.

One area to be studied in the overall design of this powerplant is the design of the controls. Therefore, an analysis has been conducted to determine the operational characteristics of this system and its control requirements. An analytical study of the operational characteristics of the primary loop of the conceptical nuclear Brayton space powerplant is reported in reference 1. In this reference, nonlinear differential equations representing the dynamics of the primary loop are presented. Also included is the transient response of the primary loop to step disturbances in reactivity, lithium flow rate in the primary loop, and argon flow rate in the power conversion loop. In reference 2, the dynamics of the primary loop are expressed in the form of transfer functions. The transfer functions were determined from the equations given in reference 1 by being linearized about given operating points.

The purpose of the work described in this report is to predict the stability of a closed-loop liquid-lithium temperature control of the primary loop of the nuclear Brayton space powerplant when either temperature dead band or control coupling backlash is present. The method of describing function analysis is applied to dead band and backlash for compatibility with the transfer-function representation of the primary loop. The operating point was varied parametrically from 20 to 120 percent of design power. From the study, the existence of limit cycles was determined and is discussed in this report. Also discussed are results obtained from a computer simulation for comparison.

SYSTEM DESCRIPTION

A simplified diagram of the conceptual nuclear Brayton space powerplant is shown in figure 1. The primary loop is coupled to the gas power conversion loop by a counterflow heat exchanger. The coolant in the primary loop is liquid lithium, and the working fluid in the gas power conversion loop is argon. The design point operating conditions for the primary loop of the nuclear Brayton space powerplant are listed in table I.

Reactor Description

The reactor is of the fast-spectrum type and is designed for 50 000 hours of operation at a power level of 2.17 megawatts. The reactor core has 253 cylindrical fuel pins, each with a diameter of 1.91 centimeters and a length of 37.6 centimeters. The pins are made of uranium nitride and are clad with a tantalum alloy with a tungsten liner between fuel and cladding. The interior of each fuel pin has a central void. The pins in the assembly are cooled by lithium, which flows through annular flow passages formed by the outside surfaces of the fuel pins and the inside surfaces of surrounding tantalum tubes. The tantalum tubes have an inside diameter of 2.11 centimeters. Six control drums are located at the periphery of the core, as shown in figure 2. The reactor power is regulated by the six control drums which, when rotated, move fuel (or poison) in or out of the core region.

Heat-Exchanger Description

The heat exchanger described in this section should not be considered as a design configuration. Rather, it is only one of many possible conceptual configurations capable of transferring the heat load from the primary loop to the gas power conversion loop.

A counterflow shell-and-tube type heat exchanger was assumed for this investigation. The conceptual heat exchanger has 331 circular tubes; the tubes have an inside diameter of 1.91 centimeters, a wall thickness of 0.127 centimeter, and a length of approximately 2.44 meters.

Argon gas is assumed to flow inside the tubes, and lithium flows countercurrently in the shell. The heat-exchanger tubes are arranged in a hexagonal-cross-sectional array

Actuator System Description

The actuator system consists of the reactor control drum, coupling, and actuator. The control drum has a moment of inertia of 0.47 joule-second^2 and weighs 159 kilograms. The control drum friction is expected to change during the 50 000-hour operating lifetime, and its value is unknown at the present time. Control drum positioning to within 0.1° is considered adequate. A 0.1° change represents a reactivity change of approximately 0.2 cent. This is expected to produce a peak transient change in reactor power of less than 1 percent and a steady-state change of less than 0.4 percent (ref. 1).

The actuator under consideration is an electric stepping motor, which rotates in discrete increments or steps. An investigation of the performance of a stepping motor as a control drum drive for a space power reactor is given in reference 3.

The coupling which connects the actuator to the control drum may be a source of backlash. It is designed to transmit torque to the control drum through the pressure vessel while providing an absolute seal for the lithium coolant.

ANALYSIS

This section is devoted to a brief discussion of describing-function analysis used to predict the stability of a system which has a dead band or backlash.

Description-Function Techniques

Frequency-respones techniques and the use of transfer functions are valuable tools in the analysis and synthesis of linear systems. It is desirable to extend these techniques to the analysis and synthesis of systems with nonlinear components such as backlash and dead band. However, to extend these techniques, it is necessary to approximate the effects of the nonlinear component by using a linear approximate transfer function or describing function.

Specifically, the describing function assumes that a pure sinusoidal signal of constant amplitude and constant frequency is applied to the input of the nonlinear component. After steady-state conditions are obtained, the output waveshape of the nonlinear component is obtained. This output waveshape is represented by its Fourier series by assuming that there are no zero-frequency components and no subharmonics. Thus, the fundamental frequency term in the Fourier series has the same frequency as the input signal but may differ in amplitude and phase. The describing function is then defined as the complex ratio of the fundamental term of the output to the input sinusoid.

When the feedback control system has been reduced to a linear system with a single nonlinear element present (such as backlash), the stability may be determined by using the describing-function method. Any of the normal graphical methods may be applied, that is, root locus, polar plot, Bode diagram, and Nichol's chart. The choice of graphical method depends in part on the purposes of the analysis. For this report, the gain-phase plot (Nichol's chart) is used for stability analysis since the gain-phase plot is very practical when the representation of the nonlinear element is simply a gain variation with phase shift such as a dead band or backlash.

From the gain-phase plot, limit cycles are determined from the intersection between the describing-function curve and the transfer-function curve. This is discussed in detail in reference 4.

Since the describing function is derived under the assumption that the input to the nonlinear element is sinusoidal and that the higher harmonics generated are neglected, the describing-function method will give accurate results only when these basic assumptions are justified. Thus, even if the existence of a limit cycle is correctly predicted by describing-function methods, the amplitude and frequency may be appreciably in error.

It should be noted that, in a situation where it becomes desirable to compensate a system to eliminate an unwanted limit cycle, the procedure may lead to erroneous results. This effort is discussed in detail in reference 5.

Describing Functions for Backlash and Dead Band

The general definition of a describing function is given in the previous section. The describing function for backlash and dead band is now presented in analyical and graphical form to indicate the general type of behavior to be expected from the nonlinear element when it is inserted into a linear system. The more general case of backlash in which inertia and friction are present simultaneously is not considered in this report. But it is assumed that either friction is present and inertia is negligible or inertia is large and friction is negligible.

Friction-controlled backlash. - For simple friction-controlled backlash the output member remains in contact with the input member until the input velocity becomes zero. Then the output member stands still until the backlash is taken up on the other side, at which time it is assumed that the output member instantaneously starts moving with the same velocity as the input member. That is, the collision takes place without bouncing. Now the output member follows the input member until the input velocity again becomes zero.

The describing function for friction-controlled backlash is derived in reference 6 and is written as

$$N(\varphi) = \frac{(2\varphi - \varphi^2)^{1/2}(1 - \varphi) + \pi - \cos^{-1}(1 - \varphi)}{\pi} + \frac{j(\varphi^2 - 2\varphi)}{\pi}$$

where

φ b/a

b backlash spacing

a amplitude of input sinusoid

Numerical values of $N(\varphi)$ are given in table II, and $N(\varphi)$ is plotted on a gain-phase plot in figure 3.

Inertia-controlled backlash. - When the friction on the output member is negligible, the backlash system is called inertia-controlled. The output member of such a system will coast with any initial velocity until contact is made with an input member. Now the output member will remain in contact with the input member as long as the acceleration is in the direction to keep the backlash spacing closed. When the acceleration becomes zero and a velocity maximum is reached, the output member will leave the input member and coast. The coasting will be at constant velocity since friction is assumed to be zero. When the coasting period is complete and driving is resumed, it is assumed that the output member instantaneously accelerates to the velocity of the input member and that there is no bouncing.

The describing function for inertia-controlled backlash is derived in reference 6 and is written as

$$N(\varphi) = \frac{(2 - \sin \varphi)\cos \varphi + (2\varphi + \pi - 2\varphi)\sin \varphi - \varphi + \frac{\pi}{2}}{\pi} + \frac{j\left[(2\varphi + \pi - 2\varphi)\cos \theta - 2 \sin \theta - 3 + \sin^2 \theta\right]}{\pi}$$

for

$$\theta = \varphi + \cos \theta - \frac{\pi}{2}$$

$$\varphi < \pi$$

and

$$N(\varphi) = 4 \sin \theta \cos \theta - j4 \sin \theta \sin \theta$$

for

$$\pi \leq \varphi < 3.72$$

Numerical values of $N(\varphi)$ are given in table III, and $N(\varphi)$ is plotted on a gain-phase plot in figure 3.

Dead band. - The describing function for the dead band is derived in reference 4 and is written as

$$N(R) = \frac{\pi - 2 \sin^{-1} R - R(1 - R)^{1/2}}{\pi}$$

where

R b/2a

b dead band spacing

a amplitude of input sinusoid

Numerical values of N(R) are given in table IV.

RESULTS

In this section, the stability of a reactor exit lithium temperature control is determined. A describing-function technique is used when a dead band or backlash is present. Some of the results are compared with a digital computer simulation of the nuclear Brayton space powerplant (NBSP) operating at design conditions (table I).

Consider the system in figure 4. In this figure the linear plant $G(s)$ is the transfer-function representation of the NBSP obtained in appendix A, and K_c is some controller gain. In order to evaluate the closed-loop stability, the transfer function $K_cG(s)$ is plotted on a gain-phase plot in figure 5 for design conditions and unity controller gain. The gain margin is the amount by which the system gain can be allowed to increase before the system reaches instability. Phase margin is the amount by which the phase angle can be allowed to decrease before the system reaches instability.

At operating conditions other than design, the gain-phase plot is very similar to figure 5 except for the amount of gain and phase margin. Figure 6 is a plot of the gain and phase margin of the liquid-lithium temperature control of the NBSP operating over a range of 20 to 120 percent of design power. Controller gain is unity. At 20 percent of design power, the gain margin and phase margin are nearly zero. However, as the power increases, the gain margin increases. The phase margin increases rapidly and peaks at about 50 percent of design power, after which the phase margin decreases slowly. Since the gain margin and the phase margin are not negative, the NBSP is stable over the range considered.

Now consider the system in figure 7. Again the linear plant G(s) is the transfer-function representation of the NBSP obtained in appendix A. And K_c is some controller gain. However, -1/N(s) is the describing-function representation of either dead band or backlash, as derived in the section ANALYSIS. In order to evaluate system stability, gain-phase plots of the transfer function and describing function will be used.

Dead-Band Control

Figure 8 is a gain-phase plot for design conditions with a temperature dead band and unity controller gain. Since the transfer function and describing function do not intersect, a limit cycle will not exist. This was also predicted by using a digital computer simulation of the NBSP, as described in reference 1.

Of special significance in figure 8 is the upper protrusion of the transfer function, as indicated by point A. It should be pointed out that the difference in phase angle between the transfer function and the describing function is only about 4°. Figure 9 is a plot of this difference in phase angle as a function of percent of design operating point. This difference in phase angle is relatively constant over the range of 50 to 120 percent of design operation. Below 50 percent of design operation, this difference decreases rapidly. Therefore, the possibility of a limit cycle will exist even if not predicted, especially if the phase angle is increased by any control compensation.

Backlash

Figure 10 is a gain-phase plot for the temperature control of the NBSP when backlash is present. The system is at design operation and unity controller gain. The describing functions for friction-controlled backlash and inertia-controlled backlash are both plotted in figure 10, since the system will operate somewhere between these limits.

In figure 10, the describing functions and transfer function intersect. Therefore, a limit cycle will exist. The amplitude of the limit cycle is a function of the backlash spacing and controller gain for both friction-controlled backlash and inertia-controlled backlash. It is given by the equation

$$\text{amplitude} = \frac{\text{backlash spacing}}{\varphi}$$

The frequency of the limit cycle is determined from the intersection of the transfer function and the describing function.

Thus, with the NBSP operating at design and with unity controller gain, a stable limit cycle is predicted at a frequency of 0.021 hertz and a φ of 0.48 for friction-controlled backlash. For inertia-controlled backlash a limit cycle is predicted at a frequency of 0.026 hertz and a φ of 0.80.

Of special significance in figure 10 is the curve of inertia-controlled backlash, since it is approximately parallel to the zero-decibel axis. Thus, the transfer function of the NBSP will always cross the describing function when inertia-controlled backlash is present. Therefore, if inertia-controlled backlash is present, the system will have a limit cycle regardless of any type of control compensation.

Figure 11 is a plot of φ and frequency of the limit cycle as functions of controller gain for friction-controlled backlash. And figure 12 is a plot of φ and frequency for inertia-controlled backlash. The NBSP is operating at design. In both figures, as the control gain is increased, the frequency of the limit cycle increases. Also, the amplitude of the limit cycle first decreases and then increases (i.e., φ increases and then decreases until reaching zero). This behavior is due to the large upper protrusion of the gain-phase plot, as indicated in figure 10.

Now if the control gain is held constant, φ and frequency of the limit cycle can be plotted as functions of the design point operation, as shown in figures 13 and 14. For this case the controller gain was set to unity. With friction-controlled backlash present, frequency and amplitude of the limit cycle increase as operation approches design. For inertia-controlled backlash, the frequency of the limit cycle is relatively constant. However, φ approches zero (amplitude approches infinity) as the design operating point approaches 20 percent.

Finally, the results were compared with a digital computer simulation of the nonlinear representation of the NBSP. The NBSP is set to operate at design with friction-controlled backlash present in the penetration device. The controller gain was set at 1.35, and the backlash spacing was 1.11 K.

For these conditions, the describing-function technique predicts that a limit cycle will exist with an amplitude of 1.78 K and a frequency of 0.024 hertz. From the simulation of the nonlinear model of the NBSP, a limit cycle also existed. The amplitude of the limit cycle was 1.0 K at a frequency of 0.019 hertz.

The difference between the analytical and computed results would be expected when linear analysis is applied to a nonlinear system. Thus, with the NBSP simulation the nonlinearities generated harmonics. The presence of these harmonics, which the describing technique neglects, introduces fundamental phase shifts which lead to errors. Despite these differences, the describing-function technique adequately predicts the existence of limit cycles for the NBSP.

SUMMARY OF RESULTS

An investigation was made of a closed-loop liquid-lithium temperature control of the primary loop of a conceptual nuclear Brayton space powerplant. The system was linearized, and the effects of temperature dead band and control coupling backlash are considered in the investigation by using the describing-function technique. The analysis predicted

1. Without a dead band or backlash, the liquid-lithium temperature control of the nuclear Brayton space powerplant is stable over the range of 20 to 120 percent of design.

2. A limit cycle does not exist for the liquid-lithium temperature control when a temperature dead-band controller is inserted into the system.

3. A limit cycle will exist when either friction-controlled backlash or inertia-controlled backlash is present in the actuator system.

A digital computer simulation of the nuclear Brayton space powerplant with friction-controlled backlash compares favorably with results predicted by linear analysis.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 21, 1972,
503-25.

APPENDIX - SYSTEM TRANSFER FUNCTIONS

The transfer function conveniently relates the response of a system to a forcing function with the aid of appropriate Laplace transforms. Since the equations representing the nuclear Brayton space powerplant are nonlinear differential equations, the equations are simplified and linearized about an operating point. The transfer functions are then obtained from the linear differential equations for small-step input disturbances. This procedure is used in reference 2 to obtain transfer functions for the NBSP operating at design.

The equation for the transfer functions of the NBSP is

$$G(s) = \frac{\text{reactor exit lithium temperature}}{\text{reactivity input}}$$

and factored values of the transfer functions for various operating points are given in the following table.

Operating point, percent of design	Gain, K_c	Value of zeros	Value of poles
		Laplace transform variable, s	
120	3.56	-1.4 -3.88 -1.16×10^{-1} -3.12×10^{-1} -1.27×10^{-2} -3.17×10^{-2} -2.327×10^{-1} $-3.429\times10^{-1}\pm j1.98\times10^{-1}$	0 -3.79 -8.37 -6.81×10^{-1} -1.24 $-2.85\times10^{-1}\pm j4.81\times10^{-1}$ -6.0×10^{-2} -1.93×10^{-1} $-1.90\times10^{-2}\pm j2.54\times10^{-2}$ -1.28×10^{-2}
100	2.96	-1.4 -3.88 -1.16×10^{-1} -3.12×10^{-1} -1.27×10^{-2} -3.17×10^{-2} -1.98×10^{-1} $-2.86\times10^{-1}\pm j1.65\times10^{-1}$	0 -3.79 -7.69 -6.28×10^{-1} -1.24 $-2.31\times10^{-1}\pm j4.06\times10^{-1}$ -1.93×10^{-1} -6.25×10^{-2} $-1.58\times10^{-2}\pm j2.34\times10^{-2}$ -1.29×10^{-2}

Operating point, percent of design	Gain, K_c	Value of zeros	Value of poles
		Laplace transform variable, s	
80	2.37	-1.4 -3.88 -1.16×10^{-1} -3.12×10^{-1} -1.27×10^{-2} -3.17×10^{-2} -1.63×10^{-1} $-2.29\times10^{-1}\pm j1.32\times10^{-1}$	0 -3.79 -7.02 -5.58×10^{-1} -1.24 $-1.80\times10^{-1}\pm j3.30\times10^{-1}$ -6.45×10^{-2} -1.94×10^{-1} $-1.28\times10^{-2}\pm j2.11\times10^{-2}$ -1.31×10^{-2}
60	1.78	-1.4 -3.88 -1.16×10^{-1} -3.12×10^{-1} -1.27×10^{-2} -3.17×10^{-2} -1.27×10^{-1} $-1.71\times10^{-1}\pm j9.90\times10^{-2}$	0 -3.79 -6.36 -4.66×10^{-1} -1.24 $-1.32\times10^{-1}\pm j2.53\times10^{-1}$ -6.62×10^{-2} -1.94×10^{-1} $-9.74\times10^{-3}\pm j1.85\times10^{-2}$ -1.33×10^{-2}
40	1.18	-1.4 -3.88 -1.16×10^{-1} -3.12×10^{-1} -1.27×10^{-2} -3.17×10^{-2} -8.92×10^{-2} $-1.14\times10^{-1}\pm j6.6\times10^{-2}$	0 -3.79 -5.72 -1.24 -1.95×10^{-1} -3.48 $-8.68\times10^{-2}\pm j1.74\times10^{-1}$ -1.37×10^{-2} -6.75×10^{-2} $-6.72\times10^{-3}+j1.52\times10^{-2}$
20	0.59	-1.4 -3.88 -1.16×10^{-1} -3.12×10^{-1} -1.27×10^{-2} -3.17×10^{-2} -4.91×10^{-2} $-5.71\times10^{-2}\pm j3.30\times10^{-2}$	0 -3.79 -5.11 -1.24 $-1.96\times10^{-1}\pm j1.06\times10^{-2}$ $-4.34\times10^{-2}\pm j9.18\times10^{-2}$ -6.81×10^{-2} -1.42×10^{-2} $-3.60\times10^{-3}\pm j1.08\times10^{-2}$

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TABLE I. - DESIGN-POINT OPERATING CONDITIONS

Reactor thermal power, Q, MW	2.17
Lithium temperature at reactor inlet (or heat-exchanger exit), K	1167
Lithium temperature at reactor exit (or heat-exchanger inlet), K	1222
Lithium flow rate, kg/sec	9.39
Argon temperature at heat-exchanger inlet, K	867
Argon temperature at heat-exchanger exit, K	1144
Argon flow rate, kg/sec	14.88

TABLE II. - DESCRIBING FUNCTION FOR

FRICTION-CONTROLLED BACKLASH

Describing function, $N(\varphi)$			Negative inverse describing function, $-1/N(\varphi)$	
Limit cycle amplitude, φ (a)	Amplitude, dB	Phase, deg	1/Amplitude, dB	Phase, deg
0	0	0	0	-180.00
.2	-.40	-6.89	.40	-173.12
.4	-1.10	-13.36	1.10	-166.64
.6	-2.00	-19.68	2.00	-160.32
.8	-3.14	-26.00	3.14	-154.00
1.0	-4.54	-32.48	4.54	-147.52
1.2	-6.33	-39.29	6.33	-140.71
1.4	-8.69	-46.66	8.68	-133.34
1.6	-12.09	-55.05	12.09	-124.95
1.8	-18.00	-65.57	18.00	-114.43

[a] $\varphi = b/a$, where b is backlash spacing and a is amplitude of input sinusoid.

TABLE III. - DESCRIBING FUNCTION FOR

INERTIA-CONTROLLED BACKLASH

Describing function, $N(\varphi)$			Negative inverse describing function, $-1/N(\varphi)$	
Limit cycle amplitude, φ (a)	Amplitude, dB	Phase, deg	1/Amplitude, dB	Phase, deg
0	0	0	0	-180.00
.5	1.35	-13.54	-1.35	-166.46
1.0	1.82	-28.34	-1.82	-151.66
1.5	2.01	-42.89	-2.01	-137.11
2.0	2.08	-57.28	-2.08	-122.72
2.5	2.10	-71.62	-2.10	-108.38
3.0	2.10	-85.94	-2.10	-94.06
3.5	1.89	-102.05	-1.89	-77.50

[a] $\varphi = b/a$, where b is backlash spacing and a is amplitude of input sinusoid.

TABLE IV. - DESCRIBING FUNCTION FOR

DEAD BAND

Describing function, N(R) (a)		1/Amplitude for negative inverse describing function 1/N(R), dB (b)
Limit cycle amplitude, R (c)	Amplitude, dB	
0	0	0
.1	-1.18	1.18
.2	-2.53	2.53
.3	-4.10	4.10
.4	-5.94	5.94
.5	-8.16	8.16
.6	-10.91	10.91
.7	-14.51	14.51
.8	-19.65	19.65
.9	-28.55	28.55

[a] Phase angle, 0°.
[b] Phase angle, 180°.
[c] $R = b/2a$, where b is backlash spacing and a is amplitude of input sinusoid.

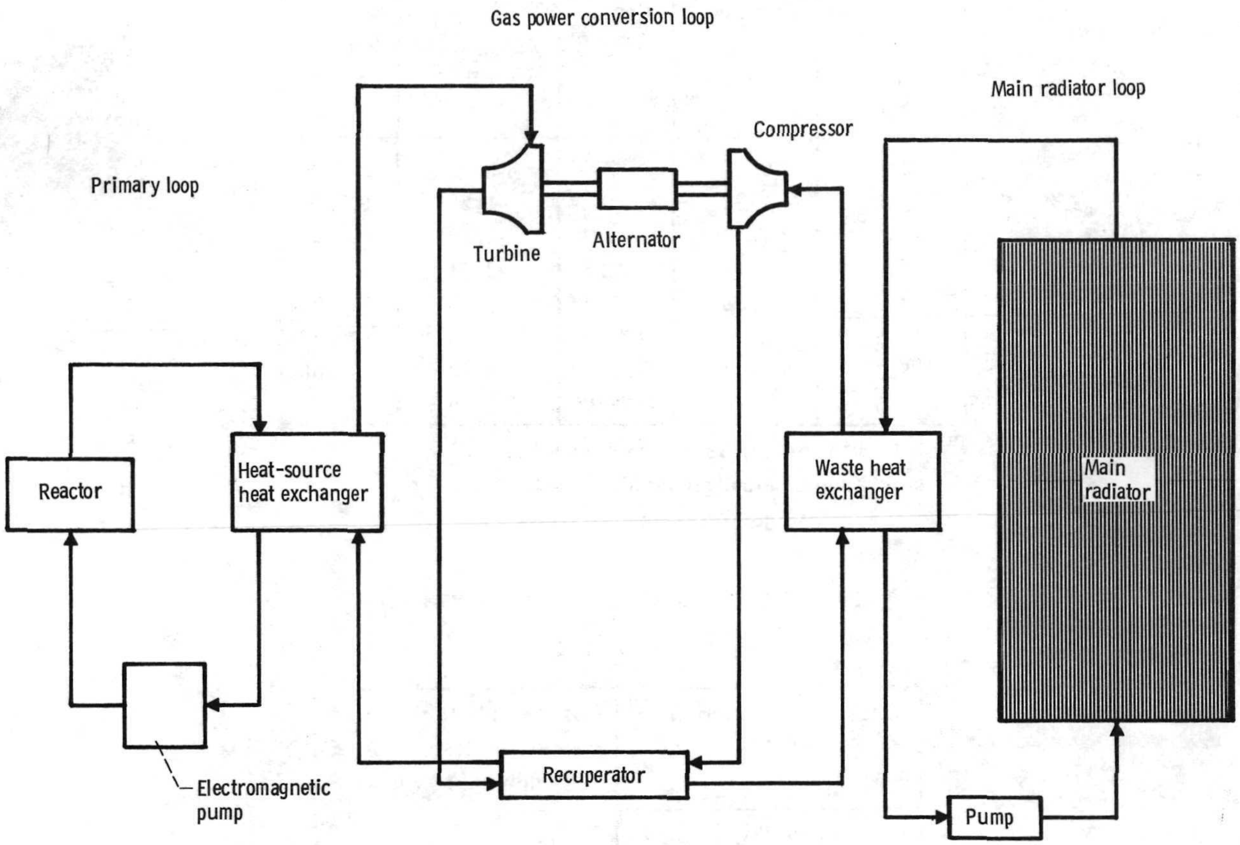

Figure 1. - Schematic diagram of conceptual nuclear Brayton space powerplant.

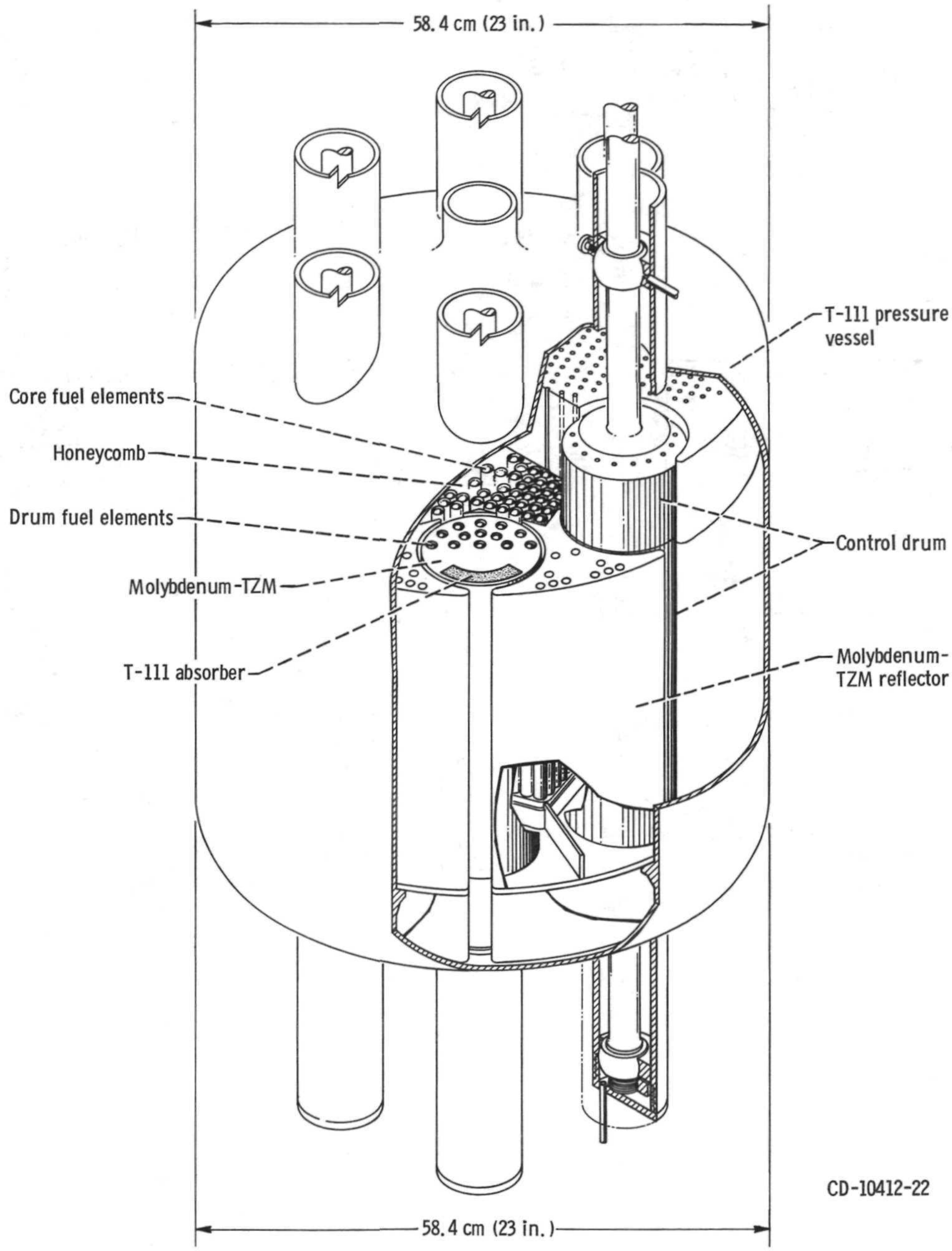

Figure 2. - Nuclear Brayton space power reactor.

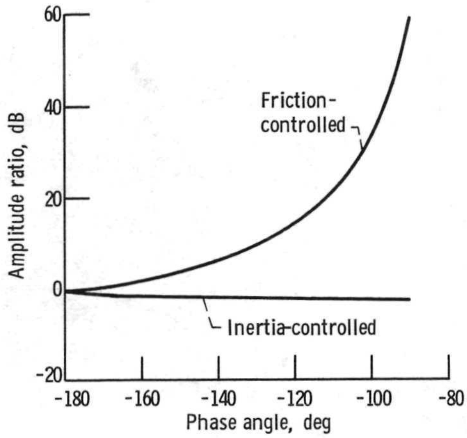

Figure 3. - Gain-phase plot for backlash describing function.

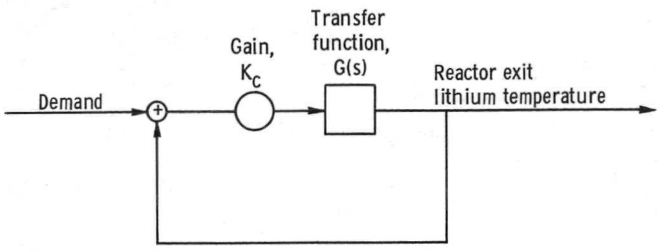

Figure 4. - Block diagram for reactor exit lithium temperature control.

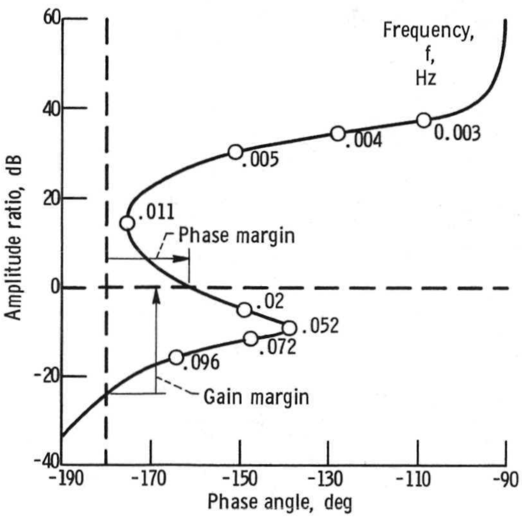

Figure 5. - Gain-phase plot for reactor exit lithium temperature control of nuclear Brayton space powerplant operating at design power.

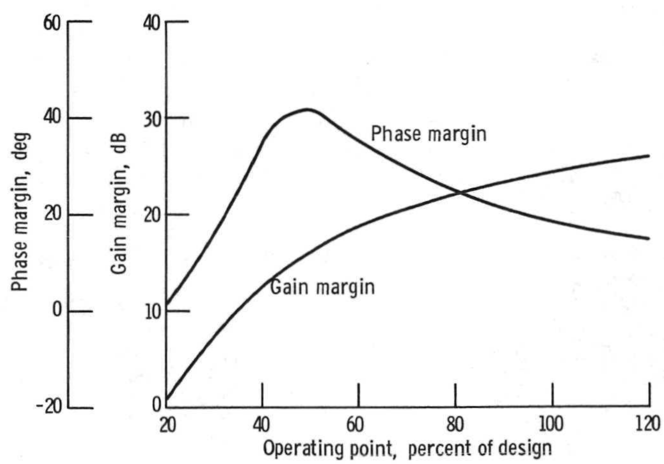

Figure 6. - Gain and phase margins of reactor exit lithium temperature control as functions of operating point.

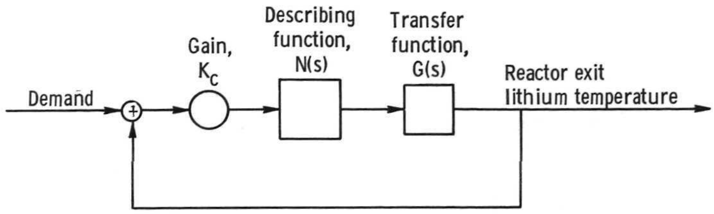

Figure 7. - Block diagram for reactor exit lithium temperature control with dead band or backlash.

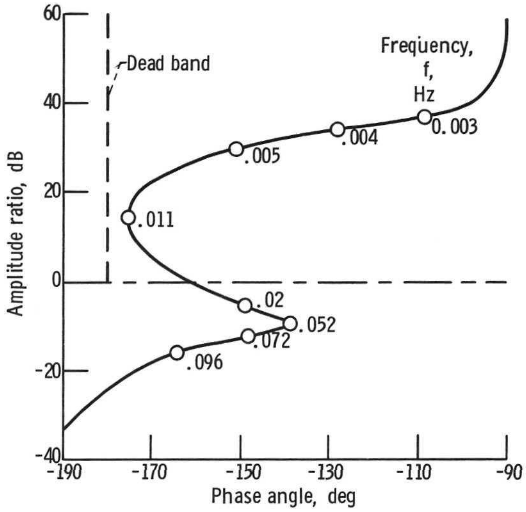

Figure 8. - Gain-phase plot for reactor exit lithium temperature control with dead band.

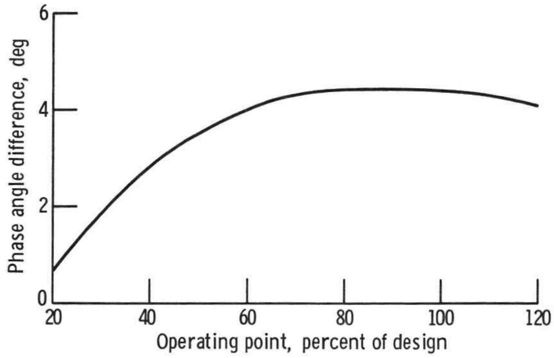

Figure 9. - Phase angle difference between transfer function and describing function as function of operating point.

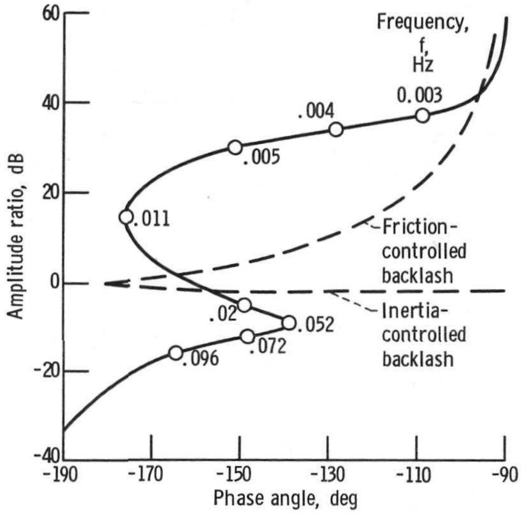

Figure 10. - Gain-phase plot for reactor exit lithium temperature control with backlash.

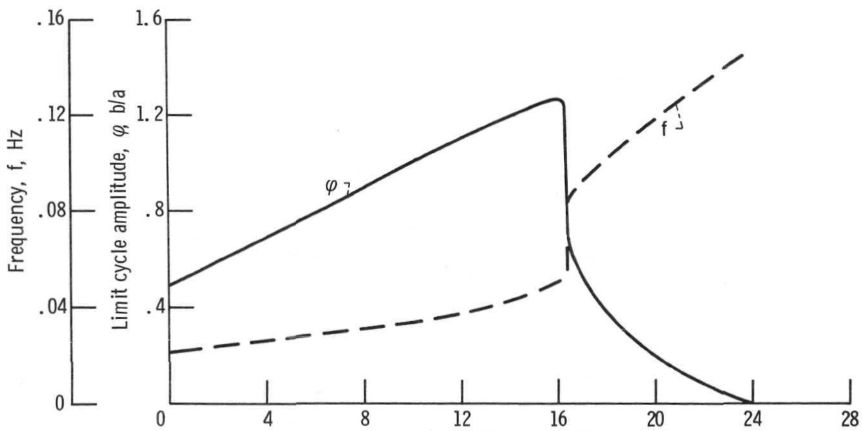

Figure 11. - Limit cycle amplitude and frequency as functions of controller gain for friction-controlled backlash.

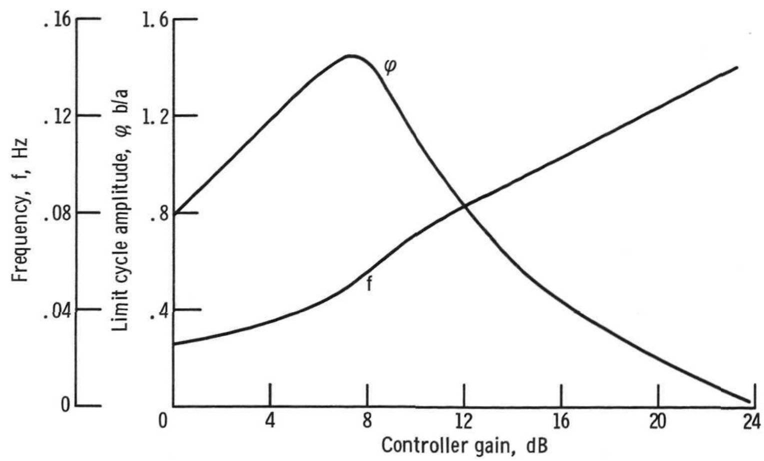

Figure 12. - Limit cycle amplitude and frequency as functions of controller gain for inertia-controlled backlash.

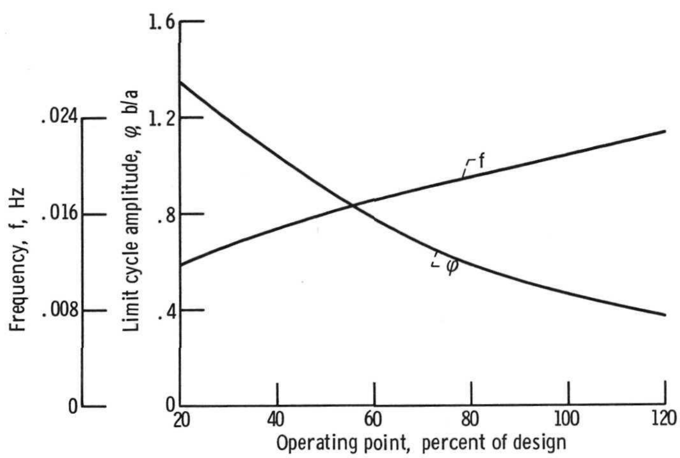

Figure 13. - Limit cycle amplitude and frequency as functions of operating point for friction-controlled backlash.

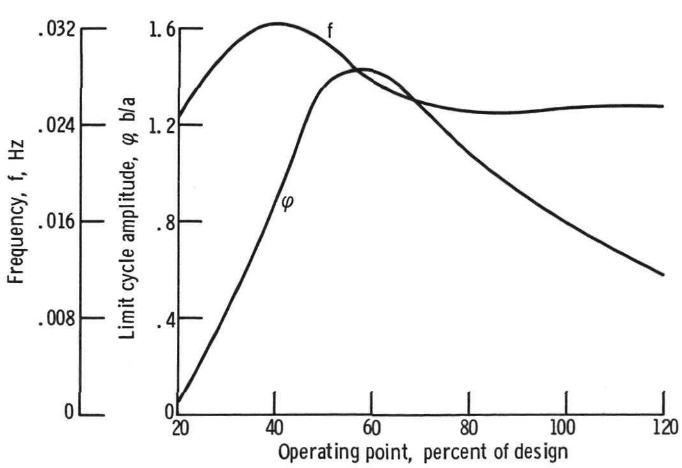

Figure 14. - Limit cycle amplitude and frequency as functions of operating point for inertia-controlled backlash.

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