The Formation of Ion Acoustic Shocks

Roscoe B. White*
Burton D. Fried
Ferdinand V. Coroniti

PPG-142
January 1973

UNIVERSITY OF CALIFORNIA
LOS ANGELES
The Formation of Ion Acoustic Shocks

Roscoe B. White*
Burton D. Fried
Ferdinand V. Coroniti

PPG-142 January 1973

Plasma Physics Group
Physics Department
University of California
Los Angeles, California 90024

*Present address: Institute for Advanced Study, Princeton University,
Princeton, New Jersey 08540
1.0 Introduction

Recent experiments performed in the UCLA double plasma (DP) device\(^1\) have verified the existence of electrostatic ion acoustic laminar shocks. These shocks were first predicted theoretically by Moiseev and Sagdeev\(^2\) from the cold ion-Boltzmann electron steady state fluid equations. In order to obtain a shock-like transition rather than solitons, Moiseev and Sagdeev\(^2\) argued that a small number of reflected ions was required. Montgomery and Joyce\(^3\) later showed that shock-like transitions were also possible if a distribution of electrons trapped in the shock potential was assumed. However, in the DP experiments there exists a third possibility for explaining the formation of laminar shocks, i.e., that the method of shock excitation, the piston, determines the resultant shock structure. In this paper we investigate the influence of the piston on the shock structure by modeling the DP device and by numerically solving the temporal and spatial evolution of the shock. In order to isolate piston effects, as opposed to kinetic theory effects such as reflected ions and trapped electrons, we model the DP plasma as a cold ion fluid with isothermal Boltzmann electrons. We show that on the time scale of the experiments laminar shock transitions with structure agreeing with DP shock experiments can be excited.
2.0 Model of DP Device

The DP device consists of two plasmas separated by a negatively biased grid whose potential greatly exceeds the electron thermal potential $T_e/e$; $T_e$ is the electron temperature in energy units and $e$ is the electronic charge. Since the electron distribution functions are essentially Maxwellian, the grid serves to electrically isolate the two plasmas so that the potential of each plasma can be varied independently. Shock excitation consists of raising the potential in one plasma (driving chamber) as a linear (ramp) function of time until a fixed potential height is achieved; after this time the potential is held constant. Ions flow into the second (target) plasma, and the resulting charge neutralization by electrons excites a large amplitude ion acoustic wave which propagates into the target plasma and steepens into a shock.

Since the physics of the sheath around the separation grid involves kinetic theory effects which we wish to avoid, we model the DP plasma using a modified Boltzmann electron equation of state

$$n_e = n_0 \exp \left( \frac{\varphi(x,t) - \varphi_1(x,t)}{T_e} \right)$$

where $\varphi(x,t)$ is the potential. $\varphi_1(x,t)$ simulates the DP grid by having different values in the two halves of the machine. For the purpose of numerical stability, the discontinuity in $\varphi$ from its value, $\varphi(t)$ at the wall of the driving chamber ($x/z \ll -1$)
to the value $\phi = 0$ at the target chamber wall ($x/z > 1$) was spread out over several Debye scale lengths, $k_D^{-1} = \frac{T_e}{4\pi n_0 e^2 \frac{1}{2}}$, giving a $\phi_1$ which is continuous and has continuous derivatives

$$\phi_1(x,t) = \frac{p(t)}{2}[1 - \tanh(x/z)]. \quad (2)$$

Here $p(t)$ has the temporal form of a truncated ramp (cf. Figure 1a) and $z$ is the effective width of the sheath, chosen to be several $k_D^{-1}$. In the target chamber and outside the sheath ($x/z > 1$), $\phi_1 = 0$ so that in the region of shock propagation a Boltzmann ansatz, $n_e \propto \exp(\phi/T_e)$ for the electrons is appropriate. Computations run with different sheath sizes ($z$) were not appreciably different, so our neglect of the exact sheath dynamics probably does not seriously affect the shock structure.

The ion equations of motion are

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(n v_i) = 0$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = - \frac{\Psi}{\lambda_x} \quad (3)$$

where spatial distance $x$ is measured in units of $k_D^{-1}$ and time is in units of the ion plasma frequency $\omega_{p_i} = \left[\frac{4\pi n_0 e^2}{m_i}\right]^{1/2}$; $\Psi = \frac{\exp(x,t)}{T_e}$; and $n$ is the ion density normalized to $n_0$. The electrons, ions, and the potential are coupled through the Poisson
All plasma motions are confined to the \( x \)-direction, and the plasma is assumed uniform in transverse directions.

In the numerical analysis, the Poisson equation was solved by iterated Fourier transforms. In order to avoid difficulties associated with the Gibbs phenomenon, we introduce the function

\[
A(x,t) = \psi(x,t) - \psi_1(x,t)
\]  

(5)

so that \( A(a,t) = A(b,t) = 0 \) where \( x = a(x = b) \) is the wall of the driving (target) chamber. \( A(x,t) \) then satisfies the differential equation

\[
\frac{d^2A}{dx^2} - A = U(A)
\]

(6)

where \( U(a) = U(b) = 0 \). After obtaining the solution to the Poisson equation, the electric field \( E(x,t) \) is calculated, and is used to step the position, velocity and density ahead in time along the Newtonian characteristics of the differential equation.
\[ x = x' + \left[ \frac{v(x', t) + v(x, t+\Delta)}{2} \right] \Delta + E(x', t)\Delta^2 \]

\[ v(x, t+\Delta) = v(x', t) + E(x', t)\Delta \]

\[ n(x) = n(x') \frac{dx'}{dx} \]

(7)

where \( \Delta \) is the time step. The procedure is then iterated.

To avoid numerical instabilities, the spatial grid size is chosen to be compatible with \( \Delta \). Numerical diffusion speeds can be shown to be negligible compared with wave propagation speeds.

3.0 Simulation of DP Shocks

To excite large amplitude ion waves in the target chamber, the potential \( p(t) \) on the driving chamber's wall is raised to various levels as a linear ramp in a time comparable to \( \omega_{pi}^{-1} \). A compressional ion acoustic wave propagates into the target chamber and steepens into a laminar shock; a rarefaction ion acoustic wave propagates into the driving chamber. Figure 1b shows the spatial profile of the potential for the steepening ion wave at various times after the start of excitation. By time \( 459 \omega_{pi}^{-1} \), the wave front has propagated a distance of \( 567 \ k^{-1}_D \) into the target chamber and has reached a quasi-steady spatial structure with a sharp leading edge of the order of \( 10 \ k^{-1}_D \) thick and a trailing wave train of slightly longer oscillation length.
The shock structure remained essentially unchanged for the rest of the run. The final Mach number (M) of the shock was M = 1.25 and the potential jump Δψ to the first maximum was Δψ = 0.46. Included in Figure 1 is an experimental DP shock from Taylor et al. with a Mach number M = 1.15 and an electron-to-ion temperature ratio of 30, chosen so as to minimize the number of reflected ions. The computed shock reproduces the essential features of the spatial structure of the experimental shocks.

By varying the height of the driving chamber potential \( p(t) \), shocks with different Mach numbers and potential jumps can be launched. Figure 2 shows Δψ vs. ramp height \( ep/T_e \) for Mach numbers 1.06 - 1.4. In Figure 3 the Mach number is plotted against Δψ. The solid points are from the computations; the lower solid curve is the theoretical M vs. Δψ relation calculated by Moiseev and Sagdeev for a steady state shock

\[
M^2 = \frac{\left[e^{\Delta \psi} - 1\right]^2}{e^{\Delta \psi} - 1 - \Delta \psi}
\]

(8)

The good agreement seen in Figure 3 indicates that the computed shocks did achieve a quasi-steady state shock flow. They are, of course, not completely steady, owing to propagation of the rarefaction wave into the driving chamber. Furthermore, this agreement indicates that our modified Boltzmann electron equation of state used to model the DP grid does not significantly affect the quasi-steady shock structure. Also shown in Figure 3
are two points (open circles) which were calculated using, instead of the Boltzmann ansatz for electrons, the trapped electron equation of state discussed by Forslund and Shonk and Forslund and Freiberg. The two differ only for electrons with kinetic energy \( mv^2/2 < mv_0^2/2 \approx e\exp(x) \). The Boltzmann ansatz assumes a distribution function \( f(x,v) = f(x,v_0) \exp\left[\frac{m(v^2-v_0^2)}{2T_e}\right] \) whereas Forslund and Freiberg assume \( f(x,v) = f(x,v_0) \). Their \( M \) vs. \( \Delta \psi \) relationship is

\[
M^2 = \frac{1}{2} \left( \frac{F - 1}{F - 1 - \Delta \psi} \right)^2
\]

\[
F = \frac{2}{\sqrt{\pi \Delta \psi}} e^{\Delta \psi} \text{erfc}\left(\sqrt{\Delta \psi}\right) + \frac{4}{3\sqrt{\pi}} (\Delta \psi)^{3/2}
\]

and the computed shocks again agree with theoretical predictions. Finally, the dashed curve in Figure 3 is the experimental \( M \) vs. \( \Delta \psi \) relation, \( M = 1.0 + 0.6\Delta \psi \), obtained by Means et al. in the DP device. Means et al. have shown that the experimental \( M \) vs. \( \Delta \psi \) relation can be explained by a combination of trapped electrons and a reflected ion distribution function which has been flattened because of unstable ion acoustic turbulence driven by the reflected ions. Since in our computations the ions were assumed cold, we do not expect to reproduce this experimental \( M \) vs. \( \Delta \psi \) relation.

In conclusion, we have shown that the spatial profile of the shock structure observed in experiments is probably a consequence of the DP ramp method of shock excitation, since quasi-steady laminar shock transitions can be formed even in the absence of reflected ions or trapped electrons. The importance
of considering the piston's influence on shock structure is further emphasized by recalling that the steady state fluid theory of Moiseev and Sagdeev\textsuperscript{2} predicted only solitons, and not shock transitions. Finally, the agreement between piston-excited and steady state theoretical $M$ vs. $\Delta\phi$ relations indicates that both Mach number and shock potential jump are relatively insensitive to the method of shock excitation.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge beneficial discussions with Professors C.F. Kennel and A.Y. Wong. The computations were performed at the UCLA On Line computer facility.

This was partially supported by the Office of Naval Research, Grant #N00014-69-A-0200-4023 and the National Aeronautics and Space Administration, Contract NGL 05-007-190.
REFERENCES

FIGURE CAPTIONS

Figure 1. a) Ramp voltage signal $p(t)$ applied to the wall of the driving chamber; b) The propagation and steepening of an ion acoustic pulse and the formation of an ion acoustic shock; c) Experimental DP shock electron density profile from Taylor et al.¹.

Figure 2. The final, quasi-steady shock potential jump $\Delta \Phi$ as a function of the maximum ramp driving potential $e\rho / T_e$.

Figure 3. The final, quasi-steady shock Mach number $M$ vs. the shock potential jump $\Delta \Phi$. The points are from the numerical computations (solid circles for Boltzmann equation of state, open circles for trapped electron equation of state). The solid curves are the theoretical relations for the Boltzmann and trapped electron
Computed DP shock
\( M = 1.25 \quad \Delta \psi = 0.46 \)

(b) \( t = 80 \quad 180 \quad 280 \quad 335 \quad 389 \quad 459 \)

Potential \( e\phi/T_e \)
Distance \( \rightarrow \)

Distance \( \rightarrow \)

Ramp voltage applied to driver chamber

(c) Experimental DP shock
\( M = 1.15 \quad T_e/T_i = 30 \quad \Delta \psi = 0.25 \)

30 \( K_D^{-1} \)
Figure 2

Shock potential jump $\Delta \psi$

Ramp height $e_p/T_e$
Experimental DP shocks — $M = 1.0 + 0.6 \Delta \psi$

Trapped electron

Boltzmann

Computed DP shocks

Mach number vs. Potential jump $\Delta \psi$
UCLA PLASMA PHYSICS GROUP REPORTS

* Published by Experimental Group
† Published by Theoretical Group

R-1 "Propagation of Ion Acoustic Waves Along Cylindrical Plasma Columns", A.Y. Wong (July 1965)*
R-2 "Stability Limits for Longitudinal Waves in Ion Beam-Plasma Interaction", B.D. Fried and A.Y. Wong (August 1965)*
R-3 "The Kinetic Equation for an Unstable Plasma in Parallel Electric and Magnetic Fields", B.D. Fried and S.L. Osakow (November 1965)†
R-5 "Effects of Collisions on Electrostatic Ion Cyclotron Waves", A.Y. Wong, D. Judd and F. Hai (December 1965)*
R-7 "Observation of Cyclotron Echoes from a Highly Ionized Plasma", D.E. Kaplan and R.M. Hill (May 1966)*
R-8 "Excitation and Damping of Drift Waves", A.Y. Wong and R. Rowberg (July 1966)*
R-9 "The Guiding Center Approximation in Lowest Order", Alfredo Baños, Jr. (September 1966)†
R-10 "Plasma Streaming into a Magnetic Field", S.L. Osakow (November 1966)†
R-11 "Cooperative Effects in Plasma Echo Phenomena", A.Y. Wong (March 1967)*
R-12 "A Quantum Mechanical Study of the Electron Gas Via the Test Particle Method", M.E. Rensink (March 1967)
R-14 "The Expansion and Diffusion of an Isolated Plasma Column", J. Hyman (May 1967)
R-17 "Parametric Coupling Between Drift Waves", F. Hai, R. Rowberg and A.Y. Wong (October 1967)*
R-18 "Cyclotron Echoes from Doppler Effects", A.Y. Wong (March 1968)
R-21 "Test Particle Theory for Quantum Plasmas", M.E. Rensink (October 1967)†
R-23 "Landau Damping of Ion Acoustic Waves in a Cesium Plasma with Variable Electron-Ion Temperature Ratio", K.B. Rajangam (October 1967)
R-24 "The Inhomogeneous Two-Stream Instability", G. Knorr (September 1967)
R-26 "Small Amplitude Waves in High Beta Plasmas", V. Formisano and C. Kennel (February 1968)†
R-27 "Low Beta-Plasma Penetration Across a Magnetic Field", B.D. Fried and S. Ossakow (March 1968)*
R-29 "The Theorist's Magnetosphere", C. Kennel (April 1968)
R-31 "Electromagnetic Echoes in Collisionless Plasmas", A.Y. Wong (April 1968)*
R-32 "Parametric Excitation of Drift Waves in a Resistive Plasma", G. Weyl and M. Goldman (June 1968)*
R-33 "Parametric Excitation from Thermal Fluctuations at Plasma Drift Wave Frequencies", A.Y. Wong, M.V. Goldman, F. Hai, R. Rowberg (May 1968)*
R-34 "Current Decay in a Streaming Plasma Due to Weak Turbulence", S.L. Ossakow and B.D. Fried (June 1968)*
R-35 "Temperature Gradient Instabilities in Axisymmetric Systems", C.S. Liu (August 1968)*
R-37 "Transverse Plasma Wave Echoes", B.D. Fried and Craig Olson (October 1968)*
R-38 "Low Frequency Interchange Instabilities of the Ring Current Belt", C.S. Liu (January 1969)*
R-39 "Drift Waves in the Linear Regime", R.E. Rowberg and A.Y. Wong (February 1969)*
R-41 "Nonlinear Oscillatory Phenomena with Drift Waves in Plasmas", F. Hai and A.Y. Wong (September 1970)
R-42 "Ion-Burst Excited by a Grid in a Plasma", H. Ikezi and R.J. Taylor (February 1969)
R-43 "Measurements of Diffusion in Velocity Space from Ion-Ion Collisions", A. Wong and D. Baker (March 1969)*
R-44 "Nonlinear Excitation in the Ionosphere", A.Y. Wong (March 1969)
R-46 "A New Representative for the Conductivity Tensor of a Collisionless Plasma in a Magnetic Field", B.D. Fried and C. Hedrick (March 1969)*
R-47 "Direct Measurements of Linear Growth Rates and Nonlinear Saturation Coefficients", A.Y. Wong and F. Hai (April 1969)*
R-49 "Auroral Micropulsation Instability", F. Coroniti and C.F. Kennel (May 1969)*
R-50 "Effect of Fokker-Planck Collisions on Plasma Wave Echoes", G. Johnston (June 1969)*
R-52 "Theory of Stability of Large Amplitude Periodic (BGK) Waves in Collisionless Plasmas", M.V. Goldman (June 1969)*
R-55 "Optical Mixing in a Magnetoactive Plasma", G. Weyl (August 1969)*
R-56 "Trapped Particles and Echoes", A.Y. Wong and R. Taylor (October 1969)*
R-84 "Observations of Parametrically Excited Ion Acoustic Waves", R. Stenzel (March 1971)
R-85 "Remote Double Resonance Coupling of Radar Energy to Ionospheric Irregularities", C.F. Kennel (January 1971)
R-86 "Ion Acoustic Waves in a Multi-Ion Plasma", B.D. Fried, R. White, T. Samec (January 1971)
R-87 "Current-Driven Electrostatic and Electromagnetic Ion Cyclotron Instabilities", D.W. Forslund, C.F. Kennel, J. Kindel (February 1971)
R-88 "Locating the Magnetospheric Ring Current", C.F. Kennel and Richard Thorne (March 1971)
R-89 "Ion Acoustic Instabilities Due to Ions Streaming Across Magnetic Field", P.J. Barrett, R.J. Taylor (March 1971)
R-90 "Evolution of Turbulent Electronic Shocks", A.Y. Wong and R. Means (July 1971)
R-91 "Density Step Production of Large Amplitude Collisionless Electrostatic Shocks and Solitons", David B. Cohen (June 1971)
R-95 "3-D Velocity Space Diffusion in Beam-Plasma Interaction without Magnetic Field", P.J. Barrett, D. Gresillon and A.Y. Wong (September 1971)
PPG-96 "Dayside Auroral Oval Plasma Density and Conductivity Enhancements due to Magnetosheath Electron Precipitation", C.F. Kennel and M.H. Rees (September 1971)
PPG-97 "Collisionless Wave-Particle Interactions Perpendicular to the Magnetic Field", A.Y. Wong, D.L. Jassby (September 1971)
PPG-98 "Magnetospheric Substorms", F.V. Coroniti and C.F. Kennel (September 1971)
PPG-100 "Structure of Ion Acoustic Solitons and Shock Waves in a Two-Component Plasma", R.B. White, B.D. Fried and F.V. Coroniti (September 1971)
PPG-101 "Solar Wind Interaction with Lunar Magnetic Field", G. Siscoe (Meteorology Dept.) and Bruce Goldstein (JPL) (November 1971)
PPG-102 "Changes in Magnetospheric Configuration During During Substorm Growth Phase", F.V. Coroniti and C.F. Kennel (November 1971)


PPG-108 "Threshold and Saturation of the Parametric Decay Instability," R. Stenzel and A. Y. Wong, November 1971


PPG-112 "Polarization of the Auroral Electrojet," F. V. Coroniti and C. F. Kennel, February

PPG-113 "Mode Coupling and Wave Particle Interactions for Unstable Ion Acoustic Waves," Pablo Martin and Burton D. Fried, February 1972


PPG-116 "Large Diameter, Quiescent Plasma in a Magnetospheric Field," Earl Ault, Thesis, April 1972


PPG-124 "Calculation of Reflection and Transmission Coefficients for a Class of One-Dimensional Wave Propagation Problems in Inhomogeneous Media," A. Banos, Jr., September 1972

PPG-125 "Electromagnetic Wave Functions for Parabolic Plasma Density Profiles," A. Banos, Jr. and D. L. Kelly, September 1972


PPG-128 "Can the Ionosphere Regulate Magnetospheric Convection?" F. V. Coroniti and C. F. Kennel, October, 1972