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DETERMINATION OF PARAMETERS RELATED TO THE INTERIOR OF MERCURY

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ABSTRACT

Bounds on (C-A)/C for Mercury as a function of the uncertainty in the value of the obliquity are determined. The high precision of 1' of arc which is required for reasonable bounds on (C-A)/C cannot be obtained by either earth based observations or the television imagery of the Mariner 73 flyby. Among other methods discussed, one involving both landers and orbiters could determine unambiguously not only (C-A)/C but also (B-A)/C and C/MR^2.

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INTRODUCTION

Cassini's laws, which describe the rotation and prescession of the moon, have been generalized to apply to Mercury (Colombo, 1966; Peale, 1969). The spin axis and orbit normal of Mercury precess around the normal to the proper or Laplacian plane while the three vectors remain coplanar. In this configuration, the angle between the spin axis and the orbit normal (the obliquity) remains constant, and the magnitude is determined by the value of the ratio (C-A)/C, where A,B,C are the principal moments of inertia in the order of increasing magnitude. A measurement of the obliquity thus determines (C-A)/C which can be used to limit "geophysical" models of the planet Mercury.

The increasing capability of planetary radar and the coming Mariner flyby in 1974 both suggest the possibility that the orientation of Mercury's spin axis may be determined. It is therefore appropriate that the necessary accuracy of the measurement be established for meaningful geophysical interpretation. The bounds on (C-A)/C as functions of the error in the determination of the obliquity are established below. The high precision of the measurement for reasonable bounds implies that neither the television imagery of the Mariner flyby nor radar will be capable of the necessary accuracy. Two alternative schemes for determining the obliquity which involve orbiters or landers are discussed. With M being the mass and R the radius of Mercury, these latter methods could also yield the value of C/MR^2, which is a geophysically important measure of the radial mass distribution.

Bounds on (C-A)/C

There are two stable positions of Mercury's spin vector which allow coplanar precession and which can be within the current errors in determination of the spin orientation (Peale, 1969). One of these is near the orbit normal and the other near the normal to the proper plane. In the first state the obliquity decreases with increasing (C-A)/C, whereas in the second the obliquity increases with (C-A)/C.

A reasonable minimum value of (C-A)/C is that appropriate to hydrostatic equilibrium [Munk & MacDonald, 1960, p. 26].

$$\frac{C-A}{C} \geq \frac{k' R^5 \omega^2}{3GC} \approx 10^{-6} \quad , \tag{1}$$

where k' is the secular Love number, G is the gravitational constant, R is the mean radius and ω is the angular velocity of the spinning planet. The numerical value is obtained by assuming that $k' = 0.96$ and $C = MR^2/3$, the values for the earth. (The secular or fluid Love number increases to 1.5 for a homogeneous planet.) A value of (C-A)/C of 10^{-6} leads to an obliquity of about 30° in the second state and is too small to allow stability of the first (see Figs. 4 and 5 of Peale, 1969). An obliquity of 30° exceeds the errors of ± 3° of the optical determination of the axis orientation (B. A. Smith, private communication 1971), which implies that Mercury cannot occupy the second state.

The optical observations in fact place Mercury in the first state where a slightly larger value of (C-A)/C stabilizes the spin vector within about 5° of the orbit normal. If $(C-A)/C \gtrsim 10^{-5}$, the stable position near the orbit normal corresponds to an obliquity $\theta \lesssim 10^{-2}$ radians. (It is of interest to note that a value of (C-A)/C for Mercury corresponding to the lunar values of internal shear stresses would be 1.3×10^{-4}. The internal stresses scale as the square of the surface gravity (Kaula, 1963).) The obliquity will thus be very small for all likely values of (C-A)/C, and we can use small angle approximations (from Peale, 1969, Eq. 17):

$$\frac{C-A}{C} = \frac{-\mu}{n}(1-e^2)^{3/2}\left[\theta^{-1}\sin\iota + \cos\iota\right] \quad ; \; B=A$$

$$\frac{C-A}{C} = \frac{-\mu}{n(1+\frac{7}{2}e+\frac{3}{2}e^2)}\left[\theta^{-1}\sin\iota + \cos\iota\right] \quad ; \; B=C \qquad (2)$$

where $\mu(<0)$ is the angular velocity of the orbit precession, n is the orbital mean motion, e is the orbit eccentricity and ι is inclination of the orbit to the proper plane. Terms of order e^3 are neglected in the second of Eqs. (2). The uncertainty in B leads to about a factor 1.2 uncertainty in (C-A)/C even if θ is known precisely.

In practice the errors in the determination of the two angular coordinates will define a region on the celestial sphere which contains the extension of the spin vector. The intersection with this region by the plane defined by the spin vector and orbit normal (which should also contain the normal to the proper plane) gives a range of obliquity and, from Eqs. (2), of the value of (C-A)/C. We write

$$\theta = \theta_o \pm \Delta\theta ,$$

where θ_o is the central value in the range and $\Delta\theta$ the uncertainty determined by the size of the measurement errors. For three values of θ_o, selected to span the most likely set of values of (C-A)/C, we have plotted in Fig. 1 the bounds on (C-A)/C as a function of the error $\Delta\theta$. The bounds are double valued because we have included the curves for both extremes of B ($A \leq B \leq C$). The contribution of the uncertainty in B and the error in measurement to the uncertainty in (C-A)/C are thus both included.

For each θ_o the upper and lower curves represent respectively upper and lower bounds on (C-A)/C. For example, if the measured value of $\theta_o = 10^{-3}$ with an error $\Delta\theta = 5 \times 10^{-4}$ then with B completely unknown

$$7.8 \times 10^{-5} \leq \frac{C-A}{C} \leq 2.7 \times 10^{-4}$$

Only a lower bound can be established if $\theta_o < \Delta\theta$. The small values of the stable obliquities for likely values of (C-A)/C are seen to require maximum errors in their measurements of less than about $1'$ of arc in order that reasonable bounds on (C-A)/C be determined.

For example, to determine that the value of (C-A)/C for Mercury is less than that for the moon would require $\Delta\theta \lesssim 2'$.

This rather extreme accuracy appears to exclude a determination of (C-A)/C by presently planned techniques. A $1'$ change in the orientation of Mercury corresponds to about a kilometer maximum displacement of a point on the surface. But 1 km is comparable to the best resolution anticipated in the Mariner 73 flyby (Murray, et al., 1971), and a relative reference such as the limb will be difficult to determine accurately from the mosaic of pictures. It is also necessary to photograph the same features several times with sufficient time intervals between photographs for significant rotation of the planet. A second pass of the planet is planned when the same features can be rephotographed, but this occurs exactly two Mercury years later when the planet has precisely the same orientation in space. The rather small rotation during each encounter and a resolution limit of 1 km implies a rather poor determination of the spin axis orientation by Mariner 73. Television imagery from a flyby spacecraft has proved inadequate even for improving the current estimate the spin axis of Mars which is rapidly rotating (Davies and Berg, 1971). Finally the long series of careful observations of the moon with ground based resolution of 0.5 km can be compared with the fleeting glimpse of Mercury by Mariner 73 with less resolution. Earth based observations still leave about a $0.'5$ uncertainty in the moon's spin axis (Jeffreys, 1961).

Current radar determinations of the spin axis are less accurate than the ± 3° from optical observations, but the increase in gain and signal to noise anticipated for the Aricebo radar in 1973 will make the radar competitive with optical observations (R. B. Dyce, 1971 private communication). Resurfacing the antenna dish and installing a new 400 kw transmitter in 1974 will allow a 5 to 10 km resolution of Mercury's surface, which gives a resolution limited precision of about 7' of arc for the spin axis (F. D. Drake, private communication, 1971). This is still an order of magnitude too large to evaluate (C-A)/C.

Finally, it should be pointed out that the small values of the stable obliquities require that the theory be considerably refined before (C-A)/C can be evaluated accurately even with a precise value for θ. Both μ and ι were assumed constant in deriving Eqs. 2. The inclination ι has a long period variation of several degrees with a period of about 2.5×10^5 years (Brouwer and Clemence, 1961). The vector $\bar{\mu}$ defines the precession of Mercury's orbit and was referred to as the normal to the proper plane. However, the dominant perturbations of Mercury's orbit are due to Venus and Jupiter whose orbit planes also precess. The resulting long period variations in $\bar{\mu}$ and ι have amplitudes which are large compared with the small stable obliquities discussed above. The time scale for relaxation to a given stable state with fixed $\bar{\mu}$ and ι is that for tidal evolution, which is long compared to the period of oscillation. These periodic oscillations in $\bar{\mu}$ and ι thus lead to comparable oscillations in θ. The value of θ used to determine $(C-A)/C$ must therefore refer to mean values of ι and $\bar{\mu}$. All perturbations of θ with amplitudes comparable to or greater than $0.'5$ must be considered, since the instantaneous amplitude of the periodic variation must be subtracted from the instantaneous value of θ.

Alternate Procedures

Neither ground based observations nor the television imagery of the Mariner flyby can determine the obliquity of Mercury with sufficient accuracy to evaluate $(C-A)/C$. The importance of this and related parameters for information about the interior of the planet encourages consideration of other technically feasible schemes. Repeated high resolution imagery from an accurately tracked orbiter might be capable of determining θ and, if the necessary refinement of the theory is sufficiently complete, $(C-A)/C$. An additional bonus from the use of an orbiter would be the harmonic coefficients J_2 and C_{22} of the expanded gravitational potential. The former is $(C = \frac{1}{2}A - \frac{1}{2}B)/MR^2$ and the latter is $(B-A)/4MR^2$. The moment B can be eliminated from these expressions and the ratio C/MR^2 determined if $(C-A)/C$ is known (Kaula, 1969).

There are two possible difficulties with the artificial satellite imagery for the determination of the moment differences and C/MR^2. The uncertainties in the theoretical development for the mean values of the orbit normal and perhaps $\bar{\mu}$ may exceed the small obliquity which we hope to determine. Also, the orbit which is necessary for precise gravitational harmonics may not be compatible with the type of photography required for finding the spin axis.

The required accuracy in the determination of the lowest order gravity coefficients is dictated by our desire to evaluate C/MR^2. One might expect a value of this ratio for Mercury near 0.33, the value for the earth, but almost certainly less than or equal to the value for a homogeneous sphere of 0.4. The determination of C/MR^2 should have a minimal precision so that the upper bound on its value is less than 0.4. If 0.33 is the central value of a determination of C/MR^2 an extreme upper bound on the error would thus be about 20%, which is essentially the sum of the errors of measurement of the three quantities J_2, C_{22} and $(C-A)/C$. As a guideline we can adopt a value of a few percent for the maximum error for each of these three parameters, if the derived value of C/MR^2 is to be meaningful.

Measurement of the secular changes of the orbit pericenter and node for a Mercury orbiter yields two weighted sums of the zonal harmonics J_2, J_4, J_6, etc. The effects of the higher harmonics relative to J_2 can be reduced by increasing the orbit parameter $p = a(1-e^2)$, where a is the orbit semimajor axis. If we adopt the lunar values $J_2 = 2.4 \times 10^{-4}$, $J_4 = -1.2 \times 10^{-5}$, $J_6 = 1 \times 10^{-6}$ (Liu and Laing, 1971) the effect of J_4 and higher harmonics on the secular changes is less than 5% of that for J_2 even for a close Mercury orbiter. To allow for possibly larger values of J_4, and any additional errors, one might wish a minimum orbit parameter p of about 2 Mercury radii to reduce the uncertainty in J_2. On the other hand, we can neglect harmonics higher than J_4 and solve for J_2 and J_4 simultaneously from the secular motions of the node and pericenter. The uncertainty is then reduced to that of J_6 and other errors provided these contributions are small compared to that of J_4. The accuracy for tracking an orbiter of 1 mm/sec and 15 meters relative to the center of mass of Mercury (J. D. Anderson, private communication, 1971) introduces an error which is negligible compared to the above uncertainties. The orbiter should be tracked at least for the period of the pericenter precession to eliminate the long period contributions in the node and pericenter motion due to J_3. We should thus be able to obtain a few percent accuracy for J_2 with a single satellite in a relatively wide range of orbits provided the tracking is maintained for several months.

The coefficient C_{22} (and S_{22} for an arbitrary orientation of coordinate axes in the equator plane) is determined by periodic perturbations of the satellite orbit. The amplitude of each periodic perturbation which contains C_{22} also includes other harmonic coefficients. For the periodic perturbations in the mean longitude which depend on C_{22}, we have (e.g. Kaula, 1966, p. 40),

$$\frac{\Delta\lambda_{22}}{\Delta\lambda_{42}} \approx -\frac{1}{4}\frac{a^2}{R^2}\frac{C_{22}}{C_{42}} \tag{3}$$

where $\Delta\lambda_{22}$ and $\Delta\lambda_{42}$ are respectively the amplitudes of the perturbation in longitude which result from the C_{22} and C_{42} terms in the potential. This implies that an orbital semimajor axis greater than 9 R is necessary to insure a contribution from C_{42} of less than a few percent of that from C_{22}. (For lunar values of C_{22} and C_{42}, $a/R \gtrsim 3.5$ leads to a few percent contribution by C_{42}.) The amplitudes of the perturbations in position and velocity for the largest term with $C_{22} = 2.4 \times 10^{-5}$, the lunar value, are

$$\begin{aligned} \Delta\lambda_{22} \Big|_{max} &\approx 10^{-4} \frac{R^2}{a^2} \\ \frac{d\Delta\lambda_{22}}{dt} \Big|_{max} &\approx 10^{-7} \frac{R^2}{a^2} \text{ sec}^{-1} \quad . \end{aligned} \tag{4}$$

These lead to amplitudes in position and velocity changes of

$$\Delta r_{22} \approx 200 \frac{R}{a} \text{ meters}$$

$$\Delta v_{22} \approx 30 \frac{R}{a} \frac{\text{cm}}{\text{sec}}$$

The velocity amplitude at a = 9 R is still well within the tracking accuracy, but the position amplitude becomes marginally determined. One may use the amplitudes of several perturbations to solve simultaneously for C_2 and C_4 etc., but terms other than the largest are not zero order in the eccentricity and may be too small.

The periodic solar perturbations of a Mercury satellite orbit will be commensurate with those of the tesseral harmonics and may dominate for the relatively distant orbit discussed here. Radiation pressure will be particularly troublesome since its magnitude will be variable. Such unknown quantities reduce confidence in a least squares analysis of the tracking data. An orbital period with a low order commensurability with the planetary spin is also an impractical means of determining C_{22} precisely, since Mercury rotates so slowly.

We should place a single orbiter relatively far from Mercury to maximize the ratio of the perturbations due to C_{22} to those due to higher harmonics. But the amplitude of the perturbation is thereby severely reduced, and the uncertainty in other perturbations becomes more critical. It will probably be necessary to track at least two satellites in distinct orbits for confidence in an error of determination of a few percent for C_{22}. Again, tracking for many orbits is necessary for the determination of the amplitude of the periodic perturbations.

Another route to the determination of all the moment differences and ratios would be to evaluate (B-A)/C from the amplitude of the forced librations in longitude about the resonant spin. If the terms in the equation of the angular motion of Mercury are expanded in the mean anomaly (see Goldreich and Peale, 1966) it is easy to show that the amplitude ϕ_o of the physical librations with the period of a Mercury year is given by

$$\phi_o = \frac{3}{2}\left(\frac{B-A}{C}\right)\left(1 - 11e^2 + \frac{959}{48}e^4 + \dots\right) \approx \frac{B-A}{C} \qquad (5)$$

where e is the orbit eccentricity. For $(B-A)/C = 10^{-4}$, the amplitude is about 20" of arc, which must be measured with an accuracy of less than 1" for a few percent error in $(B-A)/C$. This would be an easy measurement only for an observer sitting on the surface.

One can speculate about a long lived Mercury lander, perhaps a modified Viking, which could be placed near the pole to minimize temperature extremes. If a surface albedo of 0.06 is assumed for the incident solar radiation and an emissivity of unity for the infrared, the maximum surface temperature does not exceed the lunar maximum of 390°K within about 5° of the pole. A device which could periodically or continuously record the orientation of the planet relative to the stars, either by accurate tracking of a single star or by a series of long focal length images, would simultaneously yield the pole position and the amplitude of the physical librations in a time comparable to the 58 day rotation period. We could thus obtain $(B-A)/C$, $(C-A)/C$ and, with a single orbiter, the harmonic coefficient J_2 and thus C/MR^2.

If the evaluation of $(C-A)/C$ from the obliquity measurement lacks sufficient accuracy, another procedure, which would probably require at least one additional accurately tracked satellite, would replace $(C-A)/C$ with the coefficient C_{22} in the above set and all quantities including C/MR^2 can again be evaluated. Since the theoretical and data analysis procedures have already been applied to the moon, this latter method of determining the principal moments of inertia and radial mass distribution of Mercury may be the most certain of success.

Another possible technique for determining the spin axis orientation would be the Doppler tracking of a surface transmitter (J. G. Williams, 1971, private communication). However, such an instrument should be far from the poles to maximize the velocity changes during the rotation. Since tracking through several rotations with the earth at different aspects would be necessary, the instrument would experience temperatures which can exceed 700°K and would cease transmitting before sufficient data were obtained. In addition, the maximum change in velocity of a point on the equator due to the 20" libration is less than 0.5 mm/sec, which approaches the limiting accuracy of current tracking techniques (J. D. Anderson, 1971, private communication). A surface transmitter is probably not a practical means of defining the axis or measuring the librations.

SUMMARY

The importance of the moment differences and the radial mass distribution in placing limits on physical models of the planet Mercury places high priority on their determination. Neither current ground based observations nor the television imagery of the Mariner 73 flyby are capable of locating the spin axis to the precision necessary to determine (C-A)/C. Ground based radar will approach this capability after 1974 but still will not be capable of determining (C-A)/C. This leads us to the consideration of orbiters and landers which are capable in principle of yielding all the moment differences and C/MR^2. The best scheme would

B

involve a lander near a pole with instrumentation to map the orientation of the landing point relative to the stars. Such mapping yields (C-A)/C from the pole position and (B-A)/C from the amplitude of the physical librations. An orbiter gives J_2 and the three parameters can be solved for C/MR^2. Alternatively, orbiters can be used to evaluate the gravitational coefficient C_{22}, which replaces (C-A)/C in the solution for C/MR^2. In view of the uncertainties which may arise in the theoretical refinement of Cassini's laws for Mercury, the latter scheme employing a polar lander and two or more distinct satellites is most likely to yield unambiguous values of (C-A)/C, (B-A)/C and C/MR^2. The required accuracies of the measurements are within the capabilities of current technology.

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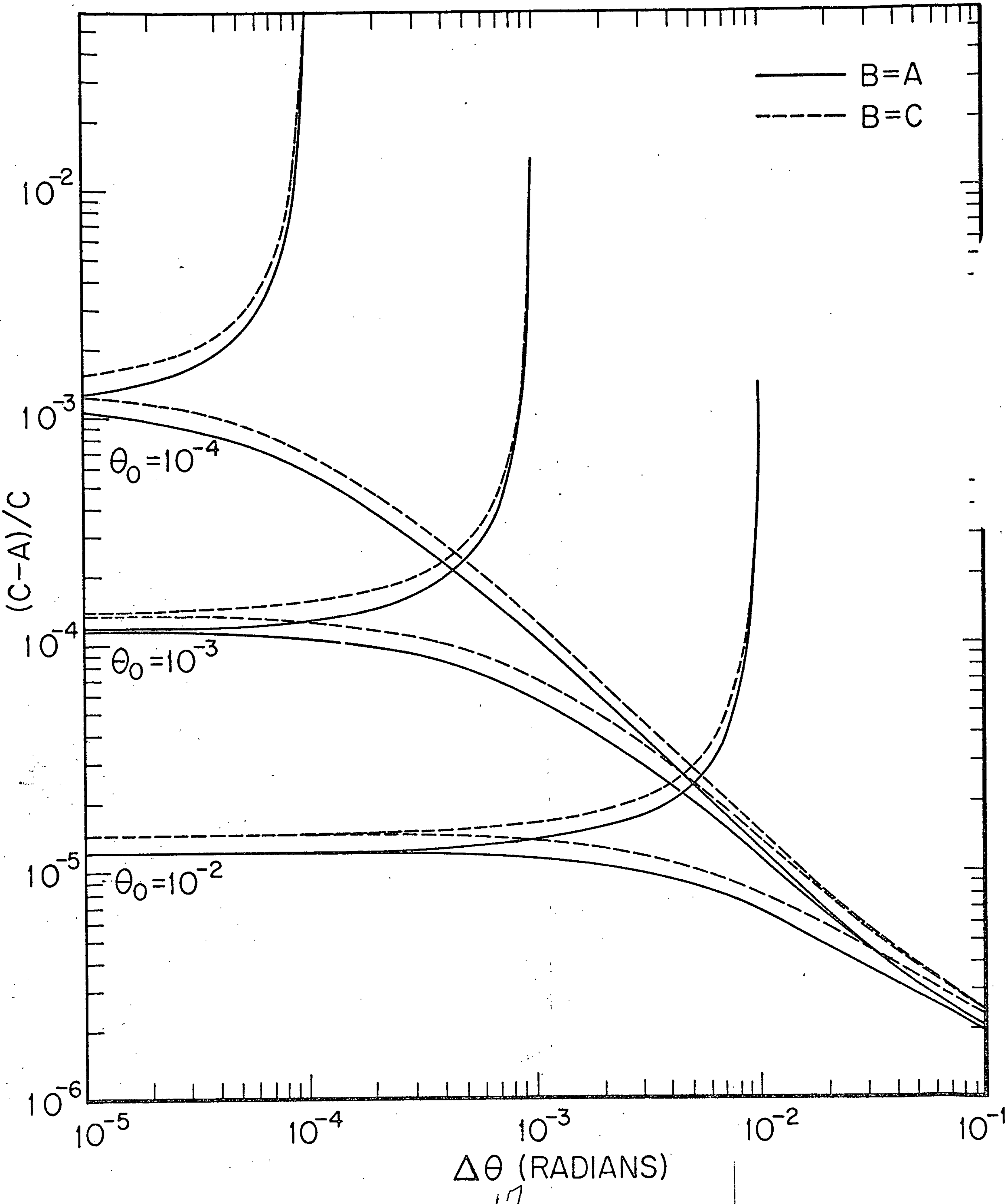
FIGURE CAPTION

Fig. 1. Bounds on (C-A)/C as a function of the uncertainty in the obliquity.



B=A
B=C
$\theta_0=10^{-4}$
$\theta_0=10^{-3}$
$\theta_0=10^{-2}$
(C−A)/C
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Δθ (RADIANS)

17