Reports of the Department of Geodetic Science
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THE FORMATION AND ANALYSIS OF A 5° EQUAL AREA BLOCK TERRESTRIAL GRAVITY FIELD

by

Richard H. Rapp

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
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OSURF Project No. 3210

The Ohio State University
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Abstract

A set of 23,355 $1^\circ \times 1^\circ$ mean free-air anomalies were used to predict a set of 5° equal area anomalies and their standard errors. Using the 1° data incorporating geophysically predicted values of ACIC, 1283 5° blocks were computed. Excluding the geophysically predicted anomalies 1249 blocks were computed. The 1° data was also used to compute covariance functions and the equatorial gravity and flattening implied by this data. The predicted anomalies were supplemented by model anomalies to form a complete 1654 global anomaly field. This data was used in a weighted least squares to determine potential coefficients to degree 15, and in a summation type formulation to determine potential coefficients to degree 25. These potential coefficients sets are compared to recent satellite determinations.

Foreword

This report was prepared by Richard H. Rapp, Professor, Department of Geodetic Science, The Ohio State University, under NASA Grant NGR 36-008-161, The Ohio State University Research Foundation Project No. 3210. The contract covering this research is administered through the Goddard Space Flight Center, Dr. David Smith, Technical Officer.

A preliminary version of this work was presented at the 53rd Annual AGU Meeting, April, 1972 under the title "A 300 n. m. Terrestrial Gravity Field." The results of this report and the AGU paper are slightly different with those of this report being the most current.

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1.0 Introduction

The combination of satellite and gravimetric data has generally taken place using $5^{\circ}\times5^{\circ}$ (Rapp, 1967) mean anomalies, or 300 n.m. (Kaula, 1966) mean anomalies. In Kaula's work, the 300 n.m. means were estimated on the basis of a set of $1^{\circ}\times1^{\circ}$ mean anomalies provided by the Aeronautical Chart and Information Center (ACIC). Using a linear regression technique Kaula computed mean anomalies for 935 near equal area blocks. The values for 934 of these anomalies were given by Gaposchkin and Lambeck (1970).

An updated set of 1470 $5^{\circ}\times5^{\circ}$ anomalies was used by Rapp (1969) in determining potential coefficients solely from gravity data. The specific anomalies were not published although they were used by other investigators (e.g. Koch (1970)).

In defining a new combination procedure (Rapp (1970)) it was necessary to determine mean gravity anomalies in 15° equal area blocks. As a first step in the evaluation of these anomalies it was decided that a set of 5° or 300 n.m. blocks should first be computed from the latest available data. This paper summarizes the data used, the procedures, the results and the analysis of the results, that are related to the determination of these anomalies.

2.0 The $1^{\circ}\times1^{\circ}$ Free-Air Anomaly Data

The starting point for these computations was a set of 19,846 $1^{\circ}\times1^{\circ}$ mean anomalies, referenced to the International Gravity Formula, their accuracy, and other pertinent information provided by ACIC (Creighton, 1971). These anomalies are a slightly updated set from the 19,164 anomalies described in a recent ACIC report (1971).

The procedures for estimating the $1^{\circ}\times1^{\circ}$ anomalies were varied but specified on the ACIC data. Of the 19,846 ACIC anomalies 2708 had been computed on the basis of gravity-geophysical correlation prediction techniques as described, for example, by Durbin (1972). These geophysically derived anomalies are, at times, treated separately in this discussion, as they are not based on actual gravity measurements.

Examination of the ACIC material revealed certain areas that were not included. It was then decided to do an updating of the ACIC material incorporating new material both as additional material and as replacement material. This updating process consisted of literature searches and correspondence with persons who could make available data that would be helpful to this compilation.

One of the biggest gaps in the ACIC data occurred in Canada. To obtain such anomalies it was necessary to resort to fast, but somewhat less than rigorous procedures. In the specific case of Canada, a tape containing 131,105 point anomalies in Canada was provided to us by J. Tanner. Using this data a set of mean Bouguer anomalies were formed for land areas by simple meaning the point values that fell within a $1^{\circ}\times 1^{\circ}$ block. These anomalies were then converted to free-air anomalies using mean elevations provided by ACIC. For ocean areas the mean free air anomalies were computed as a mean of the point free-air anomalies. Using this procedure we replaced 323 of the ACIC anomalies and added 1989 $1^{\circ}\times 1^{\circ}$ anomalies in the Canadian area.

The procedures followed for other sources depended specifically on the source. Some anomalies were obtained directly as recommended values by an investigator. Other anomalies were obtained by estimations from anomaly contour maps. In many cases we would have more than one anomaly estimate for a $1^{\circ}\times 1^{\circ}$ block. The anomaly that was selected to be used was generally the one that had the smaller standard error estimated for it. In the case of an anomaly given by ACIC and another source, the value given by ACIC was accepted if the standard errors were similar.

At the conclusion of this updating procedure we had examined 11,350 $1^{\circ}\times 1^{\circ}$ mean anomalies excluding the ACIC anomalies. Of these anomalies, 862 were used to replace values given by ACIC and 3510 were used where ACIC had given no value. Combining this material we obtained a set of 23,355 $1^{\circ}\times 1^{\circ}$ mean anomalies that form the basis for the computations described in this paper.

The accuracy of the $1^{\circ}\times 1^{\circ}$ mean anomalies was determined in several ways. The material from ACIC had specific accuracy estimates given. For non-ACIC material we automatically assigned a standard error of ± 20 mgals unless specific information was available on the number of points that were used in the estimation of the $1^{\circ}\times 1^{\circ}$ anomaly. If this information was available, a standard error was assigned on the basis of the number of points in the block. The specific criteria used are given in Table One.

Table One
$1^{\circ}\times 1^{\circ}$ Block Accuracy As a Function of the Number of Point Anomalies Used

No. of Anomalies	Accuracy
1 - 3	± 23 mgals
4 - 7	21
8 - 14	19
15 - 25	17
26 - 40	15
41 - 61	13
62 - 85	11
86 - 114	9
115 - 250	7
>250	± 5 mgals

The data for Table One was obtained by analyzing the relationship between the anomaly accuracy and number of points used in the anomaly estimation for all the ACIC anomaly data where such information was given. Such a procedure is not ideal because it does not specifically consider the accuracy of the observed data nor the distribution of the point data within a block. Such considerations would be of prohibitive expense to this study.

2.1 Gravity Formula Conversion

The $1^{\circ}\times 1^{\circ}$ anomalies described in section 2.0 were referred to the International Gravity Formula. In the computations of this paper we desired to refer our anomalies to a gravity formula more consistent with current estimates of critical earth constants. Such estimates are (Rapp, 1971):

$$a = 6378137.8 \text{ m}$$
$$f = 1/298.258$$
$$kM = 3.986013 \times 10^{14} \text{ m}^3/\text{sec}^2$$

The value of kM includes the mass of the atmosphere so that gravity values computed using this kM should (at the surface of the earth) be reduced by - 0.87 mgals (Ecker and Mittemayer, 1967) to obtain a theoretical gravity consistent with measurements taken on the surface of the earth, inside the atmosphere. The gravity formula consistent with the above constants and considerations is:

$$\gamma = \gamma_E \left(1 + .00530243 \sin^2 \varphi - .00000587 \sin^2 2\varphi \right) \qquad (1)$$

where the equatorial gravity, γ_E, is 978033.51 mgals.

In order to convert anomalies given with respect to the International Gravity Formula referred to the Potsdam system to the gravity formula given in equation (1) which is in an absolute system, we adopt a Potsdam correction of -14 mgals (IAG, 1971). If Δg_I are anomalies referred to the International Gravity Formula and Δg_1 are the anomalies referred to the gravity formula of equation (1), we have:

$$\Delta g_1 = \Delta g_I + (1.49 - 13.71 \sin^2 \varphi) \text{ mgals} \tag{2}$$

where φ is the mean latitude of the anomaly block.

Unless otherwise stated, all computations in this paper were done using anomalies that had been referred to the gravity formula given in equation (1).

3.0 Method of Computation of the 300 n.m. Mean Anomalies

The $1^\circ \times 1^\circ$ anomalies were processed using basically the same techniques and computer programs used by Kaula (1966). The $1^\circ \times 1^\circ$ means were first formed into mean anomalies for areas of 60 n.m. (in latitude) and 60 ±30 n.m. (in longitude). These anomalies were then used to estimate the mean anomalies in 300 n.m. (in latitude) and 300 ±30 n.m. (in longitude) blocks by predicting, in the 300 n.m. block any missing 60 n.m. blocks by linear regression. The 300n.m. mean anomaly block was then formed as a straight average of the 25 60 n.m. blocks in the 300 n.m. blocks. No estimations were made for a 300 n.m. block unless it contained one or more observed 60 n.m. blocks.

The specific equation used for predicting a 60 n.m. anomaly (g*) was given by Moritz (1969):

$$g^* = \underline{C}_p(\underline{C} + \underline{D})^{-1} \underline{g} \tag{3}$$

where $\underline{C}_p$ is a column vector whose elements are the covariance between the block (p) to be predicted and the observed anomalies. $\underline{C}$ is a matrix whose elements are the covariances between the observed anomalies; $\underline{D}$ is an error covariance matrix for the known blocks, and $\underline{g}$ is a column vector of the observed anomalies within the 300 n.m. blocks in which g* was situated. For these computations $\underline{D}$ is taken to be a diagonal matrix with each diagonal element being equal to $1/m_j^2$ where m_j is the accuracy estimate of the observed anomaly g_j. The procedure presented differs from that used by Kaula (1966) in that we now do not consider the $1^\circ \times 1^\circ$ data perfect.

The accuracy (m) of the 300 n. m. anomaly was computed from the following (Heiskanen and Moritz, 1967, section 7-9, Moritz, 1969, p. 11):

$$m^2 = \overline{\overline{C}} - \underline{\overline{C}}_i \, (\underline{C} + \underline{D})^{-1} \, \underline{\overline{C}}_i' \qquad (4)$$

where $\overline{\overline{C}}$ is the mean square value (or variance) of the 300 n. m. mean anomalies, and $\underline{\overline{C}}_i$ is a column vector representing the covariance between the i th observed anomaly and the 300 n. m. block in which it lies. We have:

$$\underline{\overline{C}}_i = \frac{1}{n} \sum_{k=1}^{n} C_{i,k} \qquad (5)$$

and

$$\overline{\overline{C}} = \frac{1}{n^2} \sum_{k=1}^{n} \sum_{j=1}^{n} C_{j,k} \qquad (6)$$

where n is the total number of 60 n. m. blocks within the 300 n. m. block (i. e. n = 25).

The accuracy estimation obtained through equation (4) will reflect both representation errors and errors contributed through inaccurate $1^\circ \times 1^\circ$ data.

4.0 300 n. m. Anomaly Set

4.1 Numerical Procedures and Results

We first determined the covariances from the 60 n. m. block anomalies that are needed in the evaluation of the $C_{i,k}$ and $C_{j,k}$ values of equations (5) and (6). For comparison purposes this was done using all the $1^\circ \times 1^\circ$ data and then a $1^\circ \times 1^\circ$ data set that excluded the geophysically predicted anomalies. The covariances were computed using the same procedures and program used by Kaula (1966). The results are given in Table Two.

Table Two
Short-Range Autocovariances of 60 ± 30 n. m.
Free Air Gravity Anomalies

	All Anomalies Used		Geophysically Predicted Anomalies Excluded	
Distance deg	Number of Pairs in Sample	Covariance $mgal^2$	Number of Pairs in Sample	Covariance $mgal^2$
0.00	19837	690	19245	697
0.97	22592	317	21198	309
1.60	41526	227	38956	217
2.50	55997	163	52182	147
3.46	42947	95	40318	71
4.37	24849	39	23167	17
5.34	3810	-16*	3579	-42*
6.18	110	-63*	110	-92*

*interpolated values

Using the 1° covariances where all the data was used a set of covariances $C_{i,k}$ and $\overline{C}_i$, $\overline{C}$ were computed and used to carry out the prediction of 1283 300 n. m. blocks based on all the $1^\circ \times 1^\circ$ data. The anomaly variance from the predicted anomalies was 245 $mgal^2$ while $\overline{\overline{C}}$ was 182 $mgal^2$ as computed from the 1° covariance data. This latter value is smaller than that actually found due to the fact that more anomaly variability exists in the actual anomalies than is represented by the covariance functions. In order to use the more realistic higher values of $C_{i,k}$, $\overline{C}_i$, and $\overline{\overline{C}}$ the covariance functions previously obtained for these quantities were scaled by a factor which would make $\overline{\overline{C}}$ equal to the variance of the 300 n. m. means as obtained from the first prediction tests. Using these covariances, the 1283 anomalies were predicted again, as well as a set of 1249 300 n. m. anomalies that wrer predicted excluding all geophysically predicted 1° anomalies. Maps showing the location of the 1283 anomaly set are given in Figure 1 A and 1 B where the location of the anomaly is indicated by the integer part of the rounded standard error of the prediction.

The results of these predictions are given in Table Three. The first section of the table given the 1249 anomalies predicted excluding all geophysically predicted anomalies. These anomalies are followed by 121 anomalies which were predicted from the 1° anomaly set that included the geophysically predicted anomalies. Thirty-four of the 121 values do not appear in the 1249 anomaly set. If one wishes to have a set of 300 n. m. mean anomalies based only on actual gravity observations, the 1249 set should be used. If a requirement

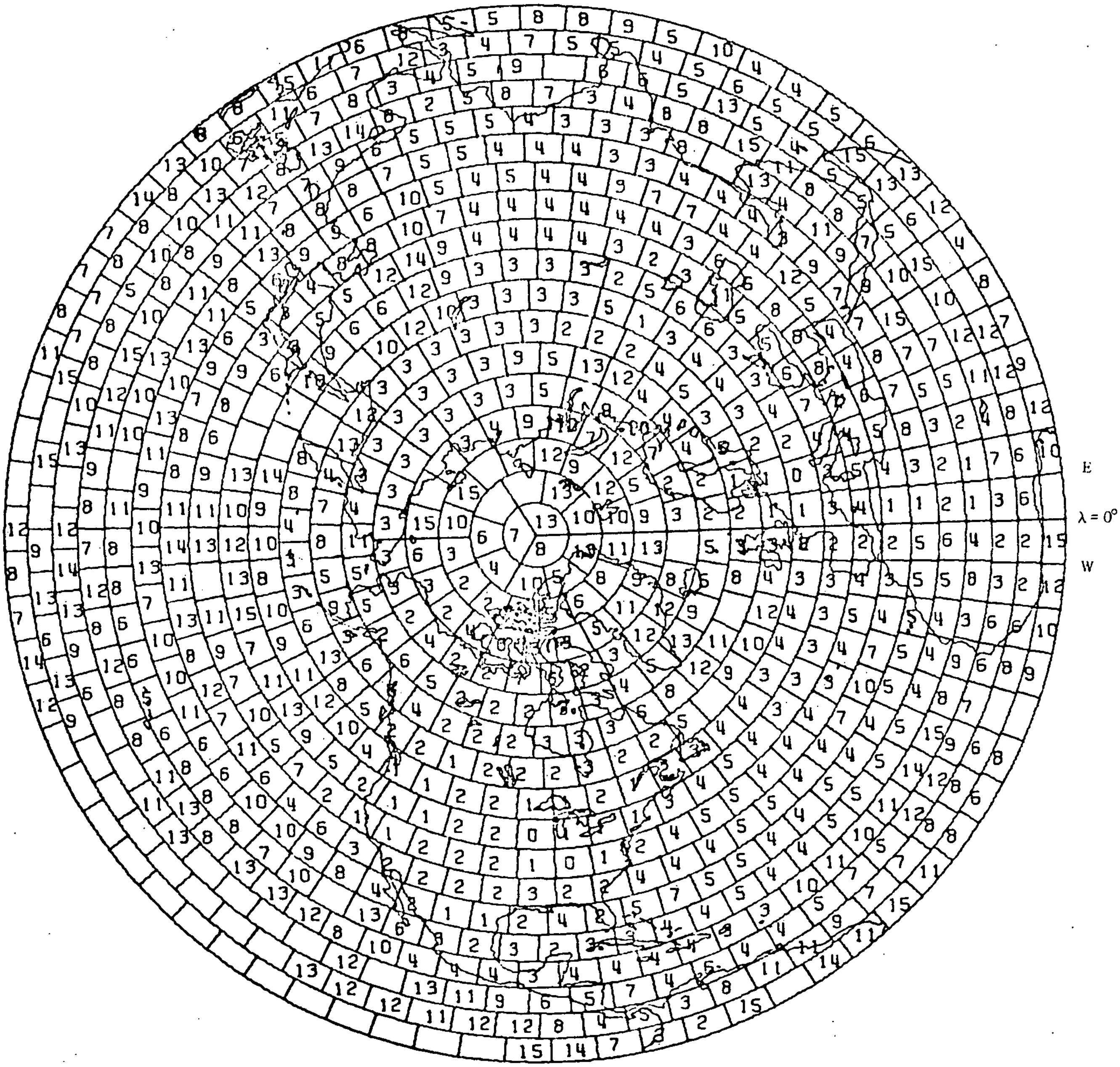

Figure 1A
Standard Errors of 1283 Predicted Anomalies
Northern Hemisphere

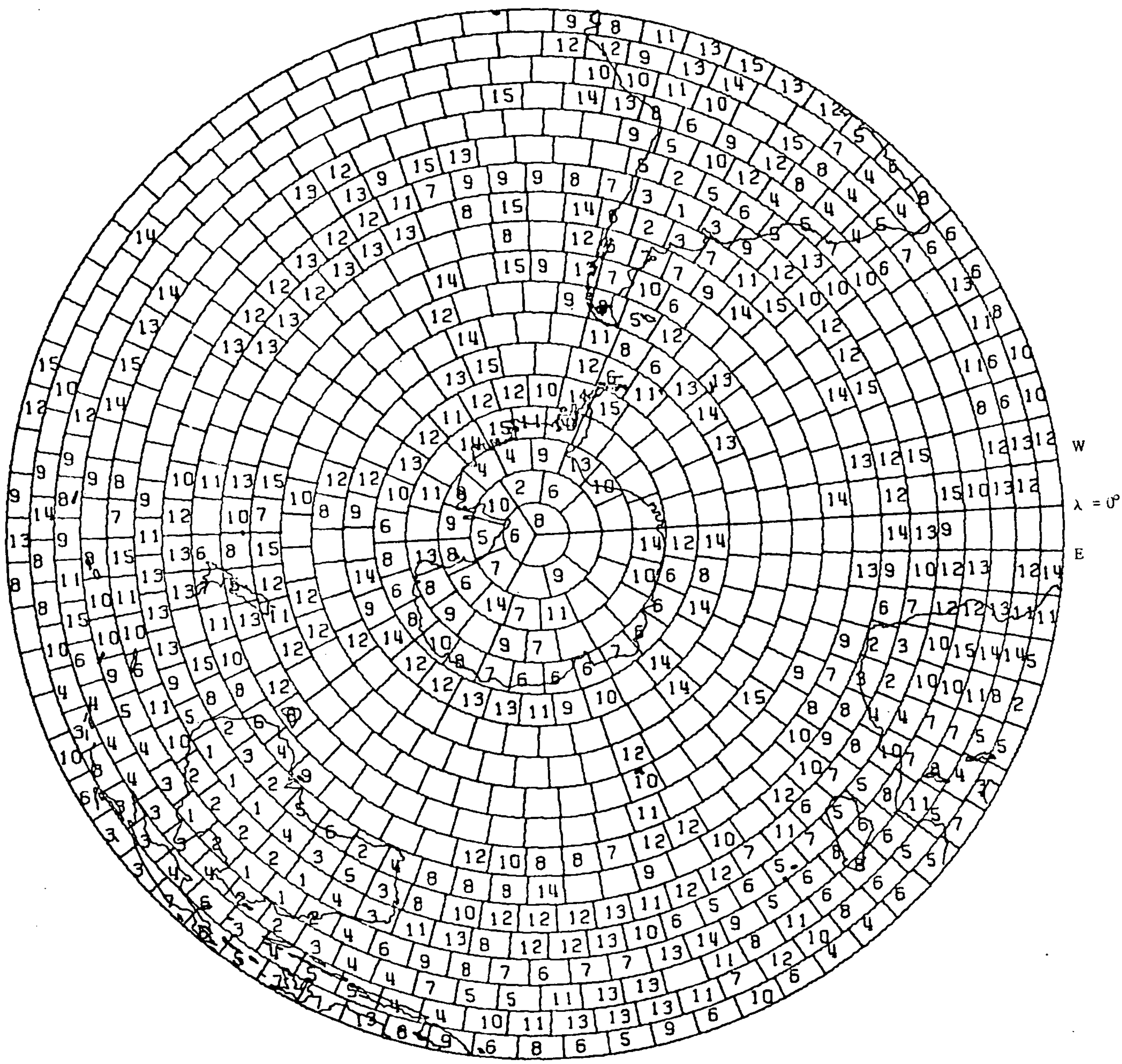

Figure 1B
Standard Errors of 1283 Predicted Anomalies
Southern Hemisphere

Table Three

1249 300 n.m. free-air mean anomalies computed using

no geophysically predicted $1^\circ \times 1^\circ$ anomalies

φ	λ	Δg	m	n	φ	λ	Δg	m	n	φ	λ	Δg	m	n	φ	λ	Δg	m	n	φ	λ	Δg	m	n
97.5	60.0	2	13	3	97.5	190.0	5	7	13	97.5	300.0	4	8	9	82.5	20.0	10	10	7	82.5	60.0	18	13	2
92.5	190.0	-1	6	16	92.5	220.0	5	4	20	32.5	260.0	6	10	7	82.5	300.0	-12	5	15	82.5	340.0	9	13	4
77.5	11.5	-8	10	9	77.5	34.0	-2	12	3	77.5	56.5	-10	9	5	77.5	79.0	-8	13	2	77.5	146.5	-15	15	1
77.5	169.0	-7	10	7	77.5	191.5	4	3	24	77.5	214.0	-12	2	25	77.5	236.5	-2	2	25	77.5	259.0	-9	2	25
77.5	281.5	1	5	18	77.5	304.0	15	6	12	77.5	326.5	-8	8	7	77.5	349.0	19	11	4	72.5	8.0	17	9	10
72.5	24.5	-3	5	19	72.5	41.0	0	12	3	72.5	57.0	-7	7	11	72.5	73.5	-2	13	2	72.5	90.0	-7	10	9
72.5	106.5	-10	12	3	72.5	172.0	-1	15	1	72.5	188.0	-4	6	16	72.5	204.5	-17	3	25	72.5	221.0	-12	2	25
72.5	237.0	-8	4	19	72.5	253.5	-19	6	13	72.5	270.0	-6	3	23	72.5	286.5	-4	3	22	72.5	303.0	-6	5	18
72.5	319.0	16	11	7	72.5	335.5	9	9	8	72.5	352.0	13	13	2	67.5	6.5	10	3	24	67.5	19.5	-1	2	24
67.5	32.5	0	7	10	67.5	45.0	-3	10	6	67.5	57.5	-8	9	7	67.5	83.5	4	13	3	67.5	186.5	-11	7	15
67.5	199.5	-3	3	22	67.5	212.5	13	2	25	67.5	225.0	4	4	21	67.5	237.5	-3	2	25	67.5	250.5	-24	2	25
67.5	263.5	-29	7	14	67.5	276.5	-35	6	15	67.5	289.5	-20	2	25	67.5	302.5	19	3	25	67.5	315.0	1	12	5
67.5	327.5	20	12	3	67.5	340.5	27	8	11	62.5	5.5	11	2	25	62.5	16.5	-10	2	24	62.5	27.5	-7	4	23
62.5	39.5	-1	4	19	62.5	49.5	1	4	20	62.5	60.0	-4	12	4	62.5	70.5	-11	13	2	62.5	81.5	-2	15	1
62.5	190.5	-2	5	19	62.5	201.5	12	5	19	62.5	212.5	23	2	24	62.5	223.5	27	6	17	62.5	234.5	10	5	18
62.5	245.5	-19	2	25	62.5	255.5	-31	2	25	62.5	267.5	-40	2	25	62.5	278.5	-28	3	25	62.5	289.5	-12	3	21
62.5	300.0	3	4	19	62.5	310.5	-12	5	17	62.5	321.5	6	13	3	62.5	332.5	27	9	9	62.5	343.5	27	5	18
62.5	354.5	16	5	19	57.5	4.5	1	2	24	57.5	13.5	0	1	25	57.5	23.0	-16	5	18	57.5	32.5	-2	3	24
57.5	41.5	2	5	18	57.5	50.5	6	4	19	57.5	60.0	-1	13	4	57.5	69.5	-8	8	7	57.5	78.5	-10	10	8
57.5	87.5	-11	9	8	57.5	97.0	-16	13	2	57.5	161.5	-6	12	4	57.5	170.5	4	7	13	57.5	190.0	-15	8	12
57.5	189.5	12	5	20	57.5	199.5	26	9	10	57.5	207.5	9	3	24	57.5	217.0	6	6	15	57.5	226.5	12	5	19
57.5	235.5	13	4	20	57.5	244.5	-6	2	25	57.5	253.5	-13	2	25	57.5	263.0	-26	2	25	57.5	272.5	-41	3	23
57.5	281.5	-36	4	21	57.5	290.5	-30	4	21	57.5	300.0	-18	6	15	57.5	309.5	7	8	9	57.5	318.5	6	12	5
57.5	327.5	24	11	5	57.5	346.5	5	8	12	57.5	355.5	16	3	24	52.5	4.0	-7	1	25	52.5	12.0	8	1	25
52.5	20.5	0	2	23	52.5	29.0	3	3	24	52.5	37.0	0	4	21	52.5	45.0	-2	4	22	52.5	53.0	-7	3	25
52.5	61.0	-1	9	10	52.5	69.5	-2	5	15	52.5	78.0	-21	3	23	52.5	86.0	-10	4	22	52.5	94.0	-4	12	4
52.5	102.0	-14	11	4	52.5	110.5	-11	10	7	52.5	119.0	3	12	4	52.5	127.0	4	10	8	52.5	159.5	-10	15	1
52.5	168.0	6	8	9	52.5	176.0	-4	4	24	52.5	184.0	-24	3	24	52.5	192.0	3	3	23	52.5	200.5	-1	6	15
52.5	209.0	7	13	3	52.5	217.0	4	8	8	52.5	225.0	-2	5	20	52.5	233.0	12	2	25	52.5	241.0	6	2	25
52.5	249.5	-3	1	25	52.5	258.0	0	2	25	52.5	266.0	-9	2	25	52.5	274.0	-26	2	25	52.5	282.0	-32	3	24
52.5	290.5	-10	2	25	52.5	299.0	-21	2	25	52.5	307.0	-1	5	20	52.5	323.0	21	9	9	52.5	331.0	12	10	7
52.5	339.5	-3	12	4	52.5	348.0	15	4	21	52.5	356.0	11	3	24	47.5	3.5	5	1	25	47.5	11.0	10	0	25
47.5	19.5	18	2	24	47.5	25.5	19	4	21	47.5	33.0	9	3	24	47.5	40.5	-4	3	25	47.5	47.5	-14	6	14
47.5	55.0	-23	6	14	47.5	62.5	-11	5	15	47.5	69.5	-10	7	13	47.5	77.0	-20	5	18	47.5	84.5	-19	11	4
47.5	92.0	2	15	1	47.5	114.0	-4	12	5	47.5	121.5	10	6	13	47.5	128.5	8	6	15	47.5	136.0	5	9	7
47.5	143.5	-1	10	10	47.5	165.5	13	14	1	47.5	172.5	5	9	9	47.5	180.0	11	10	10	47.5	197.5	15	8	10
47.5	194.5	12	10	6	47.5	202.0	6	9	8	47.5	209.5	-2	11	4	47.5	216.5	-5	12	3	47.5	224.0	-2	10	8
47.5	231.5	-10	4	23	47.5	238.5	-7	1	25	47.5	246.0	8	1	25	47.5	253.5	14	2	25	47.5	260.5	7	2	25
47.5	268.0	2	1	25	47.5	275.5	-11	1	25	47.5	283.0	-15	2	25	47.5	290.5	-7	1	25	47.5	297.5	-11	2	25
47.5	305.0	12	3	24	47.5	312.5	18	4	22	47.5	319.5	6	4	20	47.5	327.0	23	3	24	47.5	334.5	34	4	20
47.5	341.5	16	4	22	47.5	349.0	4	3	24	47.5	356.5	0	2	25	42.5	3.5	12	3	23	42.5	10.5	12	3	23
42.5	17.0	22	4	20	42.5	23.5	21	7	11	42.5	30.5	-7	11	4	42.5	37.5	8	14	2	42.5	44.5	10	5	16
42.5	51.0	-30	11	6	42.5	57.5	-7	4	17	42.5	64.5	-21	3	20	42.5	71.5	-20	6	14	42.5	78.5	-9	8	9
42.5	85.0	-6	13	3	42.5	91.5	-35	11	6	42.5	112.0	-2	14	2	42.5	118.5	4	12	4	42.5	125.5	18	5	16
42.5	132.5	4	5	13	42.5	139.5	13	3	25	42.5	146.0	-17	6	16	42.5	166.5	-2	13	3	42.5	173.5	4	10	7
42.5	180.0	1	12	3	42.5	186.5	-5	13	2	42.5	193.5	-3	15	1	42.5	200.5	-4	7	12	42.5	207.5	0	11	4
42.5	214.0	-1	13	3	42.5	220.5	-11	9	11	42.5	227.5	-8	5	20	42.5	234.5	-11	2	25	42.5	241.5	10	1	25
42.5	249.0	22	1	25	42.5	254.5	15	2	25	42.5	261.5	-3	2	25	42.5	268.5	-10	0	25	42.5	275.0	-11	1	25
42.5	281.5	-11	1	25	42.5	288.5	0	1	25	42.5	295.5	-6	3	25	42.5	302.5	-18	4	25	42.5	309.0	12	5	19
42.5	315.5	4	4	23	42.5	322.5	13	3	25	42.5	329.5	27	3	25	42.5	336.5	25	3	25	42.5	343.0	2	3	24

42.5	349.5	5	3	23	42.5	356.5	8	2	25	37.5	3.0	13	4	21	37.5	9.5	23	5	18	37.5	16.0	7	4	23
37.5	22.0	-3	7	14	37.5	29.5	-1	8	11	37.5	35.0	24	8	11	37.5	41.0	9	8	8	37.5	47.5	42	6	13
37.5	54.0	-3	8	9	37.5	60.0	-5	10	5	37.5	66.0	-16	8	9	37.5	72.5	-6	5	19	37.5	79.0	-14	8	10
37.5	85.0	-39	12	2	37.5	110.5	-13	10	6	37.5	117.0	-19	9	7	37.5	123.0	8	11	5	37.5	129.5	14	4	20
37.5	135.0	24	3	25	37.5	142.0	23	3	24	37.5	148.5	1	9	9	37.5	155.0	3	8	8	37.5	161.0	-2	6	13
37.5	167.5	-3	9	9	37.5	174.0	1	11	7	37.5	180.0	-3	13	3	37.5	192.5	-9	11	5	37.5	199.0	-8	9	8
37.5	205.0	-4	7	13	37.5	211.5	4	10	10	37.5	218.0	-20	5	21	37.5	224.0	-18	7	15	37.5	230.5	-25	2	25
37.5	237.0	-17	1	25	37.5	243.0	-1	1	25	37.5	249.5	7	2	25	37.5	256.0	9	2	25	37.5	262.0	-12	2	25
37.5	269.5	-7	1	25	37.5	275.0	-5	0	25	37.5	281.0	0	1	25	37.5	287.5	-22	2	25	37.5	294.0	-23	4	22
37.5	300.0	-17	5	19	37.5	306.0	-14	5	20	37.5	312.5	-2	4	21	37.5	319.0	16	4	25	37.5	325.0	33	4	25
37.5	331.5	40	3	25	37.5	338.0	15	4	23	37.5	344.0	5	5	17	37.5	350.5	6	4	22	37.5	357.0	11	2	25
32.5	3.0	3	1	25	32.5	9.0	-10	4	19	32.5	15.0	0	5	18	32.5	21.0	8	6	16	32.5	27.0	-36	4	24
32.5	32.5	-17	4	25	32.5	38.0	25	5	17	32.5	44.0	-4	12	5	32.5	50.0	6	5	17	32.5	56.0	17	10	7
32.5	62.0	4	14	1	32.5	69.0	4	10	8	32.5	74.0	-41	3	24	32.5	80.0	41	11	7	32.5	86.0	6	15	1
32.5	109.0	-22	10	7	32.5	115.0	-26	6	14	32.5	121.0	0	9	9	32.5	127.0	17	9	10	32.5	133.0	19	6	14
32.5	139.0	-20	5	17	32.5	145.0	-5	6	14	32.5	150.5	-5	9	9	32.5	156.0	-7	7	12	32.5	162.0	-13	8	11
32.5	168.0	-16	8	9	32.5	174.0	-11	11	5	32.5	180.0	-4	14	1	32.5	186.0	-24	11	6	32.5	192.0	-14	13	3
32.5	198.0	-4	8	9	32.5	204.0	-5	12	4	32.5	209.5	-3	7	13	32.5	215.0	-13	11	8	32.5	221.0	-9	6	17
32.5	227.0	-19	4	23	32.5	233.0	-20	6	15	32.5	239.0	-22	3	23	32.5	245.0	-12	2	25	32.5	251.0	-1	2	25
32.5	257.0	-4	2	25	32.5	263.0	-8	2	25	32.5	268.5	0	3	25	32.5	274.0	-2	2	25	32.5	290.0	-5	2	24
32.5	286.0	-39	4	22	32.5	292.0	-26	4	22	32.5	298.0	-9	4	22	32.5	304.0	-15	5	20	32.5	310.0	-4	5	20
32.5	316.0	10	4	23	32.5	322.0	27	4	24	32.5	327.5	17	7	14	32.5	333.0	5	10	10	32.5	339.0	-3	7	13
32.5	345.0	5	4	23	32.5	351.0	26	3	23	32.5	357.0	41	2	25	27.5	3.0	-12	2	24	27.5	8.5	2	5	15
27.5	14.0	6	11	5	27.5	20.0	3	11	5	27.5	25.5	2	9	8	27.5	31.0	6	7	13	27.5	36.5	1	7	12
27.5	42.0	2	10	7	27.5	48.0	-11	4	19	27.5	53.5	-16	9	10	27.5	59.0	29	10	7	27.5	65.0	5	6	19
27.5	70.5	-13	4	23	27.5	76.0	-17	3	25	27.5	91.5	-75	6	16	27.5	87.0	-32	7	13	27.5	93.0	-50	8	12
27.5	99.5	-6	11	8	27.5	104.0	-13	10	9	27.5	110.0	-20	7	14	27.5	115.5	-8	8	10	27.5	121.0	4	8	9
27.5	126.5	26	8	9	27.5	132.0	-19	13	3	27.5	138.0	16	8	9	27.5	143.5	-11	11	5	27.5	149.0	0	13	4
27.5	155.0	29	10	9	27.5	160.5	-4	13	3	27.5	166.0	-9	11	6	27.5	171.5	-9	9	10	27.5	177.0	-4	10	9
27.5	183.0	-13	13	3	27.5	188.5	-1	7	12	27.5	194.0	6	10	7	27.5	200.0	8	10	6	27.5	205.5	-2	11	7
27.5	211.0	-5	6	16	27.5	216.5	1	6	15	27.5	222.0	-13	10	10	27.5	228.0	-14	10	6	27.5	233.5	-17	9	8
27.5	239.0	-23	8	12	27.5	245.0	-19	4	21	27.5	250.5	-7	2	25	27.5	256.0	15	1	25	27.5	261.5	-4	1	25
27.5	267.0	-9	2	25	27.5	273.0	-3	4	20	27.5	278.5	3	2	25	27.5	284.0	-18	5	21	27.5	290.0	-25	7	11
27.5	295.5	-17	5	16	27.5	301.0	-29	5	20	27.5	306.5	-21	4	22	27.5	312.0	-2	4	21	27.5	318.0	8	5	18
27.5	323.5	-2	4	22	27.5	329.0	0	4	21	27.5	335.0	-10	5	20	27.5	340.5	0	5	20	27.5	346.0	14	5	18
27.5	351.5	11	5	19	27.5	357.0	-4	5	17	22.5	2.5	18	5	16	22.5	8.0	33	5	16	22.5	13.5	2	7	13
22.5	18.5	-5	15	1	22.5	29.5	4	15	1	22.5	35.0	-3	9	11	22.5	40.5	33	9	8	22.5	45.5	0	13	3
22.5	51.0	-30	4	20	22.5	56.5	-8	14	2	22.5	67.0	-6	8	11	22.5	72.5	2	3	25	22.5	78.0	1	3	25
22.5	83.5	-1	3	25	22.5	88.5	-9	4	21	22.5	94.0	-23	5	22	22.5	99.5	-12	9	12	22.5	104.5	-17	10	7
22.5	110.0	-10	7	10	22.5	115.5	-9	9	8	22.5	121.0	1	9	7	22.5	126.5	-1	7	12	22.5	131.5	1	11	6
22.5	137.0	0	9	6	22.5	142.5	1	11	4	22.5	153.0	8	13	4	22.5	158.5	-13	10	7	22.5	164.0	-10	10	5
22.5	174.5	-6	11	5	22.5	180.0	-9	8	13	22.5	185.5	2	8	10	22.5	190.5	6	6	15	22.5	196.0	10	6	13
22.5	201.5	24	5	17	22.5	207.0	5	6	18	22.5	212.5	-7	8	10	22.5	217.5	-5	8	11	22.5	223.0	-8	8	10
22.5	229.5	-12	7	13	22.5	233.5	-9	10	6	22.5	244.5	-9	13	5	22.5	250.0	-10	6	17	22.5	255.5	11	3	23
22.5	260.5	20	2	25	22.5	266.0	1	3	25	22.5	271.5	12	2	24	22.5	276.5	-5	4	22	22.5	282.0	-12	4	23
22.5	287.5	-57	4	24	22.5	293.0	-35	4	24	22.5	298.5	-28	5	21	22.5	303.5	-22	4	24	22.5	309.0	-23	4	23
22.5	314.5	4	5	19	22.5	319.5	0	5	18	22.5	325.0	0	5	19	22.5	330.5	8	5	20	22.5	336.0	-3	4	21
22.5	341.5	4	4	21	22.5	346.5	-3	4	21	22.5	352.0	-14	5	16	22.5	357.5	-5	6	15	17.5	2.5	7	1	25
17.5	7.5	5	1	25	17.5	13.0	-3	2	24	17.5	19.5	-11	5	11	17.5	23.5	5	7	12	17.5	34.0	10	10	6
17.5	39.5	4	7	13	17.5	44.5	8	8	11	17.5	49.5	-20	8	12	17.5	54.5	3	11	6	17.5	60.0	2	15	1

17.5	65.5	-11	8	9	17.5	70.5	-19	8	10	17.5	75.5	-23	4	24	17.5	80.5	-10	3	24	17.5	86.0	-13	7	13
17.5	91.5	-17	8	10	17.5	96.5	-8	6	18	17.5	101.5	-16	3	22	17.5	107.0	-13	7	12	17.5	112.5	0	15	1
17.5	117.5	14	13	2	17.5	122.5	34	9	11	17.5	127.5	5	12	4	17.5	133.0	3	11	6	17.5	138.5	1	8	9
17.5	143.5	24	8	10	17.5	148.5	17	10	8	17.5	154.0	-7	15	1	17.5	159.5	-9	12	5	17.5	164.5	-8	11	6
17.5	169.5	-7	9	11	17.5	174.5	-9	8	10	17.5	180.0	-2	7	12	17.5	185.5	0	12	5	17.5	190.5	-11	8	11
17.5	195.5	0	12	4	17.5	200.5	8	8	11	17.5	206.0	22	8	8	17.5	211.5	1	11	6	17.5	216.5	2	13	3
17.5	221.5	-11	8	9	17.5	227.0	-8	13	3	17.5	232.5	-10	13	4	17.5	237.5	-12	12	3	17.5	242.5	-15	8	8
17.5	247.5	-13	10	6	17.5	253.0	-12	4	23	17.5	258.5	-7	4	23	17.5	263.5	30	4	22	17.5	268.5	14	3	24
17.5	274.0	21	4	23	17.5	279.5	4	4	25	17.5	284.5	-3	4	24	17.5	289.5	16	4	24	17.5	294.5	-45	3	25
17.5	300.0	-37	3	25	17.5	305.5	-17	10	7	17.5	310.5	-12	11	6	17.5	315.5	-12	10	6	17.5	320.5	-16	11	4
17.5	326.0	-6	14	2	17.5	331.5	2	15	1	17.5	336.5	24	8	8	17.5	341.5	1	9	10	17.5	347.0	12	3	22
17.5	352.5	1	8	11	17.5	357.5	6	4	21	12.5	2.5	1	3	22	12.5	7.5	8	7	15	12.5	12.5	8	4	18
12.5	19.0	2	11	4	12.5	23.5	-4	12	3	12.5	28.5	5	10	5	12.5	33.5	-4	15	1	12.5	38.5	22	5	16
12.5	43.5	4	5	17	12.5	48.5	3	5	15	12.5	54.0	-4	4	21	12.5	59.5	-9	5	17	12.5	64.5	-5	13	3
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7.5	149.5	-6	7	13	7.5	154.5	-59	7	10	7.5	159.5	5	15	1	7.5	169.5	37	15	1	7.5	180.0	5	9	5
7.5	185.5	-9	13	3	7.5	190.5	-6	6	16	7.5	195.5	-1	13	3	7.5	200.5	22	9	7	7.5	240.5	-6	13	4
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7.5	322.0	-13	8	11	7.5	327.5	7	8	11	7.5	332.5	-1	6	13	7.5	342.5	7	8	8	7.5	347.5	29	6	13
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2.5	122.5	24	8	11	2.5	127.5	9	8	11	2.5	132.5	23	13	3	2.5	137.5	-1	14	2	2.5	142.5	4	7	15
2.5	147.5	10	7	15	2.5	152.5	11	8	11	2.5	157.5	2	11	5	2.5	177.5	4	12	5	2.5	182.5	3	8	12
2.5	187.5	-1	7	13	2.5	192.5	3	14	2	2.5	197.5	14	12	5	2.5	267.5	2	15	1	2.5	272.5	0	14	2
2.5	277.5	8	7	13	2.5	282.5	24	3	25	2.5	287.5	26	2	25	2.5	292.5	3	15	1	2.5	302.5	-11	14	2
2.5	307.5	-4	11	5	2.5	312.5	-2	15	1	2.5	317.5	-11	11	6	2.5	322.5	-5	8	13	2.5	327.5	10	6	18
2.5	332.5	5	8	13	2.5	342.5	-7	9	6	2.5	347.5	-8	10	6	2.5	352.5	5	12	4	2.5	357.5	2	15	1
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-2.5	82.5	-54	5	20	-2.5	87.5	-32	6	16	-2.5	92.5	-19	8	11	-2.5	97.5	4	6	15	-2.5	102.5	9	9	11

-2.5	107.5	29	6	17	-2.5	112.5	12	13	3	-2.5	117.5	8	7	13	-2.5	122.5	-3	7	16	-2.5	127.5	9	5	21
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-2.5	157.5	18	10	7	-2.5	177.5	-6	8	11	-2.5	182.5	4	13	3	-2.5	187.5	-1	9	8	-2.5	197.5	15	12	5
-2.5	202.5	6	15	1	-2.5	277.5	0	9	7	-2.5	282.5	8	8	11	-2.5	287.5	25	11	5	-2.5	292.5	15	13	3
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-7.5	22.5	-11	8	12	-7.5	27.5	-9	5	21	-7.5	32.5	5	4	23	-7.5	38.0	-1	5	20	-7.5	43.5	-35	5	19
-7.5	49.5	-18	6	14	-7.5	53.5	-2	8	9	-7.5	59.5	-6	10	8	-7.5	63.5	-8	12	5	-7.5	68.5	-5	7	12
-7.5	73.5	-5	11	6	-7.5	78.5	-15	13	3	-7.5	83.5	-13	13	4	-7.5	88.5	-6	13	2	-7.5	93.5	-11	11	5
-7.5	99.5	8	10	10	-7.5	103.5	-15	4	23	-7.5	109.0	18	4	21	-7.5	114.5	17	5	20	-7.5	119.5	-4	5	21
-7.5	124.5	-6	4	24	-7.5	129.5	-26	3	25	-7.5	134.5	9	6	16	-7.5	139.5	16	4	21	-7.5	144.5	25	3	25
-7.5	149.5	20	3	25	-7.5	154.5	36	3	25	-7.5	159.5	23	3	25	-7.5	164.5	-36	4	25	-7.5	169.5	-23	10	9
-7.5	174.5	0	8	9	-7.5	180.0	1	8	8	-7.5	185.5	-5	14	2	-7.5	190.5	-2	9	6	-7.5	200.5	22	10	6
-7.5	220.5	9	14	1	-7.5	276.5	-8	12	3	-7.5	281.5	-8	12	4	-7.5	286.5	-13	9	8	-7.5	291.5	5	13	3
-7.5	296.5	2	14	1	-7.5	311.5	-26	7	17	-7.5	316.5	-28	4	25	-7.5	322.0	-4	4	25	-7.5	327.5	-17	6	15
-7.5	332.5	-6	13	3	-7.5	337.5	-4	11	8	-7.5	342.5	-4	6	18	-7.5	347.5	3	6	18	-7.5	352.5	-4	13	2
-7.5	357.5	-4	12	3	-12.5	17.5	6	13	2	-12.5	18.0	4	14	1	-12.5	23.5	2	11	4	-12.5	28.5	-15	7	13
-12.5	33.5	-7	8	12	-12.5	38.5	-13	11	7	-12.5	43.5	-26	6	13	-12.5	48.5	11	8	10	-12.5	54.0	-5	6	14
-12.5	59.5	-28	11	4	-12.5	64.5	4	9	10	-12.5	69.5	2	11	4	-12.5	79.5	-12	13	3	-12.5	84.5	-9	13	3
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-12.5	167.5	15	6	19	-12.5	172.5	-5	15	1	-12.5	177.5	-2	11	5	-12.5	182.5	17	9	7	-12.5	187.5	14	8	11
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-12.5	295.5	32	10	6	-12.5	306.0	-16	15	1	-12.5	311.5	5	8	15	-12.5	316.5	-8	4	25	-12.5	321.5	-3	5	20
-12.5	326.5	-16	7	14	-12.5	342.0	-4	11	5	-12.5	347.5	-5	8	12	-12.5	352.5	-3	12	3	-12.5	357.5	-2	13	2
-17.5	7.5	1	13	2	-17.5	13.0	5	12	2	-17.5	18.5	4	15	1	-17.5	23.5	0	10	6	-17.5	29.5	10	7	14
-17.5	34.0	-1	7	13	-17.5	39.5	-15	8	10	-17.5	44.5	4	6	17	-17.5	49.5	18	8	12	-17.5	54.5	-8	6	14
-17.5	60.0	20	5	16	-17.5	65.5	-4	9	7	-17.5	70.5	1	14	2	-17.5	75.5	8	13	4	-17.5	80.5	-8	7	12
-17.5	86.0	-16	7	10	-17.5	91.5	-29	6	15	-17.5	96.5	-20	7	12	-17.5	101.5	-22	9	10	-17.5	107.0	-12	9	6
-17.5	112.5	-22	6	15	-17.5	117.5	-5	4	20	-17.5	122.5	11	2	25	-17.5	127.5	13	1	25	-17.5	133.0	4	1	25
-17.5	138.5	9	2	25	-17.5	143.5	20	1	25	-17.5	148.5	15	3	25	-17.5	154.0	11	4	25	-17.5	159.5	1	5	20
-17.5	164.5	18	9	10	-17.5	169.5	24	10	8	-17.5	174.5	20	10	5	-17.5	180.0	18	8	11	-17.5	190.5	6	9	7
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-17.5	279.5	-11	14	1	-17.5	284.5	4	13	3	-17.5	289.5	57	8	11	-17.5	294.5	33	6	17	-17.5	300.0	19	9	7
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-17.5	357.5	-1	10	5	-22.5	2.5	4	9	8	-22.5	9.0	-4	12	3	-22.5	13.5	4	12	3	-22.5	18.5	10	10	6
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-22.5	51.0	8	7	10	-22.5	56.5	7	5	21	-22.5	61.5	19	6	15	-22.5	67.0	30	5	16	-22.5	72.5	16	6	15
-22.5	78.0	-1	10	8	-22.5	83.5	-8	13	3	-22.5	88.5	-11	12	5	-22.5	94.0	-14	12	5	-22.5	99.5	-28	8	10
-22.5	104.5	-7	13	3	-22.5	110.0	5	11	6	-22.5	115.5	5	3	21	-22.5	121.0	-1	4	21	-22.5	126.5	-11	1	25
-22.5	131.5	-10	2	25	-22.5	137.0	8	2	25	-22.5	142.5	5	2	25	-22.5	147.5	22	2	25	-22.5	153.0	8	10	8
-22.5	159.5	7	11	6	-22.5	164.0	34	6	15	-22.5	169.5	23	10	6	-22.5	174.5	24	11	5	-22.5	180.0	4	15	1
-22.5	185.5	-21	7	15	-22.5	190.5	-19	8	7	-22.5	239.0	3	13	3	-22.5	244.5	4	12	5	-22.5	287.5	-1	9	8
-22.5	293.0	60	5	18	-22.5	298.5	-1	10	9	-22.5	303.5	0	12	4	-22.5	309.0	-19	4	22	-22.5	314.5	-4	5	19
-22.5	319.5	-13	7	15	-22.5	325.0	-14	10	6	-22.5	357.5	2	15	1	-27.5	3.0	3	13	3	-27.5	8.5	-2	10	7
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-27.5	98.5	-6	12	5
-27.5	126.5	-16	3	23
-27.5	155.0	4	5	19
-27.5	183.0	29	11	6
-27.5	261.5	-2	13	2
-27.5	312.0	-10	5	19
-32.5	9.0	5	9	8
-32.5	38.0	-3	9	8
-32.5	91.5	11	14	1
-32.5	121.0	-16	2	24
-32.5	150.5	20	2	25
-32.5	196.0	-3	12	4
-32.5	237.0	5	13	3
-32.5	263.0	-11	9	6
-32.5	292.0	29	3	23
-32.5	322.0	3	10	5
-32.5	357.0	10	12	3
-37.5	41.0	13	10	5
-37.5	91.5	-3	8	7
-37.5	148.5	18	6	15
-37.5	180.0	-17	6	18
-37.5	237.0	3	13	2
-37.5	281.0	1	14	1
-37.5	312.5	-10	11	5
-42.5	71.5	-5	11	4
-42.5	180.0	12	8	10
-42.5	288.5	35	6	15
-42.5	356.5	-7	14	1
-47.5	172.5	16	12	4
-47.5	269.0	1	15	1
-47.5	305.0	-13	6	16
-52.5	249.5	6	12	2
-52.5	223.0	25	13	3
-57.5	253.5	-4	14	2
-57.5	327.5	13	14	2
-62.5	49.5	6	14	1
-62.5	114.5	-29	13	2
-62.5	169.5	-1	14	1
-62.5	289.5	0	12	3
-67.5	32.5	25	6	13
-67.5	96.5	20	6	18
-67.5	160.5	-13	6	15
-67.5	225.0	-15	12	3
-67.5	289.5	14	11	5
-72.5	106.5	-31	9	7
-72.5	237.0	-15	14	2
-77.5	101.5	-14	7	12
-77.5	214.0	-16	9	9
-77.5	326.5	-15	10	8
-82.5	269.0	-6	2	24

-27.5	104.0	-12	10	9
-27.5	132.0	-7	4	21
-27.5	160.5	4	9	8
-27.5	188.5	-11	9	9
-27.5	290.0	55	5	18
-27.5	318.0	-12	13	2
-32.5	15.0	2	6	15
-32.5	44.0	35	10	6
-32.5	97.0	-19	8	8
-32.5	127.0	-15	6	16
-32.5	156.0	-10	8	11
-32.5	192.0	-15	10	7
-32.5	239.0	3	12	5
-32.5	268.5	-3	9	5
-32.5	298.0	11	1	25
-32.5	327.5	-12	12	2
-37.5	9.5	-3	13	3
-37.5	66.0	-1	12	4
-37.5	98.0	-11	10	5
-37.5	155.0	-10	8	8
-37.5	192.5	-24	11	8
-37.5	243.0	-3	13	2
-37.5	287.5	22	6	17
-37.5	319.0	-2	15	1
-42.5	146.0	18	8	8
-42.5	186.5	-20	10	7
-42.5	295.5	0	7	13
-47.5	40.5	11	15	1
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-47.5	275.5	10	9	9
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-52.5	282.0	4	9	8
-57.5	50.5	7	14	1
-57.5	290.5	15	11	7
-57.5	337.0	14	13	3
-62.5	70.5	30	10	4
-62.5	125.5	-18	12	3
-62.5	190.5	-10	9	5
-62.5	300.0	38	6	15
-67.5	45.0	4	6	13
-67.5	109.5	15	7	15
-67.5	173.5	-8	8	8
-67.5	237.5	-16	11	6
-67.5	302.5	7	15	1
-72.5	139.0	-17	9	8
-72.5	253.5	-2	15	1
-77.5	124.0	-2	14	1
-77.5	236.5	-6	4	20
-82.5	60.0	28	9	7
-82.5	300.0	-16	6	14

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-27.5	138.0	-10	1	25
-27.5	166.0	13	13	3
-27.5	245.0	0	13	3
-27.5	295.5	8	2	25
-27.5	323.5	-7	10	5
-32.5	21.0	20	2	25
-32.5	50.0	9	12	4
-32.5	103.0	-24	8	7
-32.5	133.0	-17	5	17
-32.5	162.0	-1	15	1
-32.5	215.0	1	13	3
-32.5	245.0	-1	12	5
-32.5	274.0	0	9	5
-32.5	304.0	19	3	24
-32.5	333.0	4	12	3
-37.5	22.0	-3	9	10
-37.5	72.5	-2	12	4
-37.5	104.0	-21	12	6
-37.5	161.0	-7	10	5
-37.5	219.0	-3	13	3
-37.5	249.5	-4	13	2
-37.5	294.0	0	2	25
-37.5	338.0	6	14	2
-42.5	152.5	-7	12	5
-42.5	193.5	-14	13	3
-42.5	332.5	-9	7	14
-47.5	59.5	15	10	5
-47.5	187.5	-19	7	14
-47.5	283.0	-19	13	2
-52.5	159.5	-1	12	6
-52.5	290.5	4	8	9
-57.5	152.5	-1	12	5
-57.5	300.0	15	8	7
-62.5	5.5	4	14	2
-62.5	81.5	12	9	10
-62.5	135.5	-5	12	3
-62.5	201.5	-4	12	2
-62.5	310.5	28	11	7
-67.5	57.5	35	7	13
-67.5	122.5	-16	8	10
-67.5	186.5	-10	6	15
-67.5	250.5	-4	12	2
-72.5	8.0	10	14	2
-72.5	155.5	-23	8	6
-72.5	270.0	9	11	5
-77.5	145.5	-36	6	13
-77.5	259.0	-14	4	22
-82.5	140.0	-32	7	11
-87.5	180.0	-19	6	14

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-27.5	143.5	-5	1	25
-27.5	171.5	5	13	2
-27.5	250.5	-2	9	9
-27.5	301.0	12	5	17
-27.5	351.5	-3	15	1
-32.5	27.0	12	3	23
-32.5	62.0	-6	10	4
-32.5	109.0	-26	8	10
-32.5	139.0	0	2	25
-32.5	174.0	25	13	2
-32.5	221.0	7	12	4
-32.5	251.0	-3	11	6
-32.5	280.0	0	8	5
-32.5	310.0	5	9	8
-32.5	339.0	4	15	1
-37.5	28.5	5	7	12
-37.5	79.0	-6	7	10
-37.5	136.0	3	9	5
-37.5	167.5	4	11	5
-37.5	224.0	-2	13	3
-37.5	262.0	-7	8	9
-37.5	300.0	11	3	22
-37.5	350.5	5	13	3
-42.5	166.5	3	13	3
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-42.5	309.0	-10	9	8
-47.5	158.0	-9	12	5
-47.5	194.5	0	15	1
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-52.5	168.0	12	12	4
-52.5	299.0	1	5	18
-57.5	189.5	-23	8	10
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-62.5	16.5	15	8	8
-62.5	92.5	-17	11	7
-62.5	147.5	-11	14	1
-62.5	245.5	3	13	2
-67.5	6.5	2	12	5
-67.5	70.5	17	6	13
-67.5	135.0	4	10	6
-67.5	199.5	-16	10	5
-67.5	263.5	15	12	2
-72.5	24.5	7	10	6
-72.5	172.0	-16	13	2
-72.5	286.5	22	10	8
-77.5	159.0	-7	8	11
-77.5	281.5	5	9	9
-82.5	180.0	-30	5	17
-87.5	330.0	-12	8	10

-27.5	121.0	-8	5	19
-27.5	149.0	12	1	25
-27.5	177.0	0	13	3
-27.5	256.0	2	15	1
-27.5	306.5	-7	6	16
-32.5	3.0	4	14	1
-32.5	32.5	17	8	11
-32.5	74.0	20	9	5
-32.5	115.0	-15	4	21
-32.5	145.0	0	3	23
-32.5	180.0	-6	13	2
-32.5	227.0	2	12	5
-32.5	257.0	-4	7	12
-32.5	286.0	-6	7	11
-32.5	316.0	0	12	4
-32.5	351.0	-3	12	3
-37.5	35.0	5	8	7
-37.5	85.0	-5	8	7
-37.5	142.0	2	4	19
-37.5	174.0	28	7	14
-37.5	230.5	3	12	3
-37.5	268.5	-1	15	1
-37.5	306.0	6	7	14
-42.5	30.5	70	9	5
-42.5	173.5	10	5	18
-42.5	281.5	0	12	3
-42.5	315.5	-4	14	2
-47.5	165.5	34	11	5
-47.5	253.5	2	14	2
-47.5	297.5	-9	10	7
-52.5	192.0	-6	10	6
-52.5	307.0	-3	12	5
-57.5	198.5	-11	12	2
-57.5	318.5	9	13	3
-62.5	27.5	0	14	1
-62.5	103.5	19	13	3
-62.5	158.5	-3	9	9
-62.5	256.5	-2	15	1
-67.5	19.5	-9	6	14
-67.5	83.5	14	6	16
-67.5	147.5	-15	8	9
-67.5	212.5	2	13	1
-67.5	276.5	-5	10	5
-72.5	90.0	10	7	12
-72.5	204.5	-17	11	3
-77.5	79.0	16	11	5
-77.5	191.5	-17	9	11
-77.5	304.0	-3	13	3
-82.5	220.0	-13	10	9

121 300 n.m. free-air mean anomalies computed from

$1^\circ \times 1^\circ$ anomalies that included geophysically predicted anomalies

φ	λ	Δg	m	n	φ	λ	Δg	m	n	φ	λ	Δg	m	n	φ	λ	Δg	m	n	φ	λ	Δg	m	n
77.5	79.0	-9	12	4	72.5	73.5	-9	10	5	72.5	90.0	-8	9	9	72.5	105.5	-11	4	20	72.5	123.0	5	3	25
67.5	83.5	-10	5	18	67.5	96.5	-16	3	25	67.5	109.5	-23	3	25	67.5	122.5	-20	3	25	67.5	135.0	3	3	25
67.5	147.5	8	3	25	67.5	160.5	13	3	25	67.5	173.5	19	3	25	67.5	185.5	2	3	25	62.5	81.5	-19	5	18
62.5	92.5	-30	3	25	62.5	103.5	-37	3	25	62.5	114.5	-27	3	25	62.5	125.5	-24	3	25	62.5	136.5	1	3	25
62.5	147.5	19	3	25	62.5	158.5	17	3	24	62.5	169.5	24	4	22	62.5	190.0	14	11	6	57.5	60.0	7	2	25
57.5	69.5	-17	2	25	57.5	79.5	-17	2	25	57.5	87.5	-17	3	25	57.5	97.0	-30	3	25	57.5	106.5	-17	3	25
57.5	115.5	-36	3	25	57.5	124.5	-33	3	25	57.5	133.5	-10	3	25	57.5	143.0	2	12	5	57.5	152.5	12	13	3
57.5	161.5	17	4	22	57.5	272.5	-40	3	25	57.5	291.5	-39	3	25	57.5	290.5	-27	3	24	57.5	300.0	-15	6	16
52.5	61.0	4	1	25	52.5	86.0	-11	3	25	52.5	94.0	-23	3	25	52.5	102.0	-33	3	25	52.5	159.5	15	8	12
47.5	69.5	-14	2	25	47.5	77.0	-27	3	25	47.5	84.5	-35	3	25	47.5	92.0	-16	3	25	47.5	99.5	-20	3	25
47.5	106.5	0	9	10	42.5	30.5	-9	6	15	42.5	37.5	-23	5	17	42.5	57.5	-7	3	20	42.5	64.5	-20	2	25
42.5	71.5	-32	3	25	42.5	73.5	-8	4	25	42.5	85.0	-20	4	25	42.5	91.5	-28	4	25	42.5	98.5	-5	4	25
42.5	105.5	4	9	10	37.5	54.0	1	6	17	37.5	60.0	-15	3	25	37.5	66.0	-26	4	23	37.5	72.5	-6	4	24
37.5	79.0	-11	4	25	37.5	95.0	-8	4	25	37.5	91.5	9	4	25	37.5	98.0	9	4	25	37.5	104.0	6	7	15
37.5	117.0	-18	8	9	37.5	123.0	8	8	10	32.5	50.0	13	4	23	32.5	56.0	16	4	25	32.5	62.0	25	7	15
32.5	68.0	12	7	14	32.5	74.0	-38	3	25	32.5	80.0	42	4	25	32.5	86.0	23	4	25	32.5	91.5	8	5	25
32.5	97.0	11	4	25	32.5	103.0	-10	5	20	27.5	3.0	-10	1	25	27.5	8.5	4	3	23	27.5	14.0	3	8	9
27.5	20.0	2	7	12	27.5	25.5	1	9	11	27.5	42.0	6	9	11	27.5	48.0	-10	3	23	27.5	53.5	-15	4	25
27.5	59.0	8	4	25	27.5	65.0	8	4	24	27.5	70.5	-10	3	25	27.5	81.5	-71	4	24	27.5	87.0	-27	4	25
27.5	93.0	-31	4	25	27.5	98.5	-7	5	25	27.5	104.0	-25	5	21	22.5	2.5	20	2	25	22.5	8.0	26	2	25
22.5	13.5	3	3	25	22.5	18.5	3	5	17	22.5	24.0	-18	7	13	22.5	45.5	-10	11	6	22.5	51.0	-31	4	21
22.5	56.5	-15	13	3	22.5	94.0	-22	5	23	22.5	99.5	-19	5	25	22.5	104.5	-19	5	20	22.5	110.0	-11	6	11
22.5	121.0	0	7	10	22.5	276.5	-1	3	24	22.5	282.0	-6	3	25	22.5	287.5	-54	4	25	17.5	86.0	-13	7	14
17.5	91.5	-21	8	11	17.5	96.5	-9	5	20	17.5	101.5	-17	2	24	17.5	107.0	-10	6	14	17.5	112.5	-2	14	2
17.5	284.5	-2	4	25																				

exists for data that incorporates the geophysically predicted anomalies, 87 of the additional 121 anomalies should be used as replacements in the 1249 set while the remaining 34 new anomalies should be added to the 1249 set.

The specific data in the table consists of the latitude and longitude of the center of the block, the anomaly (mgals), the anomaly standard error, and the number (n) of known 60 n.m. blocks used in the computation of the 300 n.m. block. All anomalies and their standard errors have been rounded to the nearest mgal. Consequently there are several blocks that have a standard error of zero printed (and given in Figure 1) since the actual standard error was less than 0.5 mgals. These blocks and the actual standard errors are: $\varphi = 47.5^\circ$, $\lambda = 11.0^\circ$, $m = \pm 0.4$ mgals; $\varphi = 42.5^\circ$, $\lambda = 268.5^\circ$, $m = \pm 0.4$ mgals; $\varphi = 37.5^\circ$, $\lambda = 275.0^\circ$, $m = \pm 0.3$ mgals.

4.2 Comparison with Kaula (1966) Anomalies

The values of 934 anomalies used by Kaula (1966) are given in Gaposchkin and Lambeck (1970). These anomalies were compared to the corresponding anomalies of the 1283 set for various ranges of accuracy of the anomalies of the 1283 set. The results of these comparisons are shown in Table Four.

Table Four
Comparison of Kaula Anomalies with
Anomalies from 1283 Set

Maximum Accuracy (mgals)	Number of Blocks	Mean Difference (1283 Set - Kaula)	Root Mean Square Difference
1	3	-1.5 mgals	±3.8 mgals
2	37	-3.7	5.6
3	116	-4.4	8.2
4	212	-2.9	12.3
10	697	-2.3	11.5
16	919	-2.0	10.9

We can see that there is a systematic anomaly difference between the Kaula results and the results of this paper. This difference probably represents the use of different reference equatorial gravity values.

These comparisons also revealed that the Kaula anomaly set contained 15 anomaly values that were not contained in the 1283 set. These values are given in Table Five.

Table Five
Kaula Anomalies Missing from 1283 Set

ϕ°	λ°	Δg (mgals)	n	m (mgals)
82.5	100.0	-4	1	± 14.5
67.5	70.5	-19	1	14.5
57.5	337.0	-9	5	11.1
52.5	143.0	-3	4	11.8
22.5	169.5	-8	5	11.1
12.5	249.5	7	8	9.1
7.5	164.5	4	4	11.8
7.5	337.5	5	8	9.1
2.5	297.5	7	1	14.5
2.5	337.5	-2	8	9.1
-7.5	210.5	14	1	14.5
-12.5	208.5	16	6	10.4
-17.5	2.5	6	3	12.7
-17.5	352.5	42	2	13.2
-87.5	60.0	-17	5	11.1

In this table n is the number of 60 n.m. blocks used in the prediction and m is the standard error of the prediction based on n as determined from corresponding predictions made in the 1283 set.

These values could be incorporated into our 1283 anomaly set by taking into account the systematic difference indicated in Table Four.

4.3 Long Range Covariances

The 300 n.m. predicted mean anomalies (with and without geophysically predicted data) were used to determine covariance functions for this block size. Using programs originally written by Kaula, these covariances were computed and are given in Table Six. These values may be compared to similiar values given in Table Two of Kaula (1966).

Table Six
Long-Range Autocovariances
of 300 n. m. Free-Air Gravity Anomalies
($mgal^2$)

Distance (deg)	All Anom. Used	Geophy. Anom. Excluded	Distance (deg)	All Anom. Used	Geophy. Anom. Excluded
0	250	245	90	-7	-8
5	139	133	95	-2	-2
9	102	99	100	2	3
13	63	63	105	7	8
18	39	40	111	10	12
23	21	20	116	11	14
29	6	5	121	11	14
34	-2	-1	126	9	10
39	-4	-4	131	6	7
44	-5	-4	136	3	2
49	-8	-7	141	0	-2
54	-10	-11	146	-3	-3
59	-12	-14	151	-2	-3
64	-9	-14	156	-1	-1
69	-9	-12	162	-3	-4
74	-8	-10	167	-10	-10
80	-7	-10	172	-18	-13
85	-8	-10	175	-20	-17

4.4 Anomaly Degree Variances from Covariance Data

The anomaly degree variances can be computed from covariance data from the following well known expression:

$$\sigma_\ell^2(\Delta g) = \frac{2\ell+1}{2} \int_0^\pi P_\ell(\cos\psi)\, C(\psi) \sin\psi \, d\psi \qquad (7)$$

In this equation ψ is the spherical arc separation, P_ℓ is a Legendre polynomial of degree ℓ, and $C(\psi)$ is the covariance function. The results of applying equation (7) to the covariance values given in Table Six are given in Table Seven along with values from the gravimetric data as given by Kaula (1966, Table Three) and values obtained from satellite derived potential coefficients as given by Gaposchkin and Lambeck (1970).

Table Seven
Anomaly Degree Variances $\sigma^2_\ell(\Delta g)$
($mgal^2$)

		Gravimetric		Satellite
		C(ψ) Table 5	C(ψ) Table 5	
ℓ	Kaula (1966)	All Data	No Geophy.	Gaposchkin & Lambeck (1970)
0	2.7	1.2	0.8	- -
1	-0.5	2.2	1.2	- -
2	6.3	12.7	13.1	7.4
3	31.8	31.3	34.0	33.3
4	18.6	13.6	13.0	19.7
5	8.4	15.1	11.4	17.5
6	22.2	19.9	20.4	14.4
7	11.0	15.5	14.8	16.4
8	9.2	7.5	8.3	8.5
9	10.1	16.1	16.3	15.1
10		9.6	9.1	17.7
11		10.8	8.9	13.7
12		3.9	3.5	8.4
13		8.2	6.2	
14		8.2	8.7	
15		8.1	8.7	

These values will be compared in a subsequent section to anomaly degree variances computed from potential coefficient solutions made using the gravity material computed in this work.

5.0 Gravity Formula Parameters

The $1^\circ \times 1^\circ$ gravity data and the covariance functions previously computed enable us to determine values of equatorial gravity and the flattening as implied by terrestrial gravity material. To determine both equatorial gravity and the flattening we perform a least squares adjustment that minimizes the sum of the squares of the weighted anomalies using as a model the gravity formula in the form of equation (1). Such a computation was carried out with the 20,662 $1^\circ \times 1^\circ$ anomalies that did not contain any geophysical anomalies using several weighting schemes and with a constraint on the flattening forcing it to be equal to 1/298.258. These solutions are described as follows:

Solution One: γ_E and f adjusted, weights $= \cos\varphi / m^2$
Solution Two: f constrained, weights $= \cos\varphi / m^2$

Solution Three: γ_E and f adjusted, weights = $\cos\varphi / 10^3$
Solution Four: f constrained, weights = $\cos\varphi / 10^3$

Solutions Three and Four assume the anomaly data has the same accuracy. In practice this reduces the dominance of the more accurately observed land areas.

In addition to using the gravity formula technique for finding γ_E, we may use the zeroth order anomaly degree variance given in Table Seven as computed from the covariance function where no geophysically predicted anomalies were used. We have (Heiskanen and Moritz, 1967):

$$\sqrt{\sigma_0^2(\Delta g)} = \Delta g_0 = (\gamma_B - \gamma_R) - \frac{1}{3}\gamma(f_B - f_R) \qquad (8)$$

where γ_B, f_B are equatorial gravity and flattening of the "best" parameters while γ_R, f_R are corresponding parameters of the gravity formula to which the anomalies have been referred. Since the anomalies, and thus the anomaly covariances and the anomaly degree variances have been referred to a gravity formula where f is considered to be accurately known, we have $f_B = f_R$. Thus:

$$\gamma_B = \gamma_R + \sqrt{\sigma_0(\Delta g)} \qquad (9)$$

This computation of equatorial gravity constitutes Solution Five.

Solution Six for γ_E and f are the values and constants inherent with equation (1). Since these constants have been derived independently of gravity information, their comparison with the gravity dependent results is of interest.

The results of these solutions are summarized in Table Eight where all equatorial gravity values have been referenced to an absolute system using a Potsdam correction of - 14 mgals.

Table Eight
Equatorial Gravity and Flattening Determinations

Solution	γ_E (mgals)	1/f
One	978034.4	298.646
Two	978035.8	298.258*
Three	978032.1	298.265
Four	978032.1	298.258*
Five	978034.4	298.258*
Six	978033.5	298.258*

Comparing solutions of the same flattening, we find solution 5, followed by solution 4 agrees best with solution 6. Solutions 2 differs by 2.3 mgal from solution 6 indicating some distortion has taken place by weighting according to actual anomaly accuracy estimates. We could conclude that within the accuracy of the data there is reasonable agreement between equatorial gravity values determined from gravity data and satellite data.

6.0 Potential Coefficient Determinations

The 300 n.m. mean anomalies computed in this paper may be used to determine potential coefficients describing the earth's gravitational field. In order to do this we first consider the relationship between point gravity anomalies, referred to a given gravity formula and potential coefficients:

$$\Delta g_r = \Delta g_0 + \frac{kM}{r^2}\left[\sum_{\ell=2}^{\infty}(\ell-1)\left(\frac{a}{r}\right)^{\ell}\sum_{m=0}^{\ell}(\bar{C}^*_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda)\,\bar{P}_{\ell m}(\sin\varphi)\right] \tag{10}$$

where Δg_0 is the mean global value of Δg_r, r is the geocentric distance to the computation point, $\bar{C}^*_{\ell m}$ and $\bar{S}_{\ell m}$ are fully normalized potential coefficients where $\bar{C}^*$ represents the difference between the actual values and reference values consistent with the flattening to which the anomalies are referrenced. $\bar{P}_{\ell m}$ is the fully normalized Legendre functions and φ is the geocentric latitude.

Generally equation (10) has been evaluated using center point block coordinates. With higher degree solutions being continually made, errors will start to exist at the higher degrees unless an integrated form of equation (10) is used that recognizes that the anomaly values are given in blocks bordered by meridians and parallels, e.g. φ_1, φ_2, λ_1, λ_2. Thus a more accurate expression relating potential coefficients to mean anomalies is as follows (Rapp, 1972, Desrochers, 1971):

$$\Delta \bar{g}_r = \Delta g_0 + \frac{kM}{\Delta\lambda(\sin\varphi_2 - \sin\varphi_1)\,\bar{r}^2}\left[\sum_{\ell=2}^{k}(\ell-1)\left(\frac{a}{\bar{r}}\right)^{\ell}\left[\Delta\lambda\,\bar{C}^*_{\ell,0}\cdot\right.\right.$$

$$\cdot\int_{\varphi_1}^{\varphi_2}\bar{P}_{\ell,0}(\sin\varphi)\cos\varphi\,d\varphi + \sum_{m=1}^{\ell}\frac{1}{m}[\bar{C}_{\ell m}(\sin m\lambda_2 - \sin m\lambda_1) - \bar{S}_{\ell m}(\cos m\lambda_2 -$$

$$\left.\left.\cos m\lambda_1)\right]\int_{\varphi_1}^{\varphi_2}\bar{P}_{\ell m}(\sin\varphi)\cos\varphi\,d\varphi\right]\right] \qquad (11)$$

where $\bar{r}$ is the average geocentric radius in the block which, in these computations, was taken at the mean geocentric latitude of the block, and k is the highest degree to which the summation is to be carried, or for which a potential coefficient solution is to be made.

Although the complications resulting in going from equation (10) to equation (11) are not justified for the lower degree solutions, it is a necessity as solutions are generated for the higher degrees, say 15 or more.

In solving for potential coefficients from gravity material it is desirable to have a global set of anomalies which, in the Kaula divisions used in this paper, will consist of 1654 blocks. To obtain such a global set the model anomalies of Uotila (1964) were used to supplement the predicted anomaly sets assigning such model anomalies a standard error of ±20 mgals as has previously been done (Rapp, 1969).

Using equation (11) as a model several weighted least squares solutions were made. In applying equation (11), the value of $\bar{r}$ was computed using the value of a = 6378137.9 m and f = 1/298.258 and all block division latitudes defined in terms of geodetic latitudes were converted to geocentric latitudes. The longitude limits on the blocks were individually computed for each block based on the specific 1° anomaly selection criteria of the anomaly prediction program.

In addition to the relationships represented by equation (10) (or (11)), potential coefficients may be directly computed from the anomaly data from the following expressions:

$$\left\{ \begin{matrix} \bar{C}^*_{\ell m} \\ \bar{S}_{\ell m} \end{matrix} \right\} = \frac{1}{4\pi\gamma(\ell-1)} \int_\sigma\int \Delta g\, \bar{P}_{\ell m}(\sin\varphi) \left\{ \begin{matrix} \cos m\lambda \\ \sin m\lambda \end{matrix} \right\} d\sigma \quad (12)$$

This formula may also be written in a manner that obtains a mean value of the kernal (excluding Δg) in a manner analogous to what was done in writing equation (11). If, in equation (10) (or (11)), r was set equal to a, kM/r^3 was set to γ, and a least squares adjustment was made with equal anomaly variances, the results for the potential coefficients from either (11) or (12) would be the same. Consequently, the application of equation (12) may be viewed as a least squares adjustment with equal anomaly variances. The computation of potential coefficients through equation (12) is considerably faster than the procedure carried out using equation (10) or (11) because in the latter case, normal equations and their solutions need to be carried out.

6.1 Solutions Made

Several potential coefficient solutions were made with the following being of primary interest:

Solution One:	Complete to degree 12 using 1283 anomalies based on predictions incorporating the 1° geophysically predicted data, plus 371 model anomalies
Solution Two:	Complete to degree 12 using 1249 anomalies based on predictions excluding the 1° geophysically predicted data, plus 405 model anomalies
Solution Three:	Complete to degree 12 using the 934 Kaula anomalies plus 720 model anomalies
Solution Four:	Complete to degree 12 as given by Rapp (1969)
Solution Five:	Complete to degree 15 using same data as solution one
Solution Six:	Complete to degree 25 using same data as solution one but using equation (12) for the computation
Solution Seven:	Complete to 15,15 using 1283 anomalies plus the 15 Kaula anomalies (Table 6) missing in the 1283 set, plus 356 model anomalies

In Solutions One, Two, Three, and Five the value of Δg_0 was computed as the mean value of the 1654 anomaly set. The potential coefficients and their standard errors from Solution Five, and the potential coefficients from Solution Six are given in Table Nine. The geoid undulations implied by the potential coefficients of Solution Five with respect to f = 1/298.258 and a best fitting "a" are given in Figure Two as computed by Method One in Rapp (1971).

6.2 Comparison of Solutions to Satellite Derived Potential Coefficients

In order to judge the accuracy of these solutions, we compare them to the recent satellite derived potential coefficients of Gaposchkin (1969) and Smith, Lerch, and Wagner (1972). The comparisons are made with respect to three quantities: 1. the root mean square coefficient difference; 2. the solution correlation (r) as defined by Rapp (1972); 3. the percentage difference (p). This latter quantity is computed as the average percentage difference of each degree, where the percentage difference of each degree (p_ℓ) is computed from the following expression:

$$p_\ell = 100 \cdot \sqrt{\frac{1}{2\ell+1} \quad \frac{\Sigma\Delta^2}{\Sigma\sigma^2}} \tag{13}$$

where σ is the value of a potential coefficient in degree ℓ of the satellite solution. These results are given in Table Ten.

Table Ten
Comparison of Potential Coefficient Sets

	Comparison Coefficients								
	Gaposchkin (1969), to 8 only			Smith et als. (1972), to 8 only			Smith et als. (1972), to 12 only		
Solution	$\Delta \times 10^6$	r	p(%)	$\Delta \times 10^6$	r	p(%)	$\Delta \times 10^6$	r	p(%)
One	.190	.937	16.0	.187	.940	14.4	.140	.928	18.6
Two	.191	.934	15.7	.190	.936	14.5	.141	.925	18.3
Three	.344	.762	24.1	.339	.790	21.8	.236	.778	22.2
Four	.239	.895	16.4	.246	.892	15.9	.173	.886	17.8
Five	.188	.938	15.3	.185	.942	13.9	.134	.934	16.3
Six	.186	.938	14.5	.180	.947	12.9	.128	.942	14.4
Seven	.192	.937	15.6	.188	.940	14.1	.136	.933	16.4

Figure Two

Geoid Undulations from Solution Five, n = 15, Reference Flattening = 1/298.258

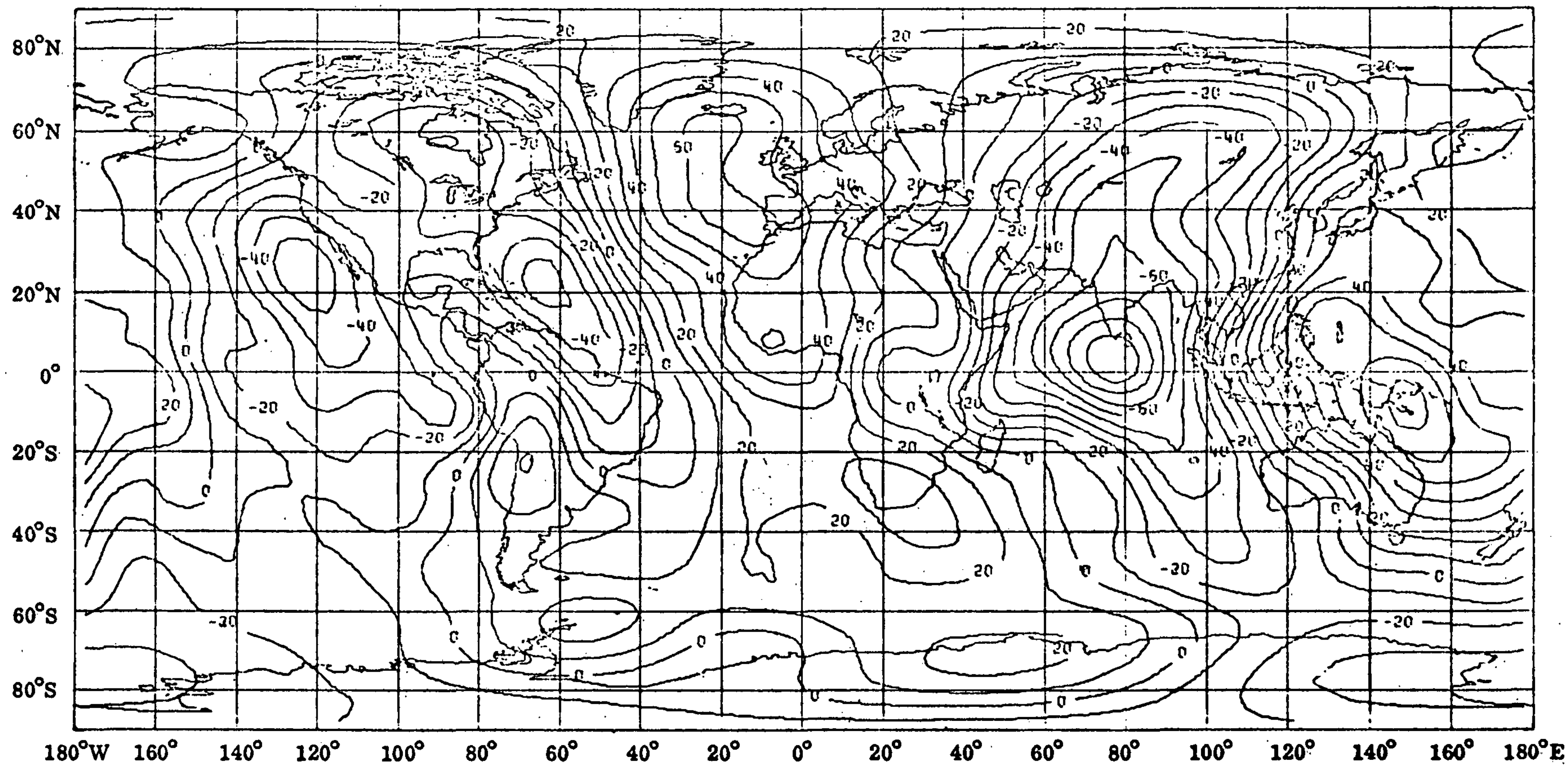

Table Nine
Potential Coefficients From Gravity Data ($\times 10^6$)

		Solution Five				Solution Six	
ℓ	m	$\bar{C}_{\ell m}$	m($\bar{C}$)	$\bar{S}_{\ell m}$	m($\bar{S}$)	$\bar{C}_{\ell m}$	$\bar{S}_{\ell m}$
2	0	-484.003	0.285			-484.400	
2	2	3.177	0.327	-1.635	0.308	2.721	-0.828
3	0	0.749	0.148			0.453	
3	1	1.290	0.154	-0.189	0.134	1.224	0.069
3	2	1.020	0.165	-0.603	0.163	0.998	-0.384
3	3	0.664	0.161	1.305	0.169	0.794	1.159
4	0	0.662	0.105			0.614	
4	1	-0.288	0.111	-0.210	0.105	-0.348	-0.217
4	2	0.339	0.115	0.244	0.123	0.413	0.281
4	3	0.836	0.104	-0.401	0.109	0.758	-0.198
4	4	-0.053	0.112	0.268	0.112	-0.084	0.237
5	0	0.001	0.083			-0.078	
5	1	-0.105	0.082	-0.027	0.074	-0.192	-0.133
5	2	0.521	0.086	-0.085	0.090	0.458	-0.067
5	3	-0.350	0.086	-0.003	0.090	-0.245	-0.143
5	4	0.086	0.079	0.003	0.079	0.025	-0.011
5	5	0.286	0.084	-0.681	0.087	0.120	-0.461
6	0	0.087	0.065			-0.003	
6	1	-0.068	0.067	-0.026	0.063	-0.059	-0.039
6	2	0.193	0.065	-0.178	0.066	0.149	-0.170
6	3	-0.099	0.071	-0.110	0.075	-0.109	-0.052
6	4	-0.205	0.069	-0.419	0.068	-0.130	-0.332
6	5	-0.394	0.062	-0.591	0.066	-0.311	-0.451
6	6	0.007	0.070	-0.230	0.068	-0.013	-0.137
7	0	0.114	0.054			0.092	
7	1	0.159	0.057	0.180	0.052	0.228	0.145
7	2	0.291	0.052	0.058	0.054	0.273	0.077
7	3	0.099	0.056	-0.123	0.060	0.134	-0.096
7	4	-0.158	0.061	-0.132	0.060	-0.143	-0.127
7	5	-0.035	0.056	0.048	0.057	-0.036	0.055
7	6	-0.240	0.053	0.157	0.053	-0.189	0.135
7	7	-0.049	0.058	-0.109	0.057	-0.011	-0.031
8	0	-0.045	0.046			-0.004	
8	1	-0.137	0.048	0.070	0.044	-0.056	0.039
8	2	0.001	0.046	0.148	0.047	0.054	0.164
8	3	0.083	0.046	0.008	0.047	0.059	0.015
8	4	-0.160	0.053	-0.002	0.051	-0.099	0.039
8	5	0.009	0.050	0.043	0.052	0.002	0.041
8	6	-0.139	0.047	0.130	0.046	-0.070	0.122
8	7	0.018	0.047	0.094	0.045	0.041	0.120
8	8	-0.103	0.050	0.047	0.049	-0.082	0.039

9	0	0.174	0.040			0.118	
9	1	0.068	0.042	0.019	0.039	0.142	-0.003
9	2	0.137	0.040	-0.145	0.041	0.061	-0.075
9	3	-0.205	0.040	-0.082	0.041	-0.162	-0.042
9	4	-0.072	0.043	-0.020	0.041	-0.043	0.023
9	5	-0.057	0.045	0.005	0.047	-0.094	0.043
9	6	0.072	0.043	0.175	0.043	0.034	0.147
9	7	-0.017	0.040	0.078	0.040	-0.022	0.028
9	8	0.262	0.041	0.023	0.040	0.201	0.024
9	9	0.052	0.043	0.036	0.044	-0.005	0.035
10	0	0.020	0.035			0.011	
10	1	0.047	0.038	-0.023	0.035	0.054	-0.056
10	2	-0.057	0.035	-0.122	0.036	-0.057	-0.048
10	3	-0.014	0.035	-0.082	0.037	-0.028	-0.069
10	4	-0.070	0.037	-0.096	0.036	-0.063	-0.102
10	5	-0.048	0.038	0.014	0.039	-0.032	-0.007
10	6	-0.006	0.040	-0.094	0.040	-0.011	-0.055
10	7	0.135	0.038	-0.070	0.037	0.082	-0.013
10	8	0.006	0.036	-0.155	0.035	0.012	-0.097
10	9	0.096	0.036	-0.010	0.036	0.133	0.004
10	10	0.150	0.038	-0.002	0.039	0.047	0.001
11	0	-0.076	0.031			-0.066	
11	1	-0.027	0.033	0.046	0.030	-0.005	0.017
11	2	-0.011	0.032	-0.131	0.033	0.000	-0.082
11	3	-0.078	0.031	-0.072	0.032	-0.075	-0.040
11	4	-0.082	0.033	-0.133	0.032	-0.060	-0.091
11	5	0.061	0.033	0.034	0.034	0.063	0.009
11	6	-0.012	0.037	-0.071	0.035	0.039	-0.045
11	7	0.042	0.036	-0.122	0.035	0.016	-0.044
11	8	-0.058	0.033	0.084	0.032	-0.022	0.062
11	9	0.011	0.031	0.030	0.031	0.009	0.060
11	10	-0.065	0.032	-0.065	0.032	-0.042	-0.046
11	11	0.136	0.035	-0.032	0.035	0.055	0.020
12	0	-0.042	0.029			-0.045	
12	1	-0.067	0.029	0.012	0.026	-0.030	-0.043
12	2	-0.067	0.029	0.038	0.030	-0.038	0.007
12	3	0.071	0.028	0.027	0.029	0.026	-0.018
12	4	-0.021	0.029	-0.020	0.028	-0.046	-0.023
12	5	0.052	0.029	-0.008	0.029	0.040	0.007
12	6	0.001	0.031	0.054	0.030	0.032	0.032
12	7	-0.079	0.032	0.023	0.031	-0.052	0.017
12	8	0.015	0.030	0.059	0.030	0.003	0.022
12	9	-0.056	0.028	0.049	0.028	-0.021	0.013
12	10	0.004	0.028	-0.058	0.027	-0.008	0.001
12	11	0.024	0.029	-0.008	0.029	-0.009	-0.004
12	12	0.020	0.032	-0.038	0.031	0.031	-0.001

13	0	0.031	0.025			0.036	
13	1	0.024	0.026	-0.037	0.023	0.009	-0.030
13	2	-0.032	0.025	-0.056	0.025	-0.022	-0.025
13	3	-0.008	0.026	0.031	0.026	0.011	0.036
13	4	0.029	0.026	-0.073	0.025	-0.007	-0.030
13	5	0.071	0.026	0.072	0.026	0.070	0.062
13	6	-0.065	0.028	0.090	0.026	-0.061	0.011
13	7	-0.093	0.028	0.071	0.028	-0.031	0.034
13	8	0.003	0.028	0.040	0.028	-0.009	-0.005
13	9	0.003	0.026	0.045	0.027	-0.002	0.058
13	10	0.020	0.024	0.029	0.024	-0.008	0.012
13	11	-0.085	0.024	-0.054	0.024	-0.013	0.006
13	12	-0.032	0.026	0.081	0.026	0.006	0.073
13	13	-0.089	0.028	0.125	0.028	-0.056	0.048
14	0	0.058	0.022			0.014	
14	1	-0.046	0.024	0.035	0.021	-0.002	0.023
14	2	-0.020	0.022	0.040	0.022	-0.024	0.009
14	3	0.039	0.024	-0.031	0.023	0.022	-0.022
14	4	-0.047	0.024	0.022	0.023	0.015	0.014
14	5	0.0	0.0	0.0	0.0	0.0	0.0
14	6	-0.015	0.025	-0.056	0.024	0.010	-0.003
14	7	0.062	0.025	0.014	0.024	0.022	-0.025
14	8	0.013	0.025	-0.060	0.025	-0.029	-0.033
14	9	0.020	0.025	0.025	0.024	-0.007	0.075
14	10	0.035	0.022	-0.074	0.022	0.048	-0.036
14	11	0.046	0.021	-0.068	0.021	0.017	-0.037
14	12	-0.023	0.021	-0.026	0.021	0.009	-0.036
14	13	0.053	0.022	0.016	0.023	0.001	0.069
14	14	-0.005	0.024	-0.001	0.024	-0.035	-0.014
15	0	0.013	0.018			0.016	
15	1	0.001	0.019	0.029	0.017	0.039	0.015
15	2	-0.050	0.018	-0.038	0.018	-0.019	-0.000
15	3	-0.012	0.018	0.084	0.019	0.012	0.038
15	4	0.016	0.018	0.021	0.018	-0.013	-0.010
15	5	-0.025	0.018	0.046	0.017	0.026	0.008
15	6	-0.008	0.018	-0.120	0.018	0.026	-0.065
15	7	0.061	0.018	0.049	0.018	0.045	0.027
15	8	-0.076	0.018	-0.017	0.018	-0.024	0.012
15	9	-0.070	0.018	0.047	0.018	0.002	0.030
15	10	-0.001	0.018	0.028	0.018	-0.017	-0.003
15	11	-0.050	0.017	-0.005	0.018	0.015	0.005
15	12	0.012	0.017	0.048	0.017	-0.012	0.046
15	13	-0.005	0.018	0.000	0.018	-0.041	-0.004
15	14	-0.022	0.019	-0.046	0.019	-0.002	-0.017
15	15	-0.092	0.020	0.046	0.021	-0.041	0.028

From the table we see the following:

1. The potential coefficient solutions based on gravity data agree slightly better with the new satellite derived solution of Smith et als. than with the Gaposchkin solution.

2. The poorest solution is Solution Three where the Kaula anomalies were used. The second poorest solution is the older solution of Rapp (Solution Four). The better agreement of the newer solutions with the satellite derived coefficients reflects the increase and improvement of the current gravity material over that used in the earlier solutions.

3. There is very little difference between the coefficient sets that include and exclude the geophysically computed 1° anomalies in their formation (Solutions One and Two).

4. The inclusion of the Kaula derived anomalies into the 1283 set (Solution Seven) appears to have slightly deteriorated the potential coefficient sets when compared to the results of Solution Six.

5. Of the weighted least squares solutions made, the best set of potential coefficients appears to be those found in Solution Five.

6. Of all solutions, the best agreement with the satellite derived solutions is with the coefficients of Solution Six. Information to be considered in the next section, however, reveals some undesirable features of Solution Six. Consequently for the results of this paper, the potential coefficients from Solution Five will be considered the best overall set.

6.3 Anomaly Degree Variances from Potential Coefficients

The anomaly degree variances discussed in section 4.4 may also be computed from potential coefficients using the following equation:

$$\sigma_\ell^2(\Delta g) = \gamma^2(\ell-1)^2 \sum_{m=0}^{\ell} (\bar{C}^{*2}_{\ell m} + \bar{S}^2_{\ell m}) \qquad (12)$$

where γ is a mean value of gravity. Such values have been computed from several solutions and are given in Table Eleven along with the anomaly degree variances given in Table Seven as computed from the covariance data obtained from the 1283 anomaly set.

Table Eleven
Anomaly Degree Variances from Potential Coefficient Solutions
($mgal^2$)

	Solutions			From Table 6
ℓ	One	Five	Six	(Col. 3)
2	11.3	12.3	7.8	12.7
3	24.5	22.3	18.5	31.3
4	11.6	10.8	9.7	13.6
5	13.2	14.8	9.0	15.1
6	22.7	21.1	12.4	19.9
7	16.6	11.3	9.9	15.5
8	9.1	6.6	4.5	7.5
9	16.2	15.3	9.3	16.1
10	14.3	10.3	5.6	9.6
11	14.9	12.3	5.4	10.8
12	15.7	5.3	2.2	3.9
13		13.0	4.9	8.2
14		7.9	4.1	8.2
15		12.9	4.0	8.1

The larger degree variances for the higher degrees for Solution One reflect the aliasing effect (Desrochers, 1971) caused by the truncation of the of the least squares adjustment. This truncation causes a distortion in the coefficients of the degrees near the truncation degree. This effect is also noticable in Solution Five, but to a lesser extent. The degree variances from Solution Six are all smaller than the ones from Solution Five and those computed from the covariance function data. The unreasonably small values of the anomaly degree variances from Solution Six indicate that solution to be undesirable as compared with other solutions (such as Five).

7.0 Summary

This paper had described the estimation of a 300 n. m. (or 5° equal area) block terrestrial free air anomaly field. The derived fields were analyzed to obtain covariance functions, anomaly degree variances, and potential coefficients. Only small and insignificant differences between solutions that did and did not incorporate the geophysically estimated $1^\circ \times 1^\circ$ into the 300 n. m. predicted set were found.

The potential coefficients as determined from this new data agree better with the satellite derived potential coefficients than the coefficients of earlier sets derived from gravity data. This improvement reflects the improvement in the gravity coverage and the accuracy of the material since the earlier solutions.

The gravity data of this paper will be used for the formation of 15° and perhaps 10° equal area block mean anomalies. In addition a combination of this gravimetric data with satellite derived potential coefficients will be carried out to determine a 300 n. m. gravity field that is consistent with a high degree potential coefficient field.

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