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ANALYTICAL STUDY OF THE PERFORMANCE OF A GUST ALLEVIATION SYSTEM FOR A STOL AIRPLANE

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16. Abstract

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SUMMARY

An analytical study has shown that a gust alleviation system for a STOL airplane in a cruise condition could reduce the root mean square of the normal acceleration of the airplane flying in random turbulence by as much as 50 percent. This alleviation is obtained by driving the flaps in response to normal acceleration and by moving the elevator in proportion to the commanded flap deflection angle and to a pitch-rate signal.

INTRODUCTION

Research by the NASA in aeronautics includes studies of methods to provide good ride qualities for airplanes. Ride improvement is particularly important for airplanes having relatively low wing loading and operating at low altitudes, where air turbulence may be expected. The present study is made to examine, analytically, the response to random turbulence of a STOL airplane during cruising flight without and with various gust alleviation systems. Five automatic control systems are modeled. These systems use a pitch-rate signal and a normal-acceleration signal to operate the elevator and flaps, respectively. In one system the elevator deflection is proportional to the commanded flap deflection angle. The control systems are assessed by their ability to alleviate the root-mean-square (rms) normal acceleration at the airplane center of gravity and at distances of 1 and 2 mean aerodynamic chords behind the center of gravity. Consideration of the rms elevator and flap deflection angles and deflection rates required for the alleviation is used in the assessments. A Von Kármán power spectral density function is used to characterize the turbulence.

SYMBOLS

A(s)	elevator or flap servo transfer function
a_n	normal acceleration at center of gravity, m/sec^2

C_m	pitching-moment coefficient, $\frac{\text{Pitching moment}}{qS_w c}$
C_Z	Z-force coefficient, $\frac{\text{Force in Z-direction}}{qS_w}$
c	wing mean aerodynamic chord, m
$\frac{d\epsilon}{d\alpha}$	change of downwash at tail per unit change of wing angle of attack
$\frac{d\epsilon}{d\delta_f}$	change of downwash at tail per unit change of flap deflection angle
F(s)	filter transfer function
g	free-fall acceleration, m/sec^2
$i = \sqrt{-1}$	
K_0	alleviation system gain, rad/g
K_2	alleviation system gain
K_5	alleviation system gain, rad/rad/sec
k_Y	radius of gyration about Y-axis, m
L	scale of turbulence, m
m	mass, kg
n_0	normal-acceleration ratio at aircraft center of gravity, $\frac{a_n}{g}$
q	dynamic air pressure, N/m^2
R_0,R_1,R_2	percent alleviation at three fuselage locations
S_w	wing area, m^2
s	Laplace variable, per second

t	time, sec
V	airspeed, m/sec
w_g	vertical component of gust velocity, m/sec
X,Y,Z	axes
$z_{11},z_{12},z_{21},z_{22}$	complex numbers appearing in matrix of equation (6)
α	angle of attack, rad
α_g	gust angle of attack, $\frac{w_g}{V}$, rad
$\dot{\alpha}$	time rate of change of angle of attack, rad/sec
δ_e	elevator deflection angle, rad
δ_f	flap deflection angle, rad
$\dot{\theta}$	pitch rate, rad/sec
$\ddot{\theta}$	pitch angular acceleration, rad/sec/sec
$\sigma_{n,0}$	rms normal-acceleration ratio at center of gravity
$\sigma_{n,1}$	rms normal-acceleration ratio 1 chord aft of center of gravity
$\sigma_{n,2}$	rms normal-acceleration ratio 2 chords aft of center of gravity
σ_{w_g}	rms vertical gust velocity, m/sec
σ_{α}	rms variation of angle of attack, rad
σ_{α_g}	rms gust angle of attack, rad
σ_{δ_e}	rms elevator deflection angle, rad
$\sigma_{\dot{\delta}_e}$	rms elevator deflection rate, rad/sec

σ_{δ_f} rms flap deflection angle, rad

$\sigma_{\dot{\delta}_f}$ rms flap deflection rate, rad/sec

$\sigma_{\dot{\theta}}$ rms variation of pitch rate, rad/sec

τ transport time lag, $\frac{\text{Tail length}}{V}$, sec

$\Phi_{\alpha_g}(\omega)$ power spectral density function for gust angle of attack, $(\text{rad})^2/\text{rad/sec}$

ω circular frequency, rad/sec

$\left(C_{Z_\alpha}\right)_o = \left(\frac{\partial C_Z}{\partial \alpha}\right)_{\text{wing + fuselage}}$ $\left(C_{m_\alpha}\right)_o = \left(\frac{\partial C_m}{\partial \alpha}\right)_{\text{wing + fuselage}}$

$\left(C_{Z_\alpha}\right)_t = \left(\frac{\partial C_Z}{\partial \alpha}\right)_{\text{tail}}$ $\left(C_{m_\alpha}\right)_t = \left(\frac{\partial C_m}{\partial \alpha}\right)_{\text{tail}}$

$C_{Z_{\delta_e}} = \frac{\partial C_Z}{\partial \delta_e}$ $C_{Z_{\delta_f}} = \frac{\partial C_Z}{\partial \delta_f}$

$C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e}$ $C_{m_{\delta_f}} = \frac{\partial C_m}{\partial \delta_f}$

Subscripts:

c commanded

0 basic airplane

ANALYSIS

Equations describing the longitudinal motion of an airplane flying at constant airspeed are used in the present investigation. A Von Kármán power spectral density function is used to represent the gust angle of attack. Application of random process theory gives the power spectral density function of the response of the airplane to the gust angle of attack. Automatic controls, actuated by feedback of an accelerometer output or of a rate gyro output, are modeled to alleviate the normal accelerations at the airplane center of gravity and at two other locations on the fuselage. The performance of the automatic controls is assessed by the percent reduction of normal acceleration obtained by operation of the controls.

Mathematical Model of the Airplane Motion

The equations of longitudinal motion are given for constant airspeed, as in reference 1. Terms are given for forces and moments contributed separately by the wing and fuselage, the horizontal tail, the elevator, and the flaps. This is done to properly account for the lag in downwash at the horizontal tail and for the effect of spatial distribution of vertical gusts. A disadvantage is incurred, however, because the resulting transport time lag leads to difficulties in the analysis of stability.

The frame of reference for the airplane motion is a system of body axes, illustrated in figure 1. The differential equations are as follows:

$$m V \dot{\alpha}(t) = qS_w \left\{ \left[\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t \right] \alpha(t) - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\alpha} \alpha(t - \tau) + \left[\frac{mV}{qS_w} + \tau \left(C_{Z_\alpha}\right)_t \right] \dot{\theta}(t) + \left(C_{Z_{\delta_e}}\right) \delta_e(t) \right.$$

$$\left. + \left(C_{Z_{\delta_f}}\right) \delta_f(t) - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\delta_f} \delta_f(t - \tau) + \left(C_{Z_\alpha}\right)_o \alpha_g(t) + \left(C_{Z_\alpha}\right)_t \left(1 - \frac{d\epsilon}{d\alpha}\right) \alpha_g(t - \tau) \right\} \tag{1a}$$

$$m k_Y^2 \ddot{\theta}(t) = qS_w c \left\{ \left[\left(C_{m_\alpha}\right)_o + \left(C_{m_\alpha}\right)_t \right] \alpha(t) - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\alpha} \alpha(t - \tau) + \tau \left(C_{m_\alpha}\right)_t \dot{\theta}(t) + \left(C_{m_{\delta_e}}\right) \delta_e(t) \right.$$

$$\left. + \left(C_{m_{\delta_f}}\right) \delta_f(t) - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\delta_f} \delta_f(t - \tau) + \left(C_{m_\alpha}\right)_o \alpha_g(t) + \left(C_{m_\alpha}\right)_t \left(1 - \frac{d\epsilon}{d\alpha}\right) \alpha_g(t - \tau) \right\} \tag{1b}$$

The Laplace transform of equations (1) is

$$\left\{ mVs - qS_w \left[\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\alpha} e^{-\tau s} \right] \right\} \alpha(s) - qS_w \left[\frac{mV}{qS_w} + \tau \left(C_{Z_\alpha}\right)_t \right] \dot{\theta}(s) - qS_w \left(C_{Z_{\delta_e}}\right) \delta_e(s)$$

$$- qS_w \left[\left(C_{Z_{\delta_f}}\right) - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\delta_f} e^{-\tau s} \right] \delta_f(s) = qS_w \left[\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t \left(1 - \frac{d\epsilon}{d\alpha}\right) e^{-\tau s} \right] \alpha_g(s) \tag{2a}$$

$$qS_w c \left[- \left(C_{m_\alpha}\right)_o - \left(C_{m_\alpha}\right)_t + \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\alpha} e^{-\tau s} \right] \alpha(s) + \left[m k_Y^2 s - \tau \left(C_{m_\alpha}\right)_t qS_w c \right] \dot{\theta}(s) - qS_w c \left(C_{m_{\delta_e}}\right) \delta_e(s)$$

$$- qS_w c \left[\left(C_{m_{\delta_f}}\right) - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\delta_f} e^{-\tau s} \right] \delta_f(s) = qS_w c \left[\left(C_{m_\alpha}\right)_o + \left(C_{m_\alpha}\right)_t \left(1 - \frac{d\epsilon}{d\alpha}\right) e^{-\tau s} \right] \alpha_g(s) \tag{2b}$$

The terms on the right-hand side represent the gust disturbance where $\alpha_g = w_g/V$.

Gust Alleviation Systems

The five gust alleviation systems examined in this study are represented by the block diagram of figure 2. Each system is defined according to which combinations of the gains K_0, K_2, and K_5 are zero. The gust alleviation systems are

1. Alleviation system using only elevator control (A pitch-rate signal actuates the elevator. $K_0 = K_2 = 0.$)
2. Alleviation system using only flap control (A normal-acceleration signal actuates the flap. $K_2 = K_5 = 0.$)
3. Alleviation system using independent flap and elevator control (A normal-acceleration signal and a pitch-rate signal actuate the flaps and elevator, respectively. $K_2 = 0.$)
4. Alleviation system using dependent elevator and flap control (A normal-acceleration signal actuates the flap, and the flap command signal actuates the elevator. $K_5 = 0.$)
5. Complete gust alleviation system (None of the gains are zero.)

In actuating the controls, a positive pitch rate causes a positive elevator deflection angle, and a positive normal acceleration (in the Z-direction) causes a positive commanded flap deflection angle $\delta_{f,c}$. When K_2 is greater than zero, a positive commanded flap deflection angle causes a positive elevator deflection angle. A positive deflection angle of the elevator and flap occurs when the trailing edge is moved down.

Notice that the normal-acceleration signal is filtered (fig. 2) and then fed back to the servos to operate the flaps and elevator. The filter was included to reduce the flap and elevator deflection rates required for alleviation.

The accelerometer signal n_0 is the ratio of the normal acceleration a_n of the airplane center of gravity to free-fall acceleration g. Thus,

$$n_0 = \frac{a_n}{g}$$

or

$$n_0 = -\frac{V}{g}(\dot{\theta} - \dot{\alpha})$$

In Laplace notation,

$$n_0(s) = -\frac{V}{g}\left[\dot{\theta}(s) - s\alpha(s)\right] \tag{3}$$

The transfer functions for the flap and elevator deflection angles can be obtained from figure 2 as follows:

$$\delta_f(s) = \delta_{f,c}(s)A(s)$$

or

$$\delta_f(s) = K_0F(s)\frac{-V}{g}\left[\dot{\theta}(s) - s\alpha(s)\right]A(s) \tag{4}$$

and

$$\delta_e(s) = \left[K_5\dot{\theta}(s) + K_2\delta_{f,c}(s)\right]A(s) \tag{5}$$

The filter and servo parameters shown in figure 2 are considered to be "practical," and no attempt has been made to obtain the best characteristics. Further research is required to obtain filter and servo characteristics that are optimum in some respect.

Frequency Response Functions

Formulation of frequency response functions for this investigation is necessary for the application of random process theory to calculate the rms response of the airplane. These frequency response functions are obtained by combining equations (2), (4), and (5), replacing s by $i\omega$, and solving the resulting equations for the transfer functions $\frac{\alpha}{\alpha_g}(i\omega)$ and $\frac{\dot{\theta}}{\alpha_g}(i\omega)$.

Equations (2), (4), and (5) may be written in complex matrix form as follows:

$$\left[Z(i\omega)\right]\begin{bmatrix}\frac{\alpha}{\alpha_g}(i\omega) \\ \frac{\dot{\theta}}{\alpha_g}(i\omega)\end{bmatrix} = \begin{bmatrix}\frac{C_Z}{\alpha_g}(i\omega) \\ \frac{C_m}{\alpha_g}(i\omega)\end{bmatrix} \tag{6}$$

where Z is a two-by-two matrix with elements

$$z_{11}(i\omega) = mVi\omega - qS_W\left\{\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}e^{-i\tau\omega}\right)\right.$$

$$+ \left(C_{Z_{\delta_e}}\right)\left(K_0K_2i\omega\frac{2}{2 + i\omega}\frac{100\pi^2}{-\omega^2 + 14\pi i\omega + 100\pi^2}\frac{V}{g}\right)$$

$$\left. + \left[\left(C_{Z_{\delta_f}}\right) - \left(C_{Z_\alpha}\right)_t\frac{d\epsilon}{d\delta_f}e^{-i\tau\omega}\right]K_0i\omega\frac{2}{2 + i\omega}\frac{100\pi^2}{-\omega^2 + 14\pi i\omega + 100\pi^2}\frac{V}{g}\right\}$$

$$z_{12}(i\omega) = -qS_w\left\{\frac{m}{qS_w}V + \tau\left(C_{Z_\alpha}\right)_t + \left(C_{Z_{\delta_e}}\right)\left(K_5 - K_2K_0\frac{2}{2 + i\omega}\frac{V}{g}\right)\frac{100\pi^2}{-\omega^2 + 14\pi i\omega + 100\pi^2}\right.$$

$$\left. - \left[\left(C_{Z_{\delta_f}}\right) - \left(C_{Z_\alpha}\right)_t\frac{d\epsilon}{d\delta_f}e^{-i\tau\omega}\right]K_0\frac{2}{2 + i\omega}\frac{100\pi^2}{-\omega^2 + 14\pi i\omega + 100\pi^2}\frac{V}{g}\right\}$$

$$z_{21}(i\omega) = qS_wc\left\{-\left(C_{m_\alpha}\right)_o - \left(C_{m_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}e^{-i\tau\omega}\right) - \left(C_{m_{\delta_e}}\right)\left(K_2K_0 i\omega\frac{2}{2 + i\omega}\frac{100\pi^2}{-\omega^2 + 14\pi i\omega + 100\pi^2}\frac{V}{g}\right)\right.$$

$$\left. - \left[\left(C_{m_{\delta_f}}\right) - \left(C_{m_\alpha}\right)_t\frac{d\epsilon}{d\delta_f}e^{-i\tau\omega}\right]K_0 i\omega\frac{2}{2 + i\omega}\frac{100\pi^2}{-\omega^2 + 14\pi i\omega + 100\pi^2}\frac{V}{g}\right\}$$

$$z_{22}(i\omega) = qS_wc\left\{\frac{mk_Y{}^2}{qS_wc}i\omega - \tau\left(C_{m_\alpha}\right)_t - \left(C_{m_{\delta_e}}\right)\left(K_5 - K_2K_0\frac{2}{2 + i\omega}\frac{V}{g}\right)\frac{100\pi^2}{-\omega + 14\pi i\omega + 100\pi^2}\right.$$

$$\left. + \left[\left(C_{m_{\delta_f}}\right) - \left(C_{m_\alpha}\right)_t\frac{d\epsilon}{d\delta_f}e^{-i\tau\omega}\right]K_0\frac{2}{2 + i\omega}\frac{100\pi^2}{-\omega^2 + 14\pi i\omega + 100\pi^2}\frac{V}{g}\right\}$$

and

$$\frac{C_Z}{\alpha_g}(i\omega) = qS_w\left[\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}\right)e^{-i\tau\omega}\right]$$

$$\frac{C_m}{\alpha_g}(i\omega) = qS_wc\left[\left(C_{m_\alpha}\right)_o + \left(C_{m_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}\right)e^{-i\tau\omega}\right]$$

The solution of equation (6), which gives the frequency response functions, is

$$\begin{bmatrix}\dfrac{\alpha}{\alpha_g}(i\omega) \\ \\ \dfrac{\dot{\theta}}{\alpha_g}(i\omega)\end{bmatrix} = \left[Z(i\omega)\right]^{-1}\begin{bmatrix}\dfrac{C_Z}{\alpha_g}(i\omega) \\ \\ \dfrac{C_m}{\alpha_g}(i\omega)\end{bmatrix} \tag{7}$$

The frequency response function for normal acceleration per g at the center of gravity of the airplane is obtained from equation (3) as

$$\frac{n_0}{\alpha_g}(i\omega) = -\frac{V}{g}\left[\frac{\dot{\theta}}{\alpha_g}(i\omega) - i\omega\frac{\alpha}{\alpha_g}(i\omega)\right] \tag{8}$$

where the functions $\frac{\dot{\theta}}{\alpha_g}(i\omega)$ and $\frac{\alpha}{\alpha_g}(i\omega)$ are obtained from equation (7). The frequency response function for normal acceleration at 0, 1, and 2 mean aerodynamic chords behind the center of gravity on the airplane X-axis is

$$\frac{n_k}{\alpha_g}(i\omega) = \frac{n_o}{\alpha_g}(i\omega) + \frac{kc}{g}\, i\omega\, \frac{\dot{\theta}}{\alpha_g}(i\omega) \tag{9}$$

where $k = 0, 1, 2$.

Response to Turbulence

The vertical component of gust velocity w_g in turbulent air is assumed to be a random variable having a normal distribution with zero mean and variance $\left(\sigma_{w_g}\right)^2$. The expression for the value of the rms angle of attack caused by the vertical gusts is

$$\sigma_{\alpha_g} = \frac{\sigma_{w_g}}{V}$$

For homogeneous isotropic turbulence having scale L, the power spectral density of the vertical gust velocity is given as a function of circular frequency ω by Von Kármán's formula (ref. 2). It then follows that the power spectral density function for the angle of attack of the wing due to the vertical gust velocity is

$$\Phi_{\alpha_g}(\omega) = \frac{L\left(\sigma_{w_g}\right)^2}{\pi V^3} \frac{1 + \frac{8}{3}\left(1.339\,\frac{L\omega}{V}\right)^2}{\left[1 + \left(1.339\,\frac{L\omega}{V}\right)^2\right]^{11/6}} \tag{10}$$

The power spectral density of the output of the airplane is related to the power spectral density of gusts by the following result from random process theory:

$$\Phi_{output}(\omega) = \left|h(i\omega)\right|^2 \Phi_{\alpha_g}(\omega) \tag{11}$$

and the variance of the output is defined as the following integral:

$$\sigma_{output}^2 = \int_0^\infty \Phi_{output}(\omega)\, d\omega \tag{12}$$

In equation (11), $\left|h(i\omega)\right|^2$ is the square of the absolute value of the frequency response function of an output variable. For instance, $h(i\omega)$ could be $\frac{\alpha}{\alpha_g}(i\omega)$, $\frac{\dot{\theta}}{\alpha_g}(i\omega)$, or $\frac{n_k}{\alpha_g}(i\omega)$ from equations (7) and (9). Thus, the rms variations of the output variables for an airplane may be calculated by using the appropriate frequency response functions.

Calculations

The airplane dimensions, mass, flight condition, and aerodynamics used in this study are presented in table I. The flight condition is for normal cruise at 3048 meters altitude. The scale for the turbulence is assumed to be 304.8 meters. Calculations are made with the relations in equations (11) and (12) to obtain the rms variation of normal acceleration per g at the center of gravity and at 1 and 2 mean aerodynamic chords behind the center of gravity on the X-axis. From these calculations, the percent reduction of normal acceleration, which is called percent alleviation, attributable to the alleviation systems is calculated. The rms variation of angle of attack σ_α, the rms variation of the pitch rate $\sigma_{\dot\theta}$, and the rms variations of the control deflection angles σ_{δ_e} and σ_{δ_f} and deflection rates $\sigma_{\dot\delta_e}$ and $\sigma_{\dot\delta_f}$ also are calculated. The rms vertical gust velocity used in the calculations is 0.3048 m/sec. This value is considered to be a unit rms vertical gust velocity. Since the rms variation of an output variable is directly proportional to the rms vertical gust velocity, the calculated values of $\sigma_{n,k}$, $\sigma_{\dot\theta}$, σ_{δ_e}, $\sigma_{\dot\delta_e}$, σ_{δ_f}, and $\sigma_{\dot\delta_f}$ can be scaled for any multiple of the unit rms gust velocity.

The response of the basic airplane (that is, $K_0 = K_2 = K_5 = 0$) is used as a basis for assessment of the gust alleviation systems. The calculated values of the rms variation of the normal acceleration per g at the center of gravity $\left(\sigma_{n,0}\right)_0$ and at 1 and 2 mean aerodynamic chords behind the center of gravity $\left(\sigma_{n,1}\right)_0$ and $\left(\sigma_{n,2}\right)_0$, respectively, for the basic airplane are compared with corresponding rms variations of normal acceleration per g when the gust alleviation systems are used. The basic airplane response is

$\left(\sigma_{n,0}\right)_0 = 0.0287$ (rms variation of normal acceleration per g at center of gravity)

$\left(\sigma_{n,1}\right)_0 = 0.0298$ (rms variation of normal acceleration per g 1 chord behind center of gravity)

$\left(\sigma_{n,2}\right)_0 = 0.0313$ (rms variation of normal acceleration per g 2 chords behind center of gravity)

$$\left(\sigma_{\dot\theta}\right)_0 = 0.00257 \text{ rad/sec}$$

and

$$\left(\sigma_\alpha\right)_0 = 0.00265 \text{ rad}$$

Percent alleviation at the three fuselage locations is calculated by

$$\left.\begin{aligned} R_0 &= 100\left[\frac{\left(\sigma_{n,0}\right)_0 - \sigma_{n,0}}{\left(\sigma_{n,0}\right)_0}\right] \\ R_1 &= 100\left[\frac{\left(\sigma_{n,1}\right)_0 - \sigma_{n,1}}{\left(\sigma_{n,1}\right)_0}\right] \\ R_2 &= 100\left[\frac{\left(\sigma_{n,2}\right)_0 - \sigma_{n,2}}{\left(\sigma_{n,2}\right)_0}\right] \end{aligned}\right\} \tag{13}$$

Values of R_0, R_1, and R_2 and the rms variations of pitch rate, control deflection angles, and control deflection rates are presented in figures 3 to 8 for values of the alleviation system gains. Values of the gains K_0, K_2, and K_5 range from 0 to 3. From the plotted values of the mean-square control deflections and control rates, the tendency of the controls to encounter saturation effects at higher values of rms gust velocities may be estimated.

Since the transport time-lag terms make stability calculations extremely difficult, the stability was checked with $e^{-\tau s}$ approximated by $1 - \tau s$. The time lag τ, which is about 0.08 second, is considered to be small enough so that if the system using the above approximation is stable, then the system using the exact expression for the time lag is also stable. All the data presented in the figures were obtained for stable systems.

RESULTS AND DISCUSSION

The alleviation system that uses only elevator control $\left(K_0 = K_2 = 0\right)$ reduced the normal acceleration at the center of gravity by as much as 4 percent and at 2 chords aft of the center of gravity by 8.5 percent (fig. 3(a)). An elevator is primarily a pitch control and does not provide the changes of force in the Z-direction that are necessary for good gust-load alleviation. The large reduction of rms variation of pitch rate $\sigma_{\dot{\theta}}$ shown in figure 3(b) is an indication of the ability of the elevator to control pitching motion. Nevertheless, only small rms variations of elevator deflection angle and deflection rate are required to obtain the given results (fig. 3(c)). Also, for this gust alleviation system, it would not be necessary for the gain K_5 to have a value greater than 0.5 rad/rad/sec.

Maximum gust alleviation with only flap control $\left(K_2 = K_5 = 0\right)$ is 40 percent, 29 percent, and 5 percent, respectively, at the center of gravity and at 1 and 2 chords behind the center of gravity $\left(K_0 = 1.4 \text{ in fig. } 4(a)\right)$. Operation of the flap produces the necessary change of the force in the Z-direction for good alleviation at the center of gravity. However, lack of control of pitching motion results in increased normal acceleration behind the center of gravity for some values of K_0. This lack of control is indicated by the large rms variation of pitch rate shown in figure 4(b). The rms variation of flap deflection angle σ_{δ_f} is small, but the rms variation of flap deflection rate $\sigma_{\dot{\delta}_f}$ is 0.044 rad/sec for K_0 of 1.4 rad/g (fig. 4(c)).

Good alleviation is obtained when only the flap control is used, and small values of rms variation of pitch rate are obtained when only the elevator control is used. The use of both controls simultaneously but acting independently $(K_2 = 0)$ gives even better alleviation with good control of pitch rate. Figure 5 shows that the alleviation is about 45 percent $(K_0 = 1.0$ rad/g and $K_5 = 1.0$ rad/rad/sec$)$ with an rms variation of pitch rate of about 0.0022 rad/sec. The rms variation of control deflection rates is less than 0.025 rad/sec. With $K_0 = 1.0$ rad/g, only slight improvement in percent alleviation and rms variation of pitch rate is obtained for K_5 greater than 0.5 rad/rad/sec.

The gust alleviation system for which the elevator control is dependent on the flap control $(K_5 = 0)$ gives about 45 percent alleviation with $K_2 = 0.5$ and $K_0 = 1.0$ rad/g, as shown in figure 6(a). The rms variation of the pitch rate is appreciably reduced by increasing the elevator command (that is, K_2 is increased) (fig. 6(b)). The rms variation of control deflection angles and the rms variation of control deflection rates are small. (See figs. 6(c) and (d).)

An instability (stability was discussed in a previous paragraph) occurs for values of the gain K_2 that are larger than 0.5. The airplane short-period mode becomes nonoscillatory and unstable when K_2 is larger than 0.5. A thorough analysis of this unstable condition would be desirable for the design of a gust alleviation system. For the purposes of this investigation, the only requirement is that the system be stable.

The complete gust alleviation system (none of the gains are zero) gives better alleviation of normal acceleration than the other systems (fig. 7). Furthermore, the rms variation of pitch rate and the rms variation of control deflection angles and deflection rates are as small or smaller than the values calculated for the other systems. The overall performance of the complete gust alleviation system is considered to be superior to the performance of the other systems.

The best theoretical performance attainable by the complete gust alleviation system is not shown by the results in figure 7. Increasing the gains K_0, K_2, and K_5 to values larger than 1.0 rad/g, 0.5, and 1.0 rad/rad/sec, respectively, should give better alleviation. The gain K_2, however, must not be greater than 0.5 in order that the airplane be stable. Furthermore, the results shown in figure 7 indicate that the gain K_5 could be limited to 0.5 rad/rad/sec without serious loss of performance. Consequently, with $K_2 = 0.5$ and $K_5 = 0.5$ rad/rad/sec, calculations were made for K_0 ranging from 0.5 to 3.0 rad/g. The results are presented in figure 8 (notice the change in the scale of the gain K_0). A maximum percent alleviation was obtained at each of the fuselage locations within the range of values of K_0. The maximum percent alleviation is 49.5 at the center of gravity, 54.5 at 1 mean aerodynamic chord behind the center of gravity, and 56.5 at 2 mean aerodynamic chords behind the center of gravity. These maximum values occurred for K_0 equal to 1.8, 1.5, and 1.4 rad/g, respectively. The values of R_0, R_1, and R_2 are very near their maximum values for $K_0 = 1.6$ rad/g. Therefore, the complete gust alleviation system with gains K_0, K_2, and K_5 equal to 1.6 rad/g, 0.5, and 0.5 rad/rad/sec, respectively, is considered to give the best alleviation. The overall performance of this system is summarized as follows:

K_0, rad/g	1.6
K_2	0.5
K_5, rad/rad/sec	0.5
R_0, percent	49.5
R_1, percent	54.5
R_2, percent	56.5
$\sigma_{\dot{\theta}}$, rad/sec	0.0018
σ_{δ_e}, rad	0.0026
$\sigma_{\dot{\delta}_e}$, rad/sec	0.017
σ_{δ_f}, rad	0.00415
$\sigma_{\dot{\delta}_f}$, rad/sec	0.04

This alleviation system produces almost the maximum load alleviation of the systems studied and a low rms variation of pitch rate and requires small rms variation of control deflection angles. To obtain an idea of the control deflections and rates required by this system in a condition of severe turbulence, consider an rms gust velocity of 9.144 m/sec. The corresponding rms values of control deflections and rates would be as follows:

σ_{δ_e}, rad	0.070
$\sigma_{\dot{\delta}_e}$, rad/sec	0.51
σ_{δ_f}, rad	0.124
$\sigma_{\dot{\delta}_f}$, rad/sec	1.20

CONCLUDING REMARKS

An analytical study has shown that a gust alleviation system for a STOL airplane in a cruise condition could reduce the root mean square of the normal acceleration of the airplane flying in random turbulence by as much as 50 percent. This alleviation is obtained by driving the flaps in response to normal acceleration and by moving the elevator in proportion to the commanded flap deflection angle and to a pitch-rate signal.

The assessment of the gust alleviation systems made in this study was based on the root-mean-square response of the airplane. Other problems, such as a poorly damped mode that shows up as a peak in the power spectrum or the presence of static instability, have not been investigated. Also the form and the parameters of the filter in the accelerometer feedback loop were not optimized. Further investigation of these problems is desirable.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 13, 1973.

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1. Phillips, William H.; and Kraft, Christopher C., Jr.: Theoretical Study of Some Methods for Increasing the Smoothness of Flight Through Rough Air. NACA TN 2416, 1951.

2. Houbolt, John C.; Steiner, Roy; and Pratt, Kermit G.: Dynamic Response of Airplanes to Atmospheric Turbulence Including Flight Data on Input and Response. NASA TR R-199, 1964.

TABLE I.- AIRPLANE MASS, DIMENSIONS, FLIGHT CONDITION, AND AERODYNAMIC CHARACTERISTICS

Mass, m, kg	5669.905
Wing area, S_W, m^2	39.019
Mean aerodynamic chord, $\bar{c}$, m	1.981
Radius of gyration about Y-axis, k_Y, m	2.572
Tail length, m	7.742
True airspeed, V, m/sec	108.893
Altitude, m	3048
Dynamic pressure, q, N/m^2	5364.030
$\left(C_{Z_\alpha}\right)_o$, per radian	-5.3243
$\left(C_{Z_\alpha}\right)_t$, per radian	-0.682
$\left(C_{m_\alpha}\right)_o$, per radian	0.576
$\left(C_{m_\alpha}\right)_t$, per radian	-2.665
$C_{Z_{\delta_e}}$, per radian	-0.459
$C_{m_{\delta_e}}$, per radian	-1.813
$C_{Z_{\delta_f}}$, per radian	-2.292
$C_{m_{\delta_f}}$, per radian	0.430
$\frac{d\epsilon}{d\alpha}$	0.2884
$\frac{d\epsilon}{d\delta_f}$	0.133

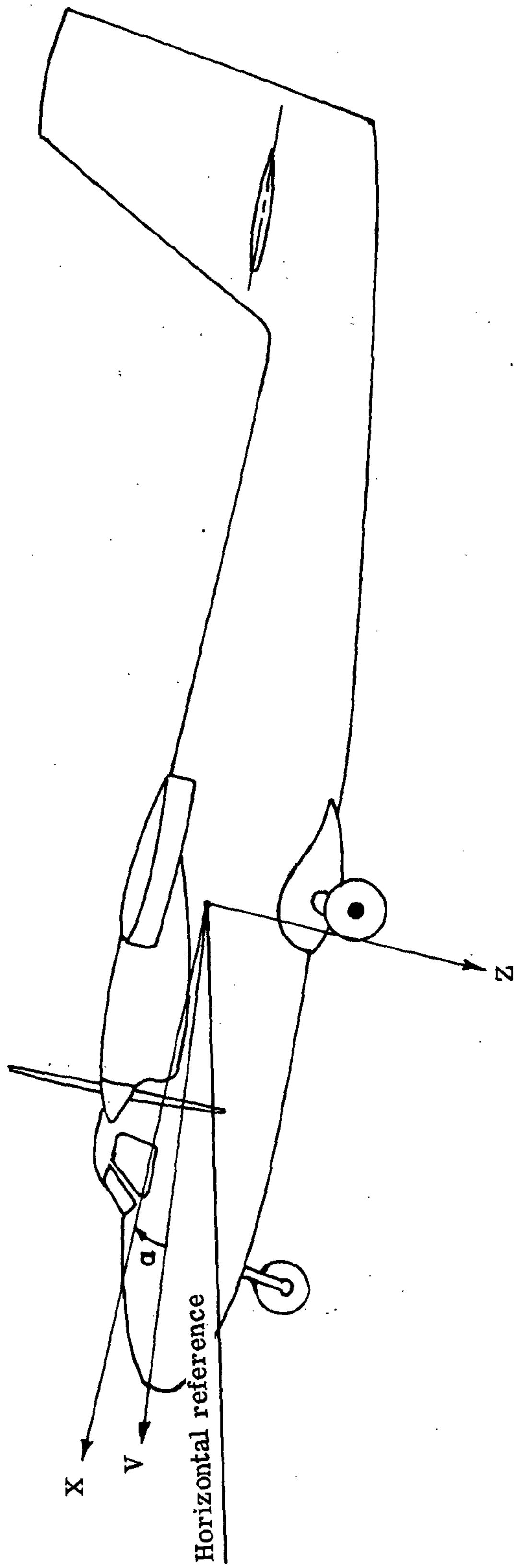


Figure 1.- Axis system.

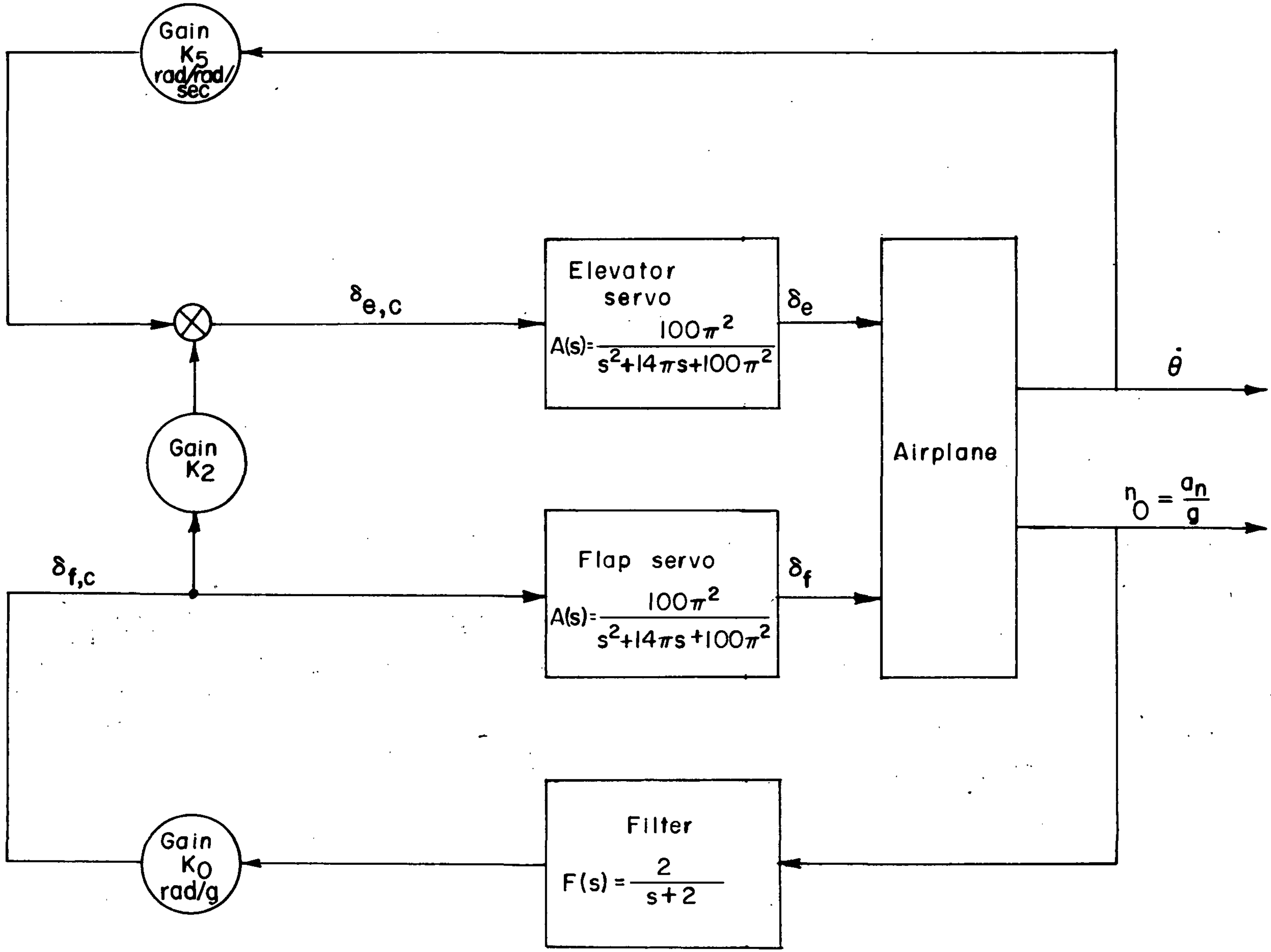
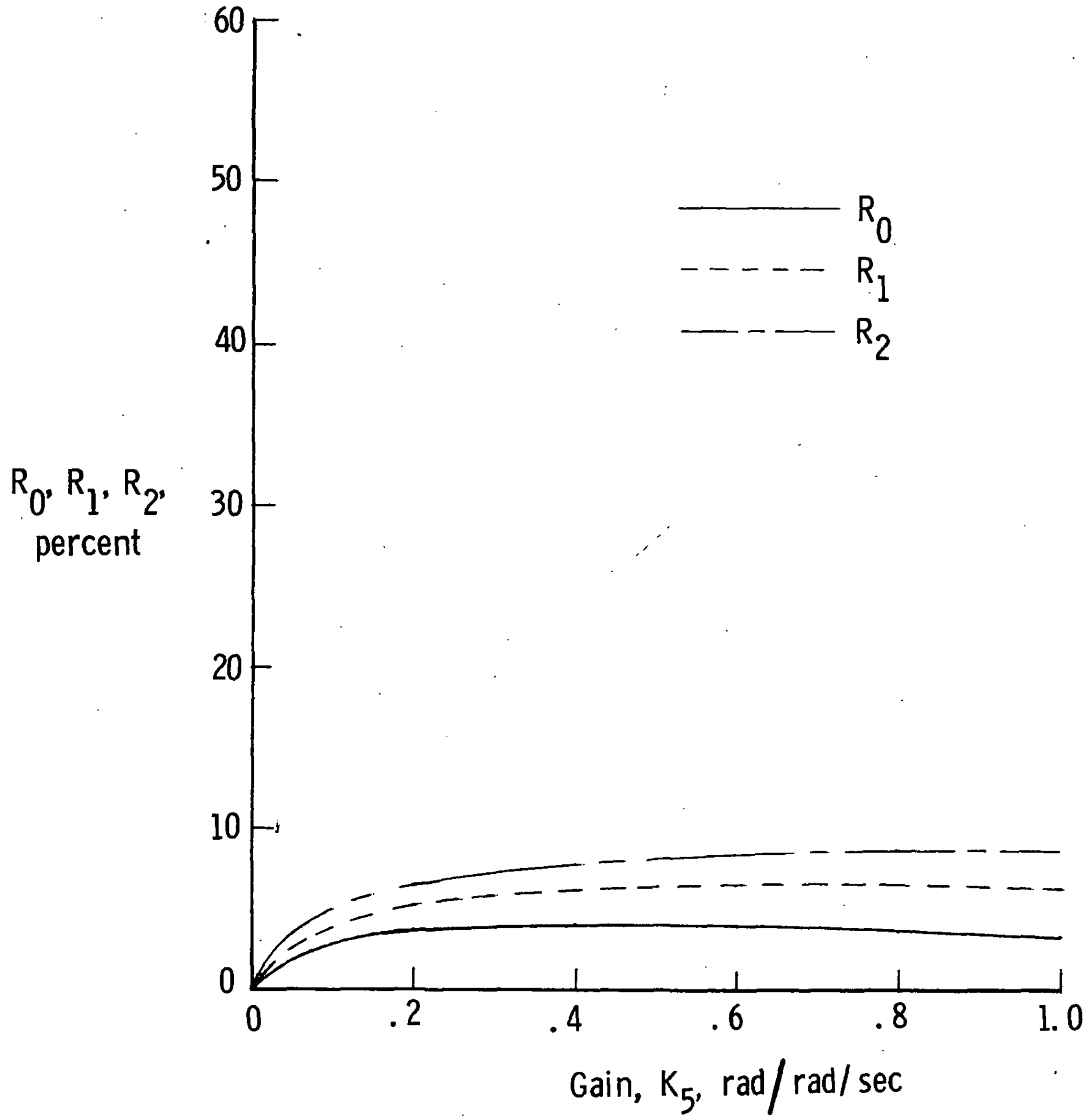
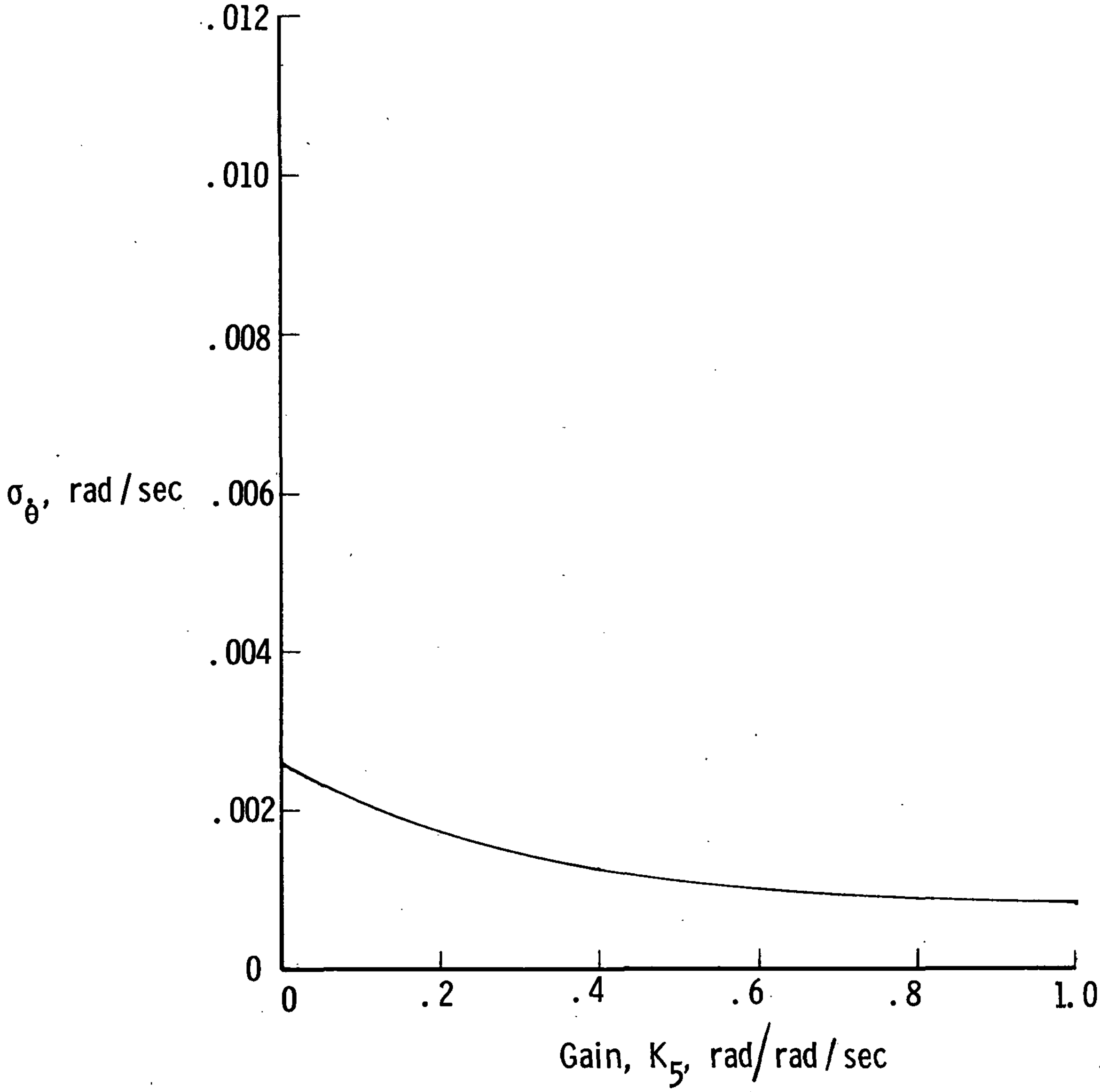


Figure 2.- Block diagram of gust alleviation systems.



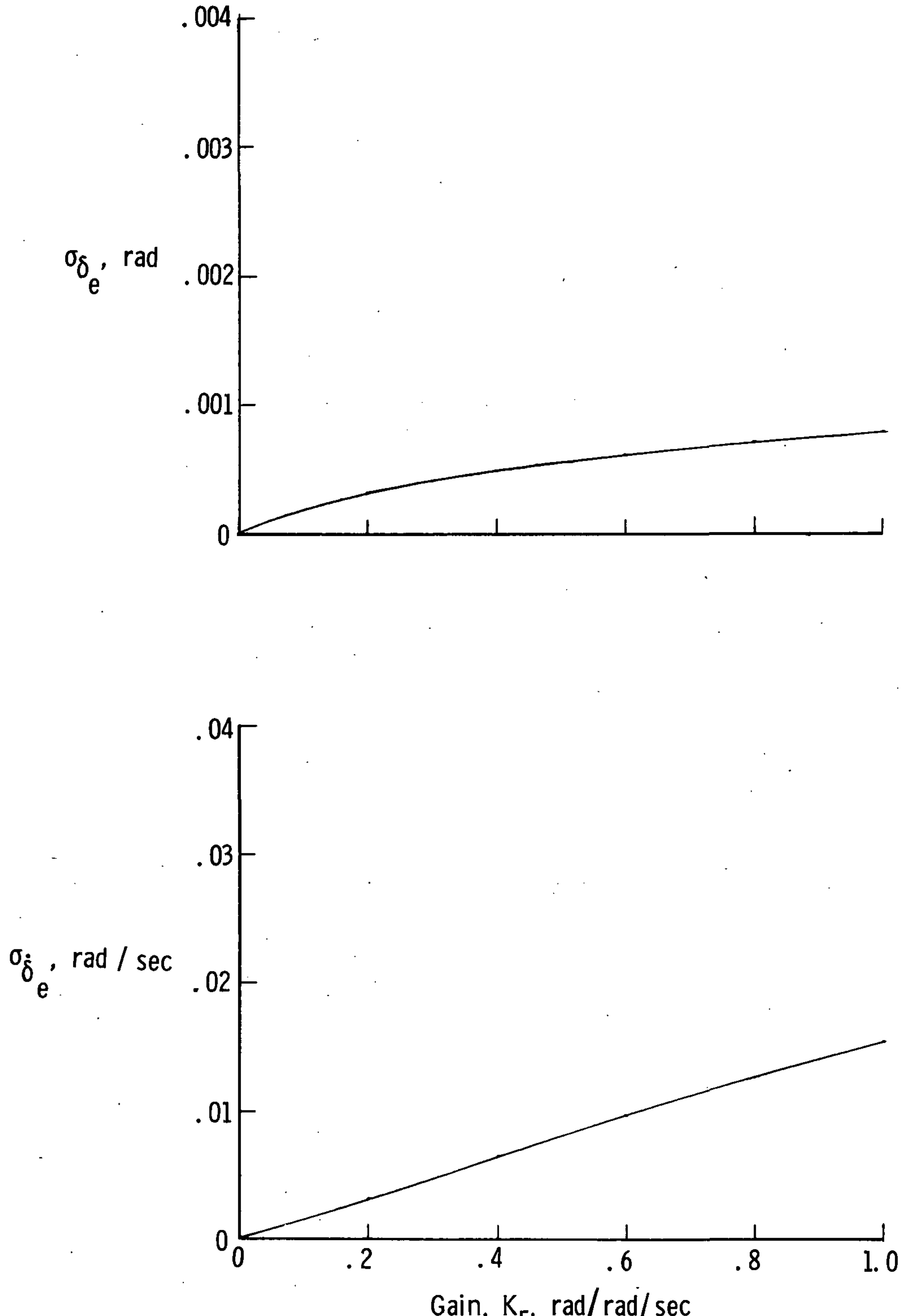
(a) Percent alleviation in normal acceleration.

Figure 3.- Alleviation system using only elevator control. $K_0 = K_2 = 0$.



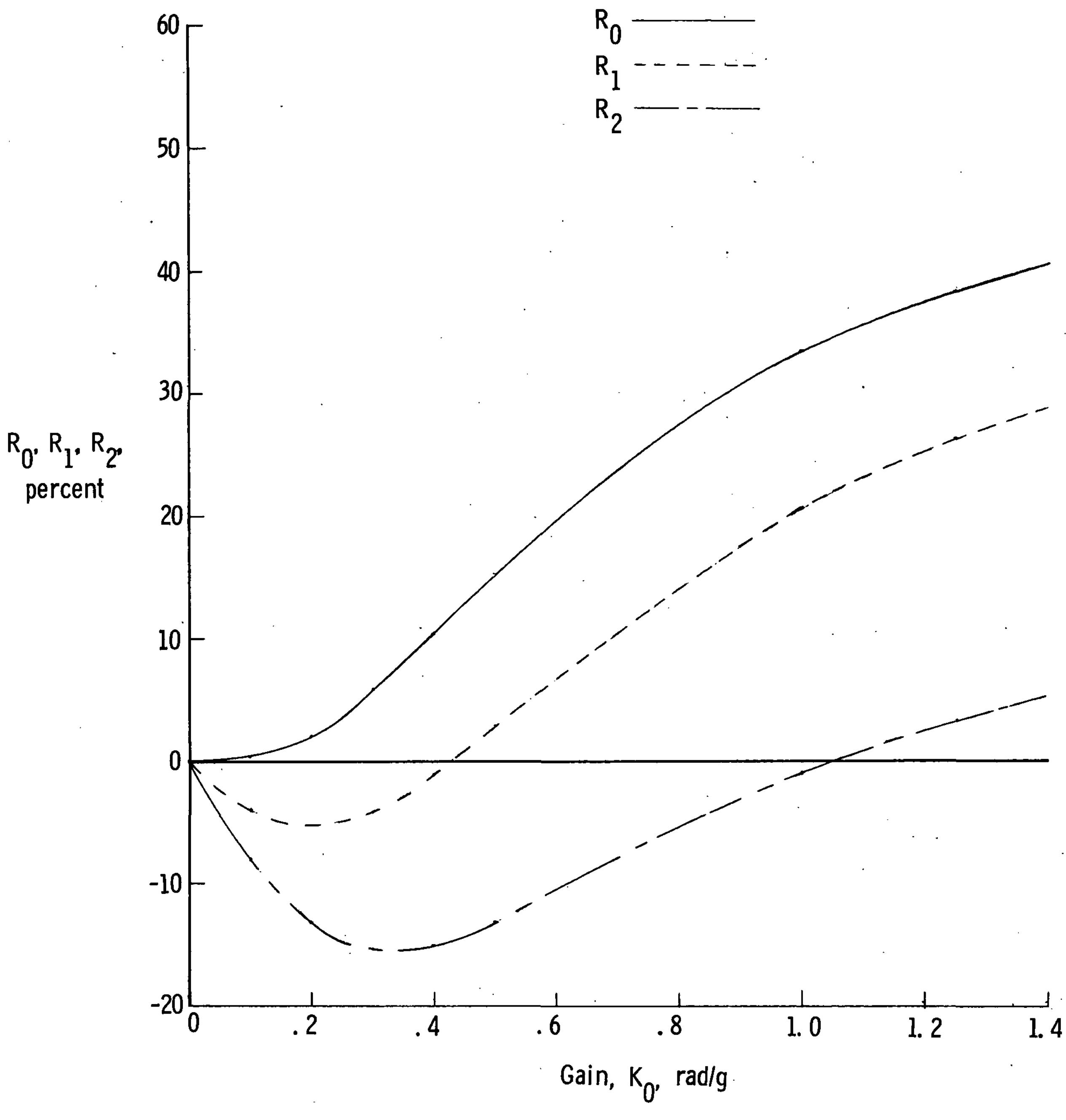
(b) rms pitch rate.

Figure 3.- Continued.



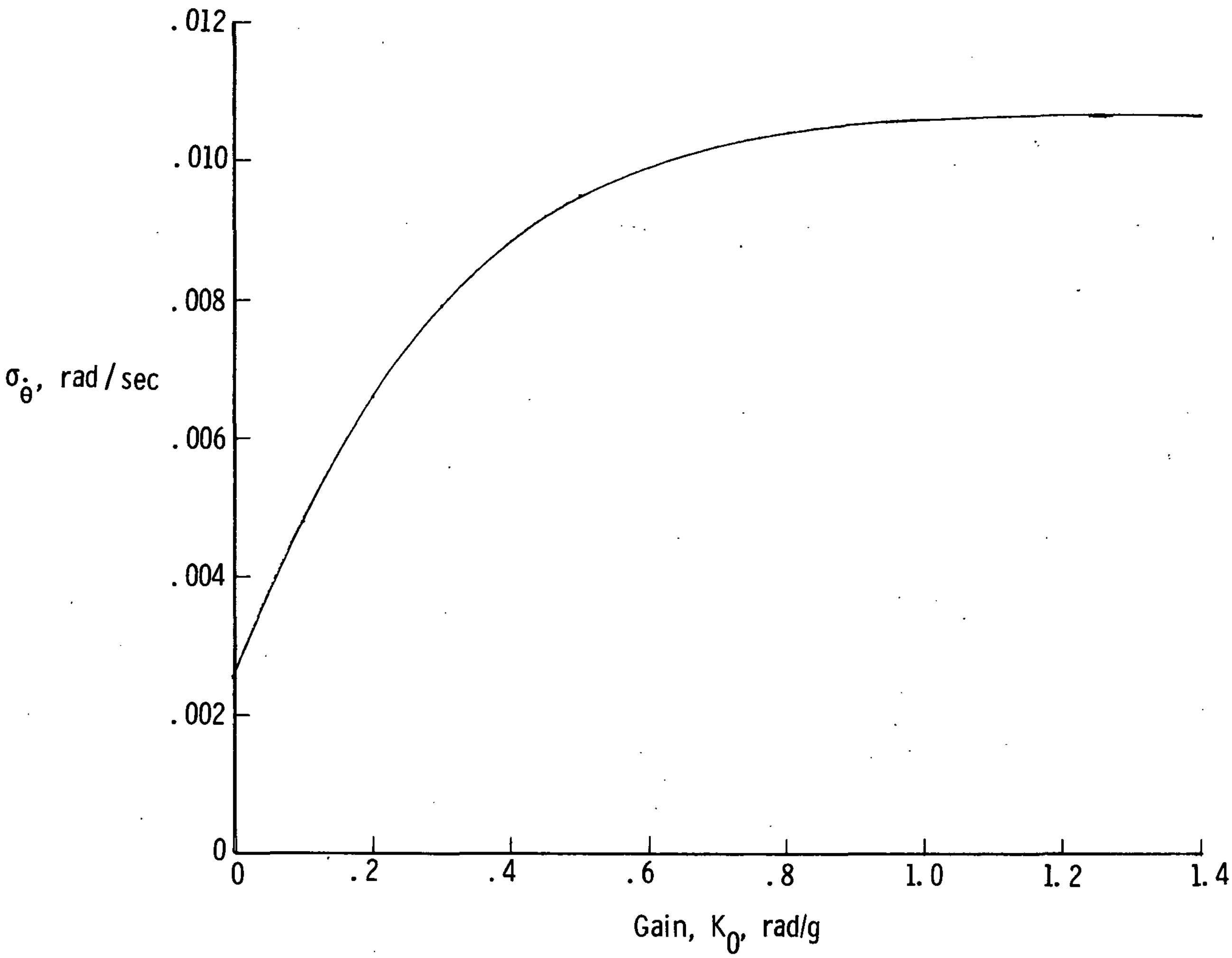
(c) rms elevator deflection angle and rms elevator deflection rate.

Figure 3.- Concluded.



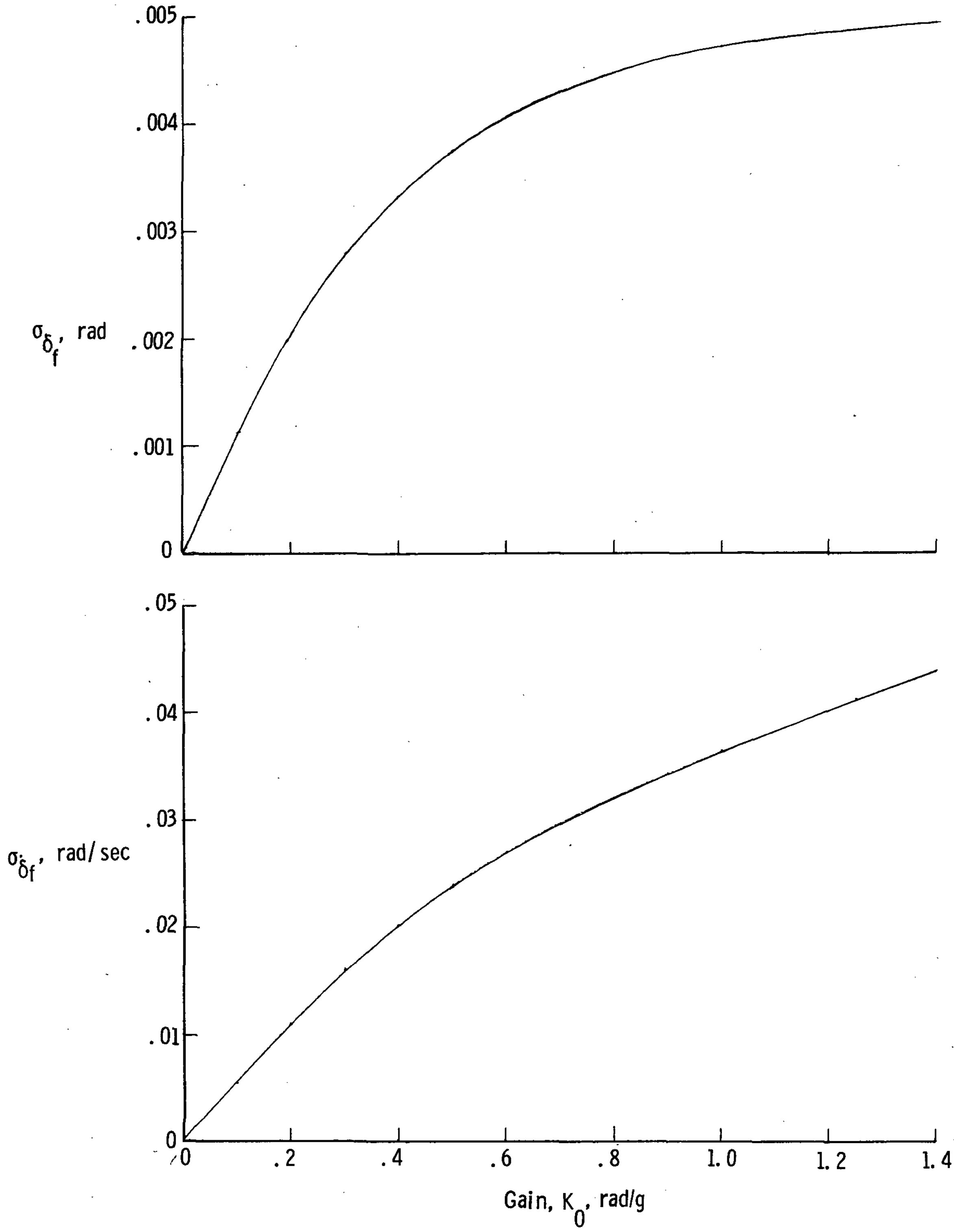
(a) Percent alleviation in normal acceleration.

Figure 4.- Alleviation system using flap control. $K_2 = K_5 = 0$.



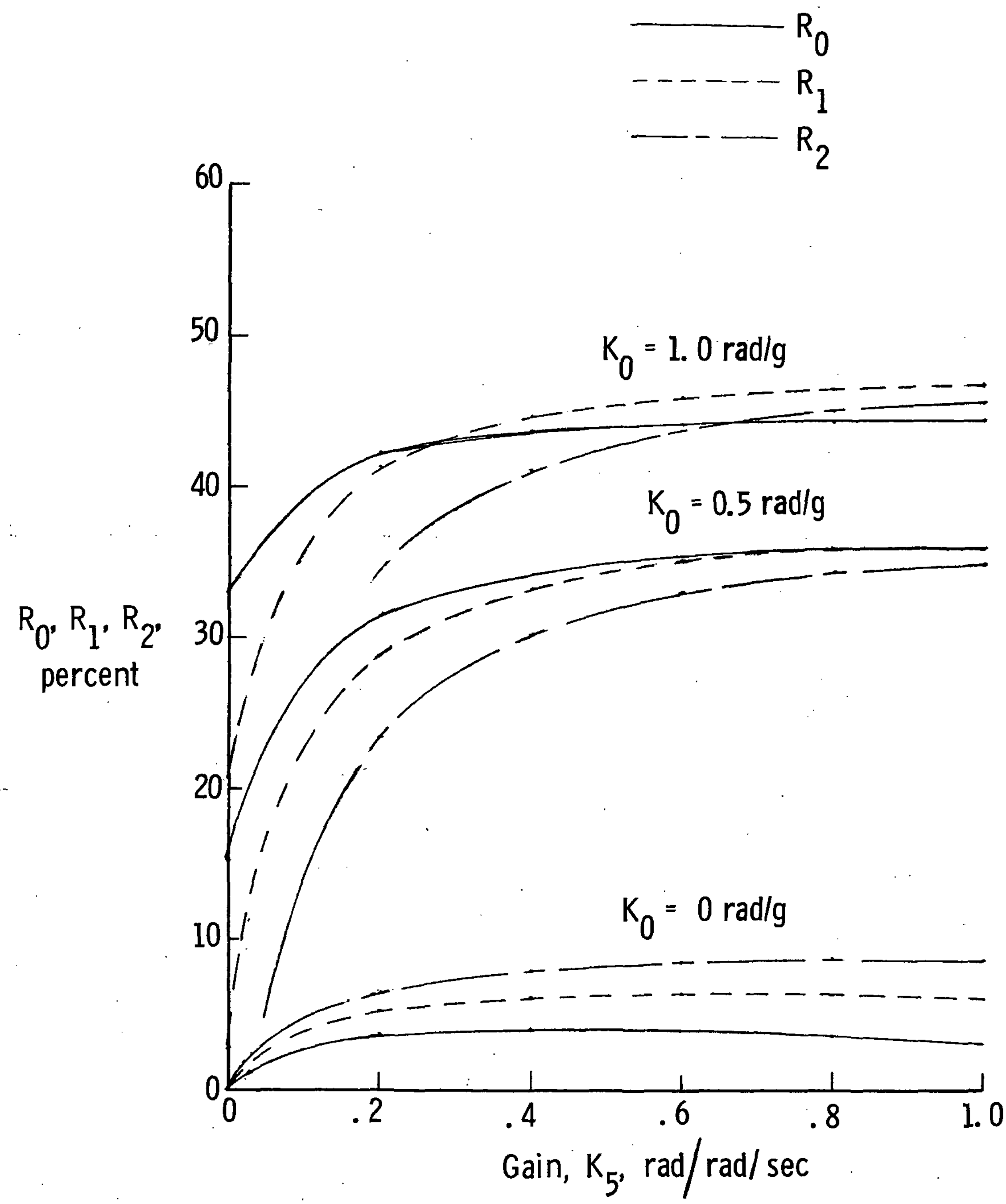
(b) rms pitch rate.

Figure 4.- Continued.



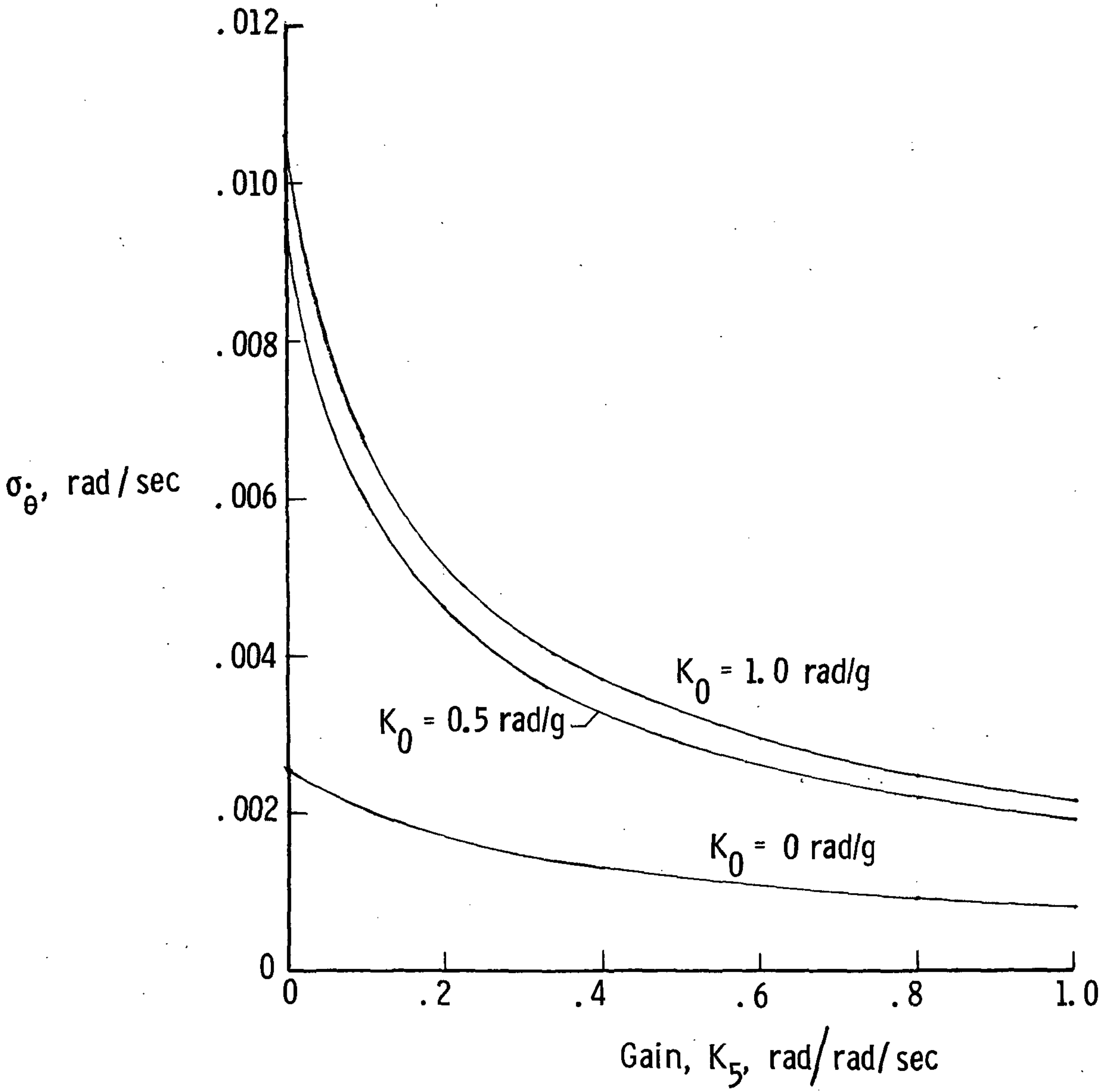
(c) rms flap deflection angle and rms flap deflection rate.

Figure 4.- Concluded.



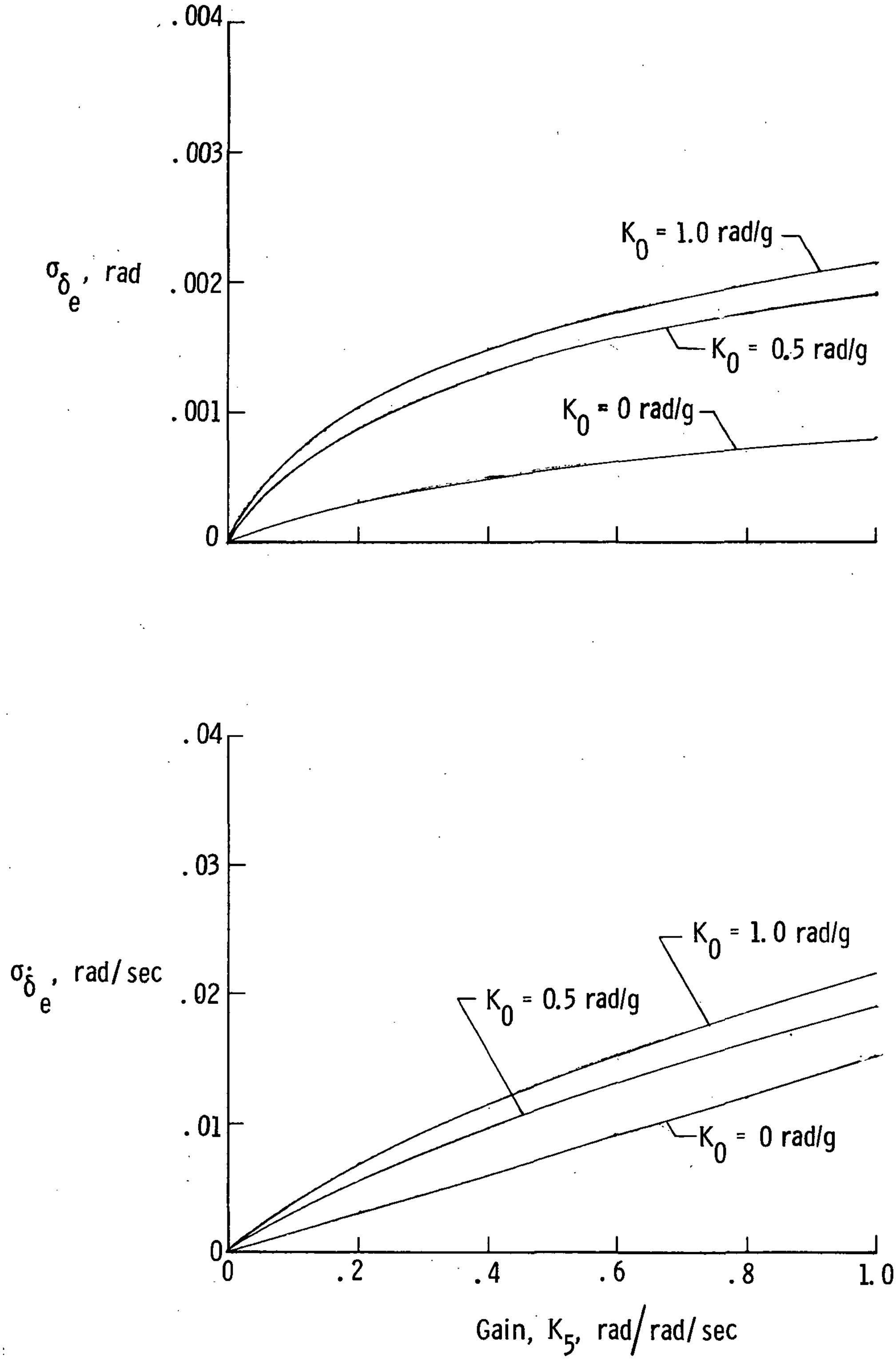
(a) Percent alleviation in normal acceleration.

Figure 5.- Alleviation system using independent flap and elevator control. $K_2 = 0$.



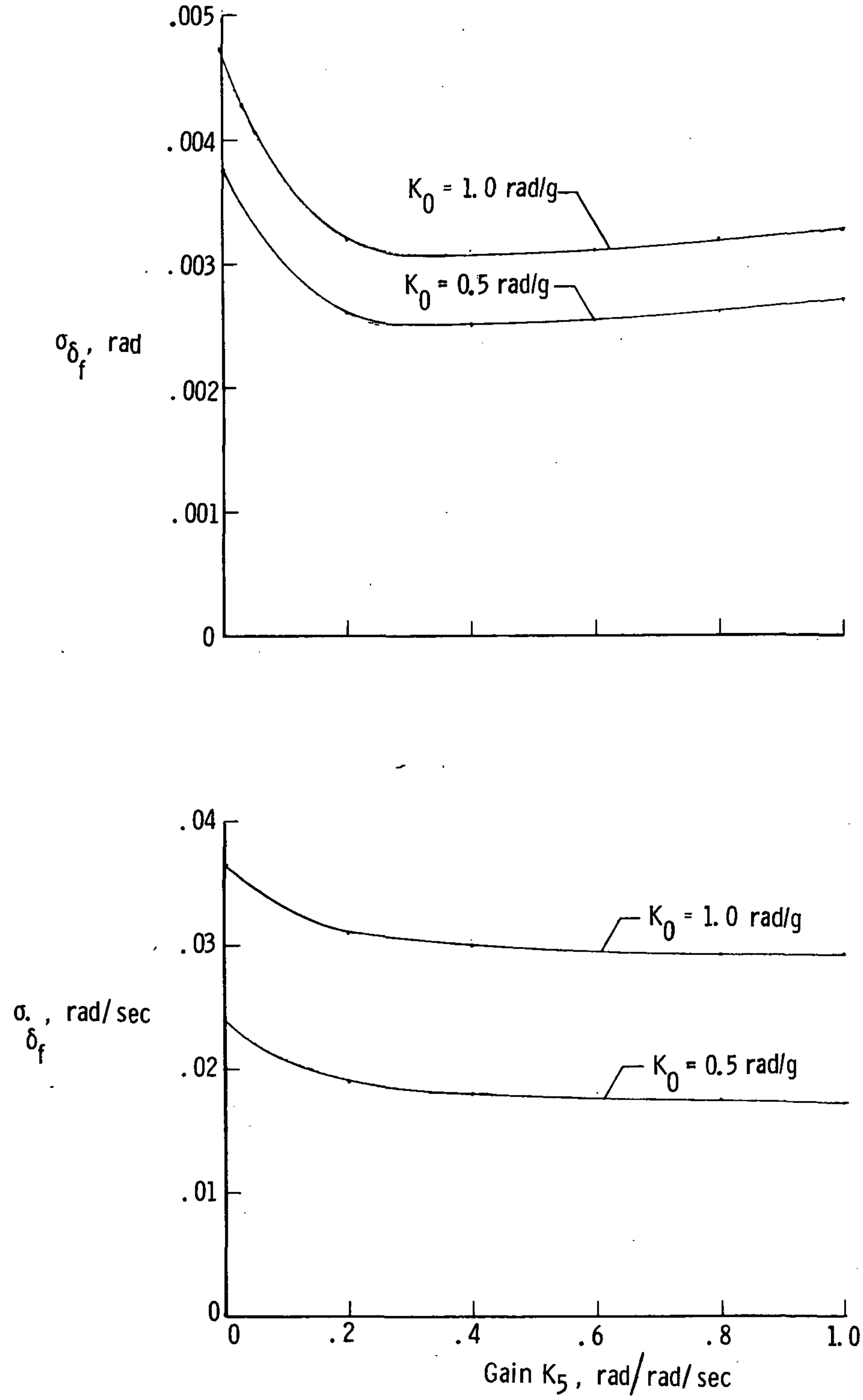
(b) rms pitch rate.

Figure 5.- Continued.



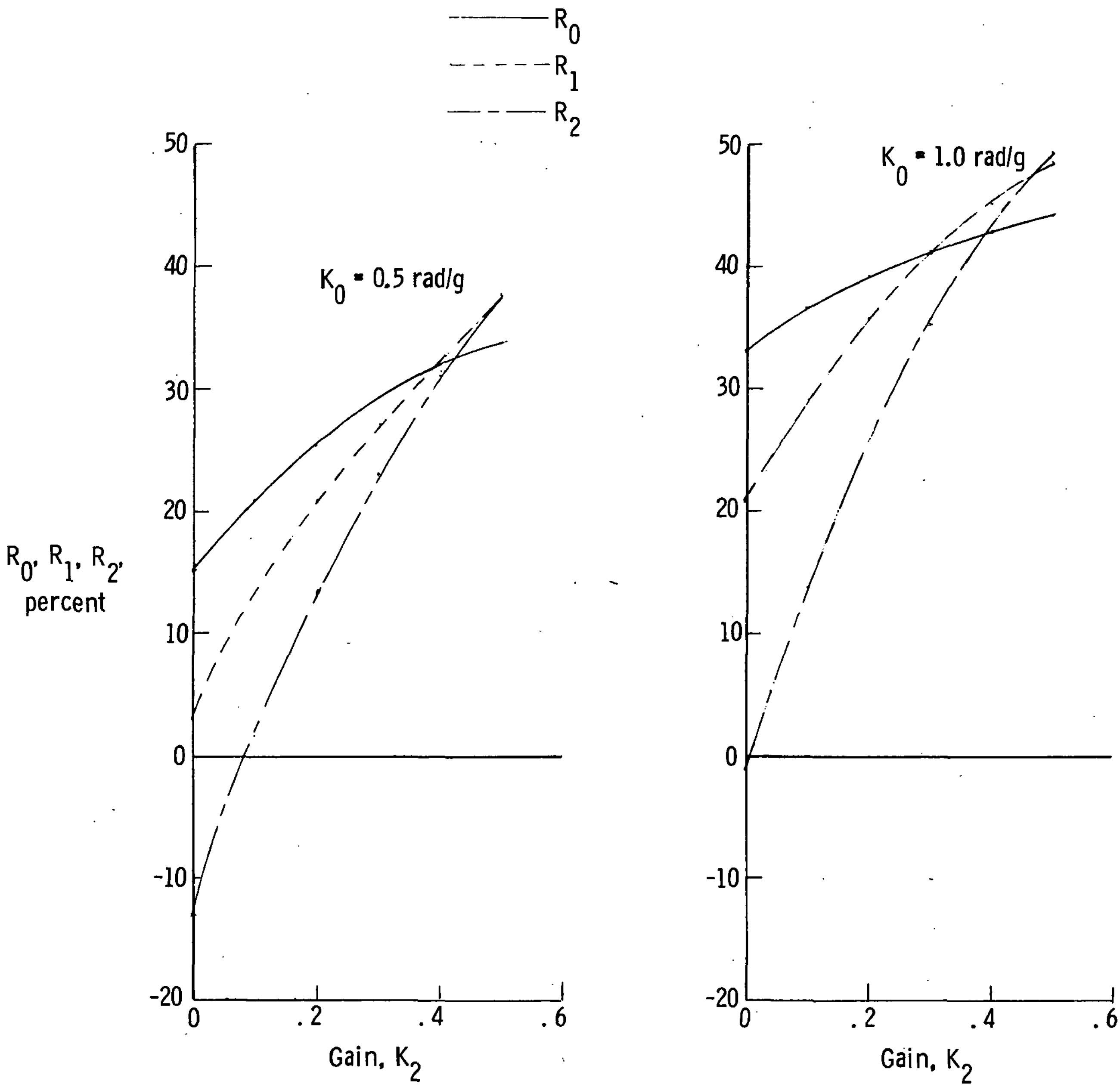
(c) rms elevator deflection angle and rms elevator deflection rate.

Figure 5.- Continued.



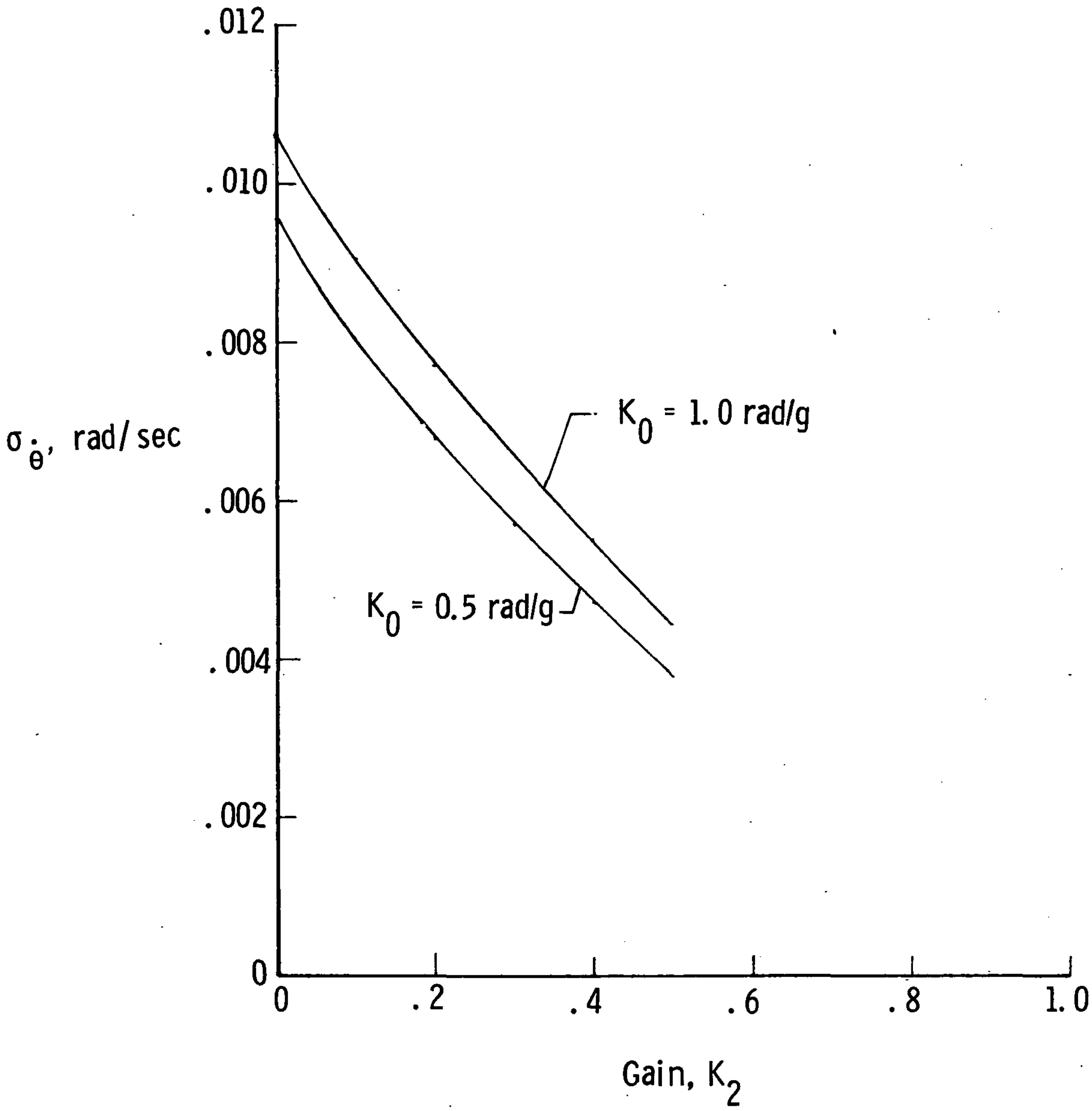
(d) rms flap deflection angle and rms flap deflection rate.

Figure 5.- Concluded.



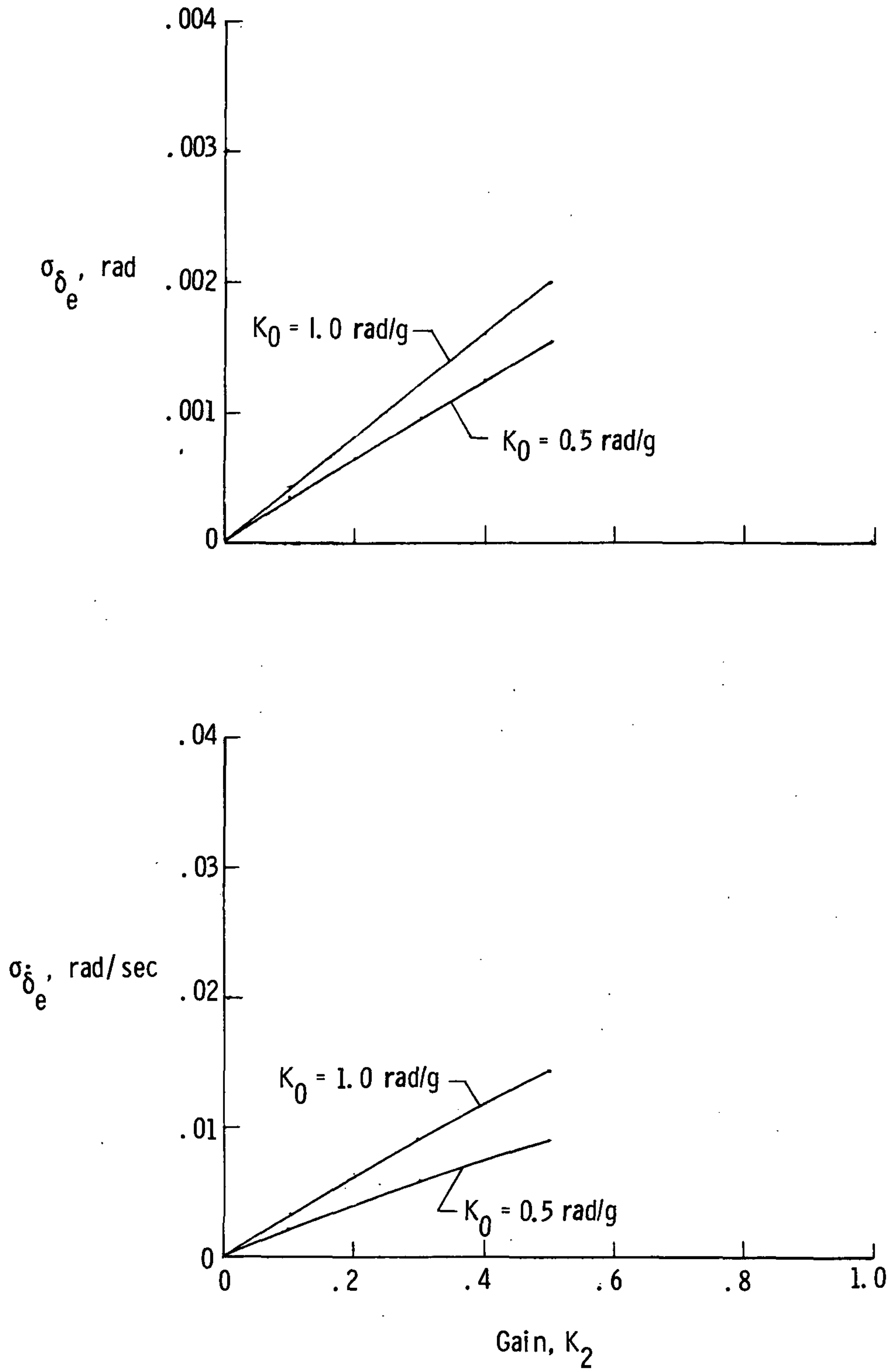
(a) Percent alleviation in normal acceleration.

Figure 6.- Alleviation system using dependent flap and elevator control. $K_5 = 0$.



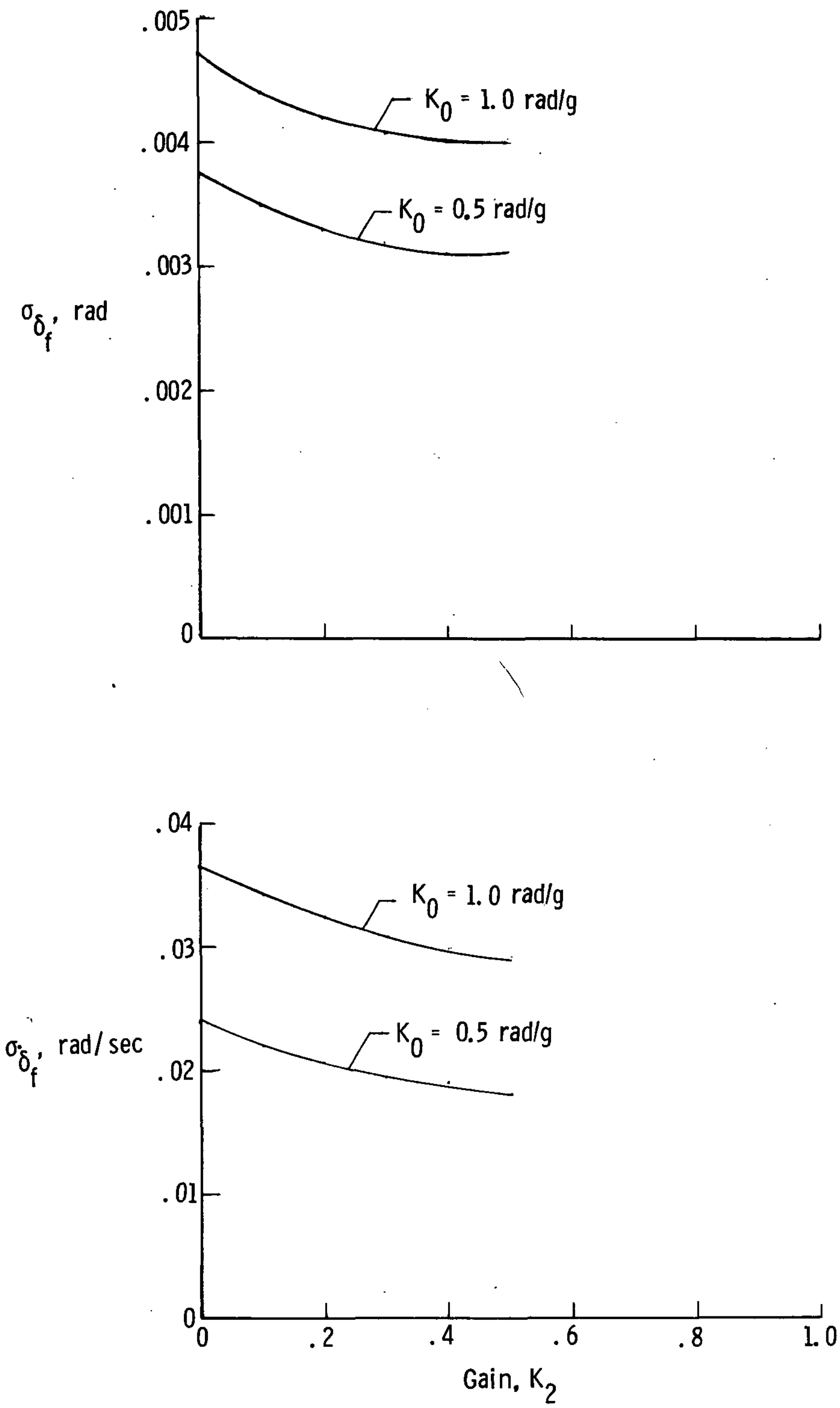
(b) rms pitch rate.

Figure 6.- Continued.



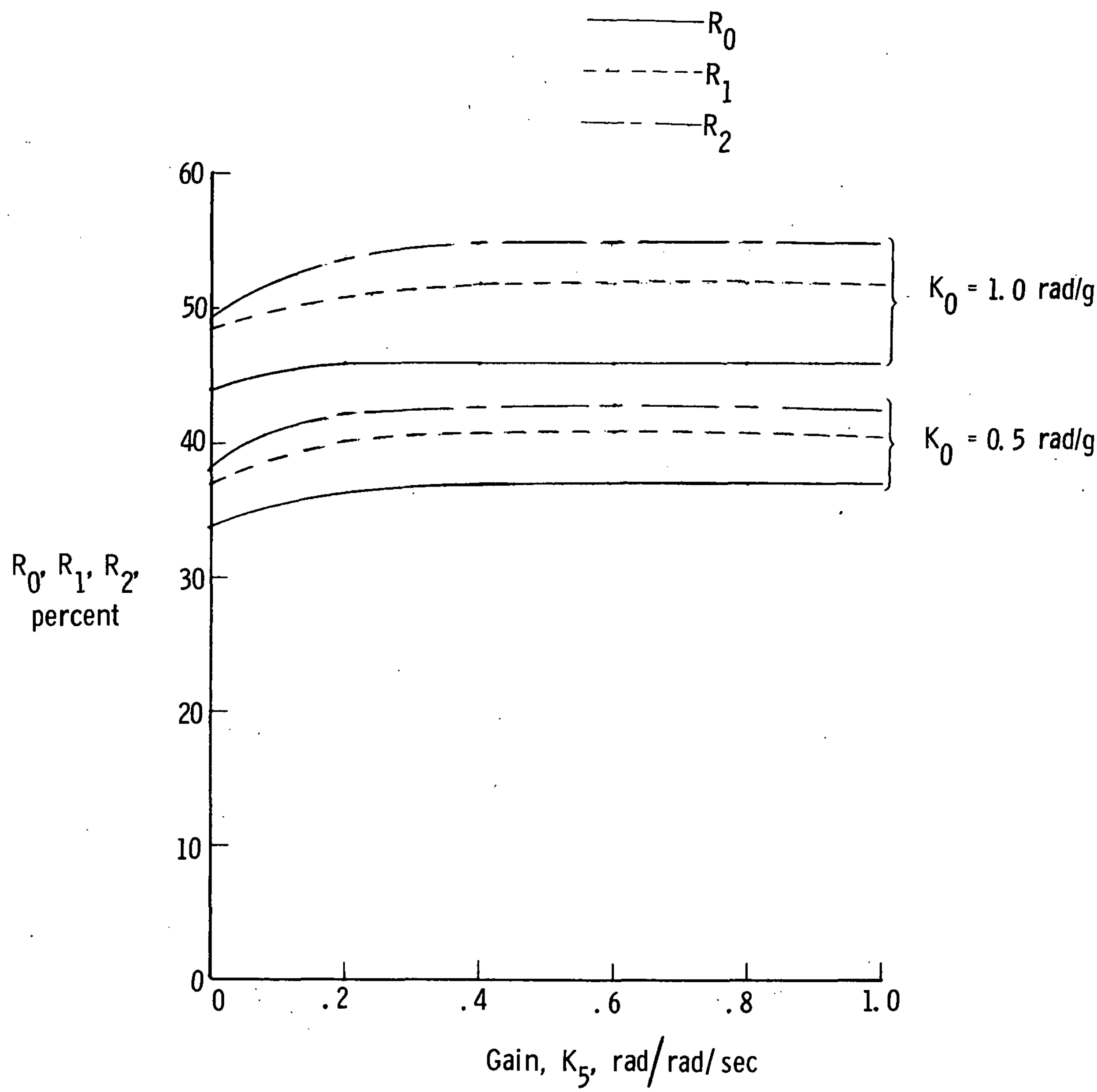
(c) rms elevator deflection angle and rms elevator deflection rate.

Figure 6.- Continued.



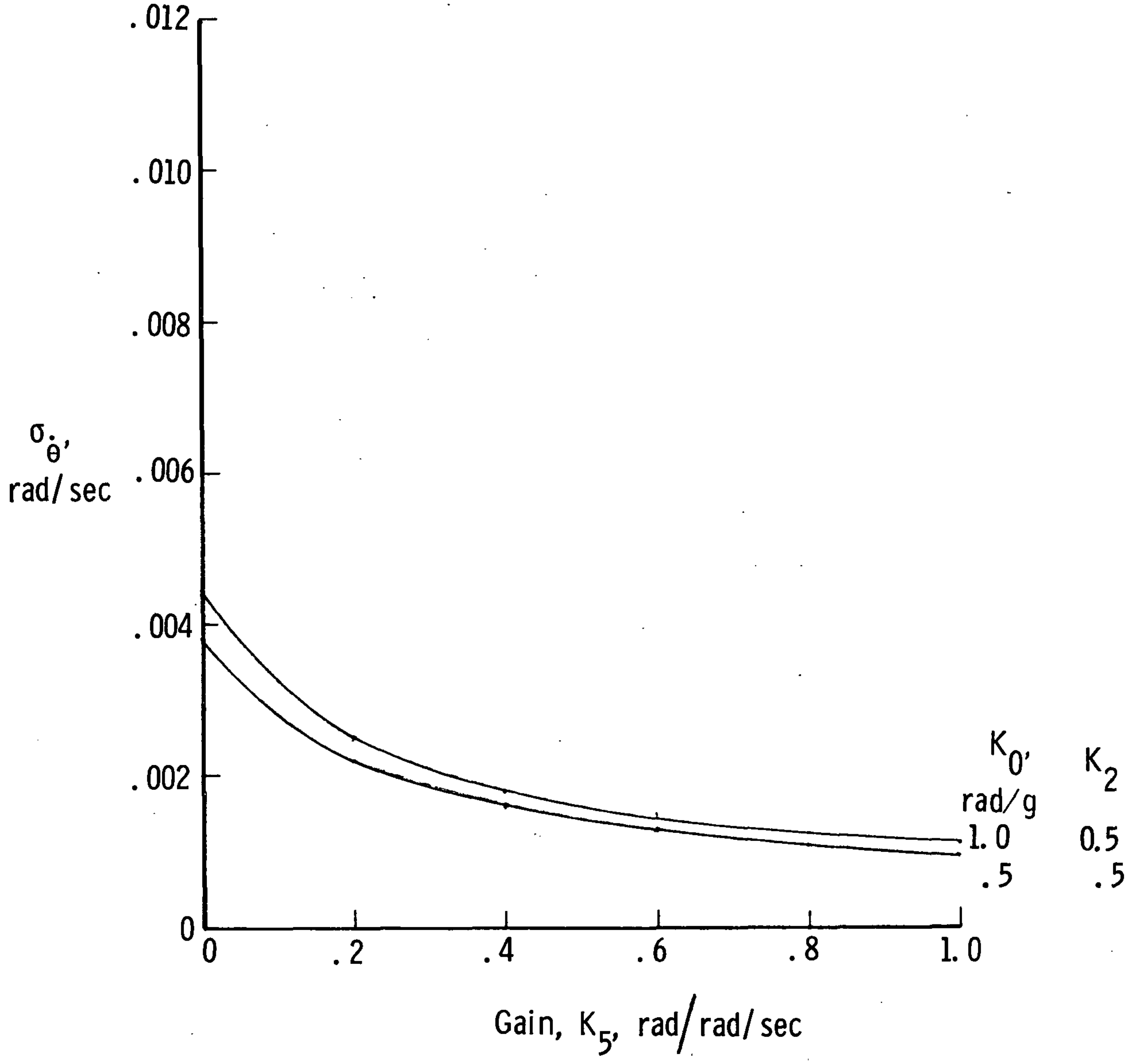
(d) rms flap deflection angle and rms flap deflection rate.

Figure 6.- Concluded.



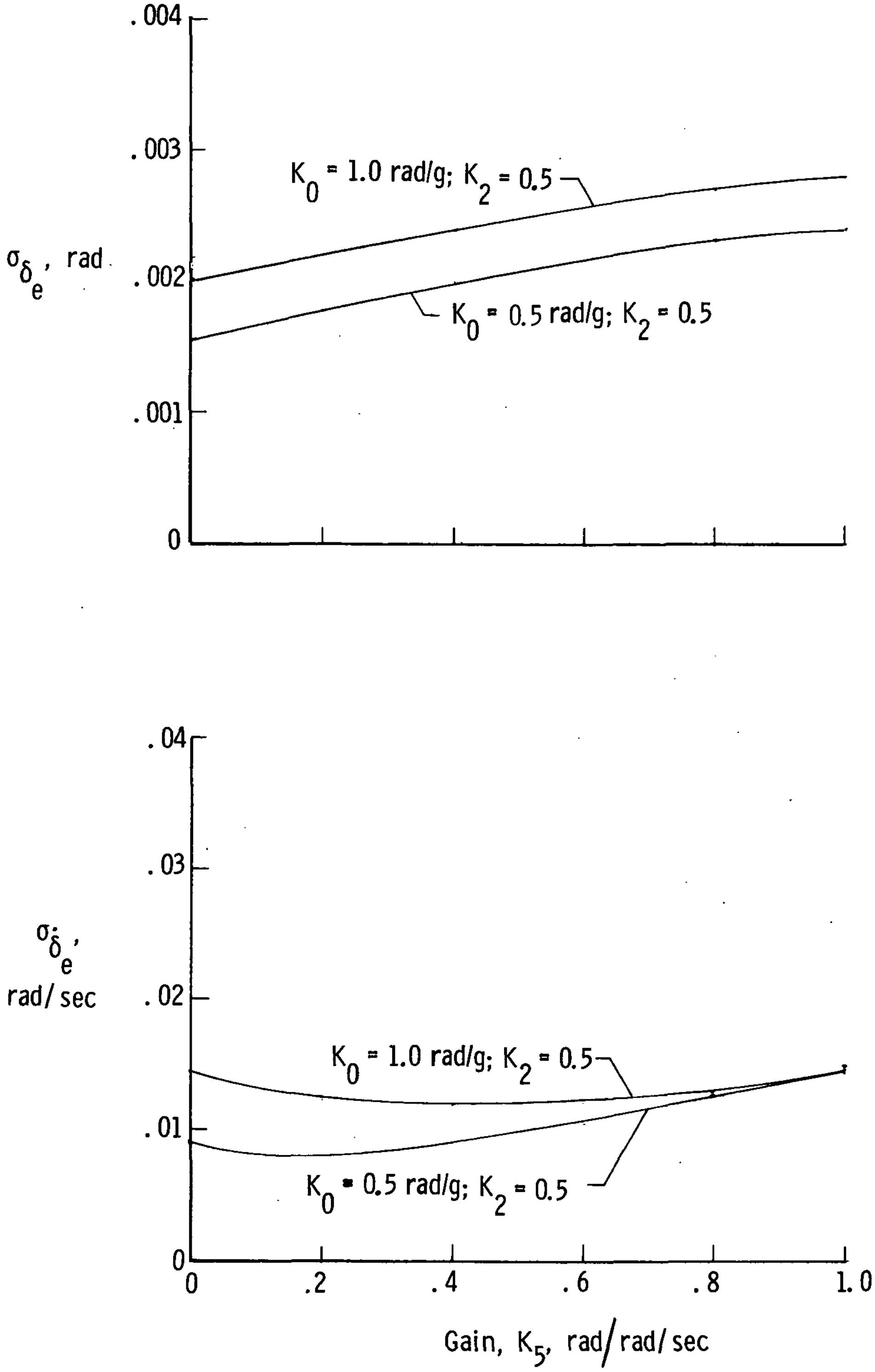
(a) Percent alleviation in normal acceleration.

Figure 7.- Complete gust alleviation system. $K_2 = 0.5$.



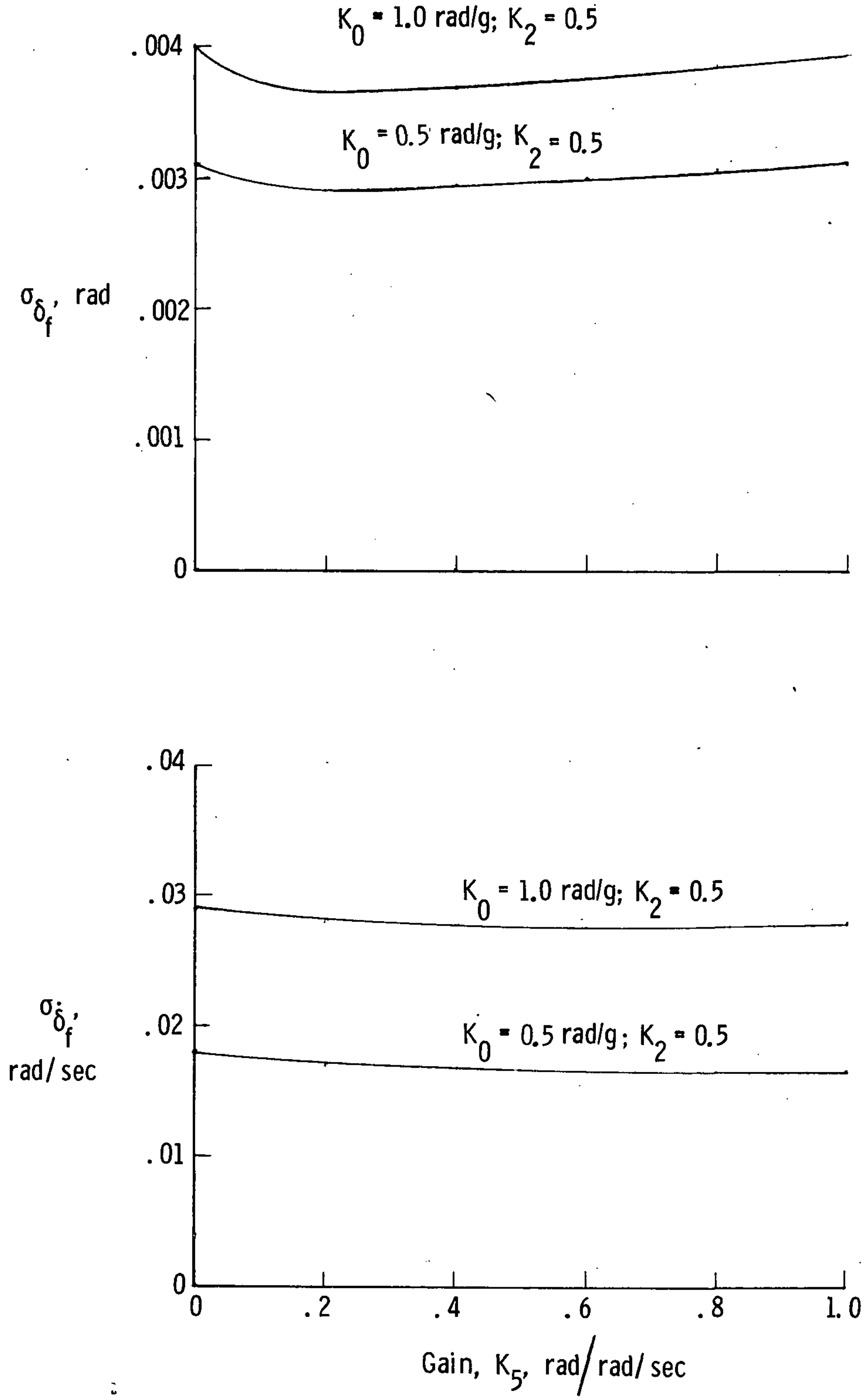
(b) rms pitch rate.

Figure 7.- Continued.



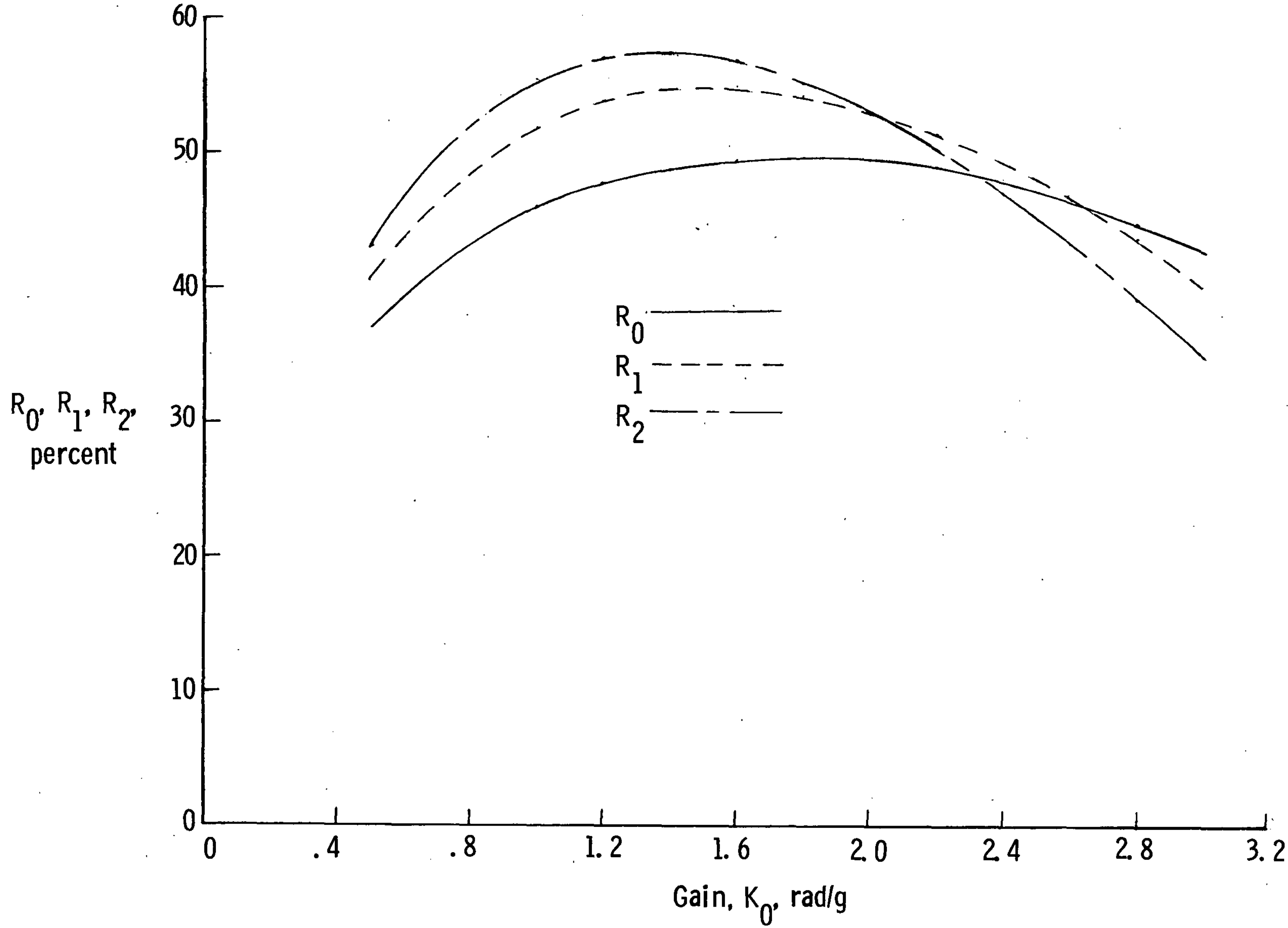
(c) rms elevator deflection angle and rms elevator deflection rate.

Figure 7.- Continued.



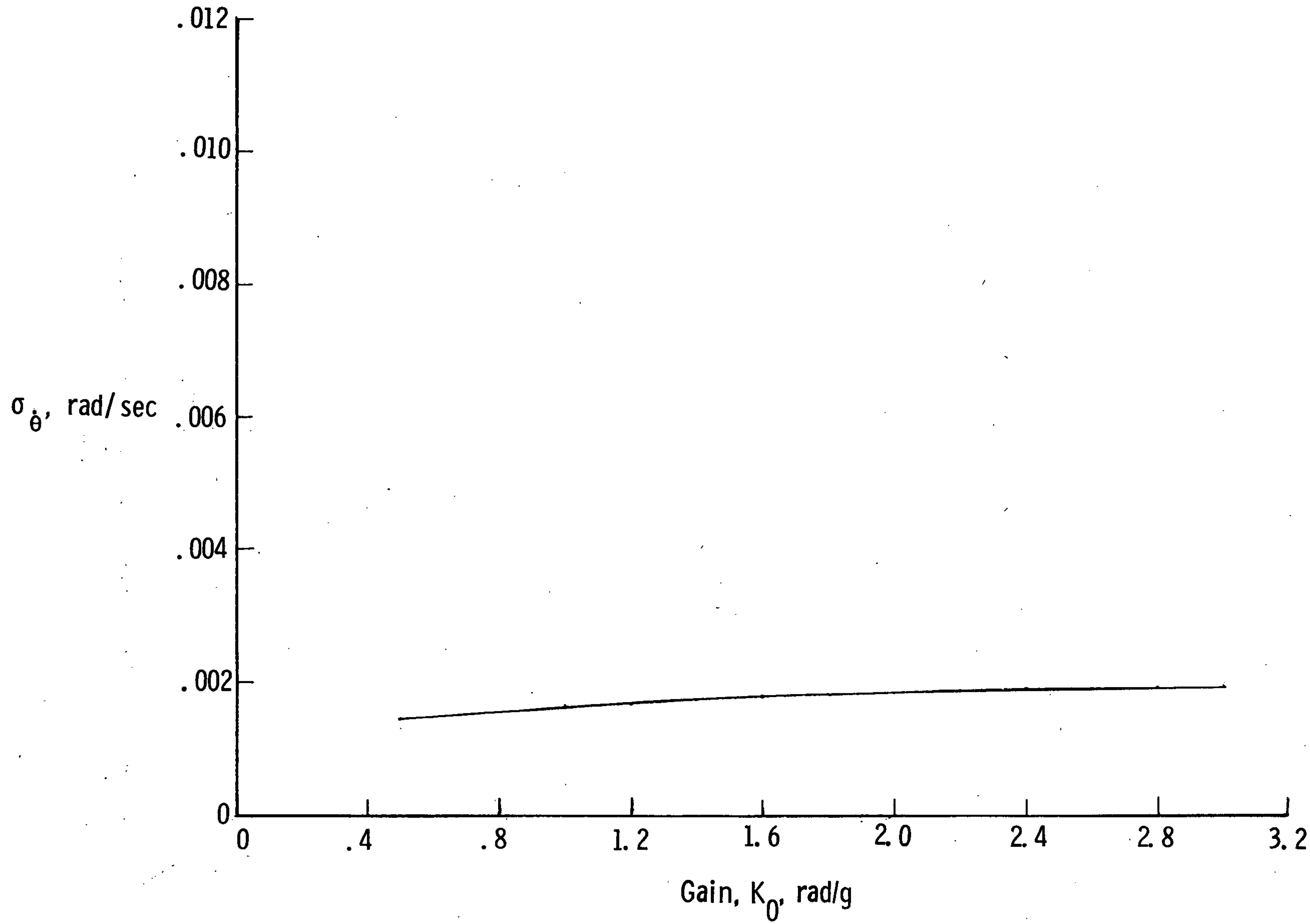
(d) rms flap deflection angle and rms flap deflection rate.

Figure 7.- Concluded.



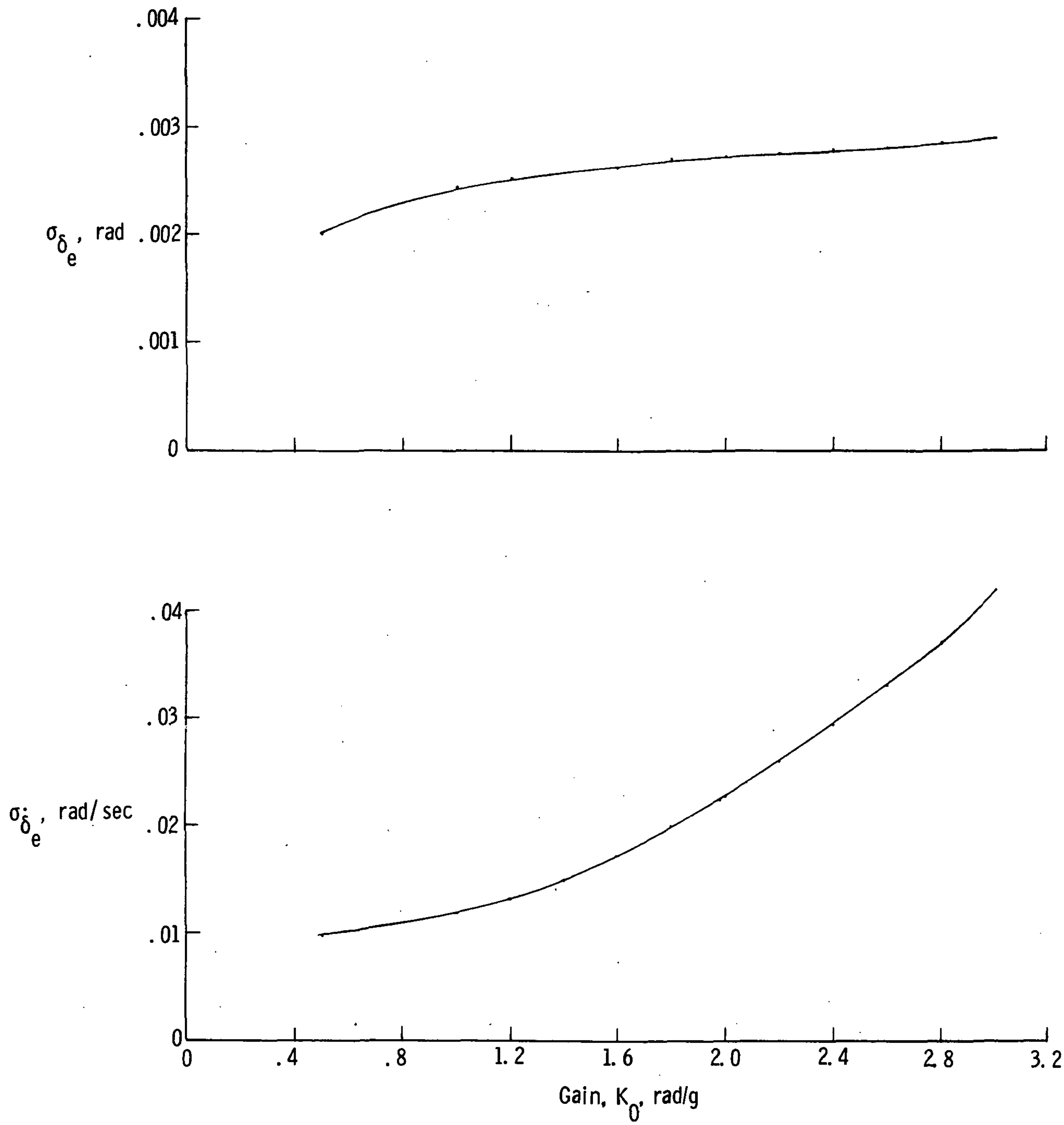
(a) Percent alleviation in normal acceleration.

Figure 8.- Complete alleviation system with selected feedback gains. $K_2 = 0.5$; $K_5 = 0.5$ rad/rad/sec.



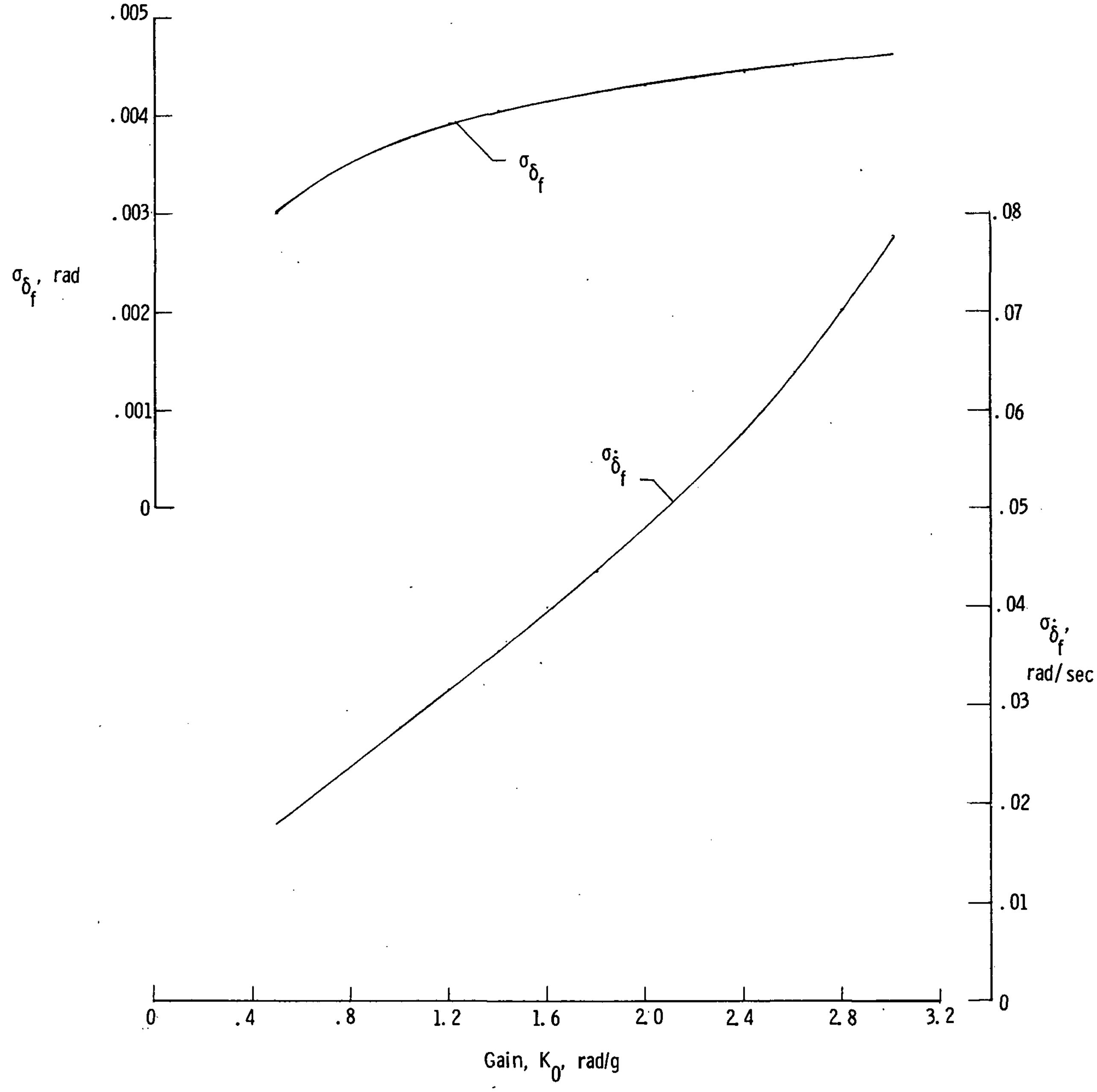
(b) rms pitch rate.

Figure 8.- Continued.



(c) rms elevator deflection angle and rms elevator deflection rate.

Figure 8.- Continued.



(d) rms flap deflection angle and rms flap deflection rate.

Figure 8.- Concluded.

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