EVALUATION OF AERODYNAMIC DERIVATIVES FROM A MAGNETIC BALANCE SYSTEM*

Boray S. Raghunath** and Hermon M. Parker†
University of Virginia, Charlottesville, Virginia

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**Received the Ph.D. in Aerospace Engineering from the University of Virginia in August 1971.

†Professor, Aerospace Engineering and Engineering Physics

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UNIVERSITY OF VIRGINIA WIND-TUNNEL COLD MAGNETIC

BALANCE SYSTEM

ABSTRACT

The dynamic testing of a model in the University of Virginia cold magnetic balance wind-tunnel facility is expected to consist of measurements of the balance forces and moments, and the observation of the essentially six degree of freedom motion of the model. The aerodynamic derivatives of the model are to be evaluated from these observations. This paper is concerned with demonstrating the basic feasibility of extracting aerodynamic information from the observation of a model which is executing transient, complex, multi-degree of freedom motion. It is considered significant that, though the problem treated here involves only linear aerodynamics, the methods used are capable of handling a very large class of aerodynamic non-linearities. The basic considerations include the effect of noise in the data on the accuracy of the extracted information. For this purpose the motion of the projected first model, a cone, is calculated with "known" aerodynamic derivatives. This data is corrupted by various amounts of noise. The aerodynamic derivatives are then evaluated from the noisy motion by two

methods: 1) the "Brute-force" method, and 2) the method of parametric differentiation. The "Brute-force" method treats the differential equations of motion as algebraic equations in the unknown aerodynamic derivatives and uses the method of least squares to average a large number of data points. In the method of parametric differentiation the equations of motion are considered in "aerodynamic derivative" space and the method of least squares is applied. Both methods extract the significant groupings of aerodynamic derivatives rather well and with only minor differences in accuracy. The relationships between noise level and the accuracy of the evaluated aerodynamic derivatives are presented.

LIST OF SYMBOLS

A	matrix of order n x n
a_{ij}	elements of matrix A
B	matrix of order 1 x M
b_j	elements of matrix B
$\underline{C}$	vector whose elements are the parameters
C_{K_w}	$\partial C_K/\partial(W/V_T)$ K = z, m
C_{K_q}	$\partial C_K/\partial(Qd/V_T)$ "
$C_{K_{Dw}}$	$\partial C_K/\partial(\dot{W}d/V^2{}_T)$ "
$C_{K_{pv}}$	$\partial C_K/\partial(PdV/V^2{}_T)$ "
$C_{K_{pDv}}$	$\partial C_K/\partial(P\dot{V}d^2/V^3{}_T)$ "
$C_{K_{pr}}$	$\partial D_K/\partial(PRd^2/V^2{}_T)$ "
D	$\frac{d}{V_T}\ (\frac{\partial}{\partial t})$
d	Base diameter
$i_A, i_B, i_C, i_D, i_E, i_F$	Nondimensionalized moments of inertia, e.g. $(A/(\frac{1}{2}\rho Sd^3))$
n	Number of parameters
N	Number of data points

LIST OF SYMBOLS (cont.)

P, Q, R	Angular velocities about body-fixed axes
p, q, r	$Pd/V_T, Qd/V_T, Rd/V_T$
S	Characteristic area
$S_j^k(\tau, \underline{C}^o)$	Parametric Coefficient
SSR	Sum of square of error
t	Time
U, V, W	Translational velocities in body-fixed axes system
u, v, w	Nondimensionalized velocities U/V_T, V/V_T, W/V_T
V_T	Tunnel speed
X, Y, Z	Tunnel fixed orthogonal system of axes X positive downward
x^k	k th element of $\underline{X}$
x, y, z	Nondimensional distance along (X,Y,Z) X/d, Y/d, Z/d
μ	Mass of model$/(\frac{1}{2}\rho S d)$
ρ	Free stream air density
σ	Standard deviation
ψ, θ, ϕ	Euler angles defining the orientation of body-fixed axes Cx, Cy, Cz (Cx being the axis of symmetry) in relation to tunnel-fixed axes

LIST OF SYMBOLS (cont.)

τ	(tV_T/d)

Subscripts and Superscripts

i,j,k,l	Integers
o	Initial conditions or values
$(\dot{\ })$	$\frac{\partial}{\partial t}$

Introduction

For several years the development of magnetic balances to support models in wind tunnels has been in progress. The ability to support a model with no physical connections to it and the ability to arrange no model motion even at a lifting incidence, allow nearly free motion, or to influence and adjust the model motion to a significant extent is considered an important advance in wind tunnel techniques. Two different but complementary systems have been or are being developed.

One of these, the MIT/French system, basically supports a cylindrical magnetic core and is effectively a five component balance. Roll control has been added but is essentially an independent subsystem. This system easily makes static tests of models in which the model is held at a fixed orientation and the forces and moments are measured. The system also can drive the model in simple oscillatory modes of motion to measure, for example, dynamic stability derivatives. Dynamic testing with the MIT/French system suffers from (1) the small magnitudes of the aerodynamic forces and moments caused by the motion in comparison with the forces and moments required to force the motion and (2) the complexities involved in dynamic calibrations of the magnetic balance system.

In the other magnetic balance, the University of Virginia system, a magnetic sphere is supported and the system is basically a three component balance. By responding to translation at frequencies below a somewhat adjustable "balance frequency", it holds the model in the test section. A model built around the magnetic sphere is nearly rotationally free at all frequencies and is essentially free in translation above the "balance frequency" (of the order of 10-15 hertz in the prototype). This sort of operation is called Quasi-6 Degree of Freedom operation and is considered to be the nearest thing to free-flight obtainable in a wind tunnel. The study of the dynamic stability of a model (a prime application of the U. Va. system) corresponds to measuring the balance forces and moments, observing the model motion, and inverting the equations of motion to obtain the aerodynamic forces and moments, static and dynamic. An obvious inconvenience with this system is that the motion of the model cannot be restricted to simple one or two degree of freedom oscillations. Thus the problem of extracting the aerodynamic information from the experimental data involves the use of 6 degree of freedom motion data. The U. Va. prototype is designed specifically for the case of spinning axisymmetrical bodies (missile types) in supersonic flow. The extension to airplane types and subsonic flow is expected to be made in the not-too-distant future.

This paper describes and summarizes the results of a first attempt to demonstrate the feasibility of extracting aerodynamic information, specifically values of aerodynamic derivatives, from six degree of freedom motion data for a reasonably realistic approximation of a model in the prototype U. Va. system. Also of considerable interest is the relationship between noise in the data and accuracy of the extracted derivatives. The two methods of solution of the inverse problem ("Brute Force" {BF} and "Parametric Differentiation" {Par Diff.}) which were used are sufficiently general that the inertial nonlinearities are retained and a quite large class of aerodynamic nonlinearities may be handled in principle as easily as linear aerodynamics. Only a transient motion case was considered.

The general procedure was as follows. The projected first model for the U. Va. prototype system, a 15° cone, was chosen as the body and a set of "best" aerodynamic derivatives was established. With suitable approximations the full six degree of freedom equations of motion were numerically integrated to produce perfect motion data, which could be corrupted with various amounts of noise. The inertia parameters, balance forces and moments (if any) and the (noisy) data were used to invert the equations of motion to recover a set of aerodynamic derivative. A comparison of the extracted derivatives with those used to calculate the perfect data and the knowledge of the amount of noise added enables one to study the relation between noise level and accuracy of the extracted derivatives.

Generation of Perfect Data

For this study the base motion case is taken to be the projected first model, a 15° cone, in the U. Va. prototype cold magnetic balance wind tunnel system. The following assumptions and approximations are made: (1) Zero-gravity; (2) Zero-drag; (3) Zero rolling moments on the model; (4) Zero balance forces and moments on the model; (5) The mass distribution of the model is nearly axially symmetrical about the cone axis, but unbalanced so that $i_D = i_E = i_F \simeq 10^{-2} i_B$, $\{i_B = i_C$; $i_A \simeq i_B/6\}$; (6) The model is assumed to have a non-zero C_{z_w}, C_{z_q}, $C_{z_{Dw}}$, $C_{z_{pv}}$, C_{m_w}, C_{m_q}, $C_{m_{Dw}}$, and $C_{m_{pv}}$ (and, of course, the other corresponding lateral derivatives which follow from axial symmetry) [6, 7]; (7) The flight is at Mach 3; (8) The center of mass is on the symmetry axis at 0.6 of the length of the model from the nose.

Subject to these assumptions, the full six-degree of freedom equations of motion are integrated by the Runge-Kutta 4 step method. The translational equations are written in the tunnel fixed frame of reference, and the rotational equations are written in the body fixed frame. Thus the perfect data consists of sets of values of x, y, z, position of the center of mass in the tunnel fixed frame and ψ, θ, ϕ the conventional orientation angles. The initial conditions are all zero except $\psi(0) = 0.1$, $\theta(0) = 0.15$, and $\dot{\phi}(0) = 0.0165$ a roll rate close to roll pitch resonance for this model. The motion of a model in the U. Va. facility is expected to be observed as a continuous record in time, which may then be reduced to discrete data points as dense as desired. About 4 cycles in θ and ψ, 100 points, are used for the positional (x,y,z,ψ,θ,ϕ) perfect data.

Inverse Problem

The inverse problem is solved by two methods: The "Brute-force" method and the method of parametric differentiation. The aerodynamic axial force and rolling moment are assumed to be zero in the generation of the perfect data and are not considered in the inverse problem. The aerodynamic derivatives are evaluated in the body-fixed coordinate system.

(1) The "Brute-force" (BF) Method

The BF method of solution uses the discrete positional data, numerically calculates velocities and accelerations and inserts them, along with other known quantities, into the equations of motion. The equations of motion are thus treated as linear algebraic equations in the "unknown" aerodynamic derivatives.The full nonlinear character of the inertial terms in the equations of motion is retained in the solution.

The model being axisymmetric, only the z force (in body frame) and pitching moment equations of motion are considered in the inverse problem. Furthermore, the force and moment equations are independent of one another.

The pitching moment equation, for example, is written as an algebraic equation in the unknown aerodynamic derivatives as follows:

$$C_{m_w} w_i + C_{m_q} q_i + C_{m_{Dw}} Dw_i + C_{m_{pv}} (pv)_i = i_B Dq_i$$
$$- (i_C - i_A) (rp)_i + i_E(p^2 - r^2)_i - i_F (qr + Dp)_i + i_D (qp - Dr)_i \qquad (1)$$
$$i = 1, 2, ----, N,$$

where N is the number of data points. This can be written as follows:

$$\sum_{j=1}^{n} a_{ij} c_j = b_i \qquad i = 1, ----, N \qquad (2)$$

where c_j's are the moment derivatives, a_{ij}'s are the kinematics of motion, and b_i's the inertial terms. Rewriting Eq. (2) as an error equation

$$SSR = \sum_{i=1}^{N} (\sum_{j=1}^{n} a_{ij} c_j - b_i)^2 \qquad (3)$$

and minimizing SSR with respect to c_j, the aerodynamic derivatives are obtained. The z force equation is treated in the same fashion.

The kinematics of motion, u, v, w, p, q, r, Du, Dv, Dw, Dp, Dq, Dr are obtained from the positional data in the following manner: In case of noisy data, the positional data at every instant are smoothed by using a quintic power series and 9-equidistant points. The velocities and accelerations in the tunnel fixed reference frame are obtained by numerically differentiating 5-equidistant points of the smoothed translational positional data. The Euler angular velocities and accelerations are obtained in a similar fashion. These are transformed to the body axis reference frame to obtain the necessary kinematics of motion.

(2) The Method of Parametric Differentiation

The motion of the model is considered as being in a parametric space of aerodynamic derivatives and the initial conditions of motion. The six equations of motion are used and are written as a set of first order nonlinear differential equations of the form

$$D\underline{X} = \underline{F}\ (\underline{X},\ \underline{C},\ \tau) \qquad (4)$$

where $\underline{X}$ is the set of twelve (12) positions and velocities and $\underline{C}$ is the twelve initial conditions, $\underline{X}(o)$, plus the aerodynamic derivatives to be evaluated. For a given set of $\underline{C}^o$ the solution is of the form

$$\underline{X} = \underline{X} \ (\tau, \ C^o) \qquad (5)$$

The change in $\underline{X}$ due to small changes in the parameters $\underline{C}$ is given by the sensitivity coefficients, defined thus:

$$S_j^k \ (\tau, \ \underline{C}^o) = \lim_{\Delta c_j \to 0} \frac{x^k \ (\tau, \ c_1^o, \ c_2^o, \ ---, \ c_j^o + \Delta c_j, \ ---, \ c_n^o) - x^k (\tau, \ \underline{C}^o)}{\Delta c_j}$$

$$= \frac{\partial x^k \ (\tau, \ \underline{C}^o)}{\partial c_j} \Big| \ c_j^o \qquad (6)$$

Differentiating Eq. (4) with respect to c_j it follows that

$$DS_j^k \ (\tau, \ \underline{C}^o) = \frac{\partial f^k}{\partial x^\ell} \ S_j^\ell \ (\tau, \ \underline{C}^o) + \frac{\partial f^k}{\partial c_j} \qquad (7)$$

These are the parametric equations, which are a set of linear differential equations with variable coefficients.

The solution of Eq. (4) is expressed in terms of Eq. (5) and Eq. (6) in the form of a truncated Taylor series:

$$x^k \ (\tau, \ \underline{C}) = x^k \ (\tau, \ \underline{C}^o) + \sum_{i=1}^{n} \ S_i^k \ (\ , \ \underline{C}^o) \ \Delta c_i . \qquad (8)$$

In the present case the parameters $\underline{C}$ are ordered sequentially in the following way:

$$x_o, \ y_o, \ z_o, \ \psi_o, \ \theta_o, \ \phi_o, \ Dx_o, \ Dy_o, \ Dz_o, \ D\psi_o, \ D\theta_o, \ D\phi_o,$$

$$C_{m_w}, \ C_{m_q}, \ C_{m_{pv}}, \ C_{m_{Dw}}, \ C_{z_w}, \ C_{z_q}, \ C_{z_{pw}}, \ C_{z_{Dw}} .$$

With this sequence it follows from consideration of Eq. (8) that

$$S_j^k (0, \underline{C}^o) = 1 \text{ if } j = k \qquad (9)$$
$$= 0 \text{ if } j \neq k$$

Equations (7) and (9) constitute the basis for the parametric study of the motion governed by Eq. (4). The observed, positional data, in tabulated form, can be denoted by

$${}^e x_i^k, \; k = 1, \text{--}6 \text{ and } i = 1, \text{---}N$$

Let $x_i^k (\tau, \underline{C}^o)$ be the values of the calculated motion obtained by solving Eq. (4) with estimated values of $\underline{C}^o$. The sum of squares of the error between the observed and calculated positions is given by the following:

$$SSR = \sum_{i=1}^{N} \sum_{k=1}^{6} [{}^e x_i^k - (x_i^k (\tau, C^o) + \sum_{j=1}^{n} S_{ji}^k \, \Delta c_j)]^2$$

Minimizing SSR with respect to Δc_j one gets the following:

$$A \quad \Delta C = B$$

where

$$a_{j\ell} = \sum_{i=1}^{N} \sum_{k=1}^{6} S_{ji}^k \quad S_{\ell i}^k$$

$$b_j = \sum_{i=1}^{N} \sum_{k=1}^{6} (x_i^k - {}^e x_i^k) \quad S_{jk}^k$$

This yields the correction to the assumed $\underline{C}^{o}$ as

$$\Delta C = A^{-1} B$$

The new set of $\underline{C}^{o}$ is used in Eq. (4) and the procedure continued until convergence in the SSR is obtained. The final set $\underline{C}$ is defined to be the "true" $\underline{C}$.

The calculated motion depends on the initial estimation of the aerodynamic derivatives and the initial conditions of the motion. The latter are estimated by obtaining the smooth positional data and the velocities by numerical differentiation of the first few data points by the methods used in the "Brute-force" method.

Modelling Problem

The modelling problem is that of determining a "best" set of aerodynamic derivatives for a body from its observed motion. It is conceivable that an important derivative could be missed, if the aerodynamic characteristics of the body are sufficiently strange and unknown.

A very preliminary attack on the general modelling problem, as well as a basic verification of the two chosen inversion methods, is accomplished in the following way. One assumes the perfect data are real, and of course very accurate, motion data for a body, the aerodynamic derivatives of which are completely unknown. The problem then is to see if one can construct a logical argument to determine a "best" set of aerodynamic derivatives for that data subject to the ignorance assumption. (Of course, in the present case, one may also make comparisons and draw conclusions using the "known" derivatives for that data.)

The process of interpretation of the results is affected by the following considerations. The aerodynamic pitching moment for the chosen model is

$$C_m = C_{m_w} w + C_{m_q} q + C_{m_{Dw}} Dw + C_{m_{pv}} pv.$$

By adding and subtracting suitable terms the same pitching moment may be written in three alternate forms:

$$C_m = C_{m_w} w + (C_{m_q} + C_{m_{Dw}}) q + (C_{m_{pv}} - C_{m_{Dw}}) pv + C_{m_{Dw}} (Dw - q + pv).$$

$$= C_{m_w} w + (C_{m_q} + C_{m_{pv}}) q + (C_{m_{Dw}} - C_{m_{pv}}) Dw + C_{m_{pv}} (Dw - q + pv).$$

$$= C_{m_w} w + (C_{m_q} + C_{m_{pv}}) pv + (C_{m_{Dw}} + C_{m_q}) Dw - C_{m_q} (Dw - q + pv).$$

For a low lift configuration, such as a 15° cone, one expects the combination (Dw-q+pv) to be small. If Dw - q + pv is a one order smaller quantity than q, Dw, or pv, then one may expect only two damping constants to be well determined. The second and third terms on the right sides are three alternate forms of these two groupings. Thus one can expect that the values of these damping derivative groupings to be of more significance than the individual derivatives. Linear combinations of any pair will give any other pair, hence there is little significance in which pair of groupings one uses. Exactly similar considerations hold for the significant groupings of force damping derivatives.

(1) The "Brute-force" Method

The number of aerodynamic derivatives, n, chosen in Eq. (3) is arbitrary. This is true also for the corresponding force equation. Table 1 shows the result of choosing n = 2, 3, 4, 5 and extracting 2, 3, 4, 5 individual force and moment derivatives. For n = 6, adding $C_{z_{pr}}$ and $C_{m_{pr}}$ to the set of derivatives, the matrix A was singular and no solution was found. Table I also shows the important groupings which follow from the fact that the quantity Dw + pv - q is very small.

An inspection of Table I from the "modelling Problem" point of view (derivatives unknown) is quite informative. For these four sets of extracted derivatives, one observes that the values of C_{m_w} and C_{z_w} are nearly constant. C_{z_q}, C_{m_q}, $C_{z_{Dw}}$, $C_{m_{Dw}}$ seem to oscillate with increasing n. The two groupings of the damping derivatives remain nearly constant for $n \geq 3$. It is apparent that, when two or more damping derivatives are extracted, $(C_{m_q} + C_{m_{pv}})$ and $(C_{m_{Dw}} - C_{m_{pv}})$, as well as the corresponding force groupings are nearly constant.

It seems clear that this particular body has two important moment damping derivatives and two important force damping derivatives which show in the groupings regardless of how many derivatives are extracted. The fact that the force derivatives behave nearly as well as the moment derivatives is somewhat surprising.

An inspection of Table I including the actual values and the percent errors verifies the above "modelling problem" conclusions. Large errors in the individual derivatives can occur, e.g., n = 4, with excellent values for the two independent groupings. One can conclude that for this body in this motion state that when two or more force and moment damping derivatives are included in the set successfully extracted from accurate data, the groups $(C_{m_q} + C_{m_{pv}})$, $(C_{m_{Dw}} - C_{m_{pv}})$ and the corresponding groupings of force derivatives are given with good accuracy.

(2) The Method of Parametric Differentiation

The same perfect data are used for extraction of derivatives by the method of parametric differentiation. Increasing the number of parameters from 15 to 20 corresponds to taking C_{m_w}, C_{m_q}, $C_{m_{pv}}$ and in order $C_{m_{Dw}}$, C_{z_w}, C_{z_q}, $C_{z_{pv}}$, $C_{z_{Dw}}$ in the inverse problem. The ordering of the aerodynamic derivatives is arbitrary but in the work reported here that order was fixed. For all the cases in Table II, except that in the last column, the starting values were the actual values. For the last case

TABLE I

Inversion of the Perfect Data by the Brute Force (BF) Method

Actual values	Derivatives and groupings	n = 2 value	n = 2 % error	n = 3 value	n = 3 % error	n = 4 value	n = 4 % error	n = 5 value	n = 5 % error
- .468	C_{m_w}	- .47246	- .95	- .46798	+ .004	- .46826	- .056	.46797	+ .006
-2.94	C_{m_q}	-3.37404	-14.7	-2.9825	-1.45	-1.90471	+35.2	-2.84637	+3.18
-1.334	$C_{m_{Dw}}$	-	-	-1.28378	+3.76	-2.3685	-77.5	-1.42018	-6.46
- .04	$C_{m_{pv}}$	-	-	-	-	-1.08702	-2620	- .13613	-240
0	$C_{m_{pDv}}$	-	-	-	-	-	-	- .28168	0
-2.98	$C_{m_q}+C_{m_{pv}}$	-3.37404	-13.2	-2.9825	- .84	-2.99173	- .39	-2.98250	+ .084
-1.294	$C_{m_{Dw}}-C_{m_{pv}}$	-	-	-1.28378	+ .79	-1.28140	+ .97	-1.28405	+ .77
-1.85	C_{z_w}	-1.85773	- .42	-1.84974	+ .014	-1.84989	+ .006	-1.84962	+ .021
-2.81	C_{z_q}	-3.56176	-26.8	-2.86333	-1.89	-2.28320	+18.8	-3.17279	-12.9
-2.34	$C_{z_{Dw}}$	-	-	-2.29004	+2.14	-2.87384	-22.8	-1.97825	+15.4
-0.052	$C_{z_{pv}}$	-	-	-	-	- .58508	-1025	+ .31279	+700
0	$C_{z_{pDv}}$	-	-	-	-	-	-	- .26605	-
-2.862	$C_{z_q}+C_{z_{pv}}$	-3.56176	-24.4	-2.86333	- .047	-2.86828	- .22	-2.86000	+ .07
-2.288	$C_{z_{Dw}}-C_{z_{pv}}$	-	-	-2.29004	- .089	-2.28876	- .033	-2.29104	- .13

TABLE II

INVERSION OF THE PERFECT DATA BY THE METHOD

OF PARAMETRIC DIFFERENTIATION (Par Diff)

Actual value	Quantity	n = 15 one iteration value	% error	n = 15 four iterations value	% error	n = 17 one iteration value	% error	n = 18 one iteration value	% error	n = 19 one iteration value	% error	n = 19 five iterations value	% error
	SSR	7.61		4.6×10^{-5}		7.51×10^{-4}		1.13×10^{-4}		4.3×10^{-7}		-7×10^{-19}	
- .468	C_{m_w}	- .46802	- .604	- .46633	+ .35	- .46410	+ .83	- .46637	+ .035	- .46798	+ .004	- .46805	- .01
-2.94	C_{m_q}	-4.21074	-43.2	- 4.8193	-64	-18.3245	-523	-8.81153	-200	-2.98348	- 1.48	-2.74136	+6.76
- .04	$C_{m_{pv}}$	+ .76626	+2020	+ 1.37206	+3530	+15.5051	+38800	5.87128	+14800	+ .00399	+110	- .24048	-500
-1.334	$C_{m_{Dw}}$					+14.1870	+1165	4.58956	+444	-1.12897	+15.4	-1.53386	-14.98
-2.98	$C_{m_q}+C_{m_{pv}}$	-3.44448	-15.6	-3.44724	-15.7	- 2.8194	+ 5.4	-2.94025	+1.33	-2.97949	+ .017	-2.98184	- .062
-1.294	$C_{m_{Dw}}-C_{m_{pv}}$	- .76626	+40.7	-1.37206	- 6.0	-1.3181	-1.86	-1.28172	+ .95	-1.13296	+12.4	-1.29338	+ .048
-1.85	C_{z_w}	-		-		-1.83938	+ .57	-1.85141	-.076	-1.84938	+ .034	-1.84935	+ .035
-2.81	C_{z_q}	-		-		-		3.62412	+229	-5.13717	-83	-5.13900	-83
- .052	$C_{z_{pv}}$	-		-		-		-		2.29429	+4520	2.29052	+4500
-2.34	$C_{z_{Dw}}$	-		-		-		-		-		-	
-2.862	$C_{z_q}+C_{z_{pv}}$	-		-		-		+3.62412	+227	-2.84288	+ .67	-2.84848	+ .47
-2.288	$C_{z_{Dw}}-C_{z_{pv}}$	-		-		-		-		-2.29429	- .274	-2.29052	- .11

(n = 19, 5 iterations) the initial estimates were $C_{m_w} = 0.4$ and $C_{z_w} = -1.8$ and the rest were zero. For the missing n = 16 case (four moment derivatives and no force derivatives) the program gave numbers which were unrealistic (magnitudes too large by 10^3 or more).

For this method an obvious and natural criterion for the "best" set of aerodynamic derivatives is: Minimum SSR corresponds to the "best" set of derivatives. It is probably no real disadvantage that this criterion loses its significance when SSR becomes sufficiently small. The Modelling Problem point of view, i.e., disregarding actual values and percent errors, again may be adopted in viewing the results presented in Table II.

A first conclusion is that a few iterations are very helpful in reducing SSR as shown by the n = 15 and n = 19 cases. Presumably a few iterations for n = 17 and 18 would have reduced SSR significantly. Secondly, the moment damping groupings change much less as n is changed than the individual moment damping derivatives. A similar conclusion about the force damping derivatives cannot be made since the force derivatives occur in so few cases.

The examination of the entire sequence suggests: (1) the translational motion is important in this motion as evidenced by n = 16 not working and the general reduction in SSR as force derivatives are added, (2) there are two important moment damping constants corresponding to the groupings, and (3) the sharp reduction in SSR in going from n = 18 to n = 19 suggests that there are (perhaps only) two important force damping constants.

Again, adding the actual values of the derivatives and the percent errors gives quantitative evidence to the above conclusions. The accuracy of the two groupings of force damping derivatives in case n = 19 is confirmed. The inclusion of translational motion in the inversion problem does increase the accuracy with which the moment damping groupings are extracted.

There is one interesting item in the comparison of the two methods. There is only one common case tried by each. Brute Force, n = 4 (Table I), corresponds to extracting the same eight non-zero derivatives used to generate the perfect data. It was successful in that values of the important force and moment groupings came out with fair accuracy, though all the percent errors were considerably worse than for the adjacent cases (n = 3,5). The Parametric Differentiation case n = 20 corresponds to extracting the same set of derivatives, and it did not work. It is tempting to conclude that Brute Force is a somewhat more stable numerical process than parametric differentiation.

Influence of Noise

In order to investigate the influence of noise in the data on the accuracy with which aerodynamic derivatives may be extracted, one case of each of the two inversion methods was chosen. These were Brute Force n = 3 and Parametric Differentiation n = 19, which seemed to give best results with the perfect data.

Noisy data were simulated by adding to the data a set of pseudo-random numbers with zero mean and a chosen standard deviation. The noise levels in the translational positions x,y, and z were chosen to be three times larger than those in the angular positions ψ and θ. This corresponds crudely to some notion of how accurately translational and rotational positions may be measured. Noise was not added to the roll angle ϕ*. At each of the noise levels five (5) experiments (independent sets of noise) were done. Different or independent sets of pseudo-random numbers with zero mean and the same standard deviation were generated. In the case of "no noise" the five experiments correspond to 5 sets of slightly altered initial conditions used to calculate the perfect data. The results are shown in Tables III and IV. ParDiff was done with 3 iterations.

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*Originally, the noise added to the perfect data was of the form

$$X_{noisy} = X_{perfect} (1+Y_{noise})$$

i.e., relative noise. For a reason which was not clear at the time, the error in the extracted aerodynamic derivatives was especially sensitive to (relative) noise in ϕ. The change from relative to absolute noise

$$X_{noisy} = X_{perfect} + Y_{noise}$$

was made because the latter seems to correspond better to reality. After approximately one half the calculations were done a case with absolute noise added to ϕ showed that the error was quite insensitive to (absolute) noise in ϕ. For consistency (and the lack of computing funds) the calculations were completed with no noise in ϕ. It is now clear that the secular character of ϕ, $\dot{\phi}$ is nearly constant, causes a small relative noise to correspond to a quite large effective absolute noise.

TABLE III

BRUTE FORCE (BF) INVERSION OF NOISY DATA

RESULTS OF 5 EXPERIMENTS, $\sigma_\phi=0$, n=3

FIRST LINE: MEAN; SECOND LINE: STD DEVIATION

$\sigma_x=\sigma_y=\sigma_z=$ $\sigma_\psi=\sigma_\theta=$		No Noise	.00075 .00025	.0015 .0005	.003 .001	.009 .003
- .468	C_{m_w}	- .46798 0	- .46859 .00078	- .46838 .00262	- .46946 .00523	- .4800 .0108
-2.98	C_{m_q} $(+C_{m_{pv}})$	-2.9825 0	-3.00017 .01400	-2.98393 .07546	-3.02285 .10107	-3.30890 .20047
-1.294	$C_{m_{Dw}}$ $(-C_{m_{pv}})$	-1.28378 .00002	-1.24490 .03350	-1.18540 .09612	-1.04045 .50244	- .93181 .90874
-4.274	$C_{m_q}+C_{m_{Dw}}$	-4.26628 0.00005	-4.24500 0.03890	-4.17370 0.11264	-4.06328 0.42568	-4.24027 0.79617
-1.85	C_{z_w}	-1.84974 0	-1.85186 0.00470	-1.85642 0.01917	-1.84748 0.02504	-1.92002 0.16211
-2.862	C_{z_q} $(+C_{z_{pv}})$	-2.86332 .00002	-2.95489 0.04436	-3.00460 0.58428	-2.76537 .71616	-6.76169 2.74812
-2.288	$C_{z_{Dw}}$ $(-C_{z_{pv}})$	-2.29004 .00008	-2.10492 0.56483	- .89314 1.85325	-4.39072 2.32419	6.08744 5.29668
-5.15	$C_{z_q}+C_{z_{Dw}}$	-5.15347 0.00027	-5.05880 0.55119	-3.89602 1.40782	-7.15709 2.59308	-0.67426 6.32919

TABLE IV

PARAMETRIC DIFFERENTIATION, n = 19 (PAR DIFF) INVERSION OF NOISY DATA

Results of 5 experiments, $\sigma_\phi = 0$, 3 iterations

First line: mean, Second line: STD Deviation

$\sigma_x = _y = _z =$ $\sigma_\psi = \sigma_\theta =$	No Noise	.00075 .00025	.0015 .0005	.003 .001	.009 .003
SSR	3.8×10^{-10}				
C_{m_w}	- .46798 .00001	- .46726 .00164	- .46808 .00347	- .46994 .00531	- .46796 .01036
C_{m_q}	-3.00352 .00101	-5.68294 6.12746	- 2.71352 12.8160	4.52998 19.789	- 2.22646 40.808
$C_{m_{pv}}$	.02414 .00098	2.72776 6.17675	- .25938 12.924	- 7.56474 19.9548	- 0.75298 41.136
$C_{m_{Dw}}$	-1.27095 .00269	1.42266 6.16222	- 1.56308 12.8950	- 8.85614 19.918	- 1.06818 40.494
$C_{m_q} + C_{m_{Dw}}$	-4.27448 .00180	-4.26068 .03488	- 4.27660 .07975	- 4.32466 .12554	- 4.32378 .26271
$C_{m_{Dw}} - C_{m_{pv}}$	-1.29500 .00180	-1.30550 .01495	- 1.30369 .02988	- 1.29140 .04034	- 1.35424 0.0720
C_{z_w}	-1.84938 .00004	-1.84862 .00178	- 1.84789 .00178	- 1.84772 .00422	- 1.84022 .01956
$C_{z_q} + C_{z_{Dw}}$	-5.13582 .00014	-5.12265 .14237	- 5.10201 0.18110	- 4.95957 .49800	- 5.71986 1.88792
$C_{z_{Dw}} - C_{z_{pv}}$	-2.29354 .00003	-2.32625 .16560	-2.44763 .20901	-2.22596 .35735	- 2.92086 1.95104

Tables III and IV present, for each derivative and important derivative grouping at each noise level, the mean of the sample of five numerical experiments and the standard deviation for the sample. In almost all cases the standard deviation is considerably larger than the error in the mean. Further, the errors in the mean show no definite trend with noise level, while the standard deviation shows a definite increase as the noise level increases. Therefore, the standard deviation, rather than error in the mean, is related to probable error in the values of extracted derivatives. It is to be noted that the confidence level is low because the number of experiments in a sample is small. The largest noise level, $\sigma_x = \sigma_y = \sigma_z = .009$ correspond to σ being about 10% of the amplitude of a typical position variable.

It is not surprising to note, that the static derivatives are extracted with greater accuracy than are the dynamic derivatives. It may be surprising to note that Parametric Differentiation recovered C_{z_w} out of the noisy data with about twice the accuracy as it recovered C_{m_w}. Table V presents the standard deviation in percent of the static derivatives as a function of the noise level for the two methods of inversion.

Figure 1 shows how the standard deviation in percent of the force and moment damping derivative groupings varies with the noise level for both methods. An overall view of Figure 1

TABLE V

STANDARD DEVIATION IN % vs NOISE LEVEL

for STATIC DERIVATIVES

NOISE LEVEL	BRUTE FORCE		PARAMETRIC DIFFERENTIATION	
$\sigma_x = \sigma_y = \sigma_z$	C_{z_w}	C_{m_w}	C_{z_w}	C_{m_w}
.00075	.254	.167	.096	.350
.0015	1.036	.560	.096	.742
.003	1.354	1.118	.229	1.135
.009	8.763	2.308	1.057	2.214

shows that Par Diff gives better results. More specifically, Par Diff recovers the force groupings about four times better than BF. Par Diff extracts $(C_{m_q} + C_{m_{Dw}})$ and $(C_{m_{Dw}} - C_{m_{pv}})$ equally well, while BF extracts the former about three times better than the latter and the former not quite as well as does Par. Diff.

The computing requirements for the two methods are significantly different. Doing 3 iterations, Par Diff requires about 25 times as much computing time as BF.

Conclusions

(1) There is no particular difficulty in extracting aerodynamic information of reasonable accuracy from reasonably noisy data describing the complicated, Quasi-Six degree of freedom motion of a 15° cone in a Mach 3 tunnel. This result is demonstrated by a quite simple and straightforward inversion technique (BF) and by a quite sophisticated and complex inversion technique (Par Diff). The interesting implication is that this conclusion is not unique to this particular motion case.

(2) Both of the inversion methods used are inherently general in that they do not depend upon linearization. BF is a linear algebraic problem as long as the aerodynamic forces and moments may be written in terms of the derivative coefficients. Par Diff involves a numerical integration of the equations of motion; the inertial nonlinearities present no difficulties, but

it is conceivable that enough higher order terms in the expressions for the aerodynamic forces and moments could be important and could cause excessive integration difficulties. Certainly most important aerodynamic non-linearities could be handled by Par Diff. The interesting possibility is that these methods, and perhaps others, coupled with magnetic balance wind tunnel systems provide a practical way of experimentally studying nonlinear aerodynamics of suitable models.

(3) The work reported here is only a first step in the attempt to understand the aerodynamic information extraction problem for the complicated quasi-six degree of freedom case. The obvious extensions which ought to be done involve: various noise levels in individual observables and their influence in combination, different models and flight conditions, and active balance forces and moments.

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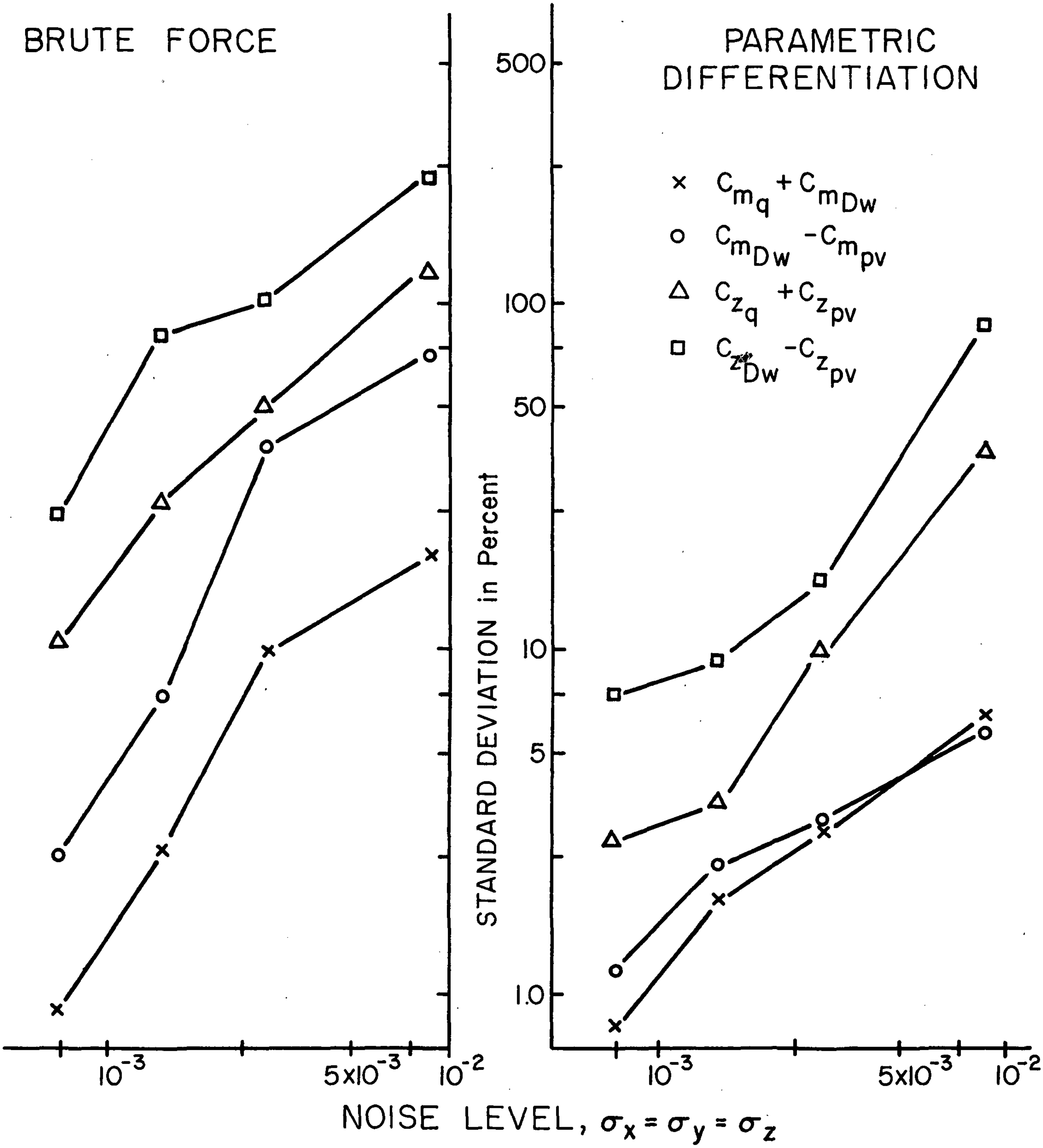

Figure 1. Standard deviation in % of the extracted derivative groupings as a function of noise level. The highest noise corresponds to σ being about 10% of the amplitude of the typical variable.