N 73 - 20312 NYU-AA-72-13

NASA CR-112304

COMPUTER PROGRAM FOR ASSESSING THE THEORETICAL PERFORMANCE OF A THREE-DIMENSIONAL INLET*

by

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ABSTRACT

A computer program for determining the theoretical performance of a three-dimensional inlet is presented. An analysis for determining the capture area, ram force, spillage force, and surface pressure force and their moment is presented along with the necessary computer program. A sample calculation is also included.

* The work reported herein was supported by the National Aeronautics and Space Administration, under Grant No. NGL-33-016-119

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LIST OF SYMBOLS

A	area normal to x-axis
A_o'	captured streamtube area at station "0"
A_o	captured streamtube area at free-stream conditions
A_{cowl}	cowl projected frontal area
A'_{cowl}	cowl projected area at station "0"
a_o	speed of sound at stagnation conditions
C_A	axial force coefficient $= F_A/q_\infty S_{ref}$
C_D	drag coefficient $= D/q_\infty S_{ref}$
C_F	force coefficient $= F/q_\infty S_{ref}$
C_L	lift coefficient $= L/q_\infty S_{ref}$
C_M	moment coefficient $= M/q_\infty S_{ref} l_{ref}$
C_N	normal force coefficient $= N/q_\infty S_{ref}$
C_p	pressure coefficient $= (p-p_\infty)/q_\infty$
D	drag force
$\vec{F}$	force
f	function u, v, w
g	gravitational constant
i,j,k	indeces
I,J,K	indeces
l	moment arm
l_{ref}	reference length
L	lift force
M	Mach number
$\mathfrak{M}$	moment $= \bar{F} \times \bar{l}$
$\dot{m}$	mass flow rate

N	normal force
p	pressure
q	dynamic pressure = $\frac{1}{2}\rho V^2$
r	radial coordinate
S	surface area
S_{ref}	reference surface area
T	temperature
u	axial component of velocity
v	radial component of velocity
w	outboard component of velocity
V	total velocity = $\sqrt{u^2+v^2+w^2}$
x	axial coordinate
y	spanwise coordinate
z	normal coordinate
α	angle of attack
β	spillage force angle
γ	ratio of specific heats
δ	flow direction
ε	ram force angle
ζ	body force angle
η	spillage force angle
θ	meridional coordinate
ξ	body force angle
ρ	density
ψ	ram force angle
ω	flow direction

Subscripts

c	cone surface
i,j,k	indeces of mesh points
o	initial or stagnation-conditions
s	streamline
w	wave
∞	free stream conditions
ref	reference conditions
p	rolling monent subscripts
q	pitching moment subscripts
r	yawing moment subscripts
ax	axial
N	normal
side	side

LIST OF FIGURES

FIGURE

INTRODUCTION

In the design of an integrated hypersonic vehicle, the various forces generated by the airbreathing propulsion system must be determined and located accurately with respect to the center of gravity to determine the interaction between the propulsive unit and the vehicle.

The supersonic combustion ramjet engine (scramjet) has been suggested as a possible airbreathing propulsion system on a hypersonic vehicle such as a launch vehicle.[1] The scramjet engine has three basic components: the inlet, combustion, and nozzle. For hypersonic flight these components tend to be of sizable dimensions with respect to the vehicle size. Therefore, the forces generated by the scramjet engine especially those of the inlet and nozzle are also large compared with other aerodynamic forces. Thus an accurate evaluation and location of these forces is essential to determine the trim requirements of the vehicle.

In this report, the pressure and momentum integrals necessary to establish the external forces generated by the inlet are evaluated in conjunction with a three-dimensional computer program[2] which determines the external flow field using the theory presented in [3]. Although the flow field analysis is inviscid in nature, the body geometry can be corrected by adding a suitable boundary layer displacement thickness to the actual body.

ANALYSIS

A typical scramjet engine installation is shown in Fig. 1. The propulsion system is surrounded by a suitable control volume to evaluate the integrals necessary in the determination of the propulsive forces. The control volume shown in Fig. 1 assumes the captured streamline in the vehicle flow field at station "0". That is, the streamline crosses the bow shock upstream of this station.This is typical of low hypersonic speeds. At hypersonic speeds, the captured streamline may have crossed the bow shock at a station downstream of station "0" as the bow shock is very shallow. The control volume changes not only with flight Mach number, but also with vehicle attitude (i.e. angle of attack, yaw, etc.). Also, here we assume the inlet is started and there is no flow separation present on the inlet ramp. The particular inlet performance evaluated here are the local capture area, ram force, spillage and body surface, forces and moments. The forces are evaluated in terms of components in the axial, normal, and side wise directions of a body fixed coordinated system.

a) Local Capture Area

To determine the inlet's capture area or mass flow, it is necessary to determine the trace at station "0" of the stream surface that impinges on the inlets cowl and sidewalls leading edges. Since the program marches forward in the axial direction, it is not known a priori through which initial point the captured streamline will pass. Therefore, a number of streamlines must be traced from the initial station. The streamlines intersecting the leading edges are then selected or interpolated at the end of the calculation. The program has been set up to trace streamlines passing through the even numbered index points in the radial direction at the initial plane (i.e. J = 2, 4,..., K=1, 2, 3,... See Fig. 2).

The trace of a generic streamline proceeds as follows. The equations of a streamline in cylindrical coordinates are

$$\frac{dx_s}{u} = \frac{dv_s}{v} = \frac{r_s d\theta_s}{w}$$

where u, v, w, are the axial, radial and outflow components of velocity in cylindrical coordinates, and the subscript s denotes streamline. To trace a streamline which at the initial station ($x = x_i$) passes through the grid point (r_o, θ_o) indexed by J,K proceed from the given initial data plane by estimating the position of the streamline at the next calculation step i.e.

$$x_{s_{i+1}} = x_i + \Delta x$$

$$r^{(1)}_{s_{i+1_{i,j,k}}} = r_{s_{i,j,k}} + \left[\frac{v}{u} (x_i, r_{s_{i,j,k}}, \theta_{s_{i,j,k}}) \right] \Delta x$$

$$\theta^{(1)}_{s_{i+1,j,k}} = \theta_{s_{i,j,k}} + \left[\frac{1}{r_{s_{i,j,k}}} \frac{w}{u} (x_i, r_{s_{i,j,k}}, \theta_{s_{i,j,k}}) \right] \Delta x$$

where the superscript (1) denotes first iteration and the subscripts i, j, k denote data at the present station x_i and r_{s_i} θ_{s_i}.

The three-dimensional program then proceeds to calculate new data at the next axial station $x_{i+1} = x_i + \Delta x$. Given this data and the first estimate of the streamline position at the new station the end slope is determined from interpolated data as explained in Appendix A. The average slope between station x_i and x_{i+1} is used to determine the location and slopes of the streamline at the new station. No additional iterations are taken as the calculated step size is sufficiently small that the flow properties do not change appreciably. The error has been found to be on the order of 10^{-4}.

The computer program subroutine outlining this procedure is presented in Appendix B. A vertical plane of symmetry is assumed in the program at the plane indexed by K = 1. In this plane the program calculates only the streamwise slope dr/dx and sets $\theta = 0$.

Additional streamlines are traced at a station downstream of the initial station to insure that the cowl leading edge is bracketed by the selected streamlines. The additional streamlines originate at odd numbered index points in the radial direction (i.e. J= 1,3,4,7,... for K=1,2,3,...). The station at which the additional streamlines are initiated is determined from the mean cowl lip radius and the conical shock angle by using an approximate formula for the streamlines through the conical flow. In the approximation the streamline is a hyperbola (Ref. 4): $r^2 = (\tan^2\theta_c)\, x^2 + C$ where C is a constant. Selecting the constant so that the streamline impinges on the cowl lip i.e. @ $x = x_c$ $r = R_c$ gives $r^2 = (R_c{}^2 - x_c{}^2 \tan^2\theta_c) + x^2\tan^2\theta_c$. The streamline crosses the conical shock $r = x \tan\theta_w$ at:

$$x = x_c \sqrt{\frac{\tan^2\theta_{cowl} - \tan^2\theta_c}{\tan^2\theta_w - \tan^2\theta_c}}$$

where θ_w is the conical shock angle

θ_c is the cone angle

and θ_{cowl} is the ray angle passing through the cowl lip = $\mathrm{Tan}^{-1}(R_c/x_c)$

The effect of spillage is to bend the streamlines further from the conical flow above. Therefore, this choice of initial station to trace the streamlines will insure that some of the selected streamlines will be spilled around the cowl. However, interpolation and/or extrapolation of these streamlines may still be necessary upstream of this station for odd-indexed captured streamlines that lie near the shock wave.

Given the above set of streamlines, the boundaries of the local captured area are determined by interpolating between the traced streamlines from the cowl leading edge to the initial station as described in Appendix A.

b) Captured Local Mass Flow Rate

Given the boundary of the local capture area at station "O", the local captured flow rate is determined by

$$\dot{m} = \iint_{A_o'} \rho u \, dA_o'$$

where A_o' is the local capture area at station "O" as shown in Fig. 1.

However, since the limits of integration are not known at the start of the program and to keep track of the accuracy of the streamlines tracing procedure, the incremental mass flow between mesh grids is calculated at every printout station, i.e.

$$\Delta m = \rho u \, \Delta A$$

or

$$\Delta\dot{m} = \left[\frac{(P/P_\infty M)}{\sqrt{(T/T_o)}} \; \frac{\gamma}{\sqrt{1+(\frac{u}{u})^2+(\frac{v}{u})^2}} \; \frac{\Delta A}{A_{ref}} \right] \left(\frac{g}{A_o}\right) P_\infty A_{ref}$$

where $\Delta A \cong r\,\Delta\theta\Delta r = \frac{1}{4}\,(r_{j,k} + r_{j+1,k} + r_{j+1,k+1} + r_{j,k+1})\ \frac{1}{2}[(r_{j+1,k} - r_{j,k}) +$

$(r_{j+1,\,k+1} - r_{j,k+1})]\ \Delta\theta$

The quantity in brackets is evaluated at the four corner points of each grid and the average ' is printed out at every print-out station. The total captured mass flow is the summation of all grids in the captured streamtube, i.e.

$$\dot{m} = \sum_{j,k}^{j_{capt},\,k_{capt}} \Delta\dot{m}$$

This quantity can be nondimensionalized with respect to the maximum local captured flow or can be converted into equivalent freestream tube capture area

$$A_o = \dot{m}/\rho_\infty V_\infty = \left\{\dot{m}\Bigg/\left[\frac{M_\infty \gamma}{\sqrt{(T/T_o)_\infty}}\right]\right\}\frac{g}{a_o}\quad P_\infty$$

and nondimensionalized with respect to the maximum freestream capture area or cowl frontal area, A_c.

c) Ram Force

The ram force in general has three components: axial, normal, and side. For zero yaw, the third component is zero due to symmetry. The individual components are calculated from the incremental ram force in the grid and then summed as in the mass flow calculation.

i) axial component

the incremental axial component of the ram force is given by

$$\Delta F_{ram_{axial}} = \Delta\dot{m}u + (p-p_\infty)\ \Delta A$$

$$\text{or} \quad \frac{\Delta F_{ram_{axial}}}{P_\infty} = \left[\left(1 + \frac{\gamma M^2}{[1+(v/u)^2 + (w/u)^2]}\right)\left(\frac{P}{P_\infty}\right) - 1\right] \Delta A$$

where ΔA is given above.

The computer program calculates this quantity at the four corners of the grid and prints out the average.

The total axial component of the ram force is the double sum of all grids in the captured streamtube. The axial component ram force coefficient is then given by

$$C_{F_{ram_{ax}}} = F_{ram_{axial}} \Big/ \tfrac{1}{2} \gamma P_\infty M_\infty^2 S_{ref}$$

ii) <u>normal-component</u>

The incremental component of the ram force normal to a plane perpendicular to the vehicle's plane of symmetry is given by

$$\Delta F_{ram_{normal}} = \Delta \dot{m} u \left(\frac{v}{u} \cos\theta - \frac{w}{u} \sin\theta\right)$$

$$\text{or} \quad \frac{\Delta F_{ram_{normal}}}{P_\infty} = \left[\frac{\gamma M^2}{[1+(v/u)^2+(w/u)^2]} \left(\frac{P}{P_\infty}\right)\left(\frac{v}{u}\cos\theta - \frac{w}{u}\sin\theta\right)\right] \Delta A$$

iii) <u>side force component</u>

The incremental side force component is given by

$$\Delta F_{ram_{side}} = \Delta \dot{m} u \left(\frac{v}{u} \sin\theta + \frac{w}{u} \cos\theta\right)$$

or $$\frac{\Delta F_{ram_{side}}}{P_\infty} = \left[\frac{\gamma M^2}{(1+(v/u)^2 + (w/u)^2)}\right] \left(\frac{P}{p_\infty}\right) \left(\frac{v}{u} \sin \theta + \frac{w}{u} \cos \theta\right) \Delta A$$

The calculation procedure to obtain the normal and side ram force components coefficient is identical to the above. The total ram force side component vanishes for zero yaw due to symmetry requirements.

iv) ram force direction

The angle which the ram force makes with the vehicle's axis is given by

$$\varepsilon = \mathrm{Tan}^{-1} \left(C_{F_{ram_{normal}}} \Big/ C_{F_{ram_{axial}}}\right)$$

and with the y-coordinate is given by

$$\psi = \mathrm{Tan}^{-1} \left(C_{F_{ram_{side}}} \Big/ C_{F_{ram_{axial}}}\right)$$

v) moment produced by the ram force

The incremental pitching moment produced by the ram force about any point such as the c.g. is given by

$$\left(\Delta C_{m_q}\right)_{F_{ram}} = \left(\frac{z-z_{c.g}}{l_{ref}}\right) \Delta C_{F_{ram_{axial}}} - \left(\frac{x-x_{c.g}}{l_{ref}}\right) \Delta C_{F_{ram_{normal}}}$$

where the subscript q denotes a pitching moment about the y-axis.

The incremental rolling moment is given by:

$$\left(\Delta C_{m_p}\right)_{F_{ram}} = \left(\frac{y-y_{c.g.}}{l_{ref}}\right) \Delta C_{F_{ram_{normal}}} - \left(\frac{z-z_{c.g.}}{l_{ref}}\right) \Delta C_{F_{ram_{side}}}$$

where x,y,z are the coordinates of the centroid of the incremental area.

The total pitching, rolling and yawing moments are calculated by summing over all grids inside the captured streamtube of the inlet, that is

$$C_{M_{F_{ram_{p,q,r}}}} = \sum_{j,k} \Delta C_{M_{F_{ram_{p,q,r}}}}$$

For a vehicle with a vertical plane of symmetry and at zero yaw only a pitching moment is produced. The computer program calculates only the pitching moment, about the origin (cone tip). The distance to the center of area is used in the calculation. The calculation is made at every print out station

vi) Point of Application

The point of application of the ram force is where the line of action of the ram force pierces the y-z plane at station "0". This point is found by locating the point where the ram force generates the same pitching, rolling and yawing moments, that is:

$$C_{M_{q_{F_{ram}}}} = \tilde{z}\, C_{F_{ram_{axial}}} - \tilde{x}\, C_{F_{ram_{normal}}}$$

$$C_{M_{p_{F_{ram}}}} = \tilde{y}\, C_{F_{ram_{normal}}} - \tilde{z}\, C_{F_{ram_{side}}}$$

$$C_{M_{r_{F_{ram}}}} = \tilde{x}\, C_{F_{ram_{side}}} - \tilde{y}\, C_{F_{ram_{axial}}}$$

Only two of the above equations are linearly independent. The third expresses the orthogonality condition between the moment vector and the force vector, i.e. $\bar{M} \cdot \bar{F} = 0$. Therefore only the line of action of the ram force can be found from the above equations, i.e.

$$\frac{\tilde{z}}{\tilde{x}} = \left(C_{M_{P_{F_{ram}}}} + \tilde{x}\; C_{F_{ram_N}}\right) \Big/ \left(C_{F_{ram_{axial}}}\; \tilde{x}\right)$$

and

$$\frac{\tilde{y}}{\tilde{x}} = \left(C_{M_{r_{ram}}} + \tilde{y}\; C_{F_{ram_{axial}}}\right) \Big/ \left(C_{F_{ram_{side}}}\; \tilde{x}\right)$$

for a vehicle with a vertical plane of symmetry and at zero yaw, the second equation is indeterminate. Further, selecting $\tilde{x} = x_o$ gives the piercing point $\tilde{z}$.

d) Surface Pressure Integral

The pressure integral on the inlet ramp has three components; axial, normal, and side components. Since the limits of integration are not known initially, i.e. the limiting streamline impinging on the sidewall leading edge intersection point with the surface (point "A" in Fig. 1), the incremental surface pressure integrals are calculated at each calculation step.

i) axial component

The incremental axial component of the inlet ramp is given by

$$\Delta C_{ax} = C_p\, \Delta S_{axial} / S_{ref} \qquad \Delta S_{ax} \approx \bar{r}\, \Delta\theta\, \Delta r$$

where C_p is the average pressure coefficient ($C_p = (P - P_\infty)/ M_\infty^2\, \frac{1}{2}\, \gamma\, P_\infty$) of the C_p's at the four corners of the grid, and ΔS_{axial} is the incremental body surface area in a typical grid (Fig. 2) ΔS. The total axial force is formed by the summation over all grids from the station "0" to the inlet's "mouth".

ii) normal component

The normal component is calculated from

$$\Delta C_N = C_p\, \Delta S_N / S_{ref} \qquad \Delta S_N \approx (\Delta x)\ (\Delta \bar{y})$$

where ΔS_N is the incremental body surface area in a typical grid projected onto a horizonaal plane normal to the plane of symmetry of the vehicle

iii) side component

The side component is calculated from

$$\Delta C_s = C_p \quad \Delta S_s / S_{ref} \qquad \Delta S_s \approx (\Delta x) \ (\Delta \bar{z})$$

where ΔS_s is the incremental body surface projected onto the vehicle's plane of plane of symmetry.

iv) direction of the body surface

The body surface force direction is given by

$$\xi = \mathrm{Tan}^{-1} \ (C_N / C_{ax})$$

and

$$\zeta = \mathrm{Tan}^{-1} \ (C_S / C_{ax})$$

v) moment produced by body surface force

The incremental pitching moment produced by the body surface force about any point is given by

$$\Delta C_{M_q} = \left(\frac{z - z_{c.g}}{l_{ref}}\right) \Delta C_{ax} - \left(\frac{x - x_{c.g.}}{l_{ref}}\right) . \quad \Delta C_N \qquad +$$

The incremental rolling moment

$$\Delta C_{M_p} = \left(\frac{x - x_{c.g.}}{l_{ref}}\right) \Delta C_s - \left(\frac{y - y_{c.g.}}{l_{ref}}\right) \Delta C_{ax}$$

The incremental yawing moment

$$\Delta C_{M_r} = \left(\frac{x - x_{c.g.}}{l_{ref}}\right) \Delta C_s - \left(\frac{y - y_{c.g.}}{l_{ref}}\right) \Delta C_{ax}$$

Total pitching, rolling, and yawing moments are obtained by summing over all the grids inside the welted surface from station "0" to the "mouth" of the inlet.

For a vehicle with a vertical plane of symmetry and at zero yaw, the rolling and yawing moments vanish. The computer program calculates only the pitching moment at every station where printout occurs.

vi) <u>point of application of body surface force</u>

The point of application of the body pressure force is where the line of action of the body surface force pierces a horizontal plane (z=0). This point is given by locating the point where the body force produces the same pitching, rolling and yawing moments, that is:

$$C_{M_q} = \bar{z}\ C_{AX} - \bar{x}\ C_N$$

$$C_{M_p} = \bar{y}\ C_N - \bar{z}\ C_S$$

$$C_{M_r} = \bar{x}\ C_S - \bar{y}\ C_{AX}$$

As before, only the line of action can be found

$$\bar{x} = (-C_{M_q} + \bar{z}\ C_{AX})/C_N$$

$$\bar{z} = (C_{M_p} + \bar{y}\ C_N)/C_S$$

$$\bar{y} = (C_{M_p} + \bar{z}\ C_S)/C_N$$

For a vehicle with a vertical plane of symmetry and at zero yaw the second equation is indeterminate. Further, selecting $\bar{z}$=0 gives the piercing point $\bar{x}$.

e) Spillage Force

The spillage force components can be calculated in analogous manner as the body surface force except that the spillage surface and the flow properties on it are used. However, this surface and its properties are not known a priori. Therefore it is more convenient to determine this force by imposing the equilibrium of a control volume consisting of the ram force at station "O", the body surface force and the ram force at the mouth of the inlet. The resultant of these is the spillage force which includes the side wall and cowl spillage. Equilibrium of moments yields the point of application and the moment of the spillage force. That is, the spillage force is given by a vectorial sum of the forces acting on the above volume:

$$-\bar{F}_{spill} = \bar{F}_{ram_{"O"}} + \bar{F}_{body} + \bar{F}_{ram_{cowl}}$$

from which the three scalar components can be obtained. For a vehicle at zero yaw and with a vertical plane of symmetry only the axial and normal components are nonvanishing and are equal to:

$$C_{F_{spill_{ax}}} = C_{F_{ram_{cowl_{axial}}}} + C_{F_{ram\ "O"_{axial}}} + C_{AX}$$

$$C_{F_{spill_{N}}} = C_{F_{ram_{cowl_{N}}}} + C_{F_{ram\ "O"_{N}}} + C_{N}$$

The pitching moments produced by the spillage force is given by

$$C_{M_{spill}} = C_{M_{q_{F_{ram_{"O"}}}}} - C_{M_{q_{body\ force}}} - C_{M_{q_{F_{ram_{cowl}}}}}$$

The line of action of the spillage force is given by

$$C_{M_{spill}} = \bar{z}\ C_{F_{spill_{ax}}} + \bar{x}\ C_{F_{spill_N}}$$

The angle which the spillage force makes with the vehicle's axis is given by

$$\beta = \mathrm{Tan}^{-1}\left(C_{F_{spill_N}} \big/ C_{F_{spill_{ax}}}\right)$$

and with the y-coordinate is given by

$$\eta = \mathrm{Tan}^{-1}\left(C_{F_{spill_s}} \big/ C_{F_{spill_{ax}}}\right)$$

SAMPLE CALCULATION

A sample calculation of the above forces was done for the inlet geometry described in Ref. 5. The free stream Mach number was equal to 6.0 and the vehicle was at zero angle of attack.

The streamlines in the vehicle's plane of symmetry and the projection of the streamlines onto a plane $\theta = 45°$ to the vehicle axis are shown in Figs. 3 and 4 respectively.

The captured streamtube at station "0" is shown in Fig. 5. The projections of the limiting streamlines are also drawn. The captured mass flow, ram force, body force, and spillage surface, directions and magnitude as well as moments are listed in Table 1 for this condition.

RESULTS

A computer program for evaluating the captured mass flow, ram, surface, and spillage forces as well as their moments and points of application for a three-dimensional inlet design have been presented along with a sample calculation.

The usefulness of this tool in determining the inlet performance and forces and moments produced by the inlet's ramp surface is appreciated in a trim analysis of a hypersonic integrated vehicle.

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APPENDIX A

1) Interpolation Scheme for Properties at a Point Inside a Grid. After locating a point inside a mesh whose corner points are indexed by (j,k), (j+1, k), (j, k+1), (j+1, k+1), (e' in Fig. 2c) the properties at the interior point are evaluated through a linear interpolation

$$f = \left[f_{j,k} + \left(\frac{f_{j+1,k} - f_{j,k}}{r_{j+1,k} - r_{j,k}}\right) (r-r_{j,k})\right] \left[1 - \left(\frac{\theta - \theta_k}{\theta_{k+1} - \theta_k}\right)\right] +$$

$$\left[f_{j,k+1} + \left(\frac{f_{j+1,\ k+1} - f_{j,\ k+1}}{r_{j+1,\ k+1} - r_{j,k+1}}\right) (r - r_{j,\ k+1})\right]\left(\frac{\theta - \theta_k}{\theta_k - \theta_k}\right)$$

Except at the plane of symmetry where the interpolation is carried out with respect to r only, i.e.

$$f = f_{j,k} + \left(\frac{f_{j+1,k} - f_{j,k}}{r_{j+1,k} - r_{j,k}}\right) (r - r_{j,k})$$

while for the surface streamlines (j = 1, k), the interpolation is carried out with respect to θ only, i.e.

$$f = f_{1,k} + \left(\frac{f_{1,k+1} - f_{1,k}}{\theta_{k+1} - \theta_k}\right) (\theta - \theta_k)$$

2) Interpolation Scheme for Captured Streamlines Given the location of the cowl leading edge piercing point (e' in Fig. 2c) with a plane x = constant, four adjacent streamlines (aa', bb', cc', dd') which enclose the point e' are used in the following interpolation scheme to determine the location e at the initial station of the streamline passing through e'.

For any general point, the location of the point e is given by

$$r_e = r_{ab} + K_\theta (r_{cd} - r_{ab})$$

$$\theta_e = \theta_{ab} + K_\theta (\theta_{cd} - \theta_{ab})$$

where

$$r_{ab} = r_a + [(r_{e'} - r_{a'})/(r_{b'} - r_{a'})] (r_b - r_a)$$

$$r_{cd} = r_c + [(r_{e'} - r_{c'})/(r_{d'} - r_{c'})] (r_d - r_c)$$

$$r_{cd} = r_c + [(r_{e'} - r_{c'})/(r_{d'} - r_{c'})] (r_d - r_c)$$

$$\theta_{ab} = \theta_a + [(r_{e'} - r_{a'})/(r_{b'} - r_{a'})] (\theta_b - \theta_a)$$

$$\theta_{cd} = \theta_c + [(r_{e'} - r_{c'})/(r_{b'} - r_{a'})] (\theta_d - \theta_c)$$

and $K_\theta = (\theta_{e'} - \theta_{a'b'})/(\theta_{c'd'} - \theta_{a'b'})$

where $\theta_{a'b'} = \theta_{a'} \div [(r_{e'} - r_{a'})/(r_{b'} - r_{a'})] (\theta_{b'} - \theta_{a'})$

$$\theta_{c'd'} = \theta_{c'} + [(r_{e'} - r_{c'})/(r_{d'} - r_{c'})] (\theta_{d'} - \theta_c)$$

in which

$r_{a'b'} = r_{c'd'} = r_{e'}$ was postulated.

In the present tracing scheme

$\theta_b = \theta_a$ and $\theta_c = \theta_d$ which implies $\theta_{ab} = \theta_a = \theta_b$

and $\theta_{cd} = \theta_c = \theta_d$. Therefore

$$\theta_e = \theta_a + K_\theta (\theta_c - \theta_a).$$

For streamlines in the plane of symmetry a simple interpolation is used, i.e.

$$r_e = r_a + \left(\frac{r_b - r_a}{r_{b'} - r_{a'}}\right) (r_{e'} - r_{a'})$$

$$\theta_e = 0.0$$

APPENDIX B - PROGRAM LISTING

```
      PROGRAM GRUM(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE7=PUNCH,TAPE8)
      DIMENSION U1(21,20),V1(21,20),W1(21,20),
     1R2(20),RX2(20),RT2(20),DR2(20),DRT2(20),RS2(20),RSX2(20),RST2(20),
     2F21(21,20,5),F22(21,20,5),F23(21,20,5),
     2U2(21,20),V2(21,20),W2(21,20),EL1B(21,20),
     3EL2B(21,20),UT2(21,20),VT2(21,20),WT2(21,20),EL2(21,20,2),
     4RSX(2),RA(2),FA(2,6),ELA(2,2),GA(2),GGA(2),DEPEP(2),CAPA(2),
     5GBAR(2),ELBAR(2),EM(21,20),CP(21,20)
*     DIMENSION XPROBE(7) ,PBAR(21,20)
      COMMON /FIK/XB(20),RB(20,20),RBP(20,20),RBPP(20,20),RBPPP(20,20),
     1UU(20),VU(20),WU(20),R1(20),RX1(20),RT1(20),DR1(20),DRT1(20),
     2RS1(20),RSX1(20),RST1(20),F11(21,20,6),F12(21,20,6),F13(21,20,6),
     2F14(21,20,6),EL1(21,20,2),T(20),P(21),
     3TOLEP,CVNST,CTST,CNX,CNNX,XEND,DT,DTD,X1,VINF2,G2,G3,G4,G5,CCP1,
     4CCP2,MI,MJ,MK,MAXM,MAXN,MODE,IPT,KPT,MIBM1,MJM1
      COMMON /F2/S(50)/F12/FPPP(50)/F13/DS/F123/F(50),FP(50),FPP(50)
     1/C1/G1,AO/MESH/NOPT,IPUNCH
      EQUIVALENCE(F11(1),U1(1)),(F11(421 ),V1(1)),(F11(841),W1(1))
      EQUIVALENCE(F21(1),U2(1)),(F22(1   ),UT2(1)),(F21(421 ),V2(1)),(
     1F22(421 ),VT2(1)),(F21(841),W2(1)),(F22(841),WT2(1)),(
     2F21(1261),EL1B(1)),(F21(1681),EL2B(1))
*     COMMON/STLIN/G,X2,RR,DRDX,DTDX,RPSAV,THSAV,  EM,CP,PBAR
*     DIMENSION DRDX(21,20), DTDX(21,20),RPSAV(21,20),THSAV(21,20),
*    1RR(21,20)
*     ICYCLE=0
*     MCYCLE=0
      MCOWW=0
C
* C   MCOWW=0 TRACE ONLY EVEN NUMBERED STREAMLINES
* C   MCOWW=1 TRACE EVERY STREAMLINE
*     READ(5,1) EMINF , (XPROBE(I),I=1,6) ,TCOWL
*   1 FORMAT( 8F10.5)
**    XC=11.
      XPRINT=XPROBE(1)
      IPROBE = 1
      ICO=0
      IJL=0
      IW=5
      CALL FIRST
*     XCOWW=XC*SQRT((TAN(TCOWL) **2-TAN(RBP(1,1))**2)/(TAN(RSX1(1))**2-
*    1  TAN(RBP(1,1))**2))
*     WRITE(6,2) XCOWW
*   2 FORMAT(///,5X,* ODD NUMBERED STREAMLINES WILL BE STARTED AT X= *,
*    1  E15.5/)
      G= G4/(G4-1.)
      G6 = 2./(G-1.)
      CONST = .5*G*EMINF**2.
      I=0
      GO TO 2201
C
C     SUBSEQUENT  INITIAL  DATA  SURFACE  (2100)
 2101 X1=X2
      DO 2104 K=1,MK
      R1(K)=R2(K)
      RX1(K)=RX2(K)
      RT1(K)=RT2(K)
      .
      .
```

```
          .
          .
          .
          .
          DO 6006 J=1,MJ
          RP=RP+DR2(K)
*         RR(J,K)=RP
          VP2=U2(J,K)**2+V2(J,K)**2+W2(J,K)**2
          A=A0-G1*VP2
          EM2=VP2/A
          EM(J,K)=SQRT(EM2)
          BETA=SQRT(EM2-1.)
          TANALF=ABS(U2(J,K))/SQRT(U2(J,K)**2-V2(J,K)**2)
          ELAMB=(BETA*TANALF+1.)/(BETA-TANALF)
          TEST=DX/DT-CTST*(RP*U2(J,K)-DX*V2(J,K))/(ELAMB*SQRT(U2(J,K)**2+
         1V2(J,K)**2))
          IF(TEST) 6005, 6005, 9002
     6005 CP(J,K)=(PIRAT*A**G4-CCP1)*CCP2
          Y = DR2(K)*FLOAT(J-1)
          EM12=EM(J,K)**2.
          PBAR(J,K) = 1. +CP(J,K)*CONST
          PT2BAR =PTRAT*(EM12/G2/(EM12+G6))**G4*(G2*(2.*G4*EM12-1.))**(1./
         1(1.-G))
          GO TO (6008,6006) NPRT
     6008 WRITE (IW,20021) J,RP,U2(J,K),V2(J,K),W2(J,K),EL2(J,K,1),
         1EL2(J,K,2),EM(J,K),CP(J,K),Y,PBAR(J,K),PT2BAR
    20021 FORMAT(I5,11F10.5)
     6006 CONTINUE
     6007 CONTINUE
*         ICYCLE=ICYCLE+1
*         IF(X2 .LT. XCOWW) GO TO 12
*         MCOWW=1
*         MCYCLE=MCYCLE+1
*      12 CONTINUE
*         GO TO (8,6)NPRT
*       8 WRITE(6,9) X2
*       9 FORMAT(///,5X,*STREAMLINE TRACE*,5X,*X2 =*,E12.5)
*       6 CONTINUE
*         CALL STREAM(NPRT,MJ,MK,ICYCLE,MCYCLE,MCOWW,DX,XCOWW,U2,V2,W2,T)
**        IF(X2.GE.9.0)NOPT=4
          .
          .
          .
          .
          .
          .
     9999 CALL EXIT
          END
```

* Cards added to original program

** Modify these cards according to particular geometry considered

```
      SUBROUTINESTREAM(NPRT,MJ,MK,ICYCLE,MCYCLE,MCOWW,DX,XCOWW,U2,V2,W2,
     1T)
      DIMENSION DRDX(21,20), DTDX(21,20),RPSAV(21,20),THSAV(21,20),
     1RR(21,20),U2(21,20),V2(21,20),W2(21,20),T(20),CP(21,20),EM(21,20)
      DIMENSION DJK(21,20),FJKA(21,20), FJKN(21,20), DMDOT(21,20),
     1 DFRAMAY(21,20), DFRAMN(21,20),DMFRAMY(21,20),CPSAV(20),RRSAV(20),
     2 PBAR(21,20),DELA(21,20)
      COMMON/STLIN/G,X2,RR,DRDX,DTDX,RPSAV,THSAV,  EM,CP,PBAR
      F(EM,G)=1./(1.+(G-1.)*EM*EM/2.)
      IF( MCOWW .EQ. 1 .AND. MCYCLE .EQ. 1) GO TO 202
      IF(ICYCLE .GT. 1) GO TO 35
C
C     INITIALIZE AT X=XI
C
      CAX   =0.
      CS    =0.0
      CN    =0.0
      WRITE(6,335) X2
  335 FORMAT(///,5X,*INITIAL STREAMLINES POSITIONS AT X= *,E15.5/)
      DO 34 K=1,MK
      WRITE(6,6001) K,T(K)
      RRSAV(K)=RR(1,K)
      CPSAV(K)=CP(1,K)
      DO 33 J=2,MJ,2
      DRDX(J,K)=V2(J,K)/U2(J,K)
      DTDX(J,K)=-W2(J,K)/(RR(J,K)*U2(J,K))
      DTDX(J,K)= 57.295779*     DTDX(J,K)
      RPSAV(J,K)=RR(J,K)
      THSAV(J,K)=T(K)
      Y1=RR(J,K)*SIN(T(K)/57.295779)
      Z1=RR(J,K)*COS(T(K)/57.295779)
      WRITE(6,162) J,Y1,Z1,RPSAV(J,K),DRDX(J,K),RR(J,K),T(K),DTDX(J,K)
   33 CONTINUE
   34 CONTINUE
      RETURN
C
C     INITIALIZE ODD NUMBERED STREAMLINES AT X= X2
C
  202 WRITE(6,335) X2
      DO 14 K=1,MK
      WRITE(6,6001) K,T(K)
      DO 13 J=1,MJ,2
      DRDX(J,K)=V2(J,K)/U2(J,K)
      DTDX(J,K)=-W2(J,K)/(RR(J,K)*U2(J,K))
      DTDX(J,K)= 57.295779*     DTDX(J,K)
      RPSAV(J,K)=RR(J,K)
      THSAV(J,K)=T(K)
      Y1=RR(J,K)*SIN(T(K)/57.295779)
      Z1=RR(J,K)*COS(T(K)/57.295779)
      WRITE(6,162) J,Y1,Z1,RPSAV(J,K),DRDX(J,K),RR(J,K),T(K),DTDX(J,K)
   13 CONTINUE
   14 CONTINUE
      GO TO 211
   35 MTEM=MK-1
      IF(MCOWW .LE. 0) GO TO 211
      MSTART=1
      MEND=1
```

```
      GO TO 212
  211 MSTART=2
      MEND=2
  212 CONTINUE
      DO 79 K=1,MTEM
      IF(NPRT.EQ.1)
     1WRITE(6,6001) K,T(K)
 6001 FORMAT(/,5X,*K=*,I2,10X,*TH=*,E12.5,/,8X,* J*,8X,* Y*,10X,*Z *,5X,
     1*RPREV*,8X,*DRDX2*,8X,*RTRAC*,8X,*TTRAC*,5X,*ERROR1*,5X,*ERROR2*/)
      DO 78 J=MSTART,MJ,MEND
      IF(RPSAV(J,K) .GE. 1000. .OR. THSAV(J,K) .GE. 1000.  ) GO TO 78
      IF(DTDX(J,K) .GE. 1000..OR. DRDX(J,K) .GE. 1000.)GO TO 78
C
C     FIRST GUESS OF STREAMLINE POSITION AT X+DX
C
      RPX2=RPSAV(J,K)+DRDX(J,K)*DX
      THX2=THSAV(J,K)+DTDX(J,K)*DX
      IF(K.EQ.1) GO TO 1001
      IF(J.EQ.1) GO TO  350
C
C  TO LOCATE KK AT WHICH THX2 LIES BETWEEN T(KK-1) AND T(KK)
C
      KK=0
   51 KK=KK+1
      IF(KK .GT. MK  ) GO TO 66
      IF(THX2-T(KK)) 53,53,51
   53 IF(KK .EQ. 1)  GO TO 1001
C
C  TO LOCATE JJ AT  T(KK-1), AND RPX2 LIES BETWEEN RR(JJ-1,KK-1) AND RR(
C
      KK=KK-1
      JJ=0
   57 JJ=JJ+1
      IF(JJ .GT. MJ) GO TO 66
      IF(RPX2 -RR(JJ,KK)) 59,59,57
   59 IF(JJ.EQ. 1) GO TO 66
C
C  TO LOCATE II AT  T(KK)   , AND RPX2 LIES BETWEEN RR(II-1,KK)   AND RR(
C
      KK=KK+1
      II=0
   60 II=II+1
      IF(II .GT. MJ) GO TO 66
      IF(RPX2-RR(II,KK)) 61,61,60
   61 IF(II .EQ. 1) GO TO 66
      CVAL1=(THX2-T(KK-1))/(T(KK)-T(KK-1))
      CVAL2=RPX2-RR(JJ-1,KK-1)
      CVAL3=RPX2-RR(II-1,KK)
      VAL1=(U2(JJ,KK-1)-U2(JJ-1,KK-1))/(RR(JJ,KK-1)-RR(JJ-1,KK-1))
      VAL2=(V2(JJ,KK-1)-V2(JJ-1,KK-1))/(RR(JJ,KK-1)-RR(JJ-1,KK-1))
      VAL3=(W2(JJ,KK-1)-W2(JJ-1,KK-1))/(RR(JJ,KK-1)-RR(JJ-1,KK-1))
      VVAL1=(U2(II,KK)-U2(II-1,KK))/(RR(II,KK)-RR(II-1,KK))
      VVAL2=(V2(II,KK)-V2(II-1,KK))/(RR(II,KK)-RR(II-1,KK))
      VVAL3=(W2(II,KK)-W2(II-1,KK ))/(RR(II,KK)-RR(II-1,KK))
      U2X2=(U2(JJ-1,KK-1)+VAL1*CVAL2)*(1.-CVAL1)+(U2(II-1,KK)+VVAL1*
     1CVAL3)*CVAL1
      V2X2=(V2(JJ-1,KK-1)+VAL2*CVAL2)*(1.-CVAL1)+(V2(II-1,KK)+VVAL2*
     1 CVAL3)*CVAL1
```

```
      W2X2=(W2(JJ-1,KK-1)+VAL3*CVAL2)*(1.-CVAL1)+(W2(II-1,KK)+VVAL3*
     1 CVAL3)*CVAL1
  350 IF(J.NE.1) GO TO  360
      U2X2=U2(1,K)+(U2(1,K+1)-U2(1,K))/(T(K+1)-T(K))*(THX2-T(K))
      V2X2=V2(1,K)+(V2(1,K+1)-V2(1,K))/(T(K+1)-T(K))*(THX2-T(K))
      W2X2=W2(1,K)+(W2(1,K+1)-W2(1,K))/(T(K+1)-T(K))*(THX2-T(K))
  360 DRDX2=V2X2/U2X2
      DTDX2=-W2X2/(RPX2*U2X2)
      DTDX2       = 57.295779*      DTDX2
      RTRAC=RPSAV(J,K)+0.5*DX*(DRDX(J,K)+DRDX2)
      TTRAC=THSAV(J,K)-0.5*DX*(DTDX(J,K)+DTDX2)
      TEST1=ABS(RTRAC-RPX2         )/RTRAC
      TEST2=ABS(TTRAC-THX2         )/TTRAC
      Y1=RTRAC*SIN(TTRAC/57.295779)
      Z1=RTRAC*COS(TTRAC/57.295779)
      GO TO 62
C
C     PLANE OF SYMMETRY
C
 1001 KK=1
      II=0
      IF(J.EQ.1) GO TO 370
 1060 II=II+1
      IF(II.GT.MJ) GO TO 66
      IF(RPX2-RR(II,KK)) 1061,1061,1060
 1061 IF(II.EQ.1)GO TO 66
      CVAL3=RPX2-RR(II-1,KK)
      VVAL1=(U2(II,KK)-U2(II-1,KK))/(RR(II,KK)-RR(II-1,KK))
      VVAL3=(W2(II,KK)-W2(II-1,KK ))/(RR(II,KK)-RR(II-1,KK))
      U2X2=U2(II-1,KK)+VVAL1*CVAL3
      V2X2= V2(II-1,KK) + VVAL2*CVAL3
      DRDX2=V2X2/U2X2
  370 IF(J.EQ.1) DRDX2=V2(1,K)/U2(1,K)
      DTDX2 = 0.0
      RTRAC=RPSAV(J,K)+0.5*DX*(DRDX(J,K)+DRDX2)
      TTRAC=0.0
      TEST1=ABS(RTRAC-RPX2         )/RTRAC
      TEST2 = 0.0
      Y1=0.
      Z1 = RTRAC
   62 CONTINUE
      GO TO(6006 , 6007) NPRT
 6006 WRITE(6,162) J,Y1,Z1,  RPSAV(J,K),DRDX2,RTRAC ,TTRAC,TEST1,TEST2
  162 FORMAT(5X,I5,9E12.5)
 6007 CONTINUE
      GO TO 68
   66 WRITE(6,67) KK,II,JJ,J,K,RPX2,THX2, RPSAV(J,K),X2
   67 FORMAT(*   CAN NOT LOCATE STREAM LINE AT THIS PT.*,5I5,4E15.5)
      RTRAC=1000.
      TTRAC=1000.
      DRDX2=1000.
      DTDX2=1000.
   68 CONTINUE
      RPSAV(J,K)=RTRAC
      THSAV(J,K)=TTRAC
      DRDX(J,K)=DRDX2
      DTDX(J,K)=DTDX2
   78 CONTINUE
```

```
   79 CONTINUE
C
C     CALCULATE INCREMENTAL BODY FORCE AND MOMENT
C
      IF(NPRT.EQ.1) WRITE(6,312)  X2
  312 FORMAT(///,5X,*BODY FORCE COMPONENTS AND MOMENTS UP TO X=*,E15.5,/
     110X,*K *,5X,*CAXIAL*,5X,*CNORMAL*,5X,*CSIDE*5X,*CMY*,/)
      MJM1=MJ-1
      MKM1=MK-1
      DO 313 K=1,MKM1
      R1AVG=(RRSAV(K+1)+RRSAV(K))/2.
      R2AVG=(RR(1,K+1)+RR(1,K))/2.
      RAVG=(R1AVG+R2AVG)/2.
      DRAVG=R2AVG-R1AVG
      DT = T(K+1)-T(K)
      DSAX = RAVG*DT*DRAVG /57.295779
      DYAVG=(DY2+DY1)/2.
      DY2=RR(1,K+1)*SIN(T(K+1)/57.295779 )-RR(1,K)*SIN(T(K)/57.295779)
      DY1=RRSAV(K+1)*SIN(T(K+1)/57.295779)-RRSAV(K)*SIN(T(K)/57.295779)
      DSN = DYAVG*DX
      DZ2=RR(1,K+1)*COS(T(K+1)/57.295779)-RR(1,K)*COS(T(K)/57.295779)
      DZ1=RRSAV(K+1)*COS(T(K+1)/57.295779 )-RRSAV(K)*COS(T(K)/57.295779)
      DZAVG=(DZ2+DZ1)/2.
      DSS=DZAVG*DX
      CPAVG=(CPSAV(K)+CPSAV(K+1)+CP(1,K)+CP(1,K+1))/4.
      DCAX=CPAVG*DSAX
      DCN=CPAVG*DSN
      DCS=CPAVG*DSS
      RRSAV(K)=RR(1,K)
      CPSAV(K)=CP(1,K)
      TBAR=(T(K+1)+T(K))/2.
      RBAR=R1AVG+DRAVG/2.
      YBAR=RBAR*SIN(TBAR/57.295779)
      ZBAR=RBAR*COS(TBAR      /57.295779)
      XBAR=X2+DX/2.
      DCMY=XBAR*DCN    -ZBAR*DCAX
      CAX    =CAX    +DCAX
      CN    =CN    +DCN
      CS    =CS    +DCS
      CMY    =CMY    +DCMY
      IF(NPRT.EQ.1) WRITE(6,320) K,CAX,CN,CS,CMY
  320 FORMAT(5X,I5,4E15.4)
  313 CONTINUE
C
C     CALCULATE INCREMENTAL MASS FLOW RATE, RAM FORCE, PITCHING MOMENT
C
      IF(NPRT.NE.1) RETURN
      DO 301 K=1,MK
      DO 302 J=1,MJ
      SQ=SQRT(1.+(V2(J,K)**2+W2(J,K)**2)/U2(J,K)**2)
      DJK(J,K)=EM(J,K)/SQRT(F(EM(J,K),G))*PBAR(J,K) /SQ
      FJKA(J,K)= (1.+G*EM(J,K)**2/SQ/SQ)*PBAR(J,K)-1.
      FJKN(J,K)= (G*EM(J,K)**2)/SQ/SQ*PBAR(J,K)*(V2(J,K)*COS(T(K)/57.295
     1779)-W2(J,K)*SIN(T(K)/57.295779))/U2(J,K)
  302 CONTINUE
  301 CONTINUE
      DO 303 K=1,MKM1
      SJM1=0.0
```

```
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      SUM5=0.0
      DO 304 J=1,MJM1
      DJKAVG = (DJK(J,K) + DJK(J+1,K)+DJK(J,K+1)+DJK(J+1,K+1))/4.
      FJKAAV=(FJKA(J,K)+FJKA(J+1,K)+FJKA(J,K+1)+FJKA(J+1,K+1))/4.
      FJKNAV=(FJKN(J,K)+FJKN(J+1,K)+FJKN(J,K+1)+FJKN(J+1,K+1))/4.
      R1AVG = (RR(J,K)+RR(J,K+1))/2.
      R2AVG=(RR(J+1,K)+RR(J+1,K+1))/2.
      RAVG=(R1AVG+R2AVG)/2.
      DRAVG= R2AVG-R1AVG
      DT=T(K+1)-T(K)
      DAJK=RAVG*DT*DRAVG/57.295779
      SUM1=SUM1+DAJK
      DELA(J,K)=SUM1
      SUM2=SUM2+DJKAVG*DAJK
      DMDOT(J,K)=SUM2
      DFRAMA=FJKAAV*DAJK
      SUM3=SUM3+DFRAMA
      DFRAMAX(J,K)=SUM3
      DFRAMNO=FJKNAV*DAJK
      SUM4=SUM4+DFRAMNO
      DFRAMN(J,K)=SUM4
      RBAR=R1AVG+DRAVG/2.
      TBAR=(T(K+1)+T(K))/2.
      YBAR = RBAR*SIN(TBAR/57.295779)
      ZBAR=RBAR*COS(TBAR/57.295779)
      SUM5=SUM5+X2*DFRAMNO-ZBAR*DFRAMA
      DMFRAMY(J,K)=SUM5
  304 CONTINUE
  303 CONTINUE
      WRITE(6,300) (K,K=1,MKM1)
  300 FORMAT(///,5X,*TABLE OF INCREMENTAL MASS FLOW RATE*,/,5X,12I10,/)
      DO 314 J=1,MJM1
  314 WRITE(6,310) J,(DMDOT(J,K),K=1,MKM1)
  310 FORMAT(5X,I5,12E10.3)
      WRITE(6,330) (K,K=1,MKM1)
  330 FORMAT(///,5X,*INCREMENTAL AREAS IN MESH  GRID*,/,5X,12I10,/)
      DO 331 J=1,MJM1
  331 WRITE(6,310) J,(DELA(J,K),K=1,MKM1)
      WRITE(6,305) (K,K=1,MKM1)
  305 FORMAT(///,5X,*AXIAL COMPONENT OF RAM FORCE*,/,5X,12I10,/)
      DO 306 J=1,MJM1
  306 WRITE(6,310) J,(DFRAMAX(J,K),K=1,MKM1)
      WRITE(6,307) (K,K=1,MKM1)
  307 FORMAT(///,5X,*NORMAL COMPONENT OF RAM FORCE*,/,5X,12I10,/)
      DO 308 J=1,MJM1
  308 WRITE(6,310) J,(DFRAMN(J,K),K=1,MKM1)
      WRITE(6,309) (K,K=1,MKM1)
  309 FORMAT(///,5X,*MOMENT PRODUCED BY  RAM FORCE*,/,5X,12I10,/)
      DO 311 J=1,MJM1
  311 WRITE(6,310) J,(DMFRAMY(J,K),K=1,MKM1)
   80 RETURN
      END
```

TABLE 1

INCREMENTAL AREAS IN MESH GRID (in^2) AT STATION "0" $x = 6.00$ IN

	1	2	3	4	5	6
1	6.504E-03	6.454E-03	6.365E-03	6.273E-03	6.200E-03	6.154E-03
2	1.324E-02	1.313E-02	1.295E-02	1.276E-02	1.262E-02	1.253E-02
3	2.019E-02	2.004E-02	1.976E-02	1.947E-02	1.925E-02	1.911E-02
4	2.738E-02	2.717E-02	2.679E-02	2.640E-02	2.611E-02	2.592E-02
5	3.479E-02	3.453E-02	3.404E-02	3.355E-02	3.318E-02	3.294E-02
6	4.243E-02	4.211E-02	4.151E-02	4.091E-02	4.047E-02	4.018E-02
7	5.030E-02	4.992E-02	4.921E-02	4.850E-02	4.797E-02	4.763E-02
8	5.840E-02	5.795E-02	5.712E-02	5.630E-02	5.569E-02	5.530E-02
9	6.672E-02	6.621E-02	6.526E-02	6.432E-02	6.365E-02	6.319E-02
10	7.527E-02	7.470E-02	7.362E-02	7.256E-02	7.179E-02	7.129E-02
11	8.405E-02	8.341E-02	8.220E-02	8.101E-02	8.017E-02	7.961E-02
12	9.305E-02	9.235E-02	9.100E-02	8.969E-02	8.876E-02	8.815E-02
13	1.023E-01	1.015E-01	1.000E-01	9.858E-02	9.757E-02	9.690E-02
14	1.117E-01	1.109E-01	1.093E-01	1.077E-01	1.066E-01	1.059E-01
15	1.214E-01	1.205E-01	1.187E-01	1.170E-01	1.158E-01	1.151E-01
16	1.313E-01	1.304E-01	1.284E-01	1.266E-01	1.253E-01	1.245E-01
17	1.415E-01	1.404E-01	1.383E-01	1.363E-01	1.350E-01	1.341E-01
18	1.519E-01	1.507E-01	1.485E-01	1.463E-01	1.449E-01	1.439E-01
19	1.625E-01	1.612E-01	1.588E-01	1.565E-01	1.550E-01	1.540E-01
20	1.733E-01	1.720E-01	1.694E-01	1.669E-01	1.653E-01	1.642E-01

captured streamtube

.1214 .1612 .1694 .1669

Free stream tube 0.0 .0156 .065

$= .6189\ in^2$

$= .0806\ in^2$

$.6995\ in^2$

$$\therefore A'_o = \left[.6189 + \frac{.0806}{(A'_o/A_\infty)_{..}}\right](16)^{*}(2)^{**}$$

* inlet model scale 1x above coordinates ** Symmetrical side

TABLE 1 Cont'd

TABLE OF INCREMENTAL MASS FLOW RATE $\dot{m}/\left(\frac{\gamma g}{a_o} P_\infty\right)$ (in^2) AT STATION "0" (x=60 inches)

	1	2	3	4	5	6
1	2.042E-01	2.031E-01	2.001E-01	1.947E-01	1.890E-01	1.859E-01
2	4.129E-01	4.110E-01	4.054E-01	3.951E-01	3.841E-01	3.781E-01
3	6.260E-01	6.235E-01	6.155E-01	6.007E-01	5.849E-01	5.761E-01
4	8.436E-01	8.405E-01	8.301E-01	8.111E-01	7.908E-01	7.796E-01
5	1.066E+00	1.062E+00	1.049E+00	1.026E+00	1.002E+00	9.880E-01
6	1.293E+00	1.288E+00	1.272E+00	1.245E+00	1.217E+00	1.201E+00
7	1.523E+00	1.517E+00	1.497E+00	1.467E+00	1.436E+00	1.419E+00
8	1.758E+00	1.749E+00	1.726E+00	1.692E+00	1.659E+00	1.640E+00
9	1.996E+00	1.985E+00	1.959E+00	1.922E+00	1.886E+00	1.866E+00
10	2.239E+00	2.225E+00	2.196E+00	2.156E+00	2.118E+00	2.096E+00
11	2.486E+00	2.471E+00	2.438E+00	2.394E+00	2.355E+00	2.331E+00
12	2.738E+00	2.722E+00	2.684E+00	2.637E+00	2.595E+00	2.570E+00
13	2.995E+00	2.976E+00	2.935E+00	2.884E+00	2.840E+00	2.813E+00
14	3.255E+00	3.235E+00	3.190E+00	3.135E+00	3.089E+00	3.061E+00
15	3.520E+00	3.497E+00	3.448E+00	3.390E+00	3.341E+00	3.311E+00
16	3.787E+00	3.762E+00	3.709E+00	3.648E+00	3.597E+00	3.565E+00
17	4.058E+00	4.031E+00	3.974E+00	3.909E+00	3.855E+00	3.822E+00
18	4.331E+00	4.302E+00	4.240E+00	4.172E+00	4.116E+00	4.081E+00
19	4.606E+00	4.575E+00	4.509E+00	4.437E+00	4.379E+00	4.343E+00
20	4.883E+00	4.850E+00	4.780E+00	4.703E+00	4.643E+00	4.605E+00

3.52 ...... 4.575 ...... 4.780 ...... 4.703 ...... $= 17.578 \ in^2$

$= 0.162 \ in^2$

Free Stream 6.0 0.0 10

$$\dot{m} = \dot{m}_\infty = \frac{M_\infty}{\sqrt{(T/T_o)_\infty}} A_o \left(\frac{\gamma g}{a_o} P_\infty\right) \quad \therefore \quad A_o = \frac{\dot{m}_\infty / (\gamma g/a_o P_\infty)}{M_\infty / \sqrt{(T/T_o)_\infty}} = \left[\frac{17.578}{6/\sqrt{.122}} + 0.162\right](16)(2)$$

$$\left.\begin{array}{l} A_o = 35.3 \ in^2 \\ A_c = 54. \ in^2 \end{array}\right\} \quad \underline{\frac{A_o}{A_c} = 0.655}$$

TABLE 1 Cont'd

AXIAL COMPONENT OF RAM FORCE $(F_{ram_{ax}}/P_{\infty})$ AT STATION "O" (x=6.0)

	1	2	3	4	5	6
1	5.897E-01	5.865E-01	5.779E-01	5.626E-01	5.466E-01	5.377E-01
2	1.196E+00	1.187E+00	1.171E+00	1.142E+00	1.111E+00	1.094E+00
3	1.810E+00	1.802E+00	1.779E+00	1.737E+00	1.692E+00	1.667E+00
4	2.440E+00	2.430E+00	2.400E+00	2.346E+00	2.289E+00	2.257E+00
5	3.084E+00	3.072E+00	3.034E+00	2.968E+00	2.900E+00	2.861E+00
6	3.742E+00	3.726E+00	3.680E+00	3.603E+00	3.524E+00	3.479E+00
7	4.411E+00	4.390E+00	4.334E+00	4.247E+00	4.160E+00	4.110E+00
8	5.091E+00	5.064E+00	4.998E+00	4.901E+00	4.807E+00	4.753E+00
9	5.782E+00	5.750E+00	5.674E+00	5.567E+00	5.467E+00	5.406E+00
10	6.488E+00	6.450E+00	6.363E+00	6.248E+00	6.141E+00	6.077E+00
11	7.208E+00	7.164E+00	7.067E+00	6.941E+00	6.828E+00	6.760E+00
12	7.941E+00	7.891E+00	7.783E+00	7.648E+00	7.528E+00	7.456E+00
13	8.687E+00	8.632E+00	8.513E+00	8.367E+00	8.240E+00	8.163E+00
14	9.446E+00	9.384E+00	9.254E+00	9.098E+00	8.964E+00	8.883E+00
15	1.022E+01	1.015E+01	1.001E+01	9.839E+00	9.699E+00	9.613E+00
16	1.099E+01	1.092E+01	1.077E+01	1.059E+01	1.044E+01	1.035E+01
17	1.178E+01	1.170E+01	1.154E+01	1.135E+01	1.120E+01	1.110E+01
18	1.258E+01	1.249E+01	1.232E+01	1.212E+01	1.196E+01	1.186E+01
19	1.338E+01	1.329E+01	1.310E+01	1.289E+01	1.272E+01	1.262E+01
20	1.419E+01	1.409E+01	1.389E+01	1.367E+01	1.350E+01	1.339E+01

10.22 13.29 13.89 13.67

= 50.93 in^2
= 4.05 in^2
54.98 in^2

Free stream $\gamma M_\infty^2 \Delta A = 1.4(36)(.0806)$

$$C'_{F_{ram_{ax}}} = \frac{F_{ram_{ax}}/P_\infty}{\frac{\gamma}{2} M_\infty^2 A_c} = \frac{54.98(16)(2)}{.7(36)(54.)} = \underline{\underline{1.30}}$$

as compared with

$$C'_{F_{ram_\infty}} = 2\,\frac{A_0}{A_c} = 2\cdot(.65) = \underline{\underline{1.30}} \quad \text{(coincidental)}$$

TABLE 1 CONT'D

NORMAL COMPONENT OF RAM FORCE (F_{ram_N}/P_∞) AT STATION "0" (x=60)

	1	2	3	4	5	6
1	8.302E-02	7.747E-02	6.525E-02	4.724E-02	2.783E-02	9.244E-03
2	1.646E-01	1.539E-01	1.299E-01	9.462E-02	5.604E-02	1.864E-02
3	2.449E-01	2.291E-01	1.940E-01	1.420E-01	8.452E-02	2.815E-02
4	3.243E-01	3.034E-01	2.575E-01	1.893E-01	1.131E-01	3.772E-02
5	4.029E-01	3.769E-01	3.201E-01	2.362E-01	1.417E-01	4.729E-02
6	4.804E-01	4.491E-01	3.814E-01	2.823E-01	1.700E-01	5.682E-02
7	5.566E-01	5.196E-01	4.413E-01	3.275E-01	1.981E-01	6.631E-02
8	6.319E-01	5.892E-01	5.002E-01	3.722E-01	2.260E-01	7.577E-02
9	7.067E-01	6.582E-01	5.586E-01	4.166E-01	2.537E-01	8.517E-02
10	7.808E-01	7.267E-01	6.166E-01	4.605E-01	2.811E-01	9.448E-02
11	8.538E-01	7.942E-01	6.738E-01	5.039E-01	3.082E-01	1.037E-01
12	9.258E-01	8.607E-01	7.300E-01	5.465E-01	3.348E-01	1.127E-01
13	9.966E-01	9.261E-01	7.854E-01	5.885E-01	3.610E-01	1.216E-01
14	1.066E+00	9.905E-01	8.398E-01	6.298E-01	3.868E-01	1.303E-01
15	1.134E+00	1.053E+00	8.931E-01	6.702E-01	4.120E-01	1.389E-01
16	1.201E+00	1.115E+00	9.451E-01	7.096E-01	4.366E-01	1.472E-01
17	1.265E+00	1.175E+00	9.957E-01	7.479E-01	4.605E-01	1.554E-01
18	1.328E+00	1.232E+00	1.045E+00	7.849E-01	4.836E-01	1.632E-01
19	1.388E+00	1.288E+00	1.092E+00	8.205E-01	5.058E-01	1.707E-01
20	1.445E+00	1.341E+00	1.137E+00	8.546E-01	5.271E-01	1.779E-01

$= 4.414 \text{ in}^2$

1.134 1.288 1.137 .855

$$C'_{F_{ram_N}} = \frac{4.414\,(16)(2)}{.7\,(36)(54.)} = 0.1043$$

$$\varepsilon = \tan^{-1}(.1043/1.30) = 4.65^\circ$$

TABLE 1

MOMENT PRODUCED BY RAM FORCE $-(\mathcal{M}/P_\infty)$ ABOUT THE ORIGIN – AT STATION "0" $x=6.0$:–

	1	2	3	4	5	6
1	4.726E-03	9.395E-03	7.842E-03	-1.028E-03	-5.063E-03	-1.944E-03
2	-2.432E-02	-1.557E-02	-1.156E-02	-1.979E-02	-1.960E-02	-7.045E-03
3	-9.328E-02	-7.449E-02	-5.840E-02	-5.697E-02	-4.448E-02	-1.552E-02
4	-2.044E-01	-1.670E-01	-1.331E-01	-1.135E-01	-8.027E-02	-2.764E-02
5	-3.472E-01	-2.941E-01	-2.369E-01	-1.908E-01	-1.281E-01	-4.378E-02
6	-5.285E-01	-4.579E-01	-3.721E-01	-2.907E-01	-1.890E-01	-6.420E-02
7	-7.482E-01	-6.500E-01	-5.380E-01	-4.127E-01	-2.628E-01	-8.081E-02
8	-1.004E+00	-8.917E-01	-7.322E-01	-5.557E-01	-3.491E-01	-1.176E-01
9	-1.296E+00	-1.159E+00	-9.547E-01	-7.200E-01	-4.480E-01	-1.508E-01
10	-1.627E+00	-1.462E+00	-1.208E+00	-9.077E-01	-5.630E-01	-1.890E-01
11	-2.001E+00	-1.805E+00	-1.494E+00	-1.120E+00	-6.927E-01	-2.325E-01
12	-2.418E+00	-2.188E+00	-1.814E+00	-1.358E+00	-8.381E-01	-2.812E-01
13	-2.879E+00	-2.611E+00	-2.168E+00	-1.621E+00	-9.993E-01	-3.353E-01
14	-3.384E+00	-3.075E+00	-2.555E+00	-1.910E+00	-1.176E+00	-3.948E-01
15	-3.933E+00	-3.579E+00	-2.977E+00	-2.225E+00	-1.370E+00	-4.597E-01
16	-4.528E+00	-4.126E+00	-3.434E+00	-2.566E+00	-1.580E+00	-5.303E-01
17	-5.169E+00	-4.714E+00	-3.927E+00	-2.934E+00	-1.806E+00	-6.064E-01
18	-5.855E+00	-5.346E+00	-4.455E+00	-3.328E+00	-2.049E+00	-6.883E-01
19	-6.588E+00	-6.020E+00	-5.019E+00	-3.750E+00	-2.309E+00	-7.758E-01
20	-7.368E+00	-6.735E+00	-5.619E+00	-4.199E+00	-2.586E+00	-8.691E-01
	-3.933	-6.020	-5.619	-4.199		

Note. a sign change was introduced in the program to conform with standard convention of positive pitching moments means a nose up moment. The equations in the report do not follow this convention as the x-axis points downstream which is unconventional.

$= -19.77$

Free stream $\quad -\frac{\gamma}{2}\gamma M_\infty^2 \Delta A = -4.05\,(0.93) \qquad = -3.76$

-23.53

area length

$$\ell_{ref} = \ell_{cone} = 24.\text{in} \qquad C_{M_{q_{F_{ram_{0,0}}}}} = \frac{\mathcal{M}_{F_{ram}}/P_\infty}{\frac{\gamma}{2} M_\infty^2 A_c\, \ell_{ref}} = \frac{-23.53\,(16.)(4)(2)}{.7\,(36.)(54.)(24.)} = -.0928 \text{ (i.e. pitching nose down)}$$

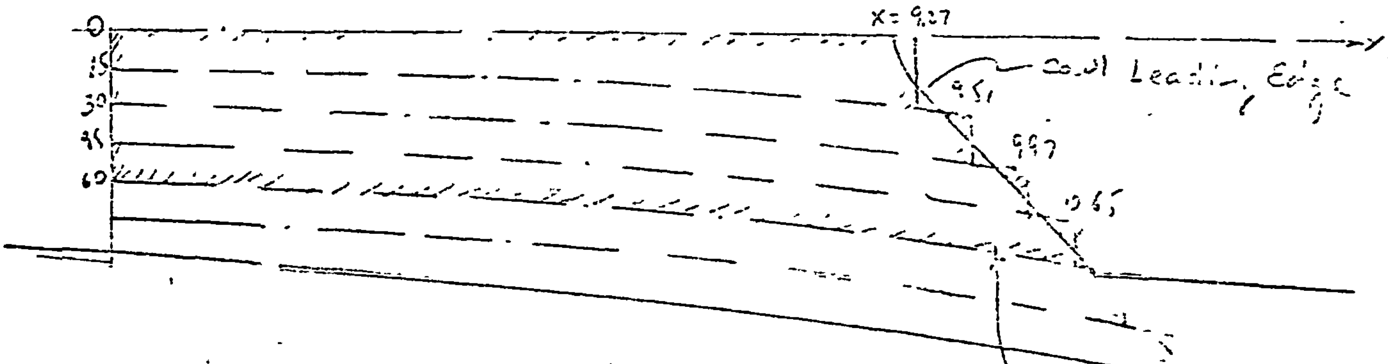
Point of Application

$$-\tilde{z} = \left(C_{M_{q_{F_{ram_{0,0}}}}} - \frac{x}{\ell_{ref}}\, C_{F_{ram_N}} \right) / C_{F_{ram_{ax}}} = \left(-.0928 + 1.(.104)\right)/1.30 = -.148$$

$\tilde{z} = 3.56$ inches (inlet model scale) (see fig 5)

Note This means that the distributed ram force (including free stream) produces no moments about this point.

INLET RAMP PRESSURE FORCE



K	θ	CA*	CN*	CS*	$C_{MY_{0,0}}$**
1	0-15°	.1767	-.202	-.214	-1.224
2	15-30°	.2045	-.222	-.241	-1.383
3	30-45°	.2712	-.260	-.300	-1.732
4	45-60°	.4053	-.325	-.405	-2.230
Σ K	0-60°	1.058	-1.007	-1.160	-6.53

* in²
** in³

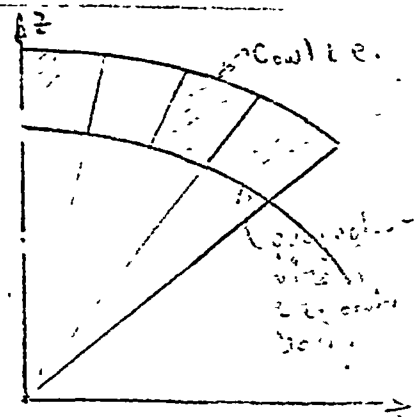
$$C_{Ax} = \frac{1.058\,(16)(2)}{(.9)(36)(54.)} = .025 \qquad C_N = -.0238 \qquad C_S = -.0274$$

i.e. in -z direction

$$C_{MY_{0,0}} = \frac{-6.53\,(16)(2)(4)}{.7\,(36)(54.)(24)} = -.026$$ i.e. nose down

RAM FORCE AT INLET MOUTH

K	θ	$C_{F_{ram_{ax}}}$	$C_{F_{ram_N}}$	$C_{MF_{ram_{y_{0,0}}}}$
1	0-15	9.755	1.981	+2.419 (nose up)
2	15-30	9.979	1.708	+1.627
3	30-45	12.290	1.573	-.457 nose down
4	45-60	17.000	1.400	-3.70
Σ	0-60°	49.024	6.567	-0.11



$$\left(C_{F_{ram_{ax}}}\right)_{\text{cowl l.e.}} = \frac{49.024\,(16)(2)}{1350.} = 1.16$$

$$\left(C_{F_{ram_N}}\right)_{\text{cowl l.e.}} = .155$$

$$\left(C_{M_{F_{ram_Y}}}\right)_{0,0} = -.434\times10^{-3}$$

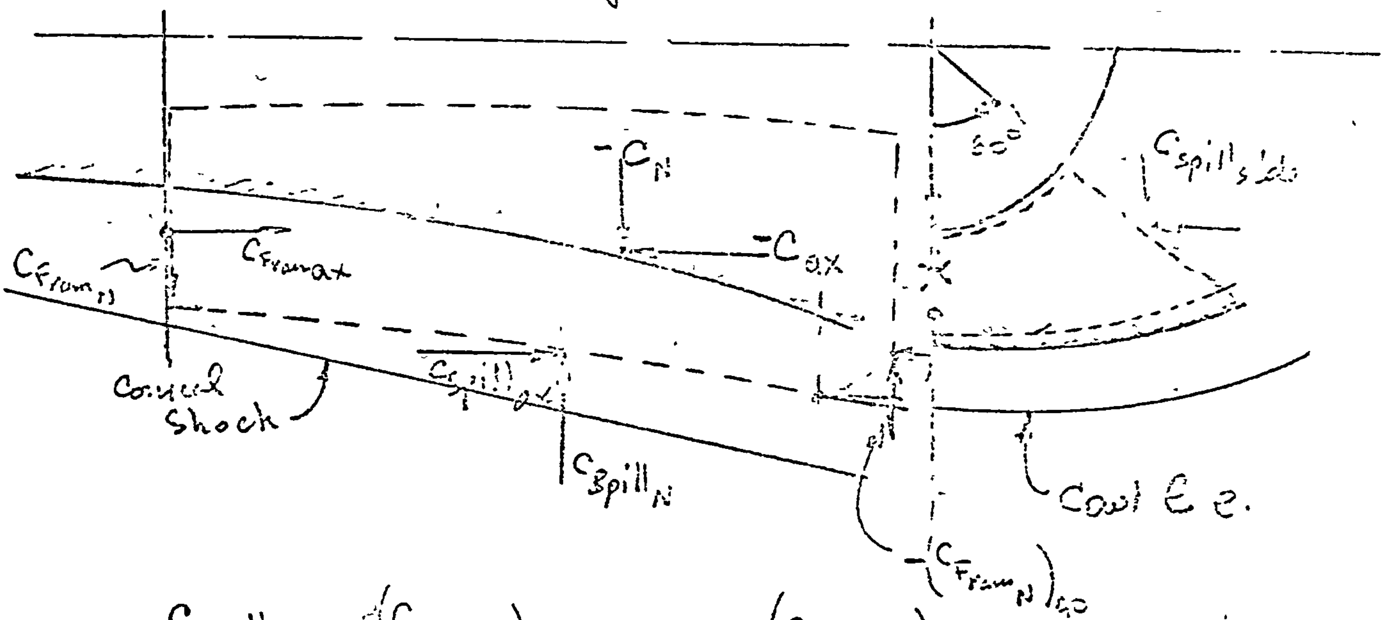
$$\varepsilon = 7.65°$$

$$\tilde{z} = \left(.434\times10^{-3} + \left(\tfrac{10}{6}\right).155\right)/1.16$$

$$\tilde{z} = .273 \times 24$$

$$\tilde{z} = 5.35 \quad @\ x = 40.0$$

INLET Spillage Force



$$C_{spill_{ax}} = (C_{F_{ram\,ax}})_{40''} + C_{AX} - (C_{F_{ram\,ax}})_{40''}$$

$$= 1.16 + .025 - 1.30$$

$$= -0.115$$

$$C_{spill_N} = +(C_{F_{ram\,N}})_{40''} + C_N - (C_{F_{ram\,N}})_{40''}$$

$$= .1043 + .0238 - .155$$

$$= -0.027$$

$$C_{M_{spill_{00}}} = C_{M_{F_{ram\,0,0}}} + C_{M_{Y_{0,0}}} - C_{M_{F_{ram\,N\,0,0}}}$$

$$= -.0923 + .026 - .434 \times 10^{-3}$$

$$= -.067$$

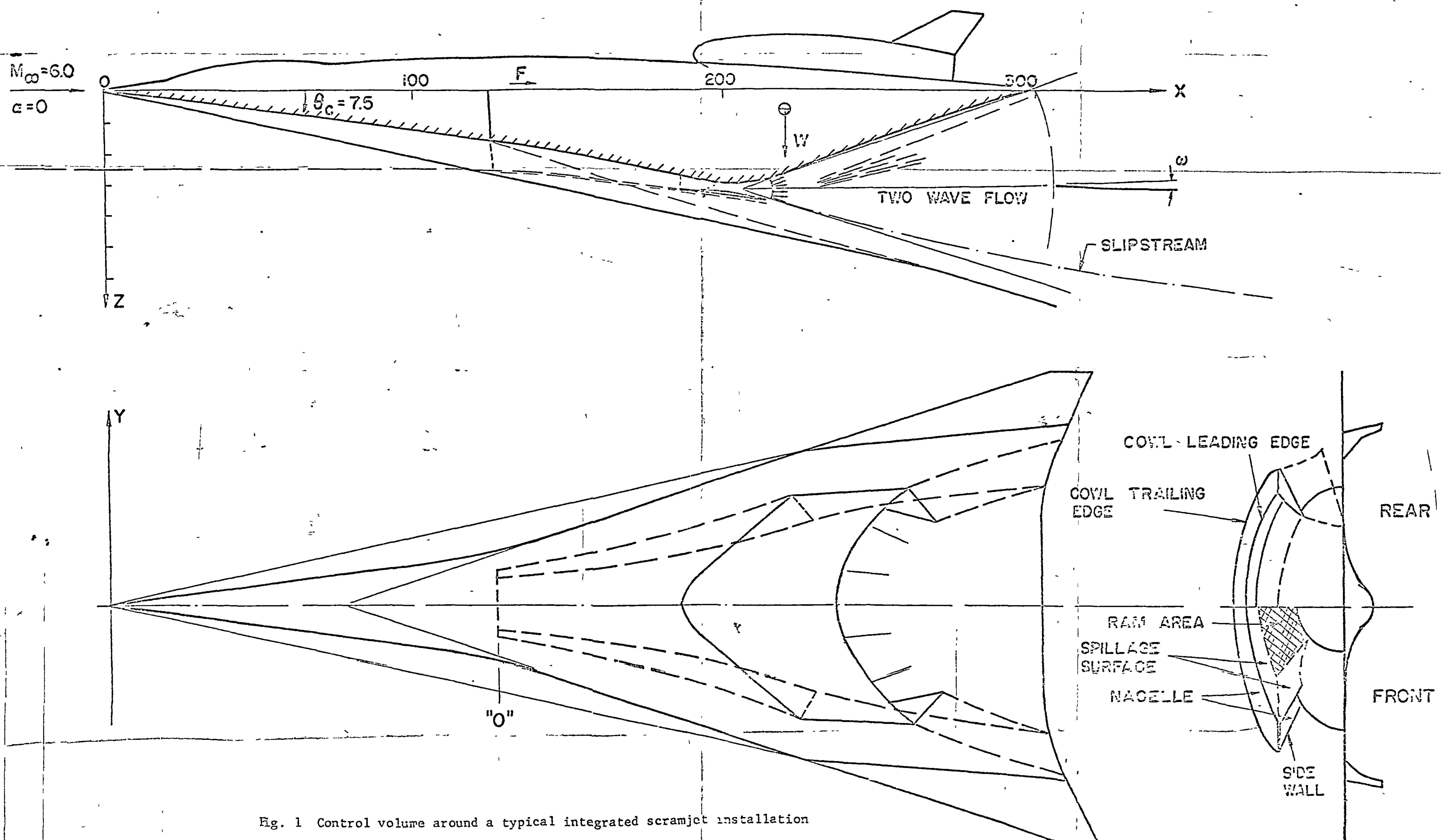


Fig. 1 Control volume around a typical integrated scramjet installation

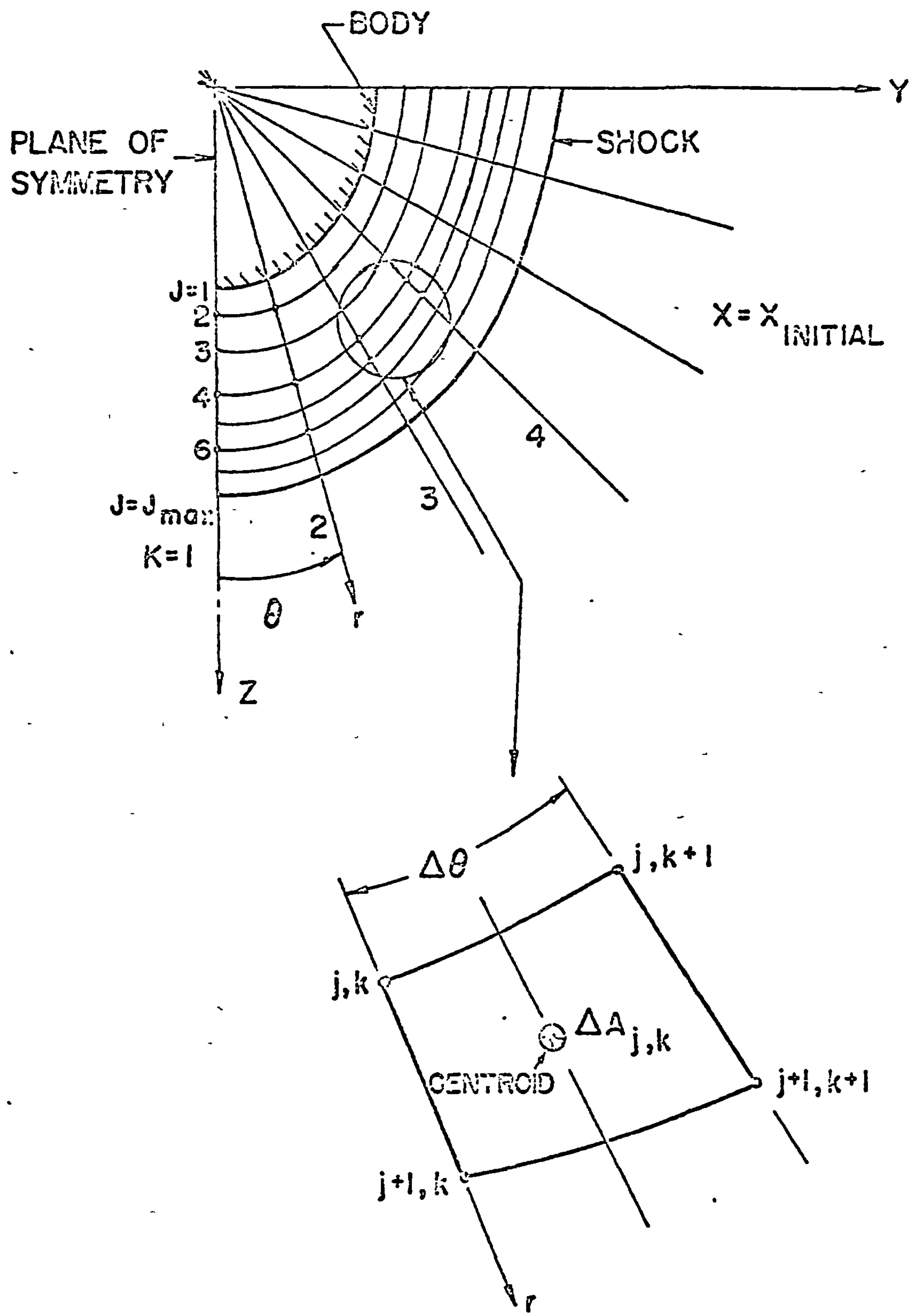


Fig. 2a Initial grid nomenclature and incremental flow area

3

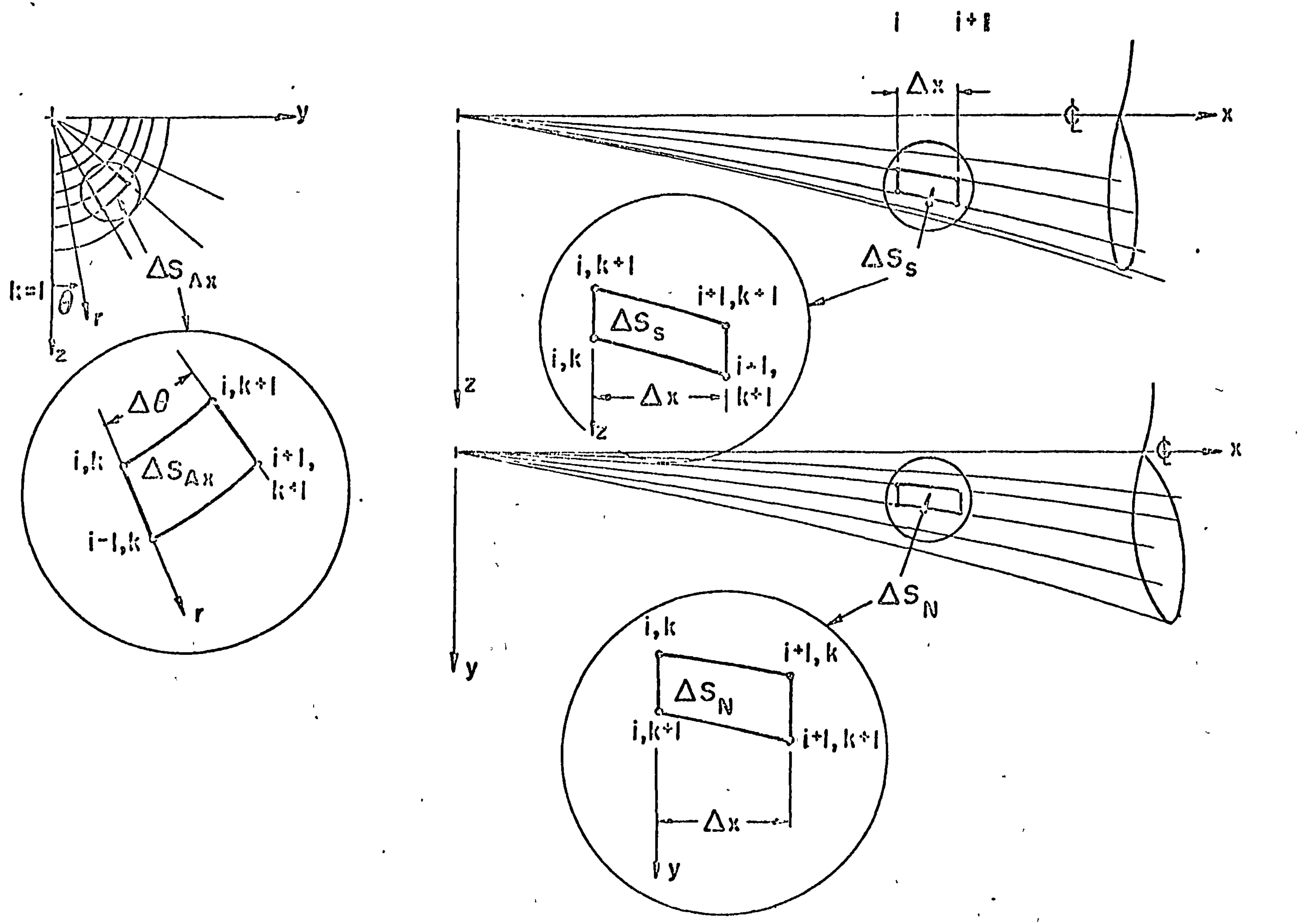


Fig. 2b Incremental body surface areas

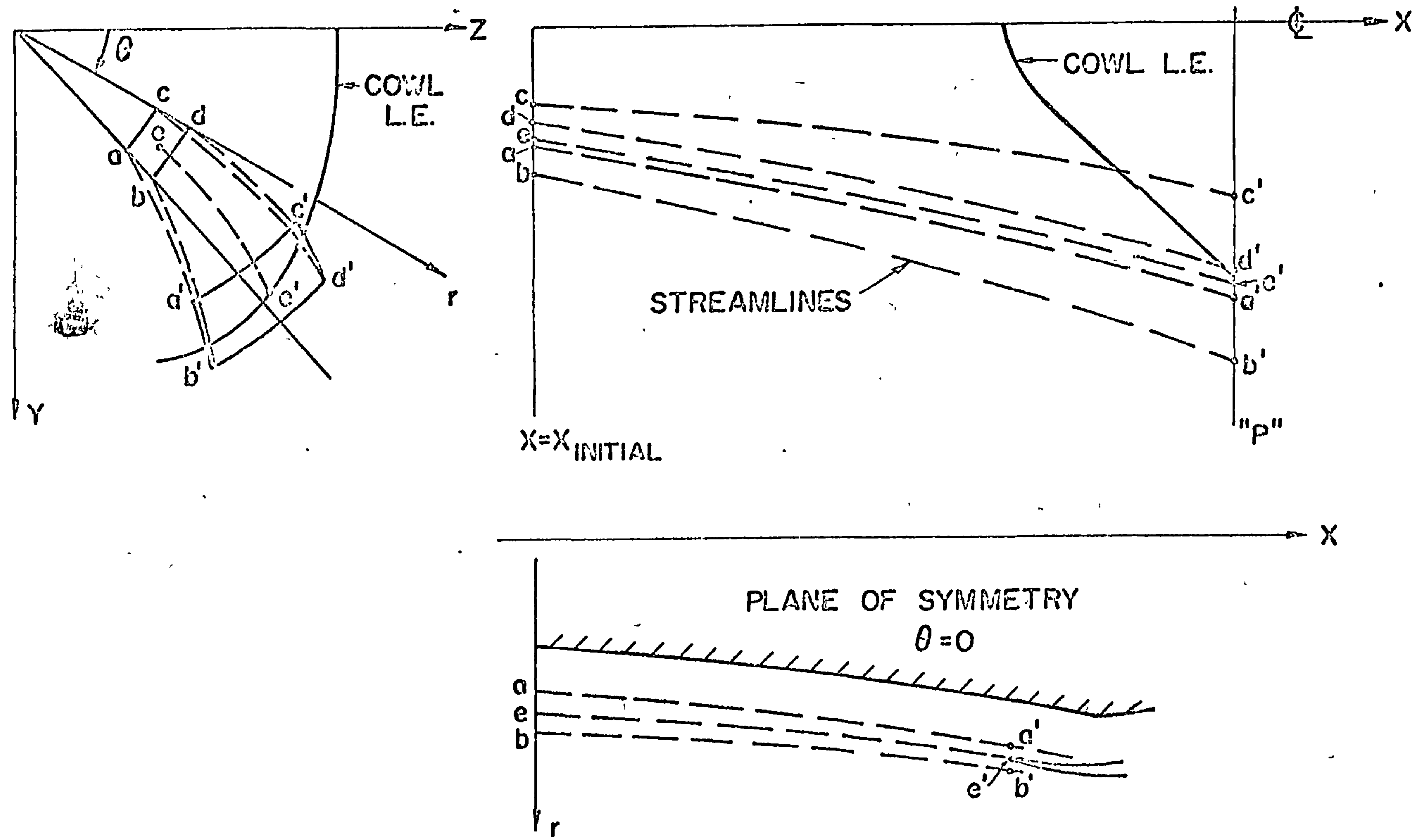


Fig. 2c Streamline interpolation scheme

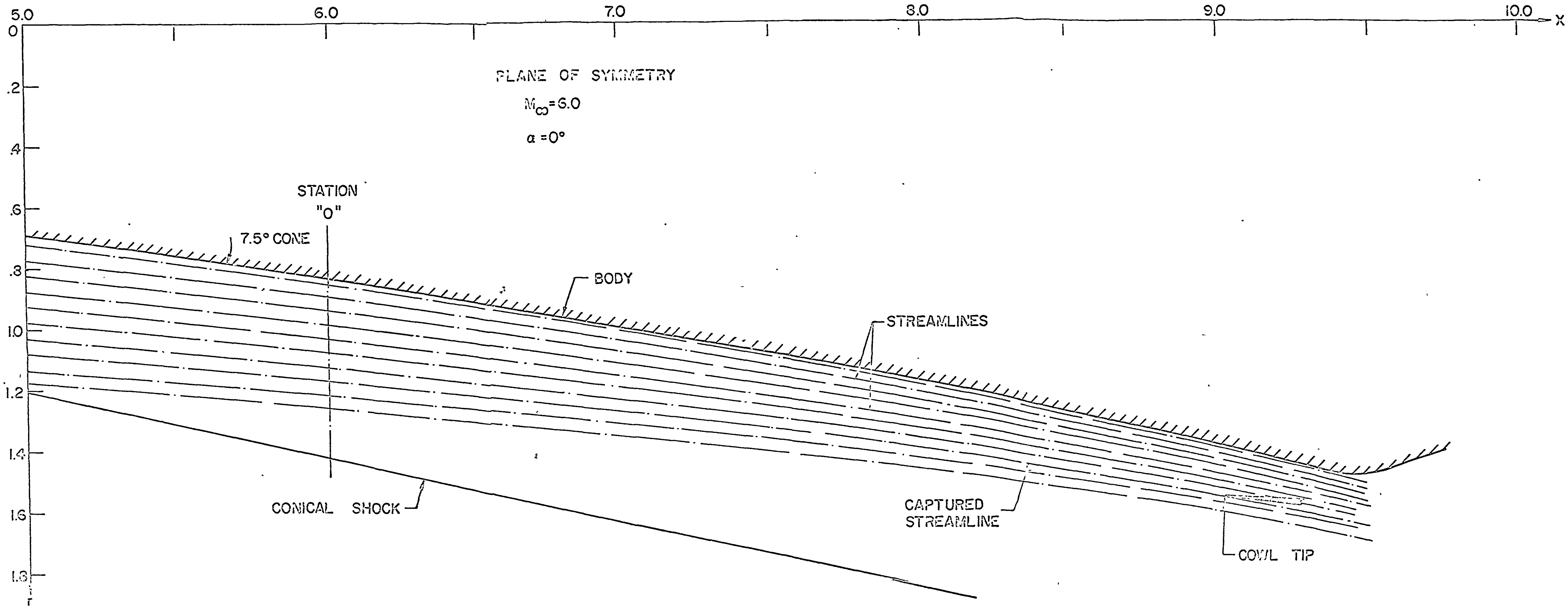


Fig. 3 Streamlines in plane of symmetry $M_\infty = 6.0$ $\alpha = 0$

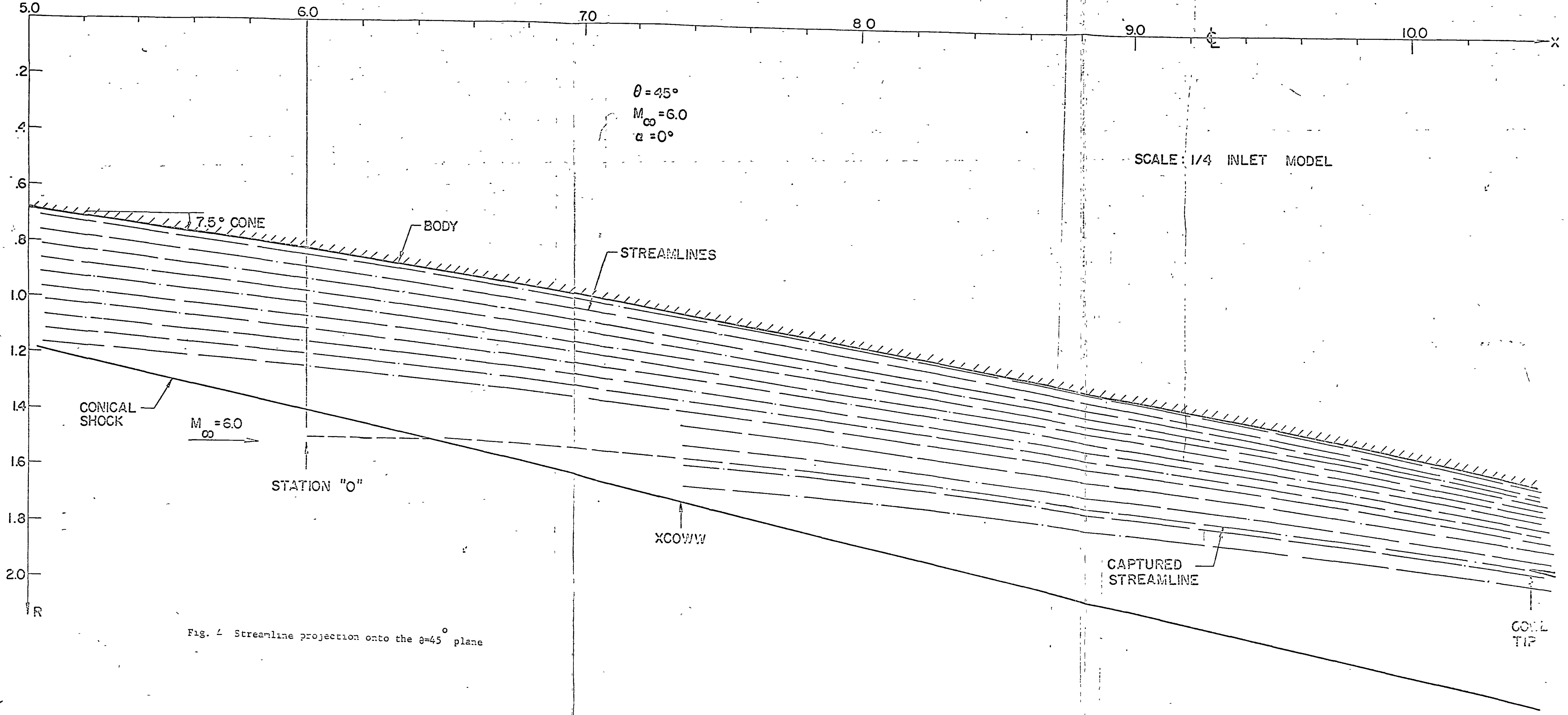


Fig. 4 Streamline projection onto the θ=45° plane

6

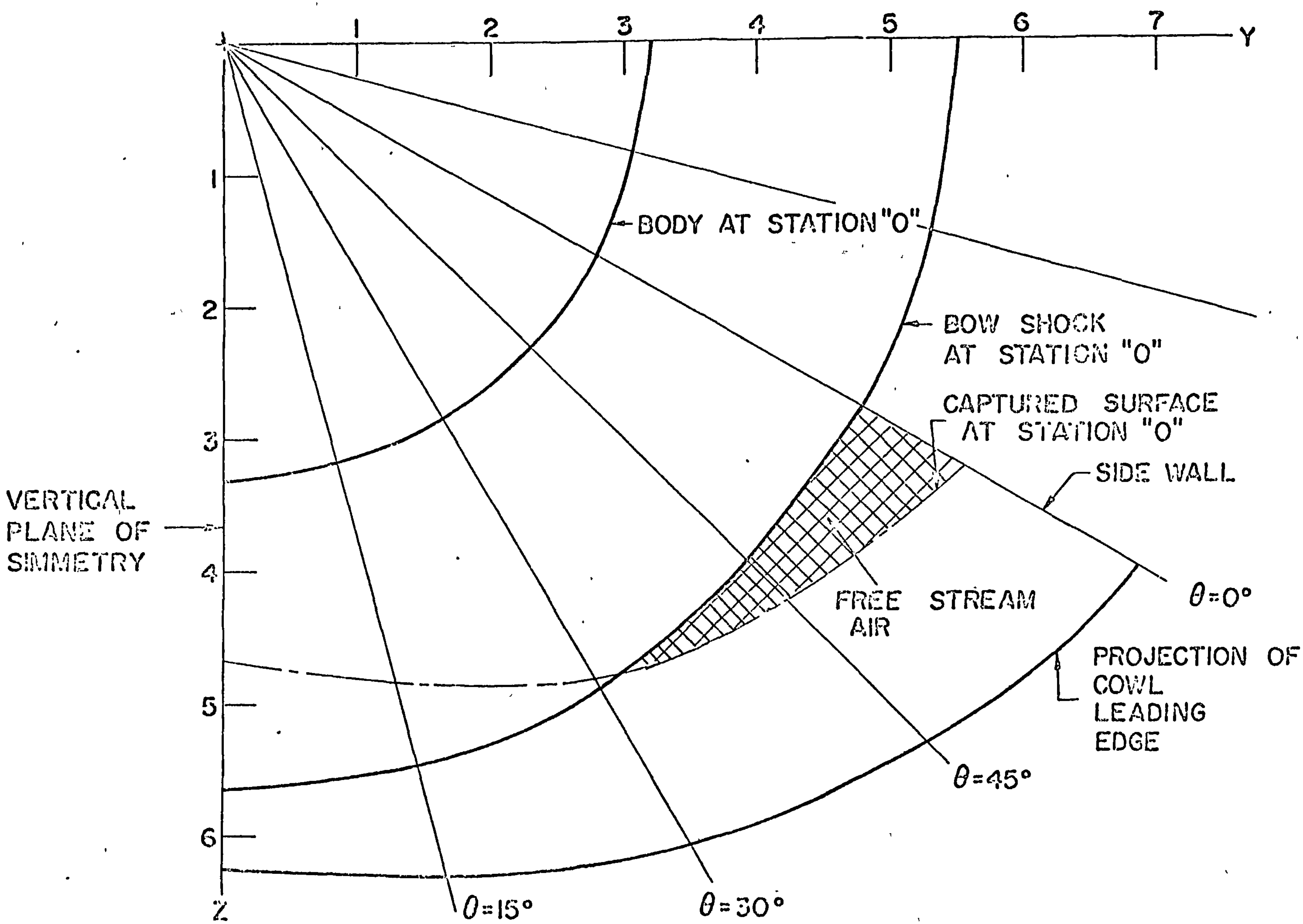


Fig. 5 Capture streamtube area