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P. W. BLUM

I. HARRIS

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ABSTRACT

The motion of the thermosphere with a rotational velocity between 10 to 20% in excess of the earth's rotational velocity has been deduced by King-Hele and his co-workers from the change of the inclination of satellite orbits. To date no completely satisfactory explanation of the observations has been presented. In this paper we shall show that in the thermosphere there exists a small diurnal mean driving force in the eastward direction. This force has not previously been considered in analyses of superrotation. This paper presents a critical review of the observations and a theoretical analysis that takes account of both equinox and solstice conditions. This work shows that the discrepancy can be resolved between observations and theoretical explanation in the lower height region where the great majority of observations were made. It is proposed that additional observational data are needed in the isothermal region for a more complete analysis.

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CONTENTS

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SOME NEW ASPECTS ON THE SUPERROTATION OF THE THERMOSPHERE

INTRODUCTION AND REVIEW OF OBSERVATIONAL EVIDENCE

The rotational velocity of the thermosphere as deduced from the changes of the inclination of the orbits of satellites during their life-time has been presented by King-Hele (1971) in his Figure 5. Based on 29 satellites that he has analysed, King-Hele's paper suggests that the ratio of the rotational velocity of the thermosphere to the rotational velocity of the earth (the superrotation ratio) increases linearly from 1.03 at 150 km to about 1.40 at 370 km and then decreases linearly from this value to a value of 0.68 at 500 km. We shall analyse the physical significance of King-Hele's deductions by dividing the 29 satellites into three groups according to height.

The division into height groups is not arbitrary but is based on the following theoretical considerations: below 270 km no or very few data are available on the diurnal variation of the thermosphere; and, therefore, an exact theoretical analysis is nearly impossible, although we may estimate theoretically the range of the superrotation ratio (Λ). Between 270 and 370 km we have a farily good knowledge of the driving and drag forces that act in the thermosphere, and, therefore, we may solve the equations of motions with fairly good reliability. Above 370 km we would expect from theoretical considerations that no further changes of velocities with height would occur due to effects of viscous drag and the generally assumed boundary condition that the vertical gradient of the velocity should go to zero.

Table 1 shows the following for three height regions: 1) the maximum and minimum values of the superrotation ratio; 2) values obtained from King-Hele's linear fit; 3) the average values of observed Λ, and 4) the chi-square value for the test of significance of the fitted values and the average values. In the height region between 140 km and 270 km there are 18 satellites; thus, for this region we may attach greatest significance to the results of the analysis. The other two height regions, the second between 270 and 370 km and the third above 370 km, contain only four and three satellites respectively; therefore, obviously any conclusions will have less significance. We suggest that King-Hele's interpretation of his analysis is not the only possible one. The following interpretation is also equally valid:

1. There is reliable evidence that in the height region between 140 km and 270 km the thermosphere rotates faster than the earth. The value of the supperotation ratio is probably between 1.1 and 1.3. The mean value deduced from observations is 1.17, but when the observations are weighted according to their observational errors, the mean is reduced to 1.14 — resulting mainly from the small error of Cosmos 1969-94b, which yielded a superrotation ratio of only 1.06. If it is

Table 1

Height Region km	No. of Satellites	Observed King-Hele			Average	Chi-square Test	
		Max	Min	Fit		King-Hele	Average
150-270	18	1.4	1.0	1.05-1.23	1.15	1.76	1.84
270-370	4	1.6	1.1	1.25-1.40	1.26	2.03	2.85
370-500	3	1.35	0.71	1.4 -0.69	1.04	—	—

assumed that the superrotation rate is not height dependent but has a constant value in this height region, a chi-square significance test shows that such an assumption does not result in any significant change in the probability of goodness of fit as compared with King-Hele's fitted linear height dependence.

2. In the height region between 270 and 370 km King-Hele has analysed 4 satellites. He deduces for this height region an increase of the superrotation ratio from about 1.2 at 270 km to 1.4 at 370 km. Again we must conclude that in this region the superrotation ratio Λ is larger than unity, probably between 1.1 and 1.4. Due to the very limited number of satellites analysed, a deduced height dependence of the superrotation ratio Λ becomes questionable from a statistical point of view, although the observations do not exclude a height dependence.

3. In the third height range above 370 km King-Hele deduces decreasing superrotation ratios. At 500 km he suggests a ratio of 0.68. Only

3 satellites were analysed in this height region. The arguments for an assumed height dependence — considering the statistics — are even weaker than in the second height region.

It is very difficult to reconcile a height dependence of the superrotation ratio above 370 km with theoretical considerations. On the other hand, we shall show that there latitudinal dependence of the superrotation ratio is possible by theoretical considerations with decreasing superrotation ratios for satellites having large inclinations. In this context it should be observed that 1967-42A (Ariel 3) resulted in a superrotation ratio of 0.7 at 500 km. This satellite has an inclination of about 80°. For these reasons we suggest that the data available so far do not justify the deduction that the superrotation ratio decreases so markedly above 370 km as suggested by King-Hele.

Figure 1 shows the observations, King-Hele's fit and our suggested mean values for the three height regions.

THE DIURNAL AVERAGE AZIMUTHAL DRIVING FORCE

The equations of horizontal motion in the thermosphere show that a diurnal mean motion of the thermosphere can result from four effects: 1) A diurnal mean-zonal driving force; 2) The variation of the drag forces, especially ion drag with local time out of phase with the zonal velocities; 3) Ion motion driven by meridional on radial electric fields; 4) Mean meridional motions that interact through the Coriolis force on the zonal motions.

Rishbeth has recently (1972) reviewed the various possibilities of explaining theoretically the observed superrotation and concludes that more data are required for a well-founded theoretical explanation.

In all previous treatments it has been assumed that the mean diurnal zonal driving force vanishes, as this is a consequence of the Jacchia model (1971). It can be shown that based on the Jacchia density distribution and the observation of thermospheric temperatures by incoherent radar back scatter measurements a small eastward diurnal average driving force results.

The zonal driving force is given by (Blum and Harris, 1973)

$$f_d = -\frac{R}{\sin\theta \; Mr\omega}\left(T\frac{\partial}{\partial\tau}\ln\rho + \frac{\partial T}{\partial\tau} - \frac{T}{M^2}\frac{\partial M}{\partial\tau}\right) \tag{1}$$

Where p is the pressure, T the temperature, ρ the density, M the mean molecular weight, τ the local time, θ the colatitude and R the universal gas constant. We shall neglect changes of the mean molecular weight with local time. Then the expression (1) reduces to

$$f_d = -\frac{R}{\sin\theta \; Mr\omega}\left(T\frac{\partial}{\partial\tau}\ln\rho + \frac{\partial T}{\partial\tau}\right) \tag{2}$$

We are interested in the diurnal mean (denoted by $<>$) of the driving force. To this mean the term $\partial T/\partial\tau$ does not contribute, therefore

$$<f_d> = -\frac{R}{\sin\theta \; M\omega r} < T\frac{\partial}{\partial\tau}\ln\rho > \tag{3}$$

Writing T and log ρ in Fourier components

$$T = \sum_{k=0}^{m} a_k \cos \omega k (\tau - p_k^{(T)})$$

$$\ln \rho = \Sigma b_k \cos \omega k (\tau - p_k^{(\rho)}) \qquad (4)$$

where $p_k^{(T)}$ and $p_k^{(\rho)}$ are the phases of the Fourier components of the temperature and log density. We obtain for the diurnal mean,

$$\langle f_d \rangle = \frac{R}{\sin\theta M r \omega} \sum_{k=0}^{m} \frac{a_k b_k}{2} k \omega \sin \omega k (p_k^{(T)} - p_k^{(\rho)}) \qquad (5)$$

Generally only the term with k = 0 has been considered and no mean diurnal zonal driving forces results. We shall estimate the contribution of higher Fourier terms by using the observationally verified phase difference of about 1.5 hours between density and temperature in the thermosphere. For this simple estimate we shall assume that the phase difference of 1.5 hours between temperature and density arises only from the first, or diurnal, Fourier components.

$$\langle f_d \rangle = \frac{R}{\sin\theta M r} a_1 b_1 \sin (22.5°) \qquad (6)$$

Substituting the numerical values for the diurnal amplitudes of temperature and $\ln \rho$ found in the thermosphere, we obtain for the mean driving force at 300 km

$$f_d/\omega = 14.5 \text{ m/sec}$$

We have calculated the value of the mean zonal driving force at a height of 300 km accurately from the Jacchia model that was modified by assuming that the temperature peaks about 1.5 hours later than the densities. This result is shown in Figure 2. It is seen that for this height the mean diurnal zonal force divided by ω, the earth's angular velocity, is at the equator about 7 m/sec. At a height of 500 km this force has increased by a factor of approximately 3. While this force induces some mean eastward velocity, it alone is insufficient to explain King-Hele's observations.

In the lower thermosphere we have no data on the phase difference between density and temperature although a much larger phase difference than 1.5 hours is probable. The maximum eastward driving force would result if the temperature peaked 6 hours after the density. This would increase the mean diurnal zonal force by a factor of 3 as compared with the forces used in our computation. As the drag forces in the lower thermosphere are small, the increase of the superrotation factor due to the possible increases of the driving forces below 250 km may be considerable. As no data on the phase difference below 250 km are available, we have also used for the lower height region a 1.5 hour phase difference.

THE EFFECT OF DAY-TO-NIGHT VARIATION OF ION DENSITIES ON THE MEAN ZONAL VELOCITY

The particularly simple case of equinox conditions at the equator lends itself to simple estimates of the possible influence of the ion density variation on the mean zonal velocities. The equation of horizontal zonal motion, $V^{(\varphi)}$, is

$$\frac{\partial V^{(\varphi)}}{\partial t} + 2\omega \cos\theta \, V^{(\theta)} - \eta \frac{\partial^2 V^{(\varphi)}}{\partial z^2} + d_{ion} \, V^{(\varphi)} = f_d \qquad (7)$$

where θ is the colatitude, η the kinematic viscosity, ω the velocity of the earth, d_{ion} the ion drag coefficient, and f_d the zonal driving force (Blum and Harris, 1973).

In the height range where ion drag dominates over viscous drag, the equation for the diurnal mean motion at the equator becomes

$$< d_{ion} \, V^{(\varphi)} > \; = \; < f_d > \qquad (8)$$

or explicitly in terms of Fourier coefficients up to the diurnal terms

$$V^{(\varphi)} = \frac{f_o}{d_o^{(\varphi)}} - \frac{d_1 V_1^{(\varphi)}}{2 \, d_o} \cos\left(p_1^{(d)} - p_1^{(V)}\right) \qquad (9)$$

where d_o is the mean diurnal ion drag, d_1, V_1 the diurnal amplitudes of ion drag and zonal velocity respectively, $p_1^{(d)}$ and $p_1^{(v)}$ the phases of the diurnal components of ion drag and zonal velocities respectively.

Based on the results of Blum and Harris (1973) for the numerical values of the ion drag coefficient and the zonal diurnal motion, we may estimate the resulting mean diurnal zonal motion. We shall use the following numerical values in Table 2 as representative. We obtain the tabulated results for the mean zonal motion. Obviously the values of Λ are too low for an explanation of the observations.

Table 2

	220 km	340 km
d_o	1.4	9.6
d_1	1.4	2.27
$\cos(p_1^{(\varphi)} - p_1^{(v)})$	~ -0.5	~ -0.5
f_o	2.2 m/sec	10.1 m/sec
$V_1^{(\varphi)}$	115 m/sec	56 m/sec
$V_o^{(\varphi)}$	30.3 m/sec	4.4 m/sec
Λ	1.07	1.01

Rishbeth (1971) has suggested that due to polarization effects in the F region the ion drag is reduced during the day by a factor R_{day} of about 0.8 and by night by a factor R_{ni} of 0.2. While Rishbeth's suggestion is not completely verified or theoretically founded, (Volland, 1971), we may easily estimate its effect on the mean diurnal motion for the above simple case. We shall have to assume that even with Rishbeth's modification the dominance of ion drag over viscous drag is maintained. This assumption may yield an overestimate of the effect.

Denoting the ion drag coefficient change by R_{day} for the daytime values and R_{ni} for the night values, it is seen that Rishbeth's modification amounts to revised ion drag coefficient (d_o^*, d_i^*) as follows:

$$d_o^* = \frac{1}{2} d_o (R_{day} + R_{ni}) + \frac{1}{\pi} d_1 (R_{day} - R_{ni})$$

$$d_1^* = \frac{2}{\pi} d_o (R_{day} - R_{ni}) + \frac{1}{2} d_1 (R_{day} + R_{ni}) \tag{10}$$

with the phases of d_1^* and d_1 equal.

We shall assume that the modified diurnal amplitudes of the zonal velocity vary inversely with the mean ion drag. Thus, we obtain for the ratio of the modified mean zonal velocity $V_o^{(R)}$ to the unmodified mean zonal velocity V_o the expression

$$\frac{V_o^{(R)}}{V_o} = \frac{2}{R_{day} + R_{ni}} \left[\frac{1}{1 + \frac{d_1}{d_o} \cdot \frac{2}{\pi} \frac{R_{day} - R_{ni}}{R_{day} + R_{ni}}} \right]^2 \left[1 + \frac{4}{\pi} \frac{d_o}{d_1} \frac{R_{day} - R_{ni}}{R_{day} + R_{ni}} \right]$$

It is seen that the ratio of modified to unmodified mean zonal velocities at the equator is not very sensitive to the particular values of R_{day} and R_{ni} but only to their difference. This difference has, according to Rishbeth's suggestion, a value of 0.6. This yields the estimate for the ratio of the revised mean zonal velocity to the unmodified mean zonal velocity (according to equation (11)) between 4 and 5. Thus, from this simple estimate, we obtain mean zonal velocities that are in the range of 20 to 30 m/sec in the isothermal region. Rishbeth's modification is not directly applicable to the lower thermosphere.

Rishbeth, who has also calculated the effects of his modification on the mean zonal velocities, obtained a factor of 25 instead of our value of 4 to 5. Thus, we cannot confirm his estimates. We shall present later the results of an accurate calculation of the mean zonal velocities with Rishbeth's modification. This exact calculation yields a ratio of the modified to the unmodified cases of about 6. As our unmodified velocities are higher than the previous estimated values of

4.4 m/sec (they are 10 m/sec), we obtain mean zonal velocities at the equator of about 55 m/sec as shown in our Figure 3.

DEFINITION OF THE SUPERROTATION RATIO

From a given distribution of the global zonal velocities we may determine the superrotation ratio Λ that is observed from the change of the inclination, i, of the satellite as analysed by King-Hele.

As the zonal force acting on the satellite is proportional to $\rho V^{(\varphi)}$, we have to calculate as a first step the weighted mean zonal motion $\overline{V}^{\varphi}$ defined by

$$\overline{V}^{(\varphi)}(\theta,Z) = \int V^{(\varphi)}(t,\theta,Z)\,\rho(t,\theta,Z)\,dt$$

where the integration is over one day.

The weighted mean motion is generally somewhat less than the mean diurnal velocity $V_o^{(\varphi)}$. In the following we shall use $V_o^{(\varphi)}$ instead of $\overline{V}^{(\varphi)}$ as the difference is not very considerable.

The observed change of the satellites inclination is accumulated at all latitudes during the satellites motion. In line with King-Hele's definition of the superrotation ratio Λ we calculate

$$\Lambda = \frac{1}{V_{ea}} \cdot \frac{\int V_o^{(\varphi)}(\theta) \cdot \sqrt{1 - \frac{\cos^2\theta}{\cos^2 i}}\, d\theta}{\int \sin\theta \sqrt{1 - \frac{\cos^2\theta}{\cos^2 i}}\, d\theta}$$

where v_{ea} is the earth's velocity at the equator, θ the colatitude, i the inclination of the satellite's orbit (identical with the maximum latitude reached by the satellite).

The integration is extended over all latitudes from -i to +i. The integral in the demoninator may be evaluated in a closed analytic form.

DISCUSSION OF RESULTS

The method of integration of the horizontal equations of motion of the thermospheric wind field has been described by Blum and Harris (1973). This method includes all the non-linear terms of the equations. The equations were integrated with Rishbeth's suggested modification of the ion drag coefficient due to F region polarization fields. The modification that was originally only made for the equator was generalized to all latitudes. The generalization assures continuity everywhere and the ion drag at the poles remaining unmodified.

Figures 3 and 4 show the results of the computations of the mean zonal velocities for equinox and solstice conditions respectively for a height of 300 km for Rishbeth's suggested values of the reduction of the ion drag by a factor of 0.8 (R_{day}) by day and a factor of 0.2 (R_{ni}) by night. Shown also are the non-linear solutions for the unmodified ion drag. No convergent non-linear solution was obtained for solstice conditions with Rishbeth's suggested modification; so only the linear solution is given. It is seen that the mean zonal velocities decrease faster than the earth's rotational velocity with increasing latitudes. An integration over latitudes, as described above, has been performed in order to obtain the superrotation ratio as a function of the inclination of satellite orbits. Thus, although relatively large mean zonal velocities are obtained at the equator (about 112 m/sec at solstice and 57 m/sec at equinox) with Rishbeth's modification, the effective values of the superrotation ratio so deduced are much smaller than the mean equatorial velocities except for very low inclination satellites.

The superrotation ratios are illustrated in Figures 5 through 8 as a function of the maximum latitude that is reached by the satellite, i.e., its inclination. It is seen that even Rishbeth's modification results in a decrease of the superrotation ratio with increasing latitudes. Around a narrow band of latitudes near the equator the superrotation ratio at 340 km is 1.13 for both solstice and equinox conditions. For higher inclination satellites it decreases to 1.09. For solstice conditions at 240 km a value of $\Lambda = 1.07$ is obtained for the equatorial zones; it decreases for higher latitudes to 1.05. At equinox at a height of 240 km Rishbeth's modification yields $\Lambda = 1.13$ for the equator, and it decreases to 1.05 as the latitude increases. The rapid decrease with latitude for the non-linear solution is partly due to the increase of the mean equatorward meridional wind (Blum and Harris, 1973) that causes through the Coriolis term a westward driving force.

Radial and meridional electric fields could cause an ion motion in the zonal direction, thus effectively changing the ion drag. They may even cause the neutrals to be dragged by the ions; i.e., the ions could have a larger velocity than the neutrals. If such fields are included in the equations of motion, they would be independent of the neutral velocity and could be included in the driving force on the right hand side of the equations of motion. Arbitrary assumptions of the direction, the magnitude and the time dependence of these fields could explain any mean zonal wind field. It seems that no evidence exists regarding the existance of electric fields that could explain the high values of the superrotation ratio found deduced by King-Hele for the heights near 370 km. In fact, Harper (1971) has deduced meridional electric fields at Aricebo that would cause a westward force acting on the thermosphere.

CONCLUSIONS

1. The superrotational velocity of the thermosphere below 270 km that has been determined by King-Hele from satellite drag analysis yields a ratio of about 1.15 to the earth's rotational velocity. The height dependence below 270 km suggested by King-Hele can neither be confirmed nor negated by statistical tests of significance.

2. Between 270 and 370 km there also exists an eastward rotation of the thermosphere. Satellite drag analysis seems to indicate a superrotation ratio of 1.25 with a maximum of 1.4 at 370 km. Statistically no definite

determination of the superrotation ratio—and even more so its height dependence—can be made as not enough data are available.

3. Above 370 km only three satellites have been analysed. The results were interpreted by King-Hele as showing a decrease of the superrotation ratio with height to a ratio less than unity at 500 km. It is suggested that the deduced decrease may be a latitudinal effect rather than a height effect.

4. A theoretical treatment involving the integration of the equations of motion for the thermosphere yields the following results:

   a. Below 270 km superrotation ratios that are compatible with the observations may be deduced.

   b. Without extremely hypothetical assumptions like strong radial electric fields no superrotation ratios in excess of 1.17 can be calculated. This makes it impossible to explain theoretically King-Hele's interpretation of the observations between 270 and 370 km.

   c. A theoretical treatment will not result in a decrease with height of the superrotation ratio above 370 km unless the theoretical assumptions regarding the transition region at the exobase are abandoned. This absence of a height dependence of the superrotation ratio above 370 km is not in conflict with the calculated mean driving forces.

These are due to the phase difference between density and temperature in the thermosphere and contribute to the observed superrotation.

d. The polarization effects in the F region suggested by Rishbeth increase the superrotation ratio by a factor 3 to 6, depending upon latitude and altitude. This is significantly less than Rishbeth's determination of the factor of 25 at the equator. This factor is about 6 at the equator but only about 3 for satellites having high inclinations.

e. In the height region between 340 and 500 km there results an insignificant height dependence of the superrotation ratio.

f. Generally the superrotation ratio is larger at solstice than at equinox. Using Rishbeth's suggested values of $R_{day} = 0.8$ and $R_{ni} = 0.2$ there results at solstice a superrotation ratio of 1.12 at 300 km. Above 400 km this ratio is increased to 1.15 while in the lower height region at 240 km it is about 1.09.

g. The effects of the non-linear terms of the equations of motion reduce the superrotation ratio, thus demonstrating that it is not a non-linear effect.

The extension of the modification of the ion drag coefficient as suggested by Rishbeth to mid and high latitudes does not yield a superrotation ratio in excess of 1.15. We can theoretically obtain superrotation ratios between 1.1 and 1.15 by taking into account the mean eastward driving forces, the modification of the ion drag due to polarization fields in the F region and the motions at solstice conditions (which generally have higher mean eastward velocities than at equinox). Higher superrotation ratios cannot be explained without additional assumptions like radial or meridional electric fields having a particular time and space distribution. Such fields are hypothetical.

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FIGURE CAPTIONS

Figure 1. King-Hele's results of the height dependence of the superrotation ratio based on 25 satellites. The fully drawn lines show the results of an assumed constant superrotation ratio separately for each of the three height regions defined in the test. The statistical reliability of such distribution of the superrotation ratios is nearly equal to that of King-Hele's suggested height dependence. The analysis gives to each observation a weight according to the observational error given by King-Hele.

Figure 2. The mean azimuthal force for summer solstice conditions as a function of latitude, for the Jacchia model with a phase difference of 1.5 hours between temperature and density, at the altitude of 300 km.

Figure 3. The mean zonal velocity as a function of latitude at the height of 300 km for equinox conditions. (____) is the linear solution with Rishbeth's modification reducing the ion drag by day by the factor $R_{day} = 0.8$, and by night with a factor $R_{ni} = 0.2$. (----) is the non-linear solution with Rishbeth's modification, and (......) is the non-linear solution without Rishbeth's modification.

Figure 4. Same as Figure 3 but for solstice conditions and no non-linear solution for Rishbeth's modification is given.

Figure 5. Superrotation ratio as a function of inclination of a satellite's orbit for equinox conditions at the height of 240 km. (______) is the result for linear solution with Rishbeth's modification, (— • — • — •) the non-linear solution with Rishbeth's modification, (— — —) is the linear solution without Rishbeth's modification, (. . . .) is the non-linear solution without Rishbeth's modification.

Figure 6. Same as Figure 5 but for a height of 340 km.

Figure 7. Same as Figure 5 but for solstice conditions.

Figure 8. Same as Figure 6 but for solstice conditions.

OBSERVED SUPERROTATION RATIOS FROM KING—HELE WITH AVERAGE VALUES FOR THREE GROUPINGS

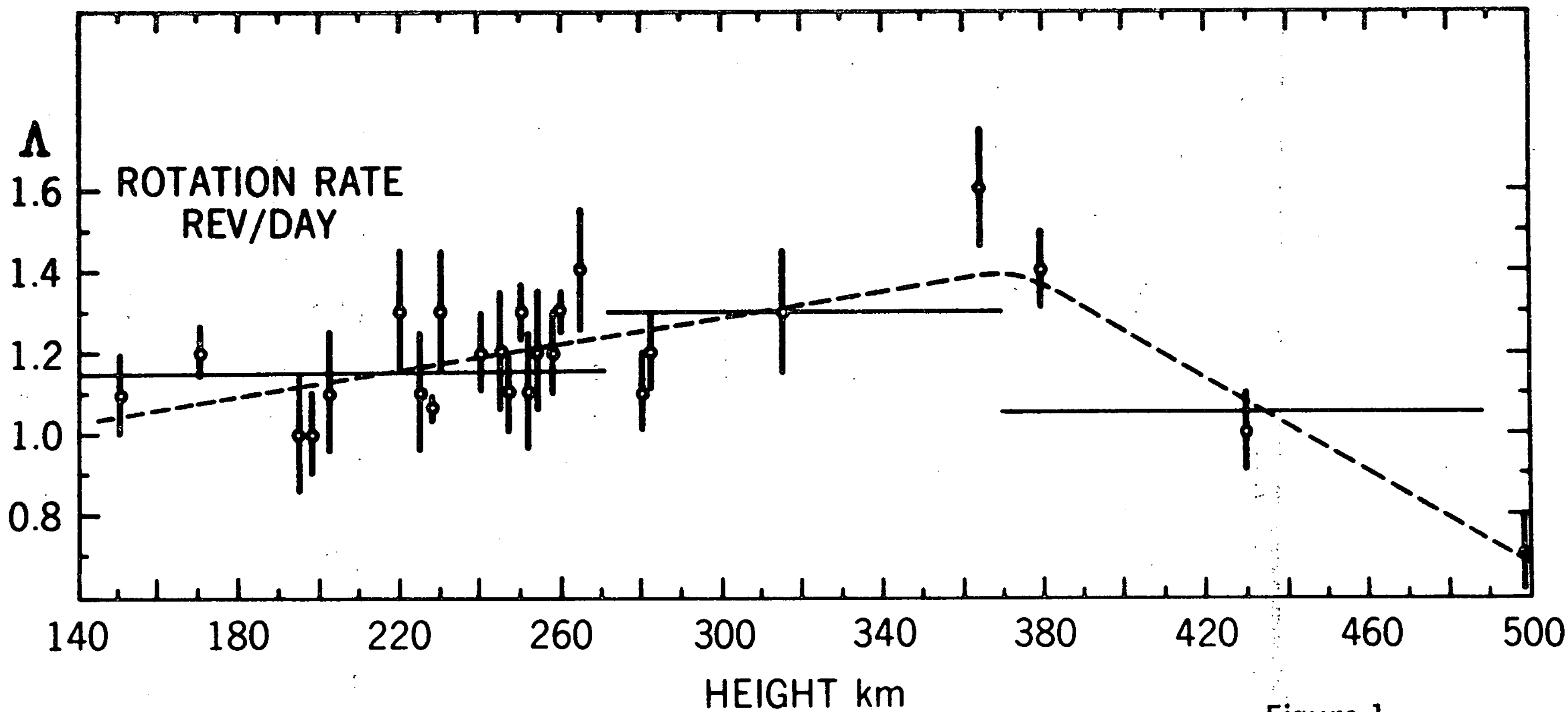

Figure 1

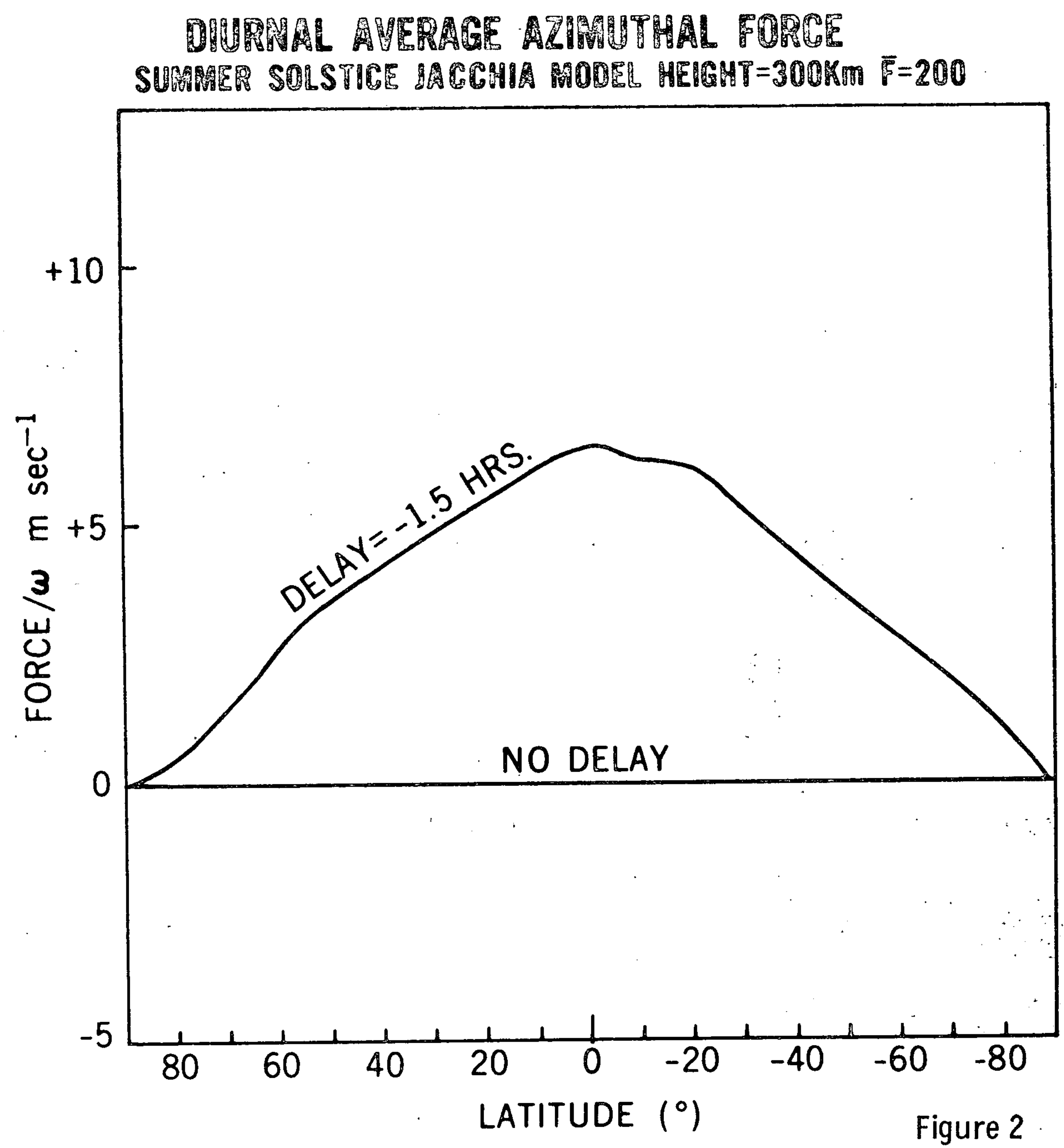
DIURNAL AVERAGE AZIMUTHAL FORCE
SUMMER SOLSTICE JACCHIA MODEL HEIGHT=300Km F̄=200
+10
+5
0
-5
FORCE/ω m sec⁻¹
DELAY= -1.5 HRS.
NO DELAY
80
60
40
20
0
-20
-40
-60
-80
LATITUDE (°)

Figure 2

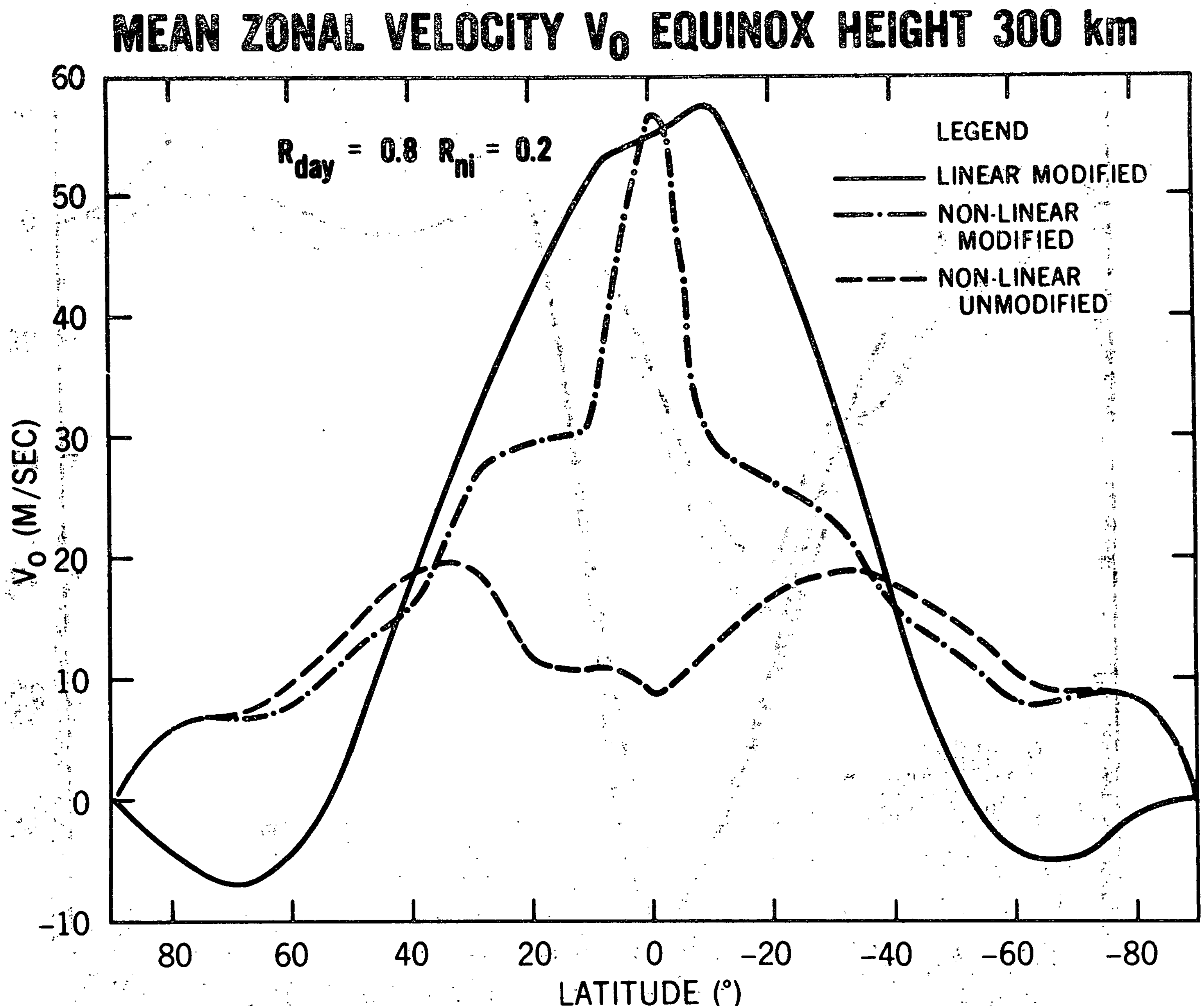
MEAN ZONAL VELOCITY V_0 EQUINOX HEIGHT 300 km
R_{day} = 0.8 R_{ni} = 0.2
LEGEND
LINEAR MODIFIED
NON-LINEAR MODIFIED
NON-LINEAR UNMODIFIED
60
50
40
30
20
10
0
-10
V_o (M/SEC)
80
60
40
20
0
-20
-40
-60
-80
LATITUDE (°)

Figure 3

MEAN ZONAL VELOCITIES SOLSTICE HEIGHT 300 km

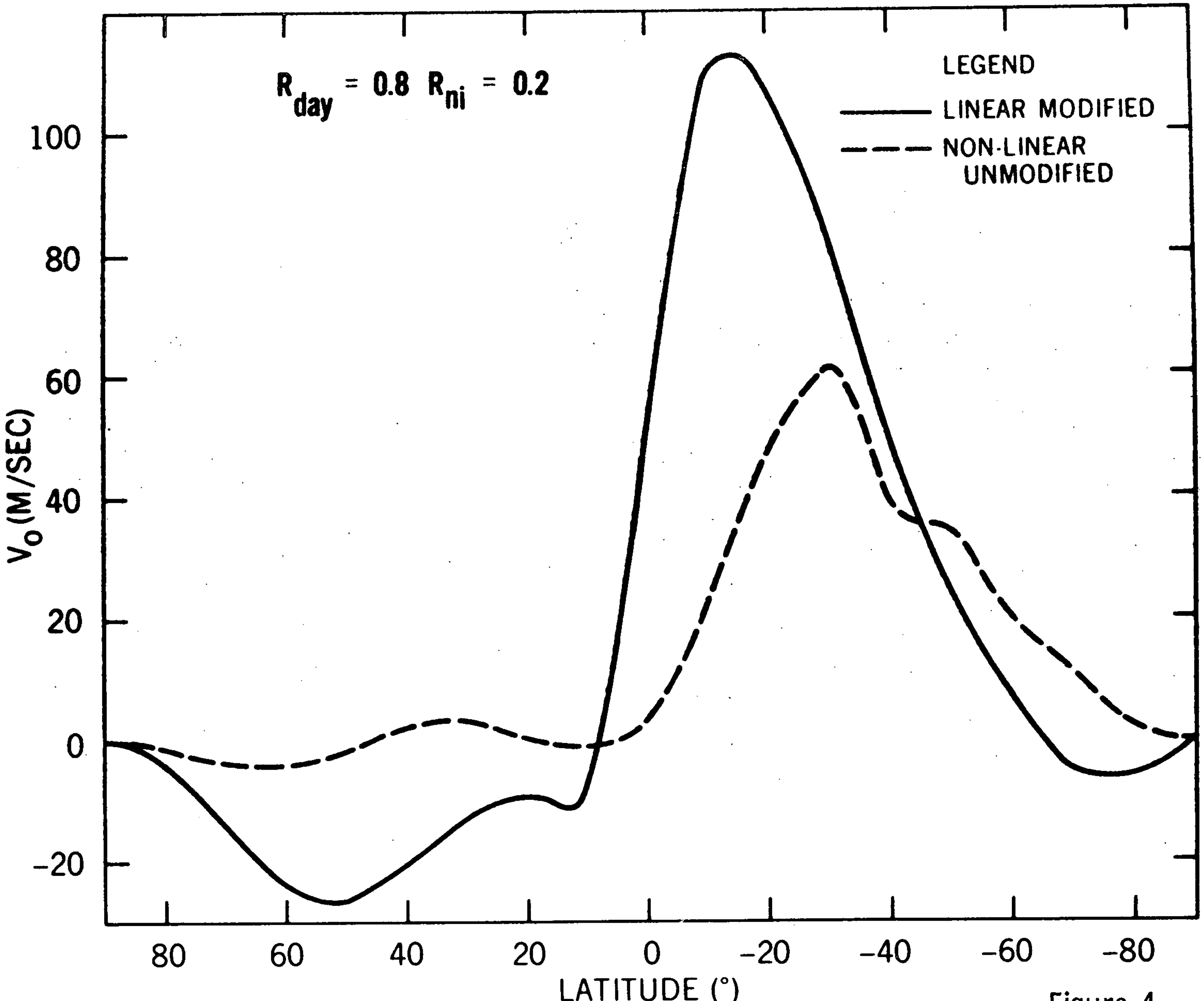

Figure 4

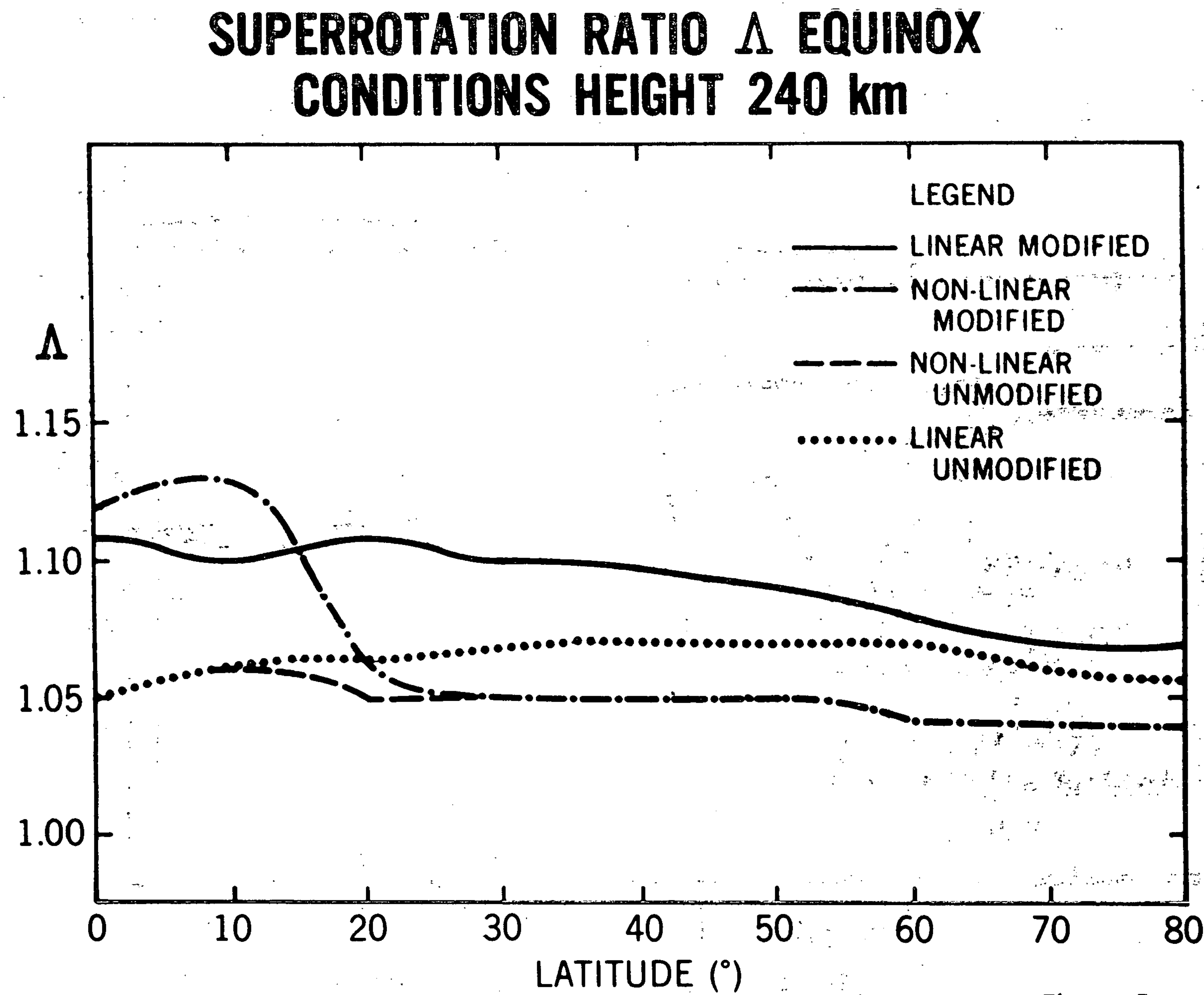
SUPERROTATION RATIO Λ EQUINOX
CONDITIONS HEIGHT 240 km
LEGEND
LINEAR MODIFIED
NON-LINEAR MODIFIED
NON-LINEAR UNMODIFIED
LINEAR UNMODIFIED
Λ
1.15
1.10
1.05
1.00
0
10
20
30
40
50
60
70
80
LATITUDE (°)

Figure 5

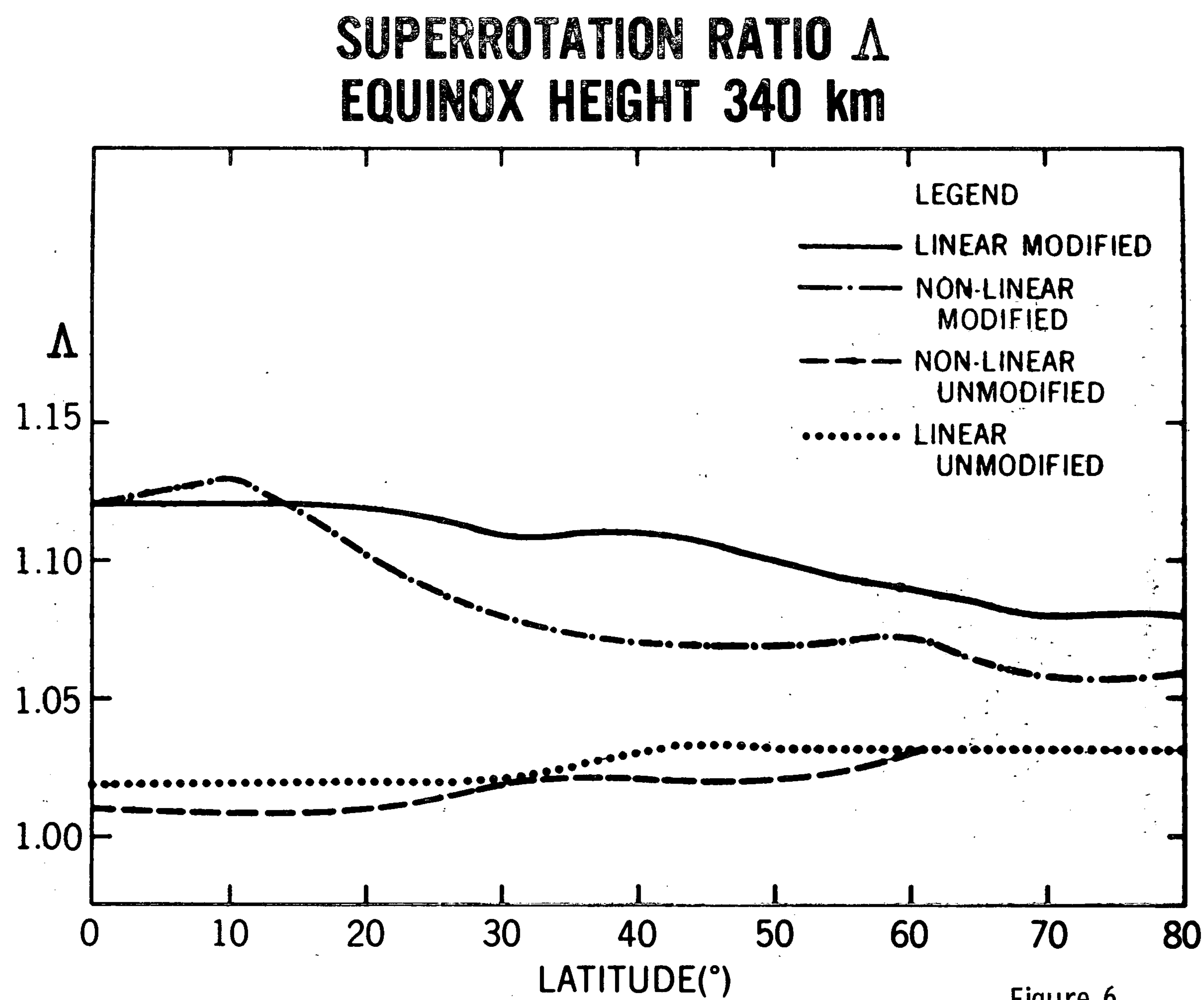
SUPERROTATION RATIO Λ
EQUINOX HEIGHT 340 km
LEGEND
LINEAR MODIFIED
NON-LINEAR MODIFIED
NON-LINEAR UNMODIFIED
LINEAR UNMODIFIED
Λ
1.15
1.10
1.05
1.00
0
10
20
30
40
50
60
70
80
LATITUDE(°)

Figure 6

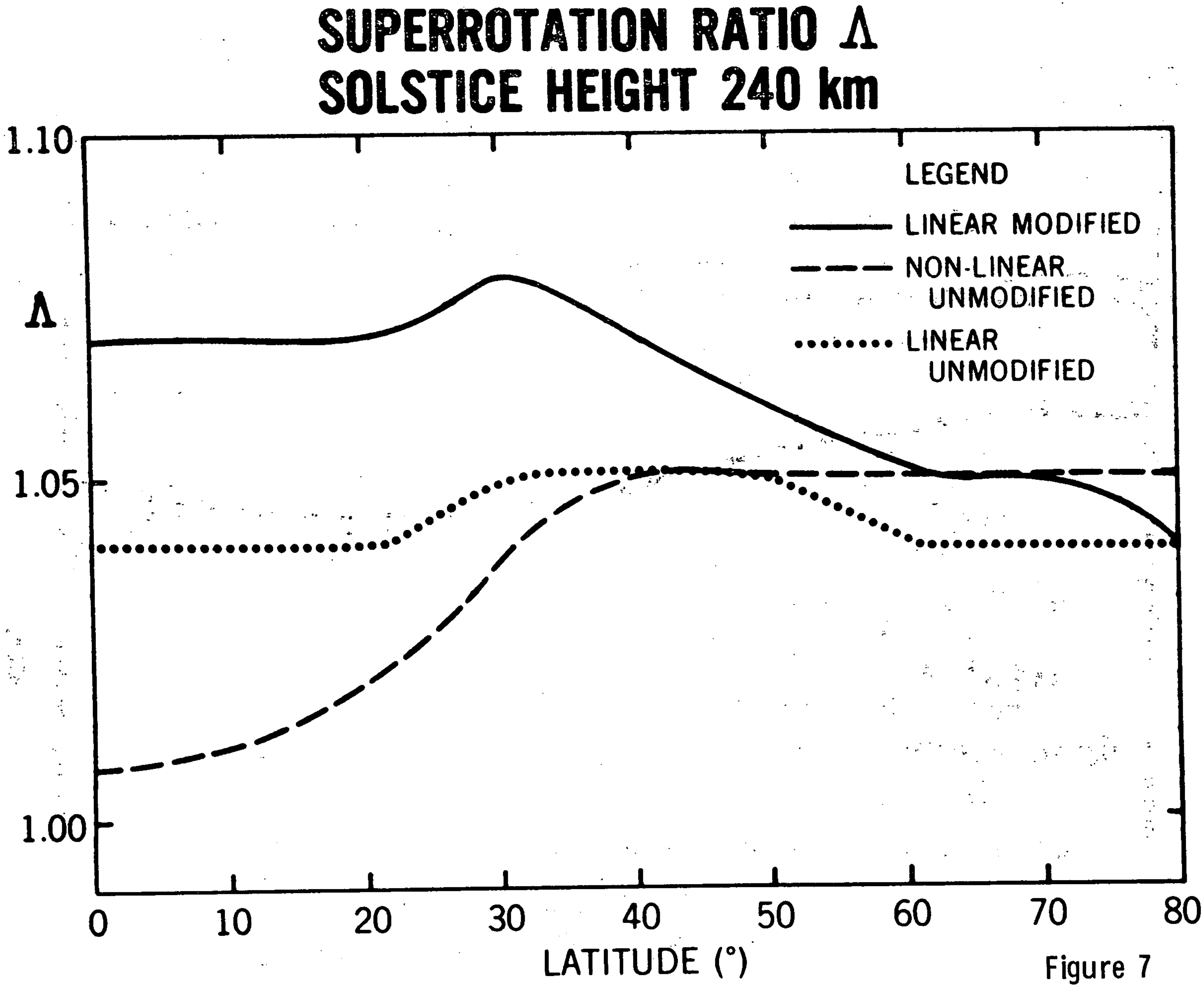
SUPERROTATION RATIO Λ
SOLSTICE HEIGHT 240 km
1.10
Λ
1.05
1.00
LEGEND
LINEAR MODIFIED
NON-LINEAR UNMODIFIED
LINEAR UNMODIFIED
0
10
20
30
40
50
60
70
80
LATITUDE (°)

Figure 7

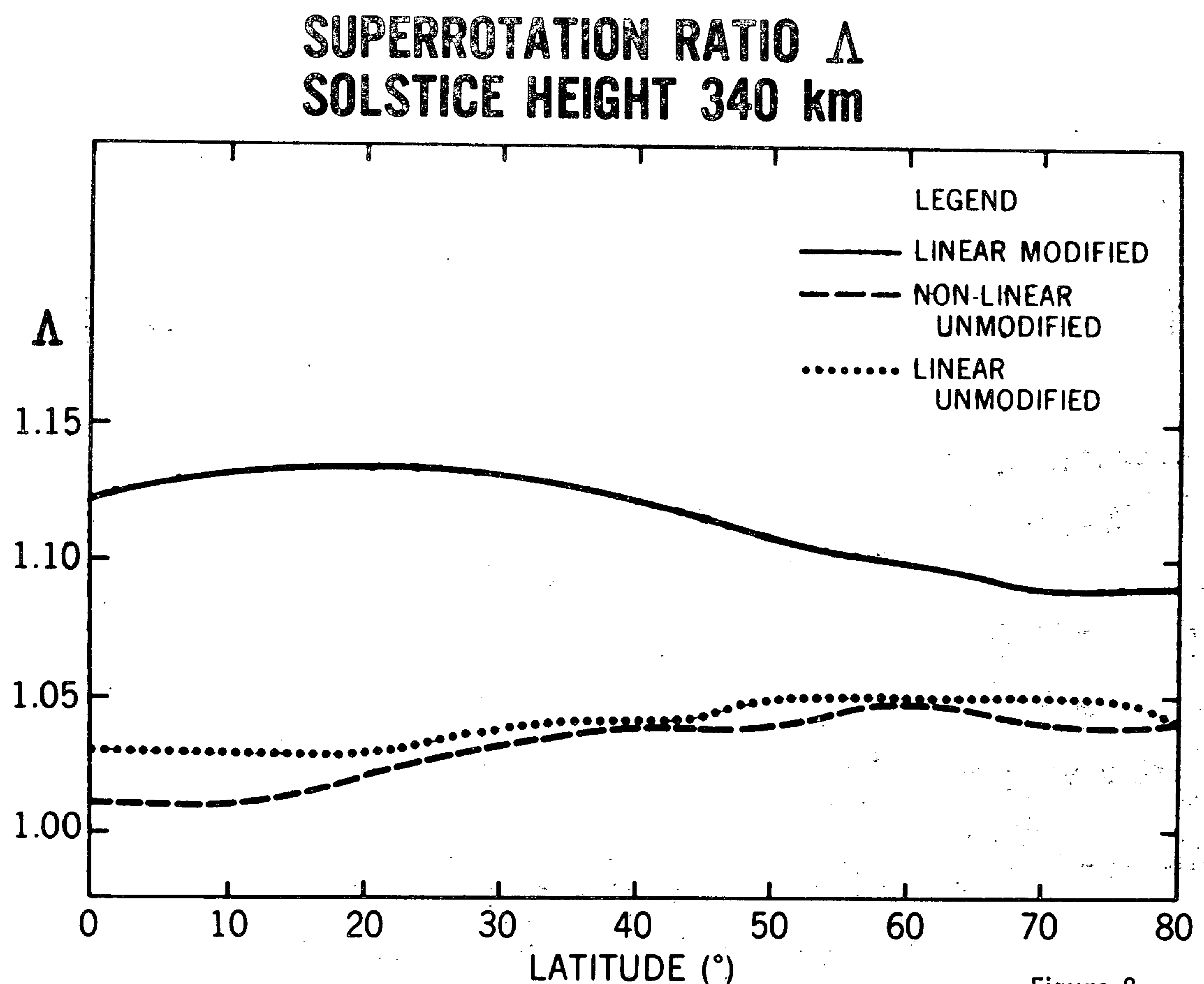

Figure 8

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