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ABSTRACT

A significant requirement for studies in Earth Physics is the resolution of trends from observations made over minimal time spans. The quantification of any surface motions with magnitudes not exceeding a few cm per year calls for the reference of such measurements to co-ordinate systems whose behaviour is unambiguous. The perturbation of these systems must therefore be capable of accurate definition.

A simple system of reference axes is defined for possible use in high precision geodetic studies over long periods of time for programs in Earth physics. The proposed system is based on the gravitational and dynamic characteristics of the axis of rotation and the Earth's center of mass as defined instantaneously at a given epoch. Techniques are outlined for its continuous representation over time intervals of significance for studies in Earth physics. The relationship between the proposed system and the representation of extra-terrestrial objects using the celestial sphere concept is also discussed.

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GEODETIC REFERENCE SYSTEMS FOR LONG PERIOD STUDIES IN EARTH PHYSICS

1. INTRODUCTION

The evolution of geodetic techniques in the past two decades can be described without excessive exaggeration, as being spectacular. The scope of geodetic science whose practical applications prior to the second world war were restricted to regional studies encompassing areas no larger than a single nation, were extended to continental extents in the first instance. The much sought for connections between the various continents were subsequently achieved by the use of artificial Earth satellites. The precision of such determinations of position based on optical techniques appears to be at the level of 1 p.p.m.

The co-ordinates (so established) for geodetic stations at the time of writing are not sufficiently sensitive for quantitative studies in Earth physics over periods of time shorter than 50 years. It is also uncertain whether the stations used are designed for unambiguous re-occupation after long periods of time. The application of geodetic techniques for the determination of changes in position for Earth physics can be summarized as follows.

Given a system of observing stations P_i at the Earth's surface whose relative positions are defined on some co-ordinate system X_j at epoch ($\tau = t$) by the co-ordinates X_{ij} on the basis of measurements made during the epoch; if the relative disposition of the same points were re-determined as χ_{ij} at some later epoch ($\tau = t + dt$) and based on a co-ordinate system χ_j, can the difference

$$\Delta X_{ij} = \chi_{ij} - X_{ij} \tag{1}$$

be interpreted in the context of relevance to the quantification of plate motion and crustal distortions? As estimates of gross motion (i.e., long wave effects) are currently placed at 10 cm yr^{-1} in sensitive areas, the quantities ΔX_{ij} are likely to be o $\{1\,\mathrm{m}\}$ if dt is approximately 1 decade. This is almost an order of magnitude smaller than the estimated precision of the best global geodetic networks available at the present time.

It is commonly held that laser ranging techniques to appropriately equipped spacecraft in near Earth orbit, and to the moon should, in the first instance, afford ranges with a precision in excess of ±10 cm. A major geodetic task in the foreseeable future is the development of techniques for the efficacious use of the data obtained from such systems for the definition of changes in position, and in a manner which lends itself to use in Earth physics with the minimum ambiguity.

The definition of the station sites on ground in order that unambiguous re-occupation is possible at the 1 - 2 cm level after an extended period of time, poses a major problem in addition to the one outlined earlier. This question falls outside the scope of the present development, and it will be assumed that any errors on this account are less than 1 cm. A secondary problem is the choice of observing station sites so that changes in its co-ordinates bear a maximum correlation to the motion characteristics of the crustal region it is supposed to represent. It would, for example, be undesirable if the changes ΔX_{ij} at one of the stations in a global network but with a limited number of ground sites, were to be dominated by local effects with short wavelengths rather than those characteristics of the appropriate part of the Earth's surface it purports to sample.

The changes ΔX_{ij} may still reflect aspects other than those of relative motion even if the stability of station locations over long time spans has been clearly established. One major problem in four dimensional geodesy is the establishment of the relationship between the systems of reference used for each of the epochs of measurement. The ensuing development shows how it is possible to use a system which is based on certain instantaneous gravitational and dynamic characteristics of the Earth, as summarized in Appendix 8.

The Earth is not a rigid body and the possibility of the constant re-distributions of mass should be taken into consideration, along with the associated dynamic consequences, when designing a system of reference for long period use. One of the tasks of four dimensional geodesy is to provide an unambiguous representation of the perturbation of co-ordinate systems with time. To this end it is necessary to define a space for which the former provide a frame of reference.

The system of reference described in the following sections refer to Euclidian space and can be related to existing concepts with minor modifications. It also has the advantage of being directly related to the Liouville equation (e.g., Munk & MacDonald 1960, p. 10).

2. EARTH SPACE

All geodetic determinations to the present day have dealt with the definition of position on a rigid Earth. Any motion the Earth may have with respect to an inertial reference system in galactic space has therefore been considered to be effectively reversed. Earth space is defined as the Euclidian space which has the same rotational and galactic motion as the Earth. All geodetic co-ordinates established to date are Earth space co-ordinates. The effect of galactic motion is of relevance only when observations to extra-terrestrial objects are involved. The resulting consequences are listed by Mather (1972, p. 4). The various aspects of galactic motion are only of indirect concern in current geodetic practice, being absorbed as proper motion effects resulting in changes in the co-ordinates of the extra-terrestrial sources on the celestial sphere.

Any geodetic determinations based on observations restricted to those between points on the Earth's surface, will result in the definition of co-ordinates in Earth space. Observations to extra-terrestrial sources have to be corrected for the effect of the Earth's rotation before use for determinations in Earth space, unless special synchronous observing procedures have been resorted to. It is not difficult to show (e.g., Ibid, p. 19) that if the Earth were a rigid body with every mass element rotating freely about a common axis of rotation, with angular velocity ω, the axis should pass through the center of mass of the Earth (geocenter). Deviations from this concept occur due to departures of the Earth from a rigid body. The variation in the rate of rotation is unlikely to exceed 1 part in 10^7.

Another factor which affects the definition of Earth space is the change in position of the axis of rotation with respect to the Earth's crust and called polar motion. The order of magnitude of this effect is 1 part in 10^6. It is therefore convenient to model the Earth by its rigid body equivalent at any epoch in time. The instantaneous geocenter for this model lies on the axis of rotation. All variations are treated as perturbations which could be either periodic or secular. For example, the existence of departures from a rigid body model manifests itself as a free nutation in the motion of the pole with respect to the Earth's crust, with a lengthening of the period over the Eulerian equivalent (e.g., Routh 1905, p. 358).

The concept of Earth space is a simple one. All the observational data available at the present time appears to indicate that the modeling of the Earth by its rigid body equivalent on lines similar to the use of a Keplerian ellipse in satellite geodesy, is an acceptable one. Deviations from this model can be treated as perturbations though further significant improvements in metrology may result in a need for some multiparameter representation as being more utilitarian than the simple concept based on the rotation axis and geocenter.

It could therefore be concluded that no ambiguity arises in defining Earth space as set out in Appendix 1 if the rotation characteristics specified are those of an instantaneous rigid body model of the Earth.

Problems arise in the formulation of a suitable system of reference for the definition of position in Earth space for long period studies in Earth physics. A dominant concern is the extraction of meaningful information about plate motion and crustal distortion from equations of the type at 1 over the shortest possible intervals of time. The expense involved also restricts the number of locations from which observations of the highest precision will be made. It would be considered over-optimistic to put this number at even 100 by the turn of the century on the basis of progress being made at the present time in this direction. The question of a suitable system of reference must therefore be assessed in the context of the above factors.

Geodetic position could be referred to an Earth space co-ordinate system defined by

<u>either</u> a system of "fixed" locations at the surface of the Earth;

<u>or</u> certain gravitational and dynamic characteristics of the Earth.

The former system will require a minimum of three or possibly four stations held fixed as the source of the frame of reference. This is uneconomic as the fixed stations will thereafter make no contribution to studies in Earth physics, apart from defining the frame of reference. The latter system based on the gravitational and dynamic characteristics of the Earth will be shown to require only 1 station to be held fixed at the surface of the Earth to obtain a complete and continuous definition of position in Earth space over long periods of time.

3. EXTRA-TERRESTRIAL REFERENCE SYSTEMS AND EARTH SPACE

The commonly used extra-terrestrial reference systems are based on the celestial sphere concept which can be summarized as follows. The angle subtended at the center of the Earth by two objects whose geocentric distance tends to infinity, is unaffected by either the solar orbit of the Earth or the motion of the objects concerned over limited periods of time. These extra-terrestrial sources are therefore treated as fixed in position on a spherical reference system and catalogued by a set of surface co-ordinates (RA, δ). The pole of the co-ordinate system coincides with the projection of the Earth's pole on the celestial sphere at some epoch of reference ($\tau = t_o$).

Observations in classical geodetic practice are oriented towards the establishment of position on a differential basis. Prior to the advent of satellite techniques, position determination was the net result of two distinct operations. The first involved observations as reduced to the local horizon, and called horizontal methods, defining as their end product, latitude and longitude, or their equivalents. The second was based on the measurement of displacements along the local vertical aimed at establishing the elevation of points at the surface of the Earth above some reference model. Horizontal survey methods give <u>differences</u> in latitude and longitude. It has been common practice to convert these differences to "absolute" values on the basis of determinations based on the extra-terrestrial reference frame provided by the celestial sphere.

Figure 1 illustrates the basic relationships involved in transferring astronomical determinations into a geodetic frame of reference. If the latter were based on a three dimensional Cartesian co-ordinate system X_i whose direction cosines were ℓ_{ij}, assuming origin — geocenter coincidence, let $P\gamma$ be the great circle of zero right ascension on the celestial sphere. The diagram referred to illustrates the case when the X_1X_3 plane coincides with that of the great circle $P\gamma$, which is equivalent to the reference meridian sidereal time being zero.

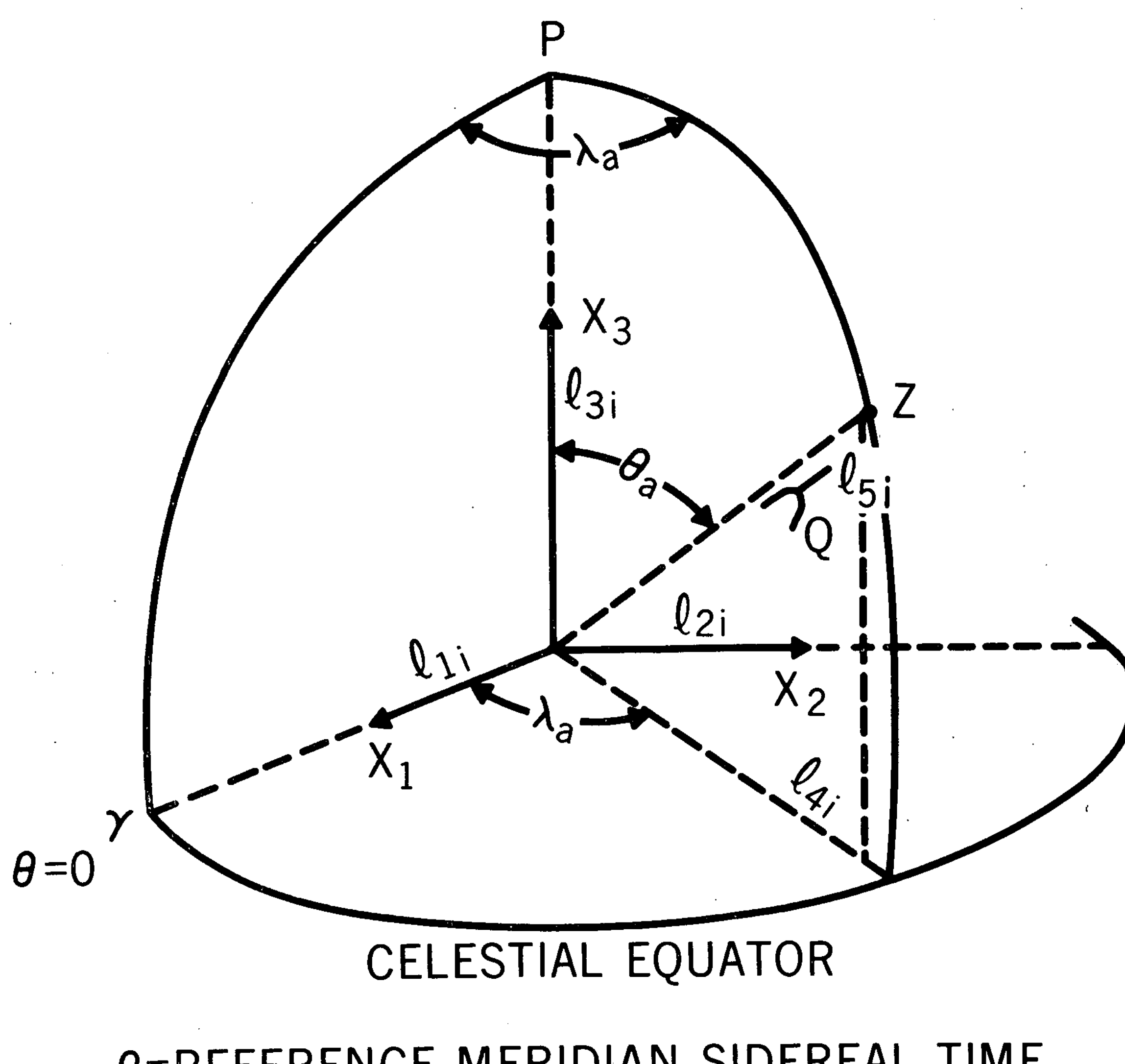


Figure 1. A Geodetic Reference System and the Celestial Sphere — 1. Determination of Co-Latitude θ_a and Longitude λ_a Give 2 Angles Linking 4 Directions; 2. 15 Direction Cosines are Linked by 12 Relations

Astronomical determinations of latitude (ϕ_a) and longitude (λ_a) are obtained from two angles defined from observations on the celestial sphere. The first is the co-latitude $\theta_a (= 1/2\pi - \phi_a)$ between the pole P and the local zenith Z, GZ in Figure 1 having the same direction cosines ℓ_{5i} as the local vertical at the observing station Q. On the basis of experience, it can be stated that θ_a is unlikely to deviate from the co-latitude θ_g on an ellipsoid which best fits the geoid by amounts much in excess of 0.3 mrad, the modal value being an order of magnitude smaller. This would also apply to λ_a which is deduced from observations to extra-terrestrial sources and a knowledge of the rotation characteristics of the Earth which are assumed to be known. The 15 direction cosines defining the five directions satisfy 12 equations given at 2 - 6.

$$\ell_{ij}\,\ell_{ij} = 1, \quad i = 1, 5, \tag{2}$$

$$\ell_{ik}\,\ell_{jk} = 0 \text{ (4 equations)}, \tag{3}$$

$$\ell_{3i}\,\ell_{5i} = \cos\theta_a, \tag{4}$$

$$\ell_{4i}\,\ell_{5i} = \sin\theta_a, \tag{5}$$

and

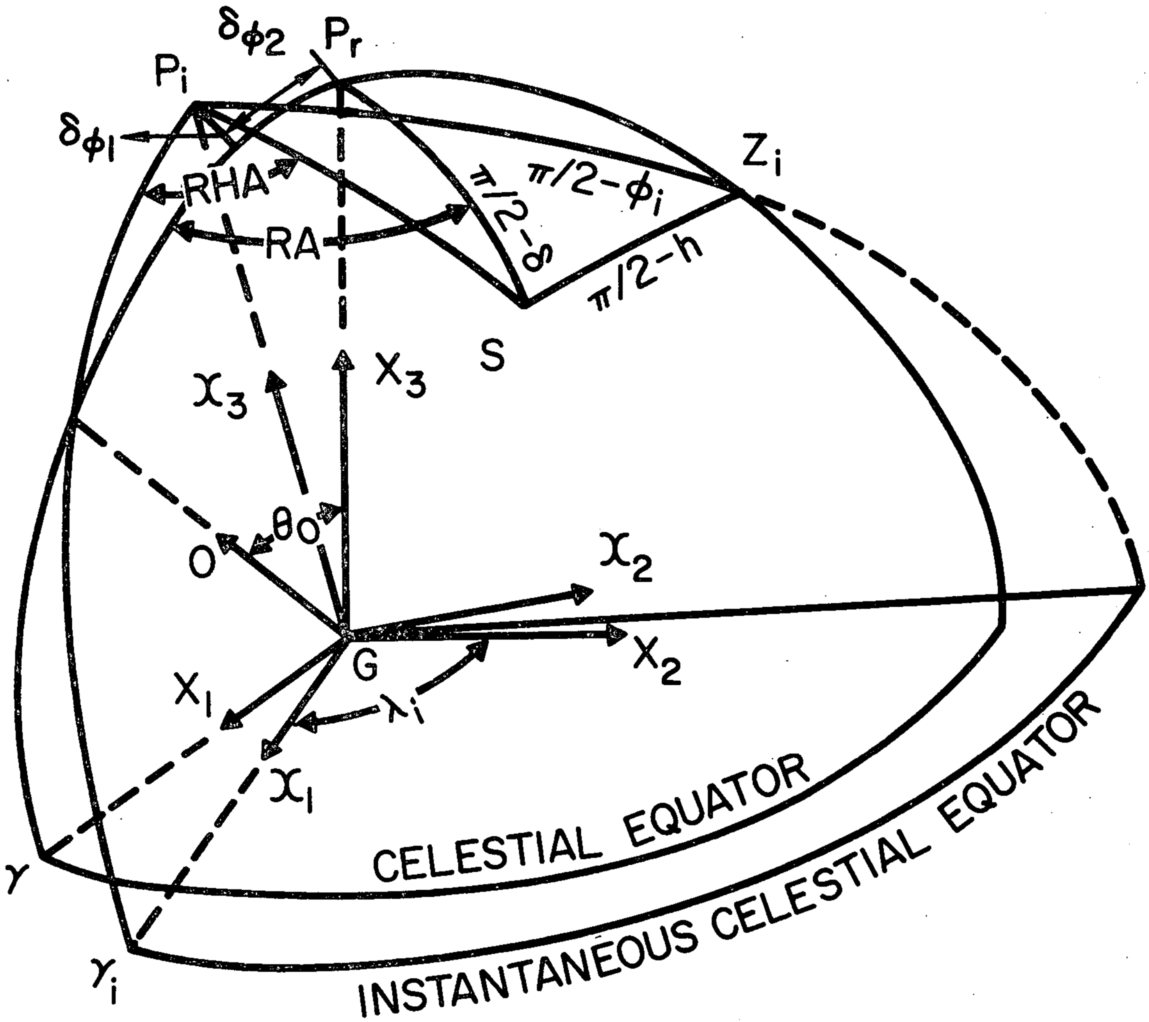
$$\ell_{1i}\,\ell_{4i} = \cos\lambda_a. \tag{6}$$

Noting that the three equations containing ℓ_{2i} can be omitted without effecting the development, the four conclusions summarized in Appendix 3 can be drawn.

The relations illustrated in Figure 1 deviate from reality. Extra-terrestrial observations are referred to the Earth's instantaneous axis of rotation (GP_i in Figure 2). The quantities observed or deduced from observations are the zenith distance SZ_i and the hour angle RHA as shown in Figure 2. The co-ordinates (RA, δ) of the extra-terrestrial source S, on the other hand, are referred to the pole P_r of the celestial sphere. Two problems arise on attempting to correlate the reference system afforded by the celestial sphere to the physical system characterized by the instantaneous axis of rotation.

1. <u>The location of the pole of the reference system in relation to the instantaneous axis of rotation.</u>

   The problem can be viewed as the definition of the vector P_iP_r on the celestial sphere, as illustrated in Figure 3. Consequently the instantaneous polar distance of S increases from $(1/2\pi - \delta)$ to $(1/2\pi - [\delta - \Delta\delta])$. The vector P_iP_r can be determined in principle if the angular distances P_iS_α were known from two points S_α of known position on the celestial sphere under the conditions normally applying to position fixes by intersection.



θ = TIME INDEX ON REFERENCE MERIDIAN

Figure 2. The Celestial Sphere, the Instantaneous Rotation System, the Meridian of Reference and Astronomical Determinations at $\theta = 0$

2. <u>The correlation of the "reference" meridian with the circle of zero right ascension.</u>

   This can only be achieved if the concept of a meridian of reference were clearly defined.

Figure 2 illustrates without unnecessary complication the problems encountered in positional astronomy. It has been assumed that the time index on the reference

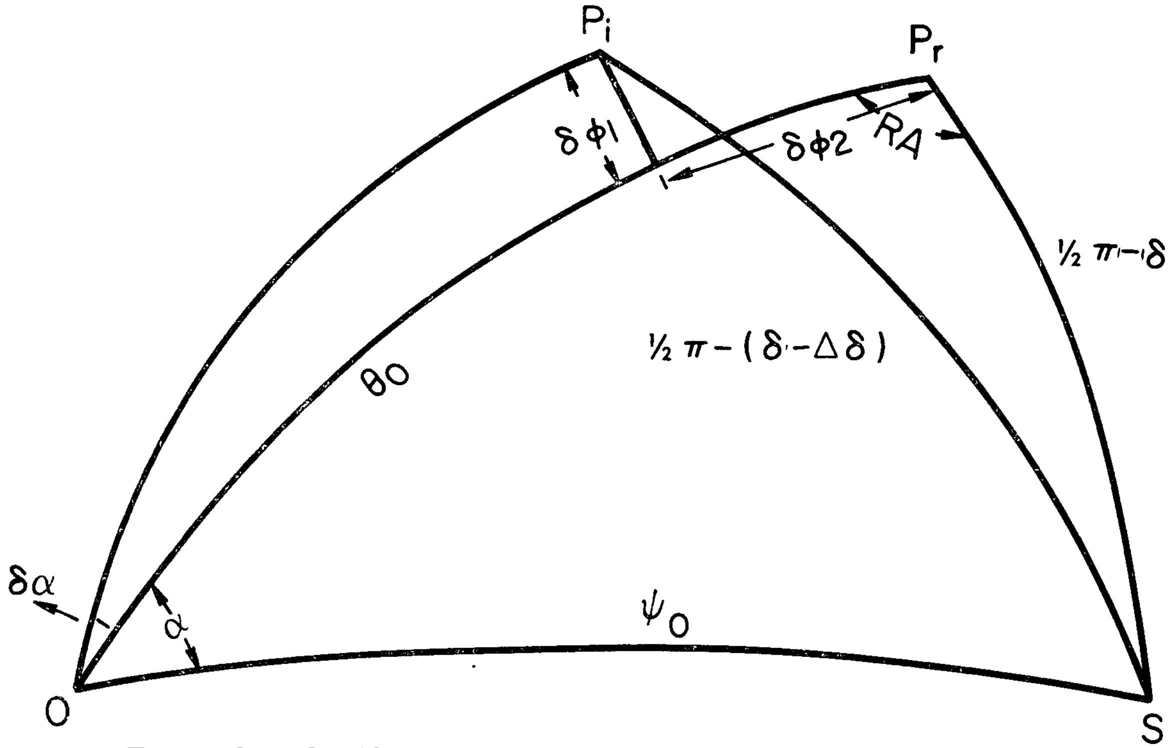


Figure 3. The Change in Declination on the Instantaneous System

meridian (θ) is 0. The extra-terrestrial reference frame provides co-ordinates of sources with respect to a pole P_r which, in the most general case, may not coincide with the instantaneous axis of rotation. The latter, of course, has measurable angular relationships on the celestial sphere to extra-terrestrial objects. The determination of these quantities is the observational basis of positional astronomy. It is conventional for P_r to be the rotation pole of some epoch of reference ($\tau = t_0$).

In the general case, $P_r P_i$ can be fully represented by two mutually perpendicular components $\delta\phi_\alpha$, in relation to the meridian of reference (as illustrated in Figure 2). If the problem were treated as one where it was necessary to define the corrections ΔRA to the right ascension RA and $-\Delta\delta$ to the declination δ of S, consider in the first instance, the question of defining the meridian of reference. If it were defined in terms of the axis of rotation and some point 0 at the Earth's surface, it becomes obvious that the meridian of reference will keep changing position in Earth space as the pole moves in relation to the Earth's crust. On the other hand, the plane so defined can be related to observations.

Alternately, the reference meridian can be defined arbitrarily. The disadvantage of an arbitrary definition is the difficulty in relating the reference system to observations. Such disadvantages could be merely conceptual rather than pragmatic if the arbitrary definition were based on the Earth space location of the rotation axis of an earlier but recent epoch. The former alternative is illustrated in Figure 2 which co-ordinates the great circle of zero right ascension to the plane of the reference meridian defined by the instantaneous axis of rotation and the zenith of O, as intersecting at O. If the co-latitude of O is θ_o with respect to P_r, it is not difficult to show that

$$\Delta RA = -\delta\phi_1 \cot\theta_o . \tag{7}$$

The correction $(-\Delta\delta)$ to the declination is given by

$$\Delta\delta = P_i S - (1/2\pi - \delta). \tag{8}$$

Figure 3 illustrates more clearly the relation between P_i, P_r and S. The evaluation of $\Delta\delta$ is based on the fact that the great circle arc OS (= ψ_o) is invariant between epochs. If P_r were the rotation pole at an earlier epoch, as mentioned earlier, at which the co-latitude of the origin O was determined as θ_o, it follows that

$$P_i O = \theta_o(t) = \theta_o(t_o) - \delta\phi_2 = \theta_o - \delta\phi_2 . \tag{9}$$

Consideration of the spherical triangle OP_rS gives

$$\psi_o = \cos^{-1}[\cos\theta_o \sin\delta + \sin\theta_o \cos\delta \cos RA]. \tag{10}$$

ψ_o is therefore a known quantity for any extra-terrestrial source represented on the celestial sphere, if the co-latitude of O were known at the epoch for which P_r was the rotation pole. Further, the angle SOP_r (= α), given by

$$\alpha = \sin^{-1}\left[\frac{\sin RA}{\sin\psi_o}\cos\delta\right] \tag{11}$$

is also a known quantity. The azimuthal angle α between the instantaneous reference meridian and the plane containing O and S changes between the epochs $(\tau = t_o)$ and $(\tau = t)$ by the angle $\delta\alpha$ given by

$$\delta\alpha = P_r\hat{O}P_i = \delta\phi_1 \operatorname{cosec}(\theta_o - \delta\phi_2) = \delta\phi_1 \operatorname{cosec}\theta_o + o\left\{10^{-10}\right\} \tag{12}$$

if the components of polar motion between the epochs were less than 2 arcsec each. Consideration of spherical triangle P_iOS gives

$$\sin(\delta - \Delta\delta) = \cos(\theta_o - \delta\phi_2)\cos\psi_o + \sin(\theta_o - \delta\phi_2)\sin\psi_o\cos(\alpha + d\alpha). \tag{13}$$

On expanding the above equation by Taylor's theorem, using the assumption that $|\delta\phi_\alpha| \ngtr 2$ arcsec, and noting the equivalent equation from spherical triangle P_rOS for δ, it follows that

$$\Delta\delta \cos\delta = \delta\phi_2 [\cos\theta_0 \sin\psi_0 \cos\alpha - \sin\theta_0 \cos\psi_0] + \delta\alpha \sin\alpha \sin\theta_0 \sin\psi_0 + o\left\{(\delta\phi_\alpha)^2\right\} \tag{14}$$

on use of Equation 13 and a little manipulation.

Equations 9, 11, 12 and 15 complete the transformation of co-ordinates on the (RA, δ) system as defined on the celestial sphere, to equivalent quantities on the instantaneous system, provided the vector P_iP_r were known, the representation being in terms of the orthogonal component $\delta\phi_\alpha$ illustrated in Figure 2. These components represent polar motion if P_r were the rotation pole at the epoch of reference.

The concepts are made more complex in practice due to

a. the rotation of the Earth; and

b. precession and nutation.

The problems involved concern the relation of the celestial sphere to the physical system at each instant in time. They are referred to (in Appendix 5) as being "non-geodetic in Earth space". It is not the intention of the present development to deal with such matters. Current practice is to adopt a non-complex model and hence linearize the system. Variations in the rate of rotation can be treated as parameters to be recovered in any adjustment of observations. The question of nutation is more complex and problems may arise in modeling the phenomenon to the required order of precision.

It is significant to note that changes in the pole position can be determined without any dependence of the result on the precision of geodetic station co-ordinates. It has been shown that a single tracking station is capable of recovering the component of changes in position with respect to the instantaneous pole (Smith et al. 1972) in the meridian of the tracking station. The analysis of such determinations at a number of stations should permit the separation of the quantities $\delta\phi_\alpha$ from other factors of significance.

It must therefore be concluded that the quantities $\delta\phi_\alpha$ can be determined without any dependence on the process used for the determination of geodetic position. Thus the X_1X_3 plane defined by the instantaneous axis of rotation and the single station O at the surface of the Earth and its position at any epoch (τ = t) can, in principle, be related to its location at any other epoch (τ = t + dt) provided the sequence of changes $\delta\phi_\alpha$ have been defined between epochs. An additional ambiguity is introduced into the definition of the X_1X_3 plane, as summarized in Appendix 8, by the fact that O is defined by its zenith and not its ground location. Consequently, any changes in the direction of the vertical due to mass redistributions will introduce an additional bias in the definition of the X_1X_3 plane if not corrected for by gravimetric means.

4. A GEODETIC SYSTEM OF REFERENCE FOR LONG PERIOD STUDIES IN EARTH PHYSICS

A three dimensional system of reference at any epoch ($\tau = t$) is obtained on defining an origin and a plane of reference defined by two orthogonal axes in the plane passing through the origin. If precession and nutation were allowed for, and as rigidity is implicit in the concept of rotation, the instantaneous axis of the Earth can be shown to pass through the geocenter. It therefore follows that only one point on the surface of the Earth is necessary to completely define the reference system.

Appendix 8 summarizes an acceptable system of reference for geodetic measurements at any epoch ($\tau = t$), all gravitational and dynamic characteristics being defined by their instantaneous locations. The position of the "natural" co-ordinate system changes with time, despite the X_1X_3 plane always passing through O. The locations of the co-ordinate systems providing a frame of reference at each epoch can be inter-related in the following manner.

1. Adopt the co-ordinate system X_i defined in terms of Appendix 8 for some initial epoch ($\tau = t_0$) as the system of reference for long period studies. The origin of this X_i system coincides with the location of the geocenter at this same epoch.

2. Geodetic measurements made during some later epoch ($\tau = t$) can be established with respect to the reference system χ_i, as described in Appendix 8, the $\chi_1\chi_3$ plane passing through the location of the geocenter for this epoch as well as the ground station O which could be a tracking station in the case of laser ranging systems or alternately a receiving antenna in the case of VLBI.

   The χ_i co-ordinates so determined are related to the reference system values X_i by the transformation

$$X_i(t) = X_{gi}(t) + \alpha_{ij}\, \chi_j(t) \tag{15}$$

where the elements α_{ij} of the transformation matrix are obtained from Figure 4 by direct resolution as

$$\begin{vmatrix} \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos\theta_3 & -\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 \cos\theta_3 & \sin\theta_1 \sin\theta_3 \\ \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2 \cos\theta_3 & -\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 \cos\theta_3 & -\cos\theta_1 \sin\theta_3 \\ \sin\theta_2 \sin\theta_3 & \cos\theta_2 \sin\theta_3 & \cos\theta_3 \end{vmatrix}$$

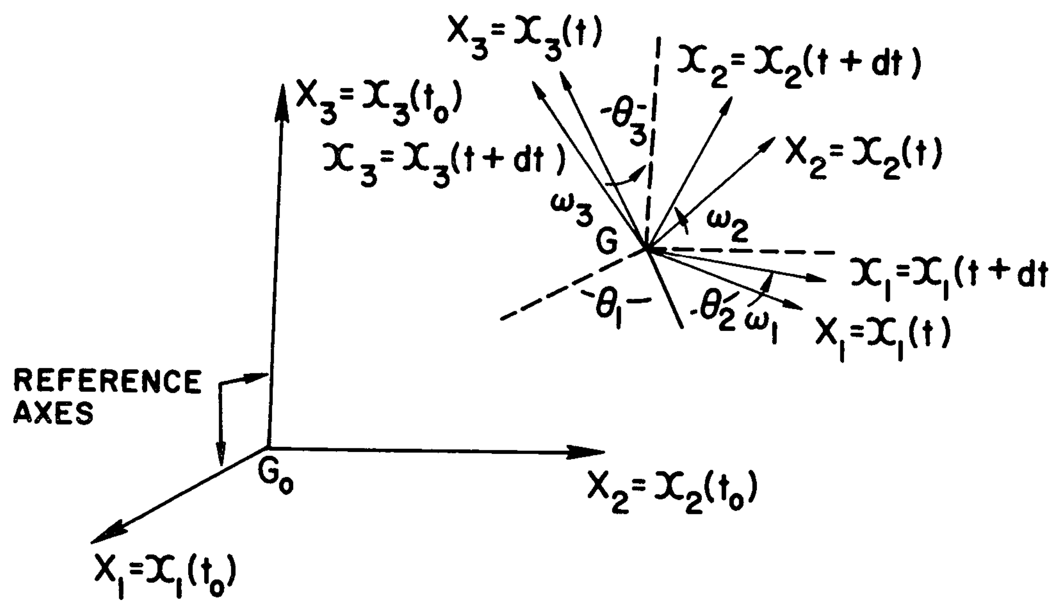


Figure 4. Rotating Axes in Euclidian Space, $x_i = \mathcal{X}_i - \epsilon_{ijk} \mathcal{X}_j \, \omega_k \, dt + x_{gi}$, x_{gi} = Co-ordinates of Geocenter at Epoch (τ = t + dt), $\omega_1 \, dt = \delta\phi_1$, $\omega_2 \, dt = \delta\phi_2$, $\omega_3 \, dt = \delta\phi_1 \cot \phi_o$, if Celestial Sphere Maintains the Rate of Rotation of the Earth

The Euler angles θ_i, illustrated in Figure 4, are a conglomeration of a series of increments $d\theta_i$ evaluated between successive epochs (τ = t) and (τ = t + dt). If $\mathcal{X}_i$ are the co-ordinates of a station in the geodetic network on the instantaneous reference system $\mathcal{X}_i$ summarized in Appendix 8 at epoch (τ = t) and x_i those of the same station on the instantaneous system x_i at epoch (τ = t + dt), the changes in position of the axes in Earth space can be completely defined by

a. 3 rotations characterized by the angular velocities ω_i about the χ_i axes;

b. the motion of the geocenter between epochs, and defined by the co-ordinates χ_{gi} on the χ_i system.

Figure 4 illustrates the relations at a. above. It can be shown (Mather 1973, Sec. 3.2) that the rotation ω_i are related to the Euler angle increments $d\theta_i$ by the first set of relations in Appendix 9. A study of Figures 2 and 4 shows that

$$\omega_\alpha \, dt = \delta\phi_\alpha \quad \alpha = 1, 2 \tag{17}$$

$$\omega_3 \, dt = \delta\phi_1 \cot \theta_o . \tag{18}$$

In addition, it can be shown that the co-ordinates x_i of a station referred to the instantaneous reference system x_i at epoch (τ = t + dt) are equivalent to the values χ_i (τ = t + dt) on the χ_i system in Earth space, the transformation being defined by the equations

$$\chi_i (t + dt) = \chi_i (t + dt) + \epsilon_{ijk} \, \omega_j \, x_k \, dt + \chi_{gi} \tag{19}$$

which is the standard expression for the inter-relation of rotating axes in Euclidian space (e.g., Mather 1972, p. 26).

Equations 15 and 19 complete the transformation of co-ordinates between those referred to the instantaneous Cartesian system x_i to values on the permanent reference system X_i. The differences

$$\Delta X_{ij} (dt) = X_{ij} (t + dt) - X_{ij} (t) \tag{20}$$

between the co-ordinates of the i-th station as determined at epochs (τ = t) and (τ = t + dt) would provide a measure of the change in position between epochs of the i-th station, which was independent of the perturbations of the co-ordinate systems with time. It has been assumed that the changes in position of the geocenter with time can be determined.

5. THE MOTION OF THE GEOCENTER

Both Equations 15 and 19 require a knowledge of the motion of the geocenter for complete definition. The geocenter is the center of mass of the solid Earth, oceans and the atmosphere. It is assumed that the gravitational effects of all extra-terrestrial sources are treated as perturbations which can be modeled and whose effects are removed.

Appendix 10 summarizes the reasons why it is considered desirable to allow for the motion of the geocenter. This represents a resultant motion of the Earth's center of mass with respect to the stations comprising the geodetic reference network at the surface of the Earth. Two methods are available for evaluating this phenomenon as summarized in Appendix 10.

The first follows from Equation 19 when the omission of the term χ_{gi} will result in the existence of a first degree effect in the difference between the co-ordinates χ_i as determined on the instantaneous system at epoch (τ = t) and x_i at (τ = t + dt). Thus if $\chi_{i'j}$ (t + dt) were defined by

$$\chi_{i'j}(t + dt) = x_{ij}(t + dt) + \epsilon_{jk\ell}\,\omega_k\, x_\ell\, dt, \tag{21}$$

the recorded difference in the station co-ordinates at the i-th station is

$$\begin{aligned} \Delta\chi_{i'j}(dt) &= \chi_{ij}'(t + dt) - \chi_{ij}(t) \\ &= \Delta\chi_{ij} - \chi_{gj}, \end{aligned} \tag{22}$$

$\Delta\chi_{ij}$ being the difference which would have been obtained had condition 2 in Appendix 8 been satisfied. The effect of the exclusion of χ_{gj} on $\Delta\chi_{ij}$ is harmonic and of first degree. This technique can only be applied to data acquisition procedures which are directly related to the natural frame of reference specified in Appendix 8. Its efficacy will depend on the progress made in modeling the various perturbing forces affecting extra-terrestrial sources in near-Earth orbit.

A second technique for evaluating χ_{gj} is from the analysis of changes in absolute gravity determined at the 1 μgal level, at a network of observing platforms spanning the globe. The method as described by Mather (1973, Sec. 3.3), is summarized in Appendix 11. It could be speculated that local effects may well be an order of magnitude larger than global effects and the success with which models can be defined for such local effects will be a critical factor in the meaningful utilization of this method.

It may in fact be more desirable, as in astro-geodesy, to trade a slight diminution (5 - 10 μgal) in observing precision for a more extensive coverage from a less complex instrument which is more economic to operate in both time and costs. The required information may well be obtained by the judicious selection of the permanent station sites in geologically inactive regions away from development.

As the problems associated with each of the techniques outlined are not similar, it may be necessary in the first instance to use both methods simultaneously in order that any problems of consequence are identified and eliminated.

6. CONCLUSION

The discussion in the preceding five sections has summarized the principles underlying the definition of the geodetic positions of a network of stations at the surface of the Earth without ambiguity over long periods of time by adopting the natural frame of reference afforded by the dynamic and gravitational characteristics of the Earth's rotation, and a single point at the Earth's surface. The sequence of development is outlined in Appendices 1 to 11.

A question that has not been covered is the scale of the space and the possibility that it may change with respect to immutable standards over long periods of time. The implications of a variable space metric in geodetic studies has been discussed by Mather (1972, p. 32). Two possible effects are the following.

1. The velocity of light c may change with respect to some immutable standards of length and time.

2. Variations may occur in the gravitational constant G.

The entire system is scaled by the instantaneous value of the velocity of light as both time and length standards are based on it. No inconsistency arises in incorporating gravity measurements into the overall context of geodetic determinations as c defines the framework for the determination of both absolute gravity and G. The main geodetic consequence of time variations in c is the occurrence of a variable set of units in defining both $d\theta_i$ described in Appendix 9, and hence in the Euler angles θ_i, as well as the Earth space co-ordinates. In the latter case, the differences $\Delta\chi_{ij}$ described in Equation 22 would contain an effect which in terms of harmonics, is of zero degree.

The possibility of variations in G at the level of 1 part in 10^{11} yr^{-1} have been widely discussed and a summary of its consequences on the determination of position is available (Ibid, p. 134). The net effect, if any, will manifest itself as a harmonic of zero degree in determinations of changes in absolute gravity, and hence χ_{gi} in Equation 19.

It can therefore be concluded that changes in Earth space co-ordinates of geodetic stations at the surface of the Earth which are of zero degree could well have nothing to do with changes in position, though there is the possibility of their occurrence as a consequence of a uniform expansion of the Earth. In view of uncertainty of the origin of such effects, it would be advantageous from all points of view to filter out zero degree changes in Earth space co-ordinates as a separate information matrix. The resulting Earth space co-ordinates would be of direct application and relevance to those studies in Earth physics concerned with plate motions and crustal distortions.

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APPENDIX 1

EARTH SPACE

DEFINITION

THE EUCLIDIAN SPACE HAVING THE SAME ROTATIONAL AND GALACTIC MOTION AS THE EARTH

ASSUMPTION

EARTH IS A RIGID BODY HAVING AN AXIS OF ROTATION

DIFFICULTIES ENCOUNTERED FOR HIGH PRECISION GEODETIC STUDIES

1. DEPARTURES FROM THE RIGID BODY MODEL
2. INSTANTANEOUS AXIS OF ROTATION HAS PERIODIC AND POSSIBLY SECULAR MOTIONS WITH RESPECT TO EARTH'S CRUST
3. THE UNAMBIGUOUS DEFINITION OF A SYSTEM OF REFERENCE FOR LONG PERIOD STUDIES

APPENDIX 2

GEODETIC PRACTICE TO 1 P.P.M. IS

1. MARGINALLY SENSITIVE TO POLAR MOTION AT 0.1 ARSEC LEVEL
2. UNAFFECTED IN A GLOBAL SENSE BY DEPARTURES OF THE EARTH FROM A RIGID BODY MODEL

TIDAL EFFECTS <60 cm
PLATE MOTIONS <10 cm yr^{-1}

IF RESTRICTED TO TIME SPANS OF LESS THAN 50 YEARS
AND IF

a. THE ROTATION OF THE EARTH

AND

b. PRECESSION AND NUTATION EFFECTS ON EXTRA-TERRESTRIAL REFERENCE SYSTEM

WERE ALLOWED FOR WHEN INCORPORATING INFORMATION DEDUCED FROM CELESTIAL SPHERE CONCEPTS INTO THE GEODETIC REFERENCE SYSTEM

APPENDIX 3

CONCLUSIONS

1. ASTRONOMICAL DETERMINATIONS PROVIDE NO SCALE
2. THREE DIRECTIONAL PARAMETERS HAVE TO BE ARBITRARILY ASSIGNED TO CONVERT THE ASTRONOMICAL DETERMINATIONS TO EQUIVALENT GEODETIC QUANTITIES
3. THE X_1X_3 PLANE IS DEFINED BY THE EARTH'S ROTATION AXIS AND POLAR GREAT CIRCLE ON CELESTIAL SPHERE WITH ZERO RIGHT ASCENSION
4. $\theta^*=0$ IMPLIES THAT THIS GREAT CIRCLE LIES WHOLLY IN THE PLANE OF THE REFERENCE MERIDIAN

*θ=REFERENCE MERIDIAN SIDEREAL TIME

APPENDIX 4

TO OBTAIN EARTH SPACE CO-ORDINATES FROM EXTRA-TERRESTRIAL CONSIDERATIONS MODELS ARE REQUIRED

FOR

1. EARTH ROTATION
2. PRECESSION AND NUTATION

IF THESE ARE ALLOWED FOR

$\delta\phi_a$ REPRESENT POLAR MOTION

HENCE ω_i ARE DEFINED

APPENDIX 5

RESULTANT PROBLEMS

NON GEODETIC IN EARTH SPACE

1. EFFECT OF PRECESSION AND NUTATION ON THE VECTOR $P_i P_r$
2. VARIATION IN THE EARTH'S RATE OF ROTATION

GEODETIC IN EARTH SPACE

1. POLAR MOTION
2. MOTION OF THE OBSERVING STATIONS
3. CHANGES IN THE DIRECTION OF LOCAL VERTICAL DUE TO MASS RE-DISTRIBUTIONS
4. DEFINITION OF THE REFERENCE "MERIDIAN"
5. MOTION OF THE GEOCENTER WITH RESPECT TO A GEODETIC REFERENCE NETWORK AT THE SURFACE OF THE EARTH

APPENDIX 6

THE INSTANTANEOUS X_1X_3 PLANE

SATELLITE RANGING SYSTEMS

1. DO NOT DIRECTLY DEPEND ON CELESTIAL SPHERE SYSTEM OF EXTRA-TERRESTRIAL REFERENCE
2. X_1X_3 PLANE IS UNAMBIGUOUSLY DEFINED IF 0 IS A STATION IN THE GEODETIC NETWORK AND VARIATIONS IN THE RATE OF ROTATION ARE ALLOWED FOR WHEN ADJUSTING OBSERVATIONS
3. ALL OBSERVATIONS REFERRED TO INSTANTANEOUS GEOCENTER

ASTRONOMICAL SYSTEMS

AN ADDITIONAL AMBIGUITY IS INTRODUCED BY CHANGES IN THE DIRECTION OF THE VERTICAL AT 0

APPENDIX 7

OPTIMUM CHARACTERISTICS IN A GEODETIC REFERENCE SYSTEM FOR LONG PERIOD STUDIES IN EARTH PHYSICS

1. OBSERVATIONS CAN BE RELATED DIRECTLY TO THE REFERENCE SYSTEM

2. A MINIMUM NUMBER OF POINTS SHOULD BE HELD FIXED AT THE SURFACE OF THE EARTH

   – PREFERABLY ONE

3. STATIONS DEFINING THE REFERENCE SYSTEM SHOULD HAVE A PERMANENT CHARACTER

   – GEODETIC OBSERVATORIES

4. THE REFERENCE SYSTEM ADOPTED SHOULD BE SUCH THAT CHANGES IN CO-ORDINATES BETWEEN EPOCHS CAN BE SIMPLY RELATED TO SECULAR VARIATIONS IN POSITION

APPENDIX 8

AN ACCEPTABLE SYSTEM IS OBTAINED AS FOLLOWS

1. AT ANY EPOCH (τ=t) REPLACE EARTH BY AN INSTANTANEOUS RIGID BODY MODEL
2. SYSTEM OF REFERENCE IS THREE-DIMENSIONAL CARTESIAN CENTERED AT GEOCENTER
3. X_3 AXIS COINCIDES WITH AXIS OF ROTATION
4. X_1X_3 PLANE IS DEFINED BY THE AXIS OF ROTATION AND ONE LOCATION AT THE EARTH'S SURFACE

APPENDIX 9

GOAL OF GEODETIC REFERENCE SYSTEM
TO PROVIDE SIMPLE INTER-RELATION OF CO-ORDINATES
BETWEEN WIDELY SEPARATED EPOCHS ($\tau = t_0$) AND ($\tau = t$)

ACHIEVE BY

1. USING A SYSTEM OF EULER ANGLES θ_i
2. DEFINING THE MOTION OF THE GEOCENTER

EULER ANGLE INCREMENTS

$$
\left\{
\begin{aligned}
d\theta &= R^{-1}\Omega\, dt \\
\Omega^T &= \left| \omega_1 \ \omega_2 \ \omega_3 \right| \\
d\theta^T &= \left| d\theta_1 \ d\theta_2 \ d\theta_3 \right| \\
R &= \begin{vmatrix} \sin\theta_2 \sin\theta_3 & 0 & \cos\theta_2 \\ \cos\theta_2 \sin\theta_3 & 0 & -\sin\theta_2 \\ \cos\theta_3 & 1 & 0 \end{vmatrix}
\end{aligned}
\right.
$$

CONVERSION OF INSTANTANEOUS CO-ORDINATES TO REFERENCE X_i SYSTEM

$$X_i(t) = X_{gi}(t) + a_{ij} x_j(t)$$

a_{ij} ARE ELEMENTS OF ARRAY

$$
\begin{vmatrix}
\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos\theta_3 & -\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 \cos\theta_3 & \sin\theta_1 \sin\theta_3 \\
\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2 \cos\theta_3 & -\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 \cos\theta_3 & -\cos\theta_1 \sin\theta_3 \\
\sin\theta_2 \sin\theta_3 & \cos\theta_2 \sin\theta_3 & \cos\theta_3
\end{vmatrix}
$$

APPENDIX 10

THE MOTION OF THE GEOCENTER

IF MOTION IS IGNORED

1. CO-ORDINATE CHANGES OF SURFACE GEODETIC NETWORKS CONTAIN INFORMATION ABOUT MOTION OF GEOCENTER, IF OF SIGNIFICANT MAGNITUDE
2. THIS INFORMATION IS BETTER EXCLUDED IF CO-ORDS ARE USED FOR REGIONAL EARTH PHYSICS STUDIES

TECHNIQUES FOR EVALUATION IN THE FUTURE

1. ANALYSIS OF CHANGES IN STATION CO-ORDINATES BETWEEN EPOCHS FOR FIRST DEGREE HARMONIC EFFECTS
2. ANALYSIS OF CHANGES IN ABSOLUTE GRAVITY AT THE $\pm 1\,\mu$GAL LEVEL BETWEEN EPOCHS FOR FIRST DEGREE HARMONIC EFFECTS

APPENDIX 11

AT EPOCH $\tau = t$

AT i-th STATION

$$\underbrace{(\Delta g_i)_t}_{\text{GRAVITY ANOMALY}} = \underbrace{g_i}_{\text{OBSERVED GRAVITY}} + \underbrace{c_i}_{\text{ELEVATION EFFECT}} - \underbrace{\gamma_i}_{\text{NORMAL GRAVITY}}$$

AT EPOCH $\tau = t+dt$

$$\underbrace{(\Delta g_i)_{t+dt}}_{\text{GRAVITY ANOMALY}} = \underbrace{(g_i+\delta g_i)}_{\text{OBSERVED GRAVITY}} + \underbrace{c_i - \gamma_i}_{\text{AS ABOVE}}$$

$$= (\Delta g_i)_t + \delta g_i$$

δg_i DUE TO

1. MOTION WITH RESPECT TO ROTATION AXIS
   $1 \mu gal \sim 1 m$
2. RADIAL MOTION RELATIVE TO GEOCENTER
   $1 \mu gal \sim 3 mm$
3. LOCAL EFFECTS e.g. INCREASE IN WATER TABLE
   $1 \mu gal \sim 2\text{-}3 cm$

GLOBAL ANALYSIS FOR A FIRST DEGREE HARMONIC AT (2) GIVES MOTION OF GEOCENTER BETWEEN EPOCHS

APPENDIX 12

SCALE

ALL MODERN GEODETIC SYSTEMS ARE SCALED BY THE VELOCITY OF LIGHT C

CONCLUSION

IT IS IMPORTANT THAT ZERO DEGREE CHANGES IN CO-ORDINATES BE REMOVED

APPENDIX 13

ADVANTAGES OF SYSTEM PROPOSED

1. UNAMBIGUOUS IF SCALE IS SPECIFIED
2. NOT COMPLEX
3. DIRECTLY RELATED TO OBSERVATIONS
4. NO SIGNIFICANT CHANGES IN CONCEPT FROM SYSTEMS CURRENTLY IN USE
5. DIRECTLY RELATED TO LIOUVILLE EQUATION

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