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Circular Carrier-Frequency Photography for Observing Phase Objects

D. E. Silva
Visibility Laboratory
University of California, San Diego
La Jolla, California 92037

Y. M. Hong
Department of Applied Physics & Information Sciences
University of California, San Diego
La Jolla, California 92037

ABSTRACT

A circular grating is photographed through a phase object which deforms the image of the grating lines. By superimposing these deformed lines with a master grating on photographic film, moiré patterns are observed. These patterns are interpreted as fringes of constant radial derivative.

In a recent paper[1], Grover et al have shown that information about the lateral slope of a phase object is obtained when a Ronchi ruling is used for modulation in a carrier-frequency experiment. This communication will show that if the Ronchi ruling is replaced by a circular grating an interferogram is obtained that yields information about the radial derivative. The setup is shown in Fig. 1. A spatially coherent beam illuminates the circular grating G. A phase object is located a distance z behind the grating and the image plane is conjugate to the grating. The plane wave impinging upon G is diffracted into conical wavefronts which radially shear the object by a constant amount $\lambda z/a$ where a is the grating period. Two exposures are made on film located in the image plane; one contains the object and the other is made with the object removed. As we shall show the resulting fringes are fringes of constant radial derivative.

Only the results of our analysis based on scalar diffraction theory will be presented here. The analysis is similar to the computation reported in an earlier paper[2]. With a test object placed a distance z from the grating the complex wavefield in the image plane is approximately,

$$v(r, \varphi) \approx \sum_{(m)} C_m \, e^{2\pi i m r/a} \, u_o(r-ms, \varphi) \qquad (1)$$

Where we have expressed the circular grating by its Fourier exponential series, (m) implies the summation is taken from $-\infty$ to $+\infty$, $u_o(r, \varphi)$ is the complex transmittance of the test object, $s = \lambda z/a$ is the constant radial shear introduced by the grating, and λ is the wavelength of the light. The intensity and therefore the transmittance of the recorded film is proportional to the modulus squared of eq. (1). Another exposure is recorded on the same film but with the object removed. Its intensity is the same as that previously recorded except with $u_o(r, \varphi)$ removed. The total recorded intensity is the sum of both recordings giving us,

$$I_T(r,\varphi) = \sum_{(m)} \sum_{(n)} C_m C_n^* e^{2\pi i(m-n)r/a} \left[1 + u_o(r-ms,\varphi)\, u_o^*(r-ns,\varphi)\right] \tag{2}$$

We now consider phase objects, $u_o(r,\varphi) = \exp i\left[\phi(r,\varphi)\right]$. The sheared object fields, $u_o(r-ms,\varphi) = \exp i\left[\phi(r-ms,\varphi)\right]$. Expanding the argument of the exponential in a Taylor series yields $u_o(r-ms,\varphi) = \exp i\left[\phi(r,\varphi) - ms\alpha + 1/2(ms)^2 \;\partial^2\phi/\partial r^2 - \ldots\right]$, where $\alpha = \partial\phi(r,\varphi)/\partial r$. By restricting our attention to objects with a phase structure such that only the first two terms in the series are significant, eq. (2) becomes,

$$I_T \approx \sum_{(m)} \sum_{(n)} C_m C_n^* \; e^{2\pi i(m-n)r/a}\left[1 + e^{-i(m-n)s\alpha}\right] \tag{3}$$

This expression can be rearranged into various sub-additions of $p = m-n = 0, \pm 1, \pm 2, \ldots \pm\infty$. Since the observational system such as the eye will attenuate the high spatial frequencies, only $p = 0, \pm 1$ are significant. A circular grating has Fourier coefficients that are real and furthermore if it has a mark-to-space ratio of 1 only the odd harmonics are non-zero. Consequently eq. (3) reduces to,

$$I_T \approx 2 \sum_{(m)} \left|C_m\right|^2 + \frac{1}{\pi}\cos(s\alpha/2)\,\cos(2\pi r/a + s\alpha/2) \tag{4}$$

The recorded intensity consists of a constant background term and a modulated term that contains information about the phase object. The latter term of eq. (4) has a carrier $\cos(2\pi r/a)$ that is amplitude modulated by $\cos(s\alpha/2)$. Dark fringes occur where the modulation is zero, that is, $\alpha = \partial\phi(r,\varphi)/\partial r = (2m+1)\,\pi/s$. As an example, a perfect spherical lens has a phase $\phi(r) = \pi r^2/\lambda f$. Thus $\alpha = 2\pi r/\lambda f$ and dark fringes occur when $r=(2m+1)\lambda f/2s$. These fringes form a set of concentric circles. Deviations from these circles for non-spherical lenses would show up the aberrations. Spatial filtering can be used to eliminate the carrier-frequency provided that the grating period a is smaller than the details that we wish to observe. We do this by illuminating the recorded film with a plane wave and passing the diffracted light through a lens. An annulus ring in the back focal plane of the lens with a mean diameter $\lambda f/a$, where f is the focal length of the lens, passes the first diffracted order thereby eliminating the grating in the final image.

An experiment to confirm these results used a helium-neon laser as a source which was expanded into a beam of 50mm, and a circular grating with a period of 0.25mm. The object was placed 1mm behind the grating and a camera was used to image the grating onto the film. An eyeglass lens was used as a test object. Figure 2 shows the result of testing the lens. Two bright fringes in the shape of concentric circles are observed. Another lens with a drop of plastic resin in its center was also tested. Figure 3 shows the lens under test.

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References

1. C. Grover, S. Mallick and M. Roblin, Opt. Comm. 3, 181 (1971).

2. D. Silva, Appl. Opt. 11, 2613 (1972).

Figure Captions

Figure 1 Experimental setup consisting of a collimated source, a circular grating G, a test object OBJ and a camera focused onto the grating. A double-exposure is made; one without the object.

Figure 2 An eyeglass lens under test. The white fringes are fringes of equal radial phase derivative. The fine concentric circular lines belong to the circular grating.

Figure 3 An eyeglass lens with a drop of resin in its center is the test object.

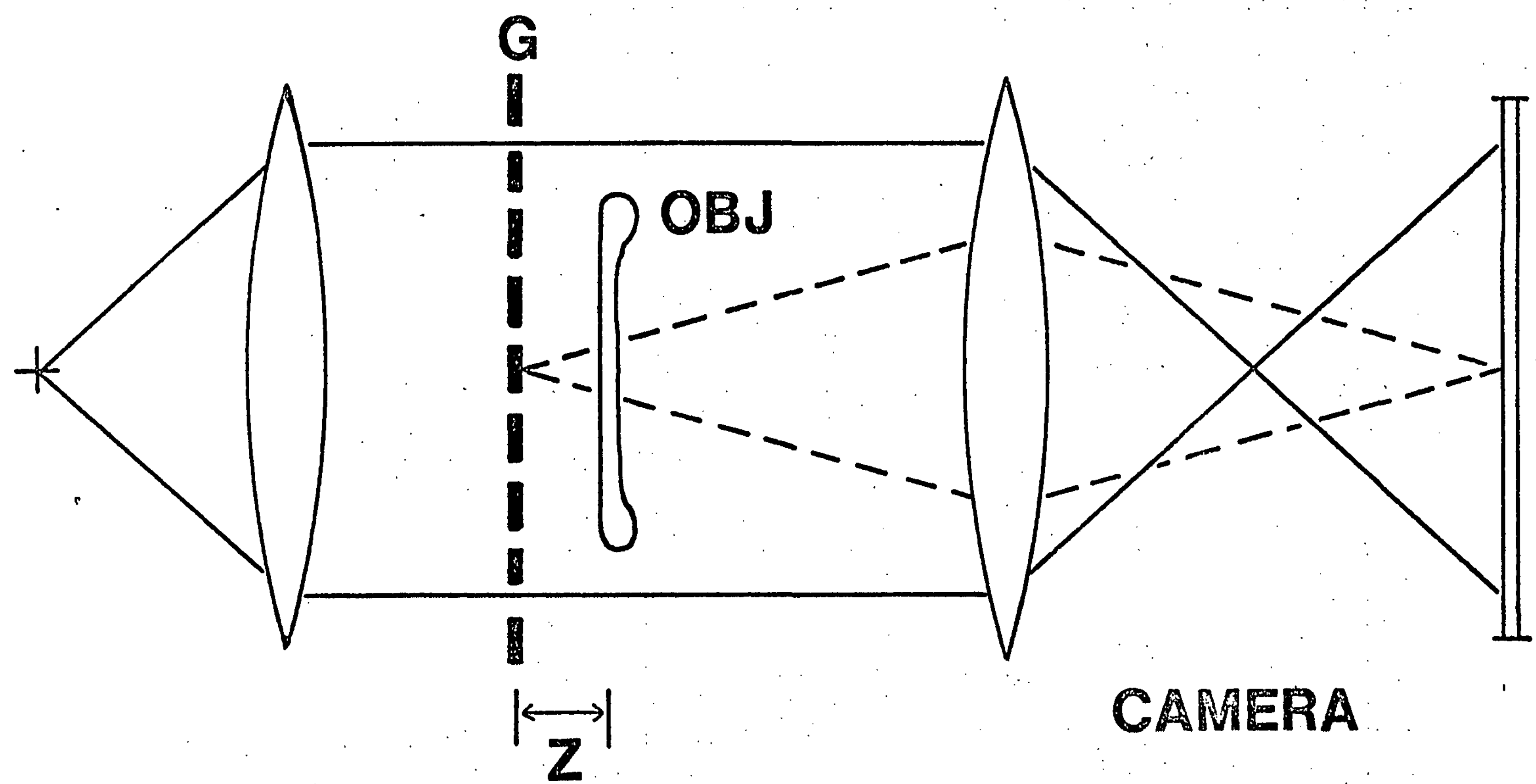

FIGURE 1

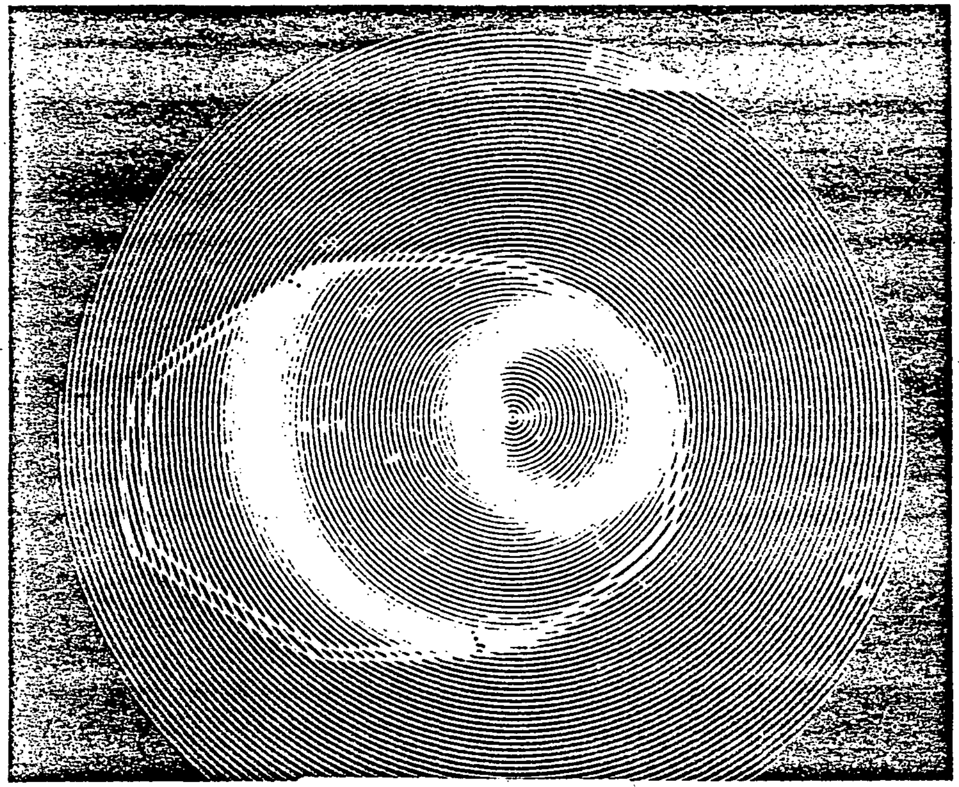

FIGURE 2

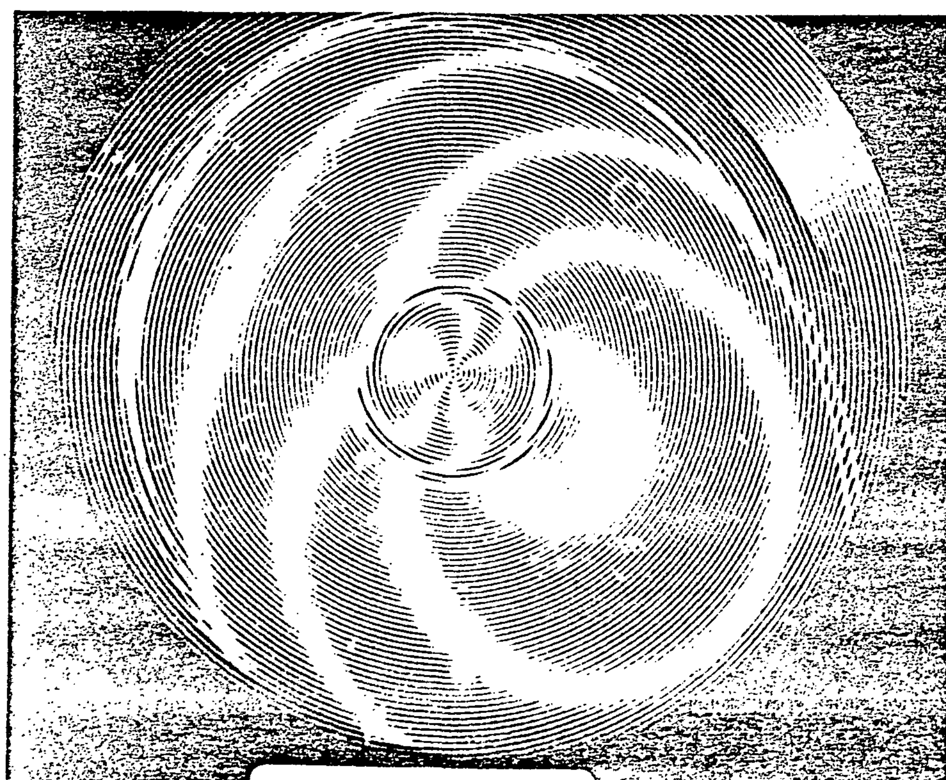

FIGURE 3