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**A PROPOSED EXPERIMENTAL METHOD FOR INTERPRETING
DOPPLER EFFECT MEASUREMENTS AND DETERMINING
THEIR PRECISION**

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A PROPOSED EXPERIMENTAL METHOD FOR INTERPRETING DOPPLER EFFECT MEASUREMENTS AND DETERMINING THEIR PRECISION

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SUMMARY

The principal problem in the measurement of the Doppler reactivity effect is separating it from the thermal reactivity effects of the expansion of the heated sample. It is shown in this proposal that the thermal effects of sample expansion can be experimentally determined by making additional measurements with porous samples having the same mass and/or volume as the primary sample. By combining these results with independent measurements of the linear temperature coefficient and the computed temperature dependence of the Doppler coefficient the magnitude of the Doppler coefficient may be extracted from the data. These additional measurements are also useful to experimentally determine the precision of the reactivity oscillator technique used to measure the reactivity effects of the heated sample.

INTRODUCTION

The only major rapid reactivity feedbacks accompanying power excursions in fast reactors are associated with the Doppler coefficient of the fuel and the cladding and with the thermal expansion of the fuel. Since both effects are small in magnitude, the principal problem in accurate measurement of the Doppler effect is in separating it from the thermal reactivity effects of the expansion of the sample. In addition it is necessary to ascertain the experimental precision in order to evaluate the uncertainty of the Doppler coefficient measurements. Usually the thermal expansion factors for heated samples in core are arrived at analytically.

In the present proposal it is shown that the reactivity contribution of the Doppler coefficient can be separated from the reactivity contribution of the sample expansion and the change in sample self-shielding cross section with temperature, in a direct experimental manner provided ancillary measurements with reduced density samples are made. These ancillary measurements also serve to estimate the precision of the reactivity data. The samples are located at the center of the core. The reactivity measurements are made by sample replacement techniques.

The total temperature reactivity effect measured in the critical assembly by the interchange of heated samples with room temperature samples consists of the contribution of the Doppler effect, the contribution of the sample expansion with temperature, and the contribution of the change in sample self-shielding average cross section with temperature.

The reactivity contribution due to sample expansion with temperature can be experimentally evaluated as the product of the measured in-pile rate of change of reactivity with sample size at each sample temperature, and the measured out-of-pile rate of change of sample size with temperature at each temperature. The rate of change of sample size with temperature can be obtained from independent measurements of the linear temperature coefficient of expansion of the sample. In this manner the contribution to the total temperature reactivity of sample expansion with temperature can be obtained entirely from measurement.

The measurement of the rate of change of reactivity with sample size above is made by measuring an additional porous sample at each temperature. This additional sample has the same mass as the full density sample but is fabricated at a reduced density. The size difference between the porous sample and the full density sample is small and is assumed to be approximately the same as the average size change occurring with temperature over the temperature range being investigated.

The contribution due to the change in sample self-shielding average cross section accompanying sample expansions with temperature is obtained by measuring in the reactor the change in reactivity resulting from the interchange of a full density sample with a reduced density sample of equal volume at each temperature. This reactivity change when multiplied by a computed mass increment equivalent to the rate of change in microscopic cross section with temperature yields the contribution to the total reactivity of sample self-shielding with temperature. Inherent in this method is the assumption that self-shielding effects may be expressed in terms of average escape probabilities so represented by the Wigner rational approximation.

Both the full density and the porous samples must be unclad and of sufficient mass to give signals which exceed the uncertainty band associated with the response and resolution of the measurement.

THEORY

A brief discussion of the analytical formulation of the multigroup perturbation equation for the reactivity of a fuel sample relative to void in the test reactor is given in this section. In the subsequent section it is shown that the analytical formulation can be separated into individual components which can be independently measured or computed. These individual parts are then combined to obtain the corrections for the reactivity effects due to thermal expansion of the samples and change in average self-shielding cross sections that are required to separate out the Doppler reactivity effect.

The explicit multigroup perturbation equation formulation of the first order centrally located fuel sample worth is (ref. 1)

$$\frac{\Delta k}{k} = \frac{Nm}{DA} \sum_i \left\{ \left[\sigma_{fi}(1 - P_{ci}) \left(\frac{1 - P_{cf}}{1 - \eta_f P_{cf}} \right) \right] v_i \phi_f^* \phi_i - [\sigma_{ai}(1 - P_{ci})] \phi_i^* \phi_i \right\} \quad (1)$$

where

N	Avagadro's number
m	mass of the sample
A	atomic mass of the sample isotope
σ_{fi}, σ_{ai}	the i^{th} group fission and absorption cross sections
$\phi_f^* = \sum_i \chi_i \phi_i^*$	the unperturbed adjoint flux for the fission neutrons
ϕ_i^*	the unperturbed adjoint flux in group i
η_f	the number of fission neutrons per neutron absorbed
ϕ_i	the scalar average flux in group i at the sample position with sample absent
P_{ci}	flat distributed source absorption probability for neutrons in the i^{th} energy group
P_{cf}	fission spectrum averaged probability that a neutron born by fission in the sample will undergo a collision in the fuel
χ_i	fractional fission neutron source in group i

$$D = \int dV_c \int dE v \Sigma_f(r, E) \phi(r, E) \phi_f^*(r)$$

is the total fission rate of the reactor.

$\Sigma_f(r, E)$ macroscopic fission cross section

V_c volume of the test reactor

The relative worths of different sized fuel samples are dependent on their self-shielding and on their self-multiplication. The self-shielding factor appears as the $(1 - P_{ci}) = P_{esci}$ the average escape probability, which increases with expansion due to the increase in the surface area, S , of the sample with temperature. By the Wigner rational

approximation (ref. 4) $P_{esci} = \frac{1}{4V\Sigma_i/S + 1}$. The term $\Sigma_i V$ remains constant with expansion. Σ_i is the total cross section in group i and V is the volume of the sample. S is the surface area of the sample. Consequently there is an increase in reactivity for the fission reaction and a decrease in reactivity for the absorption reaction due to self-shielding.

The self-multiplication appears in the factor $\frac{1 - P_{cf}}{1 - \eta P_{cf}}$. This factor arises from the consideration that $1 - P_{cf}$ is the probability that a neutron born in the sample will escape without a collision in the fuel sample. Thus

$$(1 - P_{cf}) + P_{cf}\eta(1 - P_{cf}) + P_{cf}^2\eta(1 - P_{cf}) + \dots$$

are the total number of neutrons which escape the sample. The sum of this infinite series is the self-multiplication $\left(\frac{1 - P_{cf}}{1 - \eta P_{cf}}\right)$. Since $P_{cf} = \sum_i \chi_i P_{ci} = \sum_i \chi_i (1 - P_{esci})$, P_{cf} decreases with expansion and the self-multiplication decreases with expansion if $\eta > 1$. It is interesting to note that the self-shielding decreases with the increasing energy of the neutrons, while the self-multiplication being a function of the fission spectrum is unaffected.

The terms relating to the contributions of moderation and diffusion are absent in equation (1) since in the case of Doppler measurements in fast reactors, only the effects of production and absorption need be considered (ref. 2).

PARTITION OF PERTURBATION EQUATION INTO EXPERIMENTAL COMPONENTS

Equation (1) relates the measured reactivity for a sample at the center of a core as functions of the pertinent variables and may be expressed as follows:

$$\frac{\Delta k}{k} = \int dE f \left[\sigma_f(E), \sigma_a(E), P_c(E), P_{cf}(E), \eta_f, \nu(E), \phi(E), \phi^*(E), \phi_f^*(E) \right]$$

The sum in equation (1) has been replaced by an integral and the subscripts have been dropped.

The P_c and P_{cf} are functions of $\Sigma_c \bar{R}$ and $\Sigma_{cf} \bar{R}$ where $\bar{R} = 4V/S$ is the average chord length. The Wigner rational approximation (ref. 4)

$$P_j = \frac{\Sigma_j \bar{R}}{\Sigma_j \bar{R} + 1}, \quad j = c, cf$$

is here used to relate the various capture and escape probabilities to the optical path length of the sample. However,

other approximations which involve $\Sigma_j \bar{R}$ as the variable can be used equally well.

Differentiation of the composite function f with respect to the average chord length $\bar{R}$ and then normalizing the result to the reactivity worth of the sample yields the normalized reactivity change that would be measured when a full density sample is interchanged with an expanded porous volume sample of equal mass.

$$\frac{d\left(\frac{\Delta k}{k}\right)}{d\bar{R}} d\bar{R} = \frac{\int dE \left[\frac{\partial f}{\partial P_{cf}} \frac{\partial P_{cf}}{\partial (\Sigma_{cf} \bar{R})} \frac{d(\Sigma_{cf} \bar{R})}{d\bar{R}} + \frac{\partial f}{\partial P_c} \frac{\partial P_c}{\partial (\Sigma_c \bar{R})} \frac{d(\Sigma_c \bar{R})}{d\bar{R}} \right] d\bar{R}}{\int dE f(E)}$$

Since EV is a constant for both the full and expanded volume reduced density samples, $\frac{d(\Sigma_j \bar{R})}{d\bar{R}} d\bar{R} = d\left(\frac{4V\Sigma_j}{S}\right) = -\bar{R}\Sigma_j \frac{dS}{S}$ where $j = c, cf$. Thus,

$$\begin{aligned} \frac{d\left(\frac{\Delta k}{k}\right)}{d\bar{R}} \Big|_T d\bar{R} &= \frac{\int dE \sum_j \frac{\partial f}{\partial P_j} \frac{\partial P_j}{\partial (\Sigma_j \bar{R})} \Sigma_j \Big|_T}{\int f(E) dE} \left(\bar{R} \frac{dS}{S} \right)_T \\ &= -M_T \left(\bar{R} \frac{dS}{S} \right)_T \\ &= M_T \left(\bar{R} \frac{2}{3} \frac{d\rho}{\rho} \right)_T \end{aligned}$$

where $d\rho/\rho$ is the relative density change between the full density and reduced density samples, having equal mass.

In this way $M_T \bar{R}_T$ can be obtained from the interchange of full and reduced density measurements at the series of temperatures T at which measurements are made to obtain the temperature coefficient (Doppler coefficient) of the full density sample.

$$\bar{R}_T M_T = \frac{\frac{d\left(\frac{\Delta k}{k}\right)}{d\bar{R}} d\bar{R} \Big|_T \frac{3}{2}}{\left(\frac{\Delta k}{k}\right) \frac{\Delta \rho}{\rho}}$$

or alternately

$$\bar{R}_T M_T = - \frac{\left. \frac{d\left(\frac{\Delta k}{k}\right)}{d\bar{R}} \right|_T}{\frac{\Delta k}{k}} \frac{1}{\frac{\Delta S}{S}}$$

The measured value of $\bar{R}_T M_T$ can then be used at each temperature T to correct for sample expansion of the full density sample since

$$\frac{\left. \frac{\partial\left(\frac{\Delta k}{k}\right)}{\partial \bar{R}} \right|_T}{\frac{\Delta k}{k}} \times \frac{d\bar{R}}{dT} = - \frac{\left. \frac{\partial\left(\frac{\Delta k}{k}\right)}{\partial \bar{R}} \right|_T}{\frac{\Delta k}{k}} \bar{R}_T \alpha_S = M_T \bar{R}_T \alpha_S$$

where $\bar{R}_T$ is the average chord length at room temperature T , α_S is the linear temperature coefficient of the sample

$$\frac{\left. \frac{\partial\left(\frac{\Delta k}{k}\right)}{\partial T} \right|_{\Sigma=\text{CONST}}}{\frac{\Delta k}{k}} = M_T \bar{R}_T \alpha_S \quad \text{for freely expanding cylinders}$$

The total temperature coefficient normalized to the sample worth for the full density sample at room temperature is:

$$\frac{\left. \frac{d\left(\frac{\Delta k}{k}\right)}{dT} \right|_T}{\frac{\Delta k}{k}} = \frac{\int dE \left(\frac{\partial f}{\partial \sigma_f} \frac{d\sigma_f}{dT} + \frac{\partial f}{\partial \sigma_a} \frac{d\sigma_a}{dT} + \sum_j \frac{\partial f}{\partial P_j} \frac{\partial P_j}{\partial (\Sigma_j \bar{R})} \bar{R} \frac{d\Sigma_j}{dT} \right)}{\int f(E) dE} + \frac{\left. \sum_j \frac{\partial f}{\partial P_j} \frac{\partial P_j}{\partial (\Sigma_j \bar{R})} \Sigma_j \frac{d\bar{R}}{dT} \right|_T}{\int f(E) dE}$$

or inserting the value of

$$\frac{\int dE \sum_j \frac{\partial f}{\partial P_j} \frac{\partial P_j}{\partial (\Sigma_j R)} \Sigma_j \frac{dR}{dT} \bigg|_T}{\int f(E) dE} = M_T \bar{R}_T \alpha_S$$

Thus the total temperature coefficient becomes

$$\frac{\frac{d(\frac{\Delta k}{k})}{dT} \bigg|_T}{\frac{\Delta k}{k}} = \frac{\int dE \left(\frac{\partial f}{\partial \sigma_f} \frac{d\sigma_f}{dT} + \frac{\partial f}{\partial \sigma_a} \frac{d\sigma_a}{dT} + \sum_j \frac{\partial f}{\partial P_j} \frac{\partial P_j}{\partial (\Sigma_j R)} \bar{R} \frac{d\Sigma_j}{dT} \right) \bigg|_T}{\int f(E) dE} + M_T \bar{R}_T \alpha_S \quad (3)$$

The first two terms are the changes in the sample cross sections arising from Doppler broadening of the resonances, while the third term is connected with the changes in the first flight escape probabilities (self-shielding) resulting from Doppler broadening, and the last term refers to changes in the sample size due to expansion with heating.

The change in the first flight escape probability (self-shielding) resulting from Doppler broadening of the cross sections can also be determined from a series of ancillary experiments in which a full density sample is interchanged with a reduced density sample having the same volume. The change in reactivity resulting from this interchange is:

$$\frac{\frac{d(\frac{\Delta k}{k})}{dm} dm \bigg|_T}{\frac{\Delta k}{k}} = \frac{\int dE \left[\sum_j \frac{\partial f}{\partial P_j} \frac{\partial P_j}{\partial (\Sigma_j R)} \bar{R} \frac{N}{A} \sigma_j \right] \Delta m \bigg|_T}{\int f(E) dE} = Q_T \Delta m$$

where $j = c, cf$ and m is the sample mass.

The interchange is made at each temperature, and Q_T obtained. If Q_T is multiplied by $\frac{m}{\sigma_j} \frac{d\sigma_j}{dT}$ which must be computed, the third term in equation (3) is obtained. In order to compute $\frac{1}{\sigma_j} \frac{d\sigma_j}{dT}$ the temperature dependence of σ_j due to Doppler broadening with temperature must be considered. Normally the temperature dependence of the Doppler coefficient of reactivity is expressed in the form $\frac{d\sigma_j}{dT} = AT^{-\gamma}$ (ref. 3) where T is the absolute temperature and γ is a constant whose theoretical value is $3/2$. Integration of this function gives $\sigma_j = \frac{A}{1-\gamma} T^{1-\gamma}$ and the ratio $\frac{1}{\sigma_j} \frac{d\sigma_j}{dT}$ becomes $(1-\gamma) \frac{1}{T}$ or $\frac{1}{2} \frac{1}{T}$ if the theoretical value

of γ is used.

Since $-Q_T \frac{m}{2} \frac{1}{T}$ equals the third term of equation (3), the total temperature coefficient becomes

$$\frac{d\left(\frac{\Delta k}{k}\right)}{dT} \bigg|_T = \frac{\int dE \left(\frac{\partial f}{\partial \sigma_f} \frac{d\sigma_f}{dT} + \frac{\partial f}{\partial \sigma_a} \frac{d\sigma_a}{dT} \right) \bigg|_T}{\int f(E) dE} - Q_T \frac{m}{2} \frac{1}{T} + M_T \bar{R}_T \alpha_S$$

where Q_T and $M_T \bar{R}_T$ are measured.

In this formulation the theoretical value of γ has been used. Because the temperature T appears in the denominator of the third term, this term is probably small. If it is found that this term, however, is important, the results will have to be iterated using values of γ obtained from the measurement of the temperature dependence of the Doppler coefficient.

ESTIMATE OF PRECISION OF MEASUREMENT TECHNIQUE

The precision of the proposed technique for isolating the temperature coefficient of reactivity of the standard sample can be obtained using the room temperature ancillary sample data obtained in connection with the standard Doppler sample expansion measurements described previously.

The total change in the reactivity with respect to the change in the surface area of the sample is obtained when the full and reduced density samples having equal mass are interchanged. The total change in the reactivity with respect to the change in the mass of the sample is obtained when the full and reduced density samples having equal volume are interchanged. Since

$$\frac{d\left(\frac{\Delta k}{k}\right)}{k} = \frac{\int dE \sum_j \left[\frac{\partial f}{\partial (\Sigma_j \bar{R})} \frac{\partial (\bar{R} \Sigma_j)}{\partial S} dS + \frac{\partial f}{\partial (\Sigma_j \bar{R})} \frac{\partial (\Sigma_j \bar{R})}{\partial \Sigma_j} \frac{\partial \Sigma_j}{\partial m} dm \right]}{\int f(E) dE}$$

where m is the mass of the sample and S is the surface area, the same reactivity change is obtained with the equal mass samples as with the equal volume samples when

$$\sum_j \frac{\partial f}{\partial (\Sigma_j R)} (-\bar{R} \Sigma_j) \frac{dS}{S} \bigg|_m = \sum_j \frac{\partial f}{\partial (\Sigma_j R)} \bar{R} \Sigma_j \frac{dm}{m} \bigg|_V$$

$$- \frac{dS}{S} = \frac{dm}{m}$$

Thus the normalized change $\frac{d\left(\frac{\Delta k}{k}\right)}{\frac{\Delta k}{k}} \bigg|_m$ measured with the full and reduced density equal mass samples when divided by the known change in $\frac{\Delta S}{S}$ should equal the normalized change $\frac{d\left(\frac{\Delta k}{k}\right)}{\frac{\Delta k}{k}} \bigg|_V$ measured with full and reduced equal volume samples divided by the known change in the mass $\frac{\Delta m}{m}$. The measured variation found between the values of these ratios will then indicate the precision of the measurement technique.

CONCLUSIONS

The normalized Doppler coefficient can be separated from the total temperature coefficient inferred from reactivity measurements of temperature dependent samples by obtaining correction factors by measuring reactivity of auxiliary reduced density samples at each temperature. These data are also used to measure the precision of the reactivity oscillator temperature measurement technique.

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