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**ELASTOHYDRODYNAMIC PRINCIPLES APPLIED TO  
THE DESIGN OF HELICOPTER COMPONENTS**

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# ELASTOHYDRODYNAMIC PRINCIPLES APPLIED TO THE DESIGN OF HELICOPTER COMPONENTS

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## Abstract

Elastohydrodynamic principles affecting the lubrication of transmission components are presented and discussed. Surface temperature of the transmission bearings and gears affect elastohydrodynamic film thickness. Traction forces and sliding as well as the inlet temperature determine surface temperatures. High contact ratio gears cause increased sliding and may run at higher surface temperatures. Component life is a function of the ratio of elastohydrodynamic film thickness to composite surface roughness. Lubricant starvation reduces elastohydrodynamic film thickness and increases surface temperatures. Methods are presented which allow for the application of elastohydrodynamic principles to transmission design in order to increase system life and reliability.

## Notation

E	elastic modulus, lb/in. <sup>2</sup>
E'	equivalent elastic modulus, lb/in. <sup>2</sup>
e	exponent, 2.718
f	friction coefficient
G	dimensionless material parameter, $\alpha E'$
$H_{min}$	dimensionless film parameter, $h/R$
$h_{min}$	EHD film thickness, in. or $\mu$ in.
P	pressure in contact zone, lb/in. <sup>2</sup>
$Q_m$	speed factor
R	equivalent radius, in.
$R_{1,2}$	radius of contacting rollers, in.
U	dimensionless speed parameter, $\mu_0(U_1 + U_2)E'R$
$U_{1,2}$	tangential velocity of rolling contact, in./sec
$V_s$	sliding velocity, ft/min
W	dimensionless load parameter, $W'/E'R$
$W'$	load per unit width, lb/in.
X	distance along the line of action, in.

Z	length of line of action, in.
$\alpha$	pressure viscosity coefficient, in. <sup>2</sup> /lb
$\beta$	ratio of central film thickness
$\delta$	Poisson ratio
$\theta$	gear pressure angle, deg
$\Lambda$	EHD film to roughness parameter
$\mu$	lubricant viscosity at contact pressure, lb-sec/in. <sup>2</sup>
$\mu_0$	lubricant viscosity at atmospheric pressure, lb-sec/in. <sup>2</sup>
$\sigma$	composite surface roughness, rms $\mu$ in.
$\sigma_{1,2}$	surface roughness of contacting surfaces, rms $\mu$ in.
$\phi_T$	film thickness reduction factor
$\psi$	inlet boundary
$\omega$	rotational speed, rad/sec

The lubrication of a helicopter transmission is a complex problem involving many technical areas. These include fluid hydrodynamics, materials, lubricant technology, metrology, surface chemistry, heat transfer, and kinematics. Under adverse conditions, gears may fail from scoring, pitting, wear or tooth breakage. Scoring and wear may be caused from inadequate lubrication or excessive load and is generally a short life failure. Gear tooth superficial pitting may occur early from insufficient elastohydrodynamic film; under proper lubrication however, the gear would operate a long time without a surface fatigue spall. To have long life operation in gear systems, adequate elastohydrodynamic films must be present to prevent metal-to-metal contact or, if this is not possible, a boundary lubrication additive will be needed which protects the rubbing surfaces from scoring and wear. In most gear applications, a combination of elastohydrodynamic and boundary lubrication exists.

In contrast to gears, rolling-element bearings and ball bearings will fail due either to surface fatigue spalling or wear. Elastohydrodynamic lubrication directly affects bearing life and performance. As a result, lubricant selection becomes of prime importance to transmission operation and reliability. Since operating temperature directly affects lubricant viscosity and thus elastohydrodynamic lubrication, careful consideration must be

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given to heat generation and oil cooling. In view of the aforementioned, it is the objective of this paper to describe and discuss some of the parameters which affect the lubrication of bearings and gears in a transmission system.

### What is Elastohydrodynamic Lubrication

Elastohydrodynamic lubrication (EHD) is the separation of two loaded rolling or sliding surfaces by a thin film of lubricant a few micro-inches thick. EHD lubrication combines the hydrodynamic properties of the lubricant with the elastic properties of the material. When two nonconforming lubricated surfaces roll or slide over each other, the lubricant is drawn into the contact area. The contact area is elastically deformed and the contact pressure increases the viscosity several orders of magnitude. This increase of viscosity with pressure is commonly represented as

$$\mu = \mu_0 e^{\alpha P} \quad (1)$$

where  $\mu_0$  is the atmospheric viscosity and  $\alpha$  is the pressure-viscosity coefficient, and  $P$  is the contact pressure. This increase in viscosity prevents the lubricant from being squeezed out of the contact area and also prevents the mating surfaces from coming into contact. The load is thus transmitted from one element to the other through the lubricant film.

The first EHD film thickness equation was developed by A. N. Grubin in 1949<sup>1</sup> for highly loaded elastic cylinders in contact. This model which was for line contact did not consider side leakage. The deflection under EHD conditions was assumed the same as for dry contact. The assumption is reasonably valid for low speed operation but at high speeds the contact geometry of the bodies is changed.

The Grubin model was later modified.<sup>2</sup> For line contact used in gearing and roller bearings, the minimum film thickness at the trailing edge of the contact zone is expressed as

$$h_{\min} = 1.6 \frac{G^{0.6} E'^{0.03} R^{0.43}}{W^{0.13}} [\mu_0 (V_1 + V_2)]^{0.7} \quad (2)$$

This equation when presented in dimensionless form is given as

$$H_{\min} = 1.6 \frac{G^{0.6} U^{0.7}}{W^{0.13}} \quad (3)$$

$H_{\min}$  is the dimensionless film thickness equal to  $h_{\min}/R$ ,  $G$  is the material parameter, equal to  $\alpha E'$ ;  $W$  is the load parameter, equal to  $W'/E'R$ ; and  $U$  is the speed parameter;  $\mu_0 (V_1 + V_2)/E'R$ . The equivalent elastic properties of two cylinders of the same material  $E'$  equals to  $E/(1 - \delta^2)$ . D. Dawson further modified this for line contact<sup>3</sup> as follows:

$$H_{\min} = \frac{2.65 G^{0.54} U^{0.76}}{W^{0.13}} \quad (4)$$

X-Ray measurements of film thickness<sup>4</sup> indicates that the exponent for  $W$  will be considerably larger for loads over 200 000 psi maximum Hertz stress.

Until just recently, the effects of side leakage were not considered and the solutions for the EHD film were based on two dimensional analysis. In point contact such as ball bearings, considerable reduction of the EHD film occurs at the edge because of side leakage. An analytical program has been completed which calculates the effect of side leakage in a three dimensional analysis of the point contact.<sup>5</sup>

### Geometry Considerations

Contact geometry of gears and bearings can be represented by two contacting cylinders. The geometric similarity outside the contact zone is not of importance.

When using contacting cylinders to approximate the contact of machine elements, it is useful to introduce the concept of an equivalent cylinder. It is assumed that the undeformed cylinders are separated by a minimum film thickness  $h_{\min}$  (fig. 1). A cylinder with equivalent radius  $R$  on a flat surface and with the same minimum film thickness is shown in Fig. 1(b). The equivalent radius is

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad (5)$$

If  $R_1$  and  $R_2$  lie on the same side of the common tangent, then

$$R = \frac{R_1 R_2}{R_1 - R_2} \quad (6)$$

The tangential velocities are

$$U_1 = \omega_1 R_1 \quad (7)$$

$$U_2 = \omega_2 R_2 \quad (8)$$

The geometry of an involute gear contact is shown in Fig. 2. Contact at distance  $X$  from the pitch point can be represented by two cylinders rotating at the angular velocity of the wheels. Equivalent radius, from Eq. (5), is

$$R = \frac{(R_1 \sin \theta + X)(R_2 \sin \theta - X)}{(R_1 + R_2) \sin \theta} \quad (9)$$

Contact speeds from Eqs. (7) and (8) are

$$U_1 = (R_1 \sin \theta + X) \omega_1 \quad (10)$$

$$U_2 = (R_2 \sin \theta - X) \omega_2 \quad (11)$$

### Temperature Effects

When calculating the EHD film thickness, it is

often found that the calculated film is adequate for metal separation. However, in many applications, wear and metal contact is found to occur under actual operation conditions. The primary reason for this is the temperature used to determine inlet oil viscosity and pressure viscosity coefficient for film thickness calculations. The EHD film thickness is dependent on the inlet conditions of the lubricant only and is generally not dependent on what takes place in the contact zone. This was shown by Dyson<sup>6</sup> when he shows the film thickness of two rollers under pure rolling and under rolling with sliding conditions (fig. 3). The sliding reduces the traction between the two rollers but does not reduce the EHD film thickness more than a few percent.

Cheng<sup>7</sup> has developed a method for improving the EHD film thickness calculations for high speeds. He has shown that, at high speeds, heating of the lubricant occurs at the inlet region and can considerably reduce the EHD film thickness. Figure 4 is a plot of the film reduction factor  $\phi$  versus the speed factor  $Q_m$ . This film reduction factor, however, is rarely applied to gearing since gears generally do not operate at a speed that would show any appreciable effect from inlet heating. This curve does show, however, the importance of knowing the correct inlet oil temperature. This inlet oil temperature can be assumed to be the same as the metal surface temperature entering the contact zone and not necessarily the inlet oil temperature to the bearing or gear.

#### Effect of Lubricant Traction

In designing a transmission system, temperatures of bearings and gears must be determined analytically. To analytically determine the surface temperature, the traction force (or an equivalent friction coefficient) must be known. This traction force is dependent on the viscosity in the contact zone which changes considerably with sliding speed and load. As the contact zone undergoes sliding and increased temperature, the viscosity is reduced. Naylor<sup>8</sup> presents a plot of friction against sliding speed (fig. 5) to show how the friction changes with increased sliding speed and possible reasons for this change. Data from several investigators have shown similar curves.<sup>9</sup> Niemann and Stobel<sup>10</sup> conducted a study of the friction coefficient for gears and found that the coefficient was the same over the full surface of the gear tooth. They also arrived at a method of calculating gear tooth friction using test data for a given lubricant. Figure 6 is a plot of measured and calculated gear tooth friction coefficient. However, for most of the reported values of friction coefficient for rolling and sliding contacts, the speed and loads are much less than actual conditions used in practice. For this reason, the friction coefficient in practice is usually lower for gearing than that shown. As an example, if the efficiency of a gear box with several tooth contacts is considered to be 98 percent, then the friction coefficient would have to be less than those usually given to match the losses in each gear tooth contact.

Once this coefficient of friction or viscosity of the lubricant in the contact zone is determined, the temperature of the surface can be calculated. This is done by calculating a heat input from friction coefficient, load and sliding velocity. A heat transfer program would then be used that determines the heat input to the gear and lubricant which would give the metal surface temperature. Since the sliding velocity increases from the pitch point to the end of tooth contact, it would be expected that the higher temperature would be at the ends of tooth contact. A. Gu<sup>11</sup> calculated the temperature in the gear tooth contact under light load and showed a 20° F temperature rise from pitch point to the end of tooth contact.

High-contact ratio gearing is currently being considered for advanced transmission application for reasons such as noise reduction, improved load distribution and increased gear mesh capacity. Figure 7 is a plot showing the effect of pressure angle on sliding and addendum for high-contact ratio gearing. The increased addendum for the higher pressure angle gears give higher sliding conditions at reduced loads. Since the higher sliding may increase the gear surface temperature, which has more effect on EHD film thickness than load, the high-contact ratio gearing may suffer from decreased EHD film thickness.

If an insufficient amount of lubricant reaches the contacting surfaces increased surface temperature will occur because of poor cooling. Accordingly, the EHD film will also be reduced because of lubricant starvation as illustrated in Fig. 8.<sup>12</sup> Lubricant starvation can also occur even when generous amounts of lubricant are present but are not reaching the contacting surfaces. As a result, under some conditions of high-speed and load, more than one oil jet should be used to provide good cooling of the gear surfaces.

The same lubrication principles applicable to gearing are also applicable to rolling-element bearings. Harris<sup>13,14</sup> has expanded the rolling-elements bearing analysis of Jones<sup>15-17</sup> to include elastohydrodynamic effects in rolling-element bearings. The analysis allows for the calculation of traction and bearing temperature. Additionally, life predictions are modified based upon elastohydrodynamic effects. While approximations of bearing heat generation and temperature can be obtained by assuming a constant coefficient of friction at the inner and outer race contacts, research<sup>18,19</sup> has shown that friction coefficients vary significantly with load, speed, and temperature in addition to the type of lubricant used and bearing geometry. As a result, bearing temperatures must be predicted using elastohydrodynamic theory in conjunction with calculations of the bearing dynamics.

#### Effect on Life

Surface topography is important to the EHD lubrication process. EHD theory is based on the assumption of perfectly smooth surfaces, that is,

no interaction of surface asperities. Actually, of course, this is not the case. An EHD film of several millionths of an inch can be considered adequate for highly loaded rolling elements in a high-temperature environment. However, the calculated film might be less than the combined surface roughness of the contacting elements. If this condition exists, surface asperity contact, surface distress (in the form of surface glazing and pitting), and surface smearing or deformation can occur. Extended operation under these conditions can result in high wear, excessive vibration, and seizure of mating components. A surface-roughness criterion for determining the extent of asperity contact is based upon the ratio of film thickness to a composite surface roughness. The film parameter  $\Lambda$  is

$$\Lambda = \frac{h}{\sigma} \quad (12)$$

where composite roughness  $\sigma$  is

$$\sigma = (\sigma_1^2 + \sigma_2^2)^{1/2} \quad (13)$$

and  $\sigma_1$  and  $\sigma_2$  are the rms roughness of the two surfaces in contact. Figure 9 is a plot, based upon experimental data, of percent of complete asperity or surface separation (percent film) as a function of film parameter  $\Lambda$ . At values of less than 1, surface smearing or deformation, accompanied by wear, will occur. When  $\Lambda$  is between 1 and 1.5, surface distress can occur. For values between 1.5 and 3, some surface glazing occurs. At values of 3 or greater, minimal wear can be expected.

Several investigators have shown that improved surface finish of the gear teeth will give considerable improvement in both scoring<sup>20</sup> and fatigue life.<sup>21</sup> The reason surface finish improves the fatigue life of a gear set is that it gives an increase in  $\Lambda$  which in turn affects the Hertz stress in the contact zone. The larger  $\Lambda$  value gives a lower average subsurface shearing stress that is directly related to fatigue life (life is inversely proportional to the ninth power of stress). Values of  $\Lambda$  will of course also increase with increases in viscosity, pressure viscosity coefficient and/or speed.

Tallian<sup>22</sup> and Skurka<sup>23</sup> studied the effect of values of  $\Lambda$  on roller-element bearing life. Figure 10 is a composite curve of these data showing relative life as a function of  $\Lambda$ . For most bearing applications  $\Lambda$  is generally between 1.5 to 2.0. However, in most gear applications  $\Lambda$  is generally less than 1.5. Under these conditions, boundary lubrication becomes important due to surface asperity interaction. As a result, extreme-pressure additives in the lubricant can significantly increase the load carrying capacity of gears. The extreme-pressure additives in the lubricating fluid form a film on the surfaces by chemical reaction, adsorption, and/or chemisorption. These boundary films can be less than 1  $\mu$ in. to several microinches thick.<sup>24</sup>

## Design Application

The application of elastohydrodynamic principles to helicopter transmission design and analysis is a rather new concept. Usually, gear selection and design were based upon Block's<sup>25</sup> critical temperature theory. Bearing selection and design were based upon load capacity and life predictions rather than considering the lubricant and operative environment. The designer, after making his preliminary layouts, can select a lubricant and assume for purposes of his elastohydrodynamic calculations an oil inlet temperature to the gear and bearing components. Based upon this oil temperature and the respective loading and speed of each of the components, first order EHD calculations can be made. These first order approximations can aid in recognizing those components which may be potential problems. Decisions such as changing the lubricant type or oil inlet temperature can be made. Design or component criteria such as component geometry, surface finish, and steel can also be changed. As these changes are made, additional EHD calculations will aid in the design refinement. Further, system design to maintain proper oil temperatures can be determined with reasonable assurance that the oil will adequately perform its elastohydrodynamic function.

## Concluding Remarks

The application of elastohydrodynamic technology can give the transmission designer the opportunity to increase the load capacity and improve the life and reliability of helicopter transmission systems. An analytical program should be conducted for each transmission system that will optimize the surface finish, oil inlet temperature and sliding velocities of the gears and bearings. In many systems the cost of improved surface finish may be justified by increased system reliability and life. Effort should be expended to determine surface traction and temperature in order to accurately calculate elastohydrodynamic film thickness. For a particular design improved EHD condition can be obtained by lowering the surface temperature. When adequate EHD films cannot be provided, surface protection can be obtained by using extreme pressure additives. However, reduced system life can be expected.

In gears and bearings operating at high speeds and loads, special care should be used to prevent starvation of the contacting surfaces which would result in wear and an over temperature conditions and eventual system failure.

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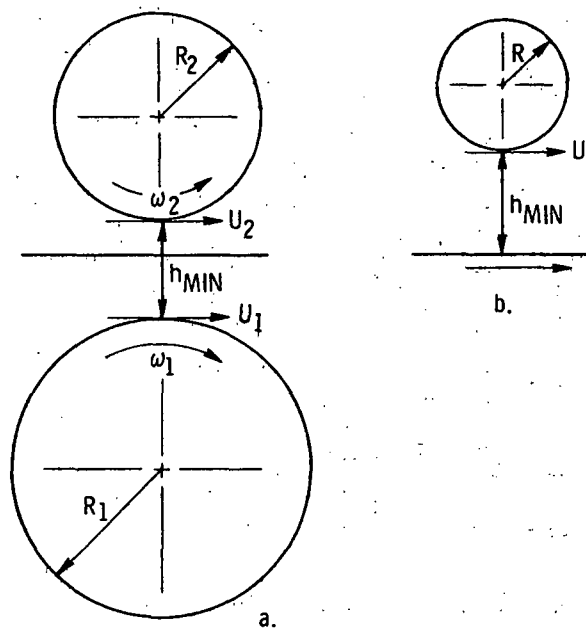


Figure 1. - Relationship between two cylinders with a lubricant film between them, a, and model of an equivalent cylinder, b (ref. 2).

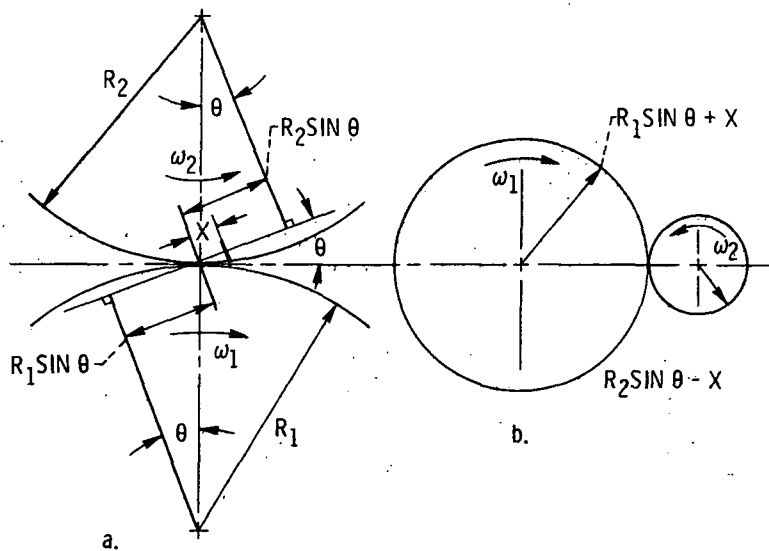


Figure 2. - Involute gears in contact, a, and equivalent cylinders, b (ref. 2).

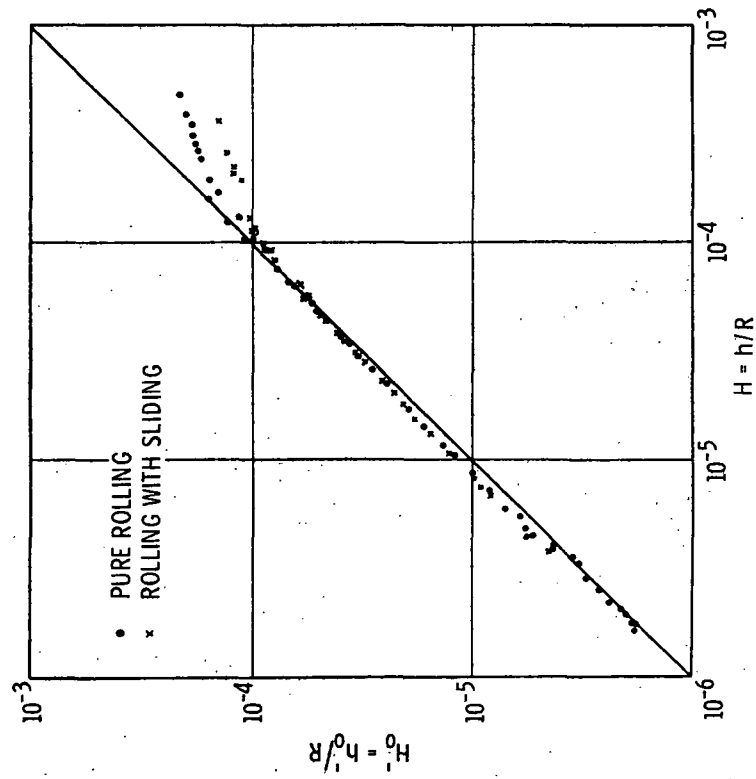


Figure 3. - Lubricant B: comparison of measured non-dimensional oil-film thickness  $H'_0 = h'_0/R$  with predicted values  $H = h/R$  (ref. 3).

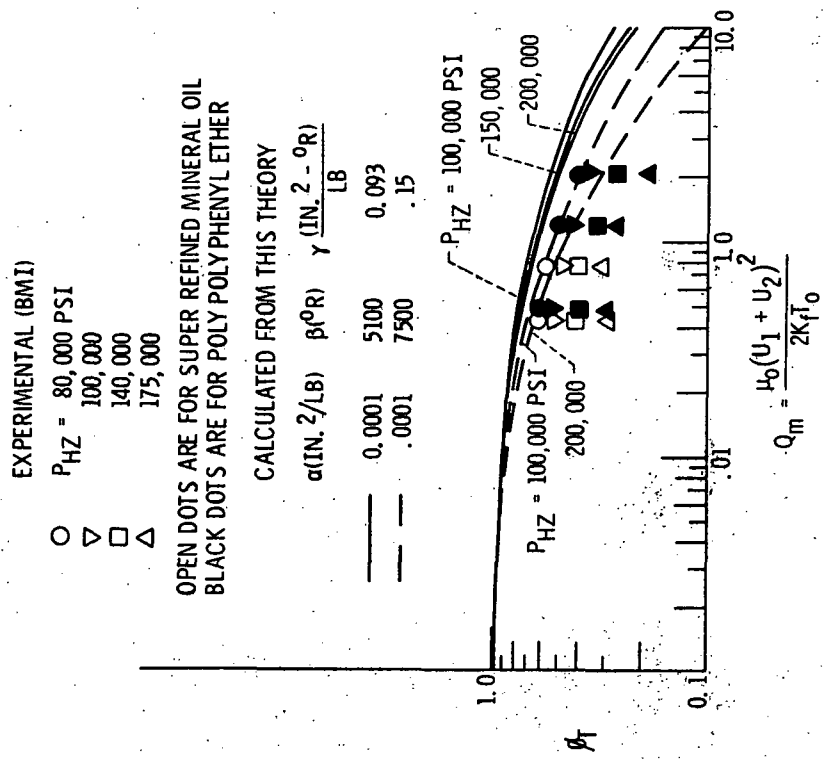


Figure 4. - Comparison of  $\phi_t$  with experimental data (ref. 7).



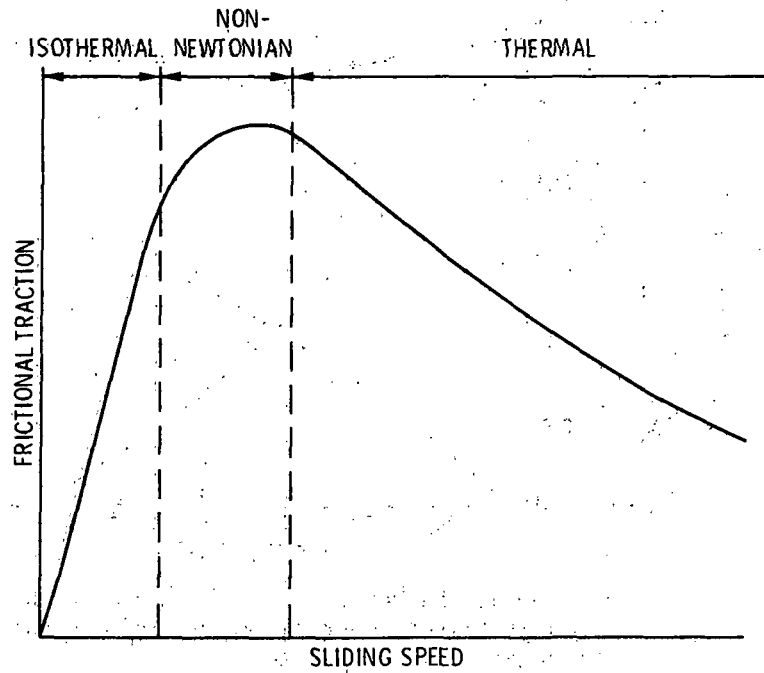


Figure 5. - Plot of friction against sliding speed (ref. 8).

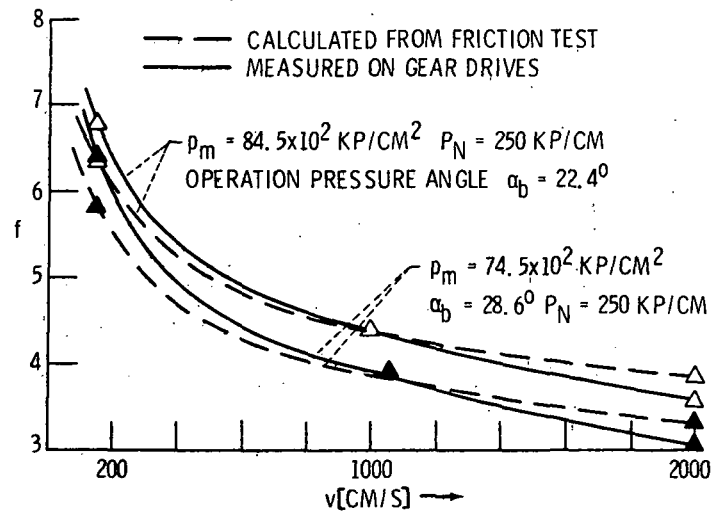


Figure 6. - Application of friction test data on spur gears (ref. 10).

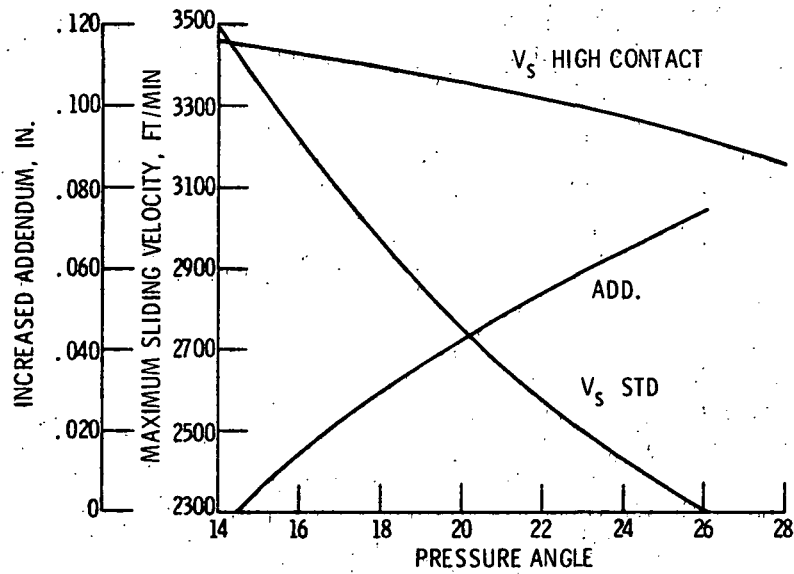


Figure 7. - Effect of increased addendum on maximum sliding velocity for high contact ratio gear and maximum sliding velocity for standard gear without increased addendum.

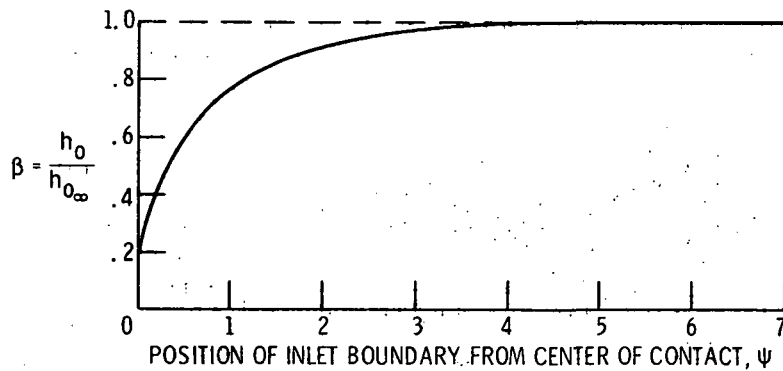


Figure 8. - The influence of the position of the inlet boundary  $\psi$  on the central film thickness  $\beta$  (ref. 12).

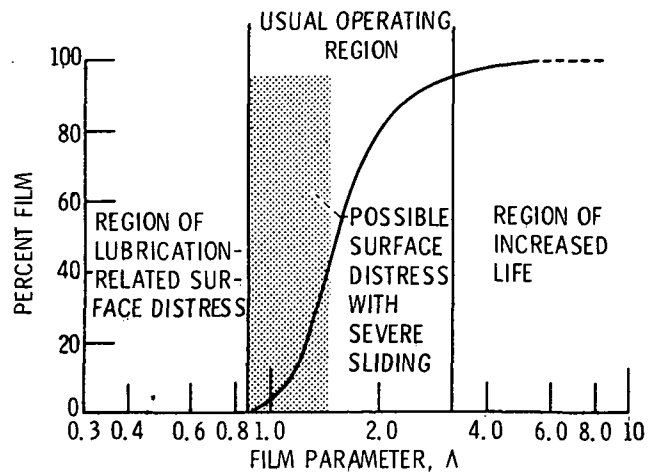


Figure 9. - Percent film as a function of film parameter.

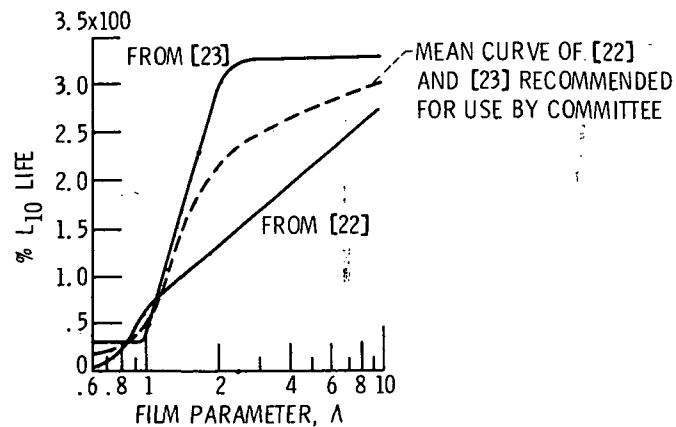


Figure 10. - Percent life, as function of film parameter,  $\Lambda$  (ref. 26).