

Gravitational-Wave Observations as a Tool for  
Testing Relativistic Gravity\*

DOUGLAS M. EARDLEY, DAVID L. LEE,<sup>†</sup> and ALAN P. LIGHTMAN

California Institute of Technology, Pasadena, California 91109

ROBERT V. WAGONER

Center for Radiophysics and Space Research,

Cornell University, Ithaca, New York 14850

CLIFFORD M. WILL<sup>‡</sup>

University of Chicago, Chicago, Illinois 60637

ABSTRACT

Gravitational-wave observations can be powerful tools in the testing of relativistic theories of gravity. Future experiments should be designed to search for six different types of polarization, and for anomalies in the propagation speed of the waves:

$|c_{\text{grav. waves}} - c_{\text{em waves}}| \gtrsim 10^{-7} c_{\text{em waves}}$ . This letter outlines the nature and implications of such measurements.

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<sup>†</sup> Imperial Oil Predoctoral Fellow.

<sup>‡</sup> Enrico Fermi Fellow.

Several viable gravitation theories now exist that differ radically when describing strong gravitational fields, but that are identical to each other and to general relativity in the "post-Newtonian limit." During the next twenty years, one will probably not be able to distinguish these theories from general relativity or from each other by means of "solar-system experiments" (gravitational redshift, perihelion shift, light deflection, time delay, gyroscope precession, lunar-laser ranging, gravimetry, Earth rotation, ...). However, gravitational-wave experiments offer hope: These theories differ in their predictions of (i) propagation speed, and (ii) polarization properties of gravitational waves.

Propagation speed: Some of the competing theories<sup>1-3</sup> predict the same propagation speed for gravitational waves ( $c_g$ ) as for light ( $c_{em}$ ). But others<sup>4,5</sup> predict a difference that, in weak gravitational fields, is typically

$$(c_g - c_{em})/c \sim (1/c^2) \times |\text{Newtonian potential}|$$

$\sim 10^{-7}$ , for waves travelling in our region of the Galaxy or in the field of the Virgo cluster. An experimental limit of  $\lesssim 10^{-8}$  would disprove most such theories and would stringently constrain future theory-building. Perhaps the most promising way to obtain such a limit is by comparing arrival times for gravitational waves and for light that come from the onset of a supernova, or from some other discrete event. If current experimental efforts continue unabated, by 1980 one may detect gravitational-wave bursts from supernovae in the Virgo cluster ( $\sim 3$  supernovae per year). Then a limit of

$$|c_g - c_{em}|/c \lesssim 10^{-9} \times (\text{time lag precision})/(1 \text{ week})$$

will be possible.

Polarization: All of the currently viable theories fall into a class called "metric theories of gravity."<sup>6,7</sup> Recently we have completed an

analysis of the polarization properties of the most general weak, plane, null gravitational wave permitted by any metric theory. (Details will be published elsewhere.<sup>8</sup> Our considerations also apply to waves which are approximately, rather than exactly, null.) We find that the most general wave is composed of six modes of polarization (general relativity has only two), as follows.

Use coordinates  $txyz$ . Let the wave propagate in the  $+z$  direction. The wave is characterized by six amplitudes which depend only on "retarded time"  $u$ , where  $u \equiv t - z/c$ . Our analysis describes these amplitudes by two real functions  $\Psi_2(u)$ ,  $\Phi_{22}(u)$  and the real and imaginary parts of two complex functions  $\Psi_3(u)$ ,  $\Psi_4(u)$ . These functions are related to those components of the Riemann tensor which determine the action of the wave on a detector<sup>9</sup> by

$$\begin{aligned}\Psi_2 &= -\frac{1}{6} R_{zozo} \\ \Psi_3 &= \frac{1}{2} (-R_{xozo} + i R_{yozo}) \\ \Psi_4 &= R_{yoyo} - R_{xoxo} + 2i R_{xoyo} \\ \Phi_{22} &= -(R_{xoxo} + R_{yoyo}).\end{aligned}$$

Figure 1 shows the action of each mode on a sphere of test bodies.  $\Psi_4$  and  $\Phi_{22}$  are purely transverse,  $\Psi_2$  is purely longitudinal, and  $\Psi_3$  is mixed.

These waves can be classified in a Lorentz-invariant manner according to the vanishing or nonvanishing of certain of the amplitudes. Imagine many observers in different Lorentz frames, some moving with respect to each other, but all measuring the same 4-momentum of the wave. The amplitudes transform between observers in a complicated way [cf. Eq. (1) below] but the waves fall into these invariant classes:

Class II<sub>6</sub>.  $\Psi_2 \neq 0$ . All observers in such Lorentz frames measure a non-zero amplitude in the  $\Psi_2$  mode, and agree on the value of this amplitude. (But they will generally disagree about the presence or absence and amplitude of all other modes.)

Class III<sub>5</sub>.  $\Psi_2 \equiv 0 \neq \Psi_3$ . All observers agree on the absence of  $\Psi_2$  and the presence of  $\Psi_3$ . (But they generally disagree about the presence or absence of  $\Psi_4$  and  $\Phi_{22}$ .)

Class N<sub>3</sub>.  $\Psi_2 \equiv 0 \equiv \Psi_3$ ;  $\Psi_4 \neq 0 \neq \Phi_{22}$ . All observers agree about the presence or absence of all modes.

Class N<sub>2</sub>.  $\Psi_2 \equiv 0 \equiv \Psi_3$ ;  $\Psi_4 \neq 0 \equiv \Phi_{22}$ . All observers agree.

Class O<sub>1</sub>.  $\Psi_2 \equiv 0 \equiv \Psi_3$ ;  $\Psi_4 \equiv 0 \neq \Phi_{22}$ . All observers agree.

Class II<sub>6</sub> is the most general; as one demands that successive amplitudes vanish identically, one descends to less and less general classes. The class of the most general permitted wave in some currently viable metric theories is: General relativity,<sup>1</sup> N<sub>2</sub>; Dicke-Brans-Jordan,<sup>2</sup> N<sub>3</sub>; Will-Nordtvedt,<sup>3</sup> III<sub>5</sub>; Ni's new theory,<sup>4</sup> II<sub>6</sub>; and Lightman-Lee,<sup>5</sup> II<sub>6</sub>. All these but Dicke-Brans-Jordan have the same post-Newtonian limit as general relativity, for a reasonable choice of cosmological model.

We see that measuring the polarization of gravitational waves provides a sharp experimental test of theories of gravity. The class of the "correct" theory is at least as general as that of any observed wave. The observation of a wave more general than N<sub>2</sub> would contradict general relativity but would be consistent with other viable theories.<sup>2-5</sup> Weber<sup>10</sup> has initiated such experiments by searching for the  $\Phi_{22}$  mode, with negative results.

To test theories, an experimenter must classify the waves that he detects. If he knows the direction of a wave a priori (e.g., from a particular supernova), he can directly extract the amplitude of each mode from his

data and determine the class. If he does not know the direction, he cannot extract the amplitudes or determine the direction without applying some further assumption to his data (e.g., that the wave is no more general than  $N_3$  and is therefore purely transverse). But he can usually say something definite about the class of the wave:

- (i) If the driving forces in his detector are not in any one plane, the wave is  $II_6$  or  $III_5$ .
- (ii) If the driving forces are in a plane and are "pure monopole" [as in Fig. 1(c)], the wave is not  $N_2$ .
- (iii) If the driving forces are in a plane and are "pure quadrupole" [as in Fig. 1(a)], the wave is not  $O_1$ .
- (iv) Otherwise the wave is either  $II_6$ ,  $III_5$ , or  $N_3$ .

We now sketch the arguments that lead to these results about polarization of gravitational waves in metric theories. Consider a weak, plane, null wave described by a linearized Riemann tensor,  $R_{\alpha\beta\gamma\delta}(u)$ , with  $\nabla u \cdot \nabla u = 0$ . Work in an approximately constant quasi-orthonormal null tetrad<sup>11</sup>  $(\underline{k}, \underline{\ell}, \underline{m}, \overline{\underline{m}})$ , where  $\underline{k} = \nabla u$ . The Bianchi identities imply that there are six functionally independent real components of the Riemann tensor; take them, in the notation of Newman and Penrose,<sup>11</sup> to be  $\Psi_2, \Psi_3, \Psi_4, \Phi_{22}$ , as above. Consider the "little group"<sup>12</sup> of Lorentz transformations of the tetrad which fix  $\underline{k}$ :  $\underline{k}' = \underline{k}, \underline{m}' = e^{i\varphi}(\underline{m} + \alpha \underline{k}), \underline{\ell}' = \underline{\ell} + \overline{\alpha} \underline{m} + \alpha \overline{\underline{m}} + \overline{\alpha} \overline{\underline{k}}$ , where  $\alpha$  is complex and  $\varphi$  is a real phase. The action of  $E(2)$  on the Riemann tensor of a wave is

$$\begin{aligned}
\psi_2' &= \psi_2 \\
\psi_3' &= e^{-i\phi}(\psi_3 + 3\bar{\alpha}\psi_2) \\
\psi_4' &= e^{-2i\phi}(\psi_4 + 4\bar{\alpha}\psi_3 + 6\bar{\alpha}^2\psi_2) \\
\phi_{22}' &= \phi_{22} + 2\alpha\psi_3 + 2\bar{\alpha}\bar{\psi}_3 + 6\bar{\alpha}\alpha\psi_2.
\end{aligned} \tag{1}$$

The Lorentz-invariant classes of waves that are defined above correspond precisely to the different representations of  $E(2)$  that can arise through Eqs. (1).

The helicity (spin) decomposition of a wave is Lorentz-invariant only for classes  $N_3$ ,  $N_2$ , and  $O_1$ . Theories in classes  $N_3$ ,  $N_2$ , and  $O_1$  provide a unitary representation of  $E(2)$  which is a direct sum of 1-dimensional massless particle representations,<sup>12-14</sup> containing at most spins 0,  $\pm 2$ . Theories in classes  $II_6$  and  $III_5$  provide a reducible representation of  $E(2)$  which is not completely reducible and is therefore nonunitary<sup>14</sup>; it is likely that such theories cannot be quantized. No other representation of  $E(2)$  (such as one with "continuous spin"<sup>13</sup>) can occur.

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# FIGURE CAPTION

Fig. 1. The six polarization modes of a weak, plane, null gravitational wave permitted in the generic metric theory of gravity. Shown is the displacement that each mode induces on a sphere of test particles. The wave is propagating in the  $+z$  direction (arrow at upper right) and has time dependence  $\cos \omega t$ . The solid line is a snapshot at  $\omega t = 0$ , the broken line one at  $\omega t = \pi$ . There is no displacement perpendicular to the plane of the figure.



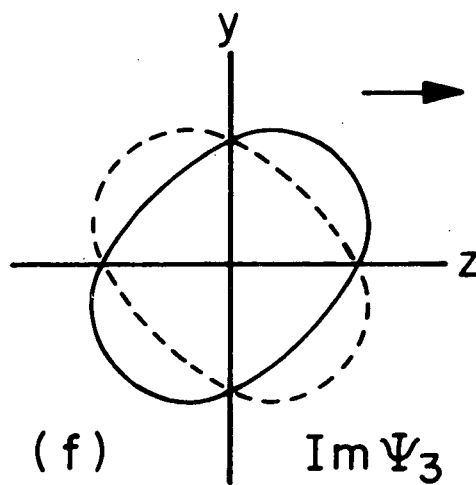
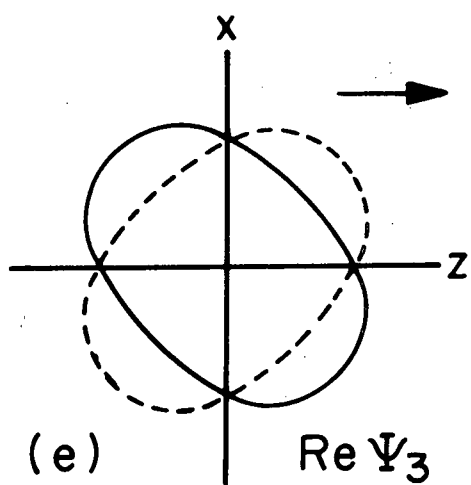
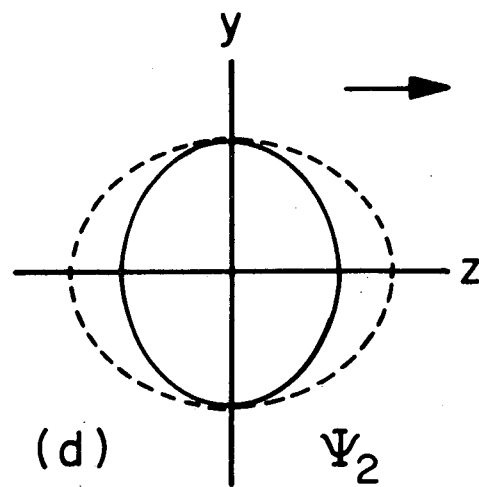
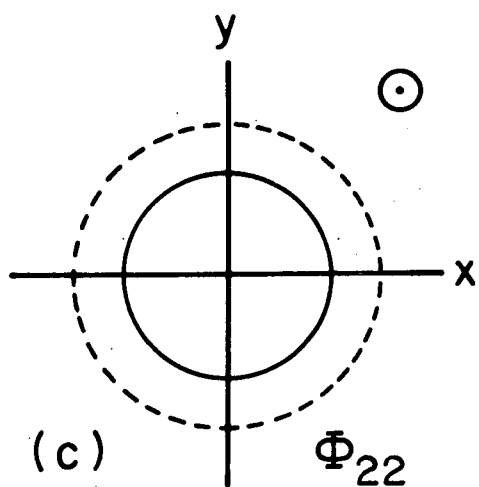
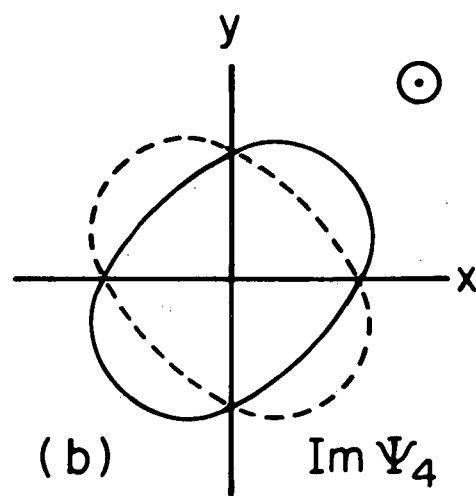
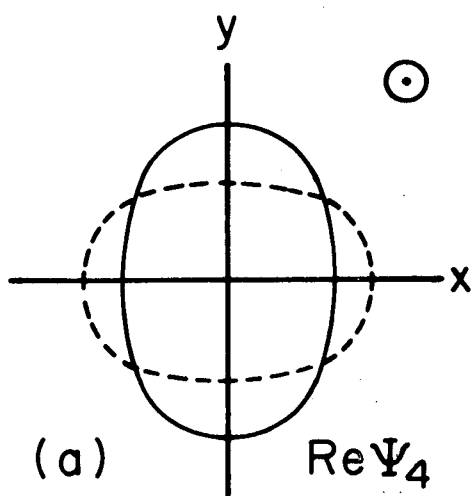


Fig. 1