NASA TECHNICAL NOTE

ANALYSIS OF NOISE PRODUCED BY AN ORDERLY STRUCTURE OF TURBULENT JETS

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1. Report No. NASA TN D-7242	2. Government Accession No.	3. Recipient's Catalog No.		
4. Title and Subtitle ANALYSIS OF NOISE PRODUCED BY AN ORDERLY STRUCTURE OF TURBULENT JETS		5. Report Date April 1973		
		6. Performing Organization Code		
7. Author(s) Jay C. Hardin		8. Performing Organization Report No. L-8843		
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23365		10. Work Unit No. 501-04-01-01		
		11. Contract or Grant No.		
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note		
		14. Sponsoring Agency Code		

16. Abstract

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17. Key Words (Suggested by Author(s)) Jet noise Toroidal vortices "Orderly" structure	18. Distribution Statement Unclassified — Unlimited			
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 29	22. Price* $3.00	

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STRUCTURE OF TURBULENT JETS[*]

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SUMMARY

The "orderly" structure which has been observed recently by numerous researchers within the transition region of subsonic turbulent jets is analyzed to reveal its noise-producing potential. For a circular jet, this structure is modeled as a train of toroidal vortex rings which are formed near the jet exit and propagate downstream. The noise produced by the model is evaluated from a reformulation of Lighthill's expression for the far-field acoustic density, which emphasizes the importance of the vorticity within the turbulent flow field. It is shown that the noise production occurs mainly close to the jet exit and depends primarily upon temporal changes in the toroidal radii. The analysis suggests that the process of formation of this regular structure may also be an important contribution to the high-frequency jet noise. These results may be helpful in the understanding of jet-noise generation and in new approaches to jet-noise suppression.

INTRODUCTION

The problems created by the noise of jet aircraft are apparent to even a casual observer and need no further documentation. Public interest has been aroused to the point where noise has actually displaced aerodynamic and economic factors as the overriding design consideration in certain applications and has recently played a part in the cancellation of a major aircraft development project. Thus, alleviation of this noise must be a prime research goal for a viable aircraft industry. This paper presents an analysis of noise production by an "orderly" structure of turbulent jets which yields new understanding of the generation of aerodynamic noise and suggests potential techniques for suppression.

The acoustic-analogy theory of aerodynamically generated sound proposed by Lighthill (refs. 1 to 3) showed that the intensity of the jet noise may be related to certain fourth-order correlations of the turbulent velocities in the jet. The theory did not provide a workable solution, however, as such correlations are extremely difficult to measure and

the coordinate system in which the measurements should be made is somewhat ambiguously defined. Thus, in most jet-noise research, the question of the turbulent structure of the jet has been in effect bypassed by attempts to relate the noise to certain gross jet parameters such as mean velocity profile or mean density. Such research, however, leads to a trial-and-error technology since the basic physics of the phenomena are not understood.

If, on the other hand, one adopts the view that the gross jet parameters act upon the turbulence, which in turn produces the noise, it is apparent that a better understanding of the turbulent structure of the jet might well provide the insight necessary to further techniques for suppression of jet noise. Fortunately, there have been some recent developments in this area. In the past few years, several research studies have been reported which indicate that a large-scale, orderly structure lies hidden within the chaotic nature of turbulent jets, particularly in the transition region where the jet is not yet fully developed, although such structure is not obvious from Eulerian measurements of the velocity field (ref. 4). One of the first such research studies was reported in a paper by Crow and Champagne (ref. 5), who used fog to photograph subsonic air jets with Reynolds numbers up to 10^5. They found that the transition region of the jet flow consisted of a series of "puffs" and stated that "The photographs led us to imagine turbulence in the transitional region of a jet as a vortex train, a train of loosely packed vortex rings only weakly dependent on the circumstances of their origin." These vortex rings are randomly generated in time and form at an average Strouhal number of 0.3 based upon vortex shedding frequency, exit velocity, and jet diameter. They are also highly structured and stable, since ring production at the Strouhal number 0.3 is shown to be favored by a nonlinear saturation mechanism.

In a second paper, Wooldridge and Wooten (ref. 6) report the results of hot-wire anemometer measurements in the initial region of a subsonic jet. They found a coherent pressure field across the core region of the jet and stated that "The potential flow fluctuations in the core, which have a scale corresponding roughly to the local thickness of the shear layer, might be produced by doughnut-like vortex rings propagating away from the jet lip." These conjectures are substantiated by the experimental results of Beavers and Wilson (ref. 7) on circular water jets with Reynolds numbers between approximately 500 and 3000. They found that for values of the Reynolds number above 470, the pattern consists of a fairly regular stream of vortex rings being generated at the sharp edge and carried downstream. These rings formed at an average Strouhal number of 0.63 based upon shedding frequency, jet diameter, and average jet velocity. If, as seems likely, the jet employed by Beavers and Wilson is supposed to have a parabolic rather than a "tophat" velocity profile, then their result can be seen to agree with that of Crow and Champagne. Reference 7 also gives a thorough historical survey of other instances of the phenomenon of vortex-ring generation in jets.

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A second fascinating feature of these studies is that the vortex rings may be responsible for most of the jet noise. Discussing some previous additional work on water jets, Crow and Champagne observe "we could see water waves radiating outward from above the region of puff formation. The chaotic turbulence further downstream did not appear to be a strong source of waves." In a further recent work, Michalke (ref. 8) has constructed a wave model of a turbulent jet. His analytical results indicate that axisymmetric modes of jet turbulence are much more efficient sound generators than nonaxisymmetric modes. He remarks that "It [the above result] suggests that a considerable reduction of jet noise could be achieved by suppressing axisymmetric pressure components in the jet."

It might be mentioned that the experimental results cited here were all obtained at Reynolds numbers at least an order of magnitude smaller than those found for commercial jets. However, there is some reason to believe this same phenomenon is present in practical jet engines as well, since the preferred Strouhal number of vortex-ring formation corresponds to the peak frequency of jet noise spectra. Crow and Champagne note that "the peak of the jet noise spectrum lies between [the Strouhal numbers of] 0.25 and 0.30 depending on angle from the jet axis. The coincidence [agreement with Strouhal number of vortex-ring formation] suggests that the vortex train is latent in jet turbulence at high Reynolds numbers and contributes to the emission of sound."

The interesting possibilities raised by these studies prompted the present investigation of the implications of a model of jet turbulence based upon a train of vortex rings. The model is developed for a circular jet and is rigidly axisymmetric. The rings are defined mathematically as toroidal vortices with finite cores and are assumed shed at random intervals. The analysis is entirely inviscid, on the basis of Lighthill's conclusion that viscosity plays a negligible role in sound generation. This leads to a description of the large-scale structure of turbulent jet flow. The sound generated by the structure is then described by a relation involving the vorticity distribution within the flow. Several conclusions about the nature of jet noise and potential methods of suppression are drawn from the analysis.

The discussion is divided into three major sections. In the first section, Lighthill's expression for the far-field acoustic density produced by a turbulent flow field is reformulated to yield the dependence of the sound upon the vorticity in the flow. In the second section, the aspects of the theory of toroidal vortices which are necessary to the model are discussed. It is also shown, by employing the expression derived in the first section, that a single vortex propagating in an inviscid, quiescent medium can produce no sound. Finally, in the last section, a model of the jet as a train of toroidal vortices is developed and the sound generated by the model determined.

SYMBOLS

A	cross-sectional area
$\vec{B}$	arbitrary vector
B	magnitude of vector $\vec{B}$
D	dimension of region containing turbulence
E	complete elliptic integral of the second kind
F(t)	time function
I_z	axial impulse of vortex
K	complete elliptic integral of the first kind
$\vec{L}$	vector, $\vec{\mathcal{L}} + \dfrac{\nabla v^2}{2}$
L_i	components of vector $\vec{L}$
$\vec{\mathcal{L}}$	Coriolis acceleration vector
$\mathcal{L}_i$	components of vector $\vec{\mathcal{L}}$
$\mathcal{L}_i^{(k)}$	components of vector $\vec{\mathcal{L}}$ contributed by kth vortex
M	Mach number of eddy
$Q_o(t)$	instantaneous source moment
$Q_i(t)$	instantaneous dipole moment
$Q_{ij}(t)$	instantaneous quadrupole moment

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$Q_{\ell mn}(t)$ source tensor

R jet radius

S surface

T self-induced kinetic energy of vortex

T_k self-induced kinetic energy of kth vortex

T_{ij} Lighthill's quadrupole moment density

U velocity

U_c self-induced convection velocity of vortex

U_o jet velocity

V volume

a_o ambient speed of sound

d distance from field point to observer point

e total kinetic energy of solenoidal field

ℓ eddy size

$\vec{n}$ outward normal vector

p_{ij} stress components

$q(\vec{y},t)$ source distribution

r radial coordinate of field point in cylindrical coordinate space

r_o radial coordinate of source point in cylindrical coordinate space

r_1 distance, $\left[(r - \eta)^2 + (z - \zeta)^2\right]^{1/2}$

r_2 distance, $\left[(r + \eta)^2 + (z - \zeta)^2\right]^{1/2}$

r_1' distance, $\left[(r - r_0)^2 + (z - z_0)^2\right]^{1/2}$

r_2' distance, $\left[(r + r_0)^2 + (z - z_0)^2\right]^{1/2}$

s   radial coordinate in polar coordinates

t   time

t_c age of vortex

u   typical turbulent velocity

$u_0(r,z;\eta,\zeta,\Gamma)$ radial velocity due to vortex

u_i velocity components

u_r radial velocity in cylindrical coordinate space

$u_{rj}(r,z)$ radial source velocity

$u_{ro}^{(k)}(r,z)$ externally induced radial velocity within kth vortex

u_θ angular velocity in cylindrical coordinate space

u_ϕ angular velocity in polar coordinate space

$\vec{v}$ solenoidal velocity vector

v   magnitude of vector $\vec{v}$

v_i components of vector $\vec{v}$

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$\vec{v}^{(k)}$ solenoidal velocity vector within kth vortex

$\vec{v}_o^{(k)}$ externally induced solenoidal velocity vector within kth vortex

$\vec{v}_s^{(k)}$ self-induced solenoidal velocity vector within kth vortex

w axial velocity in cylindrical coordinate space

$w_o(r,z;\eta,\zeta,\Gamma)$ axial velocity due to vortex

$w_o^{(k)}(r,z)$ externally induced axial velocity within kth vortex

$w_j(r,z)$ axial source velocity

$\vec{x}$ position vector

x magnitude of vector $\vec{x}$

x_i components of vector $\vec{x}$

$\vec{y}$ position vector

y_i components of vector $\vec{y}$

z axial coordinate of field point in cylindrical coordinate space

z_o axial coordinate of source point in cylindrical coordinate space

Γ circulation

Γ_k circulation of kth vortex

Δt time interval of vortex generation

Λ measure of flow complexity

ψ stream function

$\vec{\Omega}$ vorticity vector

Ω_θ angular component of vorticity vector

$$\alpha = \frac{r_2 - r_1}{r_2 + r_1}$$

γ circulation per unit length

δ core radius

δ_{ij} Kronecker delta function

ζ axial position of vortex

ζ_k axial position of kth vortex

η toroidal radius

η_k toroidal radius of kth vortex

θ angular coordinate in cylindrical coordinate space

λ wavelength of sound

$$\mu = \frac{\Omega_\theta}{r}$$

μ_k value of μ for kth vortex

ρ density

ρ_o ambient density

ρ_a acoustic density

ϕ angular coordinate in polar coordinate space

ω typical sound frequency

An asterisk denotes evaluation at $t - \dfrac{x}{a_o}$.

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AERODYNAMIC NOISE THEORY REVISITED

Lighthill, in his pioneering paper on aerodynamically generated sound (ref. 1), showed by an acoustic analogy that the density ρ in the far-field region surrounding a finite volume of turbulent flow must satisfy the equation

$$\frac{\partial^2 \rho}{\partial t^2} - a_o^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{1}$$

where a_o is the speed of sound in the ambient fluid and

$$T_{ij} = \rho u_i u_j + p_{ij} - a_o^2 \rho \delta_{ij}$$

is the quadrupole moment density of an equivalent acoustic source distribution. Here, u_i and p_{ij} are, respectively, the velocity and stress components in the general fluid motion. The basic free-space solution of equation (1) is

$$\rho_a\left(\vec{x}, t\right) = \rho - \rho_o = \frac{1}{4\pi a_o^2} \int_V \frac{\delta^2}{\delta y_i \delta y_j} \frac{T_{ij}\left(\vec{y}, t - \frac{d}{a_o}\right)}{d} \, d\vec{y} \tag{2}$$

where ρ_o and ρ_a are, respectively, the ambient and acoustic components of density, $d = |\vec{x} - \vec{y}|$, and $\delta/\delta y_i$ implies partial differentiation with respect to y_i with d held fixed.

In a recent paper, Crow (ref. 9) noted that the structure of the sound emission problem must depend upon the ratios of three lengths: the eddy size ℓ, the wavelength of the sound λ, and a dimension D of the region containing turbulence. In particular, the Mach number of the eddies $M \propto \ell/\lambda$ and a measure of the complexity of the flow $\Lambda \propto D/\ell$ were found to be critical parameters.

For $\Lambda \approx 1$ and $M \ll 1$, Crow applied the method of matched asymptotic expansions to show that Lighthill's solution was valid to order M^3 in the sound field if

$$T_{ij} \approx \rho_o v_i v_j \tag{3}$$

where v_i represents the solenoidal components of the velocity field. Thus, equation (3) replaces Lighthill's approximation $T_{ij} \approx \rho_o u_i u_j$, where u_i is the total velocity field, by

an expression which depends only upon the vorticity in the flow. Since the conditions $\Lambda \approx 1$ and $M \ll 1$ often hold in a subsonic jet, this result is employed in the following analysis to obtain an expression from which the sound generated by a turbulent jet is easily calculated when the vorticity field is known.

Note that since the divergence of a solenoidal field is zero,

$$\frac{\partial^2}{\partial y_i \partial y_j} v_i v_j = \nabla \cdot \left[\left(\vec{v} \cdot \nabla\right)\vec{v}\right] = \nabla \cdot \left(\vec{\mathcal{L}} + \nabla \frac{v^2}{2}\right) \tag{4}$$

where $\vec{\mathcal{L}} = \vec{\Omega} \times \vec{v}$, $\vec{\Omega} = \nabla \times \vec{v}$ is the vorticity, $v = |\vec{v}|$, and the vector identity

$$\left(\vec{B} \cdot \nabla\right)\vec{B} = \frac{\nabla B^2}{2} + \left(\nabla \times \vec{B}\right) \times \vec{B}$$

has been utilized. It might be noted that the vector $\vec{\mathcal{L}}$ is often referred to as the Coriolis acceleration. Thus, employing equations (3) and (4) in equation (2) yields

$$\rho_a\left(\vec{x},t\right) \approx \frac{\rho_o}{4\pi a_o^2} \int_V \frac{q\left(\vec{y}, t-\frac{d}{a_o}\right)}{d} d\vec{y} \tag{5}$$

where $q\left(\vec{y},t\right) = \nabla \cdot \vec{L}$ and $\vec{L} = \vec{\mathcal{L}} + \frac{\nabla v^2}{2}$.

Now, by expanding $\dfrac{q\left(\vec{y}, t-\frac{d}{a_o}\right)}{d}$ about $d = x$, where $x = |\vec{x}|$, Doak (ref. 10) has shown that equation (5) admits the multipole expansion

$$\rho_a\left(\vec{x},t\right) \approx \frac{\rho_o}{4\pi a_o^2} \sum_{\ell,m,n=0}^{\infty} (-1)^{\ell+m+n} \frac{\partial^{\ell+m+n}}{\partial x_1^\ell \partial x_2^m \partial x_3^n} \left[\frac{Q_{\ell mn}\left(t-\frac{x}{a_o}\right)}{x}\right] \tag{6}$$

where

$$Q_{\ell mn}(t) = \int_V \frac{y_1^\ell y_2^m y_3^n}{\ell! \, m! \, n!} q\left(\vec{y},t\right) d\vec{y}$$

By summing terms having equal values of $\ell + m + n$, this expression may also be written as

$$\rho_a(x,t) \approx \frac{\rho_o}{4\pi a_o^2}\left[\frac{Q_o\left(t-\frac{x}{a_o}\right)}{x} + \frac{\partial}{\partial x_i}\frac{Q_i\left(t-\frac{x}{a_o}\right)}{x} + \frac{\partial^2}{\partial x_i \partial x_j}\frac{Q_{ij}\left(t-\frac{x}{a_o}\right)}{x} + \ldots\right]$$ (7)

where

$$Q_o(t) = \int_V q(\vec{y},t)d\vec{y} \qquad\qquad Q_i(t) = -\int_V y_i q(\vec{y},t)d\vec{y}$$

and

$$Q_{ij}(t) = \frac{1}{2}\int_V y_i y_j q(\vec{y},t)d\vec{y}$$

are the overall, instantaneous monopole, dipole, and quadrupole moments, respectively, of the source distribution $q(\vec{y},t)$. Note that by referring all moments to the origin, this series removes all difficulties regarding differences in retarded times.

Since $q(\vec{y},t) = \nabla \cdot \vec{L}$ and the volume of turbulence is finite,

$$Q_o(t) = \int_V \nabla \cdot \vec{L}\, d\vec{y} = \int_S \vec{L} \cdot \vec{n}\, ds$$ (8)

by the divergence theorem, where $\vec{n}$ is the outward normal vector to the surface S surrounding the volume V. Now, Crow (ref. 9) has shown that $\vec{v} \to 0$ as x^{-3} as $x \to \infty$. From this result, it can be seen that $\vec{L} \propto x^{-7}$. Thus, the second integral in equation (8) is zero since S may be taken large enough to make $\vec{L}$ negligible on S and, hence, $Q_o(t) = 0$.

Likewise,

$$Q_i(t) = -\int_V y_i \nabla \cdot \vec{L}\, d\vec{y} = -\int_V \nabla \cdot \left(y_i \vec{L}\right)d\vec{y} + \int_V L_i\, d\vec{y}$$ (9)

The first of the last two integrals in equation (9) vanishes on application of the divergence theorem. Further, by equation (4),

$$\vec{L} = \left(\vec{v} \cdot \nabla\right)\vec{v} = v_i \frac{\partial \vec{v}}{\partial y_i} = \frac{\partial}{\partial y_i}\left(v_i \vec{v}\right)$$

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since $\dfrac{\partial v_i}{\partial y_i} = 0$. Thus,

$$\int_V L_i \, d\vec{y} = \int_V \frac{\partial}{\partial y_j}\left(v_i v_j\right) d\vec{y} = 0$$

since $v_i v_j$ is negligible on the surface S. Therefore, the monopole and dipole moments vanish, as is to be expected from Lighthill's theory.

The quadrupole moments are given by

$$Q_{ij}(t) = \frac{1}{2}\int_V y_i y_j \nabla \cdot \vec{L} \, d\vec{y} = \frac{1}{2}\int_V \nabla \cdot \left(y_i y_j \vec{L}\right) d\vec{y} - \frac{1}{2}\int_V y_i L_j \, d\vec{y} - \frac{1}{2}\int_V y_j L_i \, d\vec{y} \tag{10}$$

Again, the first of these integrals vanishes through application of the divergence theorem. Thus, in equation (7), the lowest term in the multipole expansion is quadrupole in nature and is given by

$$\rho_a\left(\vec{x},t\right) \approx -\frac{\rho_o}{4\pi a_o^{\,2}} \frac{\partial^2}{\partial x_i \partial x_j} \frac{1}{x} \left(\int_V y_i L_j \, d\vec{y}\right)^* \tag{11}$$

where the asterisk indicates evaluation at the retarded time $t - \dfrac{x}{a_o}$.

It should be mentioned here that the argument employed to obtain finite values for the first three orders of moments $\left(\text{namely, that } \vec{L} \to 0 \text{ as } x^{-7} \text{ as } x \to \infty\right)$ cannot be employed for multipoles of all orders. Thus, the convergence of the multipole expansion, equation (7), has not been proven. However, since the higher order multipoles are much less efficient radiators than the quadrupole, it appears that the multipole series may be interpreted as an asymptotic expansion for the sound field. From a physical standpoint, it can also be seen that viscosity would attack the higher order multipoles much more strongly. Thus, in a real, viscous atmosphere, convergence of the multipole series would be assured.

The term appearing in equation (11) may in turn be expanded by setting

$$F\left(t-\frac{x}{a_o}\right) = \left(\int_V y_i L_j \, d\vec{y}\right)^*$$

and noting that

$$\frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{F}{x}\right) = \frac{x_i x_j}{x^3}\left(\frac{1}{a_o^2}\frac{\partial^2 F}{\partial t^2} + \frac{3}{a_o x}\frac{\partial F}{\partial t} + \frac{3F}{x^2}\right) - \frac{\delta_{ij}}{x}\left(\frac{1}{a_o x}\frac{\partial F}{\partial t} + \frac{F}{x^2}\right)$$

If ω is taken to be a typical frequency of the sound produced, the relative magnitudes of the three types of terms in this expression are

$$\frac{\omega^2 F}{a_o^2} : \frac{\omega F}{a_o x} : \frac{F}{x^2}$$

or

$$(2\pi)^2 : \frac{2\pi\lambda}{x} : \frac{\lambda^2}{x^2}$$

where $\lambda = 2\pi a_o/\omega$ is the typical wavelength of the sound. Thus, for $x \gg \lambda$, the first term dominates and the acoustic density fluctuations may be written

$$\rho_a\left(\vec{x},t\right) \approx \frac{-\rho_o}{4\pi a_o^4}\frac{x_i x_j}{x^3}\left(\frac{d^2}{dt^2}\int_V y_i L_j \, d\vec{y}\right)^* \tag{12}$$

Finally, recall that $\vec{L} = \vec{\mathcal{L}} + \frac{\nabla v^2}{2}$. Thus,

$$\int_V y_i L_j \, d\vec{y} = \int_V y_i \mathcal{L}_j \, d\vec{y} + \int_V y_i \frac{\partial}{\partial y_j}\frac{v^2}{2} \, d\vec{y}$$

Now, integrating by parts and making use of the fact that $\vec{v} \to 0$ as x^{-3}, it can be shown that

$$\int_V y_i \frac{\partial}{\partial y_j}\frac{v^2}{2} \, d\vec{y} = -\delta_{ij}\int_V \frac{v^2}{2} \, d\vec{y}$$

Thus, with the total kinetic energy of the solenoidal field $\vec{v}$ defined as

$$e = \rho_o \int_V \frac{v^2}{2} \, d\vec{y} \tag{13}$$

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equation (12) may be written as

$$\rho_a\left(\vec{x},t\right) \approx \frac{-\rho_o}{4\pi a_o{}^4} \frac{x_i x_j}{x^3}\left(\frac{d^2}{dt^2}\int_V y_i \mathcal{L}_j \, d\vec{y}\right)^* + \frac{1}{4\pi a_o{}^4}\frac{1}{x}\left(\frac{d^2 e}{dt^2}\right)^* \tag{14}$$

It may be noted that equation (14), for the acoustic density fluctuations, is substantially equivalent to an expression for the acoustic particle-velocity fluctuations which was obtained by Powell (ref. 11) some years ago, although the methods of derivation differ considerably. Ffowcs Williams (ref. 12) objected to Powell's work on the grounds that it contained unresolved questions of convergence. Similar questions concerning the convergence of the multipole series expansion in this analysis are resolved by considering the multipole series to be an asymptotic expansion for the sound field.

With regard to the second term in equation (14), Powell observes that the only mechanism present that could account for a change in the total kinetic energy in a free inviscid flow is the production of acoustic energy itself. The contribution from this mechanism is of the same order as the first term, but it is known to be factored by a coefficient of order M^5. Further, as discussed in a later section, although kinetic energy associated with solenoidal velocity components is produced at a jet exit, for a steady jet this production occurs at a constant rate and therefore makes no contribution to noise generation. Thus, the second term may be safely dropped to yield the final result:

$$\rho_a\left(\vec{x},t\right) \approx \frac{-\rho_o}{4\pi a_o{}^4} \frac{x_i x_j}{x^3}\left(\frac{d^2}{dt^2}\int_V y_i \mathcal{L}_j \, d\vec{y}\right)^* \tag{15}$$

From this expression, the acoustic component of the density at any point in the far field can be calculated when the vorticity in the flow is known.

It might be mentioned that this analysis can evidently be extended to high, even supersonic, Mach numbers on the basis of the results of Doak (ref. 13). The condition $\Lambda \approx 1$ implies a compact source region which precludes the appearance of Mach wave radiation. The only apparent difficulty is obtaining an equivalent multipole expansion for the slightly more complex wave equation which appears in Doak's theory.

THEORY OF TOROIDAL VORTICES

In the previous section, it was shown that the sound generation by a turbulent field could be calculated from knowledge of the vorticity within the flow. This section is con-

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cerned with the description of the solenoidal velocity field produced by the "orderly" or "doughnut-like" structure which has been found in turbulent jets.

The solenoidal velocity fields of primary interest in this analysis are those in which the vorticity is confined to circular rings, or "toroidal vortices." Since viscosity acts as a diffusing agent, the statement that vorticity is confined tacitly involves the assumption that viscosity may be neglected. This assumption is made throughout. Such vortices have been studied for many years (refs. 14 to 16). Thus, only an outline of the derivation of the results necessary in the remaining analysis will be given.

Consider a cylindrical coordinate system r,θ,z and a toroidal vortex symmetrically placed with respect to the z-axis, as shown in figure 1. When the field is assumed
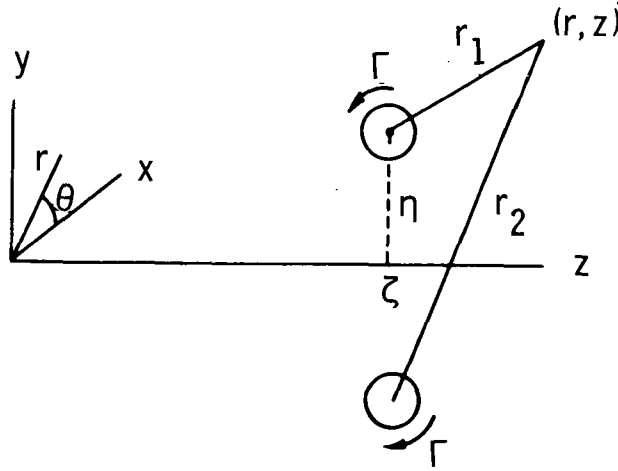

Figure 1.- Geometry of the toroidal vortex.

axisymmetric so that $u_\theta = \frac{\partial}{\partial \theta} = 0$, the velocity vector becomes $\vec{v} = [u_r, 0, w]$ and the vorticity vector is $\vec{\Omega} = [0, \Omega_\theta, 0]$.

Conservation of mass in such a flow requires that

$$\frac{1}{r} \frac{\partial}{\partial r}(r u_r) + \frac{\partial w}{\partial z} = 0 \tag{16}$$

This equation is satisfied by the introduction of a stream function ψ:

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial z} \qquad\qquad\qquad w = -\frac{1}{r} \frac{\partial \psi}{\partial r} \tag{17}$$

In terms of the stream function, the vorticity is then given by

$$\Omega_\theta = \frac{\partial u_r}{\partial z} - \frac{\partial w}{\partial r} = \frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} + \frac{1}{r} \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r^2} \frac{\partial \psi}{\partial r} \tag{18}$$

Some years ago, Lamb (ref. 14) showed that the solution to this equation could be expressed in the form

$$\psi(r,z) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} dz_0 \int_{0}^{\infty} dr_0\, \Omega_\theta(r_0,z_0)(r_1' + r_2')\left[K\left(\frac{r_2'-r_1'}{r_2'+r_1'}\right) - E\left(\frac{r_2'-r_1'}{r_2'+r_1'}\right)\right] \tag{19}$$

where K and E are the complete elliptic integrals of the first and second kinds, respectively, and

$$\left(r_1'\right)^2 = \left(r - r_0\right)^2 + \left(z - z_0\right)^2 \qquad\qquad \left(r_2'\right)^2 = \left(r + r_0\right)^2 + \left(z - z_0\right)^2$$

In order to evaluate this expression, it is necessary to determine the distribution of vorticity within the core of the ring and the shape of the core cross section. From the equations expressing conservation of momentum in such a field, it can be shown that

$$\frac{D}{Dt}\frac{\Omega_\theta}{r} = 0 \tag{20}$$

where D/Dt is the substantial derivative. Thus, the ratio Ω_θ/r is conserved as the particles move about in the flow. The simplest and most widely employed (see ref. 17) solution to this equation is

$$\frac{\Omega_\theta}{r} = \text{Constant} = \mu$$

Further, in two recent papers, Fraenkel (refs. 17 and 18) has proved the existence of toroidal vortices with such a vorticity distribution and has shown them to be stable in an inviscid fluid if their cores are small and circular to first order. Thus, since first-order results are sufficient for this analysis, the core will be assumed to be circular.
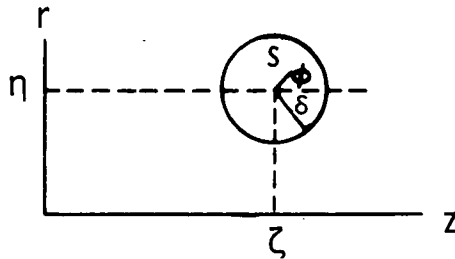

Figure 2.- Geometry of the core cross section.

Introducing polar coordinates s,ϕ within the core cross section, as shown in figure 2, leads to the relations $r = \eta + s \sin\phi$ and $z = \zeta + s \cos\phi$. In this coordinate system

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$$\Omega_\theta = \mu r = \mu\eta\left(1 + \frac{s}{\eta}\sin\phi\right) = \mu\eta\left[1 + O\left(\frac{\delta}{\eta}\right)\right] \qquad (21)$$

Thus, to first order, the vorticity is constant and the more rigorous results of Fraenkel (ref. 17) agree with those of Lamb (ref. 14). The circulation about the core is given by

$$\Gamma = \int_0^\delta ds \int_0^{2\pi} d\phi \ \Omega_\theta\left(r_0, z_0\right)s = \pi\mu\eta\delta^2 \qquad (22)$$

By Kelvin's theorem, this must be an invariant of the motion.

It is now possible to evaluate the stream function, given by equation (19), approximately in two important cases. When the point of interest (r,z) is far from the core so that r_1' and r_2' vary little over the core region,

$$\psi(r,z) \approx \frac{-\Gamma}{2\pi}\left(r_1 + r_2\right)\left[K(\alpha) - E(\alpha)\right] \qquad (23)$$

where

$$r_1^2 = (r - \eta)^2 + (z - \zeta)^2 \qquad\qquad r_2^2 = (r + \eta)^2 + (z - \zeta)^2$$

and

$$\alpha = \frac{r_2 - r_1}{r_2 + r_1}$$

The symbols r_1 and r_2 represent the shortest and longest distances, respectively, from the point of interest to the center line of the vortex. (See fig. 1.) This is exactly the stream function due to an infinitesimal vortex "filament" and agrees with the statement of Basset (ref. 15), who pointed out that a ring with a small core would have approximately the same effect upon the irrotationally moving fluid surrounding it as that of a vortex filament of the same strength which coincided with the center line of its core. From equation (23), the velocities at points not too near the core may be calculated with the aid of equation (17). Reasonably good agreement between these relations and measured velocities in real vortices has been shown experimentally.

When the point of interest is within the core of the vortex so that $r_1' \approx 0$ and $r_2' \approx 2\eta$, the argument of the elliptic integrals approaches unity. In this case, it is possible to employ expansions of the elliptic integrals (ref. 19) to show that

$$\psi(r,z) \approx \frac{-\Gamma\eta}{2\pi}\left(\ln\frac{8\eta}{\delta} - \frac{3}{2} - \frac{r_1^2}{2\delta^2}\right) \tag{24}$$

From this expression, it can be seen that the velocity within the core in the s,ϕ coordinate space is $\left(0,u_\phi\right)$, where

$$u_\phi \approx \frac{\Gamma}{2\pi\delta^2}\, s \tag{25}$$

in agreement with that of a forced two-dimensional vortex.

Now, the energy per unit mass of a single vortex ring is (Lamb, ref. 14)

$$T = \frac{1}{2}\int_V \left(u_r^2 + w^2\right)dV = -\pi\int_A \psi\Omega_\theta\, dA \tag{26}$$

where A is the cross-sectional area. Thus, employing equations (21) and (24) gives

$$T \approx \frac{\eta\Gamma^2}{2}\left(\ln\frac{8\eta}{\delta} - \frac{7}{4}\right) \tag{27}$$

which must be constant because of the absence of viscosity. Further, from Saffman's work (ref. 20), it can be seen that

$$T \approx 2U_cI_z - \frac{3}{4}\eta\Gamma^2 \tag{28}$$

where U_c is the "convection velocity" of a single vortex ring and

$$I_z = \pi\int_0^\delta ds \int_0^{2\pi} d\phi\; r^2 s\Omega_\theta \approx \pi\Gamma\eta^2 \tag{29}$$

is the only nonzero component of the impulse per unit mass. Lamb (ref. 14) has shown that the impulse must also be invariant in a quiescent field. Thus, combining equations (27), (28), and (29) yields

$$U_c = \frac{d\zeta}{dt} \approx \frac{\Gamma}{4\pi\eta}\left(\ln\frac{8\eta}{\delta} - \frac{1}{4}\right) \tag{30}$$

which is the velocity induced at one point on the vortex ring by the remainder of the ring. Note that for an infinitesimal vortex filament (i.e., $\delta = 0$), this velocity becomes infinite.

Finally, it is instructive to consider the expression for the acoustic density derived in the previous section for the case of a single vortex ring in a quiescent fluid. In a Cartesian coordinate frame,

$$\vec{\mathcal{L}} = \left[\left(U_c - u_\phi \sin \phi\right)\Omega_\theta \cos \theta, \left(U_c - u_\phi \sin \phi\right)\Omega_\theta \sin \theta, -u_\phi \Omega_\theta \cos \phi\right]$$

and

$$\vec{y} = \left[(\eta + s \sin \phi)\cos \theta, (\eta + s \sin \phi)\sin \theta, \zeta + s \cos \phi\right]$$

Therefore,

$$\int_V y_i \mathcal{L}_j \, d\vec{y} = 0 \qquad (i \neq j) \qquad (31)$$

upon integration over θ. Also,

$$\int_V y_1 \mathcal{L}_1 \, d\vec{y} = \int_V y_2 \mathcal{L}_2 \, d\vec{y} \approx \pi \Gamma U_c \eta^2 - \frac{\Gamma^2 \eta}{8} = \frac{T}{2} + \frac{\Gamma^2 \eta}{4} \qquad (32)$$

and

$$\int_V y_3 \mathcal{L}_3 \, d\vec{y} \approx \frac{-\Gamma^2 \eta}{4} \qquad (33)$$

Now, recall that the kinetic energy T and circulation Γ are invariants in the flow. Further, since the impulse must be constant, it can be seen from equation (29) that the toroidal radius η cannot vary. Thus, the time derivatives of equations (32) and (33) are zero, and a single toroidal vortex propagating in a quiescent inviscid medium can produce no sound.

These results will be employed in the next section to develop a model for the orderly structure of turbulent jets. The noise produced by this structure can then be calculated by employing the result obtained in the section entitled "Aerodynamic Noise Theory Revisited."

THE JET MODEL

Before proceeding to develop a model of the jet from which the noise can be calculated, it is instructive to examine two simpler ideal flows which bear a resemblance to the actual jet flow. These examples will be discussed in two dimensions in order not to obscure the physics by mathematical complexity. Consider first the case of an infinite region of fluid with constant velocity U_0 surrounded by ambient fluid, as shown in figure 3(a). The velocity field in this example may be completely determined by assuming the intersections between the flows to be infinitesimal vortex sheets, the upper one having circulation $\gamma = U_0$ per unit length and the lower one having the negative of that value. Each sheet travels in the positive x_1-direction with the velocity $U_0/2$ induced upon it by the other vortex sheet. Now consider the somewhat more realistic case of a semi-infinite region of fluid traveling with constant velocity U_0, again surrounded by ambient fluid, as shown in figure 3(b). For $x_1 > 0$, the upper and lower intersections can again be thought of as vortex sheets with circulations γ and $-\gamma$ per unit length, respectively, and at large positive values of x_1, the situation is very nearly that of the first example. However, at small values of x_1, a considerable difference is observed. In fact, in the moving stream at $x_1 = 0$, the velocity in the x_1-direction induced by the vortex sheets is only $U_0/2$ rather than U_0. Further, the convection velocity of the sheets is only $U_0/4$.
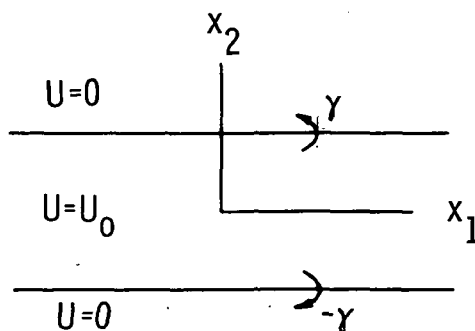
(a) Two infinite vortex sheets.
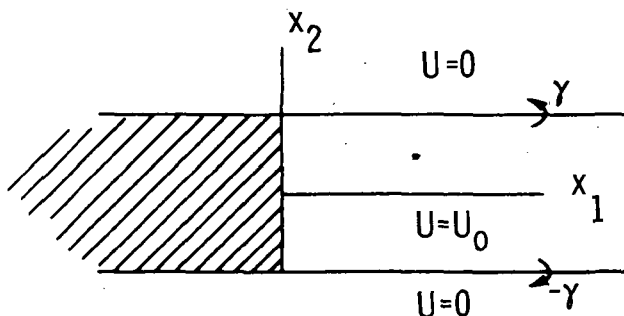
(b) Two semi-infinite vortex sheets.

Figure 3.- Simple ideal flows.

Thus, the required velocity U_0 at the origin of the flow can be attained by placing a uniform source distribution of velocity $U_0/2$ in the moving stream at the plane $x_1 = 0$. The source also produces a component of velocity which forces the vortex sheets outward, as observed for real jets. Since the conditions in the first few diameters of actual jet flows are similar to these ideal flows, the ideas developed here will be employed in constructing the jet model.

Consider a circular jet of radius R and uniform velocity U_0, as shown in figure 4. If the jet is assumed to be exhausting into the ambient atmosphere, at the edge of the jet will be formed an infinitesimal discontinuity, or cylindrical "vortex tube," across which the velocity falls from the jet velocity to zero. The circulation per unit length of this tube must again be $\gamma = U_0$. The self-induced convection velocity of such a tube unfortunately cannot be computed without assigning a finite width to the discontinuity, as can be seen in equation (30). Thus, it will be assumed to be $U_0/2$ by analogy with the two-dimensional case. From these two values, it can be seen that the rate of production of circulation by the jet orifice is given by

$$\frac{d\Gamma}{dt} = \frac{U_0^2}{2} \tag{34}$$

The velocity in the axial direction induced by the vortex tube over the whole of the jet exit is difficult to compute. However, at the center of the jet exit, it is a simple matter to show that this velocity is $U_0/2$. Thus, a distributed source must again be placed in the jet exit in order to obtain the correct mass flow from the jet.

Up to this point, the jet has been analyzed as if it produced an infinite cylindrical vortex tube. However, when the case of a starting jet is considered, it can be seen that the vortex tube will be of finite length. As can be shown analytically and as has been observed experimentally (ref. 4), such a finite tube is unstable and tends to roll up into finite circular regions of vorticity. For the purpose of this analysis, it will be assumed that these circular regions of vorticity can be represented as toroidal vortex rings. This
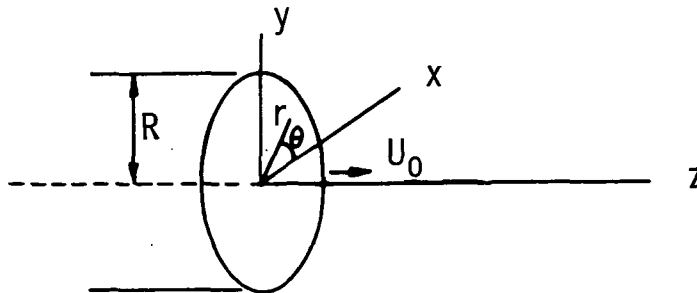
Figure 4.- Jet geometry.

leads to a model of the jet as a train of vortex rings under the influence of a distributed source, as shown in figure 5. As Crow and Champagne (ref. 5) note, these rings are randomly generated in time, and, thus, the circulations of the vortices will differ. However,
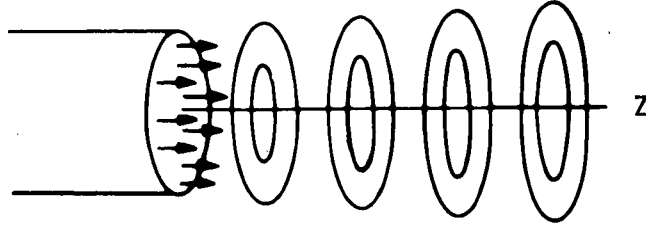

Figure 5.- Jet model.

in the absence of some mechanism through which circulation can be transferred from one vortex to another, the circulation of each individual vortex must remain constant although its size may change.

It is now possible to obtain an expression from which the sound produced by this model may be calculated. Suppose that at time t, the vortices are numbered $n = 1, 2, \ldots$; in order to evaluate equation (15), note that vorticity is present in the model only within the cores of the vortices themselves. Thus, it is necessary to determine the solenoidal velocity field only within the cores of the vortices. Since velocities may be superimposed in hydrodynamics, the solenoidal field within the core of the kth vortex is given by

$$\vec{v}^{(k)} = \vec{v}_s^{(k)} + \vec{v}_o^{(k)} \tag{35}$$

where $\vec{v}_s^{(k)}$ is the self-induced field consisting of the convection velocity U_c and the rotational component u_ϕ, and $\vec{v}_o^{(k)}$ is the field induced by all the other vortices. The self-induced components were obtained in the section entitled "Theory of Toroidal Vortices." Thus, only the externally induced components remain to be derived.

Let the radial and axial components of velocity at the point (r,z) due to a toroidal vortex of radius η and circulation Γ at the position $z = \zeta$ be $u_0(r,z;\eta,\zeta,\Gamma)$ and $w_0(r,z;\eta,\zeta,\Gamma)$, respectively. These may be calculated from equation (23) provided that the point is not too near the core of the vortex. Then, if the point (r,z) is within the core of the kth vortex, the radial and axial components of the externally induced velocity are, respectively,

$$u_{ro}^{(k)}(r,z) = \sum_{\substack{n=1 \\ n \neq k}}^{\infty} u_0\left(r,z;\eta_n,\zeta_n,\Gamma_n\right) \tag{36}$$

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and

$$w_o^{(k)}(r,z) = \sum_{\substack{n=1 \\ n \neq k}}^{\infty} w_o\left(r,z;\eta_n,\zeta_n,\Gamma_n\right) \tag{37}$$

The objective of this analysis has been to develop a model of the "orderly" structure of the jet from which the noise generation could be computed. In order to calculate the noise, consider equation (15) and note that since the vorticity is confined to the vortex cores,

$$\int_V y_i \mathcal{L}_j \, d\vec{y} = \sum_{k=1}^{\infty} \int_V y_i \mathcal{L}_j^{(k)} \, d\vec{y} \tag{38}$$

where $\mathcal{L}_j^{(k)}$ is the portion of the jth component of the vector $\vec{\mathcal{L}} = \vec{\Omega} \times \vec{v}$ contributed by the kth vortex. As in the case of a single vortex,

$$\int_V y_i \mathcal{L}_j^{(k)} \, d\vec{y} = 0 \qquad (i \neq j) \tag{39}$$

Further, if the variation of the externally induced velocity over the vortex core is neglected so that

$$u_{ro}^{(k)}(r,z) \approx u_{ro}^{(k)}\left(\eta_k,\zeta_k\right)$$

and

$$w_o^{(k)}(r,z) \approx w_o^{(k)}\left(\eta_k,\zeta_k\right)$$

then

$$\int_V y_1 \mathcal{L}_1^{(k)} \, d\vec{y} = \int_V y_2 \mathcal{L}_2^{(k)} \, d\vec{y} \approx \pi \Gamma_k \eta_k^{\,2} w_o^{(k)}\left(\eta_k,\zeta_k\right) + \frac{T_k}{2} + \frac{\Gamma_k^{\,2}\eta_k}{4} \tag{40}$$

and

$$\int_V y_3 \mathcal{L}_3^{(k)} \, d\vec{y} \approx -2\pi \Gamma_k \eta_k \zeta_k u_{ro}^{(k)}\left(\eta_k,\zeta_k\right) - \frac{\Gamma_k^{\,2}\eta_k}{4} \tag{41}$$

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where T_k is the self-induced component of the kinetic energy of the kth vortex. Thus, the integrals of interest become

$$\int_V y_1 \mathcal{L}_1 \, d\vec{y} = \int_V y_2 \mathcal{L}_2 \, d\vec{y} \approx \sum_{k=1}^{\infty} \left[\pi \Gamma_k \eta_k^2 w_0^{(k)}(\eta_k, \zeta_k) + \frac{T_k}{2} + \frac{\Gamma_k^2 \eta_k}{4} \right] \tag{42}$$

and

$$\int_V y_3 \mathcal{L}_3 \, d\vec{y} \approx - \sum_{k=1}^{\infty} \left[2\pi \Gamma_k \eta_k \zeta_k u_{ro}^{(k)}(\eta_k, \zeta_k) + \frac{\Gamma_k^2 \eta_k}{4} \right] \tag{43}$$

The noise produced by the jet depends upon the second time derivative of these integrals, as seen in equation (15). Thus, the question arises as to what parameters in these expressions have important time variation. The circulation Γ_k is known to be constant. However, in such a field of vorticity, neither the impulse nor the energy of a single vortex need be invariant. Thus, both the radius η_k and position ζ_k of the vortices may vary with time. Let $u_{rj}(r,z)$ and $w_j(r,z)$ be the radial and axial components, respectively, of the velocity produced by the steady distributed source placed in the jet exit. Then, it is possible to write the set of coupled first-order differential equations

$$\frac{d\eta_k}{dt} = u_{rj}(\eta_k, \zeta_k) + u_{ro}^{(k)}(\eta_k, \zeta_k) \tag{44}$$

and

$$\frac{d\zeta_k}{dt} = U_c(\eta_k) + w_j(\eta_k, \zeta_k) + w_0^{(k)}(\eta_k, \zeta_k) \tag{45}$$

for these variables. Further, from equation (27) and the fact that the circulation must be constant, it can be seen that

$$T_k \approx \frac{\eta_k \Gamma_k^2}{2} \left\{ \ln \left[8 \left(\frac{\pi \mu_k}{\Gamma_k} \right)^{1/2} \eta_k^{3/2} \right] - \frac{7}{4} \right\} \tag{46}$$

is a strong function of the toroidal radius. Thus, a single vortex in a field of vorticity may potentially generate noise through temporal variations in its position and radius and in its kinetic energy. There may also be variation in the induced velocities $u_{ro}^{(k)}(\eta_k, \zeta_k)$ and $w_0^{(k)}(\eta_k, \zeta_k)$.

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Some qualitative statements about the induced velocities may be made from elementary considerations. In an infinite train of coaxial vortices with equal radii, circulation, and spacing, $u_{ro}^{(k)}$ is zero and $w_o^{(k)}$ is constant. However, the model presented in this analysis differs from such a train in two respects. First, the vortices are randomly generated with different circulations and spacing. Nevertheless, at the spacing seen in experimental realizations of this phenomenon, the effect of this difference is probably second order. A more important effect is due to the fact that the train is semi-infinite. In this case, at points near the jet exit, $u_{ro}^{(k)}$ is negative because of the absence of a balancing positive contribution. However, as the vortex moves away from the exit and more vortices are generated, this velocity falls approximately to zero. On the other hand, the velocity $w_o^{(k)}$ is always positive. It is small near the jet exit but increases asymptotically to approximately its value in an infinite train as more vortices are generated. Thus, both these velocities have nonnegligible time derivatives only close to the jet exit. A computer model of the turbulent field of a starting jet which illustrates this behavior has been constructed by P. O. A. L. Davies, of the Institute of Sound and Vibration Research, University of Southampton.

When the understanding of the induced velocities obtained above is employed in equations (44) and (45), a reexamination of the behavior of the toroidal radius and velocity is possible. Note that the position of the vortex is always increasing since all three contributions to the right-hand side of equation (45) are additive. When the vortex is initiated, the toroidal radius will begin to increase provided that the positive radial velocity produced by the source distribution is large enough to overcome the negative contribution from the other vortices. This seems to be the case in real jets. As the radius is increased, the self-induced convection velocity of the vortex will decrease and the vortex will decelerate if the decrease in the self-induced velocity is larger than the increase in the externally induced velocity. Again, this seems to be the case in real jets. However, because of the fact that the vortex is moving away from the source, the effect of the source on the vortex dies away rapidly and this source-induced motion soon ceases. Then, since $u_{ro}^{(k)}$ goes to zero and $w_o^{(k)}$ becomes constant, as discussed previously, the rate of change of the toroidal radius goes to zero and the velocity of the vortex becomes constant. Thus, the radius and velocity of the vortex also have nonzero time derivatives only close to the jet exit.

In the preceding discussion, the time variation of the parameters appearing in equations (42) and (43) has been discussed in some detail. It has been shown that nearly all of the time variation takes place in the first few diameters of the jet flow. Thus, most of the jet noise must be generated in this region. In order to determine which of the parameters is most important in noise generation, it is necessary to consider the magnitude of the various terms. From equation (34), it can be seen that the circulation of a

vortex is given by $\Gamma = U_0^2 \Delta t / 2$, where Δt is the time interval in which the vortex was generated. Since the average Strouhal number of vortex generation is constant, $\Delta t \propto R U_0^{-1}$. Thus, $\Gamma \propto U_0 R$. Further, $\eta \propto R$ and $\zeta \propto U_0 t_c$, where t_c is the length of time since the vortex was generated. Finally, it seems reasonable to take $u_{ro}^{(k)}$ and $w_0^{(k)} \propto u$, where u is a "turbulent" velocity much smaller than U_0. Thus,

$$
\left.
\begin{aligned}
\Gamma_k \eta_k^2 w_0^{(k)} &\propto u U_0 R^3 \\[2ex]
T_k &\propto U_0^2 R^3 \\[2ex]
\Gamma_k^2 \eta_k &\propto U_0^2 R^3 \\[2ex]
\Gamma_k \eta_k \zeta_k u_{ro}^{(k)} &\propto u U_0^2 R^2 t_c
\end{aligned}
\right\}
\tag{47}
$$

Noting that the time variation in η_k and $u_{ro}^{(k)}$ will be quite small by the time $\zeta \propto U_0 t_c$ is large leads to the conclusion that the important terms in noise generation are the middle two of equations (47). Further, since

$$
\frac{d^2 T_k}{dt^2} = \frac{d^2 T_k}{d\eta_k^2}\left(\frac{d\eta_k}{dt}\right)^2 + \frac{dT_k}{d\eta_k}\frac{d^2\eta_k}{dt^2}
$$

and

$$
\frac{d^2}{dt^2}\left(\Gamma_k^2 \eta_k\right) = \Gamma_k^2 \frac{d^2\eta_k}{dt^2}
$$

it can be seen that the noise produced by the large-scale structure of turbulence is primarily due to changes in the toroidal radii.

CONCLUDING REMARKS

In this analysis, the noise produced by the "orderly" structure of turbulent jets has been considered. The structure was modeled as an axisymmetric train of toroidal vortices, and an expression for the noise production was derived. From the analysis, it could be seen that the noise production occurs mainly close to the jet exit and is primarily due to changes in the toroidal radii. The work supports the earlier conclusion that axi-

symmetric components in the turbulent field of the jet are significant contributors to the sound field.

One further conclusion clamors to be drawn from the model. If the cylindrical vortex tube initially produced by the jet is considered to be constructed of tightly spaced toroidal vortices, the analysis presented in this work remains essentially valid. However, there is a rolling up of this sheet to produce the orderly structure which induces large and rapid changes in the radii of these smaller vortices. Thus, the production of the orderly structure should also be a significant contribution to the high-frequency jet noise.

Although it is recognized that many jet nozzles are not circular and that the vortex structure which they form will not be circular, the fact that on the basis of the model, the orderly structure has been shown to contribute to the sound field suggests several possible new approaches to noise suppression. One such approach is to place a device in the jet flow which destroys the orderly structure. This may be part of the effectiveness of the multitube suppressor in addition to its well-known frequency shift. However, in order to alter an axisymmetric structure, the multitube arrangement with its subsequent drag loss may be unnecessary. One such device which consists of a series of concentric cylinders has recently been tried successfully by T. P. Scharton and P. H. White (J. Acoust. Soc. Amer., vol. 52, no. 1, pt. 2, 1972). A second method of suppression might attack the stability of the vortices. Vortex rings are very unstable. In particular, any variation in the axisymmetry or planarity of the ring leads to induced velocities which tend to destroy it. For this purpose, nonsymmetric nozzles might prove useful. Finally, the dependence of the noise on changes in the toroidal radii suggests that keeping the radius constant should lead to noise reduction. Mathematically, the way to achieve this feat is trivial — add a further velocity field to the flow which will introduce a term in the radial velocity so that the time rate of change of the radius is always zero. The method by which this may be accomplished on an actual engine must be a subject of experimentation.

Langley Research Center,
  National Aeronautics and Space Administration,
    Hampton, Va., March 29, 1973.

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