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SERVOMECHANISM DESIGN TECHNIQUES AND APPLICATIONS AEROSPACE PROBLEMS

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AEROSPACE PROBLEMS

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ABSTRACT

Transistorized alternating current servomechanism design offers a number of advantages over other types because of their small size, low weight, high reliability, precision accuracy, and wide range of control.

These lecture notes describe a cryogenic oxygen flow control servomechanism subsystem typical for an auxiliary power unit fuel system. Three possible types of feedback for this type of subsystem are: position feedback accomplished by gear train and synchro-transformer, position with velocity feedback accomplished by adding a rate generator and position feedback with compensating network. Servomechanisms employing any of these three types of subsystems can provide the oxygen flow control desired. The differences in performance relate primarily to steadiness of flow with the companion heat modulation. The damping ratio parameter can be used to achieve a reasonable response time and maintain a reasonable value for overshoot. Low damping ratio gives fast response but large overshoots. High damping ratios give low overshoot but slow response.

These lecture notes discuss transistorized servomechanism design for the basic position servo, plus rate feedback or network compensation. Included are design and application considerations, step-by-step design procedures, and the solution to sample design examples.

INTRODUCTION

The function of transistorized servomechanisms in aerospace systems is to maintain a variable (c) at a desired value by feeding back some function of the maintained variable (B) and comparing this signal with a reference (r) to generate an error signal (e). Increases in the error signal, cause the maintained variable to be reduced and vice versa.

A typical servomechanism with variables identified is shown in figure 1: (c) maintained variable, (B) feedback variable, (r) reference variable, and (e) error variable. The forward branch transfer function (G) is given by the solution of describing differential equations for the ratio (c/e) usually in Laplace Transformed form to simplify the algebra. Similarly, the feedback branch transfer function (H) is given by the ratio (B/c). The ratio (e/r) is an important function in this simplified servo design methodology as the denominator polynomial can be examined to determine the undamped natural frequency (ω) and the damping ratio (δ). The damping ratio is the key parameter selected for the design method presented in this paper to optimize response time and overshoot. Figure 1 shows the block diagram algebra necessary to obtain expressions for these ratios. The methods of using these simplified equations to design a transistorized oxygen flow control servomechanism are explained in these notes.

SIMPLIFIED POSITION SERVO DESIGN TECHNIQUE

The step-by-step technique which follows is recommended for the design of transistorized position servos. The design equations not derived in these notes are explained in the existing literature cited in the REFERENCES section by numbers 1 through 7.

(1) Make a preliminary selection of the synchros, amplifier, servomotor, gear train, and butterfly valve to be used in the oxygen flow control servo. Table I shows some of the salient design characteristics for the selected components. These components should be matched to each

other, and selected to have linear operating ranges about two times greater than that required to satisfy the static position accuracy requirement (see step 2). Figure 2(a) shows a typical component arrangement for the oxygen flow control servo. Figure 2(b) shows a block diagram for this component arrangement with transfer functions identified. The algebra to develop expressions for the natural frequency (ω) and the damping ratio (δ) are also shown in this figure, part 2(c) (derived in ref. 2). NOTE! Care must be used to ensure a consistent set of units for the values that are substituted into the design equations.

(2) Tabulate design values for the oxygen flow control servo (all quantities must be referred to the motor shaft):

T_o = maximum torque to drive motor, gear train, synchro, and valve

= friction plus acceleration of load torque = $T_F + T_a$

$= 0.135 + 0.197 = 0.332$ in.-oz $\qquad T_F = 0.001 + 0.002 + 0.082 + 0.050$

synchro + valve + gears + motor

$= 0.135$ in.-oz

J = inertia of motor + load

$= J_M + J_L \qquad J_M = 0.0018 \text{ lb-in.}^2$

$= 0.0019 \text{ lb-in.}^2 \qquad J_L = \dfrac{0.0033}{(470)^2} \qquad T_a = 92.5$ in.-oz at valve

$= 7.52 \times 10^{-5} \text{ in.-oz-sec}^2 \qquad \simeq 0.0001 \text{ lb-in.}^2 \qquad = \dfrac{92.5}{470} = 0.197$ in.-oz at motor

T_M = motor torque at zero speed = 2.7 in.-oz

S_M = motor speed with no load = 941 rad/sec

E = probable error = synchros + gears + valve + dead zone

$$= 22 + 20 + 10 + 2 = 54 \text{ min} = 0.9^{\circ}$$

$$= 1.57 \times 10^{-2} \text{ rad}$$

(3) Calculate the time constant (τ) of the motor plus load:

$$\tau = \frac{JS_M}{T_M} = \frac{7.52 \times 10^{-5} \text{ in.-oz-sec}^2 \times 941 \frac{\text{rad}}{\text{sec}}}{2.7 \text{ in.-oz}}$$

$$= 2.62 \times 10^{-2} \text{ sec}$$

(4) Calculate the system gain ($K = K_1K_2K_3K_4$) required to keep error to the minimum probable value:

$$K = \frac{T_o}{E} = \frac{0.332 \text{ in.-oz}}{1.57 \times 10^{-2} \text{ rad}} = 21.2 \frac{\text{in.-oz}}{\text{rad}}$$

(5) Calculate the undamped natural frequency:

$$\omega = \left(\frac{K}{J}\right)^{1/2} = \left(\frac{21.2 \frac{\text{in.-oz}}{\text{rad}}}{7.52 \times 10^{-5} \text{ in.-oz-sec}^2}\right)^{1/2} = 5.31 \times 10^2 \frac{\text{rad}}{\text{sec}}$$

(6) Calculate the damping ratio for these conditions:

$$\delta = \frac{1}{2\omega\tau} = \frac{1}{2 \times 5.31 \times 10^2 \frac{\text{rad}}{\text{sec}} \times 2.62 \times 10^{-2} \text{ sec}}$$

$$= 0.036$$

This damping ratio is quite low and the valve position will tend to oscillate about the controlled valve. Figure 3 shows a typical position response for a step input (r) of the oxygen flow control servo for three values of damping ratio. When the damping ratio is near 0. 1, large overshoot occurs with long periods required to damp out the oscillating conditions. For example, if the oxygen servo with a $\delta = 0.04$ was commanded to open 10^{o}, the valve would respond by initially opening to about 20^{o} at 3 radians; then it would close to about 5^{o} at 6 radians causing severe variations in the burning temperature for the hot gas turbine.

ADDING A RATE GENERATOR

Generally, a damping ratio of about 0. 5 is considered a reasonable compromise between speed of response and stability. Performance of the oxygen flow control servo can be improved with the addition of derivative feedback obtained by means of a rate generator. Figure 4(a) shows a typical component arrangement for an oxygen flow control servo with rate feedback. Figure 4(b) shows a block diagram for this component arrangement with transfer functions identified. The algebra to develop expressions for the natural frequency (ω') and the damping ratio (δ') are also shown in this figure, part 4(c) (derived in ref. 3).

(1) Using the tabulated design values from table I and step (2) of example (1); recalculate the design values for the oxygen flow control servo:

$T'_o = 0.145 + 0.197 = 0.342$ in.-oz

$$T_F = 0.001 + 0.002 + 0.082 + 0.050 + 0.010$$

$$\text{synchros} + \text{valve} + \text{gears} + \text{motor} + \text{generator}$$

$$= 0.145 \text{ in.-oz}$$

$$T_a = 0.197 \text{ in.-oz}$$

J' = inertia of motor + generator + load $\qquad S_M = 910$ rad/sec

$= 0.0018 + 0.0012 + 0.0001 = 0.0031$ lb-in.^{2}

$= 1.23\times10^{-4}$ in.-oz-sec^2

T_M and E are not effected by the addition of a rate generator.

(2) Calculate the range of time constant (τ') for the motor, generator, and load with a range of rate generator gain from 0.29×10^{-4} to 2.9×10^{-4} volts/rad

$$\tau' = \frac{J'}{\frac{T_M}{S_M} + K_2K_3K_5} = \frac{1.23\times10^{-4}\text{ in.-oz-sec}^2}{\frac{2.7\text{ in.-oz}}{910\text{ rad/sec}} + 1000 \times 7.5\times10^{-2} \times 0.29\times10^{-4}}$$

$$= 23.9\times10^{-3}\text{ sec}$$

for 2.9×10^{-4} volts/rad, $\tau' = 5.0\times10^{-3}$ sec.

(3) Calculate the system gain (K') required to keep error to the minimum probable value:

$$K' = \frac{T'_o}{E} = \frac{0.342\text{ in.-oz}}{1.57\times10^{-2}\text{ rad}} = 21.8\ \frac{\text{in.-oz}}{\text{rad}}$$

(4) Calculate the undamped natural frequency:

$$\omega' = \left(\frac{K'}{J'}\right)^{1/2} = \left(\frac{21.8}{1.23\times10^{-4}}\right)^{1/2} = 4.21\times10^{2}\ \frac{\text{rad}}{\text{sec}}$$

(5) Calculate the range of damping ratio using the range of τ' previously obtained:

$$\delta' = \frac{1}{2\omega'\tau'} = \frac{1}{2 \times 4.21\times10^{2}\ \frac{\text{rad}}{\text{sec}} \times 23.9\times10^{-3}\ \text{sec}}$$

$$= 0.05$$

for a value of $\tau' = 5.0\times10^{-3}$ sec, $\delta' = 0.24$.

This damping ratio range is better than case 1 and the valve position oscillations will be decreased in amplitude about the controlled value. For our previous example, if the oxygen servo with a $\delta = 0.2$ was commanded to open 10°, the valve would respond by opening with an overshoot of about 2° to 12° at 3.2 radians; then it would close to about 9° at 6.5 radians providing a more uniform burning temperature for the hot gas turbine.

ADDING LEAD NETWORK COMPENSATION

Since we have not yet achieved our goal of a damping ratio of about 0.5, perhaps by adding network compensation the oxygen flow control servo performance can be improved. Investigations (1, 2, and 4) have shown that lead network compensation between the synchro transformer and the amplifier can achieve the desired results. This third method of network compensation has an additional requirement that the other two methods do not have. This additional requirement is that the carrier frequency must not vary more than 10 percent. Figure 5(a) shows a typical component arrangement for an oxygen flow control servo with lead network compensation. Figure 5(b) shows the block for this component arrangement with the transfer functions identified. Some of the algebra to develop expressions for the natural frequency (ω'') and the damping ratio (δ'') for a third order system are also shown in this figure, part 5(c) (derived in ref. 4).

(1) Using the tabulated design values from table I and step (2) of example (1), recalculate the design values for the oxygen flow control servo in this configuration:

T_o, J, T_M, S_M, E, τ, K, and ω are not effected by the addition of a lead network: $T_o = 0.332$ in.-oz, $J = 7.52\times10^{-5}$ in.-oz-sec^2, $T_M = 2.7$ in.-oz, $S_M = 941$ rad/sec, $E = 1.57\times10^{-2}$ rad; these values result in a time constant for the motor plus load ($\tau = 2.62\times10^{-2}$ sec), a system gain (K = 21.2 in.-oz/rad) and a natural frequency ($\omega'' = 5.31\times10^2$ rad/sec).

(2) Choose the damping ratio (δ'') to be a reasonable value of 0.5. Calculate the effective gain ratio (η') for this third order system:

$$\delta'' = \frac{1}{2}\left[(\eta')^{1/2} - 1\right]$$

$$\eta' = (2\delta'' + 1)^2 = (2 \times 0.5 + 1)^2 = 4.0$$

(3) Calculate the lead network zero $\tau_6 S$ coefficient (η):

$$\eta' = \eta + \frac{4.25}{\omega\tau}$$

$$\eta = \eta' - \frac{4.25}{\omega\tau} = 4.0 - \frac{4.25}{5.31\times10^2\ \frac{\text{rad}}{\text{sec}} \times 2.62\times10^{-2}\ \text{sec}}$$

$$\eta = 3.7$$

This damping ratio is considered by many to be a reasonable compromise between speed of response and stability. Valve position oscillations will be decreased in amplitude to a minimum about the controlled value. For our previous example, if the oxygen servo with a $\delta = 0.5$ was commanded to open 10^o, the valve would respond by opening with reasonable overshoot of about 1.5^o to 11.5^o at 3.5 radians; then it would close to about 9.5^o at 7 radians providing very uniform burning temperatures for the hot gas turbine.

To complete the design for example (3), it is necessary to specify K_6, τ_6 and synthesis a network. Figure 6(a) shows one type of compensating lead network component arrangement. Equations shown in figure 6 are derived in reference 2. Figure 6(b) shows the transfer function and frequency response for the lead network.

(4) Calculate the lead network time constants τ_6, $\eta\tau_6$:

$$\tau_6 = \frac{1}{\omega''(\eta')^{3/4}}\left[1 + \frac{3}{2\omega''\tau(\eta')^{3/4}}\right]$$

$$= \frac{1}{5.31\times10^2 \times 2.83}\left[1 + \frac{3}{2 \times 5.31\times10^2 \times 2.83}\right] = 0.67 \text{ msec}$$

$$\eta\tau_6 = 3.7 \times 0.67 = 2.47 \text{ msec}$$

Generally speaking, in selecting the components for this type of network the output resistance of the network should be $\leq 10\ \text{K}\Omega$ to minimize noise susceptibility.

(5) Based on this, choose $R_1 = 1\ \text{K}\Omega/(1/2)\text{W}$ resistor and $K_6 = 0.75$; calculate values for R_2 and C_1:

$$R_2 = \frac{R_1 K}{1 - K} = \frac{(1\,\text{K})0.75}{1 - 0.75} = \frac{3\ \text{K}}{\frac{1}{2}\ \text{W resistor}}$$

$$C_1 = \frac{\eta\tau_6}{R_1} = \frac{2.47\times10^{-3}\ \text{sec}}{1\ \text{K ohm}} = \frac{2.47\ \mu\text{f}}{50\ \text{WV}}$$

Use the nearest standard components to these values.

(6) Calculate the frequency response break frequencies:

$$\frac{1}{\eta\tau_6} = \frac{1}{2.47\times10^{-3}} = 405\ \frac{\text{rad}}{\text{sec}}$$

$$\frac{1}{\tau_6} = 1490\ \frac{\text{rad}}{\text{sec}}$$

CONCLUDING REMARKS

Transistorized alternating current servomechanisms of the basic position servo with rate feedback or network compensation are described in these notes. A study of optimum methods to control position variables in aerospace applications showed that no single method was best for all cases. The optimum method was very much dependent upon available devices, generating sources and load parameters. These are constraints difficult to identify for future aerospace missions.

Past aerospace missions have used either motor, motor plus generator or motor plus compensating network servomechanisms or combinations thereof to control position variables. Table II shows some of the important typical servomechanism parameters to consider when faced with the task of choosing the type of converter for use in a given mission. The type 1 servo has the smallest size, lowest weight and parts count. Undamped frequency is high but the damping ratio is low and overshoot is high. High overshoot causes large position transients which may not be tolerable. The type 2 servo is somewhat larger in size, heavier, uses a few more parts, lowers the natural frequency and has 1 to 2 orders of magnitude increase in damping ratio with a substantial reduction in overshoot. The large reduction in damping ratio justifies its use in some cases.

Many current aerospace missions are going to some form of compensated servo. It tends to be a compromise in size, weight, circuit complexity to gain in damping ratio for a modest reduction in overshoot. Compensating networks often are dependent on constant supply frequency for proper performance. If the supply frequency varies by say 10 percent, an optimally designed system may become highly oscillatory with large amounts of transient overshoot. The presence of this frequency limitation may often render this approach useless.

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7. Anon.: Key Formulas for Servomechanism Design. Systems Designers Handbook, Brookline, Ma., 1970.

Table I

SALIENT COMPONENT CHARACTERISTICS FOR DESIGN

COMPONENT	SYNCHROS		SERVOMOTOR	RATE GENERATOR	GEAR TRAIN	VALVE	AMPLIFIER
	TRANSMITTER	TRANSFORMER					
SIZE	15CX	15CT	15	15	15	1.0"	25 WATT
ROTOR INERTIA	0.00282	0.00315	0.0018	0.0012	-----------	0.0001 LB-IN.^{2}	115 VOLT
ERROR	12	10	------------	-----------------	20	10 MINUTES	400 HERTZ
TRAVEL	2π	2π	2π	2π	2π	1.5 RADIANS	20-60 Db GAIN
GAIN	$1.0 \frac{RAD}{RAD}$	$57 \frac{VOLTS}{RAD}$	$7.5\times10^{-2} \frac{IN.\text{-}OZ}{VOLT}$	$(.29\text{-}2.9)\times10^{-4} \frac{VOLT}{RAD}$	$470{:}1 \frac{RAD}{RAD}$	.	0-40V OUT

NOTES:

1. THE BUTTERFLY VALVE SHALL HAVE 90±2.5 OZ-IN. ANTIBACKLASH SPRING TORQUE.
2. THE GEAR RATIO TO MAXIMIZE THE ACCELERATION OF THE VALVE SHAFT SHALL BE 470:1.

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Table II

OXYGEN FLOW SERVOMECHANISMS COMPARISON

TYPE	SIZE	WEIGHT	PARTS COUNT	UNDAMPED FREQUENCY	DAMPING RATIO	OVERSHOOT
1	1.0	1.0	48	531	0.036	2.0
2	1.3	1.3	51	421	0.05-0.24	1.2
3	1.1	1.1	51	531	0.5	1.15

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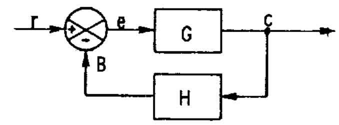

TYPICAL SERVOMECHANISM

If $H = 1,\ B = C$ $\quad \frac{C}{e} = G \quad \frac{B}{C} = H$

$r = e(1 + G)$ $\quad e = r - B$

$r = C\left(\frac{1}{G} + H\right)$

$\frac{e}{r} = \frac{1}{1 + G}$ $\quad \frac{C}{r} = \frac{G}{1 + GH}$

USEFUL DENOMINATOR POLYNOMIALS = $1 + G,\ 1 + GH$

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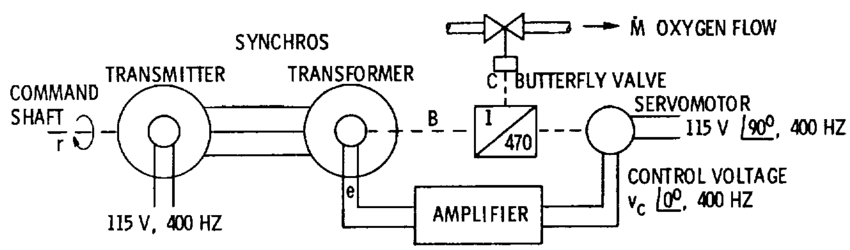
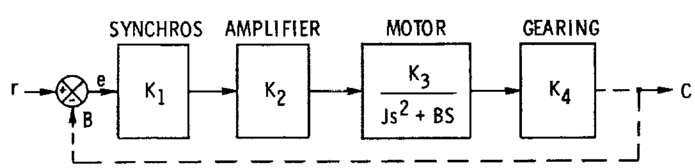
Figure 1



TYPICAL COMPONENT ARRANGEMENT FOR AN OXYGEN FLOW CONTROL SERVO



BLOCK DIAGRAM WITH TRANSFER FUNCTIONS FOR AN OXYGEN FLOW CONTROL SERVO

$$\frac{e}{r} = \frac{1}{1 + G} = \frac{Js^2 + BS}{Js^2 + BS + K_1K_2K_3K_4} \qquad \text{let } K = K_1K_2K_3K_4$$

$$\left(\frac{S}{\omega}\right)^2 + \frac{2\delta}{\omega} S + 1 \qquad \omega = \left(\frac{K}{J}\right)^{1/2}$$

$$\delta = \frac{B\omega}{2K} = \frac{1}{2\omega\tau}$$

ALGEBRA FOR NATURAL FREQUENCY AND DAMPING RATIO

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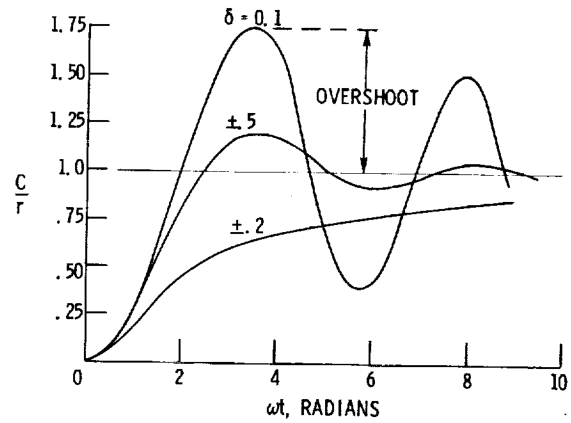
Figure 2



DESCRIBING EQUATIONS:

$$\left(\frac{S}{\omega}\right)^2 + \frac{2\delta}{\omega}S + 1$$

$$\omega = \left(\frac{K}{J}\right)^{1/2} \qquad \delta = \frac{B\omega}{2K}$$

TYPICAL POSITION RESPONSE OF THE OXYGEN FLOW CONTROL SERVO

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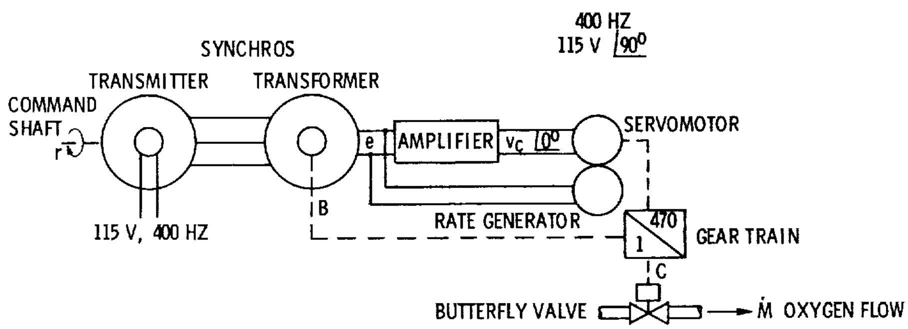
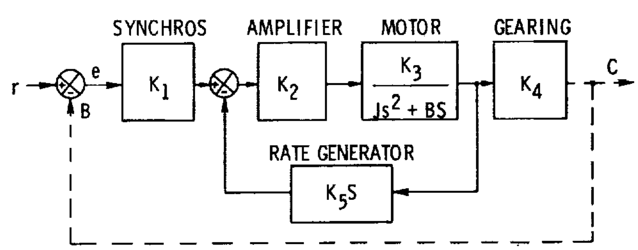
Figure 3



TYPICAL COMPONENT ARRANGEMENT FOR AN OXYGEN FLOW CONTROL SERVO WITH RATE FEEDBACK



BLOCK DIAGRAM WITH TRANSFER FUNCTIONS FOR
AN OXYGEN FLOW CONTROL SERVO WITH RATE FEEDBACK

$$\frac{e}{r} = \frac{J'S^2 + B'S}{J'S^2 + B'S + K'}$$

$$\omega' = \left(\frac{K'}{J'}\right)^{1/2} \qquad \delta' = \frac{1}{2\omega'\tau'}$$

ALGEBRA FOR NATURAL FREQUENCY AND DAMPING RATIO

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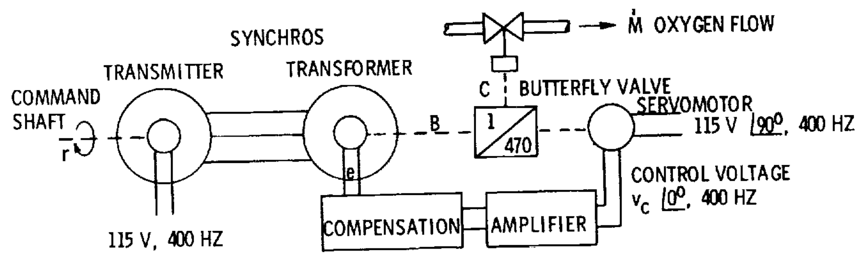
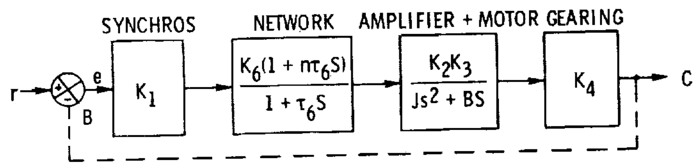
Figure 4



TYPICAL COMPONENT ARRANGEMENT FOR AN OXYGEN FLOW CONTROL SERVO WITH NETWORK COMPENSATION



BLOCK DIAGRAM WITH TRANSFER FUNCTIONS FOR
AN OXYGEN FLOW CONTROL SERVO WITH NETWORK COMPENSATION

$$\frac{e}{r} = \frac{S(\tau_6 Js^2 + (J + B\tau_6)S + B)}{\tau_6 Js^3 + (J + B\tau_6)S^2 + (B + nK''\tau_6)S + K''}$$

$$\omega'' = \left(\frac{K}{J}\right)^{1/2} \qquad \delta'' = \frac{1}{2}(n^{1/2} - 1)$$

ALGEBRA RESULTS FOR NATURAL FREQUENCY
AND DAMPING RATIO

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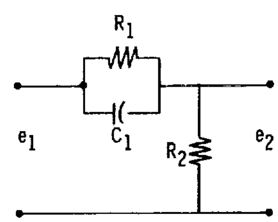
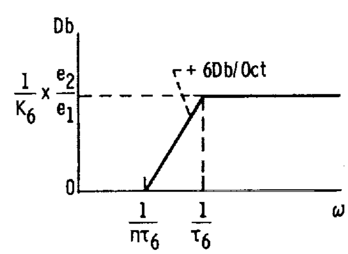
Figure 5



LEAD NETWORK COMPONENT ARRANGEMENT



$$\frac{e_2}{e_1} = \frac{K_6(1 + n\tau_6 S)}{(1 + \tau_6 S)}$$

WHERE $K_6 = \frac{R_2}{R_2 + R_1}$

$$\tau_6 = \frac{R_2 R_1 C_1}{R_2 + R_1} \qquad n\tau_6 = C_1 R_1$$

LEAD NETWORK TRANSFER FUNCTION AND FREQUENCY RESPONSE

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Figure 6