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X-592-73-130
PREPRINT

NASA TM X- 66246

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C. A. WAGNER

(NASA-TM-X-66246) ELEVENTH ORDER RESONANCE TERMS IN THE GEOPOTENTIAL FROM THE ORBIT OF VANGUARD 3 (NASA) 32 p HC $3.75 CSCL 03B

N73-24415

Unclas

G3/13 04308

MAY 1973

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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C. A. Wagner
Geodynamics Branch
Geodynamics Program Division

May 1973

Goddard Space Flight Center
Greenbelt, Maryland

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C. A. Wagner

ABSTRACT

The orbit of Vanguard 3 (1959-7A) is strongly resonant with 11th order and odd degree terms in the geopotential. It affords an excellent opportunity to determine a significant linear constraint between these terms. Tracking data on this satellite (in the form of mean Kepler elements) are analyzed over a 3 1/2 year period in the early 1960's, which ends with the orbit having just passed through perfect commensurability. The eccentricity and inclination show the deep resonance variations (up to 2×10^{-4} in e and .02° in i) with great clarity. Previous and current geopotential solutions fail to explain these perturbations. The analysis determined the following constraint for the deep resonant terms (in fully normalized harmonics):

$$
\begin{aligned}
10^7(C, S)_{\ell, 11} = (8.6 \pm 0.2,\ 7.8 \pm 0.2) = {} & (C, S)_{11,11} - 6.7(C, S)_{13,11} \\
& + 16.2(C, S)_{15,11} - 22.9(C, S)_{17,11} + 19.5(C, S)_{19,11} - 6.5(C, S)_{21,11} \\
& - 7.2(C, S)_{23,11} + 11.3(C, S)_{25,11} - 5.0(C, S)_{27,11} - 3.6(C, S)_{29,11} + \cdots
\end{aligned}
$$

If these 11th order coefficients are of the order of $10^{-5}/\ell^2$, there will be a significant contribution to this constraint for terms at least as high as (25, 11).

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11th ORDER RESONANCE TERMS IN THE GEOPOTENTIAL FROM THE ORBIT OF VANGUARD 3

INTRODUCTION

The subject of orbital resonance and its application to the determination of the geopotential has had extensive discussion in the literature (i. e., Cook, 1961; Morando, 1962; Blitzer, 1962; Allan, 1965 & 1967; Kaula, 1966; Kozai, 1966; Garfinkel, 1966; Gedeon, 1969; Gaposhkin & Lambeck, 1970, and King Hele, 1972a, among many, many other authors and papers). In fact a whole symposium was held on the subject in January, 1968 at TRW Systems Group, Redondo Beach, California. At that time there was a hope that the longitude dependent geopotential (to a fairly high degree) could be determined completely from the strong long term deviations of commensurate orbits. Indeed, there were a number of proposals (i. e., Greene, 1967; Gedeon, Private Communication, 1968) then to launch one or a series of special satellites to sample a large number of resonant orbits for just this purpose. But it was soon recognized that the magnitude of such a program was prohibitive especially in view of the many deep and near-to-resonant orbits already available for analysis (Wagner & Douglas, 1969). Indeed by the early 1970's the gravity harmonics of order 12, 13 and 14 (and past degree 20) had been so determined from near resonant, close orbits [Gaposchkin and Lambeck, 1970; Lerch et. al., 1972a]. The deviations of the distant communications satellites in deep resonant (commensurate) orbits have served as excellent independent proof that the low order and degree terms of recent comprehensive solutions (from precise orbit tracking) have considerable accuracy [Wagner, 1972a].

The possibilities of exploiting existing resonant orbits seemed exhausted by 1971. However, as Gaposchkin and Lambeck [1970, p. 20] noted, there does not exist a general theory of orbital resonance, and this should have been a warning that these possibilities were far from closed. Indeed, examination of the orbital tracking data discarded by the Smithsonian Astrophysical Observatory during preparation of their Standard Earth 2 (SAO SE 2) [Gaposchkin, 1970; Gaposchkin and Lambeck, 1970] reveals (with a shock) that all were orbits suffering from fairly strong or deep resonance effects. These were the 9th order resonant orbit of Telstar 1, and the two 11th order deeply resonant orbits of Vanguard 2 Rocket and Vanguard 3. It is ironic that the actual solution of the long period resonance problems in these orbits was not (as elaborately discussed in Gaposchkin and Lambeck, 1970) through selective editing of analytic resonant terms. The solution was to eliminate the offensive data altogether!

The other line of evidence that the possibilities are not closed stems from the remarkable new investigations by King Hele's group (i. e., Gooding, 1971a; King-Hele, 1972b) of 15th order resonances on close satellite orbits dragged (by the atmosphere) slowly through commensurabilities. These new effects were originally observed as an incidental mysterious perturbation in the inclination evolution of the orbit of Ariel 3, studied to reveal the "super-rotation" of the upper atmosphere.

The observations were of a seeming step decline in the inclination. On closer examination the evolution was actually a continuous oscillation of increasing amplitude and period, finally "reflected" at a critical point, not at mid cycle, into an oscillation of decreasing amplitude and period. The critical point (of reflection) was found to be at the time of exact commensurability for the orbit with 15th order terms in the geopotential [Gooding, 1971b]. R. R. Allan, 1971 gives the full explanation for this curious unexpected behaviour as an example of an orbit being dragged through a resonance. When the drag forces are constant and dominate the resonance forces along track during the passage through commensurability, there is an exact solution for the inclination change in terms of Fresnel integrals. This solution displays the reflection characteristics and the building and decaying oscillations observed in the Ariel data.

Using the observed inclination changes together with the observations of the satellite's longitude, Gooding (1971b) derived a linear constraint among all the 15th order and odd order terms in the geopotential from the orbit of Ariel 3. His simple theory was Allan's (1971) formulation of the appropriate inclination rates due to the resonant geopotential terms. (Kaula's [1966] formulation for the geopotential rates is completely equivalent and will be used in what follows.) Because the orbit was nearly circular only one linear constraint could be easily derived. But the way was now open for deriving many other such constraints from close orbits observed during resonance passage (King-Hele, 1972b).

In theory every satellite is a candidate for an infinite number of such passage events which could be used for geopotential determination. In practice though, the far satellites will decay too slowly to reach these points in a reasonable time, while the close ones will pass through them too fast to be of much use. However there is still a large number of medium altitude satellites, with perigees between about 500 and 600 km whose orbit decay is just right for observing at least one such strong event within 5 to 10 years from launch. And for the more eccentric orbits with these perigees, there is the possibility of observing two or more series of resonances in each passage.

The orbit of Vanguard 3 and Vanguard 2 rocket [n (the 2 body mean motion) $\approx$ 11 revolutions/day, e (orbit eccentricity) = .19, perigee height $\doteq$ 520 km] is of this multiple resonance type. These Vanguard orbits offer the only opportunity to see significant 11th order geopotential effects similar to the unique opportunities afforded the 15th order dragged resonance orbits. But this opportunity has not been properly exploited. Indeed the Smithsonian has abandoned them altogether. Thus it can be fairly said that 11th order terms, until now, remain among the most poorly determined in the geopotential (see Discussion). Unfortunately, Vanguard 3 has only been well observed through the first strong resonance (in 1963-64) which was with odd degree terms proportional to e^2 and thus relatively weak. In 1966-72 however, its orbit passed through 3 potentially stronger resonance series with even and odd degree terms (still 11th order) proportional to e and 1. And in 1973-74 it should be passing through another strong resonance (similar to the 1963-64 commensurability).

But all of this history is fine hindsight. Even the first resonance pass was unknown to me when I examined a set of 182 mean Kepler elements for Vanguard 3, calculated by T. Heuring (private communication, 1972) from the Smithsonian's precision reduced Baker-Nunn camera observations (Table 1). I knew Gaposchkin and Lambeck's (1970) difficulties with this orbit and that it was strongly affected by drag (especially during the solar cycle high of the very early 1960's). My main concern was whether the orbit program I was using [see: Wagner and Douglas, 1970] could handle the strong and variable drag on a 3 1/2 year trajectory. This program, ROAD (Rapid Orbit Analysis and Determination), integrates orbits semi-numerically using model atmospheres due (originally) to Jacchia, 1965 and 1971. In the integrator only terms not containing the satellite's mean anomaly are retained. In this sense the program generates mean elements under a wide variety of forces.

But it also can employ empirically derived secular rates and accelerations of the elements (to the 5th power). These were found necessary to keep the along-track deviations of Vanguard 3 to less than 2° over the long arc when integrating with either model atmospheres. (At this level, the longitude of the computed satellite is sufficiently accurate to obtain realistic resonant geopotential terms.) After a considerable amount of orbital experimentation in ROAD, a 3 1/2 year nonresonant trajectory (with drag and radiation effects) was computed for Vanguard 3, with less than 2° along-track variation from the observed elements. This trajectory used the Smithsonian Standard Earth 2 zonal geo-potential coefficients. My original hope was to "see" the resonance in the drag affected mean anomaly observations, but the errors in the model atmosphere were too great to allow this. However the comparison of the inclination of this computed trajectory (only slightly affected by drag) with the "observed" mean elements showed a clear but unexplained oscillation of increasing amplitude, breaking off near the middle of a cycle at the end of the data (Figure 1).

This oscillation-breakoff was, of course, reminiscent of the dragged-resonance phenomenon so well explained by Allan (1971) for near circular, uniformly dragged orbits. Could these residuals be due to the same phenomenon on an eccentric orbit subject to a more complex drag-resonance regime? The answer was supplied by simulating a Vanguard 3 resonant trajectory with all relevant terms for the geopotential harmonics (11, 11) and (12, 11) (from the Standard Earth 2) and comparing it with the non resonant trajectory computed previously. The form of the inclination residuals produced (Figure 2) emphatically showed that the resonant terms would account for the phenomenon. It remained to identify and solve for the harmonic constraint(s) that would reproduce the observed inclination variations.

ANALYSIS

Kaula [1966, p. 49] identifies geopotential resonance as occuring when the orbit longitude ψ is stationary with respect to a particular gravitational harmonic term (ℓ, m, p, q): thus

$$\dot{\psi}_{\ell,m,p,q} = 0 = (\ell - 2p)\,\dot{\omega} + (\ell - 2p + 1)\,\dot{M} + m(\dot{\Omega} - \dot{\theta}), \tag{1}$$

where ω, μ, and Ω are the orbit's argument of perigee, mean anomaly and right ascension of the ascending node and $\dot{\theta}$ is the rotation rate of the earth. For Vanguard 3, the mean motion ($\dot{M}$) is close to 11 revolutions/day, which, since $\dot{\omega}$ and $\dot{\Omega}$ are comparatively small, establishes m = 11 as the lowest order resonant term (under the specification $\ell - 2p + q = 1$). Rewriting (1) for these terms, leaves only the q index unspecified:

$$\dot{\psi}_{res.} = 0 = (\dot{\omega} + \dot{M}) - q\dot{\omega} + 11(\dot{\Omega} - \dot{\theta}),$$

from which;

$$q(res.) = [11(\dot{\Omega} - \dot{\theta}) + (\dot{\omega} + \dot{M})]/\dot{\omega}\ . \tag{2}$$

From Figure 1, commensurability occurs in the neighborhood of 38400 MJD at which time the observations show $\dot{M} \doteq 3992.4$ degrees/day, $\dot{\omega} \doteq 4.9$ degrees/day and $\dot{\Omega} \doteq 3.3$ degrees/day. The solution of (2) (using $\dot{\theta} = 360{\cdot}985$ degrees/day) is very close to q (res.) = -2. Since $\ell - 2p + q = 1$, then $\ell = 3 + 2p$ and ℓ is odd. Thus the resonant series for this commensurability passage is: (ℓ, m, p, q) = (ℓ, 11, (ℓ - 3)/2, -2), ℓ = 11, 13, 15, . . . The resonant longitude ψ for this series is simply

$$\psi_{res.} = 3\omega + M + 11(\Omega - \theta) \tag{3}$$

The observed variation of $\psi_{res.}$ (= $\psi_{11,-2}$) is shown at the bottom of Figure 1 and the stationary value is indeed near the break in the last cycle (~38360 MJD).

And since it does become stationary, I chose to base the constraint on the determination of the inclination rate ($\dot{I}$) which is always well behaved during the passage. This rate (due the geopotential) is given from the Lagrange planetary equations [Kaula, 1966, p. 29] as:

$$\dot{I} = \frac{\cos I}{na^2(1 - e^2)^{1/2} \sin I} \frac{\partial T}{\partial \omega} - \frac{1}{na^2(1 - e^2)^{1/2} \sin I} \frac{\partial T}{\partial \omega},$$

where;

$$T = \frac{\mu a_e^{\ell}}{a^{\ell+1}} \sum_{\substack{\text{relevant} \\ \ell, m, p, q}} F_{\ell mp}(I)\, G_{\ell pq}(e)\, S_{\ell mpq}(C, S, \omega, M, \Omega, \theta),$$

and;

$$S_{\ell mpq} = \begin{bmatrix} C_{\ell m} \\ \\ -S_{\ell m} \end{bmatrix}_{\ell - m \text{ ODD}}^{\ell - m \text{ EVEN}} \cos \psi_{\ell mpq} + \begin{bmatrix} S_{\ell m} \\ \\ C_{\ell m} \end{bmatrix}_{\ell - m \text{ ODD}}^{\ell - m \text{ EVEN}} \sin \psi_{\ell mpq},$$

with the orbit longitude defined (again) as:

$$\psi_{\ell mpq} = (\ell - 2p)\, \omega + (\ell - 2p + q)\, M + m(\Omega - \theta).$$

In the above expressions, μ is the earth's Gaussian gravity constant, a_e is its mean equatorial radius and a is the orbit's semimajor axis. The f functions are sinusoidal and the G functions are generally monotonic of order $e^{|q|}$ [see Kaula, 1966, p. 37]. The $C_{\ell m}$ and $S_{\ell m}$ are the usual gravitational harmonic coefficients. For the resonant series defined above, ℓ - m is even, and ψ (= $\psi_{res.}$) is the same for each term in the series. The rate due to the combination of these terms is then found to be simply:

$$\dot{I} = \left[\sum_{\substack{\text{resonant} \\ \ell}} C_{\ell,11} f_{\ell} \right] \sin \psi_{res.} + \left[\sum_{\substack{\text{resonant} \\ \ell}} - S_{\ell,11} f_{\ell} \right] \cos \psi_{res.}, \qquad (4)$$

where

$$f_{\ell} = \frac{\mu a_e^{\ell} F_{\ell,11,(\ell-3)/2} G_{\ell,(\ell-3)2,-2}}{n a^{\ell+3} \sin I (1-e^2)^{1/2}} [11 - 3 \cos I] \quad . \tag{5}$$

Clearly, only the sum of the sine and cosine terms (in brackets) in equation (4) are what can actually be determined from observations of the variations of the inclination with (the slowly changing) orbit longitude.

Calling these terms the lumped coefficients of the 11th order C_{11} and S_{11}, the two constraints which can be found from the I data are:

$$(C_{11}, S_{11}) = (\Sigma C_{\ell,11} f_{\ell}, \Sigma - S_{\ell,11} f_{\ell}) \quad . \tag{6}$$

Evaluating the f_{ℓ} from (5) for the average Vanguard 3 orbit in the resonance pass (a = 1.332 earth radii, e = 119, I = 33.3°; the μ/n is irrelevant since it does not depend on ℓ), and normalizing with respect to the (11, 11) term gives the explicit determinable constraint as:

$$\begin{aligned}(C,S)_{11} = {} & (C,S)_{11,11} - 6.7(C,S)_{13,11} + 16.2(C,S)_{15,11} \\ & - 22.9(C,S)_{17,11} + 19.5(C,S)_{19,11} - 6.5(C,S)_{21,11} - 7.2(C,S)_{23,11} \\ & + 11.3(C,S)_{25,11} - 5.0(C,S)_{27,11} - 3.6(C,S)_{29,11} + \cdots ,\end{aligned} \tag{7}$$

where the $(C, S)_{11}$ of (6) and the (C_{11}, S_{11}) of (5) differ only by an irrelevant constant factor.

At this point Gooding, (1971b) solved directly for the lumped coefficients from sets of differenced I values (with observed values of $\psi_{res.}$) for Ariel 3, using a least squares fit to Equation (4). I chose a more hazardous, though potentially more satisfying method to find these coefficients. The method was to solve for a set of (15, 11) coefficients from the same program (ROAD) which revealed the resonance effects (Figure 1). The hazard taken was in the attempt to use all the data in this determination. As I mentioned earlier, the drag model does have deficiencies which, even solving for empirical secular terms as high as the fifth power in the mean anomaly, still leaves the computed orbit longitude with peak errors of as much as 1 1/2 degrees. But this level appears adequate for a decent discrimination of these coefficients. The advantage of the method, of course, is more complete use of the tracking data. In particular, the eccentricity shows the same peculiar dragged-resonance variations as the inclination (Figure 3). This variation adds to the strength of the constraint determination set up for the inclination data [Equation (7)]. But it cannot be used to determine an independent constraint for these terms. In fact the eccentricity and inclination variations from these resonant terms are in phase and differ only by a constant orbit-factor; namely [from, Kaula, 1966, p. 40]

$$(\Delta e/\Delta I)_{res.} = \frac{(1 - e^2) \sin I \, [(1 - e^2)^{1/2} - 3]}{e \, [3 \cos I - 1]} .$$

This ratio is 0.663 for Vanguard 3. Comparison of the variations in Figures (1) and (3) gives the same ratio from the actual data.

The solution (in ROAD) for (15, 11) used the Standard Earth 2 with all its 11th order terms [through (16, 11)] having significant long term effect on the Vanguard 3 orbit. Besides the deep resonant series specified by q = -2 [adjusted by the solution for (15, 11)], the side band series with q = -1 and -3 was included; namely the terms (12, 11, 5, -1), (12, 11, 4, -3), (14, 11, 6, -1), (14, 11, 5, -3), (16, 11, 7, -1) and (16, 11, 6, -3). These terms produced effects with a maximum period of 165 days. The weights for all the "observed" elements in the differential correction process are given at the bottom of Table 1. They only represent the true "quality" of the data for a, e and I. The mean anomaly, containing unresolved effects due (most likely) to drag was downweighted so as to influence the (15, 11) solution as little as possible. A similar downweighting for the ω and Ω data was found to be necessary because of unresolved systematic errors in these arguments. The cause is probably drag again, but not directly. The computed semimajor axis shows (even in the final solution) a quadratic deviation from the observations of up to 100 m. over the 3 1/2 year data span. This is enough to explain the "cubic" Ω and ω residuals by way of errors in the computed secular rates of these arguments due to oblateness. But even with these model deficiencies, the least squares fit of the (15, 11) coefficients to the element data produced a dramatic reduction in the residuals for the inclination and eccentricity. The overall results of the differential correction is shown in Table 2. Also shown in this table are comparison solutions with the unadjusted resonance terms of the Standard Earth 2 and the Goddard Space Flight Center's (Goddard Earth Model) Gem 4 Field [Lerch et. al., 1972b] as well as with the original non resonant field. The Gem 4 included data from the deeply resonant Vanguard 3 and Vanguard 2 rocket body orbits. The significance of these test results will be discussed shortly.

RESULTS

The normalized harmonics found from the straightforward ROAD differential correction process (described in more detail by Wagner, (1972b) for a multi-arc zonal solution) was:

$$10^7(C_{15,11}, S_{15,11}) = (0.32 \pm 0.01,\ 0.73 \pm 0.01), \tag{8}$$

with a correlation coefficient of -0.51 between these terms. I was pleased to find that the correlations of (15, 11) with the empirical terms in the mean anomaly were all less than $|0.1|$. This poorly determined data was apparently not influencing the solution because of its deliberate downweighting. But the real proof of these speculations, indeed of the (too simple?) constraint theory itself, must be to derive a result from that theory and then test it against the data.

Evaluation of (7) with these adjusted (15, 11) terms and the (11, 11) and (13, 11) terms in the Standard Earth 2 (included in the trajectory) yields the following values for the Vanguard 3 resonance constraint:

$$10^7(C, S)_{11} = (8.6 \pm 0.2,\ 7.8 \pm 0.2) \tag{9}$$

Equations (7) and (9) provide the means to obtain a single additional pair of coefficients for a field. For example, the constraint using the Standard Earth 2 field requires (17, 11) to be:

$$10^7(C, S)_{17,11} = (-.457,\ .149),$$

which appears quite reasonable according to the simple rule proposed by Kaula, (1966) $[(C, S)_{\ell, m} \approx 10^{-5}/\ell^2]$. However, as can be seen from (7) this result is undoubtedly an accident because the effects of higher degree terms are far from negligible. Nevertheless this coefficient set is consistent with the Vanguard constraint and should fully explain the eccentricity and inclination residuals. Indeed the trajectory calculated in ROAD with this (17, 11) set <u>was</u> the equivalent of that calculated with the (15, 11) adjusted values, as shown also in Table 2. While this result was expected, it was still enormously gratifying as an elaborate numerical confirmation of the simple constraint.

DISCUSSION

The results of the previous section can best be summarized in a C, S diagram for the 11th order constraint (Figure 4) which emphasizes the relative superiority of the SAO SE 2, over the later Gem 4 solution. Clearly neither solution comes close to explaining the resonant eccentricity and inclination variations (see also Table 2). But why does the SAO SE 2 field, which does not include any resonant Vanguard data, provide a better fit than the Gem 4 which does? It is not so surprising when it is recalled that the 7 day arcs of the Gem 4 solution would see very little of the variation in these long term effects. Gaposchkin and Lambeck (1971, p. 21) felt that analytic term selection might solve this problem in their approach. (The idea is to use only those terms which have periods of the order of the arc length: inherently impossible with straight numerical integration as used for the orbits in the Gem's). It is interesting that an unpublished SAO 1969 satellite-only solution called B6.1, with 11th order resonance terms through 16, 11 and using Vanguard 3 data, does come closer to the mean element constraint than any other recent SAO or Goddard field. The Smithsonian analytic (really semi-analytic) solutions use 30 day arcs where possible. This is clearly an advantage where long period resonance is encountered. But apparantly it is not sufficient where some terms are considerably longer than a month, (as in this case). On the other hand, in spite of the fair success of the B6.1 solution, it is possible that SAO abandoned this data only because the solutions with it were so highly correlated that the results were unrealistic. But that result might have been anticipated and a priori constraints applied to limit the coefficient solutions. My own feeling is that since it is never a matter of just one term with a very long period, but a spectrum of terms which cannot all be absorbed properly, the resulting solution will tend to be both unrealistic and highly correlated if only the long period terms are ignored.

Another comparison solution, Gem 2, (Lerch et al., 1972a), is also shown in Figure 4. It differs from Gem 4 only in that it lacks electronic-satellite data. The impact on the Vanguard constraint is obviously not significant.

What is significant is the probable impact of the odd degree terms beyond the 15th. Using the Kaula rule to estimate the size of the neglected coefficients, and taking the root sum of squares of the estimated terms [in Equation (7)] gives the concentric circles (of uncertainty) in Figure 4. These circles can be regarded in two ways. On the one hand they say that the recent comprehensive geopotential solutions (without Vanguard 3 data) can not be expected to reproduce the Vanguard constraint because of likely contributions beyond those solved for in these models. On the other hand they show that solutions for coefficients beyond about the 25th degree will benefit strongly from this constraint.

Of course, even for the less complete solutions, the constraint could still be used (as in the previous section) to satisfy a set of "lumped" coefficients tailored for the Vanguard 3 orbit.

Beyond the 11th order commensurabilities associated with this resonance pass (in 1963-64) are series for $m = 22$, (associated with $q = -4$), $m = 33$ ($q = -6$), and so on. But the lowest degree's for these are higher (22 and 33 (etc.)) and the resonant variations are proportional to e^4 and e^6 (etc.). Thus it can be expected that their effects will be slight compared to the 11th order series. In fact I have made a solution for (22, 22) from this data and the result was an insignificant determination with no appreciable change in the residuals.

However, the effects of the even degree "side band" resonances of 11th order (terms with $q = -1$ and -3) cannot be so easily dismissed. Their frequencies are distinct enough throughout the pass to be able to discriminate two sets of even order terms. Unfortunately the data is heavily weighted on the "high" side of the commensurability. This catches the longer (and shortening) period, but lesser effects of the (previously commensurate) $q = -3$ terms. From MJD 37315 to 38602 (the span of the best data), the period of these effects falls from 165 to 75 days. The period of the potentially stronger $q = -1$ terms (commensurable later, in 1966) lengthens from only 45 to 82 days during this time span. Preliminary determinations of these constraints are promising but so far inconclusive. At any rate, these determinations are essentially uncorrelated with and do not significantly disturb the $q = -2$ constraint.

It is anticipated that a fairly strong constraint for even degree (11th order) terms will be determinable by the new mean element method from the $q = -3$ commensurability which was well observed (by precise Baker Nunn tracking) in 1959-1960. Unfortunately, only field reduced optical and X-band North American Air Defense Command Tracking is available to observe the probably much stronger $q = -1$, 0, and +1 commensurabilities in the late 1960's and early 1970's. But their greater strength should provide other sets of independent constraints for the even and odd degree terms.

CONCLUSIONS

A strong constraint has been found for 11th order and odd degree terms in the geopotential which have previously been poorly observed. This has been accomplished from analysis of mean element "observations" of the deep resonant orbit of Vanguard 3 in 1961-1964. The constraint is not satisfied by recent comprehensive geopotential solutions, some of which have used (improperly) Vanguard 3 tracking data. However the constraint is undoubtedly more strongly affected by terms above the 15th degree than those below, the latter being the only ones so far determined in these solutions. Because of this, the determined condition equations (probably) will only be useful in achieving accurate geopotential results when solutions for terms as high as the 25th degree are attempted.

Examination of independent 11th order commensurabilities for Vanguard 3 in 1959-60, 1966-71 and 1973-74 (using mean elements derived from tracking data) should provide additional strong constraints for 11th order harmonics.

ACKNOWLEDGMENT

Special thanks are due Ted Heuring for supplying the excellent mean elements which were the basis for this study, and Barbara Putney for keeping the ROAD program on the road to the solutions.

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Table 1
Mean Element Observations of Vanguard 3 (1959-7A), 1961-64

TIME (MJD)	A(e.r.)	E	INCL (°)	OMEGA (°)	NODE (°)	MEAN(°)
37315.40140000	1.33287330	.1889338	33.3590	345.4664	80.0897	8.6037
37322.07360000	1.33284930	.1891909	33.3539	18.0779	58.1833	347.4229
37329.19580000	1.33284840	.1894391	33.3509	52.8693	34.7932	321.8189
37336.04720000	1.33283000	.1895110	33.3496	86.2779	12.2916	296.0275
37343.07640000	1.33283130	.1894589	33.3514	120.5577	349.2040	259.7657
37350.01390000	1.33282210	.1891846	33.3549	154.4246	326.4215	217.9871
37357.03890000	1.33281880	.1889138	33.3576	188.7666	303.3556	165.4830
37363.07500000	1.33280260	.1887086	33.3580	218.3138	283.5362	127.6701
37370.01250000	1.33279230	.1886048	33.3604	252.2814	260.7574	86.2266
37377.31250000	1.33279500	.1885856	33.3609	288.0567	236.7840	51.2017
37384.07360000	1.33280320	.1886639	33.3578	321.1730	214.5837	26.3398
37391.01540000	1.33279760	.1888876	33.3548	355.1591	191.7913	2.0690
37398.13750000	1.33278390	.1891405	33.3478	29.9723	168.4014	338.4506
37405.07920000	1.33277590	.1893754	33.3436	63.8645	145.6000	314.6172
37412.01940000	1.33277130	.1894153	33.3431	97.7168	122.7996	285.4247
37419.04720000	1.33276740	.1893167	33.3452	132.0024	99.7116	245.4402
37426.07360000	1.33277070	.1890309	33.3525	166.3119	76.6347	199.9366
37432.01810000	1.33276780	.1888126	33.3588	195.3802	57.1124	157.6462
37439.94510000	1.33276460	.1888506	33.3631	234.0716	31.0903	108.8296
37447.24440000	1.33275400	.1885528	33.3701	269.9570	7.1108	69.6480
37452.92220000	1.33277210	.1885901	33.3620	297.7801	348.4760	43.5616
37459.77260000	1.33278200	.1888000	33.3606	331.3237	325.9800	20.1626
37466.39420000	1.33275330	.1890336	33.3568	3.5199	304.3706	356.1937
37473.29580000	1.33274840	.1892910	33.3508	37.4479	281.5719	333.1700
37480.05690000	1.33274500	.1894857	33.3471	70.4289	259.3617	309.8920
37487.08710000	1.33274570	.1894801	33.3450	104.7219	236.2660	276.1860
37494.02360000	1.33274070	.1893021	33.3461	138.5758	213.4733	236.8072
37501.05000000	1.33273330	.1890460	33.3517	172.9180	190.3912	192.2477
37508.07500000	1.33273700	.1887792	33.3563	207.2867	167.3178	142.2167
37515.01110000	1.33273860	.1886585	33.3602	241.2648	144.5386	97.4731
37522.04030000	1.33274960	.1886274	33.3622	275.6882	121.4591	64.0718
37529.07080000	1.33273430	.1887117	33.3609	310.1348	98.3707	36.2700
37536.28190000	1.33273870	.1888820	33.3581	345.4404	74.6870	9.0038
37543.13060000	1.33272870	.1891593	33.3536	18.9422	52.1832	346.4864
37550.07500000	1.33273170	.1893656	33.3501	52.8376	29.3801	324.2267
37557.01530000	1.33272910	.1894656	33.3493	86.6786	6.5821	296.5466
37564.04310000	1.33271750	.1893543	33.3511	120.9673	343.4963	258.1173
37571.06940000	1.33271720	.1890880	33.3519	155.2623	320.4175	214.1840
37578.00420000	1.33271280	.1888134	33.3552	189.1746	297.6414	164.4972
37585.03040000	1.33270560	.1885637	33.3577	223.5681	274.5616	120.7328
37592.05330000	1.33270600	.1884543	33.3593	257.9916	251.4825	82.6611
37599.08750000	1.33271720	.1884101	33.3598	292.4319	228.4019	50.2095
37606.02770000	1.33272050	.1885289	33.3566	326.4277	205.6114	23.2350
37613.02360000	1.33265570	.1887075	33.3498	1.2854	182.2281	356.8670
37620.00000000	1.33268370	.1890043	33.3464	34.7781	159.7237	335.6011
37627.03056000	1.33266410	.1891840	33.3412	69.1077	136.6239	309.4771
37634.05970000	1.33267920	.1892017	33.3430	103.3821	113.5257	277.9536
37641.17640000	1.33266380	.1890120	33.3505	138.1212	90.1496	235.7752
37648.02220000	1.33266030	.1887096	33.3566	171.5643	67.6649	193.0739
37655.04720000	1.33265630	.1884531	33.3614	205.9580	44.5979	145.3806

Table 1 (Continued)

TIME (MJD)	A(e.r.)	E	INCL (°)	OMEGA (°)	NODE (°)	MEAN (°)
37662.07220000	1.33265510	.1882510	33.3621	240.3630	21.5290	97.7260
37669.28060000	1.33266670	.1882166	33.3642	275.6804	357.8623	61.7806
37676.04030000	1.33264980	.1882481	33.3631	308.8047	335.6641	35.8259
37683.07080000	1.33266030	.1884697	33.3605	343.2391	312.5811	10.7756
37690.28190000	1.33261340	.1887299	33.3509	18.5287	288.9013	346.4631
37697.31250000	1.33263150	.1890071	33.3455	52.8539	265.8053	321.9315
37704.07220000	1.33260400	.1891149	33.3420	85.8441	243.5955	296.8415
37711.10000000	1.33261330	.1890206	33.3426	120.1283	220.5042	261.6244
37718.37710000	1.33260680	.1887608	33.3474	155.7717 ✗	196.5351 ✗	216.3240 ✗
37726.05000000	1.33259540	.1884439	33.3537	193.2082	171.3941	161.3244
37733.07500000	1.33257980	.1882851	33.3580	227.6278	148.3236	115.8753
37740.01110000	1.33257220	.1882206	33.3596	261.6148	125.5424	75.9543
37747.40000000	1.33258170	.1882799	33.3585	297.8335	101.2702	43.2836
37754.06940000	1.33253960	.1883879	33.3563	330.5086	79.3599	20.0059
37761.00970000	1.33253430	.1886698	33.3539	4.4823	56.5590	358.0155
37768.21940000	1.33253260	.1889563	33.3504	39.7267	32.8746	331.7145
37775.15830000	1.33250340	.1891580	33.3483	73.5910	10.0757	305.0169
37783.26670000	1.33249100	.1891437	33.3480	113.1629	343.4262	266.2726
37790.02220000	1.33247810	.1889744	33.3496	146.1342	321.2277	228.7380
37797.04580000	1.33247070	.1887133	33.3541	180.4825	298.1464	181.2134
37804.15370000	1.33246250	.1884728	33.3600	215.3103 ✗	274.7752 ✗	128.8071 ✗
37811.18330000	1.33245070	.1883967	33.3599	249.7420	251.6948	87.4780
37818.03060000	1.33244800	.1883543	33.3594	283.2963	229.1963	56.7958
37825.23890000	1.33244380	.1884626	33.3550	318.6285	205.5048	27.5673
37832.35830000	1.33247170	.1886858	33.3506	353.5044	182.1108	3.7076
37839.51940000	1.33240100	.1889568	33.3471	28.3407 ✗	158.7110 ✗	340.0747 ✗
37846.23610000	1.33244730	.1891887	33.3450	61.3546	136.4936	315.3730
37853.08330000	1.33243560	.1892867	33.3455	94.7671	113.9863	285.6604
37860.01940000	1.33242710	.1891747	33.3487	128.6234	91.1846	250.8955
37867.04310000	1.33240940	.1889273	33.3543	162.9326	68.0971	205.5602
37874.06530000	1.33239220	.1886395	33.3605	197.3136	45.0177	154.8537
37881.08890000	1.33239610	.1884454	33.3648	231.7266	21.9365	109.8415
37888.20420000	1.33239260	.1883868	33.3660	266.6063	358.5553	70.7903
37895.05140000	1.33240030	.1883871	33.3642	300.1706	336.0550	41.8641
37902.44030000	1.33240390	.1885764	33.3610	336.3826	311.7719	15.1551
37909.37920000	1.33240360	.1888069	33.3547	10.3585	288.9680	352.3797
37916.13750000	1.33237440	.1890677	33.3488	43.3983	266.7513	329.1064
37923.16530000	1.33238340	.1892121	33.3457	77.7238	243.6417	301.5326
37930.10140000	1.33237850	.1891764	33.3461	111.5700	220.8372	268.1942
37937.03610000	1.33237140	.1889643	33.3516	145.4472	198.0389	229.4203
37944.05830000	1.33236530	.1886530	33.3555	179.7974	174.9562	180.1127
37951.08060000	1.33235010	.1884309	33.3591	214.2089	151.8774	130.9255
37958.01530000	1.33234520	.1882890	33.3610	248.2026	129.0847	92.5291
37965.22080000	1.33235460	.1883134	33.3619	283.5429	105.4001	55.2962
37972.15830000	1.33235220	.1883963	33.3629	317.5468	82.6005	28.1765
37979.00690000	1.33235190	.1886234	33.3609	351.1043	60.0906	6.3405
37986.30560000	1.33233190	.1888628	33.3571	26.8248	36.1025	340.9922
37993.42360000	1.33235680	.1891355	33.3531	61.5918	12.7056	315.0499
38000.09030000	1.33233190	.1891865	33.3515	94.1529	350.7847	287.2729
38007.02500000	1.33232920	.1891082	33.3538	128.0050	327.9857	249.6060

Table 1 (Continued)

TIME (MJD)	A(e.r.)	E	INCL (°)	OMEGA (°)	NODE (°)	MEAN (°)
38014.04860000	1.33232990	.1888638	33.3577	162.3470	304.8959	206.8161
38021.07080000	1.33233530	.1885873	33.3611	196.7137	281.8159	158.5302
38028.00280000	1.33234000	.1884301	33.3623	230.6808	259.0340	109.7733
38035.02770000	1.33233980	.1883346	33.3621	265.1217	235.9442	72.4411
38042.23470000	1.33233590	.1884105	33.3602	300.4531	212.2558	41.4151
38049.35280000	1.33232680	.1885506	33.3588	335.3496	188.6573	15.5946
38056.02080000	1.33232410	.1888018	33.3548	8.0071	166.9372	353.5254
38063.31940000	1.33232590	.1890766	33.3504	43.7070	142.9408	328.6016
38070.16670000	1.33232850	.1892594	33.3496	77.1366	120.4296	301.8319
38077.01250000	1.33232330	.1892573	33.3512	110.5608	97.9171	269.5379
38084.03610000	1.33232400	.1890645	33.3539	144.8659	74.8247	226.9325
38091.05830000	1.33232630	.1887843	33.3596	179.2117	51.7367	178.7472
38098.17080000	1.33234010	.1885443	33.3644	214.0384	28.3594	130.7928
38105.01530000	1.33233590	.1884451	33.3672	247.5866	5.8642	92.7024
38112.13060000	1.33233720	.1883833	33.3675	282.4721	342.4831	55.7253
38119.06810000	1.33233360	.1885267	33.3626	316.4805	319.6889	29.0994
38126.45690000	1.33233310	.1887152	33.3597	352.6793	295.3962	4.3911
38133.03470000	1.33231310	.1889617	33.3527	24.8672	273.7702	341.8879
38140.15280000	1.33233820	.1891881	33.3489	59.6378	250.3647	316.1196
38147.09020000	1.33233460	.1892242	33.3479	93.4926	227.5519	289.6278
38154.02500000	1.33233000	.1890778	33.3502	127.3572	204.7532	252.0584
38161.04720000	1.33232690	.1887773	33.3511	161.6865	181.6701	203.7764
38168.06940000	1.33233210	.1885076	33.3533	196.0669	158.5857	155.4157
38175.00280000	1.33232960	.1882938	33.3564	230.0448	135.7940	112.1818
38182.02770000	1.33232890	.1882247	33.3602	264.4876	112.7054	74.8950
38189.01250000	1.33234090	.1882292	33.3632	298.9408	89.6147	43.1914
38196.08190000	1.33235380	.1883934	33.3613	333.3817	66.5227	17.0800
38203.02080000	1.33235400	.1886008	33.3572	7.3651	43.7194	356.1778
38210.04860000	1.33230060	.1888099	33.3513	41.7160	20.6216	330.1368
38217.07640000	1.33231920	.1889815	33.3483	76.0430	357.5176	304.1323
38224.01250000	1.33232160	.1889152	33.3486	109.8922	334.7184	272.2903
38231.03610000	1.33232740	.1887136	33.3515	144.2092	311.6322	229.7370
38238.05630000	1.33232240	.1884113	33.3540	178.5567	288.5545	181.5936
38245.17080000	1.33232250	.1882012	33.3540	213.3961	265.1796	133.7755
38252.01390000	1.33231940	.1880606	33.3557	246.9471	242.6902	90.4477
38259.04030000	1.33231340	.1880377	33.3547	281.3978	219.5952	59.0449
38266.33750000	1.33230800	.1880918	33.3553	317.1861	195.6168	28.8721
38273.00560000	1.33232220	.1882878	33.3525	349.8621	173.7000	7.2560
38280.12360000	1.33232800	.1885691	33.3469	24.7084	150.3082	342.0830
38287.24170000	1.33229760	.1888168	33.3432	59.4917	126.9083	317.0539
38294.26810000	1.33230200	.1889007	33.3426	93.8050	103.8057	286.3385
38301.08940000	1.33226750	.1887826	33.3444	127.2211	81.3057	249.6067
38308.04580000	1.33228840	.1885688	33.3500	161.1190	58.5117	207.7894
38315.06670000	1.33228300	.1882687	33.3566	195.4839	35.4364	155.3100
38322.09030000	1.33226540	.1881288	33.3613	229.9108	12.3563	114.2025
38329.02360000	1.33225570	.1880518	33.3619	263.9132	349.5707	73.0303
38336.05000000	1.33225060	.1881229	33.3586	298.3649	326.4754	43.6458
38343.42590000	1.33222770	.1882691	33.3544	334.5970	302.1943	16.1249
38350.01530000	1.33223630	.1885452	33.3504	6.8094	280.5666	356.2871
38357.40280000	1.33221860	.1888677	33.3430	42.9380	256.2807	329.3339

Table 1 (Continued)

TIME (MJD)	A(e.r.)	E	INCL (°)	OMEGA (°)	NODE (°)	MEAN (°)
38364.06940000	1.33224290	.1890459	33.3416	75.4874	234.3596	304.9032
38371.00410000	1.33222890	.1890407	33.3396	109.3499	211.8515	270.8312
38378.16670000	1.33220950	.1888115	33.3402	143.6613 X	188.4539 X	231.7401 X
38385.04860000	1.33220860	.1885578	33.3457	178.0137	165.3642	181.5490
38392.15970000	1.33221040	.1882899	33.3519	212.8680	141.9805	131.8222
38399.00280000	1.33220620	.1881985	33.3564	246.4287	119.4884	92.0516
38406.74720000	1.33217870	.1881853	33.3577	284.4192	94.0267	51.1773
38413.05420000	1.33215360	.1883145	33.3567	315.3439	73.2925	31.3351
38430.08190000	1.33216770	.1889216	33.3470	38.7237	17.2992	334.4366
38434.04580000	1.33218560	.1890397	33.3457	58.0989	4.2640	320.3200
38441.07080000	1.33219010	.1890891	33.3446	92.3946	341.1614	287.8939
38448.18410000	1.33217060	.1889897	33.3445	127.1437	317.7642	250.4623 X
38469.06940000	1.33216490	.1882337	33.3509	229.4007	249.0827	113.6304
38476.00280000	1.33216110	.1881533	33.3518	263.4100	226.2846	75.6520
38483.02780000	1.33217190	.1881714	33.3535	297.8811	203.1864	43.5608
38490.14440000	1.33216440	.1883088	33.3491	332.7771	179.7879	17.5849
38497.08190000	1.33216540	.1885342	33.3445	6.7804	156.9748	356.3611
38504.19860000	1.33216020	.1887957	33.3393	41.6026	133.5663	330.5612
38511.44440000	1.33214960	.1889608	33.3365	75.0650 X	111.0468 X	303.5337 X
38518.06940000	1.33215430	.1889299	33.3392	109.3747	87.9399	271.9809
38525.09170000	1.33215910	.1887574	33.3435	143.6897	64.8452	229.3736
38532.11250000	1.33215820	.1881720 X	33.3483	178.0657 X	41.7576	181.1557
38539.04310000	1.33215610	.1873837 X	33.3668 X	212.1216 X	18.9379 X	132.4016
38546.15560000	1.33214370	.1880374	33.3557	246.8997	355.9901	90.0890
38553.00000000	1.33215520	.1880127	33.3554	280.4723	333.0904	57.6032
38560.95830000	1.33215410	.1881506	33.3521	322.4778 X	304.9497 X	28.1247 X
38567.41390000	1.33214070	.1883386	33.3475	351.1806	285.6970	7.4338
38574.35000000	1.33214890	.1886468	33.3407	25.1494	262.8907	341.0947
38581.01670000	1.33214330	.1888640	33.3367	57.7366	240.9631	318.9138
38588.13190000	1.33214330	.1889669	33.3319	92.5004	217.5583	287.9681
38595.06530000	1.33214390	.1888354	33.3340	126.3759	194.7475	250.9997
38602.08750000	1.33215130	.1885981	33.3390	160.7191	171.6499	208.0981
STANDARD OBSERVATION SIGMAS	0.00001000	0.0000200	0.000500	0.020000	0.020000	1.000000

*These elements were originally determined (by T. Heuring) as osculating values from a precision tracking program (GEODYN) using the precisely reduced Baker-Nunn optical observations of the Smithsonian Astrophysical Observatory's World Wide Network. The mean elements are these osculating values less the short period terms (due to oblateness) due to D. Brouwer, 1959. The arc lengths to which these original elements apply are generally 7 days. After processing the 3-1/2 year arc of mean elements it was found necessary to discard only 48 or 4.4% of these observations. The rejected values are marked with an X.

Table 2
Results of ROAD-Vanguard 3 Orbit Determinations
(Using 3-1/2 Years of GSFC Mean Elements*)

Field Used	Overall Weighted RMS Residual	Weighted Element Residuals (rms)				
		e (2×10^{-5})	I (0.0005°)	ω (0.02°)	Ω (0.02°)	M (1.°) Unit Wts.
SAO SE2 Without Resonant Terms	3.17	2.29	6.79	0.75	1.16	0.7
SAO SE2 With Resonant Terms	2.19	1.44	4.49	1.06	0.89	0.8
GEM 4 With Resonant Terms	2.78	1.83	5.92	1.07	1.03	0.8
SAO SE2 With Resonant Terms + (15, 11) Adjusted From data	1.13	0.76	1.59	1.13	0.73	0.7
SAO SE2 With Res. Terms + (17, 11) Solved from Constraint	1.14	0.73	1.59	1.19	0.75	0.7

*All orbit determinations used all six mean elements as observed data and adjusted the six initial elements, a radiation pressure and drag coefficient and 5 mean anomaly "Secular" terms. Included are short and long period luni-solar terms to the 4th degree in their disturbing potentials, as well as all significant long period zonal variations in the given fields and the effects of precession and nutation of the earth's polar axis.

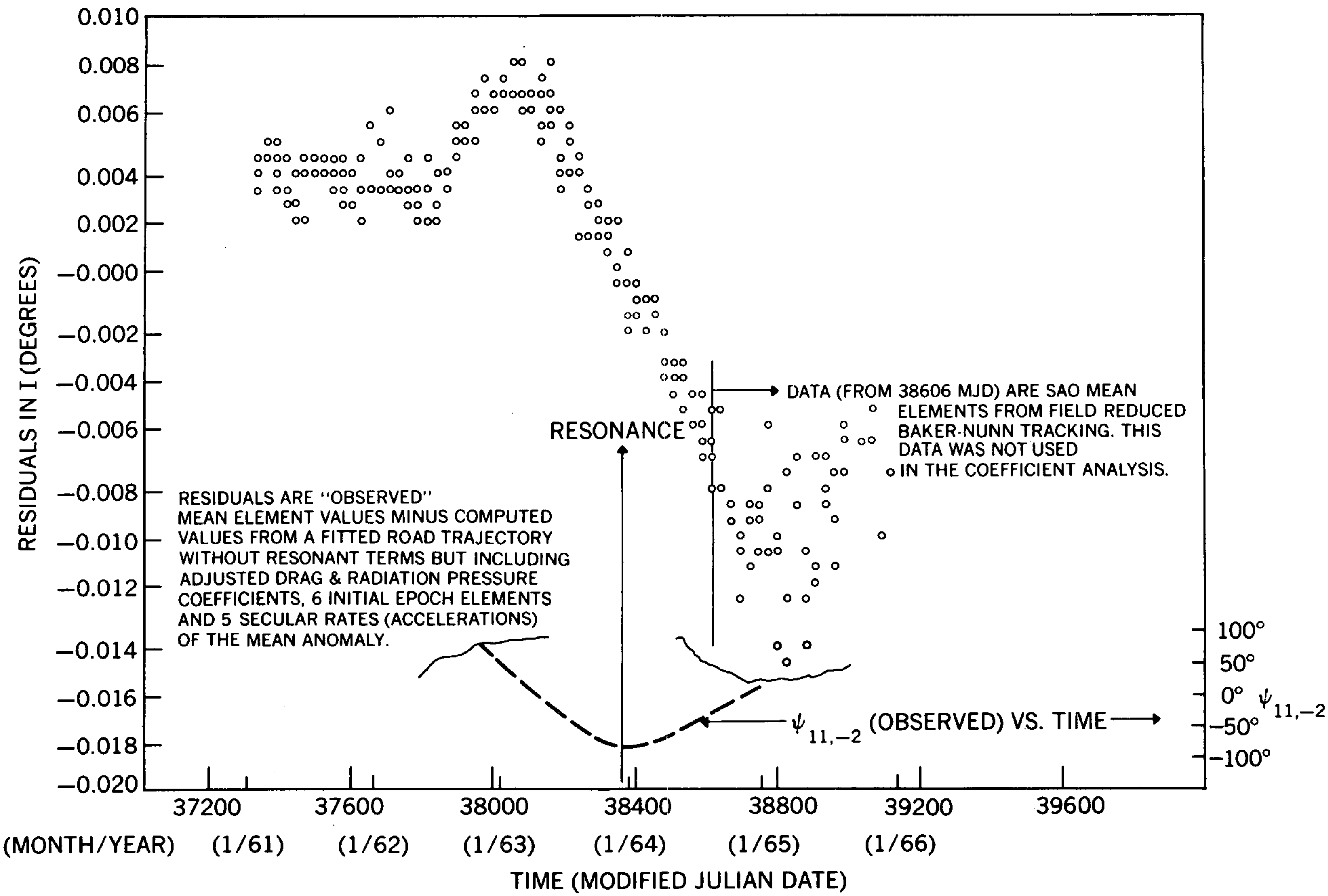

Figure 1. Measurement Residuals In Inclination From A Five Year Nonresonant Vanguard 3 Trajectory

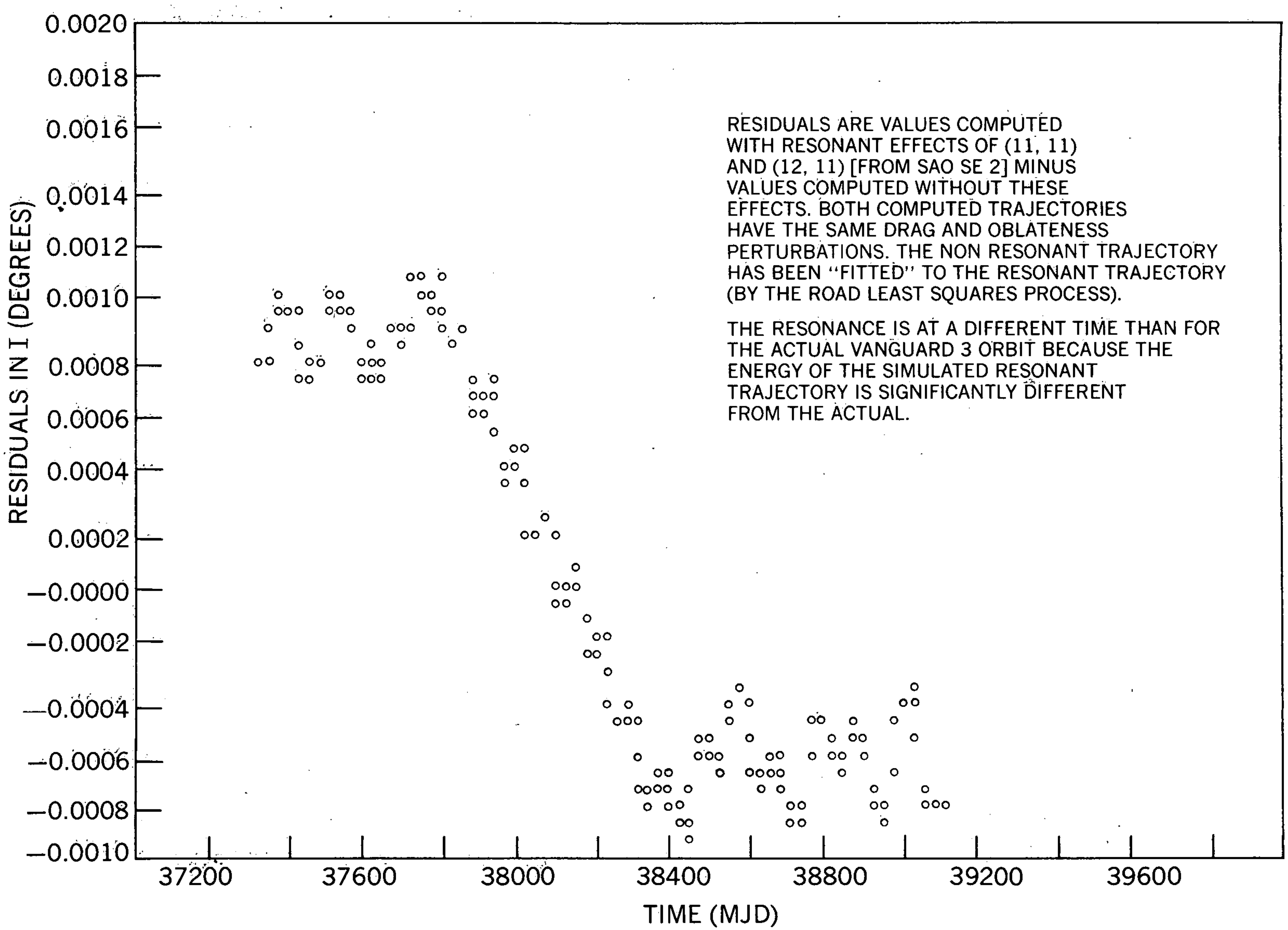

Figure 2. Inclination Residuals From Two 5 Year Simulated Vanguard 3 Trajectories

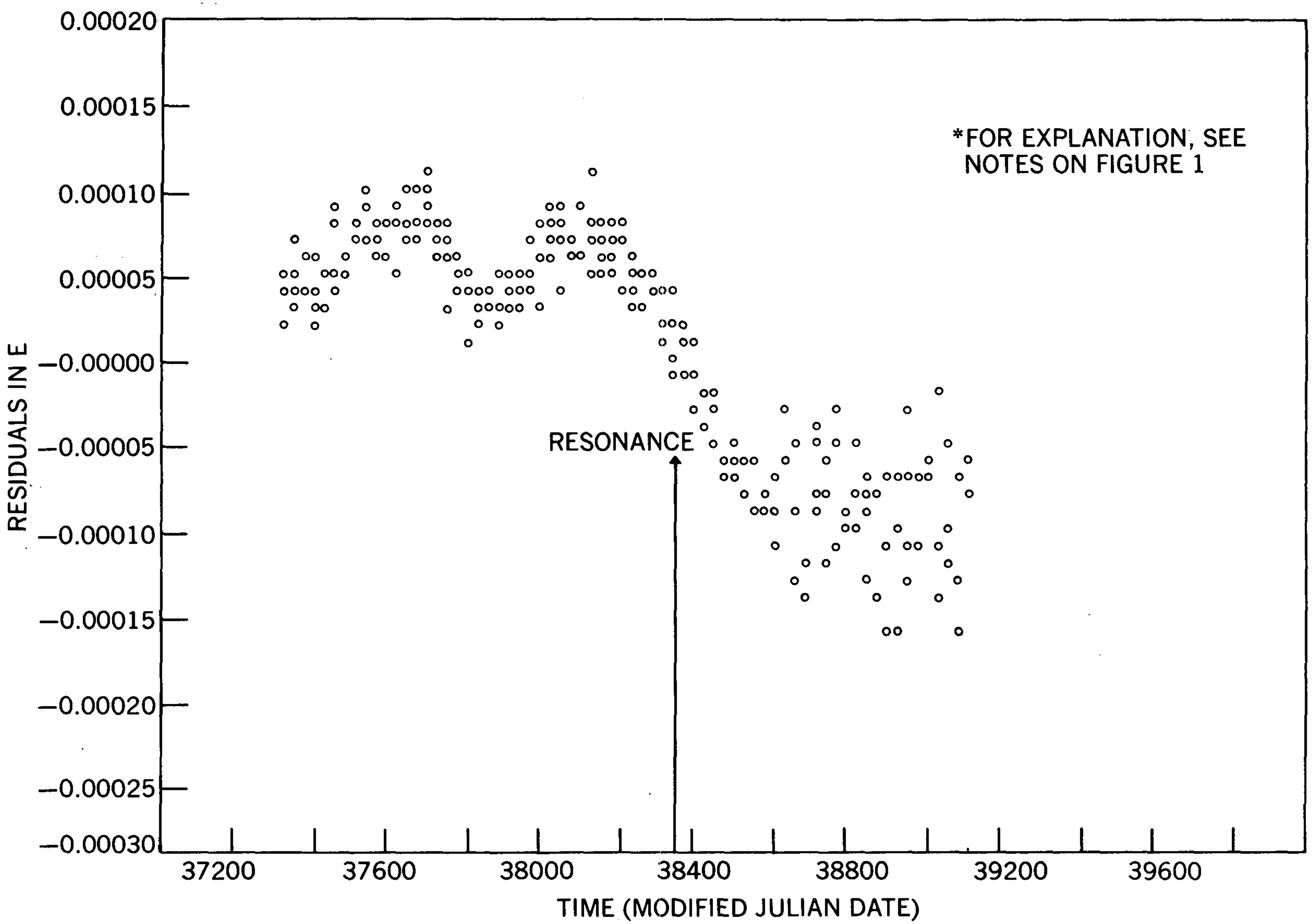

Figure 3. Measurement Residuals In Eccentricity From a 5 Year Nonresonant Vanguard 3 Trajectory*

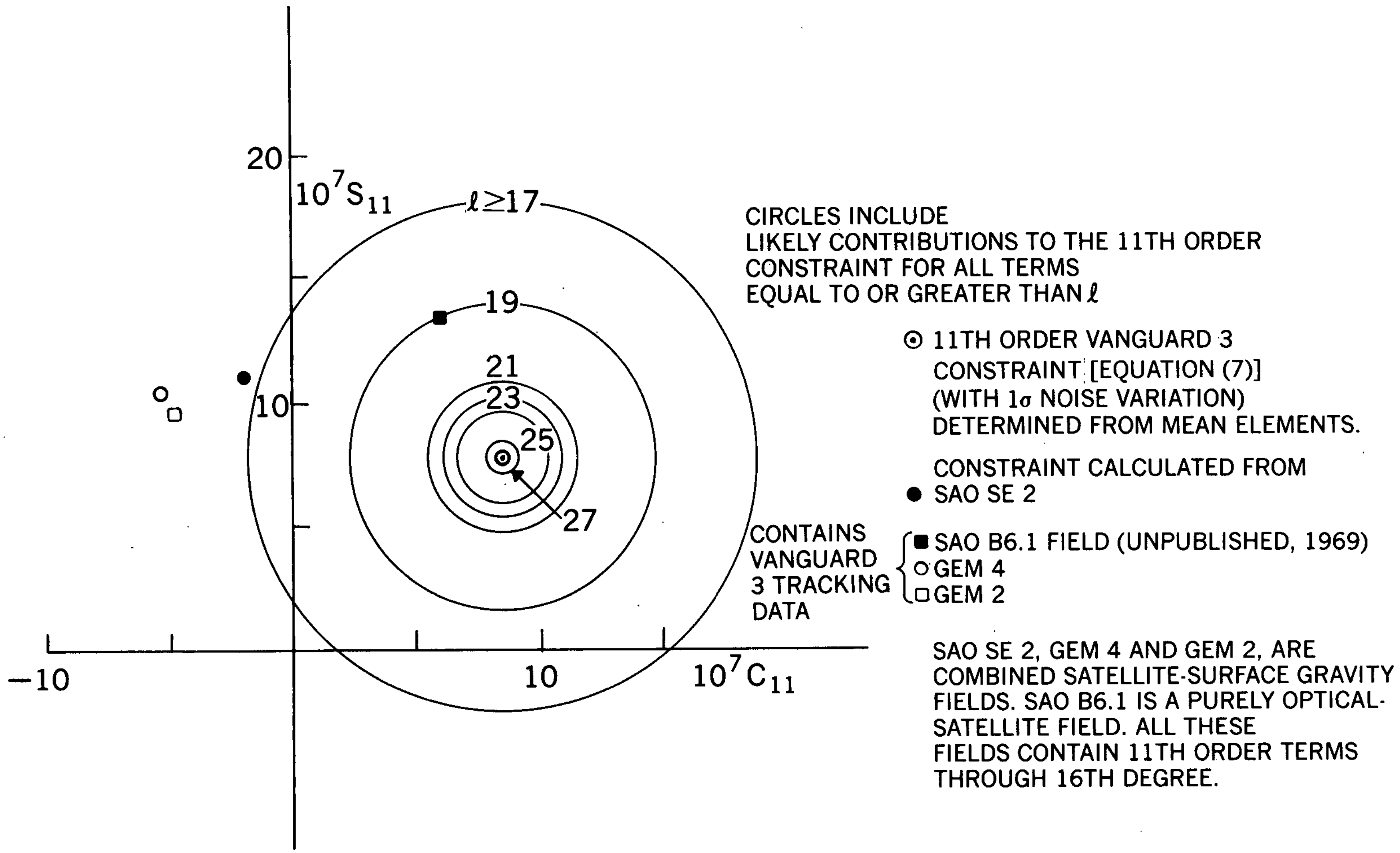

Figure 4. Vanguard 3 Constraint From Various Fields