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ENGINEERING OPERATIONS REPORT

VIBRATION COMPUTER PROGRAMS
E13101, E13102, E13104, AND E13112
AND APPLICATION TO THE NERVA PROGRAM

PROJECT 187 - METHODOLOGY DOCUMENTATION

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I. ABSTRACT

Basically, these programs are the analyses of the free or forced, undamped vibrations of one or two elastically-coupled lumped parameter teams. The whirl analysis of a rotor-bearing-casing system is facilitated by the assumptions that the rotor, casing, and bearing stiffness characteristics are axially symmetric and that the shaft executes circular orbits. Bearing nonlinearities, casing and rotor distributed mass and elasticity, rotor imbalance, forcing functions, gyroscopic moments, rotary inertia, and shear and flexural deformations are all included in the system dynamics analysis.

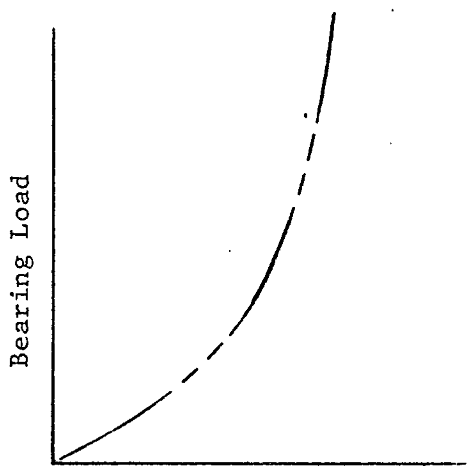
The analysis is based upon a lumped mass parameter model using a modified Myklestad-Thomson transfer matrix technique. Bearings are characterized as springs which can have constant spring rates or load-dependent values defined by

$$K = A \cdot P^B \text{ or a table of P vs K}$$

points, where A and B are constants and P is the load transmitted through the spring.

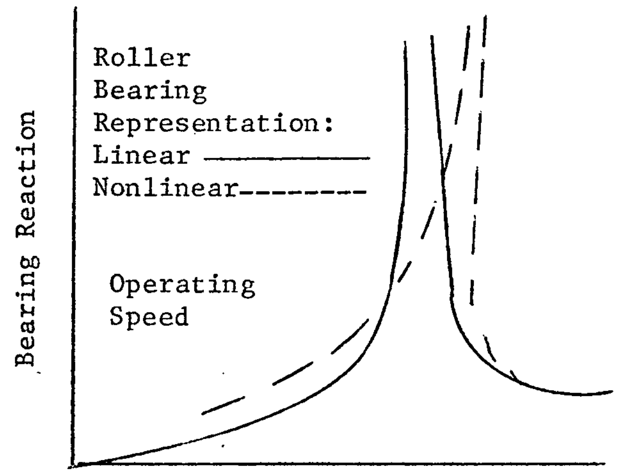
All bearings have nonlinear load displacement characteristics, the solution is achieved by iteration. Rotor imbalances allowed by such considerations as pilot tolerances and runouts as well as bearing clearances (allowing conical or cylindrical whirl) determine the forcing function magnitudes. The computer programs first obtain a solution wherein the bearings are treated as linear springs of given spring rates. Then, based upon the computed bearing reactions, new spring rates are predicted and another solution of the modified system is made. The iteration is continued until the changes to bearing spring rates and bearing reactions become negligibly small.

If the machine operating speed is near a critical speed, the magnified bearing reaction is of interest for comparison to the bearing capacity. The nonlinear treatment of the bearings by this method shows that bearing reaction predictions, based upon a linear representation of the bearings, can be unconservative (see (b) below).



(a)

Typical Load-Spring Rate Curve for a Roller Bearing



(b)

Response Prediction Influenced by Bearing Representation

These are some of the preferred types of computer programs used for the analysis of rolling contact supported rotors. These programs, which are fully described in the users manuals included with the Turbopump Shafts and Couplings dossier, are available from COSMIC, the University of Georgia, through the National Aeronautics and Space Administration.

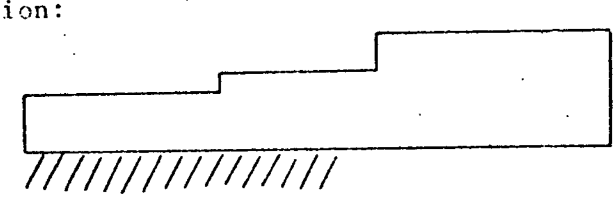
II. TECHNICAL DISCUSSION

All four programs can be summarized in a simple tabular form. They all compute natural frequencies, mode shapes, and the amplitudes of the shears, moments, slopes and deflections produced in a lumped parameter beam system by harmonic forces and/or moments.

PROGRAMS	FORCED VIBRATION	FREE VIBRATION	SINGLE BEAM	DOUBLE BEAM	CONSTANT SPRING	NON-LINEAR SPRING
E13101		X	X		X	
E13102		X		X	X	
E13104	X			X		X
E13112	X		X			X

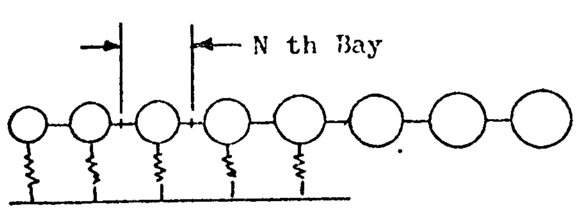
The method of analysis is a modified Myklestad-Thomson type and is described below.

To describe the model, first consider the following beam on a discontinuous foundation:

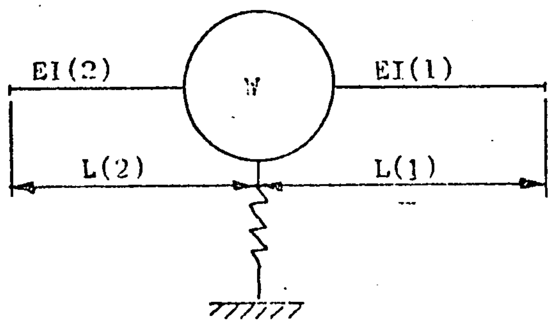


k (force/unit length)

We might represent this structure for the vibration analysis as:



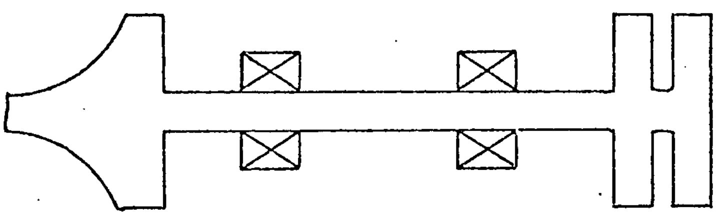
We see that a typical element, which is called the N th bay in the above sketch, is:



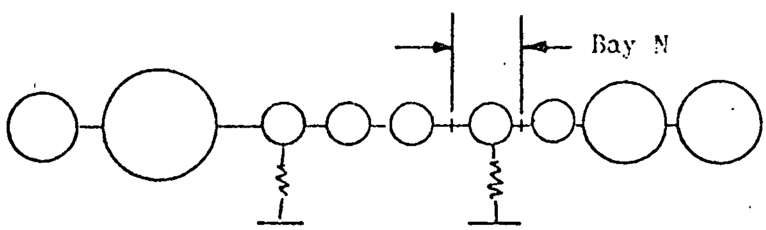
The weight of the bay, W, is lumped at point N. The beam lengths, which need not be equal, are designated L(1) and L(2). The associated bending rigidities are EI(1) and EI(2). To represent the elastic foundation acting on the bay, a spring constant K is utilized. Note that in this example K would equal $\left[L(1) + L(2)\right]$ k.

One can see that the physical beam rigidities are quite well represented in the analysis whereas the mass distribution and elastic foundation representation depend on the number of lumping stations used.

As a second example to describe the lumped parameter model, consider the following two-bearing shaft with overhung rotors:

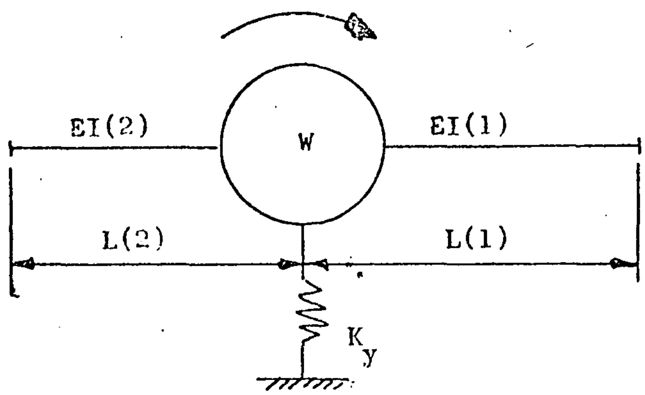


The lumped model is:



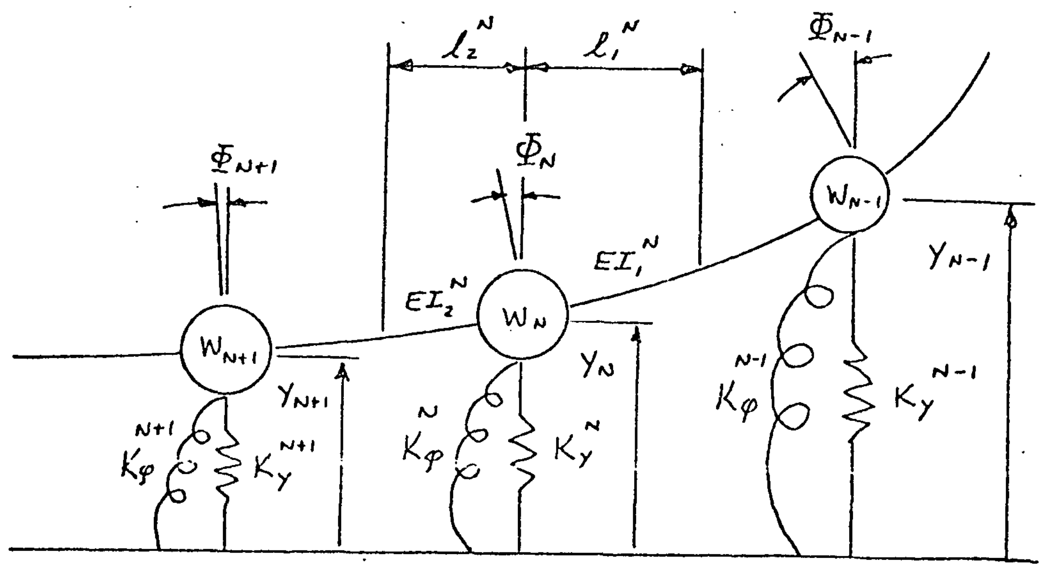
Enlarging the sketch of Bay N:

$$\Delta M = (I_j - I_x)\,\omega^2 \Phi = -J\omega^2 \Phi$$



The value ΔM accounts for the rotary inertia and gyroscopic effects of the mass (of prime importance for rotors). It is shown as a D'Alembert moment. Good discussions of these effects and derivation of the above formula can be found in References (2), (3), and (4). The parameter I_x is the mass moment of inertia of the mass about a diametral line and I_j is its mass polar moment of inertia. Understanding of the lumped parameter model is best obtained through a knowledge of the computational formulas used in the program. Therefore, the following section sets forth the theory and derivation of equations used in this vibration analysis.

1. LUMPED PARAMETER MODEL

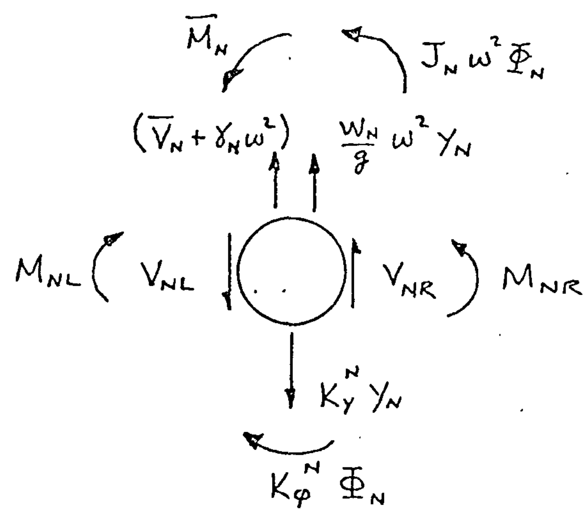


2. STATE VECTOR

$$\{\Delta_N\} = \begin{Bmatrix} V_N \\ M_N \\ \Phi_N \\ Y_N \\ 1 \end{Bmatrix}$$

The state vector Δ_N is defined as the column array of the shear, moment, slope, and deflection in the beam at the end of bay N. The fifth element of the state vector is the constant one which permits the inclusion of the load constants in the transfer matrices.

3. MASS TRANSFER MATRIX



$$V_{NL} = V_{NR} + \frac{W_N}{g}\omega^2 Y_N - K_y^N Y_N + \overline{V}_N + \gamma_N \omega^2$$

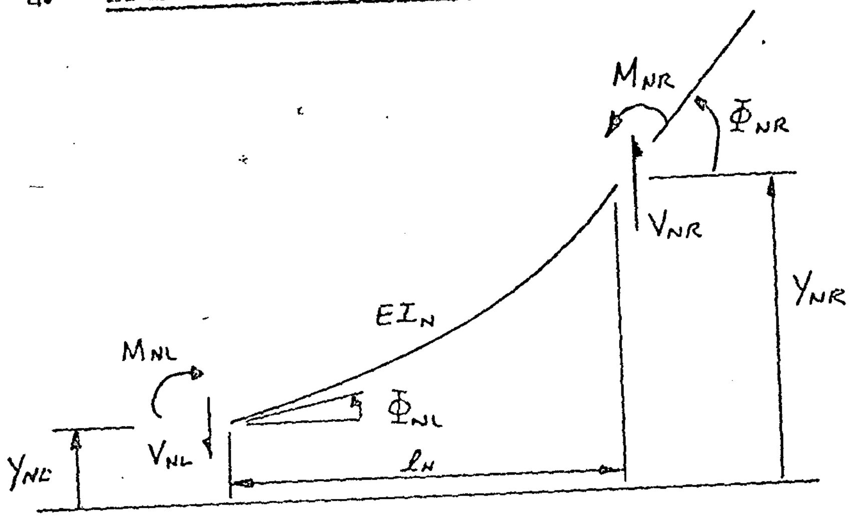
$$M_{NL} = M_{NR} + J_N \omega^2 \Phi_N - K_\varphi^N \Phi_N + \overline{M}_N$$

$$\Phi_{NL} = \Phi_{NR} \quad ; \quad Y_{NL} = Y_{NR}$$

$$\begin{Bmatrix} V_{NL} \\ M_{NL} \\ \Phi_{NL} \\ Y_{NL} \\ \hline 1 \end{Bmatrix} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & \left(\frac{W_N}{g}\omega^2 - K_y^N\right) & \overline{V}_N + \gamma_N \omega^2 \\ 0 & 1 & \left(J_N \omega^2 - K_\varphi^N\right) & 0 & \overline{M} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} V_{NR} \\ M_{NR} \\ \Phi_{NR} \\ Y_{NR} \\ \hline 1 \end{Bmatrix}$$

OR $\{\Delta_{NL}\} = [F_N]\{\Delta_{NR}\}$

4. ELASTICITY TRANSFER MATRIX



$$\Phi_{NL} = \Phi_{NR} - \frac{l_N^2}{2EI_N} V_{NR} - \frac{l_N}{EI_N} M_{NR}$$

$$Y_{NL} = Y_{NR} - \Phi_{NR}\, l_N + \left(\frac{l_N^3}{6EI_N} - \frac{C_N l_N}{G_N}\right) V_{NR} + \frac{l_N^2}{2EI_N} M_{NR}$$

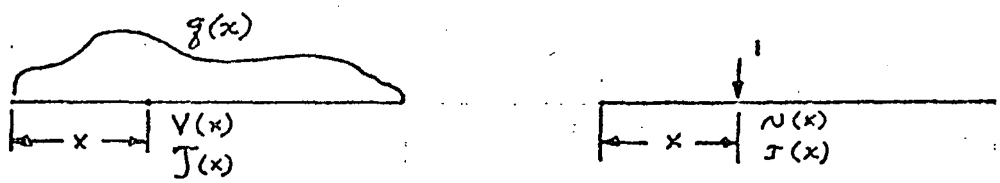
$$V_{NL} = V_{NR} \quad ; \quad M_{NL} = M_{NR} + V_{NR}\, l_{NR}$$

$$\begin{Bmatrix} V_{NL} \\ M_{NL} \\ \Phi_{NL} \\ Y_{NL} \\ \hline 1 \end{Bmatrix} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ l_N & 1 & 0 & 0 & 0 \\ -\frac{l_N^2}{2EI_N} & -\frac{l_N}{EI_N} & 1 & 0 & 0 \\ \left(\frac{l_N^3}{6EI_N} - \frac{C_N l_N}{G_N}\right) & \frac{l_N^2}{2EI_N} & -l_N & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} V_{NR} \\ M_{NR} \\ \Phi_{NR} \\ Y_{NR} \\ \hline 1 \end{Bmatrix}$$

OR $$\{\Delta_{NL}\} = [E_N]\{\Delta_{NR}\}$$

This transfer matrix symbolically represents both spans of bay M and for each the appropriate l_N, EI_N, C_N, and G_N must be used, where C_N is the shear deflection coefficient as described on the following two pages.

SHEAR DEFLECTION COEFFICIENT FOR BEAMS

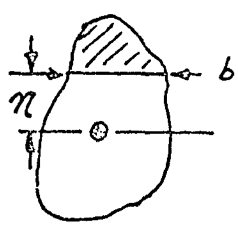


EQUATING WORK DONE –

$$\frac{1 \cdot \delta_s}{2} = \int_V \frac{1}{2}\,\mathcal{T}\left(\frac{\tau}{G}\right) dV$$

$$\therefore \quad \delta_s = \int_V \frac{\mathcal{T}\cdot\tau}{G}\, dV$$

GENERAL CROSS SECTION –



ASSUME $\mathcal{T} = \dfrac{VQ}{Ib}$, $\tau = \dfrac{\nu Q}{Ib}$

$$\delta_s = \int_0^L \frac{V\nu}{G}\left[\frac{1}{I^2}\int_A \frac{Q^2}{b^2}\, dA\right] dx$$

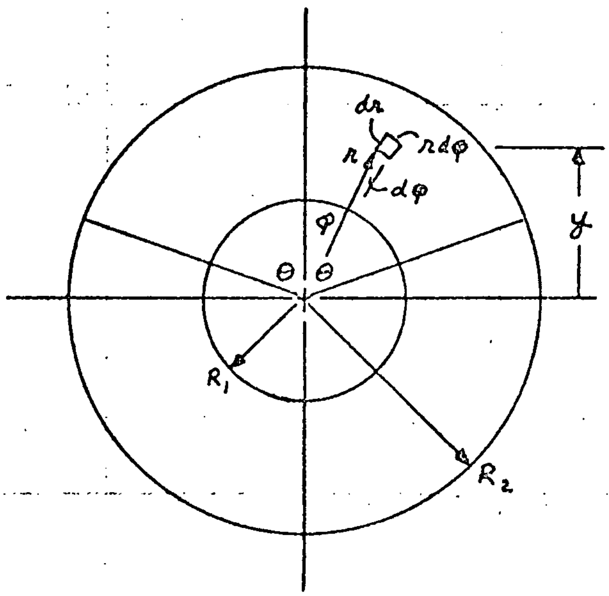
$$\therefore \quad \delta_s = \int_0^L k\,\frac{V\nu}{GA}\, dx$$

WHERE $k = \dfrac{A}{I^2}\displaystyle\int_A \frac{Q^2}{b^2}\, dA$

IF $dA = b\, d\eta$

$$\boxed{k = \frac{A}{I^2}\int_h \frac{Q^2\, d\eta}{b}}$$

HOLLOW CIRCULAR SECTION



$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2}\, dA$$

$$R_1 = a R_2$$

$$A = \pi (R_2^2 - R_1^2) = \pi R_2^2 (1-a^2)$$

$$I = \frac{\pi}{4} (R_2^4 - R_1^4) = \frac{\pi R_2^4}{4} (1-a^4)$$

$$C = \frac{k}{A}$$

$$dA = r\, d\varphi\, dr \quad , \quad y = r \cos\varphi$$

$$Q(\theta) = \int_{-\theta}^{\theta} \int_{R_1}^{R_2} y\, dA$$

$$= \int_{-\theta}^{\theta} \int_{R_1}^{R_2} r \cos\varphi \; r\, d\varphi\, dr$$

$$Q(\theta) = \frac{2}{3} (R_2^3 - R_1^3) \sin\theta = \frac{2}{3} R_2^3 (1-a^3) \sin\theta$$

$$b(\theta) = 2 (R_2 - R_1) = 2R_2 (1-a)$$

$$k = \frac{\pi R_2^2 (1-a^2) \; \frac{4}{9} R_2^6 (1-a^3)^2}{\frac{\pi^2}{16} R_2^8 (1-a^4)^2 \; 4R_2^2 (1-a)^2} \int_{-\pi}^{\pi} \int_{R_1}^{R_2} \sin^2\theta \; r\, d\theta\, dr$$

$$= \frac{16}{9\pi R_2^2} \frac{(1-a^2)(1-a^3)^2}{(1-a^4)^2 (1-a)^2} \; \frac{1}{2} (R_2^2 - R_1^2) \quad (\pi)$$

$$k = \frac{8}{9}\left[\frac{(1-a^2)(1-a^3)}{(1-a^4)(1-a)}\right]^2$$

$$k = \frac{8}{9}\left[\frac{(1+a+a^2)}{(1+a^2)}\right]^2$$

FOR THIN CIRCULAR SECTION $a \to 1$

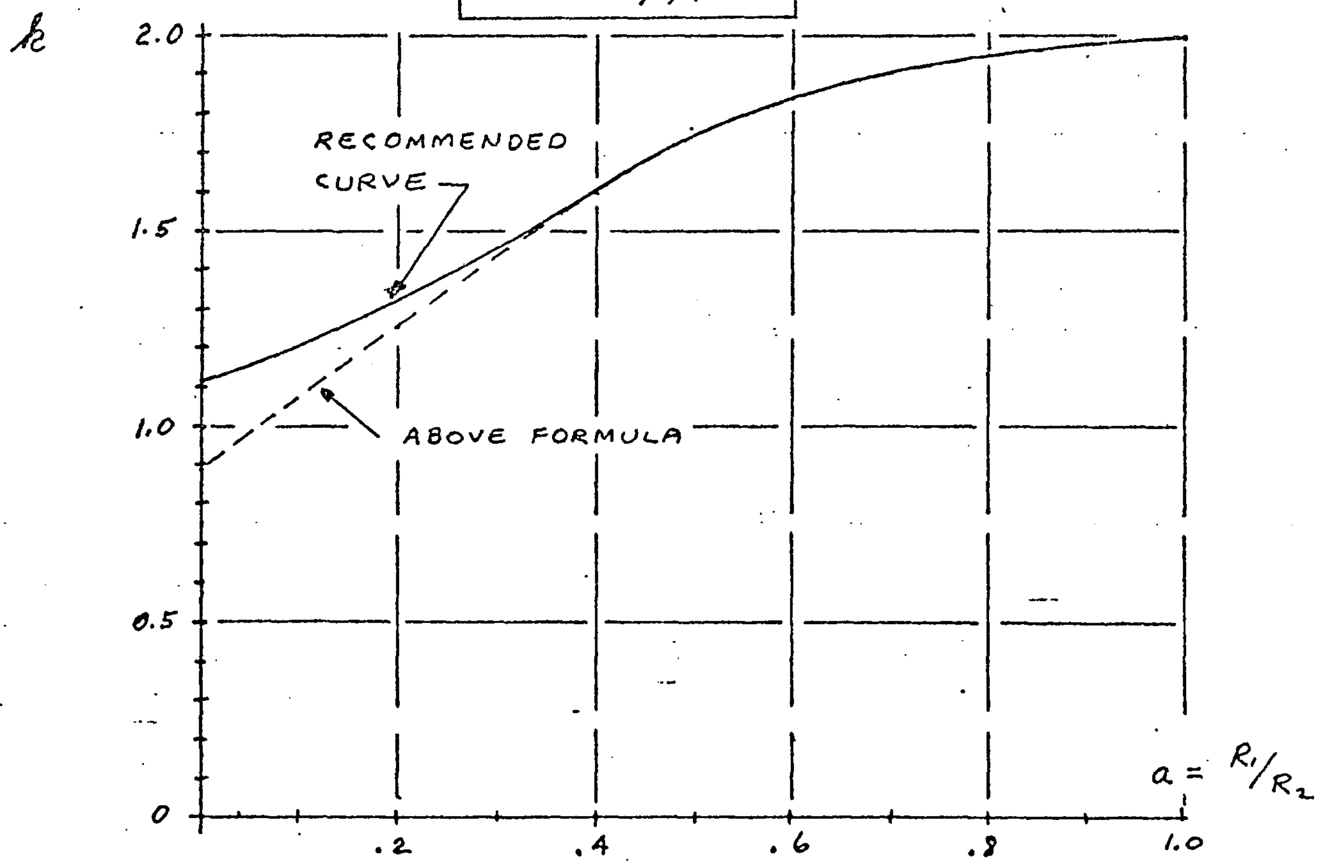
$$k = \frac{8}{9}\left(\frac{3}{2}\right)^2 = 2 \qquad \text{OK}$$

FOR SOLID CIRCULAR SECTION $a \to 0$

$$k = \frac{8}{9} \neq \frac{10}{9} \qquad \text{(NOT CORRECT)}$$

REF: ROARK, "FORMULAS FOR STRESS & STRAIN," p. 120.

$C = k/A$



5. SOLUTION PROCEDURE FOR A SET OF LINEAR SPRINGS

At the start, N = 0, thus, $\{\Delta_0\}$

Going across the first elasticity,

$$\{\Delta_1''\} = [E_1^1]\{\Delta_0\}$$

And across the first mass,

$$\{\Delta_1'\} = [F_1]\{\Delta_1''\} = [F_1][E_1^1]\{\Delta_0\}$$

Next across the second elasticity.

$$\{\Delta_1\} = [E_1^2]\{\Delta_1'\} = [E_1^2][F_1][E_1^1]\{\Delta_0\} = [C_1]\{\Delta_0\}$$

In like manner, transformations can be made across each bay, expressing each state vector in terms of the previous state vector, and thus in terms of the starting vector.

$$\{\Delta_{NSTA}\} = \prod_{N=1}^{NSTA}[C_N]\{\Delta_0\} = [D]\{\Delta_0\}$$

Expanding we get,

$$\begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ \hline 1 \end{Bmatrix}_{NSTA} = \left[\begin{array}{cccc|c} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{45} \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ \hline 1 \end{Bmatrix}_0$$

The fourth order nature of the governing beam equation requires the specification of two boundary conditions at each end. The program requires that two of the four variables of the state vector be zero at the beginning and at the end. Other homogeneous boundary conditions such as elastic restraints can be obtained by inputting a zero length so as to put the mass point at the boundary and then setting V and M equal to zero with appropriate lateral and/or moment springs. Likewise, concentrated end forces and moments can be inputted as $\overline{V}$ and $\overline{M}$ with the boundary condition that V = M = 0.

Let
$$\begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ 1 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ 1 \end{Bmatrix}$$

Further, let

M = subscript of 1st zero variable at end of last bay

N = subscript of 2nd zero variable at end of last bay

R = subscript of 1st non-zero variable at start of 1st bay

S = subscript of 2nd non-zero variable at start of 1st bay

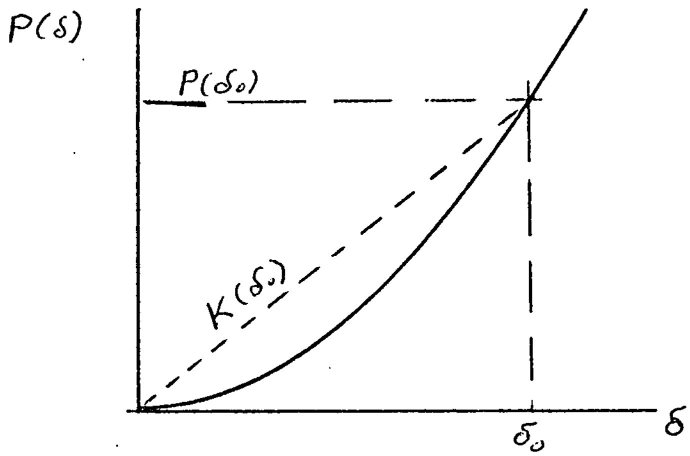
Considering the two equations associated with the zero variables at the end, and dropping those terms multiplied by the zero variables at the start:

$$\begin{Bmatrix} -d_{M5} \\ -d_{N5} \end{Bmatrix} = \begin{vmatrix} d_{MR} & d_{MS} \\ d_{NR} & d_{NS} \end{vmatrix} \begin{Bmatrix} Q_R \\ Q_S \end{Bmatrix}$$

These can be readily solved for Q_R and Q_S, the two unknowns at the start of the bearm. Thus knowing the initial state vector, all succeeding state vectors can be found by "walking through" the system.

6. SOLUTION PROCEDURE FOR A SET OF NON-LINEAR SPRINGS

The previous section described the explicit procedure for determining the response of an elastically supported beam to a harmonic force of moment input of frequency . If the beam support (for example, bearings) has a non-linear load-deflection relation, the secant line intersecting the curve is not a constant, but is instead a function of the deformation of the beam.



The elastic analysis described in the previous section yields an elastic spring force which is simply the product of the spring constant times the deformation. If this elastic force equals the non-linear force for the same deformation (as determined from the non-linear force-deformation relation as typified above), then the linear spring used was the correct secant. Adjusting the choice of secants until this agreement is achieved for all the non-linear springs supporting the beam leads to the solution of the problem. After a given unsuccessful iteration, the initial and final secant values are averaged to obtain the trial spring for the next iteration.

The first trial spring for the first frequency investigated must be input to the program. When the response to a harmonic load of successive frequencies is desired, the converged secant value for the first frequency is used as the first trial spring for the second frequency. Likewise, the converged secant value for the second frequency is used as the first trial spring for the third frequency. After this, a parabola is fitted through the three previously converged frequency springs, and the first trial spring for the next frequency is extrapolated along this curve.

$$K_{\omega_i}^{(I)} = 3\left(K_{\omega_{i-1}}^{(F)} - K_{\omega_{i-2}}^{(F)}\right) + K_{\omega_{i-3}}^{(F)}$$

7. DETERMINATION OF THE NON-LINEAR FORCE

As described in the previous section, the forced vibration analysis for a set of linear springs yields deformations and associated elastic forces in the springs. In order to test for convergence, the non-linear force associated with that deformation must be determined. At present, the program permits two types of non-linear springs. The appropriate flag must be set in the input.

a) FLAG(N) = 1. Angular Contact Ball Bearing: The pertinent bearing data is input after all the station data, and the exact load-deflection equations for ball bearings are utilized, including the interaction of thrust with the lateral response.

b) FLAG(N) = 2. $P = A\ y^{B}$ where A and B are constants input to the program.

At present, roller bearing load-deflection curves are fitted by a form b). However, it is easily possible to introduce a third flag alternate and incorporate the exact load-deflection equations for roller bearings.

8. ANGULAR CONTACT BALL BEARINGS

The explicit analytical load-deflection relations for angular contact ball bearings have been derived and are extensively treated by A. B. Jones of New Departure Ball Bearing Company.

The outer race of the bearing is assumed fixed in space. The inner race has three degrees of freedom with respect to the fixed outer race. It may move axially, laterally, and may rotate. It is also capable of transmitting three force resultants between shaft and bearing support (i.e., lateral force, axial force, and moment). Each of these force resultants can be expressed explicitly in terms of the three deformations. These functions are explicit, but non-linear. Their inverse cannot be explicitly stated, that is, the deformations cannot be expressed in terms of the three force resultants, nor can mixed functions of forces and deformations be expressed.

$$H = f_1 (\Delta_A , \Delta_R , \theta)$$
$$V = f_2 (\Delta_A , \Delta_R , \theta)$$
$$M = f_3 (\Delta_A , \Delta_R , \theta)$$

The significant parts of the derivation have been reproduced on the following pages. The value of "K" referred to on p 22 by Jones*, and DKK in the program, is not computed internally by the program, though it could be, but is computed by IBM Job 773A. Since this number is a constant, it need be computed only once. Job 773A essentially programs Jones' equations in an iterative scheme to yield deformations as a function of input loads.

* "New Departure - Analysis of Stresses and Deflections", Vol. 1 and 2, by A. B. Jones, New Departure Division, General Motors Corporation, 1946.

A. DERIVATION OF THE EQUATIONS (PER JONES)

I. Basic Geometric Relations.

The operating characteristics of a ball bearing depend to a great extent upon the internal fitup. Internal fitup is generally measured by the diametral clearance of the bearing.

Fig. 1 shows a cross section through a radial, single row bearing. Diametral clearance is denoted by P_D From Fig. 1:

$$P_D = D_o - D_i - 2d \qquad \text{Eq. 1}$$

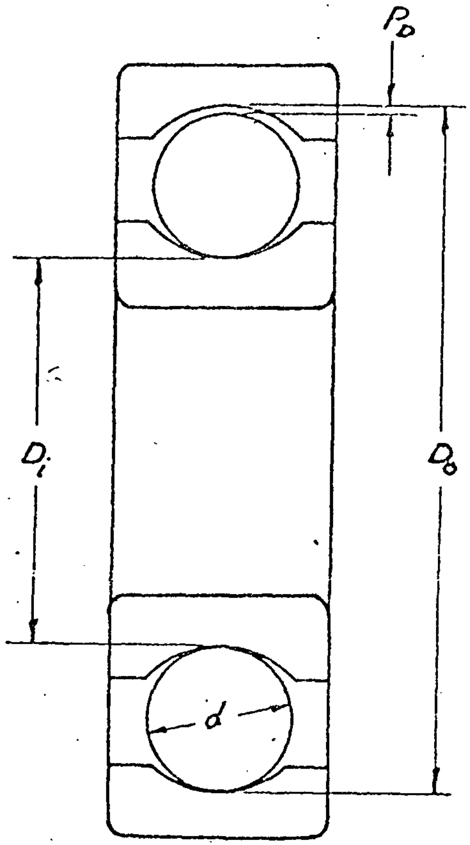


Fig. 1

Although diametral clearance is generally used in connection with single row, radial bearings, Eq. 1 is applicable to angular contact bearings as well since there is a definite relation between diametral clearance, race curvatures and free contact angle (See Eq. 8 p.).

The value of P_D from Eq. 1 may be positive or negative. Loose bearings have positive diametral clearance. Tight bearings have negative values of P_D.

Diametral clearance in loose, single row, radial bearings is sometimes called radial clearance, radial play, radial shake, diametral play or diametral slackness.

For loose, single row, radial bearings diametral clearance may be defined as the maximum distance one race may move diametrally with respect to the other without the application of measureable force when both races lie in the same plane.

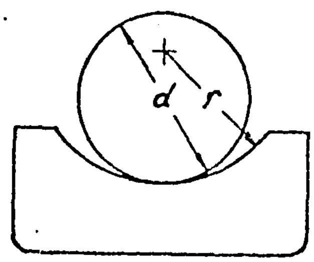


Fig. 2

Race curvature is a measure of the conformity of the race to the ball in a plane passing through the bearing axis and transverse to the raceway. It is expressed as a percentage or a decimal. Throughout this text decimal notation will be used.

The curvature of a race is defined as: (See Fig. 2)

$$f = \frac{r}{d} \qquad \text{Eq. 2}$$

Thus, if the curvature and ball diameter are known, the radius of curvature is:

$$r = fd \qquad \text{Eq. 3}$$

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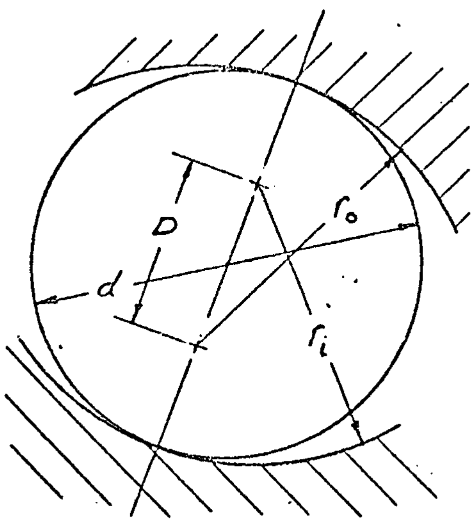

Fig. 3

The distance between the centers of curvatures of two races in line and line contact with a ball is of great importance. This distance is indicated by D in Fig. 3 and is a fixed quantity depending on race radii and ball diameter. Denoting quantities referred to the outer race by the subscript, $_o$, and quantities referred to the inner race by the subscript, $_i$, we have from Fig. 3:

$$D = r_o + r_i - d \qquad \text{Eq. 4}$$

Since both r_o and r_i may be expressed in terms of outer and inner race curvatures, respectively, by Eq. 3, we have:

$$D = (f_o + f_i - 1)\,d \qquad \text{Eq. 5}$$

Letting:

$$B = (f_o + f_i - 1) \qquad \text{Eq. 6}$$

$$D = Bd \qquad \text{Eq. 7}$$

The quantity B in Eq. 7 is known as the total curvature and is a measure of the conformity of both outer and inner races to the ball. Upon it depend all bearing deflection computations.

- -

Free contact angle is the angle made by a line passing through the points of contact of the ball and both raceways with a plane perpendicular to the axis of the bearing when both races are centered with respect to each other and one race is axially displaced with respect to the other without the application of measureable force.

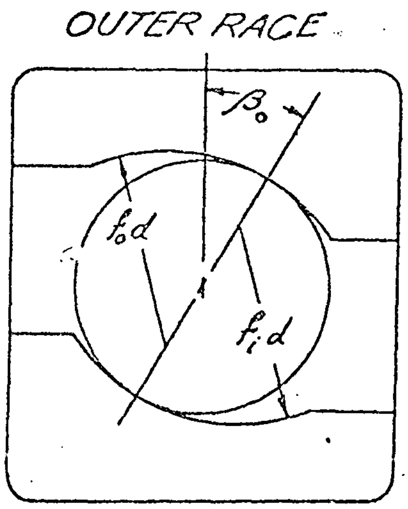

Fig. 4

The centers of curvature of both outer and inner races lie on the line defining the free contact angle. Free contact angle is denoted by β_o and is illustrated in Fig. 4.

Free contact angle is determined by diametral clearance, P_o, and total curvature, B, as:

$$\cos\beta_o = \frac{2Bd - P_o}{2Bd} \qquad \text{Eq. 8}$$

or:

$$P_o = 2Bd(1-\cos\beta_o) \qquad \text{Eq. 9}$$

In the case of radially tight bearings the value of P_o is negative and the value of $\cos\beta_o$ from Eq. 8 becomes greater than 1. Mathematically, this is an imaginary condition. However, the value of $\cos\beta_o$ for radially tight bearings obtained from Eq. 8 is of importance in certain deflection computations and has a definite physical significance.

Therefore, radially tight bearings may be considered as having an imaginary contact angle whose sine is zero and whose cosine is greater than 1 as defined by Eq. 8.

- -

The effect of interference mounting fits on free contact angle is important. Due to the interference fit there is a change in diameter of the press fitted raceway and a corresponding reduction in diametral clearance. Hence the free contact angle is reduced by press fitting.

If ΔP_D is the total reduction in diametral clearance due to press fitting one or both race members, the initial mounted contact angle, β_o', is:

$$\cos \beta_o' = \frac{2Bd - P_D + \Delta P_D}{2Bd} \qquad \text{Eq. 10}$$

or:

$$\cos \beta_o' = \cos \beta_o + \frac{\Delta P_D}{2Bd} \qquad \text{Eq. 11}$$

For the effect of interference fits on ring dimensions see Chapter XVII p. 161.

- -

Free endplay is the maximum possible relative axial movement of inner race with respect to the outer, when both races are coaxially centered, without the application of measureable force. It is denoted by P_E

In practice, endplay is measured under a definite gauging load and is known as gauged endplay. Gauged endplay is always greater than free endplay because of the deflection of the bearing under the gauging load. See Chapter XV, p.152 for the relation between gauged endplay and diametral clearance.

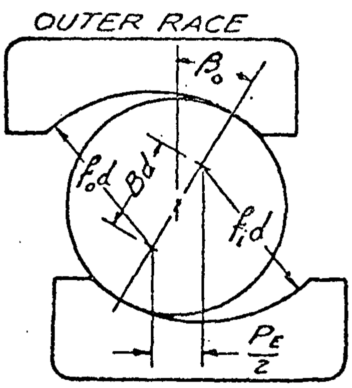

Fig. 5

Free endplay depends on total curvature and contact angle as shown in Fig. 5.

$$P_E = 2Bd \sin \beta_o \qquad \text{Eq. 12}$$

or:

$$\sin \beta_o = \frac{P_E}{2Bd} \qquad \text{Eq. 13}$$

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The relation between free endplay and diametral clearance is obtained by eliminating β_o between Eqs. 8 and 13.

$$P_D = 2Bd - \sqrt{(2Bd)^2 - P_E^2} \qquad \text{Eq. 14}$$

$$P_E = \sqrt{4Bd\,P_D - P_D^2} \qquad \text{Eq. 15}$$

- - - - - - - - - - - - - - - - - - - -

II. Solid Elastic Bodies In Contact.

When two, solid, elastic, curved bodies are pressed together under load a certain amount of flattening occurs in the neighborhood of the contact point. Due to the flattening there is produced an elliptical pressure area over which the total load is distributed. The relations governing the shape and size of the pressure area and the distribution of stress over the pressure area were mathematically investigated by Heinrich Hertz in 1881. These relations show good agreement with test results except where the dimensions of the projected pressure area are large in comparison to the principal radii of curvature of the contacting bodies. Good agreement is shown for conformities generally used in ball bearings.

Although Hertz's work was limited to an analysis of the distribution of stress at the pressure surface, more recent investigators have determined the nature and distribution of the stresses occurring beyond the pressure surface and have substantiated their results by photo-elastic tests.

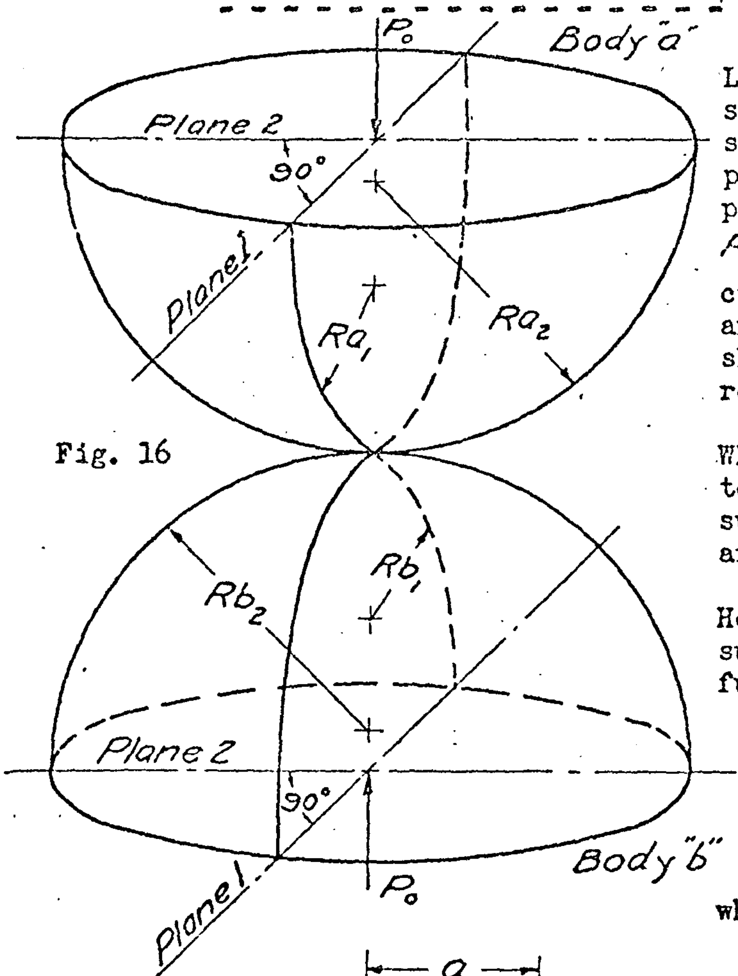


Fig. 16

Let the bodies be denoted by the subscripts "a" and "b", respectively, as shown in Fig. 16. Also, let the principal radii of curvature at the contact point be Ra_1 and Ra_2 for body "a" and Rb_1 and Rb_2 for body "b". The radii of curvature are measured in two planes, 1 and 2, at right angles to one another as shown in Fig. 16, the subscripts 1 and 2 referring to the respective planes.

When body "a" and body "b" are pressed together by the normal load, P_o, the resulting pressure area whose semi-axes are a and b is shown in Fig. 17.

Hertz gives the dimensions of the pressure area in terms of the transcendental functions μ and ν, as:

$$a = \mu g \qquad \text{Eq. 53}$$

$$b = \nu g \qquad \text{Eq. 54}$$

where:

$$g = \sqrt[3]{\frac{3P_o(\vartheta_a + \vartheta_b)}{8\left(\frac{1}{Ra_1} + \frac{1}{Ra_2} + \frac{1}{Rb_1} + \frac{1}{Rb_2}\right)}} \qquad \text{Eq. 55}$$

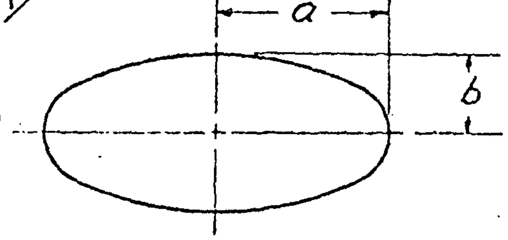


Fig. 17

$$\vartheta_b = \frac{4(1-\sigma_b^2)}{E_b} \qquad \text{Eq. 57}$$

If both bodies are of steel with modulus of elasticity 29×10^6 #/sq. in. and with Poisson's ratio 1/4, the value of g from Eq. 55 is:

$$g = .0045944 \sqrt[3]{\frac{P_o}{\frac{1}{Ra_1} + \frac{1}{Ra_2} + \frac{1}{Rb_1} + \frac{1}{Rb_2}}} \qquad \text{Eq. 58}$$

The values of the principal radii of curvature, Ra_1, Ra_2, Rb_1 and Rb_2 are taken in accordance with Fig. 16.

The principal radii of curvature may be either positive or negative, depending on whether the centers of curvature lie within or without the body as shown in Fig. 18.

In addition, planes 1 and 2 should be so chosen that:

$$\frac{1}{Ra_1} + \frac{1}{Rb_1} > \frac{1}{Ra_2} + \frac{1}{Rb_2} \qquad \text{Eq. 59}$$

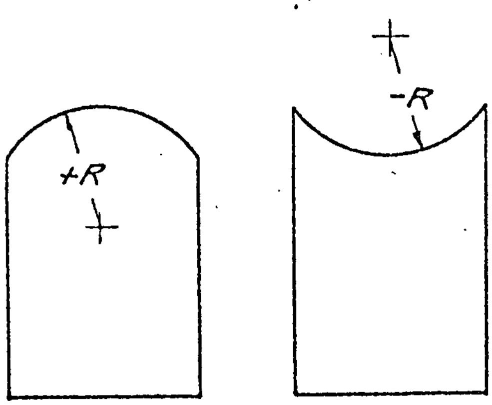


Fig. 18

Plane 1 then determines the direction of the semi-minor axis of the pressure area and plane 2 the direction of semi-major axis of the pressure area.

The values of the functions μ and ν for use in Eqs. 53 and 54 depend on the conformity of the contacting bodies in the vicinity of the pressure area as determined by the auxiliary angle, τ.

$$\cos \tau = \frac{\frac{1}{Ra_1} - \frac{1}{Ra_2} + \frac{1}{Rb_1} - \frac{1}{Rb_2}}{\frac{1}{Ra_1} + \frac{1}{Ra_2} + \frac{1}{Rb_1} + \frac{1}{Rb_2}} \qquad \text{Eq. 60}$$

Note that the denominator in the expression for cos τ is the same as that occurring under the radical in Eq. 55 and 58.

μ and ν are related by another auxiliary angle, ϵ, which depends on the shape of the pressure ellipse.

$$\cos \tau = 1 - \frac{2\left[K(\epsilon) - E(\epsilon)\right] \cot^2 \epsilon}{E(\epsilon)} \qquad \text{Eq. 61}$$

$$\nu = \sqrt[3]{\frac{2E(\epsilon)\cos \epsilon}{\pi}} \qquad \text{Eq. 62}$$

where:

$$\cos \epsilon = \frac{\nu}{\mu} = \frac{b}{a} \qquad \text{Eq. 63}$$

$K(\epsilon)$ and $E(\epsilon)$ are the complete elliptic integrals of the first and second order, having the modulus sin ϵ

$$K(\epsilon) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - \sin^2 \epsilon \sin^2 \varphi}} \qquad \text{Eq. 64}$$

$$E(\epsilon) = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2 \epsilon \sin^2 \varphi}\, d\varphi \qquad \text{Eq. 65}$$

Since accurate tables of $K(\epsilon)$ and $E(\epsilon)$ are not always available, values of $K(\epsilon)$ and $E(\epsilon)$ correct to ten decimal places are given on Charts 5 and 6. Four place tables may also be found in Jahnke and Emde's "Funktionentafeln" 1943 edition.

By assuming a series of values of the modulus, sin ϵ, corresponding values of cos τ, μ and ν may be calculated by Eqs. 61, 62 and 63.

Values of μ computed in this manner are plotted against corresponding values of cos τ in Charts 7 through 21. Values of ν are plotted against corresponding values of cos τ in Charts 22 through 31.

It must be emphasized that the semi-axes of the pressure ellipse, a and b, are the projected semi-axes and are not measured along the curvature of the pressure surface.

- -

IV. Load Distribution And Deflection In Ball Bearings - Generalized Solution.

A ball bearing derives its load carrying ability from the forces produced at the contact points of balls and races. These loads, called normal ball loads and designated by P_o, result from the elastic deformations of the contacting bodies.

Fig. 27

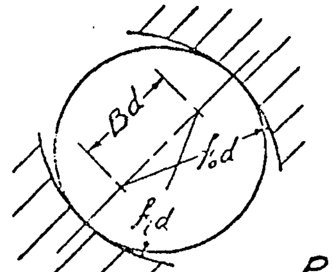


Fig. 28

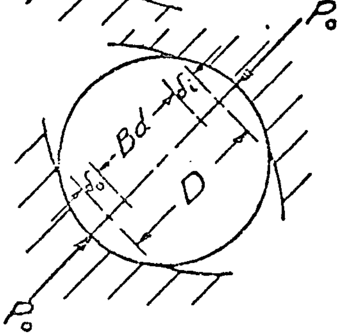


Fig. 27 shows a ball between two curved races. When the ball is in point (no load) contact with both races, the centers of curvature are separated by the distance Bd (see P.2) which depends on curvatures and ball diameter.

If the races are displaced with respect to each other so that the ball is compressed between them, the external force causing the compression is resisted by an elastic force (normal ball load), P_o, which acts along the line passing through the displaced centers of curvature of the two races as shown in Fig. 28.

The elastic deformations at the points of contact are δ_o and δ_i and the sum of these two equals the normal approach of the two races. Since the curvature centers are fixed with respect to their races and move with them, the original distance between race curvature centers, Bd, is increased by the normal approach of the two races. Calling the normal approach of the two races δ_N, the distance between the displaced curvature centers is:

$$D = Bd + \delta_N \qquad \text{Eq. 146}$$

or:

$$\delta_N = D - Bd \qquad \text{Eq. 147}$$

The relation between normal ball load and normal approach is:

$$P_o = K_N \delta_N^{3/2} \qquad \text{Eq. 148}$$

where the value of K_N is, from Eq. 143:

$$K_N = \frac{d^{1/2} \times 10^9}{\left[7.8107 (C_{\delta_o} + C_{\delta_i})\right]^{3/2}} \qquad \text{Eq. 149}$$

C_{δ_o} and C_{δ_i} are obtained from Chart 56.

K_N may be more conveniently expressed in terms of the axial deflection constant, K, by the relation:

$$K_N = \frac{K d^{1/2}}{B^{3/2}} \qquad \text{Eq. 150}$$

Values of K may be obtained from Chart 57. See P. 49

In a complete ball bearing which involves a number of balls symmetrically disposed around a pitch circle, the normal load on any ball and the contact angle at which it acts may be completely determined and evaluated in terms of the following relative displacements of inner and outer races.

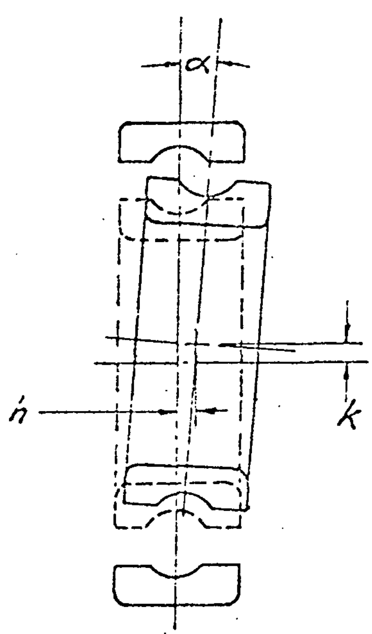


Fig. 29

1) A relative axial displacement, h, of inner and outer races.

2) A relative radial displacement, k, of inner and outer races.

3) A relative angular misalignment, α, of inner and outer races.

Fig. 29 shows these displacements. They are measured with reference to the relative position of inner and outer rings when all parts of the bearing are in symmetric, geometric contact under zero thrust load.

Some of the dimensions used in the following discussion are:

The radius of the locus of the center of curvature of inner race:

$$R_i = \frac{E}{2} + (f_i - .5) d \cos \beta_o \qquad \text{Eq. 151}$$

where: E = pitch circle diameter.

The radius of the locus of the center of curvature of the outer race:

$$R_o = R_i - Bd\, Cos\, \beta_o \qquad \text{Eq. 152}$$

and are also connected by the relations:

$$R_i - R_o = Bd\, Cos\, \beta_o \qquad \text{Eq. 153}$$

and:

$$R_i - R_o = Bd - \frac{P_D}{2} \qquad \text{Eq. 154}$$

where: P_D = Diametral Clearance

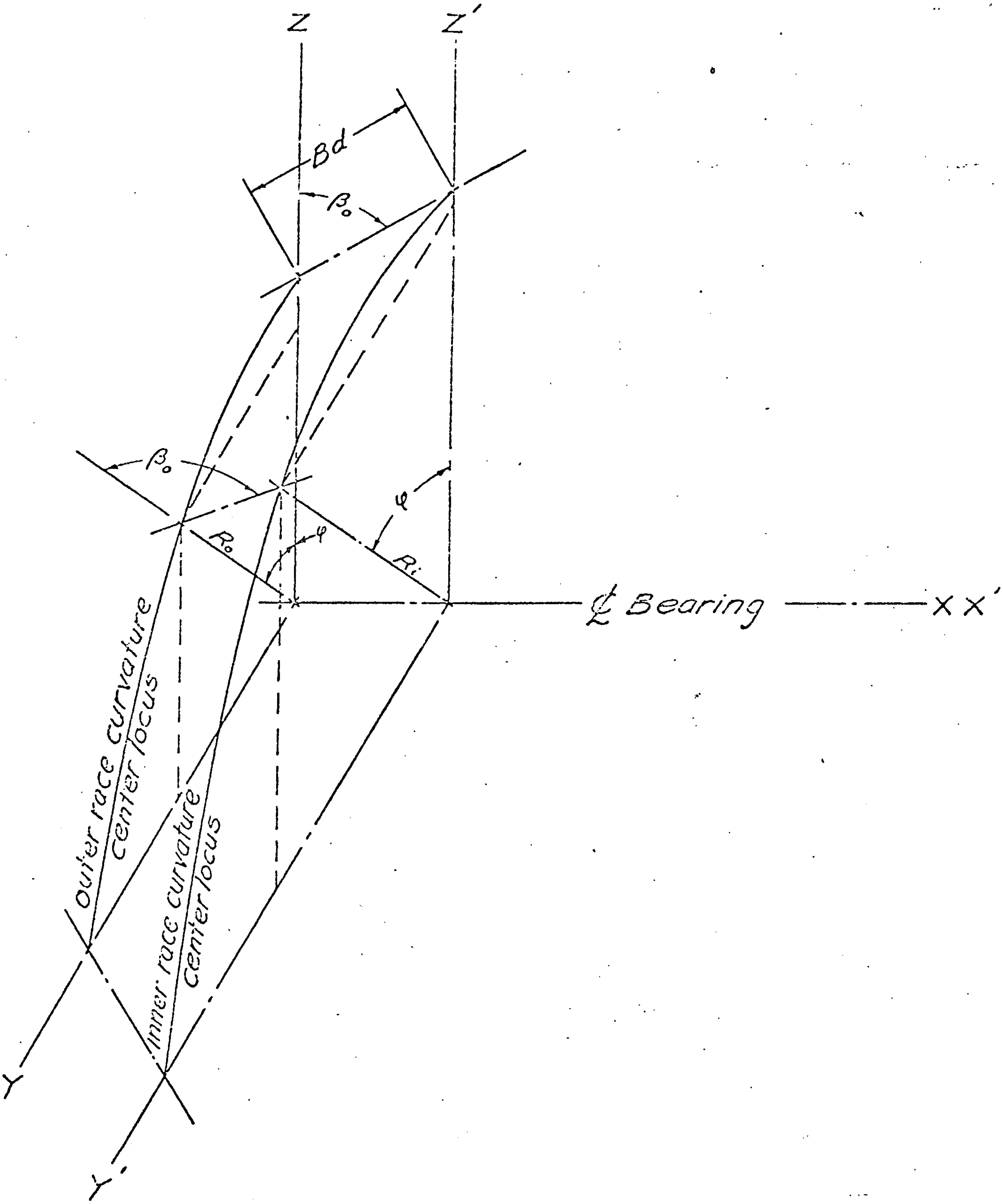
In order to express the normal ball loads and operating contact angles developed within the bearing in terms of the relative displacements of the inner race with respect to the outer, the following system is used.

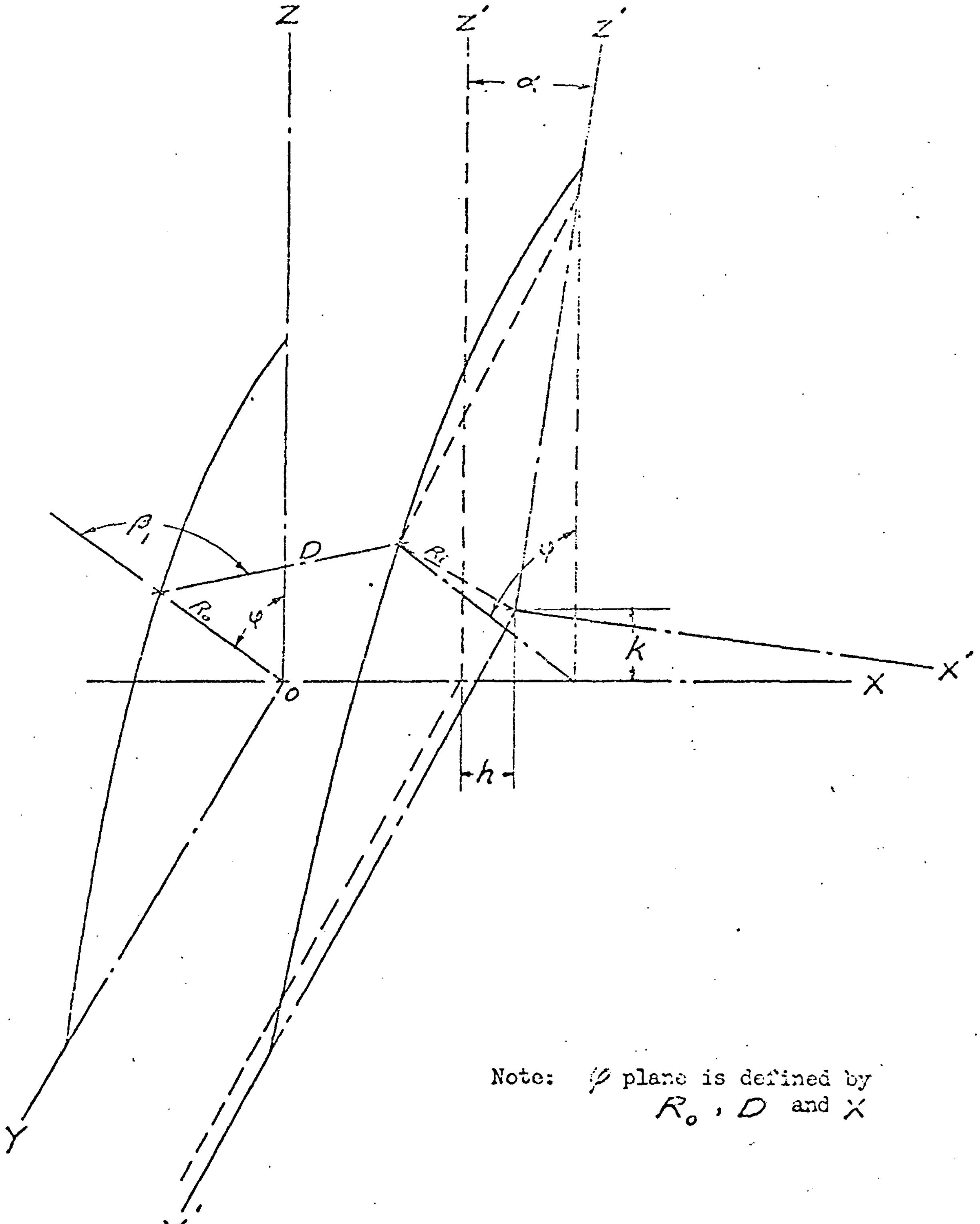
The outer race is assumed to be fixed in space while the inner race is allowed to move with respect to the outer as shown in Fig. 29. The normal ball load and operating contact angle for a ball at any angle, φ, measured around the pitch circle from the heaviest loaded ball, are obtained by evaluating the change in distance between inner and outer race curvature centers in terms of the displacements shown in Fig. 29.

Fig. 30 shows the relative position of inner and outer race curvature center loci before displacement. The locus of the outer race curvature centers is a circle in space and is referred to a fixed, three dimensional coordinate system, X, Y, Z. The locus of the inner race curvature centers is also a circle in space and is referred to the movable, three dimensional coordinate system, X', Y', Z'.

Now, assume that the origin of the movable coordinate system is displaced the amounts h and k and misaligned the amount α as shown in Fig. 31. These displacements are those previously shown in Fig. 29.

In Fig. 31, the heaviest loaded ball lies in the X, Z plane. We are interested in the normal ball load, P_o, and operating contact angle, β_1, of a ball lying in the φ plane. This is determined by the relative positions of the intersection of the two race curvature loci with the plane.


Z
Z'
Bd
β_o
β_o
φ
φ
R_o
R_i
℄ Bearing
X X'
outer race curvature center locus
inner race curvature center locus
Y
Y'



Note: φ plane is defined by R_o, D and X

The distance, D, Fig. 31, between the centers of curvature of the inner and outer races after displacement and measured in the φ plane is:

$$D = Bd\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} \qquad \text{Eq. 155}$$

where:

$$h' = \frac{h}{Bd} \qquad \text{Eq. 156}$$

$$k' = \frac{k}{Bd} \qquad \text{Eq. 157}$$

$$\alpha' = \frac{\alpha}{Bd} \qquad \text{Eq. 158}$$

h, k and α being the three displacements of inner race with respect to the outer, Fig. 29. α is measured in radians. β_o is the free contact angle of the mounted bearing before load application.

The normal approach of the races, δ_N, is, from Eq. 147:

$$\delta_N = Bd\left[\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right] \qquad \text{Eq. 159}$$

The normal ball load, P_o, is, from Eq. 148:

$$P_o = K_N (Bd)^{3/2}\left[\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} \qquad \text{Eq. 160}$$

where K_N is the normal deflection constant from Eq. 149.

The normal ball load may be more conveniently expressed in terms of the axial deflection constant, K, as:

$$P_o = Kd^2\left[\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} \qquad \text{Eq. 161}$$

Values of K may be obtained from Chart 57.

The operating contact angle β_1 of a ball positioned in the φ plane is:

$$\sin\beta_1 = \frac{\sin\beta_0 + h' + \alpha' R_i \cos\varphi}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k'\cos\varphi)^2}} \qquad \text{Eq. 162}$$

$$\text{or: } \cos\beta_1 = \frac{\cos\beta_0 + k'\cos\varphi}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k'\cos\varphi)^2}} \qquad \text{Eq. 163}$$

If the normal ball load, P_0, which acts at the contact angle β_1 (along the line D in Fig. 31) is projected onto the XZ plane in Fig. 31, it may be resolved into two components. One is a thrust force, H, parallel to the X axis. The other is a vertical component, V, parallel to the Z axis.

The thrust component, H, is:

$$H = P_0 \sin\beta_1 \qquad \text{Eq. 164}$$

or

$$H = \frac{K d^2\left[\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k'\cos\varphi)^2} - 1\right]^{3/2}(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k'\cos\varphi)^2}} \qquad \text{Eq. 165}$$

The vertical component, V, is:

$$V = P_0 \cos\beta_1 \cos\varphi \qquad \text{Eq. 166}$$

or

$$V = \frac{K d^2\left[\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k'\cos\varphi)^2} - 1\right]^{3/2}(\cos\beta_0 + k'\cos\varphi)\cos\varphi}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k'\cos\varphi)^2}} \qquad \text{Eq. 167}$$

If it is assumed that the pitch circle radius does not appreciably change during the deformations, the moment of the thrust component about an axis through the center of the pitch circle and parallel to the Y axis in Fig. 31 is:

$$M = \frac{P_0 E}{2} \sin\beta_1 \cos\varphi \qquad \text{Eq. 168}$$

where E is the pitch circle diameter.

or

$$M = \frac{EKd^2}{2} \frac{\left[\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} (\sin\beta_o + h' + d'R_i \cos\varphi)\cos\varphi}{\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2}} \quad \text{Eq. 169}$$

In order that the bearing be in equilibrium after displacement, the following conditions must be satisfied:

$$\sum H = Kd^2 \sum \frac{\left[\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} (\sin\beta_o + h' + d'R_i \cos\varphi)}{\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2}} \quad \text{Eq. 170}$$

$$\sum V = Kd^2 \sum \frac{\left[\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} (\cos\beta_o + k'\cos\varphi)\cos\varphi}{\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2}} \quad \text{Eq. 171}$$

$$\sum M = \frac{EKd^2}{2} \sum \frac{\left[\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} (\sin\beta_o + h' + d'R_i \cos\varphi)\cos\varphi}{\sqrt{(\sin\beta_o + h' + d'R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2}} \quad \text{Eq. 172}$$

where $\sum H$ and $\sum V$ are respectively the thrust and radial components of the externally applied load and $\sum M$, the moment of the external load about the center of the pitch circle. The $\sum$ in the right hand sides of the above equations indicates that the computations must be performed for each ball position in the bearing and the sum taken.

The equations of equilibrium, Eqs. 170, 171, and 172, above, are statically indeterminate; that is, a direct solution for the displacements in terms of the externally applied load is not possible without further reduction of the equations.

III. NERVA APPLICATION

The E13101 computer program was originally used for parametric analysis of various models. If the rotating mass is less than 1/10 of the stationary mass of the pump, the natural frequency predictions will be within 5%, if the bearing spring rate is properly modified for housing support flexibility. Once the model is selected one can use E13112, where the bearing spring rate can be varied due to preload on the bearing, and the bearing load can be calculated at operating speed. E13102 can be utilized for parametric studies of light housings where the rotating mass is greater than 1/10 of the stationary mass. The final analysis should be performed using E13104 which is the most expensive, but the most accurate program. The bearing program that is attached to this program E13112 is old, and a better and a newer version of R. B. Jones program should be used. The spring rate should be input in the $K = A(P)^B$ form.

Once the model is built one should run the non-rotating vibration test and compare the results to the output from the computer run of the same model, only omitting gyroscopic effects. If these results are close, then the operational predictions will also be close.

Example:

Figure 3-1 presents the critical frequencies versus bearing spring rate for the dual beam on spring support model.

Figures 3-2 through 3-7 present the associated normalized mode shapes for frequencies presented in Figure 3-1.

Based on these figures the following can be concluded. The first two frequencies are primarily housing modes and are below the possible operating speed. The third and fourth critical frequencies are bearing related rotor critical speeds. The fifth frequency is a coupled rotor housing mode and the sixth frequency is primarily the classical rotor bending critical speed.

Table 3.1 presents a comparison of the rotor only model that was used for preliminary evaluation and dual beam model used for the final analysis.

TABLE 3.1

COMPARISON OF CRITICAL SPEED PREDICTIONS

E13101 ROTOR ONLY MODEL	E13104 DUAL BEAM MODEL
	2360*
	3420*
40210	38060
41810	42480
	48470**
58720	58420

* Denotes frequencies that are primarily mount related and do not produce bearing loading.

** Denotes frequencies that are coupled motor housing mode shapes and do produce bearing loading.

DYNAMIC ANALYSIS SUMMARY - TPA CRITICAL SPEED

G. Mironenko
May 1972

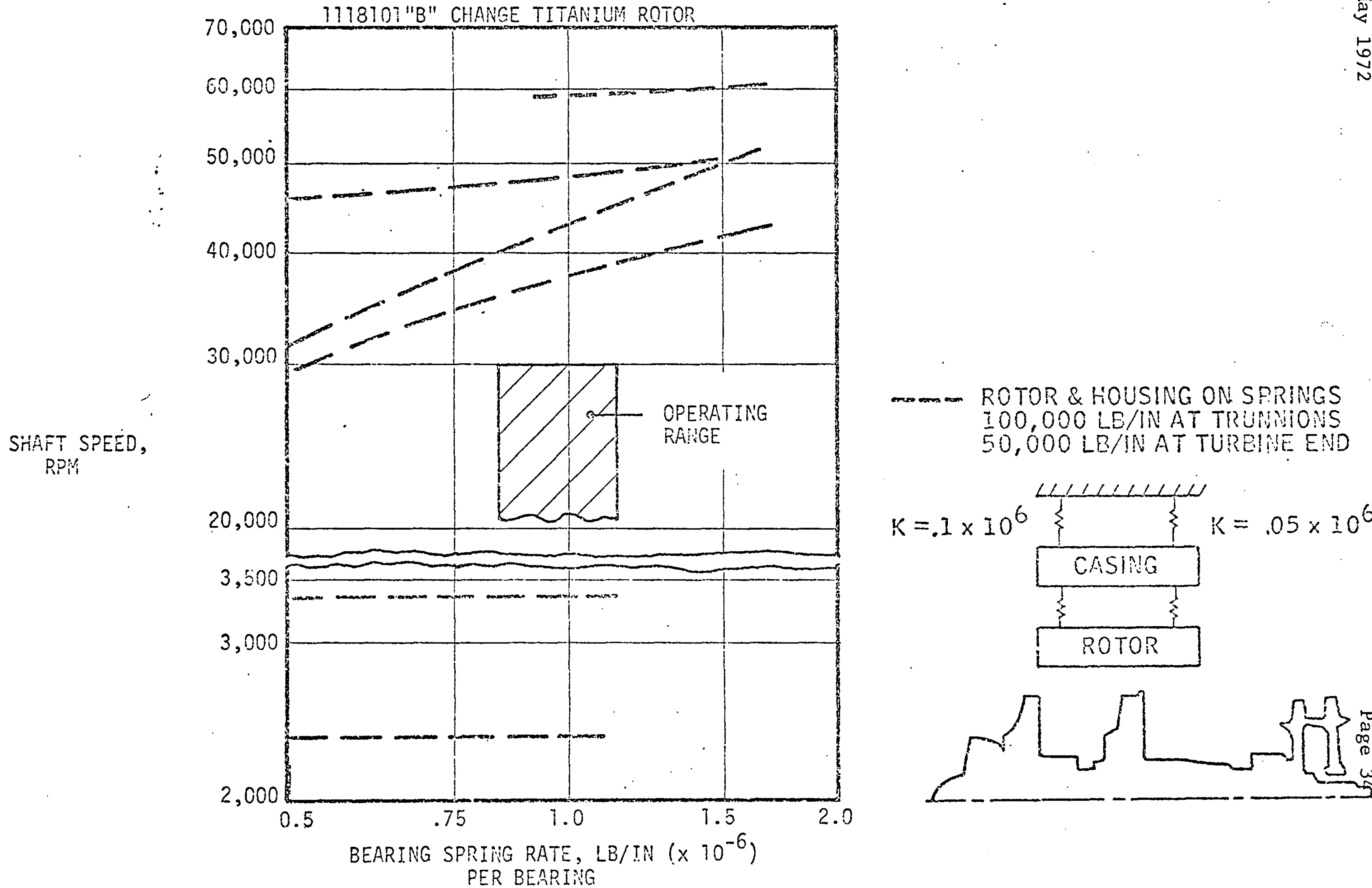
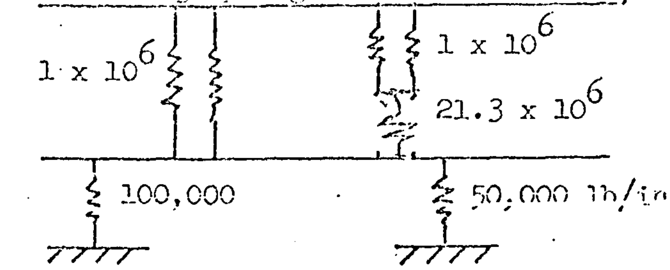


FIGURE 3.1

1118101, "B" change Titanium Rotor
normalized displacement near resonant speed 2400 rpm
Unbalance Forcing = .01 gm-in./lb out of phase forward circular whirl



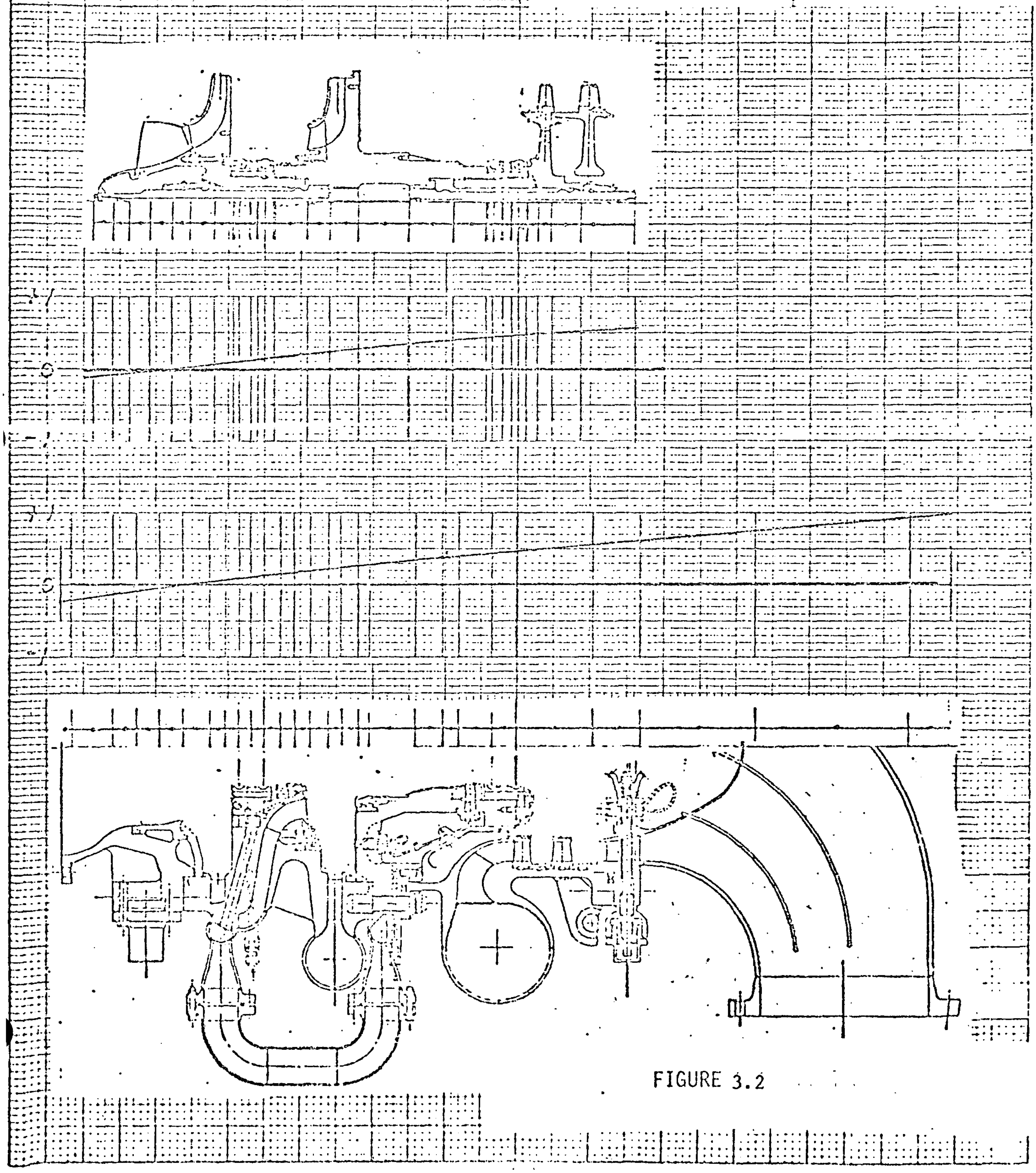
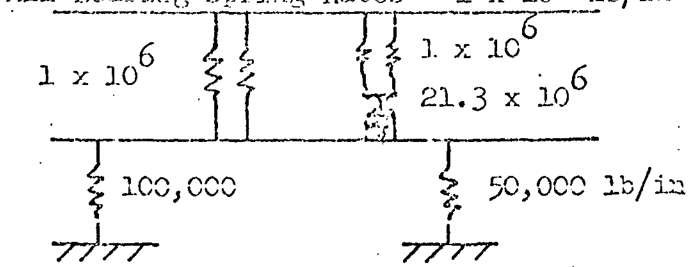


FIGURE 3.2

NERVA TPA

1118101, "B" change Titanium Rotor normalized displacement near resonant speed 3600 rpm
Unbalance Forcing = .01 gm-in./lb out of phase forward circular whirl



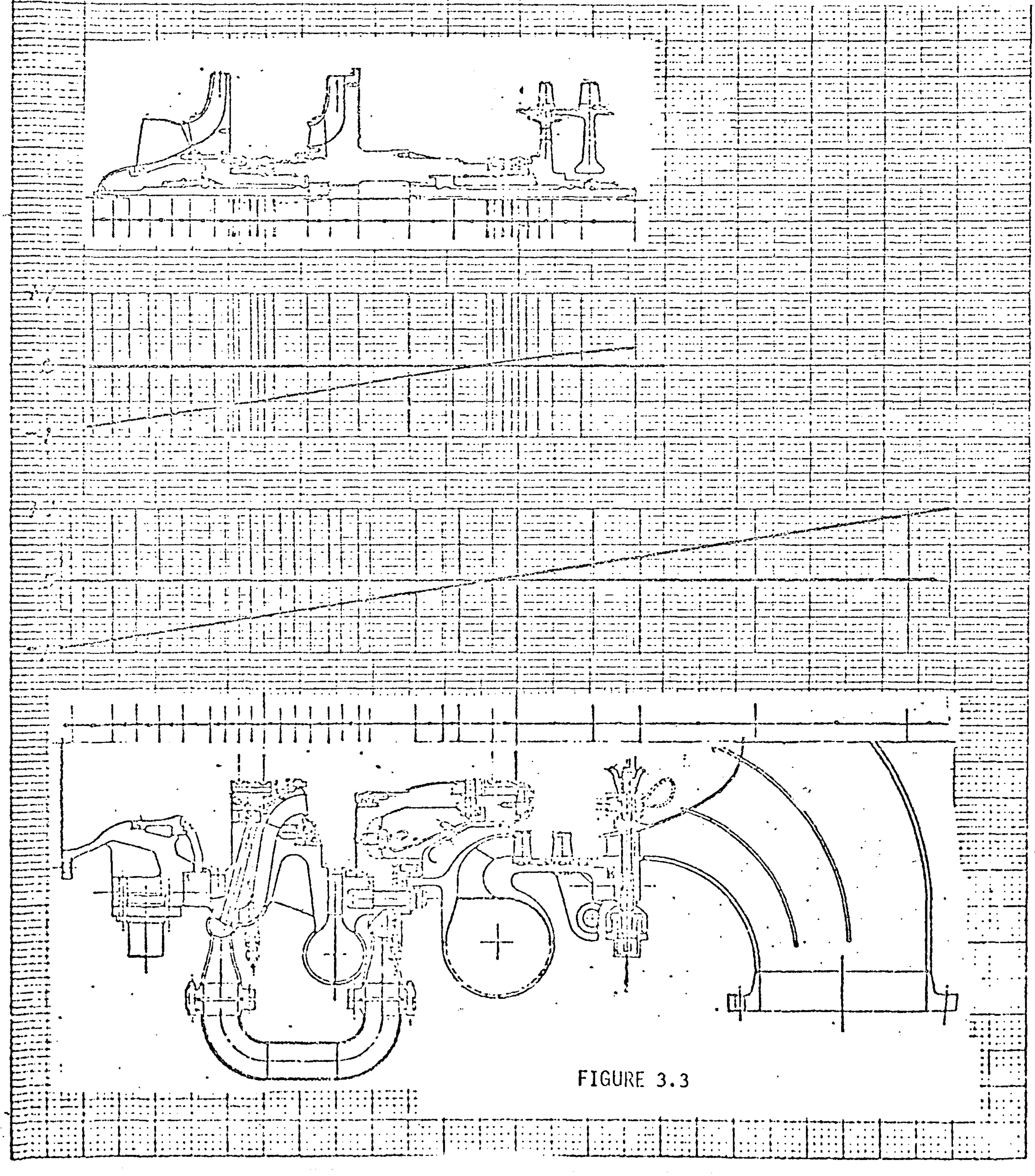
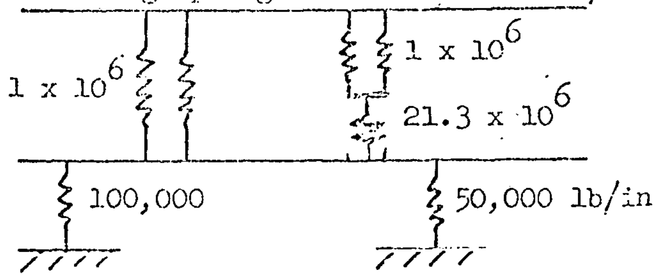


FIGURE 3.3

1118101, "B" change Titanium Rotor
normalized displacement near resonant speed 38,400 rpm
Unbalance Forcing = .01 gm-in./lb out of phase forward circular whirl

All Bearing Spring Rates = 1 x 10^6 lb/in



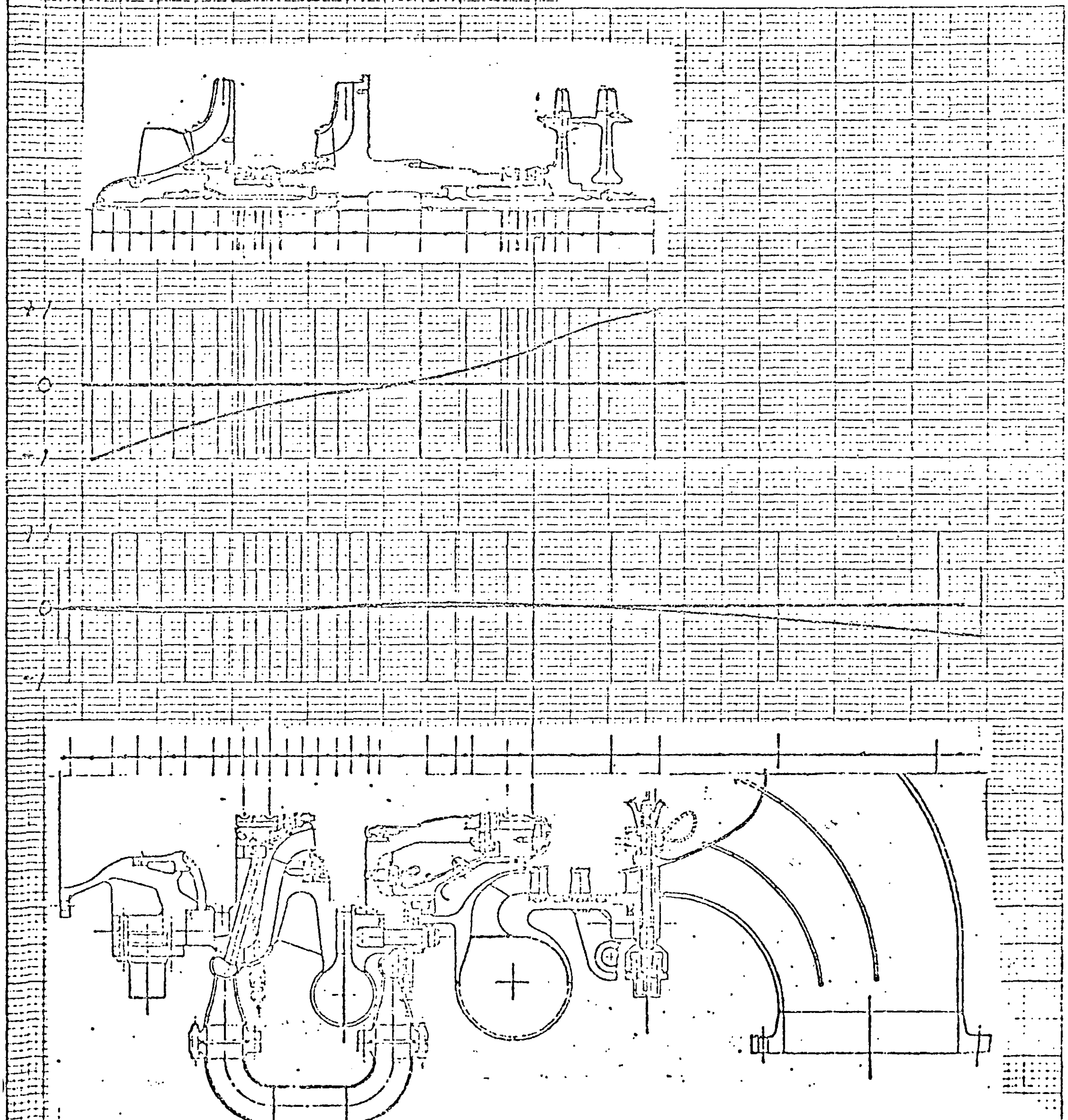
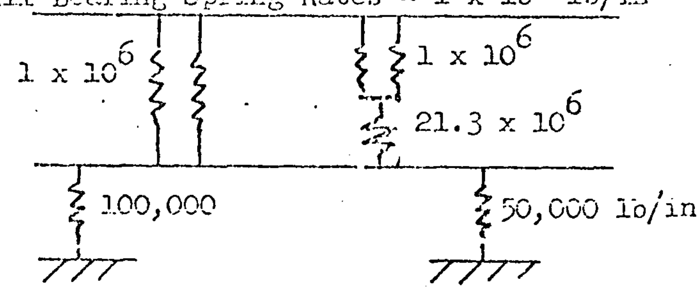


FIGURE 3.4

1118101, "B" change Titanium Rotor
normalized displacement near resonant speed 42,000 rpm
Unbalance Forcing = .01 gm-in./lb out of phase forward circular whirl



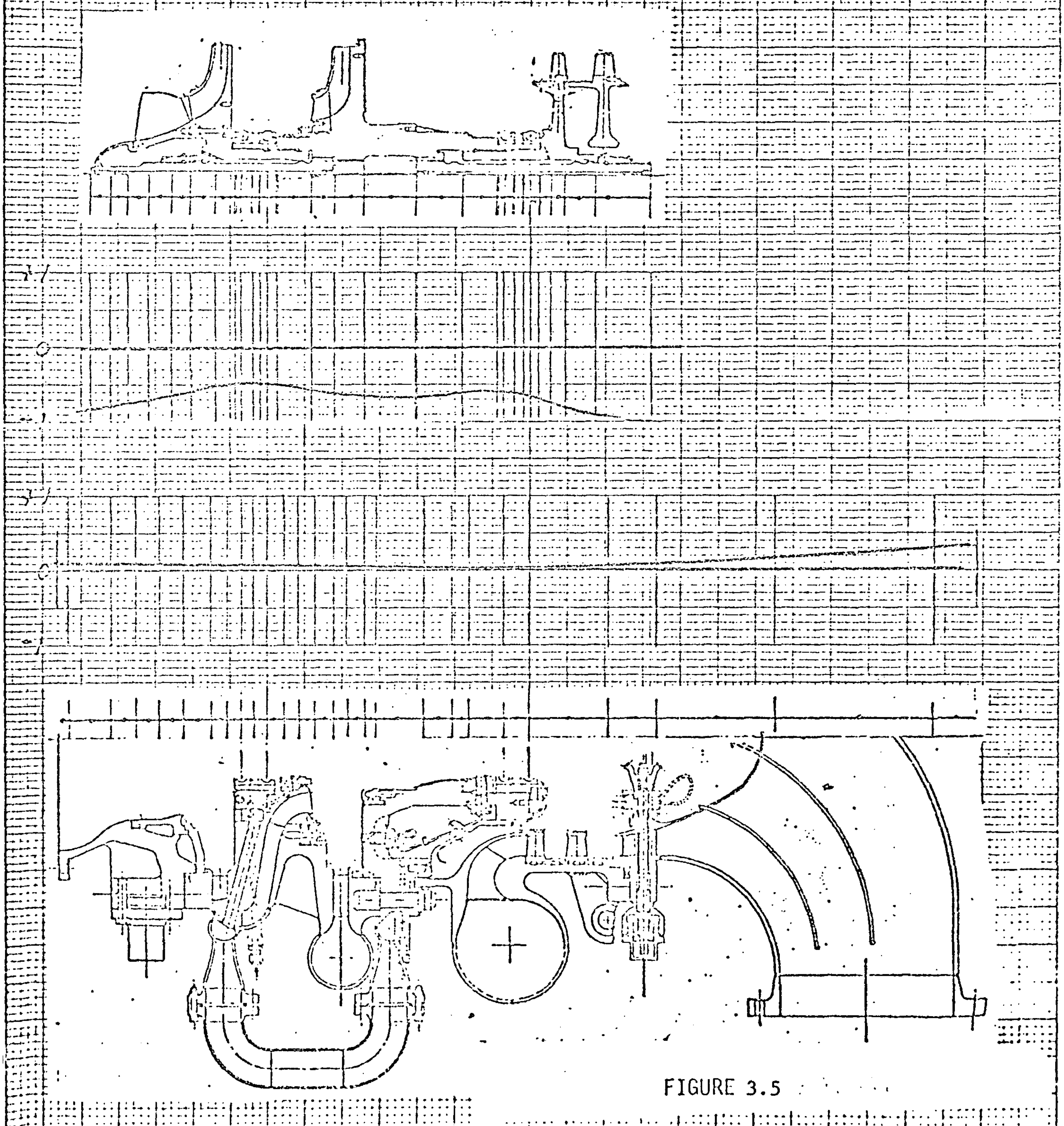
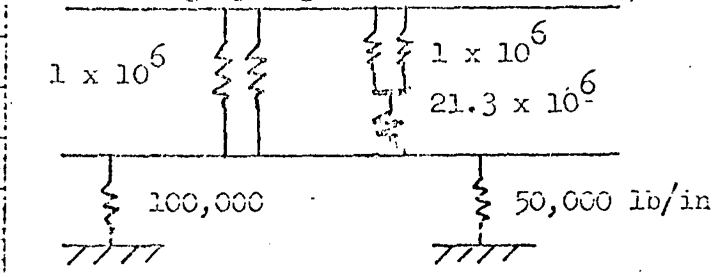


FIGURE 3.5

NERVA TPA

1118101, "B" change Titanium Rotor normalized displacement near resonant speed 48,600 rpm
Unbalance Forcing = .01 gm-in./lb out of phase forward circular whirl



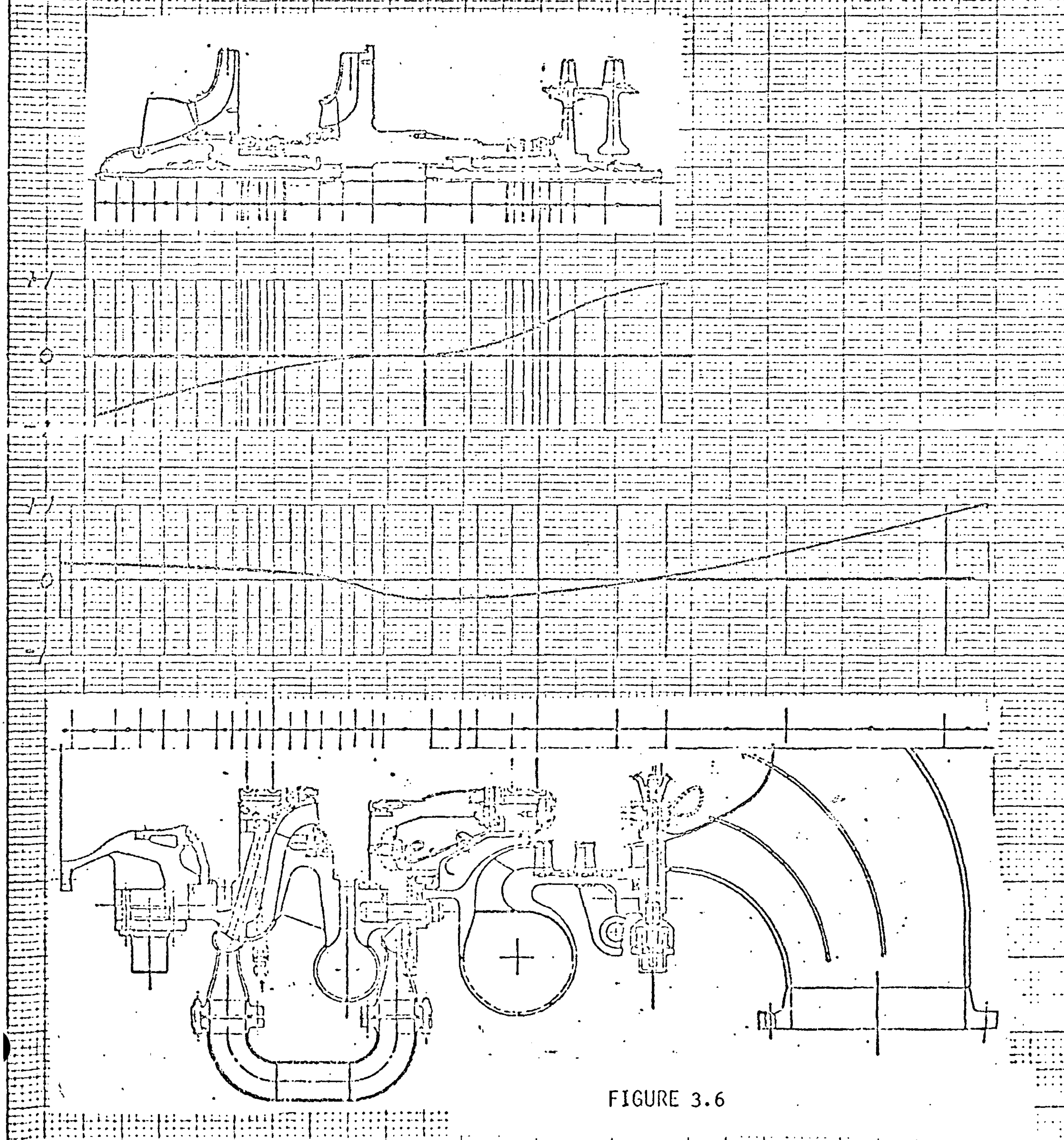
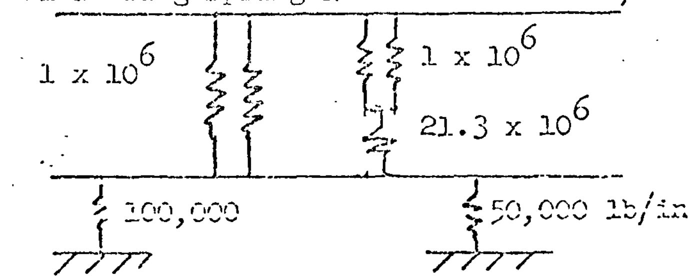


FIGURE 3.6

NERVA TPA

1118101, "B" change Titanium Rotor
normalized displacement near resonant
speed 58,200 rpm
Unbalance Forcing = .01 gm-in./lb out
of phase forward circular whirl



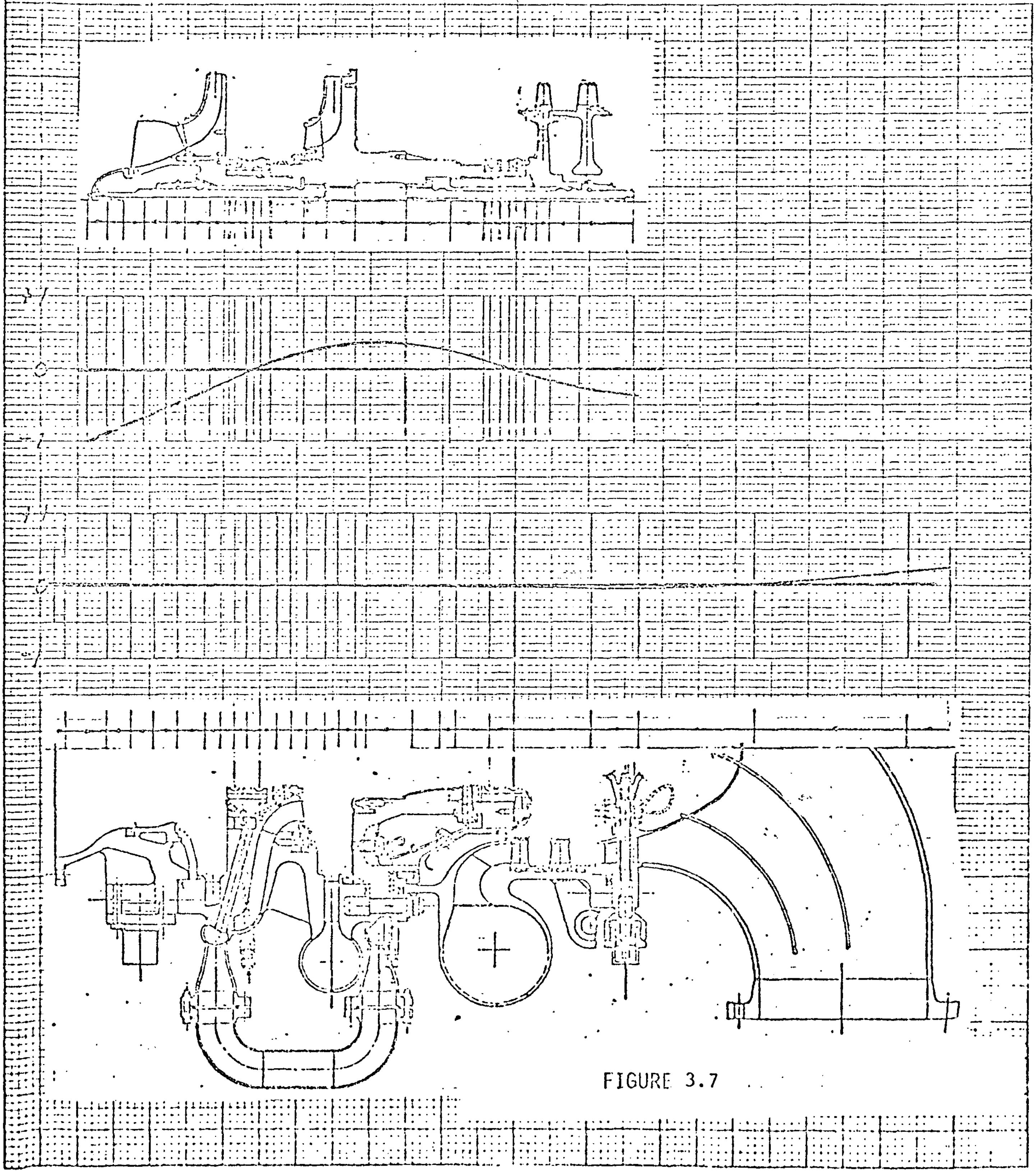


FIGURE 3.7

APPENDIX A

PROGRAM E13101, VIBRATION ANALYSIS (FREE VIBRATION ANALYSIS OF A SINGLE, UNDAMPED LUMPED PARAMETER BEAM) USERS' MANUAL AND SAMPLE OF INPUT/OUTPUT

VIBRATION ANALYSIS

Program E13101
(Formerly Programs 14009)
& 14029

Aerojet-General Corporation
Computing Sciences Division
Sacramento, California

APPROVED:

R. D. Glaus, Manager
Engineering Analysis
and Programming

by

J. A. Budzenski

12 December 1967

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Aerojet-General Corporation
Computing Sciences Division
Sacramento, California

VIBRATION ANALYSIS

Program E13101

(Formerly Programs 14009)
& 14029

I. INTRODUCTION

E13101, a 360 FORTRAN program, replaces 14009 without changes in logic.

A. Problem to be Solved

The purpose of this program is to compute the natural modes and frequencies set up in a uniform shaft rotating at a constant speed. Studies can be made of shaft performance at various speeds.

B. Initial Information

Geometrical and mechanical properties of the shaft are provided.

C. Problem Solution

This program solves for the Eigenvalues of a 4 x 4 matrix using the Frequency Iteration Method.

D. Restrictions

The number of stations must not exceed 50.

II. PROBLEM SOLUTION

$$\{\Delta\} = \begin{Bmatrix} V \\ M \\ \phi \\ Y \end{Bmatrix}$$

$$[E]_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \ell_1 & 1 & 0 & 0 \\ -\dfrac{\ell_1^2}{2EI_1} & -\dfrac{\ell_1}{EI_1} & 1 & 0 \\ \dfrac{\ell_1^3}{GEI_1} - \dfrac{C_1\ell_1}{G_1} & \dfrac{\ell_1^2}{2EI_1} & -\ell_1 & 1 \end{bmatrix}$$

$$[E]_L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \ell_2 & 1 & 0 & 0 \\ -\dfrac{\ell_2^2}{2EI_2} & -\dfrac{\ell_2}{EI_2} & 1 & 0 \\ \dfrac{\ell_2^3}{GEI_2} - \dfrac{C_2\ell_2}{G_2} & \dfrac{\ell_2^2}{2EI_2} & -\ell_2 & 1 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 1 & 0 & 0 & \dfrac{W_N}{g}\omega^2 - K_N \\ 0 & 1 & (I_J - I_X)_{\omega^2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[D] = \prod_{n=1}^{m} [E]_{nL}\,[F]\,[E]_{nR}$$

Compute [D] for successive values of ω^2

until
$$\begin{vmatrix} d_{mr} & d_{ms} \\ d_{nr} & d_{ns} \end{vmatrix} = 0$$

$$\{\Delta\} = \begin{Bmatrix} 0 \\ 0 \\ -\dfrac{d_{14}}{d_{13}} \\ 1 \end{Bmatrix}$$

$$\{\Delta\}_1 = [E]_{1L}\,[F]_1\,[E]_{1R}\,\{\Delta\}_0$$

$$\{\Delta\}_n = [E]_{nL}\,[F]_n\,[E]_{nR}\,\{\Delta\}_{n-1}$$

III. INPUT/OUTPUT

A. Input Format

1. See sample input sheets.

2. Input Instructions

CARD 1:

Columns	1-70	May be used for case identification, or may be left blank, as desired.
	71-72	Contain the number of stations. A right adjusted integer not to exceed 50.

CARD 2:

Columns	1-2	Contain the number of modes desired from this run; a right adjusted integer.
	3-14	Contain the value of the trail mode in floating point format (see note below).
	15-26	Contain the value of the step size ($\Delta\omega$), in floating point format.
	29	Contain 1 if ϕ = 1.0 2 if Y = 1.0
	32	Contain value of the subscript m.
	35	Contain value of the subscript n.
	38	Contain value of the subscript r.
	41	Contain value of the subscript s.

CARD 3:

Columns	1-12	Contain the value of L_1.
	13-24	Contain the value of L_2.
	25-36	Contain the value of EI_1.
	37-48	Contain the value of EI_2.
	49-60	Contain the value of G_1.
	61-72	Contain the value of G_2.

CARD 4:

Columns	1-12	Contain the value of C_1.
	13-24	Contain the value of C_2.
	25-36	Contain the value of I_J.
	37-48	Contain the value of I_X.
	49-60	Contain the value of W_N.
	61-72	Contain the value of K_N.

Cards 3 and 4, 5 and 6, etc., are taken in pairs; one pair for each station, and are all floating point numbers identical in format.

NOTE: Floating Point Format: A number of the form ±XXXXXXXE ± XX with the decimal point assumed immediately to the right of the sign. The sign position may be left blank if the number is positive.

B. Output Format

1. See sample output sheet.

C. Restrictions

1. The number of stations must not exceed 50.

D. Timing

1. Running time is largely a function of the number of stations and the number of iterations required for convergence but should not exceed 0.5 minute per root.

CAUTION: Values for K_N should be carefully chosen. Large values may cause overflow which will lead to erroneous results.

ADDENDUM

This addendum documents changes to the input for Program E13101.

Card 4 — K_φ (Moment spring constant in units of lbs.in/rad) must be input at the former location of I_J.

I_X-I_J must be input at the former location of I_X.

Cards 3 & 4 — A repeating-card option has been added in columns 73-74. If the data at succeeding stations is identical the number of stations can be punched in columns 73-74 (right-adjust integer format) and the card will be repeatedly read and the data stored in locations corresponding to consecutive stations.

SUPPLEMENT

NOTE: The modification described in this supplement supercedes and obsoletes Program 14029. Program 14029 is now included in Program E13101.

I. INTRODUCTION

This report describes a modification to Program E13101 which allows the user to specify optional sets of input data.

II. PROGRAM MODIFICATION

The essential feature of the modification is the addition of SUBRØUTINE CØNVER. Heretofore input data consisted of incremental shaft lengths (ΔL, L_1, L_2), stiffness (EI_1, EI_2), shear modulus (G_1, G_2), geometric terms (C_1, C_2), inertial term ($I_J - I_X$), weight term (W_N), moment spring constant (K_φ) and the bearing spring rate constant (K_N). CØNVER computes these data from a set of input consisting of ΔL, R_o, R_I, ρ, E, $(I_J)_{EFF}$ and K_N, where:

ΔL	= Incremental shaft length (inches)
R_o	= Outer radius of shaft (inches)
R_I	= Inner radius of shaft (inches)
ρ	= Density of shaft (lbs/in^3)
E	= Young's modulus (lbs/in^2)
$(I_J)_{EFF}$	= Effective rotatory inertia ($lb \cdot in \cdot sec^2$) = $I_X - I_J$
K_N	= Bearing spring rate constant (lbs/in)
ΔW_L	= Weight term for cantilevered shafts (lbs)
K_φ	= Moment spring constant (lbs·in/rad)

This is accomplished by solution of the following equations:

$$A = R_I/R_O \quad , \quad AA = \pi(R_O^2 - R_I^2)$$

1.

$$B_I = .89[(1+A+A^2)/(1+A^2)]^2 + .73(.3 \cdot A) \qquad A \leq .3$$

$$B_I = .89[(1+A+A^2)/(1+A^2)]^2 \qquad A > .3$$

2. $C_1 = C_2 = B_I/AA$

3. $L_1 = L_2 = \Delta L/2$

4. $W_N = \rho \Delta L \pi (R_O^2 - R_I^2) + \Delta W_L$

5. $EI_1 = EI_2 = E\pi(R_O^4 + R_I^4)/4$

6. $G_1 = G_2 = E/2(1+\nu)$ where $\nu = .3$

III. INPUT

A. CARD 1

Columns 1-72 Identical to that described in E13101.

73-74 Optional input flag.

If the input data is in the form described in References 1 and 2, leave columns 73-74 blank.

If the input data is in the modified form, place an integer "1" in column 74.

B. CARD 2

Identical to that described in E13101.

C. CARD 3

If Card 1, Column 74, is blank, then the input for Card 3 is identical to that described in E13101.

If the optional input is desired (i.e., Card 1, Column 74=1), then the format is (See Section II for symbol definitions):

Columns	1-9	ΔL
	10-18	R_o
	19-27	R_I
	28-36	ρ
	37-45	E
	46-54	$(I_J)_{EFF}$
	55-63	K_N
	64-72	ΔW_L
	73-80	K_φ

The input format for the parameters ΔL to ΔW_L is:

<u>±XXXXE±XX</u>

The format for the variable K_φ is:

<u>±XXXXE±X</u>

Decimals are assumed immediately to the right of the sign.

D. <u>Remaining Cards</u>

If the optional input is desired, a card identical to Card 3 must be input for each of the stations specified on Card 1.

IV. <u>OUTPUT</u>

The output format is identical to that described in Reference 1. Changes in the output parameters are noted below:

$(I_J)_{EFF}$ is output under the heading J.
W_N is output under the heading W.
K_N is output under the heading K(Y)

V. <u>RESTRICTIONS</u>

The program is limited to materials having a Poisson's ratio of 0.3. Other restrictions are noted in E13101.

CUSTOMER INSTRUCTIONS

1. [illegible] SPACES PROVIDED
2. [illegible] 1 vs I, 0 vs O, Z vs 2, U vs V, S vs 5
3. [illegible] repeated in pairs for each station. [illegible] succeeding stations is identical, the number of [illegible] in columns 73-74 (right adjusted integer) [illegible] repeatedly read and the data stored in [illegible] to consecutive stations.

KEYPUNCH INSTRUCTIONS

- X PUNCH 1 CARD PER HAND POSTED LINE ITEM
- PUNCH ALL * LINES WHETHER POSTED OR NOT. IF NECESSARY PROVIDE BLANK CARDS
- PUNCH ALL * LINES THAT ARE HAND POSTED
- PAS INCLUDING SPACES
- ALL SPACES MAY BE IGNORED
- ALL SPACES MAY BE IGNORED EXCEPT ON 1 CARD
- ALL SPACES MAY BE IGNORED EXCEPT (Specify cols.)
- ALL SIGNS AND XP LINES MUST BE PUNCHED
- DO NOT PUNCH PRE-PRINTED SIGNS SHOWN AFTER LAST MANUAL ITEM VALUE ENTRY
- X USE 360 SYMBOLS

CUSTOMER	DATE

JOB NO.	PROGRAMMER
E13101	J. A. Budzinski

TITLE: Vibration Analysis

FORM APPROVED (KEY PUNCH)	DATE

CASE IDENTIFICATION

NO. MODES	TRIAL ω	ω	SHAPE MODE	m	n	r	s
2							

ℓ_1	ℓ_2	EI_1	EI_2	G_1	G_2	REPEAT OPT
C_1	C_2	K_Z	I_X-I_J	M_N	K_N	REAC OPT

PLEASE PRINT CLEARLY - USE BLACK PENCIL

CUSTOMER INSTRUCTIONS		KEYPUNCH INSTRUCTIONS
1. [illegible]	X	PUNCH 1 CARD PER HAND POSTED LINE ITEM
2. [illegible]		PUNCH ALL LINES WHETHER POSTED OR NOT. IF NECESSARY PROVIDE BLANK CARDS
3. [illegible] for each Station specified on Card 1.		PUNCH ALL LINES THAT ARE HAND POSTED
		PAS INCLUDING SPACES
		ALL SPACES MAY BE IGNORED
		ALL SPACES MAY BE IGNORED EXCEPT ON 1 CARD
		ALL SPACES MAY BE IGNORED EXCEPT (Specify cols.)
		ALL SIGNS AND XP LINES MUST BE PUNCHED
		DO NOT PUNCH PRE-PRINTED SIGNS SHOWN AFTER LAST HANDWRITTEN VALUE ENTRY
	X	USE 360 SYMBOLS

CUSTOMER	DATE
JOB NO. E13101	PROGRAMMER J. A. Budzenski
TITLE Vibration Analysis	
FORM APPROVED (KEY PUNCH)	DATE

OPTIONAL INPUT

CASE IDENTIFICATION — NO STA

NO. MODES	TRIAL ω	ω	SHAPE MODE	m	n	r	s

ΔL	Z_C	R_I	ρ	E	$(I_J)_{EFF}$	K_H	ω_L	K_ϕ

PLEASE PRINT CLEARLY - USE BLACK PENCIL

TASK 3 14029 CHECK CASE 6-17-66 15 1

	0.	50.	2	4			
.68	.78	.56	.285	29.E&06		3.5E&06	
.12	.78	.48	.285	29.E&06			
2.06	.80	.30	.285	29.E&06			
1.14	.90	.0	.285	29.E&06			
.42	1.40	.0	.285	29.E&06			
1.56	1.50	.40	.285	29.E&06			
.42	1.40	.75	.285	29.E&06			
.72	1.10	.75	.285	29.E&06		4.5E&06	
1.10	1.10	.75	.285	29.E&06			
.25	1.30	.75	.285	29.E&06			
.48	1.50	1.00	.285	29.E&06			
.18	1.50	1.0	.285	29.E&06			
.68	1.50	1.0	.285	29.E&06	.1142		8.1
.76	1.50	1.0	.285	29.E&06			
.6	1.50	1.0	.285	29.E&06	.11		7.8

E

JOB E13101 VIBRATION ANALYSIS

TASK 3 14029 CHECK CASE 6-17-66 NUMBER OF STATIONS 15

NUMBER OF MODES 1 TRIAL OMEGA 0.0 DELTA OMEGA 0.50000000D 02

KK	M	N	R	S	SKIP
2	1	2	3	4	0

L(1)	L(2)	EI(1)	EI(2)	G(1)	G(2)
0.34000000D 00	0.34000000D 00	0.61907930D 07	0.61907930D 07	0.11153847D 08	0.11153847D 08
0.90000000D-01	0.90000000D-01	0.72216777D 07	0.72216777D 07	0.11153847D 08	0.11153847D 08
0.10300000D 01	0.10300000D 01	0.91447835D 07	0.91447835D 07	0.11153847D 08	0.11153847D 08
0.57000000D 00	0.57000000D 00	0.14943692D 08	0.14943692D 08	0.11153847D 08	0.11153847D 08
0.21000000D 00	0.21000000D 00	0.87498382D 08	0.87498382D 08	0.11153847D 08	0.11153847D 08
0.78000000D 00	0.78000000D 00	0.11472319D 09	0.11472319D 09	0.11153847D 08	0.11153847D 08
0.21000000D 00	0.21000000D 00	0.80291740D 08	0.80291740D 08	0.11153847D 08	0.11153847D 08
0.36000000D 00	0.36000000D 00	0.26140500D 08	0.26140500D 08	0.11153847D 08	0.11153847D 08
0.55000000D 00	0.55000000D 00	0.26140500D 08	0.26140500D 08	0.11153847D 08	0.11153847D 08
0.12500000D 00	0.12500000D 00	0.57845453D 08	0.57845453D 08	0.11153847D 08	0.11153847D 08
0.24000000D 00	0.24000000D 00	0.92529721D 08	0.92529721D 08	0.11153847D 08	0.11153847D 08
0.90000000D-01	0.90000000D-01	0.92529721D 08	0.92529721D 08	0.11153847D 08	0.11153847D 08
0.34000000D 00	0.34000000D 00	0.92529721D 08	0.92529721D 08	0.11153847D 08	0.11153847D 08
0.37000000D 00	0.37000000D 00	0.92529721D 08	0.92529721D 08	0.11153847D 08	0.11153847D 08
0.32500000D 00	0.32500000D 00	0.92529721D 08	0.92529721D 08	0.11153847D 08	0.11153847D 08

C(1)	C(2)	K(PHI)	J	W	K(Y)
0.20845838D 01	0.20845638D 01	0.0	0.0	0.17948623D 00	0.35000000D 07
0.15658630D 01	0.15658630D 01	0.0	0.0	0.60919880D-01	0.0
0.90830701D 00	0.90830701D 00	0.0	0.0	0.10144360D 01	0.0
0.43576580D 00	0.43576580D 00	0.0	0.0	0.82676980D 00	0.0
0.18008689D 00	0.18008689D 00	0.0	0.0	0.73705534D 00	0.0
0.21490168D 00	0.21490168D 00	0.0	0.0	0.29192119D 01	0.0
0.40609525D 00	0.40609525D 00	0.0	0.0	0.52552798D 00	0.0
0.93841828D 00	0.93841828D 00	0.0	0.0	0.41741399D 00	0.45000000D 07
0.93841828D 00	0.93841828D 00	0.0	0.0	0.63771582D 00	0.0
0.51520961D 00	0.51520961D 00	0.0	0.0	0.25237788D 00	0.0
0.48351293D 00	0.48351293D 00	0.0	0.0	0.53721234D 00	0.0
0.48351293D 00	0.48351293D 00	0.0	0.0	0.20145463D 00	0.0
0.48351293D 00	0.48351293D 00	0.0	0.11420000D 00	0.88610508D 01	0.0
0.48351293D 00	0.48351293D 00	0.0	0.0	0.82820236D 00	0.0
0.48351293D 00	0.48351293D 00	0.0	0.11000000D 00	0.85274750D 01	0.0

OMEGA = 0.0 DETERM = -0.66134880D 15

OMEGA = 0.50000000D 02 DETERM = -0.65187035D 15

OMEGA = 0.10000000D 03 DETERM = -0.62359766D 15

OMEGA = 0.15000000D 03 DETERM = -0.57701656D 15

OMEGA = 0.20000000D 03 DETERM = -0.51292974D 15

OMEGA = 0.25000000D 03 DETERM = -0.43244636D 15

OMEGA = [illegible] 03 DETERM = [illegible]

```
OMEGA =  0.35000000D 03   DETERM = -0.22816774D 15
OMEGA =  0.40000000D 03   DETERM = -0.10797321D 15
OMEGA =  0.45000000D 03   DETERM =  0.21464220D 14
OMEGA =  0.44170865D 03   DETERM = -0.54216572D 12
OMEGA =  0.44191292D 03   DETERM = -0.23699450D 10
OMEGA =  0.44191382D 03   DETERM =  0.26877400D 06
OMEGA =  0.44191382D 03   DETERM =  0.10000000D 02
OMEGA =  0.44191382D 03   DETERM = -0.30000000D 01
OMEGA =  0.44191382D 03   DETERM = -0.20000000D 01
OMEGA =  0.44191382D 03   DETERM =  0.0
                       OMEGA =  0.44191382D 03
```

V	M	PHI	Y
0.0	0.0	0.10000000D 01	-0.86118093D 00
0.41978276D 07	0.14279414D 07	0.96078854D 00	-0.18036102D 01
0.41974645D 07	0.21836977D 07	0.91577724D 00	-0.20788826D 01
0.41333648D 07	0.10764452D 08	-0.54632361D 00	-0.34900106D 01
0.40814345D 07	0.15446887D 08	-0.15466721D 01	-0.25138741D 01
0.40499663D 07	0.17154293D 08	-0.16249245D 01	-0.18756930D 01
0.40128723D 07	0.23442605D 08	-0.19010377D 01	0.74228431D 00
0.42265992D 07	0.25130474D 08	-0.20280756D 01	0.15056362D 01
-0.57552745D 07	0.24507431D 08	-0.27359190D 01	0.32741231D 01
-0.56692911D 07	0.18212920D 08	-0.36343797D 01	0.73316502D 01
-0.55498730D 07	0.16795524D 08	-0.37100252D 01	0.83152997D 01
-0.57504192D 07	0.14107455D 08	-0.37901492D 01	0.10232426D 02
-0.55077894D 07	0.13112216D 08	-0.38166228D 01	0.10960207D 02
-0.33728299D 07	0.67100978D 07	-0.38880949D 01	0.13713338D 02
-0.30712554D 07	0.43442866D 07	-0.39321122D 01	0.16710538D 02
0.67523887D-07	0.12246892D-06	-0.39456180D 01	0.19316359D 02

END OF CASE

APPENDIX B

PROGRAM E13101 LISTING

i

```
@9 RUN FYEE,428999,2,200   LIST E13101                25 APR 72   14:46:09.459

@ CTL UN=E13101                                       25 APR 72   14:46:09.459

@R     ASG X=AN4149                                   25 APR 72   14:46:09.540
       AN4149 ASSIGNED UNIT  2

@N     HDG                                            25 APR 72   14:46:09.553
```

2

```
0.    XQT CUR                                              25 APR 72   14:46:09.554
  1.      PEF X                                                        14:46:09
  2.      IN X                                                         14:46:10
   END OF FILE -- UNIT X
  3.      LIST 1                                                       14:46:11
```

```
@ ELT AERO/STEVE,1,710407, 52140

                    TASK 3  14029 CHECK CASE 6-17-66                                15 1
   1         0.          50.        2  1  2  3  4
   .68       .78         .56       .285        29.E+06                3.5E+06
   .18       .78         .48       .285        29.E+06
  2.06       .80         .30       .285        29.E+06
  1.14       .90         .0        .285        29.E+06
   .42      1.40         .0        .285        29.E+06
  1.56      1.50         .40       .285        29.E+06
   .42      1.40         .75       .285        29.E+06
   .72      1.10         .75       .285        29.E+06                4.5E+06
  1.10      1.10         .75       .285        29.E+06
   .25      1.30         .75       .285        29.E+06
   .48      1.50        1.00       .285        29.E+06
   .18      1.50        1.0        .285        29.E+06
   .68      1.50        1.0        .285        29.E+06 .1142                  8.1
   .74      1.50        1.0        .285        29.E+06
   .65      1.50        1.0        .285        29.E+06 .11                    7.8
```

```
@ ELT CONVER,1,710407, 52141

      SUBROUTINE CONVER(NSTA,DC2,DC1,DKN,DL2,DL1,DWN,DIX,DEI2,DEI1,DG2,
     1DG1,DIJ,*)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION DC2(50),DC1(50),DKN(50),DL2(50),DL1(50),DWN(50),DIX(50),
     1DEI2(50),DEI1(50),DG2(50),DG1(50),DIJ(50)
      PI = 3.141592653589793
      DO 50 N=1,NSTA
      READ (5,10) DELTAL,RO,RI,RHO,E,EFFIJ,AIKN,DLTAWL,AKPHI
   10 FORMAT (8E9.4,E8.4)
      R1=RO**2+RI**2
      R2=RO**2-RI**2
      D=R1/R2
      A=RI/RO
      AA=PI*R2
       B = 0.8888888888888889*((1.D0+(1.0D0+A)*A)/(1.0D0+A**2))**2
      IF(A.GT.0.3D0)GOTO20
      B = B+ 0.7333333333333333*(0.3D0-A)
   20 DC2(N)=B/AA
      DC1(N)=DC2(N)
      DKN(N)=AIKN
      DL2(N)=DELTAL/2.0D0
      DL1(N)=DL2(N)
      DWN(N)=RHO*DELTAL*R2*PI+DLTAWL
      DIJ(N)=AKPHI
      DIX(N)=EFFIJ
      DEI2(N)=E*PI*0.25D0*R1*R2
      DEI1(N)=DEI2(N)
      DG2(N)=0.3846154D0*E
   50 DG1(N)=DG2(N)
      RETURN 14
      END
```

5

```
@ ELT MAIN,1,710507, 61552

C                                JOB 14009     VIBRATION ANALYSIS
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION TITLE(12)
      DIMENSION DL1(50),DL2(50),DEI1(50),DEI2(50),DG1(50),DG2(50),DC1(50
     1),DC2(50),DIJ(50),DIX(50),DWN(50),DKN(50),E1MTRX(4,4),E2MTRX(4,4),
     2AMATRX(4,4),BMATRX(4,4),CMATRX(4,4),FMATRX(4,4),DLMTRX(4,1),SHMTRX
     3(4,1),NREP(2)
      COMMON DL1,DL2,DEI1,DEI2,DG1,DG2,DC1,DC2,DIJ,DIX,DWN,DKN,E1MTRX,
     1 E2MTRX,AMATRX,BMATRX,CMATRX,FMATRX,DLMTRX,SHMTRX,SUMG,OMGSQ
    1 FORMAT (11A6,A4,2I2,L1)
    2 FORMAT (I2,2D12.7,5I3,29X,I2)
    3 FORMAT (74H1                                               JOB E13101
     1 VIBRATION ANALYSIS///)
    4 FORMAT (6E12.7)
    5 FORMAT (36H  100 ITERATIONS AND NO ROOTS FOUND )
    6 FORMAT (1H )
    7 FORMAT (14H0 END OF CASE )
    8 FORMAT (60H0                                                  OME
     1GA = E15.8///)
    9 FORMAT (35H0                      OMEGA = E15.8,12H   DETERM =
     1 E15.8)
   10 FORMAT (15H               E15.8,14H              E15.8,14H
    1      E15.8,14H              E15.8)
      LOGICAL DIJFLG
   11 FORMAT (110H                            V                        M
     1                  PHI                              Y)
   22 FORMAT (6(6H      E15.8))
  101 FORMAT (1H ,8X,11A6,A4,4X20HNUMBER OF STATIONS  I2  )
  102 FORMAT (1H0,92X26HKK   M   N   R   S       SKIP     )
  103 FORMAT (1H ,4X16HNUMBER OF MODES I2,5X13HTRIAL OMEGA  E15.8,
     1    5X13HDELTA OMEGA  E15.8,3X5I3,8XI2 )
 4098 FORMAT (1H1)
 4097 FORMAT (1H )
C
C      DIJ  =  K(PHI)
C      DIX  =  I(X) - I(J)
C
C
C PROGRAM STARTS HERE ---- INPUT HEADER AND NUMBER OF STATIONS
C
      PI2 = 6.283185307179586
   30 READ (5,1,END=1000) TITLE,NSTA,INFLAG,DIJFLG
C
C INPUT NUMBER OF ROOTS DESIRED,TRIAL ROOT,STEP SIZE, AND SUBSCRIPTS
C
      READ (5,2) NOMODE,TROMGA,DELOMG,KK,KM,KN,KR,KS,LSKIP
C
C PRINT TITLE
C
      WRITE (6,4098)
      LINE=1
      DO 555 II=1,LSKIP
      WRITE (6,4097)
  555 LINE=LINE+1
```

```
      WRITE (6,3)
      LINE=LINE+3
C
C PRINT HEADER AND NUMBER OF STATIONS
C
      WRITE (6,101) TITLE,NSTA
C
C PRINT SUBSCRIPTS CARD
C
      WRITE (6,102)
      WRITE (6,103) NOMODE,TROMGA,DELOMG,KK,KM,KN,KR,KS,LSKIP
      LINE=LINE+4
      IF (INFLAG.EQ.1) CALL CONVER(NSTA,DC2,DC1,DKN,DL2,DL1,DWN,DIX,
     1  DEI2,DEI1,DG2,DG1,DIJ,$100)
C
C INPUT REMAINING DATA
C
      DO 50 N=1,NSTA
      CALL REPEAT(DL1(N-1),DL1(N),DL2(N-1),DL2(N),DEI1(N-1),DEI1(N),DEI2
     1(N-1),DEI2(N),DG1(N-1),DG1(N),DG2(N-1),DG2(N),NREP(1))
   50 CALL REPEAT(DC1(N-1),DC1(N),DC2(N-1),DC2(N),DIJ(N-1),DIJ(N),DIX(N-
     11),DIX(N),DWN(N-1),DWN(N),DKN(N-1),DKN(N),NREP(2))
C
C PRINT STATION DATA
C
100   CALL STAOUT(DL1,DL2,DEI1,DEI2,DG1,DG2,1,1,36HL(1)  LL(2) EI(1) EI(
     12) G(1)  G(2)  ,NSTA,LINE,LSKIP)
      CALL STAOUT(DC1,DC2,DIJ,DIX,DWN,DKN,1,1,36HC(1)  C(2)  K(PHI)J
     1 W     K(Y)  ,NSTA,LINE,LSKIP)
      TMODE=TROMGA*PI2
      DELMOD=DELOMG*PI2
C
C INITIALIZE E1, E2, AND F MATRICES
C
      DO 51 I=1,4
      DO 51 J=1,4
      IF(I-J)55,56,55
   56 E1MTRX(I,J)=1.0D0
      E2MTRX(I,J)=1.0D0
      FMATRX(I,J)=1.0D0
      GO TO 51
   55 E1MTRX(I,J)=0.0D0
      E2MTRX(I,J)=0.0D0
      FMATRX(I,J)=0.0D0
   51 CONTINUE
      OMGWRK=TMODE-DELMOD
      DO 95 MM=1,NOMODE
      DELOMG=DELMOD
      OMGWRK=OMGWRK+DELOMG
      EE = 2.0D-12
      NCNTRL=0
      CALL ROOT(50)
 1050 CONTINUE                                                          *NEW
      IF(NCNTRL-100)83,822,822
   83 NCNTRL=NCNTRL+1
      DO 58 I=1,4
      DO 58 J=1,4
      IF(I-J)53,54,53
   54 CMATRX(I,J)=1.D0
      GO TO 58
```

```
   53 CMATRX(I,J)=0.D0                                                     00001140
   58 CONTINUE                                                             00001150
      OMGSQ=OMGWRK*OMGWRK                                                  00001160
      SUMG=OMGSQ/386.04D0                                                  00001170
      DO 69 N=1,NSTA                                                       00001180
      CALL MATELM(N,DIJFLG)                                                00001190
      CALL MATMPY(E1MTRX,CMATRX,AMATRX,4,4,4,4,4)                          00001200
      CALL MATMPY(E2MTRX,FMATRX,BMATRX,4,4,4,4,4)                          00001210
      CALL MATMPY(BMATRX,AMATRX,CMATRX,4,4,4,4,4)                          00001220
   69 CONTINUE                                                             00001230
      DETNOW=CMATRX(KM,KR)*CMATRX(KN,KS)-CMATRX(KM,KS)*CMATRX(KN,KR)       00001240
      OMGPRT=OMGWRK/PI2                                                    00001250
      IF(LINE-(80-LSKIP))558,556,556                                       00001260
  556 WRITE (6,4098)                                                       00001270
      LINE=1                                                               00001280
      DO 557 II=1,LSKIP                                                    00001290
      WRITE (6,4097)                                                       00001300
  557 LINE=LINE+1                                                          00001310
  558 WRITE (6,9) OMGPRT,DETNOW                                            00001320
      LINE=LINE+2                                                          00001330
      CALL ROOTB(OMGWRK,DELOMG,DETNOW,EE,KKK)                              00001340
      IF (.FALSE.) GO TO 1050                                                  *NEW
      IF(KKK)822,82,822                                                    00001350
  822 WRITE (6,5)                                                          00001360
      GO TO 30                                                             00001370
   82 DLMTRX(1,1)=0.0D0                                                    00001380
      DLMTRX(2,1)=0.0D0                                                    00001390
      DLMTRX(3,1)=0.0D0                                                    00001400
      DLMTRX(4,1)=0.0D0                                                    00001410
      IF(KK-1)666,666,667                                                  00001420
  666 DLMTRX(KR,1)=-CMATRX(KM,KS)/CMATRX(KM,KR)                            00001430
      DLMTRX(KS,1)=1.0D0                                                   00001440
      GO TO 668                                                            00001450
  667 DLMTRX(KR,1)=1.0D0                                                   00001460
      DLMTRX(KS,1)=-CMATRX(KM,KR)/CMATRX(KM,KS)                            00001470
      OMGPRT=OMGWRK/PI2                                                    00001480
  668 WRITE (6,8) OMGPRT                                                   00001490
      WRITE (6,11)                                                         00001500
      WRITE (6,6)                                                          00001510
      WRITE (6,10) ( DLMTRX(II,1),II=1,4 )                                 00001520
      WRITE (6,6)                                                          00001530
      LCTR=0                                                               00001540
      DO 95 N=1,NSTA                                                       00001550
      LCTR=LCTR+1                                                          00001560
      CALL MATELM(N,DIJFLG)                                                00001570
      CALL MATMPY(E2MTRX,FMATRX,AMATRX,4,4,4,4,4)                          00001580
      CALL MATMPY(AMATRX,E1MTRX,BMATRX,4,4,4,4,4)                          00001590
      CALL MATMPY(BMATRX,DLMTRX,SHMTRX,4,4,4,4,1)                          00001600
      WRITE (6,10) ( SHMTRX(II,1),II=1,4 )                                 00001610
      DLMTRX(1,1)=SHMTRX(1,1)                                              00001620
      DLMTRX(2,1)=SHMTRX(2,1)                                              00001630
      DLMTRX(3,1)=SHMTRX(3,1)                                              00001640
      DLMTRX(4,1)=SHMTRX(4,1)                                              00001650
      IF(LCTR-5)95,94,94                                                   00001660
   94 LCTR=0                                                               00001670
      WRITE (6,6)                                                          00001680
   95 CONTINUE                                                             00001690
      WRITE (6,7)                                                          00001700
      GO TO 30                                                             00001710
 1000 STOP                                                                 00001720
```

000177 END 00001730

@ ELT MATELM,1,710407, 52145

```
      SUBROUTINE MATELM(N,DIJFLG)
      IMPLICIT REAL*8 (A-H,O-Z)
      LOGICAL DIJFLG
      DIMENSION DL1(50),DL2(50),DEI1(50),DEI2(50),DG1(50),DG2(50),DC1(50
     1),DC2(50),DIJ(50),DIX(50),DWN(50),DKN(50),E1MTRX(4,4),E2MTRX(4,4),
     2AMATRX(4,4),BMATRX(4,4),CMATRX(4,4),FMATRX(4,4),DLMTRX(4,1),SHMTRX
     3(4,1)
      COMMON DL1,DL2,DEI1,DEI2,DG1,DG2,DC1,DC2,DIJ,DIX,DWN,DKN,E1MTRX,
     1 E2MTRX,AMATRX,BMATRX,CMATRX,FMATRX,DLMTRX,SHMTRX,SUMG,OMGSQ
   61 E1MTRX(2,1)=DL1(N)
      E2MTRX(2,1)=DL2(N)
   62 E1MTRX(4,2)=0.5D0*DL1(N)*DL1(N)/DEI1(N)
      E2MTRX(4,2)=0.5D0*DL2(N)*DL2(N)/DEI2(N)
   63 E1MTRX(3,1)=-E1MTRX(4,2)
      E2MTRX(3,1)=-E2MTRX(4,2)
   64 E1MTRX(3,2)=-DL1(N)/DEI1(N)
      E2MTRX(3,2)=-DL2(N)/DEI2(N)
   65 E1MTRX(4,1)=(DL1(N)*((DL1(N)*DL1(N))/(6.D0*DEI1(N))-DC1(N)/DG1(N))
     1)
      E2MTRX(4,1)=(DL2(N)*((DL2(N)*DL2(N))/(6.D0*DEI2(N))-DC2(N)/DG2(N))
     1)
      E1MTRX(4,3)=-DL1(N)
      E2MTRX(4,3)=-DL2(N)
   66 FMATRX(1,4)=DWN(N)*SUMG-DKN(N)
      FMATRX(2,3)=DIX(N)*OMGSO-DIJ(N)
      IF(.NOT.DIJFLG)RETURN
      FMATRX(2,3)=DIX(N)*OMGSO
      FMATRX(3,2)=1.0D0/DIJ(N)
      RETURN
      END
```

```
@ ELT MATMPY,1,710407, 52146

      SUBROUTINE MATMPY(A,B,C,K1,M1,K,M,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(20),B(20),C(20)
      DO 10 I=1,K
      DO 10 J=1,N
      II=(J-1)*K1+I
      C(II)=0.0D0
      DO 10 L=1,M
      JJ=(L-1)*K1+I
      KK=(J-1)*M1+L
   10 C(II)=C(II)+A(JJ)*B(KK)
      RETURN
      END
```

```
@ ELT REPEAT,1,710407, 52147

      SUBROUTINE REPEAT(A,AA,B,BB,C,CC,D,DD,E,EE,F,FF,NR)
      IMPLICIT REAL*8 (A-H,O-Z)
C**********************************************************************
C     REPEAT READS IN A STATION CARD OR SIMULATES A REPEATED CARD BY
C     MOVING DATA.
C     A,B,C,D,E,F   OLD        AA,BB,CC,DD,EE,FF   NEW
C                   NR = NUMBER OF REPEATS FOR A PARTICULAR CARD
C**********************************************************************
      IF(NR-1)400,100,100
  400 READ (5,3002)        AA,BB,CC,DD,EE,FF,NR
 3002 FORMAT ( 6D12.6,I3)
      GO TO 700
  100 AA=A
      BB=B
      CC=C
      DD=D
      EE=E
      FF=F
      NR=NR-1
  700 RETURN
      END
```

```
@ ELT ROOT,1,710407, 52149

         .
         .        CALL ROOT(N)
         .          N= NUMBER OF SEARCH ITERATIONS
         .
         $(1).
                  REGNAM
         ROOT*.
                  S         B11,SVB11          . SAVE B11 FOR RETURN
                  DL        A0,0,B11           . GET THE CALLING SEQUENCE
                  DS        A0,CALSEQ          . PUT AWAY
                  LMJ       B11,ROOTF          . GO INITIALIZE ROOTB
         CALSEQ   RES       2                  . CALLING SEQUENCE
         RETAGN*.                              . ITERATION RETURN ENTRY
                  L         B11,SVB11
                  J         2,B11
         .
         .
         .
         SVB11    +         0
                  END
```

```
@ ELT ROOTB,1,710507, 61553

C
C
C
      SUBROUTINE ROOTF(NN)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)                              *NEW
C
C        NN = SEARCH ITERATION LIMIT
C
      N = NN
      FLGA = 0
      FLGB = 0
      RETURN
C
C
C
      ENTRY ROOTB(X,DX,F,E,K)
C
C        X= X VALUE
C        DX = SEARCH INCREMENT
C        F = F(X)
C        E = ERROR LIMIT
C        K = TERMINATION STATUS FLAG
C
      IF (F)  100, 9000, 200
C
C  F<0
C
100   CONTINUE
      XMINUS = X
      FMINUS = F
      FLGB = F
      IF (FLGA .NE.0) GO TO 1000
      GO TO 300
C
C  F>0
C
200   CONTINUE
      XPLUS = X
      FPLUS = F
      FLGA = F
      IF (FLGB .NE. 0)  GO TO 1000
C
C  TRY A NEW X VAKUE TO BRACKET THE ROOT
C
300   CONTINUE
      XLAST = X
      FLAST = F
      X = X+DX
      N = N-1
      IF (N .GE. 0)  CALL RETAGN
      K = N
      RETURN
C
C  DO LINEAR INTERPOLATION TO APPROXIMATE THE ROOT
C
1000  CONTINUE
```

```
      IF (F-FLAST .EQ. 0)  GO TO 2000
      X1 = (F*XLAST-X*FLAST)/(F-FLAST)
      FLAST = F
      XLAST = X
      X = X1
      I10 = 1
C
C  SEE IF NEW X IS IN THE PROPER INTERVAL
C
1100  CONTINUE
      IF((X-XMINUS)*(XPLUS-X))  1200, 9000, 1300
1200  CONTINUE
      GO TO (2000, 9000), I10
C
C  TEST TO SEE IF CLOSE ENOUGH
C
1300  CONTINUE
      IF (ABS(X-XLAST)-E .LE. 0)  GO TO 9000
      CALL RETAGN
C
C  INTERPOLATE USING THE INTERVAL BOUNDARIES
C
2000  CONTINUE
      X = (XMINUS*FPLUS-XPLUS*FMINUS)/(FPLUS-FMINUS)
      I10 = 2
      GO TO 1100
C
C  NORMAL RETURN
C
9000  K = 0
      RETURN
      END
```

```
@ ELT STAOUT,1,710407, 57004

      SUBROUTINE STAOUT(AA,BB,CC,DD,EE,FF,KODE,L,BCD,NSTA,LINE,LSKIP)    00002560
      IMPLICIT REAL*8 (A-H,O-Z)                                          00002570
      DIMENSION AA(50,10),BB(50,10),CC(50,10),DD(50,10),EE(50,10),       00002580
     1    FF(50,10),BCD(6)                                                00002590
      REAL BCD                                                                *NEW
C************************************************************************00002600
C     STAOUT WILL PRINT 1 TO 6 HEADINGS. IT ALSO PRINTS NSTA VALUES      00002610
C     BELOW THE HEADING.    LINE = LAST LINE USED                        00002620
C************************************************************************00002630
      N=1                                                                00002640
      IF(LINE-(73-LSKIP))31,30,30                                        00002650
   30 WRITE (6,4098)                                                     00002660
      LINE=1                                                             00002670
      DO 55 II= 1,LSKIP                                                  00002680
      WRITE (6,4097)                                                     00002690
   55 LINE=LINE+1                                                        00002700
   31 WRITE (6,20) (BCD(II),II=1,6)                                      00002710
 20   FORMAT (1H0,10X,A6,5(15X,A6) //)                                        *NEW
      LINE=LINE+3                                                        00002730**-1
      LCTR=0                                                             00002740
   56 GO TO (1,2,3,4,5),KODE                                             00002750
C                                                                        00002760
    1 WRITE (6,21) AA(N,L),BB(N,L),CC(N,L),DD(N,L),EE(N,L),FF(N,L)       00002770
   21 FORMAT (1H ,5X, 6(E15.8,6X)  )                                     00002780
      GO TO 575                                                          00002790
C                                                                        00002800
    2 WRITE (6,22) AA(N,L),BB(N,L),CC(N,L),DD(N,L)                       00002810
   22 FORMAT (1H ,5X, 4(E15.8,6X)  )                                     00002820
      GO TO 575                                                          00002830
C                                                                        00002840
    3 WRITE (6,23) CC(N,L),DD(N,L),FF(N,L)                               00002850
   23 FORMAT (1H,47X, 2(E15.8,6X),21X,E15.8)                             00002860
      GO TO 575                                                          00002870
C                                                                        00002880
    4 WRITE (6,24) AA(N,L),BB(N,L),CC(N,L)                               00002890
   24 FORMAT (1H ,5X, 3(E15.8,6X)  )                                     00002900
      GO TO 575                                                          00002910
C                                                                        00002920
    5 WRITE (6,25) AA(N,L)                                               00002930
   25 FORMAT (1H ,5X,E15.8)                                              00002940
C                                                                        00002950
  575 LINE=LINE+1                                                        00002960
      LCTR=LCTR+1                                                        00002970
      IF(LCTR-5)33,32,32                                                 00002980
   32 WRITE (6,4097)                                                     00002990
      LINE=LINE+1                                                        00003000
   33 N=N+1                                                              00003010
      IF(N-NSTA)34,34,57                                                 00003020
   34 IF(LINE-(80-LSKIP))56,30,30                                        00003030
   57 RETURN                                                             00003040
 4097 FORMAT (1H )                                                       00003050
 4098 FORMAT (1H1)                                                       00003060
C                                                                        00003070
C     73 LEAVES MIN 5 STA. AT BOTTOM. THE HEADERS TAKE 3 LINES.          00003080
C     ASSUME 80 PRINT LINES AVAILABLE. LINE=LAST LINE USED.              00003090
      END                                                                00003100
```

4.   TRI X   14:46:12

END CUR

```
<***Δ****1****Δ****2****Δ****3****Δ****4****Δ****5****Δ****6****Δ****7****Δ****8****Δ****9****Δ****0****Δ****1****Δ****2****Δ****3*
*****************************************ISD-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972******************************************
***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ
[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ

25 APR 72 P 14:46:13    IDENT=FYEE    ACCOUNT=428999  CARDS IN=    9, OUT=    0

  PAGES=   16, LINES=   475, TIME=00:00:04 (HMS)
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* *

*** USER NOTICES - APRIL 20, 1972 ***

(1) ISD 1108 TERMINAL SERVICE IS SCHEDULED AS FOLLOWS

MON :	07:00 - 24:00
TUE - FRI :	00:00 - 04:00 ; 07:00 - 24:00
SAT :	00:00 - 22:00
SUN :	04:00 - 22:00

(2) LARGE-CORE (LCR) PRODUCTION JOBS ARE NOW BEING RUN ON AN OVERNIGHT BASIS STARTING AT 04:00 EACH DAY.

(3) ISD NOW HAS AVAILABLE REMOTE-BATCH JOB ENTRY VIA LOW-SPEED TELETYPE COMPATIBLE TERMINALS USING DIAL-UP COMMUNICATION LINES. THIS SERVICE HAS BEEN IN USE FOR OVER TWO MONTHS AND IS CALLED RON/I. THE DIAL-UP TELEPHONE NUMBERS AND TRANSMISSION RATES ARE LISTED BELOW.

10 CHAR/SEC   415-562-4035, 415-562-4036, 415-562-5186
30 CHAR/SEC   415-562-4716 ** EFFECTIVE 4/24/72 THIS NUMBER WILL BE CHANGED TO 415-562-4294 **

(4) ISD'S SECOND PUBLIC TERMINAL IN SAN FRANCISCO IS LOCATED AT # 1 CALIFORNIA ST., ROOM 2555.

(5) BEGINNING 4/24/72 AND AFFECTIVE MONDAY - FRIDAY TURNAROUND TIME SHOULD BE REDUCED BETWEEN THE HOURS OF 10:30 - 11:30 AND 14:00 - 16:00 FOR USERS SUBMITTING NON-TAPE JOBS WITH RUN TIMES ESTIMATED AT LESS THAN 6 MINUTES.

ADDITIONAL INFORMATION ON (2) & (3) IS NOW AVAILABLE TO ALL INTERESTED USERS BY CONTACTING YOUR SALESMAN AT 415-562-4204.

* *

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<***Δ****1****Δ****2****Δ****3****Δ****4****Δ****5****Δ****6****Δ****7****Δ****8****Δ****9****Δ****0****Δ****1****Δ****2****Δ****3*
*****************************************ISD-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972*****************************************
***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ
[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ

25 APR 72 P 14:46:13   IDENT=FYEE    ACCOUNT=428999  CARDS IN=     9, OUT=     0

  PAGES=   16, LINES=   475. TIME=00:00:04 (HMS)
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\\EOF\

APPENDIX C

PROGRAM E13102 LATERAL (FREE) VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED, UNDAMPED, LUMPED PARAMETER BEAMS, USERS' MANUAL AND SAMPLE OF INPUT/OUTPUT

i

LATERAL VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED, UNDAMPED, LUMPED PARAMETER BEAMS

Program E13102

(Formerly Program 14034)

Aerojet-General Corporation
Computing Sciences
Sacramento, California

ii

Converted to OS/360

by

J. A. Budzenski

25 April 1968

APPROVED

R. D. Glauz

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Engineering Analysis
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Aerojet-General Corporation
Computing Sciences
Sacramento, California

LATERAL VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED, UNDAMPED, LUMPED PARAMETER BEAMS

Program E13102

(Formerly Program 14034)

Converted to OS/360 by
J. A. Budzenski
25 April 1968

INTRODUCTION

This program, originally 14034, a FORTRAN IV program, has been converted to 360 FORTRAN Level H without changes in logic.

This program provides the capacity to analyze the free undamped lateral vibrations of two elastically coupled, lumped parameter beams. Natural frequencies, mode shapes, and associated shear and moment distributions can be computed. Shear deflections, rotary inertia, and gyroscopic effects (for rotating shaft analyses) are included in the program capability. Each beam has four state variables (i.e., shear, moment, slope, and deflection) of which two at each end for each beam must equal zero.

This program is essentially an extension of IBM Job 14009 which is a free lateral vibration analysis of a single, undamped lumped parameter beam. For a more extensive discussion of the capabilities of lumped parameter beam models, consult the user's manual for that job. The principle application of these programs has been to analyze rotor-stator models to determine critical speeds of turbomachinery.

For any additional information concerning the analysis of this program, contact Laverne K. Severud, Dept. 3252, Bldg. 2019.

RESTRICTIONS

1. The maximum number of bays is 50.
2. The two specified boundary conditions at each end for each beam must equal zero.
3. To analyze a single beam, input a fictitious cantilever for the other beam, with zero lengths and weights.

4. Always input finite values of G and EI.

5. To input data in exponential form, put E+xx or E-xx flush right in the field of 12, where xx is the two digit exponent and a plus sign will be understood.

6. When either fixed point variables N or NSTA are only one digit, it must be right-adjusted in the field of two.

NOMENCLATURE

L - Length of elasticity element (in)

E - Modulus of Elasticity (psi)

I - Area Moment of Inertia of Cross Section (in^4)

C - Shape Constant for Shear Deflection (in^{-2})

G - Modulus of Rigidity (psi)

W - Weight of Lumped Mass (#)

I_J - Polar Mass Moment of Inertia (# in sec^2)

I_x - Diametral Mass Moment of Inertia (# in sec^2)

K - Spring Constant (#/in)

ω - Natural Frequency (cps)

$\Delta\omega$ - Increment in Frequency (cps)

c - Damping Coefficient (# sec/in)

DX - Offset between corresponding stations in two beams (in)

γ - Forcing Shear Coefficient of w^2 (# sec^2)

η - Forcing Shear Constant (#)

β - Forcing Moment Coefficient of w^2 (# in sec^2)

V - Shear (#)

M - Moment (in #)

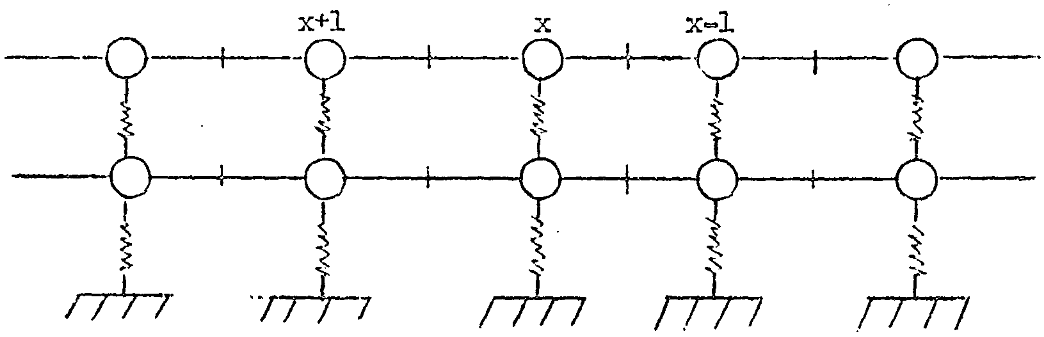
ϕ - Slope (rad)

Y - Deflection (in)

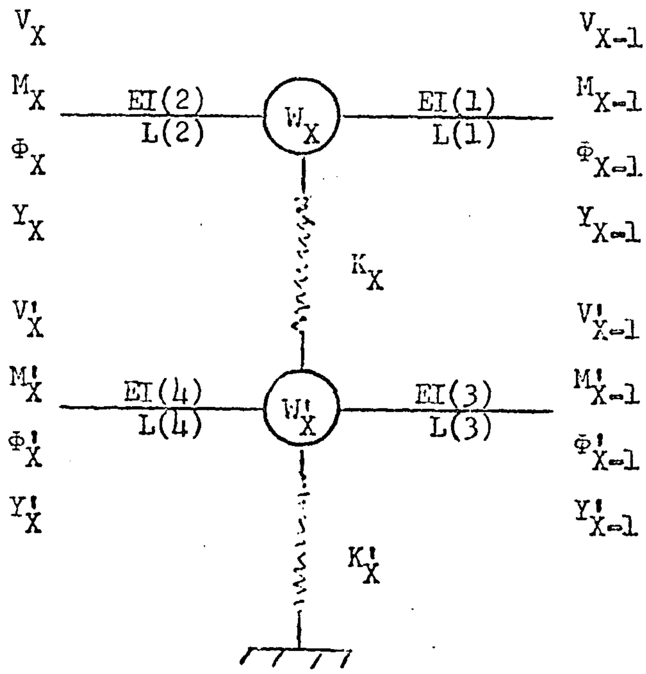
A, B, C, D, M, N, O, P (See p. 9)

Note: All unprimed quantities refer to top beam and springs between the beams. All primed quantities refer to the bottom beam and springs between it and ground.

I. LUMPED PARAMETER MODEL



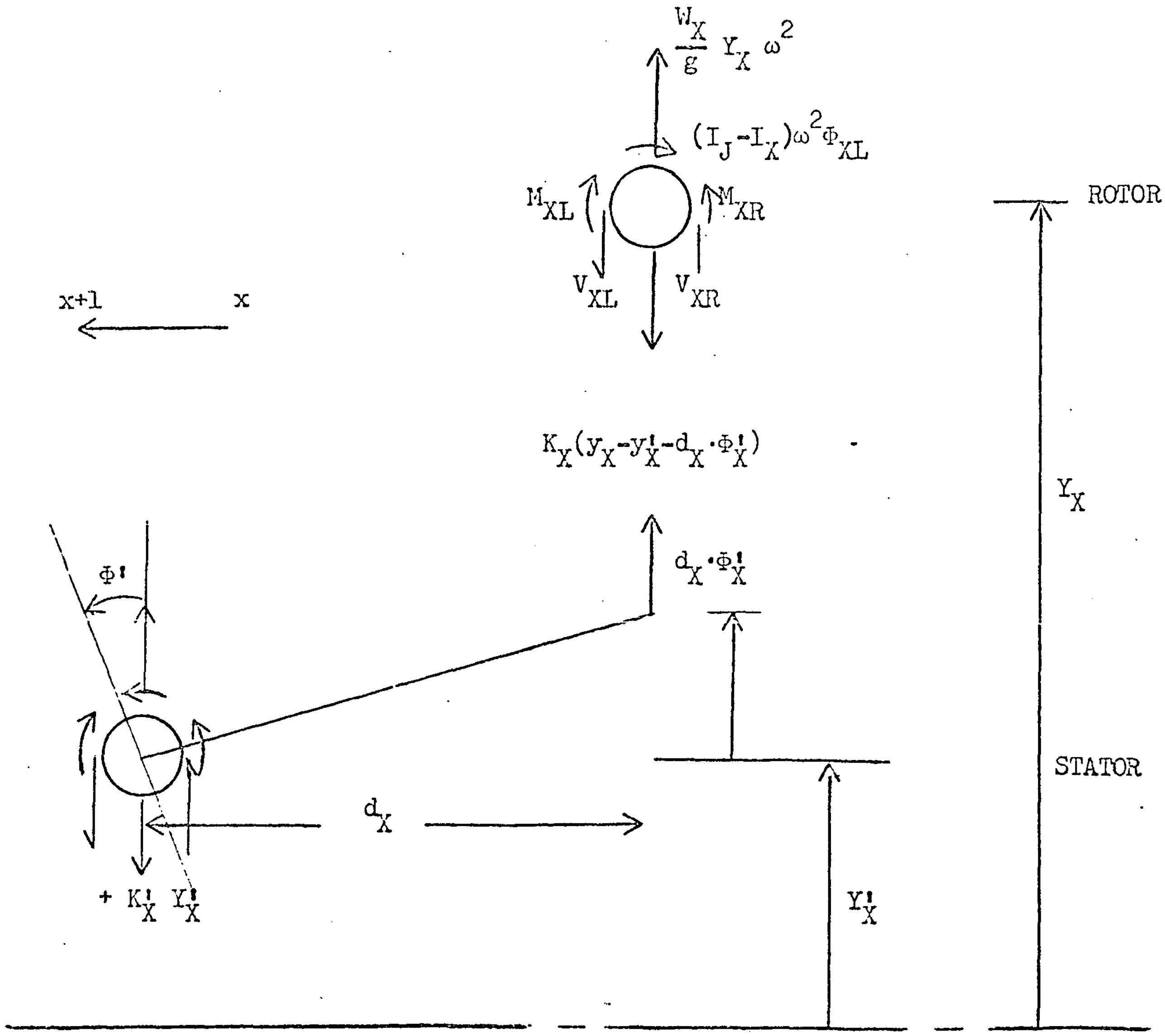
TYPICAL ELEMENT CALLED BAY X



II. DEFINITION

$$\{\Delta\} = \begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ V' \\ M' \\ \Phi' \\ Y' \end{Bmatrix} = \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \end{Bmatrix}$$

III. TRANSFORMATION ACROSS IDEALIZED MASS



1) DX is positive when direction of offset from rotor to stator is in direction of increasing station numbers.

2) Since DX is infinitely stiff, K_X must be the equivalent stiffness of the actual offset arm and the bearing in series.

$$V_{XL} = V_{XR} + \frac{W_X}{g} Y_X \omega^2 - K_X(Y_X - Y'_X - d_X \Phi'_X)$$

$$M_{XL} = M_{XR} - (I_J - I_X)\,\omega^2\,\Phi_X$$

$$\Phi_{XL} = \Phi_{XR} \quad ; \quad Y_{XL} = Y_{XR}$$

$$V'_{XL} = V'_{XR} + \frac{W'_X}{g} Y'_X \,\omega^2 + K_X(Y_X - Y'_X - d_X \Phi'_X) - K''_X Y'_X$$

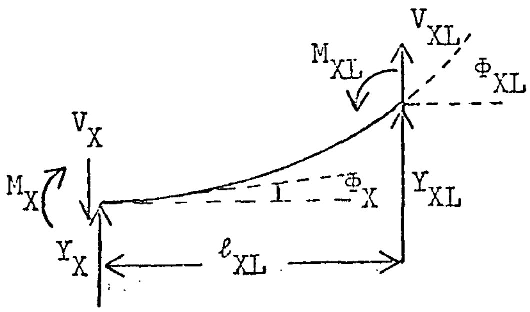
$$M'_{XL} = M'_{XR} + I'_X\,\omega^2\,\Phi'_X + d_X K_X(Y_X - Y'_X - d_X \Phi'_X)$$

$$\Phi'_{XL} = \Phi'_{XR} \quad ; \quad Y'_{XL} = Y'_{XR}$$

$$[F_N] = \left[\begin{array}{cccc:cccc}
1 & 0 & 0 & \frac{W_X}{g}\omega^2 - K_X & 0 & 0 & d_X K_X & K_X \\
0 & 1 & -(I_J - I_X)\omega^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & K_X & 1 & 0 & -d_X K_X & \frac{W'_X}{g}\omega^2 - K_X - K'_X \\
0 & 0 & 0 & d_X K_X & 0 & 1 & I'_X\omega^2 - d_X^2 K_X & -d_X K_X \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]$$

Stator may not have any I'_J; if it needs to be included, let

$$I'_X = I'_X - I'_J$$



$$\Phi_X = \Phi_{XL} - \frac{V_{XL}\,\ell_{XL}^2}{2(EI)_{XL}} - \frac{M_{XL}\,\ell_{XL}}{(EI)_{XL}}$$

$$Y_X = Y_{XL} - \Phi_{XL}\ell_{XL} + \frac{V_{XL}\,\ell_{XL}^3}{6(EI)_{XL}} + \frac{M_{XL}\,\ell_{XL}^2}{2(EI)_{XL}} - \frac{V_{XL}C\,\ell_{XL}}{G}$$

$$V_X = V_{XL} \quad ; \quad M_X = M_{XL} + V_{XL}\,\ell_{XL}$$

Similarly for ℓ'_{XL}

$$\therefore \quad \{\Delta\}_X = [E]_{XL}\{\Delta\}_{XL}$$

$$[E]_{XL} = \left[\begin{array}{cccc|cccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\ell & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\ell^2}{2EI} & -\frac{\ell}{EI} & 1 & 0 & 0 & 0 & 0 & 0 \\
\left(\frac{\ell^3}{6EI} - \frac{C\ell}{G}\right) & \frac{\ell^2}{2EI} & -\ell & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \ell' & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -\left(\frac{\ell^2}{2EI}\right)' & -\left(\frac{\ell}{EI}\right)' & 1 & 0 \\
0 & 0 & 0 & 0 & \left(\frac{\ell^3}{6EI} - \frac{C\ell}{G}\right)' & \left(\frac{\ell^2}{2EI}\right)' & -\ell' & 1
\end{array}\right]_{XL}$$

In like manner

$$\{\Delta\}_{XR} = [E]_{XR}\{\Delta\}_{X-1}$$

V. SOLUTION PROCEDURE

Thus,

$$\{\Delta\}_X = [E]_{XL}\,\{\Delta\}_{XL} = [E]_{XL}\,[F]_X\,\{\Delta\}_{XR}$$

$$= [E]_{XL}\,[F]_X\,[E]_{XR}\,\{\Delta\}_{X-1}$$

$$\{\Delta\}_X = [C]_X\,\{\Delta\}_{X-1}$$

Continuing from span to span

$$\{\Delta\}_X = [C]_X\,[C]_{X-1}\,\{\Delta\}_{X-2} \quad \text{etc.}$$

and

$$\{\Delta\}_N = \prod_{i=1}^{N} [C]_i\,\{\Delta\}_O$$

$$\{\Delta\}_N = [D]\,\{\Delta\}_O$$

For generalized zero boundary conditions

At N $\quad \delta_a, \delta_b, \delta_c, \delta_d \quad$ are zero

At O $\quad \delta_m, \delta_n, \delta_o, \delta_p \quad$ are non-zero

$$\therefore \quad \begin{Bmatrix} \delta_a \\ \delta_b \\ \delta_c \\ \delta_d \end{Bmatrix} = \begin{bmatrix} d_{am} & d_{an} & d_{ao} & d_{ap} \\ d_{bm} & d_{bn} & d_{bo} & d_{bp} \\ d_{cm} & d_{cn} & d_{co} & d_{cp} \\ d_{dm} & d_{dn} & d_{do} & d_{dp} \end{bmatrix} \begin{Bmatrix} \delta_m \\ \delta_n \\ \delta_o \\ \delta_p \end{Bmatrix} = 0$$

For non-trivial solution, determinant must equal zero.

Iterate with trial values of ω until roots are found.

VI. NORMALIZATION OF MODE SHAPE

When ω is determined, set $\delta_i = 1$ ($i = m, n, o$ or p)

Substitute into first 3 equations and solve for the remaining 3 δ's at point O.

$$\therefore \quad \{\Delta\}_o \text{ is known}$$

Calculate remaining $\{\Delta\}_X$ from relation

$$\{\Delta\}_X = \prod_{i=1}^{X} [C]_i \{\Delta\}_o$$

One check of computational sensitivity is to see if the zero variables at end N do in fact calculate to zero.

VII. SPECIAL INPUT PARAMETERS

$a, b, c, d \Rightarrow \delta_a, \delta_b, \delta_c, \delta_d$ (Zero at end N)

$m, n, o, p \Rightarrow \delta_m, \delta_n, \delta_o, \delta_p$ (Non-zero at end O)

$i \Rightarrow \delta_i$ (i either m,n,o,p - normalizing parameter)

CUSTOMER INSTRUCTIONS	KEYPUNCH INSTRUCTIONS
1. ENTER DATA LEGIBLY WITHIN SPACES PROVIDED	X PUNCH 1 CARD PER HAND POSTED LINE ITEM
2. DISTINGUISH BETWEEN 1 vs I, 0 vs O, 2 vs Z, U vs V, 7 vs S	PUNCH ALL LINES WHETHER POSTED OR NOT. IF NECESSARY PROVIDE BLANK CARDS
3. A right-adjusted integer in col. 73-75 will repeat a card of station data and store the data in locations corresponding to consecutive stations.	PUNCH ALL LINES THAT ARE HAND POSTED
	PAS INCLUDING SPACES
	ALL SPACES MAY BE IGNORED
	ALL SPACES MAY BE IGNORED EXCEPT ON T CARD
	ALL SPACES MAY BE IGNORED EXCEPT (Specify cols.)
	ALL SIGNS AND KP LINES MUST BE PUNCHED
	DO NOT PUNCH PRE-PRINTED SIGNS SHOWN AFTER LAST HANDWRITTEN VALUE ENTRY
	X USE 360 SYMBOLS

CUSTOMER | DATE

JOB NO. E13102 | PROGRAMMER

TITLE Lateral Vibration Analysis of Two Elastically Coupled Undamped, Lumped Parameter Beams

FORM APPROVED (KEY PUNCH) | DATE

N	OMEGA(CPS)	DELTA OMEGA(CPS)	K K	A	B	C	D	M	N	O	P

Col. 1–12	Col. 13–24	Col. 25–36	Col. 37–48	Col. 73
L(1)	L(2)	L(3)	L(4)	OPTION
EI(1)	EI(2)	EI(3)	EI(4)	OPTION
G(1)	G(2)	G(3)	G(4)	OPTION
C(1)	C(2)	C(3)	C(4)	OPTION
I_{J1}	I_{J2}	I_{X1}	I_{X2}	OPTION
W_{N1}	W_{N2}	K_{N1}	K_{N2}	OPTION

Lateral Vibration Analysis of Two Elastically Coupled, Undamped, Lumped Parameter Beams

Program E13102 Input Sheet Page 1 of 1

AGC3 0143-9 (REV 10/66)

PLEASE PRINT CLEARLY - USE BLACK PENCIL

8080 LISTING   COUNT

```
GEMINI GEARBOX FORCED VIBRATION  RUN 1                              21
 2   100.      100.        1  1  2  5  6  3  4  5  6  1  8
.30         .30                                                    L1
3.70     E&063.70    E&061.0        1.0                           EI1
11.0     E&0611.0    E&061.0        1.0                            G1
3.73        3.73                                                   C1
.000092                                                            I1
.211                                                               W1
0.0         0.0                                                    L2
1.0         1.0         1.0         1.0                           EI2
1.0         1.0         1.0         1.0                            G2
0.0         0.0                               1000.                C2
0.0                                                                I2
0.0                     2.0      E6           36.5    E&04.250     W2
.25         .25                                                    L3
7.05     E&067.05    E&061.0        1.0                           EI3
11.0     E&0611.0    E&061.0        1.0                            G3
2.16        2.16                                                   C3
.000104                                                            I3
.29                                                                W3
.30         .30                                                    L4
81.8     E&0681.8    E&061.0        1.0                           EI4
11.0     E&0611.0    E&061.0        1.0                            G4
.056        .056                                                   C4
.00128                                                             I4
1.07                                                               W4
.35         .35                                                    L5
81.8     E&0681.8    E&061.0        1.0                           EI5
11.0     E&0611.0    E&061.0        1.0                            G5
.056        .056                                                   C5
.00146                                                             I5
1.25                                                               W5
.50         .50                                                    L6
81.8     E&0681.8    E&061.0        1.0                           EI6
11.0     E&0611.0    E&061.0        1.0                            G6
.071        .071                                                   C6
.00191                                                             I6
1.71                                                               W6
.275        .275                                                   L7
12.7     E&0612.7    E&061.0        1.0                           EI7
11.0     E&0611.0    E&061.0        1.0                            G7
.208        .208                                                   C7
.000436                                                            I7
.56                                                                W7
0.0         0.0                                                    L8
1.0         1.0         1.0         1.0                           EI8
1.0         1.0         1.0         1.0                            G8
0.0         0.0                               1000.                C8
0.0                                                                I8
0.0                     .7       E6           4.60    E&04.413     W8
.275        .275                                                   L9
12.7     E&0612.7    E&061.0        1.0                           EI9
```

Elastically Coupled, Undamped, Lumped Parameter Beams

Test Case Input
Page 1 of 3

8080 LISTING						COUNT
11.0	E&0611.0	E&061.0	1.0			G9
.208	.208					C9
.000436						I9
.56						W9
.40	.40					L10
9.35	E&069.35	E&061.0	1.0			&I10
11.0	E&0611.0	E&061.0	1.0			G10
.248	.248					C10
.000465						I10
.738						W10
.45	.45					L11
9.35	E&069.35	E&061.0	1.0			EI11
11.0	E&0611.0	E&061.0	1.0			G11
.248	.248					C11
.000495						I11
.83						W11
.35	.35					L12
12.7	E&0612.7	E&061.0	1.0			EI12
11.0	E&0611.0	E&061.0	1.0			G12
.208	.208					C12
.00053						I12
.715						W12
0.0	0.0					L13
1.0	1.0	1.0	1.0			EI13
1.0	1.0	1.0	1.0			G13
0.0	0.0			1000.		C13
0.0	0.0					I13
0.0		.8	E6	4.60	E&04.413	W13
.30	.30					L14
12.7	E&0612.7	E&061.0	1.0			EI14
11.0	E&0611.0	E&061.0	1.0			G14
.208	.208					C14
.000382						I14
.552						W14
.15	.15					L15
6.78	E&066.78	E&061.0	1.0			EI15
11.0	E&0611.0	E&061.0	1.0			G15
.303	.303					C15
.000679						I15
.51						W15
.35	.35					L16
22.0	E&0622.0	E&061.0	1.0			EI16
11.0	E&0611.0	E&061.0	1.0			G16
.106	.106					C16
.000465						I16
.638						W16
.15	.15					L17
22.0	E&0622.0	E&061.0	1.0			EI17
11.0	E&0611.0	E&061.0	1.0			G17
.106	.106					C17
.000155						I17
.238						W17

8080 LISTING				COUNT
.20	.20			L18
111.1	E&06111.1	E&061.0	1.0	EI18
11.0	E&0611.0	E&061.0	1.0	G18
.047	.047			C18
.000188				I18
.133				W18
.475	.475			L19
231.	E&06231.	E&061.0	1.0	EI19
11.0	E&0611.0	E&061.0	1.0	G19
.131	.131			C19
.1029		.78	E-04.0702	I19
8.30				W19
.425	.425			L20
93.5	E&0693.5	E&061.0	1.0	EI20
11.0	E&0611.0	E&061.0	1.0	G20
.456	.456			C20
.0019				I20
.969				W20
.30	.30			L21
231.	E&06231.	E&061.0	1.0	EI21
11.0	E&0611.0	E&061.0	1.0	G21
.131	.131			C21
.0992		.780	E-04	I21
7.73				W21

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JOB E13102 VIBRATION ANALYSIS

GEMINI GEARBOX FORCED VIBRATION RUN 1 NUMBER OF STATIONS 21

NUMBER OF ROOTS 2 TRIAL OMEGA 100.000 DELTA OMEGA 100.000 1 2 5 6 3 4 5 6

L(1)	L(2)	L(3)	L(4)
0.30000000D 00	0.30000000D 00	0.0	0.0
0.0	0.0	0.0	0.0
0.25000000D 00	0.25000000D 00	0.0	0.0
0.30000000D 00	0.30000000D 00	0.0	0.0
0.35000000D 00	0.35000000D 00	0.0	0.0
0.50000000D 00	0.50000000D 00	0.0	0.0
0.27500000D 00	0.27500000D 00	0.0	0.0
0.0	0.0	0.0	0.0
0.27500000D 00	0.27500000D 00	0.0	0.0
0.40000000D 00	0.40000000D 00	0.0	0.0
0.45000000D 00	0.45000000D 00	0.0	0.0
0.35000000D 00	0.35000000D 00	0.0	0.0
0.0	0.0	0.0	0.0
0.30000000D 00	0.30000000D 00	0.0	0.0
0.15000000D 00	0.15000000D 00	0.0	0.0
0.35000000D 00	0.35000000D 00	0.0	0.0
0.15000000D 00	0.15000000D 00	0.0	0.0
0.20000000D 00	0.20000000D 00	0.0	0.0
0.47500000D 00	0.47500000D 00	0.0	0.0
0.42500000D 00	0.42500000D 00	0.0	0.0
0.30000000D 00	0.30000000D 00	0.0	0.0

EI(1)	EI(2)	EI(3)	EI(4)
0.37000000D 07	0.37000000D 07	0.10000000D 01	0.10000000D 01
0.10000000D 01	0.10000000D 01	0.10000000D 01	0.10000000D 01
0.70500000D 07	0.70500000D 07	0.10000000D 01	0.10000000D 01
0.81800000D 08	0.81800000D 08	0.10000000D 01	0.10000000D 01
0.81800000D 08	0.81800000D 08	0.10000000D 01	0.10000000D 01
0.81800000D 08	0.81800000D 08	0.10000000D 01	0.10000000D 01
0.12700000D 08	0.12700000D 08	0.10000000D 01	0.10000000D 01
0.10000000D 01	0.10000000D 01	0.10000000D 01	0.10000000D 01
0.12700000D 08	0.12700000D 08	0.10000000D 01	0.10000000D 01
0.93500000D 07	0.93500000D 07	0.10000000D 01	0.10000000D 01
0.93500000D 07	0.93500000D 07	0.10000000D 01	0.10000000D 01
0.12700000D 08	0.12700000D 08	0.10000000D 01	0.10000000D 01
0.10000000D 01	0.10000000D 01	0.10000000D 01	0.10000000D 01
0.12700000D 08	0.12700000D 08	0.10000000D 01	0.10000000D 01
0.67800000D 07	0.67800000D 07	0.10000000D 01	0.10000000D 01
0.22000000D 08	0.22000000D 08	0.10000000D 01	0.10000000D 01
0.22000000D 08	0.22000000D 08	0.10000000D 01	0.10000000D 01
0.11110000D 09	0.11110000D 09	0.10000000D 01	0.10000000D 01
0.23100000D 09	0.23100000D 09	0.10000000D 01	0.10000000D 01
0.93500000D 08	0.93500000D 08	0.10000000D 01	0.10000000D 01
0.23100000D 09	0.23100000D 09	0.10000000D 01	0.10000000D 01

G(1)	G(2)	G(3)	G(4)
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.10000000D 01	0.10000000D 01	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.10000000D 01	0.10000000D 01	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.10000000D 01	0.10000000D 01	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01
0.11000000D 08	0.11000000D 08	0.10000000D 01	0.10000000D 01

C(1)	C(2)	C(3)	C(4)
0.37300000D 01	0.37300000D 01	0.0	0.0
0.0	0.0	0.0	0.0
0.21600000D 01	0.21600000D 01	0.0	0.0
0.56000000D-01	0.56000000D-01	0.0	0.0
0.56000000D-01	0.56000000D-01	0.0	0.0
0.71000000D-01	0.71000000D-01	0.0	0.0
0.20800000D 00	0.20800000D 00	0.0	0.0
0.0	0.0	0.0	0.0
0.20800000D 00	0.20800000D 00	0.0	0.0
0.24800000D 00	0.24800000D 00	0.0	0.0
0.24800000D 00	0.24800000D 00	0.0	0.0
0.20800000D 00	0.20800000D 00	0.0	0.0
0.0	0.0	0.0	0.0
0.20800000D 00	0.20800000D 00	0.0	0.0
0.30300000D 00	0.30300000D 00	0.0	0.0
0.10600000D 00	0.10600000D 00	0.0	0.0
0.10600000D 00	0.10600000D 00	0.0	0.0
0.47000000D-01	0.47000000D-01	0.0	0.0
0.13100000D 00	0.13100000D 00	0.0	0.0
0.45600000D 00	0.45600000D 00	0.0	0.0
0.13100000D 00	0.13100000D 00	0.0	0.0

I SUB J1	DX	I SUB X1	I SUB X2
0.92000000D-04	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.10400000D-03	0.0	0.0	0.0
0.12800000D-02	0.0	0.0	0.0
0.14600000D-02	0.0	0.0	0.0
0.19100000D-02	0.0	0.0	0.0
0.43600000D-03	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.43600000D-03	0.0	0.0	0.0
0.46500000D-03	0.0	0.0	0.0
0.49500000D-03	0.0	0.0	0.0
0.53000000D-03	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.38200000D-03	0.0	0.0	0.0
0.67900000D-03	0.0	0.0	0.0
0.46500000D-03	0.0	0.0	0.0
0.15500000D-03	0.0	0.0	0.0
0.18800000D-03	0.0	0.0	0.0
0.10290000D 00	0.0	0.78000000D-04	0.70200000D-01
0.19000000D-02	0.0	0.0	0.0
0.99200000D-01	0.0	0.78000000D-04	0.0

W SUB N1	W SUB N2	K SUB N1	K SUB N2
0.21100000D 00	0.0	0.0	0.0
0.0	0.0	0.20000000D 07	0.0
0.29000000D 00	0.0	0.0	0.0
0.10700000D 01	0.0	0.0	0.0
0.12500000D 01	0.0	0.0	0.0
0.17100000D 01	0.0	0.0	0.0
0.56000000D 00	0.0	0.0	0.0
0.0	0.0	0.70000000D 06	0.0
0.56000000D 00	0.0	0.0	0.0
0.73800000D 00	0.0	0.0	0.0
0.83000000D 00	0.0	0.0	0.0
0.71500000D 00	0.0	0.0	0.0
0.0	0.0	0.80000000D 06	0.0
0.55200000D 00	0.0	0.0	0.0
0.51000000D 00	0.0	0.0	0.0
0.63800000D 00	0.0	0.0	0.0
0.23800000D 00	0.0	0.0	0.0
0.13300000D 00	0.0	0.0	0.0
0.83000000D 01	0.0	0.0	0.0
0.96900000D 00	0.0	0.0	0.0
0.77300000D 01	0.0	0.0	0.0

```
OMEGA =   0.10000000D 03      DETERM = -0.91066693D 14
OMEGA =   0.20000000D 03      DETERM = -0.60910163D 14
OMEGA =   0.30000000D 03      DETERM = -0.99336849D 13
OMEGA =   0.40000000D 03      DETERM =  0.62775871D 14
OMEGA =   0.31366214D 03      DETERM = -0.13039628D 13
OMEGA =   0.31541904D 03      DETERM = -0.16465589D 12
OMEGA =   0.31567295D 03      DETERM =  0.55851808D 09
OMEGA =   0.31567209D 03      DETERM = -0.23768456D 06
OMEGA =   0.31567209D 03      DETERM = -0.68750000D 00
OMEGA =   0.31567209D 03      DETERM = -0.50000000D 00
OMEGA =   0.31567209D 03      DETERM =  0.20000000D 01
OMEGA =   0.31567209D 03      DETERM =  0.12500000D 00

                           OMEGA =  0.31567209D 03
```

V	M	PHI	Y
0.0	0.0	0.10000000D 01	0.24524882D 01
0.46283165D 04	0.10265686D 04	0.99997306D 00	0.18520186D 01
-0.36994090D 07	0.10265698D 04	0.99997306D 00	0.18520186D 01
-0.36941419D 07	-0.18477770D 07	0.10654840D 01	0.17040745D 01
-0.36789979D 07	-0.40651285D 07	0.10871778D 01	0.10703509D 01
-0.36746671D 07	-0.10324976D 08	0.12367182D 01	-0.84973203D 00
-0.36815293D 07	-0.12350454D 08	0.17277012D 01	-0.16226723D 01
-0.25456587D 07	-0.12350454D 08	0.17277012D 01	-0.16226723D 01
-0.25577685D 07	-0.13757331D 08	0.22929912D 01	-0.26990289D 01
-0.25856986D 07	-0.15820030D 08	0.35580939D 01	-0.49812972D 01
-0.26426369D 07	-0.18181246D 08	0.51939023D 01	-0.88495749D 01
-0.27208851D 07	-0.20070380D 08	0.62477036D 01	-0.12812547D 02
0.75291526D 07	-0.20070380D 08	0.62477036D 01	-0.12812547D 02
0.74459106D 07	-0.15587922D 08	0.70897308D 01	-0.16909303D 02
0.73522113D 07	-0.13388030D 08	0.77306362D 01	-0.19195924D 02
0.72094275D 07	-0.83059502D 07	0.80753702D 01	-0.24786558D 02
0.71463381D 07	-0.61575418D 07	0.81739526D 01	-0.27245439D 02
0.71071856D 07	-0.33128897D 07	0.81909941D 01	-0.30530949D 02
0.41922412D 07	-0.12627822D 07	0.81989793D 01	-0.38380233D 02
0.37781035D 07	0.20633179D 07	0.81949405D 01	-0.45490194D 02
-0.40046871D-07	0.14435500D-06	0.81915249D 01	-0.50419156D 02

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.76818710D 07	0.0	0.0	0.0
0.76818710D 07	0.0	0.0	0.0
0.11385908D 08	0.0	0.0	0.0
0.11385908D 08	0.0	0.0	0.0
0.11385908D 08	0.0	0.0	0.0
0.11385908D 08	0.0	0.0	0.0
0.11385908D 08	0.0	0.0	0.0
0.11385908D 08	0.0	0.0	0.0
0.10250038D 08	0.0	0.0	0.0
0.10250038D 08	0.0	0.0	0.0
0.10250038D 08	0.0	0.0	0.0
0.10250038D 08	0.0	0.0	0.0
0.10250038D 08	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0
0.47264621D-07	0.0	0.0	0.0

C2

```
OMEGA =  0.41567209D 03        DETERM =  0.76205397D 14
OMEGA =  0.51567209D 03        DETERM =  0.17524387D 15
OMEGA =  0.61567209D 03        DETERM =  0.29775437D 15
OMEGA =  0.71567209D 03        DETERM =  0.44368832D 15
OMEGA =  0.81567209D 03        DETERM =  0.61187551D 15
OMEGA =  0.91567209D 03        DETERM =  0.79955366D 15
OMEGA =  0.10156721D 04        DETERM =  0.10018808D 16
OMEGA =  0.11156721D 04        DETERM =  0.12114526D 16
OMEGA =  0.12156721D 04        DETERM =  0.14178492D 16
OMEGA =  0.13156721D 04        DETERM =  0.16072384D 16
OMEGA =  0.14156721D 04        DETERM =  0.17620635D 16
OMEGA =  0.15156721D 04        DETERM =  0.18608473D 16
OMEGA =  0.16156721D 04        DETERM =  0.18781427D 16
OMEGA =  0.17156721D 04        DETERM =  0.17846626D 16
OMEGA =  0.18156721D 04        DETERM =  0.15476222D 16
OMEGA =  0.19156721D 04        DETERM =  0.11313235D 16
OMEGA =  0.20156721D 04        DETERM =  0.49801276D 15
OMEGA =  0.21156721D 04        DETERM = -0.39096155D 15
OMEGA =  0.20716931D 04        DETERM =  0.33966739D 14
OMEGA =  0.20752086D 04        DETERM =  0.20150642D 13
OMEGA =  0.20754303D 04        DETERM = -0.11526043D 11
OMEGA =  0.20754291D 04        DETERM =  0.38733650D 07
OMEGA =  0.20754291D 04        DETERM =  0.37000000D 02
OMEGA =  0.20754291D 04        DETERM =  0.67000000D 02
OMEGA =  0.20754291D 04        DETERM = -0.41000000D 02
OMEGA =  0.20754291D 04        DETERM =  0.30000000D 02
OMEGA =  0.20754291D 04        DETERM = -0.22000000D 02

                   OMÉGA =  0.20754291D 04
```

V	M	PHI	Y
0.0	0.0	0.10000000D 01	-0.72759217D 01
-0.70423640D 06	-0.22691548D 06	0.10098335D 01	-0.78063284D 01
0.14908420D 08	-0.22691548D 06	0.10098335D 01	-0.78063284D 01
0.13786040D 08	0.69298670D 07	0.76716511D 00	-0.96803392D 01
0.91071394D 07	0.13638020D 08	0.68915879D 00	-0.10154631D 02
0.33800930D 07	0.17853633D 08	0.55012628D 00	-0.10612697D 02
-0.48081336D 07	0.16998057D 08	0.32458382D 00	-0.11044500D 02
-0.75362644D 07	0.13605510D 08	-0.34621446D 00	-0.10967628D 02
0.14107558D 06	0.13605510D 08	-0.34621446D 00	-0.10967628D 02
-0.25310951D 07	0.12995797D 08	-0.93018304D 00	-0.10603028D 02
-0.58144159D 07	0.97733950D 07	-0.19323587D 01	-0.93646994D 01
-0.88635374D 07	0.33652653D 07	-0.25977192D 01	-0.71316988D 01
-0.10801331D 08	-0.32788100D 07	-0.26094475D 01	-0.51580662D 01
-0.66748779D 07	-0.32788100D 07	-0.26094475D 01	-0.51580662D 01
-0.77329283D 07	-0.74382125D 07	-0.23600385D 01	-0.35758560D 01
-0.84524674D 07	-0.96140056D 07	-0.19839709D 01	-0.28551120D 01
-0.90599930D 07	-0.15600443D 08	-0.15845234D 01	-0.15360956D 01
-0.91956046D 07	-0.18299944D 08	-0.13534537D 01	-0.10580978D 01
-0.92420607D 07	-0.21945314D 08	-0.12810134D 01	-0.52501357D 00
-0.87875438D 07	-0.89626668D 07	-0.12172360D 01	0.75591069D 00
-0.81821980D 07	-0.15797430D 08	-0.11041054D 01	0.20457365D 01
0.29585790D-05	0.94801653D-05	-0.10819954D 01	0.27275627D 01

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.27416450D 08	0.0	0.0	0.0
0.27416450D 08	0.0	0.0	0.0
0.11803793D 08	0.0	0.0	0.0
0.11803793D 08	0.0	0.0	0.0
0.11803793D 08	0.0	0.0	0.0
0.11803793D 08	0.0	0.0	0.0
0.11803793D 08	0.0	0.0	0.0
0.11803793D 08	0.0	0.0	0.0
0.41264530D 07	0.0	0.0	0.0
0.41264530D 07	0.0	0.0	0.0
0.41264530D 07	0.0	0.0	0.0
0.41264530D 07	0.0	0.0	0.0
0.41264530D 07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0
-0.79628080D-07	0.0	0.0	0.0

END OF CASE

APPENDIX D

PROGRAM E13102 LISTING

i

```
@9 RUN FYEE,428999,2,200     LIST E13102            25 APR 72   14:46:14.697

@ CTL UN=E13102                                     25 APR 72   14:46:14.697

@R     ASG X=AN4150                                 25 APR 72   14:46:14.770
       AN4150 ASSIGNED UNIT  1

@N     HDG                                          25 APR 72   14:46:14.778
```

```
$   XQT CUR                                         25 APR 72   14:46:14.780
  1.      REF X                                                 14:46:15
  2.      IN X                                                  14:46:15
    END OF FILE -- UNIT X
  3.      LIST 1                                                14:46:17
```

```
@ ELT EXPAND,1,710422, 35969

      SUBROUTINE EXPAND
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BLO (250)
      COMMON/ARRAY/BLO/ARRAYZ/BHI
      BHI = 0.0
      RETURN
C***********************************************************************
C     EXPAND SHOULD PRECEDE 1ST SIMST USE.
C     EXPAND SHOULD ONLY BE CALLED FOR THE 7094.
C     TO DIMENSION BLO,ESTIMATE NEEDED STORAGE.
C     PUT DIM BLO(1) AND NAMED COMMON IN EVERY SIMST-USING ROUTINE.
C     EXPAND ONLY NEED BE CHANGED IF AVAILABLE STORAGE CHANGES.
C***********************************************************************
      END
```

```
@ ELT E13102,1,710423, 57304

     TURBOPUMP ROTOR 12-29-70 LIGH.TUR.&BEAR CASE 6 F.C.W. FIXED MIRONENKO 40
     99   10.          10.           1  2  5  6  3  4  5  6  0   7
               .51          .34          .0            .0
              2.0E6        4.0E6        1.0E9         1.0E9
              6.1E6        6.1E6       11.7E6        11.7E6
              4.7          3.3          1.            1.
              +.0                       .97E-8        .0
               .17          .0
               .37          .36          .0            .0
              6.0E6       10.0E6        1.0E9         1.0E9
              6.1E6        6.1E6       11.7E6        11.7E6
              2.6          1.2          1.            1.
              +.0                       .81E-8        .0
               .14          .0
               .46          .32
             20.0E6       26.0E6       17.4E9        17.4E9
              6.1E6        6.1E6       11.7E6        11.7E6
              1.25          .97          .05           .05
            +.0066                      3.6E-8         .225
               .63         6.3
               .68          .62
             37.0E6       66.0E6        4.0E9         3.7E9
              6.1E6        6.1E6       11.7E6        11.7E6
               .82          .61          .16           .14
             +.007                     14.5E-8         .2
              2.54         7.24
               .28          .32
            120.0E6      190.0E6       32.0E9        32.0E9
              6.1E6        6.1E6       11.7E6        11.7E6
               .39          .27          .05           .05
             +.059                      7.0E-8         .89
              1.22        16.78
               .67          .33          .55           .5
            260.0E6      260.0E6       33.0E9        33.0E9
              6.1E6        6.1E6       11.7E6        11.7E6
               .21          .21          .05           .05
             +.094                     22.6E-8        1.01
              3.97        20.91
               .41          .3           .5            .5
            260.0E6      260.0E6       79.1E9        79.1E9
              6.1E6        6.1E6       11.7E6        11.7E6
               .21          .21          .01           .01
             +.036                     38.0E-8        2.2
              6.64        45.73
               .07          .07          .64           .65
             55.0E6       55.0E6       40.1E9        40.1E9
              8.6E6        8.6E6       11.7E6        11.7E6
               .3           .3           .03           .03
              +.0                       1.7E-8        1.7
               .3         34.01
               .02          .02          .36           .26
             47.3E6       47.3E6       37.0E9        37.0E9
             11.7E6       11.7E6       11.7E6        11.7E6
              1.22         1.22          .02           .02
              +.0                        .0E-8         .935
               .04        13.08
```

000057	.17	.17	.38	.22
000058	59.3E6	27.3E6	74.0E9	97.6E9
000059	11.7E6	11.7E6	11.7E6	11.7E6
000060	.96	1.96	.02	.01
000061	+.0		2.5E-8	1.07
000062	.43	22.61		
000063	.0	.0	.0	.0
000064	10.0E6	10.0E6	10.0E9	10.0E9
000065	11.7E6	11.7E6	11.7E6	11.7E6
000066	1.	1.	1.	1.
000067	+.0		.0E-8	.0
000068	.01	.01	1.0E6	
000069	.164	.164	.24	.28
000070	27.3E6	24.7E6	97.6E9	97.6E9
000071	11.7E6	11.7E6	11.7E6	11.7E6
000072	1.96	2.15	.01	.01
000073	+.0		2.1E-8	1.3
000074	.37	29.53		
000075	.02	.02	.24	.278
000076	16.9E6	16.9E6	36.2E9	36.2E9
000077	11.7E6	11.7E6	11.7E6	11.7E6
000078	3.14	3.14	.01	.01
000079	+.0		.0E-8	.82
000080	.04	22.17		
000081	.151	.151	.0	.0
000082	24.7E6	24.7E6	1.0E9	1.0E9
000083	11.7E6	11.7E6	11.7E6	11.7E6
000084	2.15	2.15	1.	1.
000085	+.0		2.1E-8	.0
000086	.37	.0		
000087	.02	.02	.0	.0
000088	16.9E6	16.9E6	1.0E9	1.0E9
000089	11.7E6	11.7E6	11.7E6	11.7E6
000090	3.14	3.14	1.	1.
000091	+.0		.0E-8	.0
000092	.04	.0		
000093	.164	.164	.0	.0
000094	24.7E6	27.3E6	1.0E9	1.0E9
000095	11.7E6	11.7E6	11.7E6	11.7E6
000096	2.15	1.96	1.	1.
000097	+.0	.	2.1E-8	.0
000098	.37	.0		
000099	.0	.0	.0	.0
000100	10.0E6	10.0E6	10.0E9	10.0E9
000101	11.7E6	11.7E6	11.7E6	11.0E6
000102	1.	1.	1.	1.
000103	+.0		.0E-8	.0
000104	.01	.0	1.0E6	
000105	.17	.17	.32	.3
000106	27.3E6	59.3E6	35.0E9	35.0E9
000107	11.7E6	11.7E6	11.7E6	11.7E6
000108	1.96	.96	.02	.02
000109	+.0		2.5E-8	.6
000110	.43	16.75		
000111	.02	.02	.71	.72
000112	47.3E6	47.3E6	2.0E9	2.4E9
000113	11.7E6	11.7E6	11.7E6	11.7E6
000114	1.22	1.22	.1	.1
000115	+.0		.0E-8	.35
000116	.04	27.64		

000117	.84	.64	.38	.4
000118	61.0E6	61.0E6	82.2E9	82.2E9
000119	6.1E6	6.1E6	11.7E6	11.7E6
000120	.44	.44	.02	.02
000121	+.0048		13.6E-8	1.8
000122	2.37	32.49		
000123	.51	.45	.29	.29
000124	102.0E6	102.0E6	13.0E9	13.0E9
000125	6.1E6	6.1E6	11.7E6	11.7E6
000126	.21	.21	.	.
000127	+.0596		32.E-8	.33
000128	5.58	25.66		
000129	.72	.47	.32	.28
000130	222.0E6	222.0E6	73.5E9	73.5E9
000131	6.1E6	6.1E6	11.7E6	11.7E6
000132	1.	1.	1.	1.
000133	+.165		73.5E-8	1.08
000134	12.84	16.51		
000135	1.14	1.13	.28	.24
000136	218.0E6	218.0E6	46.5E9	46.5E9
000137	6.1E6	6.1E6	11.7E6	11.7E6
000138	.12	.12	.03	.03
000139	+.0085		24.2E-8	1.13
000140	4.24	26.82		
000141	.85	1.09	1.2	.93
000142	115.0E6	105.0E6	51.6E9	46.5E9
000143	6.1E6	6.1E6	11.7E6	11.7E6
000144	.2	.2	.02	.03
000145	+.0046		17.9E-8	3.9
000146	3.13	66.72		
000147	.71	.80	.6	.6
000148	105.0E6	51.0E6	10.0E9	2.6E9
000149	6.1E6	6.1E6	11.7E6	11.7E6
000150	.2	.4	.03	.1
000151	+.002		-11.2E-8	1.0
000152	1.97	29.3		
000153	.02	.02	.25	.45
000154	47.3E6	47.3E6	2.2E9	1.9E9
000155	11.7E6	11.7E6	11.7E6	11.7E6
000156	1.22	1.22	.1	.1
000157	+.0		-.0E-8	3.7
000158	.04	37.75		
000159	.17	.17	.77	.78
000160	59.3E6	27.3E6	2.9E9	2.9E9
000161	11.7E6	11.7E6	11.7E6	11.7E6
000162	.96	1.96	.08	.08
000163	+.0		-2.5E-8	1.42
000164	.43	20.12		
000165	.0	.0	.0	.0
000166	10.0E6	10.0E6	10.0E9	10.0E9
000167	11.7E6	11.7E6	11.7E6	11.7E6
000168	1.	1.	1.	1.
000169	+.0		-.0E-8	.0
000170	.01	.01	.92E6	
000171	.164	.164	.45	.588
000172	27.3E6	24.7E6	2.9E9	2.9E9
000173	11.7E6	11.7E6	11.7E6	11.7E6
000174	1.95	2.15	.08	.08
000175	+.0		-2.1E-8	2.23
000176	.37	41.96		

000177	.02	.02	.0	.0
000178	16.9E6	16.9E6	1.0E9	1.0E9
000179	11.7E6	11.7E6	11.7E6	11.7E6
000180	3.14	3.14	1.	1.
000181	+.0		-.0E-8	.0
000182	.04	.0		
000183	.151	.151	.0	.0
000184	24.7E6	24.7E6	1.0E9	1.0E9
000185	11.7E6	11.7E6	11.7E6	11.7E6
000186	2.15	2.15	1.	1.
000187	+.0		-2.1E-8	.0
000188	.37	.0		
000189	.02	.02	.0	.0
000190	16.9E6	16.9E6	1.0E9	1.0E9
000191	11.7E6	11.7E6	11.7E6	11.7E6
000192	3.14	3.14	1.	1.
000193	+.0		-.0E-8	.0
000194	.04	.0		
000195	.164	.164	.0	.0
000196	24.7E6	27.3E6	1.0E9	1.0E9
000197	11.7E6	11.7E6	11.7E6	11.7E6
000198	2.15	1.96	1.	1.
000199	+.0		-2.1E-8	.0
000200	.37	.0		
000201	.0	.0	.0	.0
000202	10.0E6	10.0E6	10.0E9	10.0E9
000203	11.7E6	11.7E6	11.7E6	11.7E6
000204	1.	1.	1.	1.
000205	+.0		-.0E-8	.0
000206	.01	.01	.92E6	
000207	.17	.17	1.2	2.1
000208	27.3E6	59.3E6	7.8E9	7.8E9
000209	11.7E6	11.7E6	11.7E6	11.7E6
000210	1.96	.96	.11	.11
000211	+.0		-2.4E-8	4.55
000212	.43	42.2		
000213	.02	.02	.84	.76
000214	47.3E6	47.3E6	39.3E9	39.3E9
000215	11.7E6	11.7E6	11.7E6	11.7E6
000216	1.22	1.22	.03	.03
000217	+.0		-.0E-8	1.88
000218	.04	44.23		
000219	.25	.20	1.9	3.1
000220	70.0E6	90.0E6	2.4E9	2.4E9
000221	6.1E6	6.1E6	11.7E6	11.7E6
000222	.24	.2	.31	.31
000223	+.002		-5.2E-8	.49
000224	.91	17.43		
000225	.26	.34	4.15	2.27
000226	1230.0E6	1230.0E6	2.4E9	2.4E9
000227	6.1E6	6.1E6	11.7E6	11.7E6
000228	.16	.16	.31	.31
000229	+.088		-36.E-8	.72
000230	6.3	17.86		
000231	.55	.57	.72	1.28
000232	530.0E6	530.0E6	.1E9	.1E9
000233	6.1E6	6.1E6	11.7E6	11.7E6
000234	.16	.16	.31	.31
000235	+.039		-11.8E-8	.0
000236	2.07	1.2		

000237	.42	.60	.0	.0
000238	1230.0E6	1230.0E6	1.0E6	1.0E6
000239	6.1E6	6.1E6	11.7E6	11.7E6
000240	.16	.16	.0	.0
000241	+.104		-40.E-8	.0
000242	7.0	.0		

ELT MAIN,1,710427, 53665

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      C                                                                 13102128
      C     LATERAL VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED,UNDAMPED  13102129
      C           LUMPED PARAMETER BEAMS        JOB 14034   L.VAN TRIEST    13102130
      C                                                                 13102131
            IMPLICIT REAL*8 (A-H,O-Z)
            COMMON /ARRAY/BLO/ARRAYZ/BHI                                13102133
            DIMENSION BLO(1)
            DIMENSION DL1(50),DL2(50),DL3(50),DL4(50),DEI1(50),DEI2(50),  13102134
           1         DEI3(50),DEI4(50),DG1(50),DG2(50),DG3(50),DG4(50),   13102135
           2          DC1(50),DC2(50),DC3(50),DC4(50),DIJ1(50),DIJ2(50),  13102136
           3         DIX1(50),DIX2(50),DWN1(50),DWN2(50),DKN1(50),DKN2(50) 13102137
            DIMENSION E1MTRX(8,8),E2MTRX(8,8),AMATRX(8,8),BMATRX(8,8),  13102138
           1          CMATRX(8,8),FMATRX(8,8),DLMTRX(8,1),SMATRX(8,1),  13102139
           2          DUMMY(8),DETERM(4,4),C(4),ID(4),X(3),KID(3),DXXX(4), 13102140
           3          STORE(8,50)                                       13102141
            DIMENSION TITLE(11),NREP(7)                                 13102142
            COMMON DL1,DL2,DL3,DL4,DEI1,DEI2,DEI3,DEI4,DG1,DG2,DG3,DG4, 13102143
           1       DC1,DC2,DC3,DC4,DIJ1,DIJ2,DIX1,DIX2,DWN1,DWN2,DKN1,DKN2, 13102144
           2       E1MTRX,E2MTRX,AMATRX,BMATRX,CMATRX,FMATRX,DLMTRX,SMATRX, 13102145
           3       DUMMY,KM,KN,KO,KP,KA,KB,KC,KD,DETERM,B                13102146
            DO 999 II=1,7                                               13102147
        999 NREP(II)=0                                                  13102148
            LINE=6                                                      13102149
         30 READ (5,3000,END=5000) TITLE,NSTA                           13102150
            READ (5,3001)            NROOT,TOMGA,DOMGA,K<,KA,KB,KC,KD,KM,KN,KO,13102151
           1     KP                                                     13102152
            DO 35 N=1,NSTA                                              13102153
            CALL REPEAT(DL1(N-1),DL1(N),DL2(N-1),DL2(N),DL3(N-1),DL3(N),DL4(N-13102154
           11),DL4(N),X,X,X,X,NREP(1))                                  13102155
            CALL REPEAT(DEI1(N-1),DEI1(N),DEI2(N-1),DEI2(N),DEI3(N-1),DEI3(N),13102156
           1DEI4(N-1),DEI4(N),X,X,X,X,NREP(2))                          13102157
            CALL REPEAT(DG1(N-1),DG1(N),DG2(N-1),DG2(N),DG3(N-1),DG3(N),DG4(N-13102158
           11),DG4(N),X,X,X,X,NREP(5))
            CALL REPEAT(DC1(N-1),DC1(N),DC2(N-1),DC2(N),DC3(N-1),DC3(N),DC4(N-13102160
           11),DC4(N),X,X,X,X,NREP(3))
            CALL REPEAT(DIJ1(N-1),DIJ1(N),DIJ2(N-1),DIJ2(N),DIX1(N-1),DIX1(N),13102162
           1DIX2(N-1),DIX2(N),X,X,X,X,NREP(4))                          13102163
            CALL REPEAT(DWN1(N-1),DWN1(N),DWN2(N-1),DWN2(N),DKN1(N-1),DKN1(N),13102164
           1DKN2(N-1),DKN2(N),X,X,X,X,NREP(6))                          13102165
         35 CONTINUE                                                    13102166
            WRITE (6,4000)                                              13102167
            WRITE (6,4001)            TITLE,NSTA                        13102168
            WRITE (6,4021)            NROOT,TOMGA,DOMGA,KA,KB,KC,KD,KM,KN,KO, 13102169
           1   KP                                                       13102170
            LINE=LINE+NSTA+4                                            13102171
            IF(LINE-55)39,39,38                                         13102172
         38 WRITE (6,4022)                                              13102173
            LINE=NSTA                                                   13102174
         39 WRITE (6,4002)                                              13102175
            DO 40 N=1,NSTA                                              13102176
         40 WRITE (6,4008)            DL1(N),DL2(N),DL3(N),DL4(N)       13102177
            LINE=LINE+NSTA+4                                            13102178
            IF(LINE-55)44,44,43                                         13102179
         43 WRITE (6,4022)                                              13102180
            LINE=NSTA                                                   13102181
         44 WRITE (6,4003)                                              13102182
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B

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      DO 45 N=1,NSTA                                                    13102183
   45 WRITE (6,4008)          DEI1(N),DEI2(N),DEI3(N),DEI4(N)           13102184
      LINE=LINE+NSTA+4                                                  13102185
      IF(LINE-55)49,49,48                                               13102186
   48 WRITE (6,4022)                                                    13102187
      LINE=NSTA                                                         13102188
   49 WRITE (6,4004)                                                    13102189
      DO 50 N=1,NSTA                                                    13102190
   50 WRITE (6,4008)          DG1(N),DG2(N),DG3(N),DG4(N)               13102191
      LINE=LINE+NSTA+4                                                  13102192
      IF(LINE-55)54,54,53                                               13102193
   53 WRITE (6,4022)                                                    13102194
      LINE=NSTA                                                         13102195
   54 WRITE (6,4005)                                                    13102196
      DO 55 N=1,NSTA                                                    13102197
   55 WRITE (6,4008)          DC1(N),DC2(N),DC3(N),DC4(N)               13102198
      LINE=LINE+NSTA+4                                                  13102199
      IF(LINE-55)59,59,58                                               13102200
   58 WRITE (6,4022)                                                    13102201
      LINE=NSTA                                                         13102202
   59 WRITE (6,4006)                                                    13102203
      DO 60 N=1,NSTA                                                    13102204
   60 WRITE (6,4008)          DIJ1(N),DIJ2(N),DIX1(N),DIX2(N)           13102205
      LINE=LINE+NSTA+4                                                  13102206
      IF(LINE-55)64,64,63                                               13102207
   63 WRITE (6,4022)                                                    13102208
   64 WRITE (6,4007)                                                    13102209
      DO 65 N=1,NSTA                                                    13102210
   65 WRITE (6,4008)         DWN1(N),DWN2(N),DKN1(N),DKN2(N)            13102211
      LINE=NSTA                                                         13102212
      WRITE (6,4022)                                                    13102213
      PI2 = 6.283185307179586
      TMOD = TOMGA * PI2
      DMOD = DOMGA * PI2
C                                                                       13102216
C  INITILIZE E1,E2,AND F MATRICES                                       13102217
C                                                                       13102218
  100 DO 110 I=1,8                                                      13102219
      DO 110 J=1,8                                                      13102220
      IF(I-J)105,106,105                                                13102221
  105 E1MTRX(I,J)=0.0D0                                                 13102222
      E2MTRX(I,J)=0.0D0                                                 13102223
      FMATRX(I,J)=0.0D0                                                 13102224
      GO TO 110                                                         13102225
  106 E1MTRX(I,J)=1.0D0                                                 13102226
      E2MTRX(I,J)=1.0D0                                                 13102227
      FMATRX(I,J)=1.0D0                                                 13102228
  110 CONTINUE                                                          13102229
      OMGW=TMOD-DMOD                                                    13102230
      DO 150 MMM=1,NROOT                                                13102231
      DOMG=DMOD                                                         13102232
      OMGW=OMGW+DOMG                                                    13102233
      NN=100                                                            13102234
      CALL ROOT(NN)                                                     13102235
      DO 130 I=1,8                                                      13102236
      DO 130 J=1,8                                                      13102237
      IF(I-J)125,126,125                                                13102238
  125 CMATRX(I,J)=0.0D0                                                 13102239
      GO TO 130                                                         13102240
  126 CMATRX(I,J)=1.0D0                                                 13102241
```

```
  130 CONTINUE                                                  13102242
      DO 135 N=1,NSTA                                           13102243
      CALL SETUP(OMGW,N)                                        13102244
      CALL MATMPY(E1MTRX,CMATRX,AMATRX,8,8,8,8,8)               13102245
      CALL MATMPY(E2MTRX,FMATRX,BMATRX,8,8,8,8,8)               13102246
      CALL MATMPY(BMATRX,AMATRX,CMATRX,8,8,8,8,8)               13102247
  135 CONTINUE                                                  13102248
      CALL SETUP1(DETNOW)                                       13102249
      OPRT=OMGW/PI2                                             13102250
      IF(LINE-56)136,136,137                                    13102251
  136 WRITE (6,4009)         OPRT,DETNOW                        13102252
      LINE=LINE+1                                               13102253
      GO TO 138                                                 13102254
  137 WRITE (6,4009)         OPRT,DETNOW                        13102255
      LINE=1                                                    13102256
      WRITE (6,4022)                                            13102257
  138 EE = 2.D-12
      CALL ROOTB(OMGW,DOMG,DETNOW,EE,KKK)                       13102259
      IF(KKK)822,140,822                                        13102260
  822 WRITE (6,4020)                                            13102261
      GO TO 30                                                  13102262
  140 DO 1140 I=1,8                                             13102263
 1140 DLMTRX(I,1)=0.0D0                                         13102264
      CALL SIMSZ
      ID(1)=1                                                   13102265
      ID(2)=2                                                   13102266
      ID(3)=3                                                   13102267
      ID(4)=-1                                                  13102268
      IF (KK .EQ. 0) KK = 1                                        *NEW
      GO TO (141,142,143,144),KK                                13102269
  141 DLMTRX(KM,1)=1.0D0                                        13102270
C                                                               13102271
C     SOLVE FOR DELTA(N),DELTA(O),DELTA(P)                      13102272
C                                                               13102273
      C(1)=CMATRX(KB,KN)                                        13102274
      C(2)=CMATRX(KB,KO)                                        13102275
      C(3)=CMATRX(KB,KP)                                        13102276
      C(4)=-CMATRX(KB,KM)                                       13102277
      CALL SIMST(C,ID,4,BLO,BHI)                                13102278
      C(1)=CMATRX(KC,KN)                                        13102279
      C(2)=CMATRX(KC,KO)                                        13102280
      C(3)=CMATRX(KC,KP)                                        13102281
      C(4)=-CMATRX(KC,KM)                                       13102282
      CALL SIMST(C,ID,4,BLO,BHI)                                13102283
      C(1)=CMATRX(KD,KN)                                        13102284
      C(2)=CMATRX(KD,KO)                                        13102285
      C(3)=CMATRX(KD,KP)                                        13102286
      C(4)=-CMATRX(KD,KM)                                       13102287
      CALL SIMST(C,ID,4,BLO,BHI)                                13102288
      CALL SIMSD(X,KID,DXXX(1),KERR,ITEGN)                      13102289
      IF(KERR)310,145,310                                       13102290
  142 DLMTRX(KN,1)=1.0D0                                        13102291
C                                                               13102292
C     SOLVE FOR DELTA(M),DELTA(O),DELTA(P)                      13102293
C                                                               13102294
      C(1)=CMATRX(KA,KM)                                        13102295
      C(2)=CMATRX(KA,KO)                                        13102296
      C(3)=CMATRX(KA,KP)                                        13102297
      C(4)=-CMATRX(KA,KN)                                       13102298
      CALL SIMST(C,ID,4,BLO,BHI)                                13102299
```

```
      C(1)=CMATRX(KC,KM)
      C(2)=CMATRX(KC,KO)
      C(3)=CMATRX(KC,KP)
      C(4)=-CMATRX(KC,KN)
      CALL SIMST(C,ID,4,BLO,BHI)
      C(1)=CMATRX(KD,KM)
      C(2)=CMATRX(KD,KO)
      C(3)=CMATRX(KD,KP)
      C(4)=-CMATRX(KD,KN)
      CALL SIMST(C,ID,4,BLO,BHI)
      CALL SIMSD(X,KID,DXXX(1),KERR,ITEQN)
      IF(KERR)310,145,310
  143 DLMTRX(KO,1)=1.0D0
C
C  SOLVE FOR DELTA(M),DELTA(N),DELTA(P)
C
      C(1)=CMATRX(KA,KM)
      C(2)=CMATRX(KA,KN)
      C(3)=CMATRX(KA,KP)
      C(4)=-CMATRX(KA,KO)
      CALL SIMST(C,ID,4,BLO,BHI)
      C(1)=CMATRX(KB,KM)
      C(2)=CMATRX(KB,KN)
      C(3)=CMATRX(KB,KP)
      C(4)=-CMATRX(KB,KO)
      CALL SIMST(C,ID,4,BLO,BHI)
      C(1)=CMATRX(KD,KM)
      C(2)=CMATRX(KD,KN)
      C(3)=CMATRX(KD,KP)
      C(4)=-CMATRX(KD,KO)
      CALL SIMST(C,ID,4,BLO,BHI)
      CALL SIMSD(X,KID,DXXX(1),KERR,ITEQN)
      IF(KERR)310,145,310
  144 DLMTRX(KP,1)=1.0D0
C
C  SOLVE FOR DELTA(M),DELTA(N),DELTA(O)
C
      C(1)=CMATRX(KA,KM)
      C(2)=CMATRX(KA,KN)
      C(3)=CMATRX(KA,KO)
      C(4)=-CMATRX(KA,KP)
      CALL SIMST(C,ID,4,BLO,BHI)
      C(1)=CMATRX(KB,KM)
      C(2)=CMATRX(KB,KN)
      C(3)=CMATRX(KB,KO)
      C(4)=-CMATRX(KB,KP)
      CALL SIMST(C,ID,4,BLO,BHI)
      C(1)=CMATRX(KC,KM)
      C(2)=CMATRX(KC,KN)
      C(3)=CMATRX(KC,KO)
      C(4)=-CMATRX(KC,KP)
      CALL SIMST(C,ID,4,BLO,BHI)
      CALL SIMSD(X,KID,DXXX(1),KERR,ITEQN)
      IF(KERR)310,145,310
  145 CONTINUE
      GO TO (1141,1142,1143,1144),KK
 1141 KSUB=KID(1)
      DLMTRX(KN,1)=X(KSUB)
      KSUB=KID(2)
      DLMTRX(KO,1)=X(KSUB)
```

```
      KSUB=KID(3)
      DLMTRX(KP,1)=X(KSUB)
      GO TO 146
 1142 KSUB=KID(1)
      DLMTRX(KM,1)=X(KSUB)
      KSUB=KID(2)
      DLMTRX(KO,1)=X(KSUB)
      KSUB=KID(3)
      DLMTRX(KP,1)=X(KSUB)
      GO TO 146
 1143 KSUB=KID(1)
      DLMTRX(KM,1)=X(KSUB)
      KSUB=KID(2)
      DLMTRX(KN,1)=X(KSUB)
      KSUB=KID(3)
      DLMTRX(KP,1)=X(KSUB)
      GO TO 146
 1144 KSUB=KID(1)
      DLMTRX(KM,1)=X(KSUB)
      KSUB=KID(2)
      DLMTRX(KN,1)=X(KSUB)
      KSUB=KID(3)
      DLMTRX(KO,1)=X(KSUB)
  146 WRITE (6,4012)        OPRT
      DO 147 I=1,8
  147 DUMMY(I)=DLMTRX(I,1)
      DO 149 N=1,NSTA
      CALL SETUP(OMGW,N)
      CALL MATMPY(E2MTRX,FMATRX,AMATRX,8,8,8,8,8)
      CALL MATMPY(AMATRX,E1MTRX,BMATRX,8,8,8,8,8)
      CALL MATMPY(BMATRX,DLMTRX,SMATRX,8,8,8,8,1)
      DO 149 I=1,8
      STORE(I,N)=SMATRX(I,1)
      DLMTRX(I,1)=SMATRX(I,1)
  149 CONTINUE
      WRITE (6,4013)          (DUMMY(K),K=1,4)
      DO 1149 I=1,NSTA
 1149 WRITE (6,4008)          (STORE(N,I),N=1,4)
      WRITE (6,4014)          (DUMMY(K),K=5,8)
      DO 1150 I=1,NSTA
 1150 WRITE (6,4008)          (STORE(N,I),N=5,8)
      WRITE (6,4022)
      LINE=1
  150 CONTINUE
C
C  END OF CASE
C
      WRITE (6,4011)
      GO TO 30
  310 WRITE (6,4023) KERR
      WRITE (6,4024)
      CALL PRINTM (CMATRX,8,8,8,12H  CMATRX    )
      GO TO 30
 3000 FORMAT (11A6,4X,I2)
 3001 FORMAT (I2,2E12.6,9I3)
 3002 FORMAT (4E12.6)
 4000 FORMAT (1H1,50X30HJOB E13102  VIBRATION ANALYSIS///)
 4001 FORMAT (1H 11A6,4X,19HNUMBER OF STATIONS I2//)
 4002 FORMAT (1H013X4HL(1),29X4HL(2),29X4HL(3),29X4HL(4)//)
 4003 FORMAT (1H012X5HEI(1),28X5HEI(2),28X5HEI(3),28X5HEI(4)//)
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      4004 FORMAT (1H013X4HG(1),29X4HG(2),29X4HG(3),29X4HG(4)//)          13102420
      4005 FORMAT (1H013X4HC(1),29X4HC(2),29X4HC(3),29X4HC(4)//)          13102421
      4006 FORMAT (1H012X8HI SUB J1,25X8H   DX   ,25X8HI SUB X1,25X8HI SUB X213102422
          1//)                                                            13102423
      4007 FORMAT (1H012X8HW SUB N1,25X8HW SUB N2,25X8HK SUB N1,25X8HK SUB N213102424
          1//)                                                            13102425
      4008 FORMAT (9XE15.8,3(18XE15.8))                                   13102426
      4009 FORMAT (1H ,26X8HOMEGA = E15.8,5X9HDETERM = E15.8)             13102427
      4011 FORMAT (14H0 END OF CASE )                                     13102428
      4012 FORMAT (1H0,53X8HOMEGA = E15.8///)                             13102429
      4013 FORMAT (1H1,15X1HV,32X1HM,31X3HPHI,31X1HY///9XE15.8,3(18XE15.8)) 13102430
      4014 FORMAT (1H0,11X7HV PRIME,26X7HM PRIME,25X9HPHI PRIME,25X7HY PRIME/13102431
          1//9XE15.8,3(18XE15.8))                                         13102432
      4020 FORMAT (36H0 100 INTERATIONS AND NO ROOTS FOUND)               13102433
      4021 FORMAT (19H   NUMBER OF ROOTS I3,5X,13H TRIAL OMEGA F8.3,      13102434
          1   5X,13H DELTA OMEGA F8.3,5X,8I4///)                          13102435
      4022 FORMAT (1H1)                                                   13102436
      4023 FORMAT (1H0,41X24HFAILURE IN SIMST--KERR =  I2,3X,18HGOING TO NEXT13102437
          1 CASE  )                                                       13102438
      4024 FORMAT (1H0,8X,100HTHE COEFFICIENTS OF THE SIMULTANEOUS EQUATIONS 13102439
          1WHICH WERE NOT SOLVED FOR THE INITIAL STATE VECTOR       / 5X    13102440
          2 69HWERE EXTRACTED FROM THE FOLLOWING MATRIX VIA THE BOUNDARY COND13102441
          3ITIONS.    )                                                   13102442
      5000 STOP
           END                                                            13102443
```

```
@ ELT MATMPY,1,710422, 35976

      SUBROUTINE MATMPY(A,B,C,K1,M1,K,M,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(20),B(20),C(20)
      DO 10 I=1,K
      DO 10 J=1,N
      II=(J-1)*K1+I
      C(II)=0.0D0
      DO 10 L=1,M
      JJ=(L-1)*K1+I
      KK=(J-1)*M1+L
   10 C(II)=C(II)+A(JJ)*B(KK)
      RETURN
      END
```

```
@ ELT PRINTM,1,710427, 53667

      SUBROUTINE PRINTM(A,NR,NC,MAXR,TITLE)                             13102101
      IMPLICIT REAL*8 (A-H,O-Z)
C                                                                       13102102
C FORTRAN IV PRINTM                                                     13102103
C SUBROUTINE TO PRINT ANY MATRIX WITH 2-WORD TITLE                      13102104
C     CALL PRINTM (CMATRX,8,8,8,12H  CMATRX     )  EXAMPLE CALL UP      13102105
C                                                                       13102106
      DIMENSION A(1),NHED(8),TITLE(2)                                   13102107
C                                                         MATRIX TITLE  13102108
      DATA B /'   COL'/
      WRITE (6,22)TITLE                                                 13102109
   22 FORMAT (1H0,52X,2A6)                                              13102110
C                                                                       13102111
      DO 50 I=1,NC,8                                                    13102113
      II=NC-I+1                                                         13102114
      IF(II-8)20,20,10                                                  13102115
   10 II=8                                                              13102116
   20 DO 30 J=1,II                                                      13102117
   30 NHED(J)=I+J-1                                                     13102118
      WRITE (6,120) (B,NHED(J),J=1,II)                                  13102119
      DO 50 J=1,NR                                                      13102120
      KL=J+(I-1)*MAXR                                                   13102121
      KH=KL+(II-1)*MAXR                                                 13102122
   50 WRITE (6,130)          (J, A(K),K=KL,KH,MAXR )                    13102123
      RETURN                                                            13102124
  120 FORMAT (1H0,9X,10(A6,I4,4X))                                      13102125
  130 FORMAT (4H ROW,I3,5X,1P8D14.7)                                        *NEW
      END                                                               13102127**-1
```

```
@ ELT REPEAT,1,710422, 35979

      SUBROUTINE REPEAT(A,AA,B,BB,C,CC,D,DD,E,EE,F,FF,NR)              13102 81
      IMPLICIT REAL*8 (A-H,O-Z)
C***********************************************************************13102 82
C     REPEAT READS IN A STATION CARD OR SIMULATES A REPEATED CARD BY    13102 83
C     MOVING DATA.                                                      13102 84
C     A,B,C,D,E,F   OLD         AA,BB,CC,DD,EE,FF    NEW                13102 85
C                NR = NUMBER OF REPEATS FOR A PARTICULAR CARD           13102 86
C***********************************************************************13102 87
    1 IF(NR-1)400,100,100                                               13102 88
  400 READ (5,3002)          AA,BB,CC,DD,EE,FF,NR                       13102 89
 3002 FORMAT ( 6E12.6,I3)                                               13102 90
      GO TO 700                                                         13102 91
  100 AA=A                                                              13102 92
      BB=B                                                              13102 93
      CC=C                                                              13102 94
      DD=D                                                              13102 95
      EE=E                                                              13102 96
      FF=F                                                              13102 97
      NR=NR-1                                                           13102 98
  700 RETURN                                                            13102 99
      END                                                               13102100
```

```
@ ELT ROOT,1,710429, 57628

          .
          .       CALL ROOT(N)
          .         N= NUMBER OF SEARCH ITERATIONS
          .
          S(1).
                  REGNAM
          ROOT*.
                  S          B11,SVB11              . SAVE B11 FOR RETURN
                  DL         A0,0,B11               . GET THE CALLING SEQUENCE
                  DS         A0,CALSEQ              . PUT AWAY
                  LMJ        B11,ROOTF              . GO INITIALIZE ROOTB
          CALSEQ  RES        2                      . CALLING SEQUENCE
          RETAGN*.                                  . ITERATION RETURN ENTRY
                  L          B11,SVB11
                  J          2,B11
          .
          .
          .
          SVB11   +          0
                  END
```

```
@ ELT ROOTB,1,710429, 57629

      C
      C
      C
            SUBROUTINE ROOTF(NN)
      C
      C        NN = SEARCH ITERATION LIMIT
      C
            IMPLICIT DOUBLE PRECISION (A-H,O-Z)                          *NEW
            N = NN
            FLGA = 0
            FLGB = 0
            RETURN
      C
      C
      C
            ENTRY ROOTB(X,DX,F,E,K)
      C
      C        X= X VALUE
      C        DX = SEARCH INCREMENT
      C        F = F(X)
      C        E = ERROR LIMIT
      C        K = TERMINATION STATUS FLAG
      C
            IF (F)  100, 9000, 200
      C
      C  F<0
      C
      100   CONTINUE
            XMINUS = X
            FMINUS = F
            FLGB = F
            IF (FLGA .NE.0) GO TO 1000
            GO TO 300
      C
      C  F>0
      C
      200   CONTINUE
            XPLUS = X
            FPLUS = F
            FLGA = F
            IF (FLGB .NE. 0)  GO TO 1000
      C
      C  TRY A NEW X VAKUE TO BRACKET THE ROOT
      C
      300   CONTINUE
            XLAST = X
            FLAST = F
            X = X+DX
            N = N-1
            IF (N .GE. 0)  CALL RETAGN
            K = N
            RETURN
      C
      C  DO LINEAR INTERPOLATION TO APPROXIMATE THE ROOT
      C
      1000  CONTINUE
```

```
      IF (F-FLAST .EQ. 0)  GO TO 2000
      X1 = (F*XLAST-X*FLAST)/(F-FLAST)
      FLAST = F
      XLAST = X
      X = X1
      I10 = 1
C
C  SEE IF NEW X IS IN THE PROPER INTERVAL
C
1100  CONTINUE
      IF((X-XMINUS)*(XPLUS-X))  1200, 9000, 1300
1200  CONTINUE
      GO TO (2000, 9000), I10
C
C  TEST TO SEE IF CLOSE ENOUGH
C
1300  CONTINUE
      IF (ABS(X-XLAST)-E .LE. 0)  GO TO 9000
      CALL RETAGN
C
C  INTERPOLATE USING THE INTERVAL BOUNDARIES
C
2000  CONTINUE
      X = (XMINUS*FPLUS-XPLUS*FMINUS)/(FPLUS-FMINUS)
      I10 = 2
      GO TO 1100
C
C  NORMAL RETURN
C
9000  K = 0
      RETURN
      END
```

@ ELT SETUP,1,710422, 35981

```
      SUBROUTINE SETUP(OMGG,NN)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION DL1(50),DL2(50),DL3(50),DL4(50),DEI1(50),DEI2(50),          13102  1
     1          DEI3(50),DEI4(50),DG1(50),DG2(50),DG3(50),DG4(50),          13102  3
     2           DC1(50),DC2(50),DC3(50),DC4(50),DIJ1(50),DIJ2(50),         13102  4
     3          DIX1(50),DIX2(50),DWN1(50),DWN2(50),DKN1(50),DKN2(50)       13102  5
      DIMENSION E1MTRX(8,8),E2MTRX(8,8),AMATRX(8,8),BMATRX(8,8),            13102  6
     1           CMATRX(8,8),FMATRX(8,8),DLMTRX(8,1),SMATRX(8,1),           13102  7
     2           DUMMY(8),DETERM(4,4),C(4),ID(4),X(3),KID(3),DXXX(4),       13102  8
     3           STORE(8,50)                                                13102  9
      DIMENSION TITLE(11),NREP(7)                                           13102 10
      COMMON DL1,DL2,DL3,DL4,DEI1,DEI2,DEI3,DEI4,DG1,DG2,DG3,DG4,           13102 11
     1       DC1,DC2,DC3,DC4,DIJ1,DIJ2,DIX1,DIX2,DWN1,DWN2,DKN1,DKN2,       13102 12
     2       E1MTRX,E2MTRX,AMATRX,BMATRX,CMATRX,FMATRX,DLMTRX,SMATRX,       13102 13
     3       DUMMY,KM,KN,KO,KP,KA,KB,KC,KD,DETERM,B                         13102 14
      N    = NN
      OMG  = OMGG
      E1MTRX(2,1)=DL1(N)                                                    13102 15
      E1MTRX(4,2)=DL1(N)**2*.5D0/DEI1(N)                                    13102 16
      E1MTRX(3,1)=-E1MTRX(4,2)                                              13102 17
      E1MTRX(3,2)=-DL1(N)/DEI1(N)                                           13102 18
      E1MTRX(4,1)=DL1(N)**3/6.D0/DEI1(N)-DC1(N)*DL1(N)/DG1(N)               13102 19
      E1MTRX(4,3)=-E1MTRX(2,1)                                              13102 20
      E1MTRX(6,5)=DL3(N)                                                    13102 21
      E1MTRX(8,6)=DL3(N)**2*.5D0/DEI3(N)                                    13102 22
      E1MTRX(7,5)=-E1MTRX(8,6)                                              13102 23
      E1MTRX(7,6)=-DL3(N)/DEI3(N)                                           13102 24
      E1MTRX(8,5)=DL3(N)**3/6.D0/DEI3(N)-DC3(N)*DL3(N)/DG3(N)               13102 25
      E1MTRX(8,7)=-E1MTRX(6,5)                                              13102 26
      E2MTRX(2,1)=DL2(N)                                                    13102 27
      E2MTRX(4,2)=DL2(N)**2*.5D0/DEI2(N)                                    13102 28
      E2MTRX(3,1)=-E2MTRX(4,2)                                              13102 29
      E2MTRX(3,2)=-DL2(N)/DEI2(N)                                           13102 30
      E2MTRX(4,1)=DL2(N)**3/6.D0/DEI2(N)-DC2(N)*DL2(N)/DG2(N)               13102 31
      E2MTRX(4,3)=-E2MTRX(2,1)                                              13102 32
      E2MTRX(6,5)=DL4(N)                                                    13102 33
      E2MTRX(7,5)=-DL4(N)**2*.5D0/DEI4(N)                                   13102 34
      E2MTRX(7,6)=-DL4(N)/DEI4(N)                                           13102 35
      E2MTRX(8,5)=DL4(N)**3/6.D0/DEI4(N)-DC4(N)*DL4(N)/DG4(N)               13102 36
      E2MTRX(8,6)=DL4(N)**2*.5D0/DEI4(N)                                    13102 37
      E2MTRX(8,7)=-E2MTRX(6,5)                                              13102 38
      FMATRX(1,4)=DWN1(N)*OMG/386.04D0*OMG-DKN1(N)                          13102 39
      FMATRX(2,3)=-(DIJ1(N))*OMG**2                                         13102 40
      IF (DIX1(N).NE.0.0D0) GO TO 1
      FMATRX(7,6) = 0.0D0
      GO TO 2
    1 FMATRX(7,6)=-1.D0/DIX1(N)                                             13102 41
    2 FMATRX(1,7)=DIJ2(N)*DKN1(N)                                           13102 42
      FMATRX(1,8)=DKN1(N)                                                   13102 43
      FMATRX(5,4)=DKN1(N)                                                   13102 44
      FMATRX(5,7)=-FMATRX(1,7)                                              13102 45
      FMATRX(5,8)=DWN2(N)*OMG/386.04D0*OMG-DKN1(N)-DKN2(N)                  13102 46
      FMATRX(6,4)=FMATRX(1,7)                                               13102 47
      FMATRX(6,7)=DIX2(N)*OMG**2-DIJ2(N)**2*DKN1(N)                         13102 48
      FMATRX(6,8)=FMATRX(5,7)                                               13102 49
      RETURN                                                                13102 50
```

22

000057 END 13102 51

```
@ ELT SETUP1,1,710422, 35983

      SUBROUTINE SETUP1(DET)                                            13102 52
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION DL1(50),DL2(50),DL3(50),DL4(50),DEI1(50),DEI2(50),      13102 53
     1         DEI3(50),DEI4(50),DG1(50),DG2(50),DG3(50),DG4(50),       13102 54
     2          DC1(50),DC2(50),DC3(50),DC4(50),DIJ1(50),DIJ2(50),      13102 55
     3         DIX1(50),DIX2(50),DWN1(50),DWN2(50),DKN1(50),DKN2(50)    13102 56
      DIMENSION E1MTRX(8,8),E2MTRX(8,8),AMATRX(8,8),BMATRX(8,8),        13102 57
     1          X(8,8),FMATRX(8,8),DLMTRX(8,1),SMATRX(8,1),             13102 58
     2          DUMMY(8),DETERM(4,4),C(4),ID(4),Z(3),KID(3),DXXX(4),    13102 59
     3          STORE(8,50)                                             13102 60
      DIMENSION TITLE(11)                                               13102 61
      COMMON DL1,DL2,DL3,DL4,DEI1,DEI2,DEI3,DEI4,DG1,DG2,DG3,DG4,       13102 62
     1       DC1,DC2,DC3,DC4,DIJ1,DIJ2,DIX1,DIX2,DWN1,DWN2,DKN1,DKN2,   13102 63
     2       E1MTRX,E2MTRX,AMATRX,BMATRX,X,FMATRX,DLMTRX,SMATRX,        13102 64
     3       DUMMY,KM,KN,KO,KP,KA,KB,KC,KD,DETERM,B                     13102 65
      DUMMY(1)=X(KA,KM)*(X(KB,KN)*X(KC,KO)*X(KD,KP)+X(KB,KO)*X(KC,KP)*X(13102 66
     1KD,KN)+X(KB,KP)*X(KD,KO)*X(KC,KN)-X(KD,KN)*X(KC,KO)*X(KB,KP)-X(KC,13102 67
     2KN)*X(KB,KO)*X(KD,KP)-X(KB,KN)*X(KC,KP)*X(KD,KO))                 13102 68
      DUMMY(2)=X(KA,KN)*(X(KB,KM)*X(KC,KO)*X(KD,KP)+X(KB,KO)*X(KC,KP)*X(13102 69
     1KD,KM)+X(KB,KP)*X(KD,KO)*X(KC,KM)-X(KD,KM)*X(KC,KO)*X(KB,KP)-X(KC,13102 70
     2KM)*X(KB,KO)*X(KD,KP)-X(KB,KM)*X(KC,KP)*X(KD,KO))                 13102 71
      DUMMY(3)=X(KA,KO)*(X(KB,KM)*X(KC,KN)*X(KD,KP)+X(KB,KN)*X(KC,KP)*X(13102 72
     1KD,KM)+X(KB,KP)*X(KD,KN)*X(KC,KM)-X(KD,KM)*X(KC,KN)*X(KB,KP)-X(KC,13102 73
     2KM)*X(KB,KN)*X(KD,KP)-X(KB,KM)*X(KC,KP)*X(KD,KN))                 13102 74
      DUMMY(4)=X(KA,KP)*(X(KB,KM)*X(KC,KN)*X(KD,KO)+X(KB,KN)*X(KC,KO)*X(13102 75
     1KD,KM)+X(KB,KO)*X(KD,KN)*X(KC,KM)-X(KD,KM)*X(KC,KN)*X(KB,KO)-X(KC,13102 76
     2KM)*X(KB,KN)*X(KD,KO)-X(KB,KM)*X(KC,KO)*X(KD,KN))                 13102 77
      DET=DUMMY(1)-DUMMY(2)+DUMMY(3)-DUMMY(4)                           13102 78
      RETURN                                                            13102 79
      END                                                               13102 80
```

```
@ ELT SIMEQ,1,710422, 36801

      SUBROUTINE SIMEQ(A,B,NN,MM,NA ,ITEM,DD,NND,KERR)                  61210002
C*****                                                                  61210003
C     SOLVES MATRIX EQUATIONS - AX = B                                  61210004
C     GAUSS ELIMINATION WITH COMPLETE PIVOTING ON ABSOLUTE LARGEST      61210005
C     ELEMENT TO FORM TRIANGULAR MATRIX,WITH BACK SUBSTITUTION FOR      61210006
C     SOLUTION VECTORS.                                                 61210007
C******                                                                 61210008
C                                                                       61210009
C     CALL SIMEQ (A,B,NN,MM,NA ,ITEM,DD,NND,KERR )                      61210010
C     A          =    A(1,1) OF INPUT MATRIX                            61210011
C     B          =    INPUT VECTORS                                     61210012
C     NN         = NUMBER OF SIMULTANEOUS EQUATIONS.                    61210013
C     MM         = NUMBER OF B-VECTORS.                                 61210014
C     NA         = DIMENSION OF MATRIX A,THAT IS, A(NA,--)              61210015
C     ITEM       = TEMPORARY STORAGE (FOR PERMUTATION VECTOR)           61210016
C                  WITH DIMENSION - ITEM(NA)                            61210017
C     DD         = DETERMINANT                                          61210018
C     NND        = POWER OF TEN TO MULTIPLY DETERMINANT                 61210019
C     KERR       = ERROR CODE,   =K, SINGULAR RANK , =-1 SOLVED EQUATIONS 61210020
      DOUBLE PRECISION A(NA,NA),B(NA,1 ),PIVOT,XTEM,D,DD                61210021
      DIMENSION ITEM(2)                                                 61210022
C                                                                       61210023
      D          = 1.0D0                                                61210024
C     ND         = POWERS OF TENS FACTOR FOR DETERMINANT.               61210025
      ND         = 0                                                    61210026
      N=NN                                                              61210027
      M=MM                                                              61210028
C                                                                       61210029
C     SET-UP THE PERMUTATION VECTOR.                                    61210030
      DO 1       I=1,N                                                  61210031
    1 ITEM(I)    = I                                                    61210032
      N1         = N-1                                                  61210033
      DO 60      K=1,N                                                  61210034
C                                                                       61210035
C     SEARCH AND SET THE ABSOLUTE LARGEST ELEMENT AS THE PIVOT.         61210036
C                                                                       61210037
      PIVOT      = 0.D0                                                 61210038
      DO 10      I=K,N                                                  61210039
      DO 9       J=K,N                                                  61210040
      XTEM       = A(I,J)                                               61210041
      IF(DABS(XTEM) .LE. DABS(PIVOT))  GO TO 9                          61210042
      PIVOT      = XTEM                                                 61210043
      IS         = I                                                    61210044
      IT         = J                                                    61210045
    9 CONTINUE                                                          61210046
   10 CONTINUE                                                          61210047
C     COMPUTE DETERMINANT AND TEST FOR SINGULAR MATRIX.                 61210048
C                                                                       61210049
C                                                                       61210050
      D          = D*PIVOT                                              61210051
      IF(D.NE.0.D0)          GO TO 11                                   61210052
C     IF MATRIX IS SINGULAR,SET THE RANK OF MATRIX A IN KERR    AND EXIT61210053
      KERR       =    K-1                                               61210054
      GO TO 100                                                         61210055
   11 XTEM       =    DABS(D)                                           61210056
      IF(XTEM.LE.1.D0)       GO TO 13                                   61210057
```

```
      D       = D/10.D0
      ND      = ND+1
      GO TO 11
   13 IF(XTEM.GE.0.1D0)          GO TO 14
      D       = D*10.D0
      ND      = ND-1
      GO TO 11
   14 CONTINUE
      IF(K.EQ.IS)                GO TO 30
C
C     IF THE PIVOT IS NOT IN THE RIGHT ROW,INTERCHANGE ROWS.
C
      DO 20   J=1,N
      XTEM    = A(IS,J)
      A(IS,J) = A(K,J)
      A(K ,J) = XTEM
   20 CONTINUE
      DO 21   J=1,M
      XTEM    = B(IS,J)
      B(IS,J) = B(K ,J)
      B(K ,J) = XTEM
   21 CONTINUE
      D       = -D
   30 IF(K.EQ.IT)                GO TO 40
C
C     IF THE PIVOT IS NOT IN THE RIGHT COL.,EXCHANGE COLS AND RECORD
C     THIS IN THE PERMUTATION VECTOR.
C
      DO 31   I=1,N
      XTEM    = A(I,IT)
      A(I,IT) = A(I,K )
      A(I,K ) = XTEM
   31 CONTINUE
      D       = -D
C
C     SET PERMUTATION VECTOR
C
      I       = ITEM(IT)
      ITEM(IT) = ITEM(K )
      ITEM(K ) = I
C
   40 CONTINUE
      K1      = K+1
      IF(K1.GT.N)                GO TO 60
C
C     MULTIPLY THE K-TH ROW BY -A(I,K)/PIVOT AND ADD TO THE I-TH ROW
      DO 50   I=K1,N
      DO 50   J=K1,N
      A(I,J)  = A(I,J) - A(K,J)/PIVOT * A(I,K)
   50 CONTINUE
      DO 51   I=K1,N
      DO 51   J=1,M
      B(I,J)  = B(I,J) - A(I,K)/PIVOT*B(K,J)
   51 CONTINUE
   60 CONTINUE
C
C     BACKSUBSTITUTION FOLLOWS.
C
      DO 70   J=1,M
      B(N,J)  = B(N,J)/A(N,N)
```

```
   [illegible] CONTINUE
       I       = N
       DO 73   K=2,N
       I1      = I
       I       = I-1
       PIVOT   = A(I,I)
       DO 72   IT=1,M
       XTEM    = 0.D0
       DO 71   J=I1,N
   71  XTEM    = A(I,J)*B(J,IT) + XTEM
   72  B(I,IT) = (B(I,IT) - XTEM)/PIVOT
   73  CONTINUE
  C
  C    USE PERMUTATION VECTOR TO EXCHANGE ROWS OF B-MATRIX.
  C
       DO 81   I=1,N
   79  IF(ITEM(I).EQ.I)        GO TO 81
       K       = ITEM(I)
       DO 80   J=1,M
       XTEM    = B(K,J)
       B(K,J)  = B(I,J)
       B(I,J)  = XTEM
   80  CONTINUE
       ITEM(I) = ITEM(K)
       ITEM(K) = K
       GO TO 79
   81  CONTINUE
       KERR=-1
       DD      =    D
       NND     =    ND
  100  RETURN
       END
```

@ ELT SIMST,1,710422, 35985

```
      SUBROUTINE SIMST (C,K,M,BLO,BHI )
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION C(4),K(4),X(4),ITEM(10),KI(3),A(3,3),XX(3)
      KER = 0
      IR   = IR+1
      GO TO (10,20,30,50),IR
   10 A(1,1)   = C(1)
      A(1,2)   = C(2)
      A(1,3)   = C(3)
      XX(1)  = C(4)
      GO TO 40
   20 A(2,1)   = C(1)
      A(2,2)   = C(2)
      A(2,3)   = C(3)
      XX(2)  = C(4)
      GO TO 40
   30 A(3,1)   = C(1)
      A(3,2)   = C(2)
      A(3,3)   = C(3)
      XX(3)  = C(4)
   40 RETURN
   50 KER = 4
      GO TO 40
      ENTRY SIMSD (X,KI,DET,KERR,IDUM)
      IR   = 0
      KI(1)  = 1
      KI(2)  = 2
      KI(3)  = 3
      D = 0.D0
      ND = 0
      IF (KER-4) 55,65,55
   55 CONTINUE
      CALL SIMEQ (A, XX,3,1,3,ITEM,D,ND,KER )
      X(1)   = XX(1)
      X(2)   = XX(2)
      X(3)   = XX(3)
      IF(D) 56,58,56
   56 CONTINUE
      DET    = D*10.D0**ND
   58 CONTINUE
      IF (KER) 70,65,65
   65 KER = 0
   70 KERR = KER + 1
   45 RETURN
      ENTRY SIMSZ
      IR   = 0
      RETURN
      END
```

4.   TRI X   14:46:19

END CUR

<**Δ****1****Δ****2****Δ****3****Δ****4****Δ****5****Δ****6****Δ****7****Δ****8****Δ****9****Δ****0****Δ****1****Δ****2****Δ****3*
ISD-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972
Δ1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ
[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ

25 APR 72 P 14:46:20 IDENT=FYEE ACCOUNT=428999 CARDS IN= 9, OUT= 0

PAGES= 27, LINES= 1055. TIME=00:00:06 (HMS) ,

* *

*** USER NOTICES - APRIL 20, 1972 ***

(1) ISD 1108 TERMINAL SERVICE IS SCHEDULED AS FOLLOWS

MON :	07:00 - 24:00
TUE - FRI :	00:00 - 04:00 ; 07:00 - 24:00
SAT :	00:00 - 22:00
SUN :	04:00 - 22:00

(2) LARGE-CORE (LCR) PRODUCTION JOBS ARE NOW BEING RUN ON AN OVERNIGHT BASIS STARTING AT 04:00 EACH DAY.

(3) ISD NOW HAS AVAILABLE REMOTE-BATCH JOB ENTRY VIA LOW-SPEED TELETYPE COMPATIBLE TERMINALS USING DIAL-UP COMMUNICATION LINES. THIS SERVICE HAS BEEN IN USE FOR OVER TWO MONTHS AND IS CALLED RON/I. THE DIAL-UP TELEPHONE NUMBERS AND TRANSMISSION RATES ARE LISTED BELOW.

10 CHAR/SEC 415-562-4035, 415-562-4036, 415-562-5186
30 CHAR/SEC 415-562-4716 ** EFFECTIVE 4/24/72 THIS NUMBER WILL BE CHANGED TO 415-562-4294 **

(4) ISD'S SECOND PUBLIC TERMINAL IN SAN FRANCISCO IS LOCATED AT # 1 CALIFORNIA ST., ROOM 2555.

(5) BEGINNING 4/24/72 AND AFFECTIVE MONDAY - FRIDAY TURNAROUND TIME SHOULD BE REDUCED BETWEEN THE HOURS OF 10:30 - 11:30 AND 14:00 - 16:00 FOR USERS SUBMITTING NON-TAPE JOBS WITH RUN TIMES ESTIMATED AT LESS THAN 6 MINUTES.

ADDITIONAL INFORMATION ON (2) & (3) IS NOW AVAILABLE TO ALL INTERESTED USERS BY CONTACTING YOUR SALESMAN AT 415-562-4204.

* *

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<***Δ****1****Δ****2****Δ****3****Δ****4****Δ****5****Δ****6****Δ****7****Δ****8****Δ****9****Δ****0****Δ****1****Δ****2****Δ****3*
************************************ISD-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972*******************************************
***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ
[ ]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[ ]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[ ]#Δ

25 APR 72 P 14:46:20   IDENT=FYEE    ACCOUNT=428999  CARDS IN=    9, OUT=    0
  PAGES=   27, LINES=  1055. TIME=00:00:06 (HMS)
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APPENDIX E

PROGRAM E13104 FORCED UNDAMPED LATERAL VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED BEAMS - VARIABLE MASS AND ELASTICITY USERS' MANUAL AND SAMPLE OF INPUT/OUTPUT

i

FORCED UNDAMPED LATERAL VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED BEAMS - VARIABLE MASS AND ELASTICITY

Program E13104

(Formerly Program 14043)

Aerojet-General Corporation
Computing Sciences
Sacramento, California

by

J. A. Budzenski

18 June 1968

APPROVED:

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Engineering Analysis
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Computing Sciences
Sacramento, California

FORCED UNDAMPED LATERAL VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED BEAMS - VARIABLE MASS AND ELASTICITY

Program E13104
(Formerly Program 14043)

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ABSTRACT

A Fortran Program for the analysis of shaft whirl critical speeds and bearing loads considering bearing nonlinearities and housing coupling in the shaft dynamics is presented. Included are descriptions of the structural idealization, method of analysis, and the program input and output.

For any additional information concerning the analysis of this program contact Laverne K. Severud, Dept. 3252, Bldg. 2019A.

NOTATION

- V - Shear (lb)
- M - Moment (in.-lb)
- φ - Slope (rad)
- Y - Deflection (in.)
- $[E]$ - Elasticity Transfer Matrix
- $[F]$ - Mass Transfer Matrix
- $[\Delta]$ - State Vector
- L - Length of Elasticity Element (in.)
- E - Modulus of Elasticity (psi)
- I - Area Moment of Inertia of Cross Section (in.^{4})
- C - Shape Constant for Shear Deflection (in.$^{-2}$)
- G - Modulus of Rigidity (psi)
- W - Weight of Lumped Mass (lb)
- I_J - Polar Mass Moment of Inertia (lb-in.-sec^2)
- I_X - Diametral Mass Moment of Inertia (lb-in.-sec^2)
- K - Spring Constant (lb/in.)
- ω - Shaft Whirl Frequency (cps)
- $\Delta\omega$ - Increment in Frequency (cps)

d - Offset between corresponding stations in two beams (in.)

γ - Forcing function coefficient of ω^2 (lb-sec^2) for static imbalance

β - Forcing function coefficient of ω^2 (in-lb-sec^2) for dynamic imbalance

η - Constant applied lateral load (lbs)

P - Bearing load (lbs)

NOTE: All unprimed quantities refer to top beam and springs between the beams. All primed quantities refer to the bottom beam and springs between it and ground.

I. INTRODUCTION

The classical techniques of calculating shaft critical speeds and bearing loads have been shown by experience to many times yield very crude estimates. The need for high-performance, lightweight turbomachinery has greatly increased in the aircraft and aerospace industries, and, as a result, accurate prediction tools are required for turbomachinery dynamics. In accordance with this need, the computer program presented herein was developed.

This program presents a computerized method of analysis for predicting bearing loads, shaft deflections, and critical speeds for shafts coupled by rolling contact bearings to the machine housing. The bearing nonlinearities, casing as well as rotor dynamics, and rotor-imbalance forcing functions are all included in the system dynamics analysis.

Basically, it has the capability for analyzing the forced-undamped, lateral vibrations of two elastically coupled lumped parameter beams. The program computes the amplitudes of the shears, moments, slopes, and deflection attributable to harmonic forcing functions. Shear deflections, rotary inertia, and gyroscopic effects for rotating shaft analyses are also included.

The analysis is facilitated by a lumped-parameter model using a modified Mkylestad-Thompson transfer-matrix technique.

The bearing is characterized as a spring which may be input as either constant values or load dependent functions defined by

$$K = A \cdot P^B$$

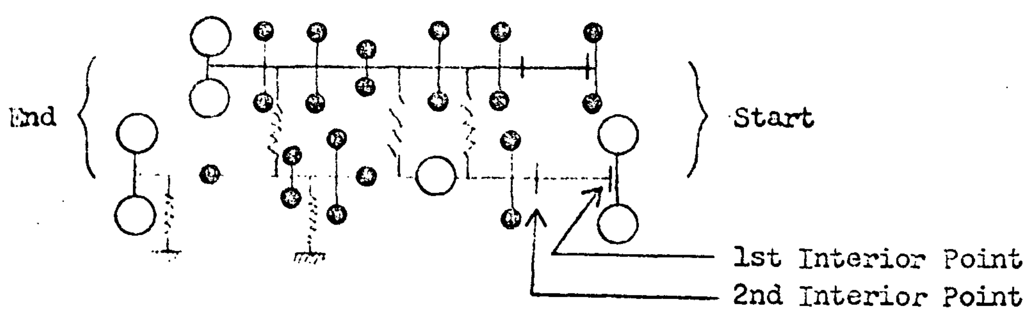
where A and B are constants and P is applied load, or by a table of P vs. K points.

As the bearings have nonlinear load-displacement characteristics, the solution is achieved by iteration. Rotor imbalances allowed by such factors as pilot tolerances and runouts and bearing clearances (allowing conical or cylindrical whirl) determine the forcing-function magnitudes. The computer program initially obtains a solution in which the bearings are treated as linear springs of given spring rates. Then, on the basis of computed bearing reactions, new spring rates are predicted, and another solution of the modified system is made. The iteration is performed a specified number of times and then solution for the next speed level is undertaken. It has been found that about five to eight iterations result in changes in bearing spring rates and bearing reactions that are negligibly small.

In order to facilitate the analysis the machine is characterized as a lumped mass parameter model. Figures 1 and 2 are typical models. Further breakdown of the model into bays is accomplished. A typical bay is shown in Figure 3. The bays consist of massless beam elasticity elements that connect to lumped mass points. More discussion on this type of idealization for dynamic analysis can be found in References 1 through 4.

II. METHOD OF ANALYSIS

Analyses of complex multi-degree-of-freedom systems, of which the rotor-stator system is one, are commonly undertaken using matrix transfer techniques and the methods used herein are based upon this technique. The system is first reduced to an idealized mass-elastic model such as shown in Figure 2 and, then subdivided into bays of the type shown in Figure 3.



Then a column matrix containing all the types of load and deflection variables which can occur in the system is made. This column matrix is called the "state vector." At the start, the state vector consists of the boundary conditions, both known and unknown. Next, a matrix equation is written which transforms the variables of the state vector from their values at the start to their values at the first interior point in the system. Further relating the conditions at the second interior point to the first interior point intern relates the second point to the start. Thus far, two matrix transformation equations are required: the first is for a transformation of variables across the idealized mass (Figures 4 and 5) and the second is for transformation of variables across the idealized elasticity (Figures 6 and 7). The procedure is continued until the last interior point and also the start is related to the end point. Then, by utilizing the boundary conditions at the end, the unknown conditions at the start and at the end can be evaluated. Once all the boundary conditions at the start are known, all interior conditions can be evaluated by re-walking through the system to the end.

At the start of the first bay N = 0, thus

$$\{\Delta_N\} = \{\Delta_o\}$$

Assuming the model starts with elastic elements we have going across the first elements in bay 1.

$$\{\Delta_1''\} = [E_1'] \{\Delta_o\}$$

And across the first lumped masses in bay 1

$$\{\Delta_1'\} = [F_1] \{\Delta_1''\} = [F_1] [E_1'] \{\Delta_o\}$$

Next, across the second elasticity

$$\{\Delta_1\} = [E_1^2] \{\Delta_1'\} = [E_1^2] [F_1] [E_1'] \{\Delta_o\} = [C_1] \{\Delta_o\}$$

In like manner, transformations can be made across each bay, expressing each state vector in terms of the previous state vector, and thus in terms of the initial state vector.

$$\{\Delta_{N_{STA}}\} = \left(\prod_{N=1}^{N=N_{STA}} [C_N]\right) \{\Delta_o\} = [D] \{\Delta_o\}$$

In expanded form we get,

$$\begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ V' \\ M' \\ \Phi' \\ Y' \\ 1 \end{Bmatrix}_{N_{STA}} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} & d_{17} & d_{18} & d_{19} \\ d_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d_{28} & d_{29} \\ d_{31} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d_{38} & d_{39} \\ d_{41} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d_{48} & d_{49} \\ d_{51} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d_{58} & d_{59} \\ d_{61} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d_{68} & d_{69} \\ d_{71} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d_{78} & d_{79} \\ d_{81} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d_{88} & d_{89} \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ V' \\ M' \\ \Phi' \\ Y' \\ 1 \end{Bmatrix}_{0}$$

The resulting above simultaneous equation are reduced to four simultaneous equations by virtue of the four known boundary conditions at each of stations N = 0 and N = NSTA. Then, solving simultaneously the remaining boundary conditions at station N = 0 are evaluated.

For instance, if

$$\{\Delta_{N_{STA}}\} = \begin{Bmatrix} 0 \\ 0 \\ \Phi \\ Y \\ 0 \\ 0 \\ \Phi' \\ Y' \\ 1 \end{Bmatrix}_{N_{STA}} \quad \text{and} \quad \{\Delta_{o}\} = \begin{Bmatrix} 0 \\ 0 \\ \Phi \\ Y \\ 0 \\ 0 \\ \Phi' \\ Y' \\ 1 \end{Bmatrix}_{0}$$

The following system of equations are solved:

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} d_{13} & d_{14} & d_{17} & d_{18} & d_{19} \\ d_{23} & d_{24} & d_{27} & d_{28} & d_{29} \\ d_{53} & d_{54} & d_{57} & d_{58} & d_{59} \\ d_{63} & d_{64} & d_{67} & d_{68} & d_{69} \end{bmatrix} \begin{Bmatrix} \Phi \\ Y \\ \Phi' \\ Y' \\ 1 \end{Bmatrix}_0$$

After the entire conditions of state at N = 0 are known, all other state vectors are evaluated by repeating the chain multiplication.

All $[F_N]$ have elements containing ω. Thus to obtain the dynamic response over the entire shaft speed range of interest, the aforementioned procedure is accomplished first for an initial given shaft whirl frequency ω. Then the procedure is repeated for the additional number of frequencies, separated by the increment $\Delta\omega$, required to define the response of the system in the range of interest.

III. THEORY AND DERIVATION OF EQUATIONS

A. STATE VECTOR

The state vector $[\Delta]_N$ is defined as the column matrix of the shear, moment, slope, and deflection of the beam or beams at the end of bay N. The ninth element of the state vector is the constant one which permits the inclusion of the load constant in the transfer matrices.

$$\{\Delta\}_N = \begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ V' \\ M' \\ \Phi' \\ Y' \\ 1 \end{Bmatrix}$$

B. MASS TRANSFER MATRIX

Figure 4 illustrates a free body diagram of the lumped masses at bay N and the forces and moments which act upon the same. The corresponding equations of equilibrium and compatibility are presented in the same figure.

Some of the terms may require more explanation than is given in the nomenclature. The term $\gamma_N \omega^2$ represents the "forcing function" caused by imbalance or conical whirling mode of motion and its associated centrifugal forces. The stator has an element labeled d_N; this element has infinite stiffness and permits the lumped masses of the rotor and stator at bay N to be located at positions other than immediately above or below the other. The term $(I_{JN} - I_{XN})\ \Phi_N \omega^2$ accounts for what is often called the "gyroscopic effect." This term is largest in the bays that contain inducer or turbine wheels.[1]

The transfer matrix across the rotor and stator mass at bay N is given in Figure 5.

C. ELASTICITY TRANSFER MATRIX

A free body diagram of the elastic elements that connect the adjacent lumped masses is shown in Figure 6. The resulting equations of equilibrium and deformation are also included in Figure 6.

The terms in the equation are straightforward with the possible exception of the term

$$\frac{C_N L_N}{G_N} \cdot V_{NR}$$

This component expresses the deflection resulting from shear which may be of importance in short stubby shafts.

The transfer matrix across the rotor and stator elastic element is illustrated in Figure 7.

D. PROCEDURE FOR NON-LINEAR LOAD-DEFLECTION BEARING SUPPORTS

In applying this program to the lateral vibrations of turbomachinery, the rotor is represented as one beam and the housing as a second beam. The bearings connecting them are represented as springs. However, the

[1] Den Hartog, J. P., Mechanical Vibrations, New York, McGraw-Hill 1956, 4th ed., pp. 252-265 and pp. 270-373.

load deflection relationships of typical turbomachinery bearings are not linear. One relation given by Palmgren [2] for roller bearings is of the form.

$$\delta = C_1 (P^{.9}/\ell^{.8})$$

For a given effective length, ℓ,

$$K = \frac{P}{\delta} = C_2 P^{0.1}$$

which is a non-linear function of P. The force on the bearing, P, is a function of the unbalance in the systems, and is magnified greatly in the neighborhood of resonance. As bearing loads increase, the value of K, or stiffness, increases. The effect upon a plot of bearing load versus shaft speed is to cause a leaning-over of the curve [3].

The computer program treats this effect by calculating a spring rate

$$K = AP^B$$

where A and B are constants. A value of P is assumed (P_o), K is calculated from the above equation and then a forced-vibration analysis is performed. From the resulting deflections, the load in the bearing is calculated P = KY or, P = KY-KY' if working with a flexible housing. This value of P will, in general, not agree with the value P_o upon which K was based. Thus, a new K is calculated and the cycle repeated until the resulting P agrees with the assumed P_o. All of this iteration and convergence is based upon a single frequency ω. Once convergence on K is achieved for a given ω, the frequency is changed until the range of interest is investigated. The projected P_o for subsequent speeds is given by (starting with the 4th ω)

$$P_o{}^i = 3.0(P^{i-1} - P^{i-2}) + P^{i-3}$$

This prediction equation was found to be needed to obtain convergence in a sufficiently small number of iterations as the ω approaches resonant ω.

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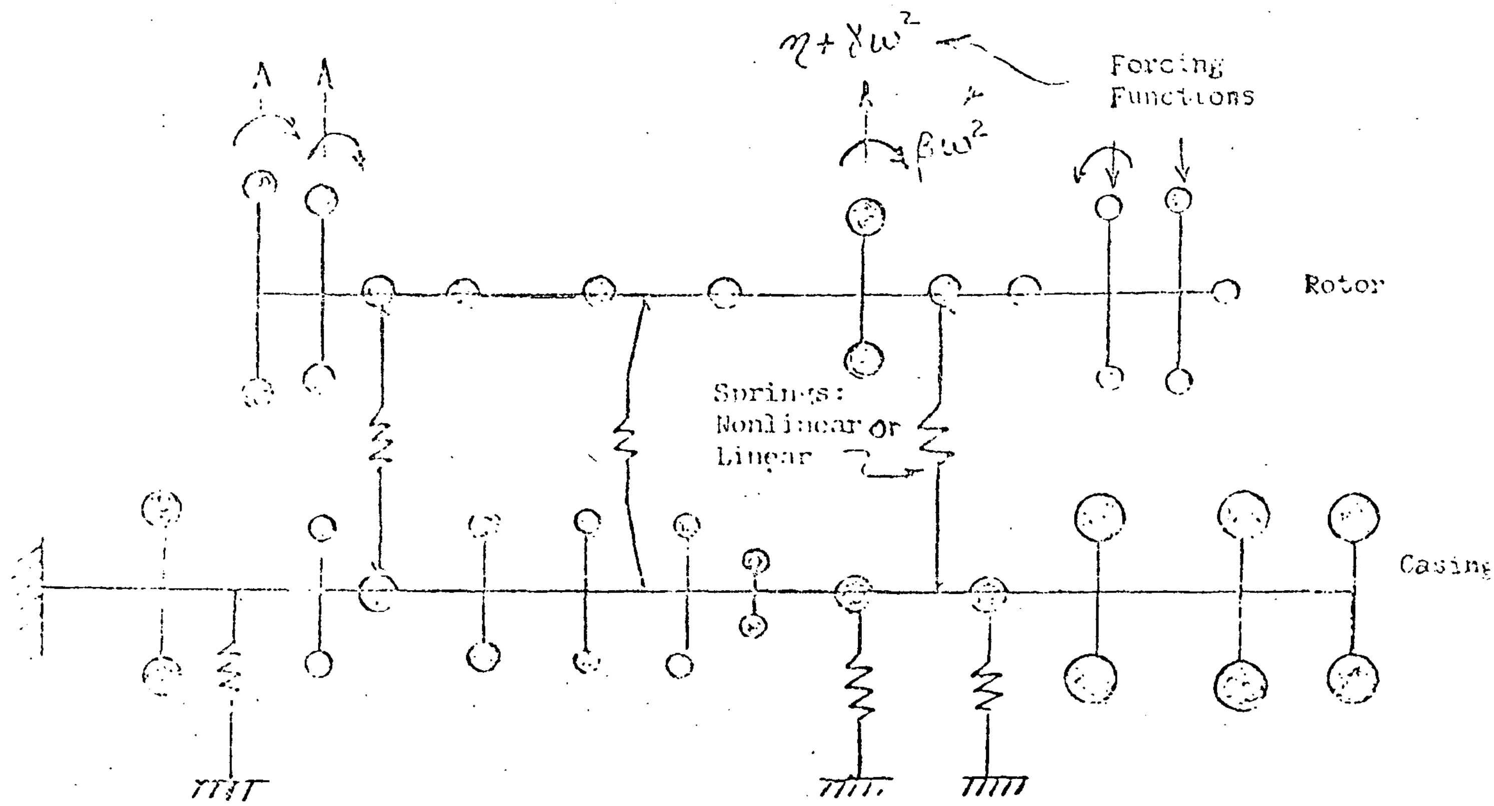
[2] Palmgren, A. Ball and Roller Bearing Engineering, SKF Industries, Inc. 3rd Edition, 1959.

[3] Den Hartog, J. P., Op. Cit.



1. Typical Lumped Mass Rotor-Casing Model

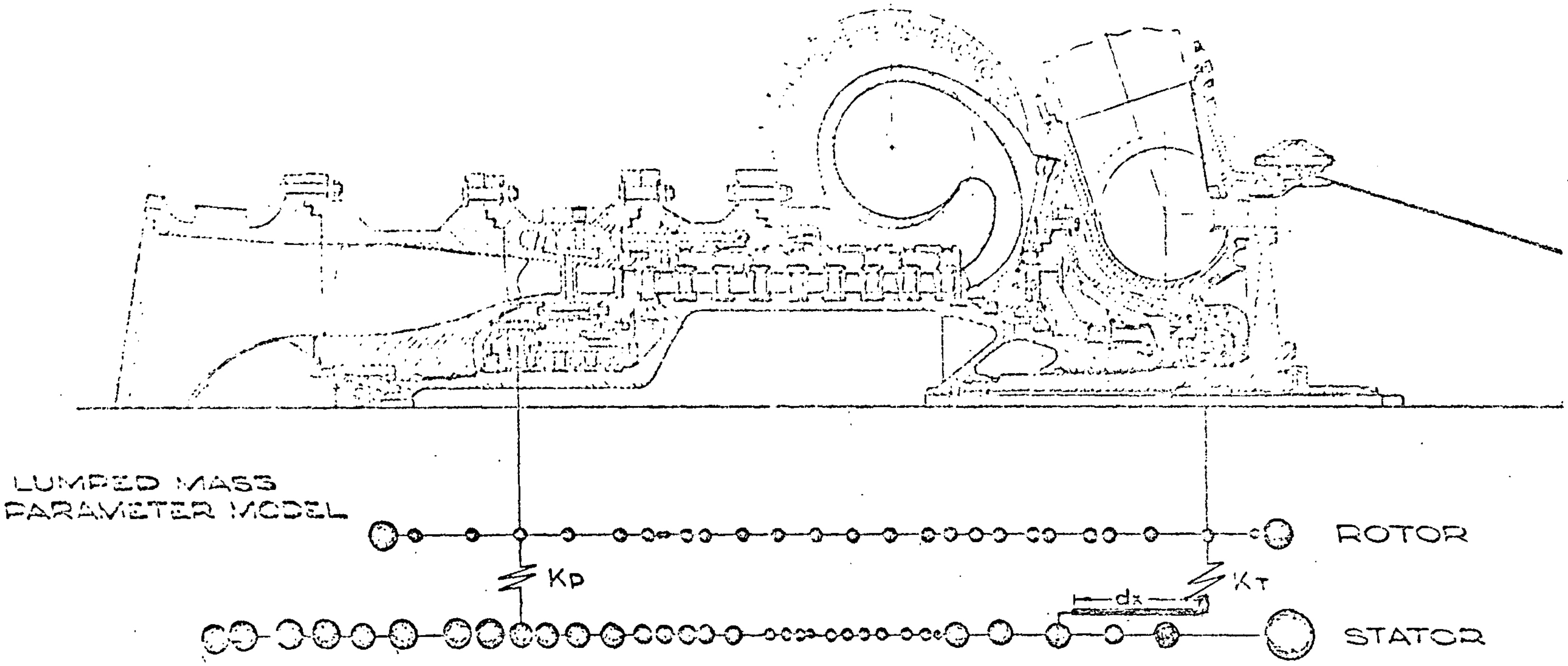


Figure 2

Cross-Section of a Turbopump
And a Typical Lumped-Mass Parameter Model

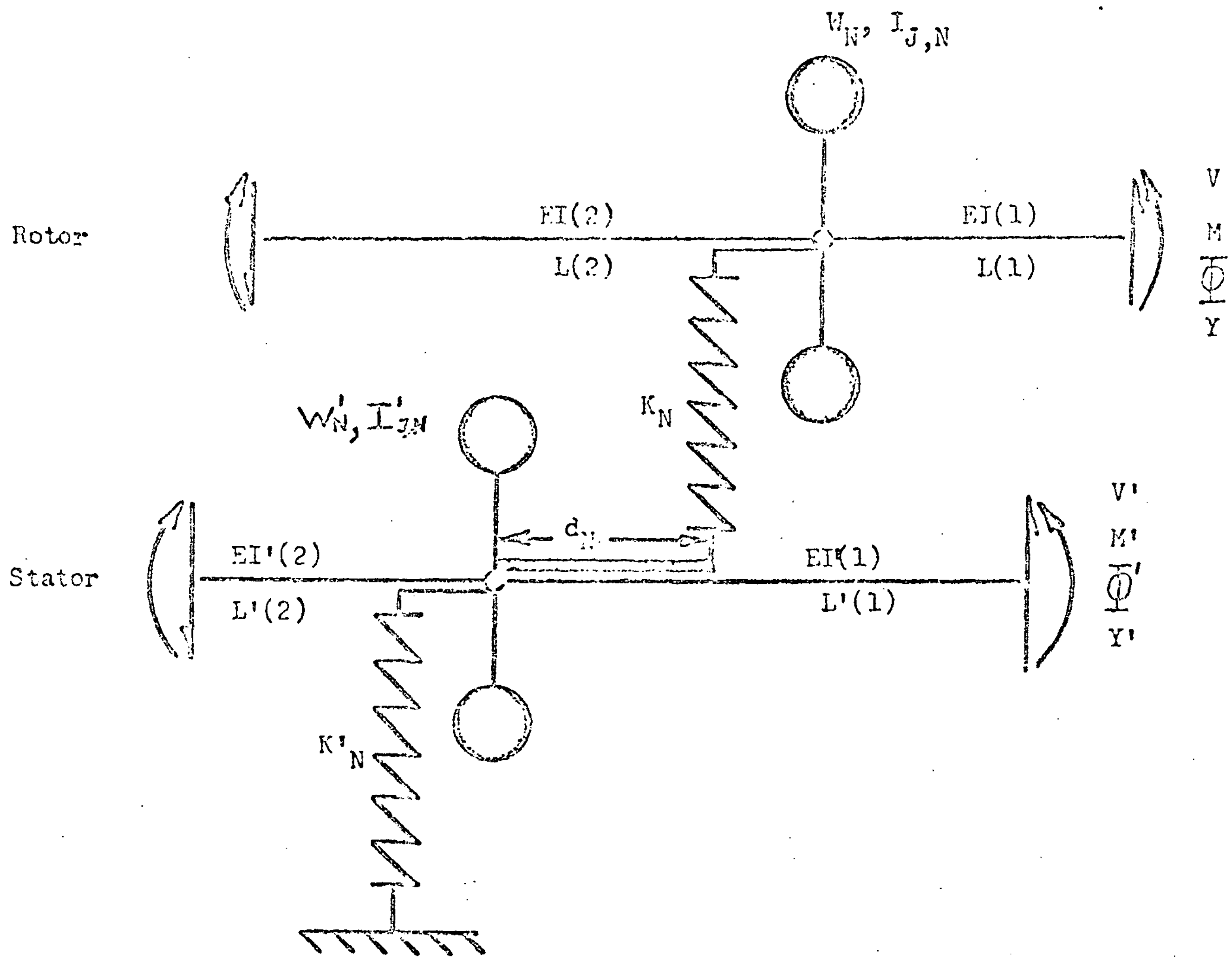
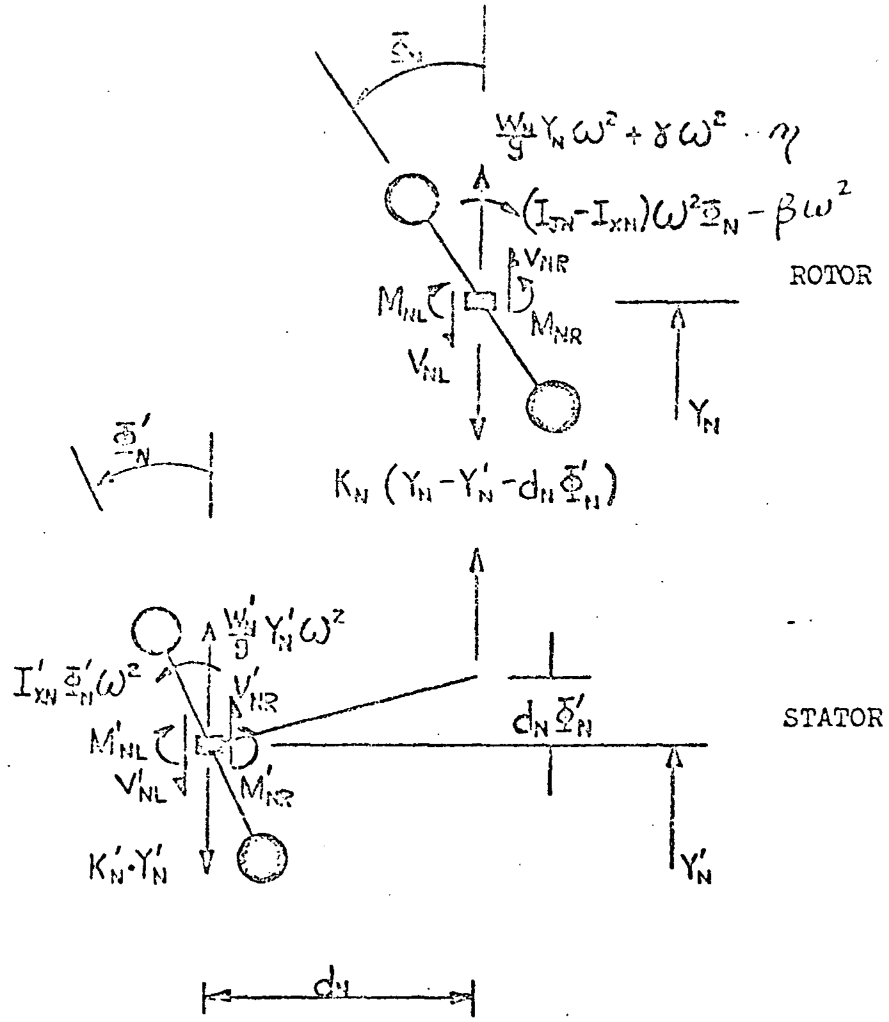


Figure 3

Typical Bay of Lumped Mass Parameter Model



$$V_{NL} = V_{NR} + \frac{W_N}{g} Y_N \omega^2 + \gamma_N \omega^2 - K_N (Y_N - Y'_N - d_N \bar{\Phi}'_N) + \eta$$

$$M_{NL} = M_{NR} - (I_{JN} - I_{XN}) \bar{\Phi}_N \omega^2 + \beta \omega^2$$

$$\bar{\Phi}_{NL} = \bar{\Phi}_{NR} \quad ; \quad Y_{NL} = Y_{NR}$$

$$V'_{NL} = V'_{NR} + \frac{W'_N}{g} Y'_N \omega^2 + K_N (Y_N - Y'_N - d_N \bar{\Phi}'_N) - K'_N Y'_N$$

$$M'_{NL} = M'_{NR} + I'_{XN} \bar{\Phi}'_N \omega^2 + d_N K_N (Y_N - Y'_N - d_N \bar{\Phi}'_N)$$

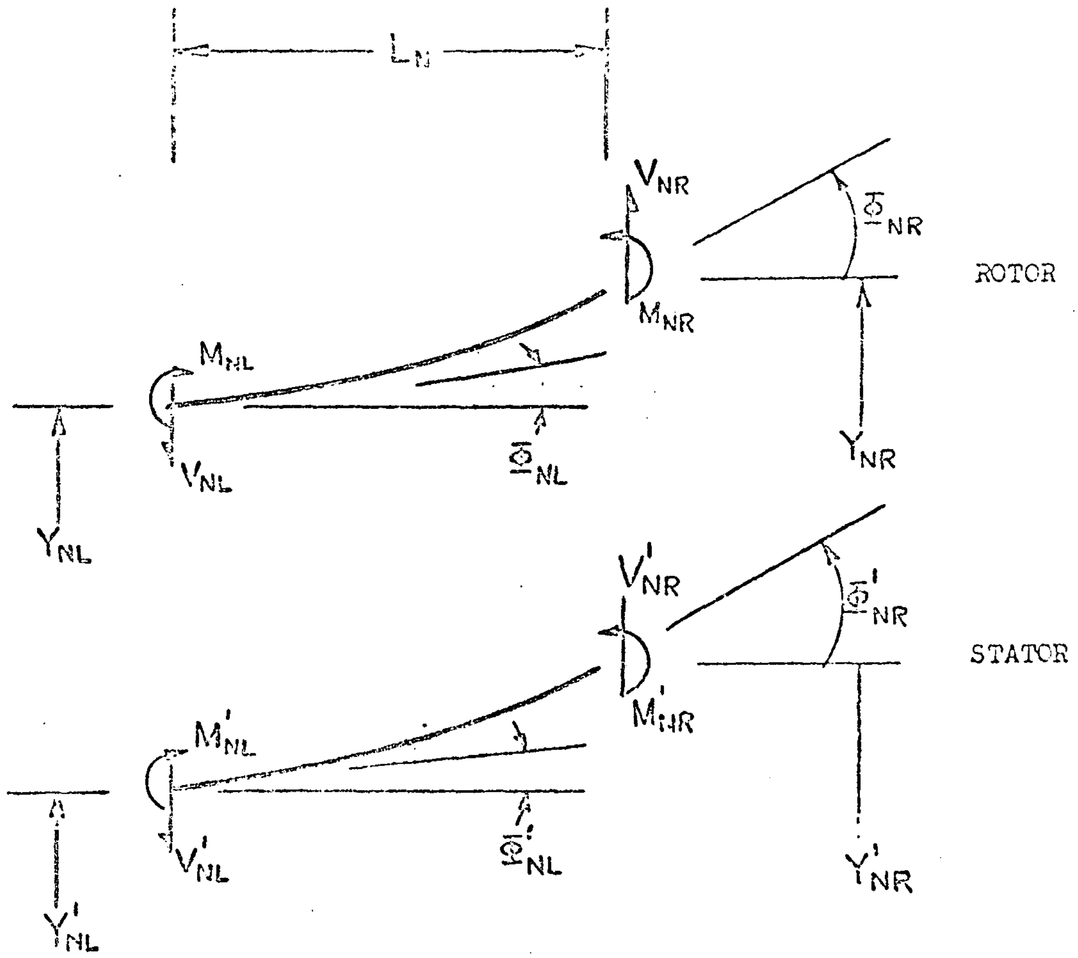
$$\bar{\Phi}'_{NL} = \bar{\Phi}'_{NR} \quad ; \quad Y'_{NL} = Y'_{NR}$$

Figure 4
MASS ELEMENT TRANSFER EQUATION

$$
\begin{bmatrix} V \\ M \\ \bar{\Phi} \\ Y \\ V' \\ M' \\ \bar{\Phi}' \\ Y' \\ 1 \end{bmatrix}_L
=
\begin{bmatrix}
1 & 0 & 0 & \frac{W_N}{g}\omega^2 - K_N & 0 & 0 & d_N K & K_N & \alpha\omega^2\gamma \\
0 & 1 & -(I_{PN} - I_{XN})\omega^2 & 0 & 0 & 0 & 0 & 0 & \beta\omega^2 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & K_N & 1 & 0 & -d_N K_N & \frac{W_N}{g}\omega^2 - K_N - K_N' & 0 \\
0 & 0 & 0 & d_N K_N & 0 & 1 & (I_{XN}'\omega^2 - d_N^2 K_N) & -d_N K_N & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} V \\ M \\ \bar{\Phi} \\ Y \\ V' \\ M' \\ \bar{\Phi}' \\ Y' \\ 1 \end{bmatrix}_R
$$

which may be written more compactly $\left[\Delta\right]_L = \left[T_M\right] \left[\Delta\right]_R$

Figure 5
MASS ELEMENT TRANSFER MATRIX



$$V_{NL} = V_{NR} \; ; \; M_{NL} = M_{NR} + V_{NR} L_N$$

$$\Phi_{NL} = \Phi_{NR} - \frac{L_N^2}{2(EI)_N} V_{NR} - \frac{L_N}{(EI)_N} M_{NR}$$

$$Y_{NL} = Y_{NR} - \Phi_{NR} L_N + \left(\frac{L_N^3}{6(EI)_N} - \frac{C_N L_N}{G_N} \right) V_{NR} + \frac{L_N^2}{2(EI)_N} M_{NR}$$

$$V'_{NL} = V'_{NR} \; ; \; M'_{NL} = M'_{NR} + V'_{NR} L'_N$$

$$\Phi'_{NL} = \Phi'_{NR} - \frac{(L'_N)^2}{2(EI)'_N} V'_{NR} - \frac{L'_N}{(EI)'_N} M'_{NR}$$

$$Y'_{NL} = Y'_{NR} - \Phi'_{NR} L'_N + \left[\frac{(L'_N)^3}{6(EI)'_N} - \frac{C'_N L'_N}{G'_N} \right] V'_{NR} + \frac{(L'_N)^2}{2(EI)'_N} M'_{NR}$$

Figure 6

ELASTICITY ELEMENT TRANSFER EQUATIONS

$$
\begin{bmatrix} V \\ M \\ \Phi \\ Y \\ V' \\ M' \\ \Phi' \\ Y' \\ 1 \end{bmatrix}_L
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_N & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{L_N^2}{2(EI)_N} & -\frac{L_N}{(EI)_N} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{L_N^3}{6(EI)_N} - \frac{C_N L_N}{G_N} & \frac{L_N^2}{2(EI)_N} & -L_N & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L'_N & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{(L'_N)^2}{2(EI)'_N} & -\frac{L'_N}{(EI)'_N} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{(L'_N)^3}{6(EI)'_N} - \frac{C'_N L'_N}{G'_N} & \frac{(L'_N)^2}{2(EI)_N} & -L_N & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} V \\ M \\ \Phi \\ Y \\ V' \\ M' \\ \Phi' \\ Y' \\ 1 \end{bmatrix}_R
$$

which may be written more compactly $\left[\Delta\right]_L = \left[E_N\right] \left[\Delta\right]_R$,

Figure 7

ELASTICITY ELEMENT TRANSFER MATRIX

INPUT

The following cards are needed for the input to the program:

1. Title Card - Columns 1 through 70

   Columns 71 - 72: Number of Stations or Bays

2. Control Card

   Columns 1 - 2: Number of Shaft Speeds at Which Response is to be evaluated.

   3 - 14: Initial shaft speed, ω, Revs per second.

   15 - 26: Increment in shaft speeds, $\Delta\omega$, RPS

   32: Value of a
   35: Value of b
   38: Value of c
   41: Value of d
   — Subscripts of zero quantities of State Vector $\{\Delta_{N_{sta.}}\}$

   44: Value of m
   47: Value of n
   50: Value of O
   53: Value of P
   — Subscripts of non-zero quantities of State Vector $\{\Delta_o\}$.

$$\{\Delta\} = \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ 1 \end{Bmatrix} = \begin{Bmatrix} V \\ M \\ \varphi \\ Y \\ V' \\ M' \\ \varphi' \\ Y' \\ 1 \end{Bmatrix}$$

   56: Spring representation flag (SF) $\begin{cases} 0 = \text{Linear} \\ 1 = K = AP^B \\ -1 = \text{Table of P vs K} \end{cases}$

   59 - 60: Number (NF) of Iterations Desired

3. Bay Data Card No. 1

   Columns 1 - 12: Value of L(1)

   13 - 24: Value of L(2)

   25 - 36: Value of L'(1)

   37 - 48: Value of L'(2)

4. Bay Data Card No. 2

Columns 1 - 12: Value of EI(1)
13 - 24: Value of EI(2)
25 - 36: Value of EI'(1)
37 - 48: Value of EI'(2)

5. Bay Data Card No. 3

Columns 1 - 12: Value of G(1)
13 - 24: Value of G(2)
25 - 36: Value of G'(1)
37 - 48: Value of G'(2)

6. Bay Data Card No. 4

Columns 1 - 12: Value of C(1)
13 - 24: Value of C(2)
25 - 36: Value of C'(1)
37 - 48: Value of C'(2)

7. Bay Data Card No. 5

Columns 1 - 12: Value of I_J
13 - 24: Value of d
25 - 36: Value of γ
37 - 48: Value of I'_X
49 - 60: Value of η
61 - 72: Value of β

8. Bay Data Card No. 6

Columns 1 - 12: Value of W
13 - 24: Value of W'
25 - 36: Value of K
37 - 48: Value of K'

9. Table Input Control Card (only if P vs K Table is to be input)

Columns 1 - 2: Number of Tables

6 - 7: Bay for first P vs K Table data

9 - 10: Bay for second P vs K Table data

12 - 13: Bay for third P vs K Table data

15 - 16: Bay for fourth P vs K Table data

10. Table Input Title Card (only if Table is to be input)

Columns 1 - 70: Title

71 - 72: Number of P vs K Points

11. Table P vs k Data Cards (one card per P vs K Point)

Columns 1 - 12: Value of P (lbs)

13 - 24: Value of K (lbs/inch)

THE ORDER OF THE CARDS IN THE INPUT DECK ARE:

Cards 1 and 2;

Cards 3 through 8 for first bay data;

Repeat Cards 3 through 8 for each additional bay;

If Tables of P vs K input are given, add card 9 and cards of type 10 and 11 as required.

LIMITATIONS

Maximum number of bays: 50

Maximum number of P vs K Tables: 10

OUTPUT

The output consists of the following:

1. Print out of: Title Card, number of roots or shaft speeds at which response is evaluated, initial value of shaft speed ω, $\Delta\omega$, and boundary condition control numbers.

2. Print out of all bay input data

   $L(1) = L(1)$

   $L(2) = L(2)$

   $L(3) = L'(1)$

   $L(4) = L'(2)$

   ETC.

3. For each shaft speed;

   Print out of K_X, P_{oX}, P_X at bearing stations for each iteration, print our value of and characteristic determinant, print out values of V. M, φ, Y, V', M', φ', Y' starting at the top of the print out with values for the "start" of the first bay, then the end of the first bay, then the end of the second bay, etc.

JOB E13104 VIBRATION ANALYSIS

1375.FT/SEC. M.B. SPEED CASE 2 ROTOR SK8705-52 1.85 1.85 1.8 EI=5 NUMBER OF STATIONS 20

NUMBER OF ROOTS 8 OMEGA 300.000 DELTA OMEGA 15.000 1 2 5 6 3 4 7 8 1 7

L(1)	L(2)	L(3)	L(4)
0.0	0.0	0.35000000D 01	0.35000000D 01
0.0	0.0	0.30000000D 01	0.30000000D 01
0.10600000D 01	0.10600000D 01	0.30000000D 01	0.30000000D 01
0.10500000D 01	0.10500000D 01	0.35000000D 01	0.35000000D 01
0.10400000D 01	0.10400000D 01	0.65000000D 00	0.65000000D 00
0.60000000D 00	0.60000000D 00	0.40000000D 00	0.40000000D 00
0.93000000D 00	0.93000000D 00	0.15000000D 01	0.12500000D 01
0.0	0.0	0.0	0.0
0.26600000D 01	0.16600000D 01	0.15500000D 01	0.55000000D 00
0.92000000D 00	0.92000000D 00	0.0	0.0
0.96000000D 00	0.96000000D 00	0.0	0.0
0.12100000D 01	0.12100000D 01	0.0	0.0
0.0	0.0	0.0	0.0
0.27000000D 00	0.27000000D 00	0.14000000D 01	0.14000000D 01
0.30000000D 00	0.61000000D 00	0.14000000D 01	0.14000000D 01
0.40000000D 00	0.55000000D 00	0.14000000D 01	0.14000000D 01
0.40000000D 00	0.40000000D 00	0.75000000D 00	0.21500000D 01
0.50000000D 00	0.50000000D 00	0.15000000D 01	0.15000000D 01
0.50000000D 00	0.50000000D 00	0.50000000D 00	0.50000000D 00
0.30000000D 00	0.50000000D 00	0.37000000D 01	0.37000000D 01

EI(1)	EI(2)	EI(3)	EI(4)
0.0	0.0	0.77000000D 10	0.77000000D 10
0.0	0.0	0.77000000D 10	0.77000000D 10
0.65700000D 08	0.65700000D 08	0.52000000D 10	0.52000000D 10
0.10000000D 09	0.18600000D 09	0.21000000D 10	0.21000000D 10
0.76100000D 09	0.24000000D 10	0.50000000D 07	0.18300000D 12
0.31100000D 11	0.31100000D 11	0.47500000D 11	0.47500000D 11
0.60300000D 09	0.35900000D 09	0.42500000D 12	0.16200000D 12
0.60000000D 09	0.60000000D 09	0.19500000D 12	0.19500000D 12
0.26600000D 09	0.24000000D 09	0.14200000D 12	0.27600000D 12
0.18100000D 09	0.28800000D 09	0.44000000D 11	0.44000000D 11
0.23200000D 09	0.23200000D 09	0.52000000D 11	0.91000000D 11
0.25800000D 09	0.25800000D 09	0.17000000D 11	0.15000000D 11
0.25800000D 09	0.25800000D 09	0.17000000D 11	0.15000000D 11
0.37600000D 09	0.37600000D 09	0.16500000D 11	0.16500000D 11
0.24400000D 09	0.82500000D 09	0.16500000D 11	0.16500000D 11
0.15750000D 11	0.15750000D 11	0.16500000D 11	0.16500000D 11
0.65000000D 10	0.65000000D 10	0.62000000D 11	0.16000000D 11
0.65000000D 10	0.65000000D 10	0.12000000D 11	0.86000000D 11
0.65000000D 10	0.65000000D 10	0.10300000D 11	0.10300000D 11
0.65000000D 10	0.65000000D 10	0.60000000D 11	0.60000000D 11

Forced Undamped Lateral Vibration Analysis of Two Elastically Coupled Beams - Variable Mass and Elasticity

G(1)	G(2)	G(3)	G(4)
0.0	0.0	0.11500000D 08	0.11500000D 08
0.0	0.0	0.11500000D 08	0.11500000D 08
0.62000000D 07	0.62000000D 07	0.11500000D 08	0.11500000D 08
0.62000000D 07	0.62000000D 07	0.11600000D 08	0.11600000D 08
0.62000000D 07	0.62000000D 07	0.11600000D 08	0.11600000D 08
0.62000000D 07	0.62000000D 07	0.11500000D 08	0.11500000D 08
0.62000000D 07	0.62000000D 07	0.11600000D 08	0.11600000D 08
0.62000000D 07	0.62000000D 07	0.11600000D 08	0.11600000D 08
0.62000000D 07	0.62000000D 07	0.11600000D 08	0.11600000D 08
0.62000000D 07	0.62000000D 07	0.11600000D 08	0.11600000D 08
0.62000000D 07	0.62000000D 07	0.11600000D 08	0.11600000D 08
0.90000000D 07	0.90000000D 07	0.11600000D 08	0.11600000D 08
0.90000000D 07	0.90000000D 07	0.11600000D 08	0.11600000D 08
0.11500000D 08	0.11500000D 08	0.11600000D 08	0.11600000D 08
0.11500000D 08	0.11500000D 08	0.11500000D 08	0.11500000D 08
0.11500000D 08	0.11500000D 08	0.11500000D 08	0.11500000D 08
0.11500000D 08	0.11500000D 08	0.11500000D 08	0.11500000D 08
0.11500000D 08	0.11500000D 08	0.11500000D 08	0.11500000D 08
0.11500000D 08	0.11500000D 08	0.11500000D 08	0.11500000D 08
0.11500000D 08	0.11500000D 08	0.11500000D 08	0.11500000D 08

C(1)	C(2)	C(3)	C(4)	P SUB X	FLEX
0.0	0.0	0.30000000D 00	0.30000000D 00	0.0	0.0
0.0	0.0	0.30000000D 00	0.30000000D 00	0.0	0.0
0.85000000D 00	0.85000000D 00	0.30000000D 00	0.30000000D 00	0.0	0.0
0.57000000D 00	0.24000000D 00	0.30000000D 00	0.30000000D 00	0.0	0.0
0.80000000D-01	0.80000000D-01	0.20000000D-01	0.20000000D-01	0.0	0.0
0.10000000D-01	0.10000000D-01	0.80000000D-01	0.80000000D-01	0.0	0.0
0.10000000D 00	0.15000000D 00	0.10000000D-01	0.20000000D-01	0.0	0.0
0.10000000D 00	0.10000000D 00	0.20000000D-01	0.20000000D-01	0.10000000D 04	0.35700000D-07
0.19000000D 00	0.20000000D 00	0.20000000D-01	0.20000000D-01	0.0	0.0
0.22000000D 00	0.22000000D 00	0.10000000D 00	0.10000000D 00	0.0	0.0
0.28000000D 00	0.28000000D 00	0.80000000D-01	0.50000000D-01	0.0	0.0
0.25000000D 00	0.25000000D 00	0.26000000D 00	0.27000000D 00	0.0	0.0
0.25000000D 00	0.25000000D 00	0.26000000D 00	0.27000000D 00	0.15000000D 04	0.68500000D-07
0.17500000D 00	0.17500000D 00	0.20000000D-01	0.25000000D-01	0.0	0.0
0.23000000D 00	0.10000000D 00	0.28000000D-01	0.28000000D-01	0.0	0.0
0.20000000D-01	0.20000000D-01	0.25000000D-01	0.32000000D-01	0.0	0.0
0.90000000D-01	0.90000000D-01	0.17000000D-01	0.27000000D 00	0.0	0.0
0.90000000D-01	0.90000000D-01	0.29000000D 00	0.40000000D-01	0.0	0.0
0.90000000D-01	0.90000000D-01	0.30000000D 00	0.30000000D 00	0.0	0.0
0.90000000D-01	0.90000000D-01	0.70000000D-01	0.70000000D-01	0.0	0.0

I SUBJ1	DX	GAMMA(X)	I SUB J2	ETA	BETA
0.0	0.0	0.0	0.13700000D 01	0.0	0.0
0.0	0.0	0.0	0.13700000D 01	0.0	0.0
0.47200000D-01	0.0	0.0	0.78000000D 00	0.0	0.0
0.91300000D-01	0.0	0.0	0.27000000D 00	0.0	0.0
0.32020000D 00	0.0	-0.82400000D-04	0.63900000D 01	0.0	0.0
0.21317000D 01	0.0	-0.38800000D-04	0.10400000D 01	0.0	0.0
0.37700000D-01	0.0	0.0	0.21750000D 02	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.52400000D-01	0.0	0.0	0.59700000D 01	0.0	0.0
0.26600000D-01	0.0	0.0	0.0	0.0	0.0
0.25300000D-01	0.0	0.0	0.0	0.0	0.0
0.39000000D-01	0.0	0.0	0.0	0.0	0.0
0.0	-0.84000000D 01	0.0	0.0	0.0	0.0
0.12900000D-01	0.0	0.0	0.93000000D 01	0.0	0.0
0.24200000D-01	0.0	0.0	0.49800000D 01	0.0	0.0
0.22150000D 01	0.0	0.98000000D-04	0.29500000D 01	0.0	0.0
0.20000000D 00	0.0	0.0	0.34000000D 01	0.0	0.0
0.21600000D 01	0.0	0.11300000D-03	0.30000000D 01	0.0	0.0
0.20000000D 00	0.0	0.0	0.30000000D 00	0.0	0.0
0.21600000D 01	0.0	0.13800000D-03	0.57000000D 01	0.0	0.0

W SUB N1	W SUB N2	K SUB N1	K SUB N2	A SUB X	B SUB X
0.0	0.70000000D 02	0.0	0.0	0.0	0.0
0.0	0.80000000D 02	0.0	0.0	0.0	0.0
0.43600000D 01	0.80000000D 02	0.0	0.0	0.0	0.0
0.86900000D 01	0.50000000D 02	0.0	0.0	0.0	0.0
0.15180000D 02	0.50000000D 02	0.0	0.0	0.0	0.0
0.40400000D 02	0.35000000D 03	0.0	0.0	0.0	0.0
0.73000000D 01	0.13700000D 03	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.11930000D 06	0.31100000D 01
0.14170000D 02	0.34700000D 02	0.0	0.0	0.0	0.0
0.65300000D 01	0.0	0.0	0.0	0.0	0.0
0.61700000D 01	0.0	0.0	0.0	0.0	0.0
0.10430000D 02	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.11930000D 06	0.31100000D 01
0.27300000D 01	0.67900000D 02	0.0	0.0	0.0	0.0
0.48500000D 01	0.58500000D 02	0.0	0.0	0.0	0.0
0.51200000D 02	0.14550000D 03	0.0	0.0	0.0	0.0
0.81000000D 01	0.17600000D 03	0.0	0.0	0.0	0.0
0.46000000D 02	0.65000000D 02	0.0	0.0	0.0	0.0
0.98000000D 01	0.40000000D 02	0.0	0.0	0.0	0.0
0.45000000D 02	0.10000000D 03	0.0	0.0	0.0	0.0

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011201D 08	0.10000000D 04	0.11879500D 04
13	0.14598540D 08	0.15000000D 04	0.99026628D 03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011202D 08	0.10939750D 04	0.11879500D 04
13	0.14598540D 08	0.12451331D 04	0.99026631D 03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011202D 08	0.11409625D 04	0.11879500D 04
13	0.14598540D 08	0.11176997D 04	0.99026633D 03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.11644562D 04	0.11879500D 04
13	0.14598539D 08	0.10539830D 04	0.99026635D 03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.11762031D 04	0.11879500D 04
13	0.14598539D 08	0.10221247D 04	0.99026636D 03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.11820765D 04	0.11879500D 04
13	0.14598539D 08	0.10061955D 04	0.99026636D 03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.11850132D 04	0.11879500D 04
13	0.14598539D 08	0.99823094D 03	0.99026636D 03

OMEGA = 0.30000000D 03 DETERM = -0.25951035D 35

V	M	PHI	Y
0.0	0.0	-0.22547333D-04	-0.28229644D-03
0.0	0.0	-0.22547333D-04	-0.28229644D-03
0.0	0.0	-0.22547333D-04	-0.28229644D-03
-0.10369138D 02	-0.72100015D 01	-0.22519673D-04	-0.23298822D-03
-0.27037398D 02	-0.39224715D 02	-0.22245509D-04	-0.18385643D-03
-0.34221856D 03	-0.39802528D 03	-0.22077326D-04	-0.13277708D-03
-0.52440085D 03	-0.75085525D 03	-0.22056215D-04	-0.10545726D-03
-0.52962366D 03	-0.17283491D 04	-0.16682742D-04	-0.48418364D-04
0.65832630D 03	-0.17283491D 04	-0.16682742D-04	-0.48418364D-04
0.64881486D 03	0.11013498D 04	-0.12047922D-04	-0.91935184D-04
0.64286535D 03	0.22915067D 04	-0.25538388D-04	-0.99560130D-04
0.63730272D 03	0.35237309D 04	-0.49612489D-04	-0.84550564D-04
0.63392247D 03	0.50713288D 04	-0.89932243D-04	0.38634926D-04
-0.19248298D 04	0.50713288D 04	-0.89932243D-04	0.38634926D-04
-0.19230383D 04	0.40366847D 04	-0.96472399D-04	0.10484063D-03
-0.19165197D 04	0.22993876D 04	-0.10321323D-03	0.21854911D-03
-0.14452431D 04	0.15505695D 04	-0.10332992D-03	0.31937445D-03
-0.14180135D 04	0.47874940D 03	-0.10345447D-03	0.41105993D-03
-0.81823313D 03	0.15467155D 03	-0.10349166D-03	0.52328259D-03
-0.76607800D 03	-0.56394437D 03	-0.10345917D-03	0.63296623D-03
0.22140512D-09	-0.64329697D-09	-0.10342784D-03	0.71751209D-03

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	-0.36342773D-05	-0.37224619D-04
-0.15787613D 02	-0.72947175D 02	-0.36136779D-05	-0.10371924D-04
-0.14570638D 02	-0.18142893D 03	-0.35138591D-05	0.13432207D-04
0.37961990D 01	-0.22316548D 03	-0.32645448D-05	0.34637430D-04
0.24542494D 02	-0.12676562D 03	-0.26208163D-05	0.52486318D-04
0.47106288D 02	0.21091259D 03	0.12821099D-04	0.40644424D-04
0.16109465D 03	0.34156238D 03	0.12816639D-04	0.29809814D-04
0.17418014D 03	0.17912605D 04	0.12802026D-04	-0.60085476D-05
-0.10137698D 04	0.17912605D 04	0.12802026D-04	-0.60085476D-05
-0.10211575D 04	-0.70398768D 02	0.12790630D-04	-0.29198324D-04
-0.10211575D 04	-0.70398768D 02	0.12790630D-04	-0.29198324D-04
-0.10211575D 04	-0.70398768D 02	0.12790630D-04	-0.29198324D-04
-0.10211575D 04	-0.70398768D 02	0.12790630D-04	-0.29198324D-04
0.15375949D 04	-0.21563918D 05	0.12790630D-04	-0.29198324D-04
0.15050635D 04	-0.16824109D 05	0.16045863D-04	-0.78019172D-04
0.14476828D 04	-0.12382668D 05	0.18520606D-04	-0.13665920D-03
0.12231197D 04	-0.84393092D 04	0.20273982D-04	-0.20030264D-03
0.87376249D 03	-0.53972961D 04	0.21222001D-04	-0.30576429D-03
0.65174363D 03	-0.28765098D 04	0.21873445D-04	-0.40728203D-03
0.49463253D 03	-0.22798660D 04	0.22121848D-04	-0.44423733D-03
-0.63835159D-10	0.97293196D-09	0.22206010D-04	-0.61958089D-03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.11864816D 04	0.13558367D 04
13	0.14598539D 08	0.99424865D 03	0.14816161D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.12711591D 04	0.13558366D 04
13	0.14598540D 08	0.12379324D 04	0.14816160D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.13134979D 04	0.13558366D 04
13	0.14598540D 08	0.13597742D 04	0.14816160D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.13346673D 04	0.13558366D 04
13	0.14598540D 08	0.14206951D 04	0.14816160D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.13452520D 04	0.13558366D 04
13	0.14598540D 08	0.14511556D 04	0.14816160D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.13505443D 04	0.13558366D 04
13	0.14598540D 08	0.14653858D 04	0.14816160D 04

STA X K SUB X P SUB OX P SUB X

	[illegible]	[illegible]	0.13558366D 04
13	0.14598540D 08	0.14740009D 04	0.14816160D 04

OMEGA = 0.315000000D 03 DETERM = -0.31223011D 35

V	M	PHI	Y
0.0	0.0	-0.27882655D-04	-0.34492796D-03
0.0	0.0	-0.27882655D-04	-0.34492796D-03
0.0	0.0	-0.27882655D-04	-0.34492796D-03
-0.13952712D 02	-0.96345391D 01	-0.27846521D-04	-0.28378716D-03
-0.36286979D 02	-0.52490747D 02	-0.27479669D-04	-0.22283499D-03
-0.38892247D 03	-0.46036309D 03	-0.27270294D-04	-0.16016188D-03
-0.59970915D 03	-0.82591734D 03	-0.27246698D-04	-0.12649397D-03
-0.60658957D 03	-0.19440030D 04	-0.21237504D-04	-0.57370183D-04
0.74924708D 03	-0.19440030D 04	-0.21237504D-04	-0.57370183D-04
0.73789171D 03	0.12763089D 04	-0.16354150D-04	-0.96321878D-04
0.73114288D 03	0.26303805D 04	-0.31894977D-04	-0.99912024D-04
0.72517614D 03	0.40328304D 04	-0.59478736D-04	-0.77191675D-04
0.72334267D 03	0.57978287D 04	-0.10558897D-03	0.70503620D-04
-0.22948601D 04	0.57978287D 04	-0.10558897D-03	0.70503620D-04
-0.22918410D 04	0.45649543D 04	-0.11303003D-03	0.14845561D-03
-0.22821500D 04	0.24964974D 04	-0.12058056D-03	0.28190711D-03
-0.17259089D 04	0.16810785D 04	-0.12070765D-03	0.39975895D-03
-0.16886375D 04	0.40989263D 03	-0.12083587D-03	0.50707541D-03
-0.97801230D 03	0.98982393D 02	-0.12086135D-03	0.63835617D-03
-0.90814233D 03	-0.74941496D 03	-0.12080997D-03	0.76658270D-03
0.60509819D-09	0.15361366D-08	-0.12076909D-03	0.86533658D-03

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	-0.33907286D-05	-0.34699300D-04
-0.16217570D 02	-0.74958270D 02	-0.33695570D-05	-0.95129905D-05
-0.14746951D 02	-0.18572740D 03	-0.32671317D-05	0.12867025D-04
0.44508759D 01	-0.22623180D 03	-0.30128494D-05	0.32538890D-04
0.25778519D 02	-0.12323041D 03	-0.23682068D-05	0.48438748D-04
0.48599028D 02	0.23957208D 03	0.12561810D-04	0.36758784D-04
0.16082497D 03	0.37450914D 03	0.12556828D-04	0.26128404D-04
0.17067630D 03	0.18987860D 04	0.12541252D-04	-0.89668129D-05
-0.11851603D 04	0.18987860D 04	0.12541252D-04	-0.89668129D-05
-0.11940432D 04	-0.30189731D 03	0.12530499D-04	-0.30987079D-04
-0.11940432D 04	-0.30189731D 03	0.12530499D-04	-0.30987079D-04
-0.11940432D 04	-0.30189731D 03	0.12530499D-04	-0.30987079D-04
-0.11940432D 04	-0.30189731D 03	0.12530499D-04	-0.30987079D-04
0.18241595D 04	-0.25654800D 05	0.12530499D-04	-0.30987079D-04
0.17866738D 04	-0.20067789D 05	0.16407765D-04	-0.81526370D-04
0.17203493D 04	-0.14806730D 05	0.19362876D-04	-0.14377491D-03
0.14590950D 04	-0.10118415D 05	0.21462220D-04	-0.21204035D-03
0.10510251D 04	-0.64770006D 04	0.22600172D-04	-0.33039881D-03
0.78415374D 03	-0.34502845D 04	0.23381700D-04	-0.44372671D-03
0.59513640D 03	-0.27329762D 04	0.23679565D-04	-0.48525400D-03
-0.49766413D-09	-0.89954942D-08	0.23780203D-04	-0.67448670D-03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011203D 08	0.13545136D 04	0.15477879D 04
13	0.14598540D 08	0.14778085D 04	0.20774395D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.14511507D 04	0.15477879D 04
13	0.14598540D 08	0.17776240D 04	0.20774395D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.14994693D 04	0.15477879D 04
13	0.14598540D 08	0.19275317D 04	0.20774395D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.15236286D 04	0.15477879D 04
13	0.14598540D 08	0.20024856D 04	0.20774395D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.15357082D 04	0.15477879D 04
13	0.14598540D 08	0.20399625D 04	0.20774395D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.15417481D 04	0.15477879D 04
13	0.14598540D 08	0.20587010D 04	0.20774395D 04

STA X K SUB X P SUB OX P SUB X

13	0.14598540D 08	0.20680702D 04	0.20774395D 04

OMEGA = 0.33000000D 03 DETERM = -0.36607681D 35

V	M	PHI	Y
0.0	0.0	-0.34006594D-04	-0.41686753D-03
0.0	0.0	-0.34006594D-04	-0.41686753D-03
0.0	0.0	-0.34006594D-04	-0.41686753D-03
-0.18491075D 02	-0.12699846D 02	-0.33959813D-04	-0.34208324D-03
-0.47983691D 02	-0.69260900D 02	-0.33475753D-04	-0.26749079D-03
-0.44147417D 03	-0.53239147D 03	-0.33215776D-04	-0.19153896D-03
-0.68530330D 03	-0.90416606D 03	-0.33189472D-04	-0.15060453D-03
-0.69426714D 03	-0.21820929D 04	-0.26487048D-04	-0.67636315D-04
0.85352074D 03	-0.21820929D 04	-0.26487048D-04	-0.67636315D-04
0.83999466D 03	0.14862719D 04	-0.21475723D-04	-0.10090780D-03
0.83239557D 03	0.30284154D 04	-0.39445210D-04	-0.99453594D-04
0.82613370D 03	0.46264368D 04	-0.71132898D-04	-0.67323742D-04
0.82672055D 03	0.65423482D 04	-0.12398096D-03	0.10938136D-03
-0.27443996D 04	0.66423482D 04	-0.12398096D-03	0.10938136D-03
-0.27396946D 04	0.51687685D 04	-0.13246189D-03	0.20124841D-03
-0.27257416D 04	0.26985478D 04	-0.14092165D-03	0.35801481D-03
-0.20670517D 04	0.18138519D 04	-0.14105953D-03	0.49583037D-03
-0.20166491D 04	0.30173432D 03	-0.14118910D-03	0.62152549D-03
-0.11722333D 04	0.18268792D 02	-0.14119748D-03	0.77519224D-03
-0.10794240D 04	-0.98617083D 03	-0.14112124D-03	0.92517453D-03
0.52011728D-09	0.75067419D-09	-0.14106825D-03	0.10405718D-02

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	-0.30832912D-05	-0.31599541D-04
-0.16221244D 02	-0.74934641D 02	-0.30621333D-05	-0.85649322D-05
-0.14583990D 02	-0.18515822D 03	-0.29598416D-05	0.11958250D-04
0.48439527D 01	-0.22390325D 03	-0.27070319D-05	0.29746235D-04
0.26084819D 02	-0.11837806D 03	-0.20746105D-05	0.43479858D-04
0.48378445D 02	0.26552219D 03	0.12211566D-04	0.32044404D-04
0.15371982D 03	0.40095110D 03	0.12206131D-04	0.21714782D-04
0.15861445D 03	0.19709945D 04	0.12189866D-04	-0.12380286D-04
-0.13891734D 04	0.19709945D 04	0.12189866D-04	-0.12380286D-04
-0.13998205D 04	-0.63950840D 03	0.12180610D-04	-0.32923240D-04
-0.13998205D 04	-0.63950840D 03	0.12180610D-04	-0.32923240D-04
-0.13998205D 04	-0.63950840D 03	0.12180610D-04	-0.32923240D-04
-0.13998205D 04	-0.63950840D 03	0.12180610D-04	-0.32923240D-04
0.21712996D 04	-0.30636918D 05	0.12180610D-04	-0.32923240D-04
0.21282151D 04	-0.24031807D 05	0.16816610D-04	-0.85454888D-04
0.20515856D 04	-0.17779092D 05	0.20359650D-04	-0.15200484D-03
0.17473560D 04	-0.12184773D 05	0.22883969D-04	-0.22582393D-03
0.12699152D 04	-0.78073984D 04	0.24256001D-04	-0.35997172D-03
0.94774063D 03	-0.41570190D 04	0.25197776D-04	-0.48774488D-03
0.71932571D 03	-0.32907413D 04	0.25556545D-04	-0.53487353D-03
-0.26875568D-09	-0.45080242D-08	0.25677411D-04	-0.74091384D-03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.17617747D 04	0.17698419D 04
13	0.14598540D 08	0.27790878D 04	0.28106816D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.17658083D 04	0.17698419D 04
13	0.14598540D 08	0.27948847D 04	0.28106816D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.17678251D 04	0.17698419D 04
13	0.14598540D 08	0.28027832D 04	0.28106816D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.17688335D 04	0.17698419D 04
13	0.14598540D 08	0.28067324D 04	0.28106816D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.17693377D 04	0.17698419D 04
13	0.14598540D 08	0.28087070D 04	0.28106816D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.17695898D 04	0.17698419D 04
13	0.14598540D 08	0.28096943D 04	0.28106816D 04

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8	0.28011204D 08	0.17697159D 04	0.17698419D 04
13	0.14598540D 08	0.28101880D 04	0.28106816D 04

OMEGA = 0.34500000D 03 DETERM = -0.41824292D 35

V	M	PHI	Y
0.0	0.0	-0.41111221D-04	-0.50042359D-03
0.0	0.0	-0.41111221D-04	-0.50042359D-03
0.0	0.0	-0.41111221D-04	-0.50042359D-03
-0.24244970D 02	-0.16581653D 02	-0.41051012D-04	-0.40973974D-03
-0.62793179D 02	-0.90492393D 02	-0.40418511D-04	-0.31923921D-03
-0.50106279D 03	-0.61634238D 03	-0.40096043D-04	-0.22791752D-03
-0.78339313D 03	-0.98553466D 03	-0.40066773D-04	-0.17857616D-03
-0.79498392D 03	-0.24466963D 04	-0.32604368D-04	-0.79584506D-04
0.97485799D 03	-0.24466963D 04	-0.32604368D-04	-0.79584506D-04
0.95875393D 03	0.17431535D 04	-0.27655782D-04	-0.10576113D-03
0.95026082D 03	0.35042915D 04	-0.48555602D-04	-0.97924745D-04
0.94388125D 03	0.53303835D 04	-0.85125569D-04	-0.54134160D-04
0.94811488D 03	0.76403758D 04	-0.14594544D-03	0.15748054D-03
-0.32981403D 04	0.76403758D 04	-0.14594544D-03	0.15748054D-03
-0.32911238D 04	0.58704343D 04	-0.15564667D-03	0.26609778D-03
-0.32714322D 04	0.29059744D 04	-0.16514386D-03	0.45084570D-03
-0.24873719D 04	0.19487768D 04	-0.16529299D-03	0.61245884D-03
-0.24196987D 04	0.14137217D 03	-0.16542078D-03	0.76011981D-03
-0.14115075D 04	-0.95627467D 02	-0.16540491D-03	0.94051661D-03
-0.12887930D 04	-0.12903653D 04	-0.16529593D-03	0.11164477D-02
0.39324277D-09	0.44639137D-09	-0.16522746D-03	0.12516663D-02

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	-0.26845108D-05	-0.27644679D-04
-0.15548917D 02	-0.71702797D 02	-0.26642871D-05	-0.74615960D-05
-0.13896290D 02	-0.17695148D 03	-0.25664430D-05	0.10579303D-04
0.48010443D 01	-0.21322539D 03	-0.23251606D-05	0.25989435D-04
0.24938176D 02	-0.11165497D 03	-0.17249600D-05	0.37277894D-04
0.45558957D 02	0.28657081D 03	0.11735582D-04	0.26202531D-04
0.13665140D 03	0.41679311D 03	0.11729813D-04	0.16309174D-04
0.13421513D 03	0.19881560D 04	0.11713286D-04	-0.16401148D-04
-0.16356268D 04	0.19881560D 04	0.11713286D-04	-0.16401148D-04
-0.16483723D 04	-0.11253034D 04	0.11706760D-04	-0.35051156D-04
-0.16483723D 04	-0.11253034D 04	0.11706760D-04	-0.35051156D-04
-0.16483723D 04	-0.11253034D 04	0.11706760D-04	-0.35051156D-04
-0.16483723D 04	-0.11253034D 04	0.11706760D-04	-0.35051156D-04
0.25978829D 04	-0.36793847D 05	0.11706760D-04	-0.35051156D-04
0.25484386D 04	-0.28947728D 05	0.17281897D-04	-0.89918653D-04
0.24598226D 04	-0.21477821D 05	0.21555165D-04	-0.16166703D-03
0.21046613D 04	-0.14765514D 05	0.24609263D-04	-0.24225483D-03
0.15442064D 04	-0.94711079D 04	0.26274076D-04	-0.39601992D-03
0.11528397D 04	-0.50405114D 04	0.27416192D-04	-0.54172347D-03
0.87505890D 03	-0.39875890D 04	0.27851078D-04	-0.59581664D-03
-0.48402171D-09	-0.98007149D-08	0.27997150D-04	-0.82249472D-03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.20250164D 04	0.20307005D 04
13	0.14598540D 08	0.36908483D 04	0.37297354D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.20278585D 04	0.20307005D 04
13	0.14598540D 08	0.37102918D 04	0.37297354D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.20292795D 04	0.20307005D 04
13	0.14598540D 08	0.37200136D 04	0.37297354D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.20299900D 04	0.20307005D 04
13	0.14598540D 08	0.37248745D 04	0.37297354D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.20303452D 04	0.20307005D 04
13	0.14598540D 08	0.37273049D 04	0.37297354D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.20305229D 04	0.20307005D 04
13	0.14598540D 08	0.37285201D 04	0.37297354D 04

STA X K SUB X P SUB OX P SUB X

STA X	K SUB X	P SUB OX	P SUB X
	[illegible]	[illegible]	0.20307005D 04
13	0.14598540D 08	0.37291278D 04	0.37297354D 04

OMEGA = 0.36000000D 03 DETERM = -0.46511111D 35

V	M	PHI	Y
0.0	0.0	-0.49471429D-04	-0.59890224D-03
0.0	0.0	-0.49471429D-04	-0.59890224D-03
0.0	0.0	-0.49471429D-04	-0.59890224D-03
-0.31577613D 02	-0.21525206D 02	-0.49394162D-04	-0.48942712D-03
-0.81642867D 02	-0.11752023D 03	-0.48572621D-04	-0.38011132D-03
-0.56934594D 03	-0.71533158D 03	-0.48172312D-04	-0.27074766D-03
-0.89706391D 03	-0.10699661D 04	-0.48139766D-04	-0.21154095D-03
-0.91199082D 03	-0.27435438D 04	-0.39837578D-04	-0.93748742D-04
0.11187097D 04	-0.27435438D 04	-0.39837578D-04	-0.93748742D-04
0.10995030D 04	0.20647130D 04	-0.35249846D-04	-0.11099260D-03
0.10900834D 04	0.40857080D 04	-0.59765042D-04	-0.94954875D-04
0.10838658D 04	0.61829042D 04	-0.10226820D-03	-0.36437912D-04
0.10934953D 04	0.88443175D 04	-0.17271738D-03	0.21805143D-03
-0.39923208D 04	0.88443175D 04	-0.17271738D-03	0.21805143D-03
-0.39821207D 04	0.67030115D 04	-0.18388070D-03	0.34723547D-03
-0.39546440D 04	0.31197396D 04	-0.19458625D-03	0.56624471D-03
-0.30143001D 04	0.20859005D 04	-0.19474718D-03	0.75681229D-03
-0.29236755D 04	-0.89915296D 02	-0.19486889D-03	0.93126420D-03
-0.17113938D 04	-0.25456770D 03	-0.19481908D-03	0.11442337D-02
-0.15492546D 04	-0.16855910D 04	-0.19466672D-03	0.13517528D-02
0.44377657D-09	0.13289991D-08	-0.19457820D-03	0.15110665D-02

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	-0.21535539D-05	-0.22421253D-04
-0.13808443D 02	-0.63424827D 02	-0.21357084D-05	-0.61104273D-05
-0.12395908D 02	-0.15677829D 03	-0.20490895D-05	0.85339848D-05
0.40437680D 01	-0.18960844D 03	-0.18350243D-05	0.20861155D-04
0.21524178D 02	-0.10223532D 03	-0.12976335D-05	0.29343281D-04
0.38780451D 02	0.29932660D 03	0.11082543D-04	0.18792765D-04
0.10489346D 03	0.41575328D 03	0.11076633D-04	0.95291422D-05
0.91783466D 02	0.19202559D 04	0.11060514D-04	-0.21252734D-04
-0.19389170D 04	0.19202559D 04	0.11060514D-04	-0.21252734D-04
-0.19541889D 04	-0.18221661D 04	0.11058516D-04	-0.37435443D-04
-0.19541889D 04	-0.18221661D 04	0.11058516D-04	-0.37435443D-04
-0.19541889D 04	-0.18221661D 04	0.11058516D-04	-0.37435443D-04
-0.19541889D 04	-0.18221661D 04	0.11058516D-04	-0.37435443D-04
0.31316272D 04	-0.44543022D 05	0.11058516D-04	-0.37435443D-04
0.30749007D 04	-0.35156707D 05	0.17817548D-04	-0.95084213D-04
0.29721590D 04	-0.26165485D 05	0.23014541D-04	-0.17322814D-03
0.25556920D 04	-0.18048282D 05	0.26741276D-04	-0.26221697D-03
0.18942409D 04	-0.11590114D 05	0.28779059D-04	-0.44080802D-03
0.14147180D 04	-0.61654280D 04	0.30176281D-04	-0.60918176D-03
0.10739453D 04	-0.48743451D 04	0.30708057D-04	-0.67209539D-03
-0.13943691D-09	-0.46612172D-08	0.30886123D-04	-0.92458595D-03

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.23289095D 04	0.23434596D 04
13	0.14598540D 08	0.48297451D 04	0.49087490D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.23361846D 04	0.23434596D 04
13	0.14598540D 08	0.48692471D 04	0.49087490D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.23398221D 04	0.23434596D 04
13	0.14598540D 08	0.48889981D 04	0.49087490D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.23416408D 04	0.23434596D 04
13	0.14598540D 08	0.48988735D 04	0.49087490D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.23425502D 04	0.23434596D 04
13	0.14598540D 08	0.49038113D 04	0.49087490D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.23430049D 04	0.23434596D 04
13	0.14598540D 08	0.49062802D 04	0.49087490D 04

STA X K SUB X P SUB OX P SUB X

13	0.14598540D 08	0.49075146D 04	0.49087490D 04

OMEGA = 0.37500000D 03   DETERM = -0.50242161D 35

V	M	PHI	Y
0.0	0.0	-0.59498798D-04	-0.71725636D-03
0.0	0.0	-0.59498798D-04	-0.71725636D-03
0.0	0.0	-0.59498798D-04	-0.71725636D-03
-0.41018406D 02	-0.27888565D 02	-0.59399593D-04	-0.58514862D-03
-0.10588497D 03	-0.15229252D 03	-0.58334746D-04	-0.45315561D-03
-0.64874186D 03	-0.83391002D 03	-0.57836195D-04	-0.32220831D-03
-0.10308239D 04	-0.11574243D 04	-0.57799988D-04	-0.25120128D-03
-0.10500402D 04	-0.30810589D 04	-0.48558937D-04	-0.11093914D-03
0.12934194D 04	-0.30810589D 04	-0.48558937D-04	-0.11093914D-03
0.12704062D 04	0.24784784D 04	-0.44800445D-04	-0.11678456D-03
0.12600449D 04	0.48154083D 04	-0.73899588D-04	-0.89989778D-04
0.12544273D 04	0.72428315D 04	-0.12380691D-03	-0.12429275D-04
0.12720067D 04	0.10334748D 05	-0.20619452D-03	0.29607448D-03
-0.48824219D 04	0.10334748D 05	-0.20619452D-03	0.29607448D-03
-0.48677870D 04	0.77174561D 04	-0.21915612D-03	0.45114845D-03
-0.48296044D 04	0.33416591D 04	-0.23130700D-03	0.71317565D-03
-0.36898158D 04	0.22255219D 04	-0.23148048D-03	0.93988766D-03
-0.35681981D 04	-0.42056212D 03	-0.23159006D-03	0.11478573D-02
-0.20956971D 04	-0.47659223D 03	-0.23149227D-03	0.14015447D-02
-0.10807056D 04	-0.22078456D 04	-0.23128210D-03	0.16485125D-02
0.32127900D-09	0.15367618D-08	-0.23116718D-03	0.18378797D-02

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	-0.14269021D-05	-0.15288751D-04
-0.10363280D 02	-0.47124160D 02	-0.14137256D-05	-0.43724741D-05
-0.96199008D 01	-0.11764046D 03	-0.13490971D-05	0.55094276D-05
0.21143880D 01	-0.14566903D 03	-0.11870331D-05	0.13722967D-04
0.14531343D 02	-0.88833600D 02	-0.75997930D-06	0.18921656D-04
0.25877371D 02	0.29837091D 03	0.10173410D-04	0.91371059D-05
0.51025266D 02	0.38785561D 03	0.10167673D-04	0.78654940D-06
0.22398547D 02	0.17199408D 04	0.10153006D-04	-0.27277627D-04
-0.23210611D 04	0.17199408D 04	0.10153006D-04	-0.27277627D-04
-0.23394288D 04	-0.28278564D 04	0.10158220D-04	-0.40174828D-04
-0.23394288D 04	-0.28278564D 04	0.10158220D-04	-0.40174828D-04
-0.23394288D 04	-0.28278564D 04	0.10158220D-04	-0.40174828D-04
-0.23394288D 04	-0.28278564D 04	0.10158220D-04	-0.40174828D-04
0.38149998D 04	-0.54525057D 05	0.10158220D-04	-0.40174828D-04
0.37498323D 04	-0.43182657D 05	0.18444701D-04	-0.10120568D-03
0.36301419D 04	-0.32245608D 05	0.24837566D-04	-0.18740198D-03
0.31383394D 04	-0.22321684D 05	0.29438308D-04	-0.28706416D-03
0.23513231D 04	-0.14352091D 05	0.31962310D-04	-0.49781309D-03
0.17568738D 04	-0.76312566D 04	0.33691971D-04	-0.69552613D-03
0.13338547D 04	-0.60291971D 04	0.34349966D-04	-0.76987414D-03
-0.18758328D-09	-0.27848728D-08	0.34569596D-04	-0.10554174D-02

STA X	K SUB X	P SUB 0X	P SUB X
8	0.28011204D 08	0.27078485D 04	0.27289271D 04
13	0.14598540D 08	0.63465356D 04	0.64674468D 04

STA X	K SUB X	P SUB 0X	P SUB X
8	0.28011204D 08	0.27183878D 04	0.27289271D 04
13	0.14598540D 08	0.64069912D 04	0.64674468D 04

STA X	K SUB X	P SUB 0X	P SUB X
8	0.28011204D 08	0.27236574D 04	0.27289271D 04
13	0.14598540D 08	0.64372190D 04	0.64674468D 04

STA X	K SUB X	P SUB 0X	P SUB X
8	0.28011204D 08	0.27262922D 04	0.27289271D 04
13	0.14598540D 08	0.64523329D 04	0.64674468D 04

STA X	K SUB X	P SUB 0X	P SUB X
8	0.28011204D 08	0.27276097D 04	0.27289271D 04
13	0.14598540D 08	0.64598898D 04	0.64674468D 04

STA X	K SUB X	P SUB 0X	P SUB X
8	0.28011204D 08	0.27282684D 04	0.27289271D 04
13	0.14598540D 08	0.64636683D 04	0.64674468D 04

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STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.27285977D 04	0.27289271D 04
13	0.14598540D 08	0.64655575D 04	0.64674468D 04

OMEGA = 0.39000000D 03 DETERN = -0.52540202D 35

V	M	PHI	Y
0.0	0.0	-0.71844750D-04	-0.86332870D-03
0.0	0.0	-0.71844750D-04	-0.86332870D-03
0.0	0.0	-0.71844750D-04	-0.86332870D-03
-0.53384290D 02	-0.36225087D 02	-0.71716785D-04	-0.70324707D-03
-0.13760713D 03	-0.19781892D 03	-0.70333249D-04	-0.54320731D-03
-0.74298652D 03	-0.97911478D 03	-0.69708252D-04	-0.38575904D-03
-0.11916490D 04	-0.12479169D 04	-0.69667883D-04	-0.30026182D-03
-0.12164883D 04	-0.34723425D 04	-0.59358799D-04	-0.13245267D-03
0.15124388D 04	-0.34723425D 04	-0.59358799D-04	-0.13245267D-03
0.14846249D 04	0.30293026D 04	-0.57180502D-04	-0.12344720D-03
0.14733343D 04	0.57627721D 04	-0.92293168D-04	-0.82151234D-04
0.14690115D 04	0.86055122D 04	-0.15175672D-03	0.20795579D-04
0.14983992D 04	0.12242045D 05	-0.24944677D-03	0.39959149D-03
-0.60579644D 04	0.12242045D 05	-0.24944677D-03	0.39959149D-03
-0.60370312D 04	0.89963542D 04	-0.26469572D-03	0.58831373D-03
-0.59838092D 04	0.35750305D 04	-0.27863622D-03	0.90612015D-03
-0.45816377D 04	0.23683653D 04	-0.27882332D-03	0.11794558D-02
-0.44171747D 04	-0.89620413D 03	-0.27891189D-03	0.14307455D-02
-0.26028108D 04	-0.79068280D 03	-0.27874724D-03	0.17370201D-02
-0.23152351D 04	-0.29150820D 04	-0.27845665D-03	0.20348916D-02
0.26909674D-09	0.14636612D-08	-0.27830608D-03	0.22629957D-02

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	-0.40043411D-06	-0.51984593D-05
-0.41341707D 01	-0.17763726D 02	-0.39564823D-06	-0.20244097D-05
-0.47895993D 01	-0.47712985D 02	-0.37052087D-06	0.98476718D-06
-0.17711101D 01	-0.68982161D 02	-0.30058459D-06	0.35242588D-05
0.17515194D 01	-0.69343283D 02	-0.59767846D-07	0.47882523D-05
0.32380793D 01	0.27465689D 03	0.88798854D-05	-0.38644522D-05
-0.37183978D 02	0.31651764D 03	0.88748390D-05	-0.10871962D-04
-0.88615668D 02	0.13089065D 04	0.88632934D-05	-0.35029974D-04
-0.28175427D 04	0.13089065D 04	0.88632934D-05	-0.35029974D-04
-0.28398015D 04	-0.43021032D 04	0.88798578D-05	-0.43428615D-04
-0.28398015D 04	-0.43021032D 04	0.88798578D-05	-0.43428615D-04
-0.28398015D 04	-0.43021032D 04	0.88798578D-05	-0.43428615D-04
-0.28398015D 04	-0.43021032D 04	0.88798578D-05	-0.43428615D-04
0.47165621D 04	-0.67775558D 05	0.88798578D-05	-0.43428615D-04
0.46414275D 04	-0.53872998D 05	0.19197091D-04	-0.10869181D-03
0.45008782D 04	-0.40371270D 05	0.27185227D-04	-0.20533891D-03
0.39138774D 04	-0.28053117D 05	0.32956068D-04	-0.31898054D-03
0.29661723D 04	-0.18061046D 05	0.36133091D-04	-0.57265703D-03
0.22173836D 04	-0.95991507D 04	0.38309077D-04	-0.80959204D-03
0.16837447D 04	-0.75787857D 04	0.39136481D-04	-0.89912573D-03
-0.34509640D-09	-0.60354068D-08	0.39411752D-04	-0.12283028D-02

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.31869054D 04	0.32225428D 04
13	0.14598540D 08	0.84045425D 04	0.86125704D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.32047241D 04	0.32225428D 04
13	0.14598540D 08	0.85085565D 04	0.86125704D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.32136335D 04	0.32225428D 04
13	0.14598540D 08	0.85605635D 04	0.86125704D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.32180881D 04	0.32225428D 04
13	0.14598540D 08	0.85865669D 04	0.86125704D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.32203155D 04	0.32225428D 04
13	0.14598540D 08	0.85995687D 04	0.86125704D 04

STA X	K SUB X	P SUB OX	P SUB X
8	0.28011204D 08	0.32214291D 04	0.32225428D 04
13	0.14598540D 08	0.86060696D 04	0.86125704D 04

STA X K SUB X P SUB OX P SUB X

	[illegible]	[illegible]	0.32225428D 04
13	0.14598540D 08	0.86093200D 04	0.86125704D 04

OMEGA = 0.40500000D 03 DETERN = -0.52896449D 35

V	M	PHI	Y
0.0	0.0	-0.87615633D-04	-0.10504439D-02
0.0	0.0	-0.87615633D-04	-0.10504439D-02
0.0	0.0	-0.87615633D-04	-0.10504439D-02
-0.70031767D 02	-0.47454733D 02	-0.87448842D-04	-0.85450413D-03
-0.18027765D 03	-0.25910196D 03	-0.85636135D-04	-0.65849298D-03
-0.85829092D 03	-0.11626519D 04	-0.84843513D-04	-0.46729083D-03
-0.13911525D 04	-0.13415407D 04	-0.84798285D-04	-0.36333044D-03
-0.14235862D 04	-0.39392950D 04	-0.73241545D-04	-0.16051315D-03
0.17989566D 04	-0.39392950D 04	-0.73241545D-04	-0.16051315D-03
0.17648454D 04	0.37951758D 04	-0.73893008D-04	-0.13153626D-03
0.17526839D 04	0.70480650D 04	-0.11724901D-03	-0.69943816D-04
0.17507941D 04	0.10435961D 05	-0.18960046D-03	0.68236656D-04
0.17981052D 04	0.14791626D 05	-0.30778165D-03	0.54248921D-03
-0.76734536D 04	0.14791626D 05	-0.30778165D-03	0.54248921D-03
-0.76432996D 04	0.10682638D 05	-0.32607141D-03	0.77682611D-03
-0.75682637D 04	0.38259419D 04	-0.34233182D-03	0.11700938D-02
-0.58066218D 04	0.25159416D 04	-0.34253416D-03	0.15062193D-02
-0.55808836D 04	-0.15953402D 04	-0.34258804D-03	0.18159479D-02
-0.32989783D 04	-0.12467214D 04	-0.34232553D-03	0.21931168D-02
-0.29082086D 04	-0.39071790D 04	-0.34192156D-03	0.25595606D-02
0.35191761D-09	0.97963948D-09	-0.34172110D-03	0.28397965D-02

V PRIME	M PRIME	PHI PRIME	Y PRIME
0.0	0.0	0.11081137D-05	0.96858462D-05
0.68190028D 01	0.33696989D 02	0.10982211D-05	0.13205937D-05
0.34855966D 01	0.74201682D 02	0.10542346D-05	-0.59608696D-05
-0.90333733D 01	0.62651676D 02	0.96444697D-06	-0.11577546D-04
-0.20761306D 02	-0.40079990D 02	0.89262102D-06	-0.15182103D-04
-0.35541301D 02	0.21215017D 03	0.69793904D-05	-0.22119498D-04
-0.18121040D 03	0.17244033D 03	0.69759064D-05	-0.27098463D-04
-0.26699122D 03	0.54936105D 03	0.69702510D-05	-0.45468370D-04
-0.34895340D 04	0.54936105D 03	0.69702510D-05	-0.45468370D-04
-0.35168660D 04	-0.65233247D 04	0.70048463D-05	-0.47471869D-04
-0.35168660D 04	-0.65233247D 04	0.70048463D-05	-0.47471869D-04
-0.35168660D 04	-0.65233247D 04	0.70048463D-05	-0.47471869D-04
-0.35168660D 04	-0.65233247D 04	0.70048463D-05	-0.47471869D-04
0.59546929D 04	-0.86084419D 05	0.70048463D-05	-0.47471869D-04
0.58674489D 04	-0.68693009D 05	0.20132295D-04	-0.11824604D-03
0.57002888D 04	-0.51672238D 05	0.30335175D-04	-0.22902170D-03
0.49882357D 04	-0.36051539D 05	0.37736113D-04	-0.36173416D-03
0.38266038D 04	-0.23243243D 05	0.41825668D-04	-0.67505635D-03
0.28621792D 04	-0.12348077D 05	0.44625144D-04	-0.96623056D-03
0.21737595D 04	-0.97423199D 04	0.45689139D-04	-0.10770953D-02
-0.34077630D-09	-0.40890882D-08	0.46041926D-04	-0.14662627D-02

END OF CASE

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CUSTOMER INSTRUCTIONS	KEYPUNCH INSTRUCTIONS	CUSTOMER
1. ENTER DATA LEGIBLY WITHIN SPACES PROVIDED	X PUNCH 1 CARD PER HAND POSTED LINE ITEM	
2. DISTINGUISH BETWEEN I vs 1, Ø vs 0, Z vs 2, U vs V, S vs 5	PUNCH ALL LINES WHETHER POSTED OR NOT. IF NECESSARY PROVIDE BLANK CARDS	
3. Repeat cards 3 through 8 for each bay or station. If tables of P vs X input are given, add card 9 and cards of type 10 and 11 as required.	PUNCH ALL LINES THAT ARE HAND POSTED	JOB NO. E13104
	PAS INCLUDING SPACES	PROGRA
	ALL SPACES MAY BE IGNORED	
	ALL SPACES MAY BE IGNORED EXCEPT ON T CARD	
	ALL SPACES MAY BE IGNORED EXCEPT (Specify cols.)	TITLE
	ALL SIGNS AND KP LINES MUST BE PUNCHED	
	DO NOT PUNCH PRE-PRINTED SIGNS SHOWN AFTER LAST HANDWRITTEN VALUE ENTRY	
	X USE 360 SYMBOLS	FORM APPROVED (KEY PUN

1. TITLE INFORMATION N

2. N OMEGA(RPS) DELTA OMEGA(RPS) a b c d m n ø p SF NØ.

3. L(1) L(2) L'(1) L'(2)

4. EI(1) EI(2) EI'(1) EI'(2)

5. G(1) G(2) G'(1) G'(2)

6. C(1) C(2) C'(1) C'(2)

7. I_J D Y I'_X η β

8. W W' K

9. N BAY 1 BAY 2 BAY 3 BAY 4

10. TITLE

11. P(LBS) K(LBS/IN)

AGCS 0140-9 (REV 10/66) PLEASE PRINT CLEARLY - USE BLACK PENCIL

CUSTOMER INSTRUCTIONS	KEYPUNCH INSTRUCTIONS	CUSTOMER
1. ENTER DATA LEGIBLY WITHIN SPACES PROVIDED	X PUNCH 1 CARD PER HAND POSTED LINE ITEM	
2. DISTINGUISH BETWEEN I vs 1, Ø vs O, Z vs 2, U vs V, S vs 5	PUNCH ALL * LINES WHETHER POSTED OR NOT. IF NECESSARY PROVIDE BLANK CARDS	
3. Repeat cards 3 through 8 for each bay or station. If tables of P vs K input are given, add card 9 and cards of type 10 and 11 as required.	PUNCH ALL * LINES THAT ARE HAND POSTED	JOB NO. E13104
	PAS INCLUDING SPACES	PROGRAM
	ALL SPACES MAY BE IGNORED	
	ALL SPACES MAY BE IGNORED EXCEPT ON T CARD	
	ALL SPACES MAY BE IGNORED EXCEPT (Specify cols.)	TITLE
	ALL SIGNS AND KP LINES MUST BE PUNCHED	
	DO NOT PUNCH PRE-PRINTED SIGNS SHOWN AFTER LAST HANDWRITTEN VALUE ENTRY	
	X USE 360 SYMBOLS	FORM APPROVED (KEY PUNCH)

Card	Fields
1.	TITLE INFORMATION
2.	N, OMEGA(RPS), DELTA OMEGA(RPS), a, b, c, d, m, n, ø, p, SF, NØ.
3.	L(1), L(2), L'(1), L'(2)
4.	EI(1), EI(2), EI'(1), EI'(2)
5.	G(1), G(2), G'(1), G'(2)
6.	C(1), C(2), C'(1), C'(2)
7.	I_J, D, Y, I'_X, η, β
8.	W, W', K
9.	N, BAY 1, BAY 2, BAY 3, BAY 4
10.	TITLE
11	P(LBS), K(LBS/IN)

ACCS 0140-9 (REV 10/66)

PLEASE PRINT CLEARLY - USE BLACK PENCIL

3

CUSTOMER INSTRUCTIONS	KEYPUNCH INSTRUCTIONS
1. ENTER DATA LEGIBLY WITHIN SPACES PROVIDED	X PUNCH 1 CARD PER HAND POSTED LINE ITEM
2. DISTINGUISH BETWEEN I vs 1, Ø vs 0, Z vs 2, U vs V, S vs 5	PUNCH ALL * LINES WHETHER POSTED OR NOT. IF NECESSARY PROVIDE BLANK CARDS
3. Repeat cards 3 through 8 for each bay or station. If tables of P vs K input are given, add card 9 and cards of type 10 and 11 as required.	PUNCH ALL * LINES THAT ARE HAND POSTED
	PAS INCLUDING SPACES
	ALL SPACES MAY BE IGNORED
	ALL SPACES MAY BE IGNORED EXCEPT ON T CARD
	ALL SPACES MAY BE IGNORED EXCEPT (Specify cols.)
	ALL SIGNS AND KP LINES MUST BE PUNCHED
	DO NOT PUNCH PRE-PRINTED SIGNS SHOWN AFTER LAST HANDWRITTEN VALUE ENTRY
	X USE 360 SYMBOLS

CUSTOMER

JOB NO. E13104

PROGRAM

TITLE

FORM APPROVED KEY PUNCH

1. TITLE INFORMATION N

2. N | OMEGA(RPS) | DELTA OMEGA(RPS) | a | b | c | d | m | n | ø | p | SF | Nø.

3. L(1) | L(2) | L'(1) | L'(2)

EI(1) | EI(2) | EI'(1) | EI'(2)

5. G(1) | G(2) | G'(1) | G'(2)

6. C(1) | C(2) | C'(1) | C'(2) | P_x | ∫ LEV

7. I_J | D (c/in) | Y | I_X | η | β

8. W | W' | K_{N1} | K_{N2} | A_x | B_x

9. N | BAY 1 | BAY 2 | BAY 3 | BAY 4

10. TITLE

11. P(LBS) | K(LBS/IN)

AGCS 0140-9 (REV 10/56) PLEASE PRINT CLEARLY - USE BLACK PENCIL

```
1375.FT/SEC. M.B. SPEED CASE 2 ROTOR  SK8705-52  1.85 1.85 1.8  EI=5 20
8  300.        15.         1  2  5  6  3  4  7  8  1  7
.0          .0          3.5         3.5
                        7.7         E97.7       E9
                        11.5        E611.5      E6
                        .3          .3
                                    +1.37
            70.
.0          .0          3.          3.
                        7.7         E97.7       E9
                        11.5        E611.5      E6
                        .3          .3
                                    +1.37
            80.
1.06        1.06        3.          3.
65.7        E665.7      E65.2       E95.2       E9
6.2         E66.2       E611.5      E611.5      E6
.85         .85         .3          .3
+.0472                              +.78
4.36        80.
1.05        1.05        3.5         3.5
100.        E6186.      E62.1       E92.1       E9
6.2         E66.2       E611.6      E611.6      E6
.57         .24         .3          .3
+.0913                              +.27
8.69        50.0
1.04        1.04        .65         .65
761.        E62400.     E6.005      E9183.      E9
6.2         E66.2       E611.6      E611.6      E6
.08         .08         .02         .02
+.3202                  -82.4       E-6+6.39
15.18       50.
.6          .6          .4          .4
31.1        E931.1      E947.5      E947.5      E9
6.2         E66.2       E611.5      E611.5      E6
.01         .01         .08         .08
+2.1317                 -38.8       E-6+1.04
40.4        350.
.93         .93         1.5         1.25
603.        E6359.      E6425.      E9162.      E9
6.2         E66.2       E611.6      E611.6      E6
.1          .15         .01         .02
+.0377                              +21.75
7.3         137.
.0          .0          .0          .0
600.        E6600.      E6195.      E9195.      E9
6.2         E66.2       E611.6      E611.6      E6
.1          .1          .02         .02          1000.       .0357    E-6
+.0                                 +.0
.0                                               .1193       E63.11
2.66        1.66        1.55        .55
266.        E6240.      E6142.      E9276.      E9
6.2         E66.2       E611.6      E611.6      E6
.19         .2          .02         .02
+.0524                              +5.97
14.17       34.7
```

Forced Undamped Lateral Vibration Analysis of Two Elastically Coupled Beams - Variable Mass and Elasticity

Program EL3104
Sample Input
Page 1 of 3

Forced Undamped Lateral Vibration Analysis of Two Elastically Coupled Beams - Variable Mass and Elasticity

.92	.92	.0	.0			
181.	E6288.	E644.	E944.	E9		
6.2	E66.2	E611.6	E611.6	E6		
.22	.22	.1	.1			
+.0266			.0			
6.53	.0					
.96	.96	.0	.0			
232.	E6232.	E652.	E991.	E9		
6.2	E66.2	E611.6	E611.6	E6		
.28	.28	.08	.05			
+.0253						
6.17						
1.21	1.21	.0	.0			
258.	E6258.	E617.	E915.	E9		
9.	E69.	E611.6	E611.6	E6		
.25	.25	.26	.27			
+.039						
10.43						
.0	.0	.0	.0			
258.	E6258.	E617.	E915.	E9		
9.	E69.	E611.6	E611.6	E6		
.25	.25	.26	.27	1500.	.0685	E-6
.0	-8.4		+.0			
.0				.1193	E63.11	
.27	.27	1.4	1.4			
376.	E6376.	E616.5	E916.5	E9		
11.5	E611.5	E611.6	E611.6	E6		
.175	.175	.02	.025			
+.0129			+9.3			
2.73	67.9					
.3	.61	1.4	1.4			
244.	E6825.	E616.5	E916.5	E9		
11.5	E611.5	E611.5	E611.5	E6		
.23	.1	.028	.028			
+.0242			+4.98			
4.85	58.5					
.4	.55	1.4	1.4			
15.75	E915.75	E916.5	E916.5	E9		
11.5	E611.5	E611.5	E611.5	E6		
.02	.02	.025	.032			
+2.215		98.	E-6+2.95			
51.2	145.5					
.4	.4	.75	2.15			
6.5	E96.5	E962.	E916.	E9		
11.5	E611.5	E611.5	E611.5	E6		
.09	.09	.017	.27			
+.2			3.4			
8.1	175.					
.5	.5	1.5	1.5			
6.5	E96.5	E912.	E986.	E9		
11.5	E611.5	E611.5	E611.5	E6		
.09	.09	.29	.04			
+2.16		113.	E-6+3.0			
46.	65.					
.5	.5	.5	.5			
6.5	E96.5	E910.3	E910.3	E9		

3

11.5	E611.5	E611.5	E611.5	E6
.09	.09	.3	.3	
+.2			+.3	
9.8	40.			
.3	.5	3.7	3.7	
6.5	E96.5	E960.	E960.	E9
11.5	E611.5	E611.5	E611.5	E6
.09	.09	.07	.07	
+2.16		138.	E-6+5.7	
45.	100.			

Forced Undamped Lateral Vibration
Analysis of Two Elastically Coupled
Beams - Variable Mass and Elasticity

Program E3104
Sample Input
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APPENDIX F

PROGRAM E13104 LISTING

```
@RUN F7&2,423,59,2,200   LIST E13104          25 APR 72   14:42:25.[illegible]

@ CTL UN=E13104                               25 APR 72   14:42:25.849

@R     ASG X=AN4151                           25 APR 72   14:42:25.926
       AN4151 ASSIGNED UNIT  4

@N     HDG                                    25 APR 72   14:42:25.935
```

```
> .T CUR                                          25 APR 72   14:42:25.937
 1.      PEF X                                                14:42:26
 2.      IN X                                                 14:42:26
   END OF FILE -- UNIT X
 3.      LIST 1                                               14:42:28
```

ELT EXPAND,1,710420, 59936

```
      E13104                                                                BILL
      E13104                                                                BILL
      C
      C              PLACE ON PRODUCTION 19 FEBRUARY 1970 BY F. YEE
      C
            SUBROUTINE EXPAND                                          13104  1
            IMPLICIT REAL*8 (A-H,O-Z)
            DIMENSION BLO(250)                                         13104  2
            COMMON /ARRAY/BLO/ARRAYZ/BHI                               13104  3
            BHI=0.0D0                                                  13104  4
            RETURN                                                     13104  5
      C*****************************************************************13104  6
```

```
[illegible],2,200        LIST E13104                    BO    25 APR 72   14:42:25.849

CTL UN=E13104                                                 25 APR 72   14:42:25.849

OR     ASG X=AN4151                                           25 APR 72   14:42:25.926
       AN4151 ASSIGNED UNIT  4

@N     HDG                                                    25 APR 72   14:42:25.935
```

5

```
XQT CUR                                   25 APR 72   14:42:25.937
  1.      PEF X                                       14:42:26
  2.      IN X                                        14:42:26
     END OF FILE -- UNIT X
  3.      LIST 1                                      14:42:28
```

```
@ ELT EXPAND,1,710420, 59936

        E13104                                                              BILL
        E13104                                                              BILL
        C
        C                    PLACE ON PRODUCTION 19 FEBRUARY 1970 BY F. YEE
        C
              SUBROUTINE EXPAND                                          13104  1
              IMPLICIT REAL*8 (A-H,O-Z)
              DIMENSION BLO(250)                                         13104  2
              COMMON /ARRAY/BLO/ARRAYZ/BHI                               13104  3
              BHI=0.0D0                                                  13104  4
              RETURN                                                     13104  5
        C************************************************************************13104  6
        C     EXPAND SHOULD PRECEDE 1ST SIMST USE.                       13104  7
        C     EXPAND SHOULD ONLY BE CALLED FOR THE 7094.                 13104  8
        C     TO DIMENSION BLO,ESTIMATE NEEDED STORAGE.                  13104  9
        C     PUT DIM BLO(1) AND NAMED COMMON IN EVERY SIMST-USING ROUTINE. 13104 10
        C     EXPAND ONLY NEED BE CHANGED IF AVAILABLE STORAGE CHANGES.  13104 11
        C************************************************************************13104 12
              END                                                        13104 13
```

```
@ ELT E13104,1,710428, 32300

000001   1375.FT/SEC. M.B. SPEED CASE 2 ROTOR  SK8705-52  1.85 1.85 1.8  EI=5 20        **-8
000002   8  300.        15.          1  2  5  6  3  4  7  8  1   7
000003   .0        .0        3.5        3.5
000004                       7.7        E97.7       E9
000005                       11.5       E611.5      E6
000006                       .3         .3
000007                                  +1.37
000008             70.
000009   .0        .0        3.         3.
000010                       7.7        E97.7       E9
000011                       11.5       E611.5      E6
000012                       .3         .3
000013                                  +1.37
000014             80.
000015   1.06      1.06      3.         3.
000016   65.7      E665.7    E65.2      E95.2       E9
000017   6.2       E66.2     E611.5     E611.5      E6
000018   .85       .85       .3         .3
000019   +.0472                         +.78
000020   4.36      80.
000021   1.05      1.05      3.5        3.5
000022   100.      E6186.    E62.1      E92.1       E9
000023   6.2       E66.2     E611.6     E611.6      E6
000024   .57       .24       .3         .3
000025   +.0913                         +.27
000026   8.69      50.0
000027   1.04      1.04      .65        .65
000028   761.      E62400.   E6.005     E9183.      E9
000029   6.2       E66.2     E611.6     E611.6      E6
000030   .08       .08       .02        .02
000031   +.3202              -82.4      E-6+6.39
000032   15.18     50.
000033   .6        .6        .4         .4
000034   31.1      E931.1    E947.5     E947.5      E9
000035   6.2       E66.2     E611.5     E611.5      E6
000036   .01       .01       .08        .08
000037   +2.1317             -38.8      E-6+1.04
000038   40.4      350.
000039   .93       .93       1.5        1.25
000040   603.      E6359.    E6425.     E9162.      E9
000041   6.2       E66.2     E611.6     E611.6      E6
000042   .1        .15       .01        .02
000043   +.0377                         +21.75
000044   7.3       137.
000045   .0        .0        .0         .0
000046   600.      E6600.    E6195.     E9195.      E9
000047   6.2       E66.2     E611.6     E611.6      E6
000048   .1        .1        .02        .02          1000.      .0357    E-6
000049   +.0                            +.0
000050   .0                                          .1193     E63.11
000051   2.66      1.66      1.55       .55
000052   266.      E6240.    E6142.     E9276.      E9
000053   6.2       E66.2     E611.6     E611.6      E6
000054   .19       .2        .02        .02
000055   +.0524                         +5.97
000056   14.17     34.7
```

000057	.92	.92	.0	.0				
000058	181.	E6288.	E644.	E944.	E9			
000059	6.2	E66.2	E611.6	E611.6	E6			
000060	.22	.22	.1	.1				
000061	+.0266			.0				
000062	6.53	.0						
000063	.96	.96	.0	.0				
000064	232.	E6232.	E652.	E991.	E9			
000065	6.2	E66.2	E611.6	E611.6	E6			
000066	.28	.28	.08	.05				
000067	+.0253							
000068	6.17							
000069	1.21	1.21	.0	.0				
000070	258.	E6258.	E617.	E915.	E9			
000071	9.	E69.	E611.6	E611.6	E6			
000072	.25	.25	.26	.27				
000073	+.039							
000074	10.43							
000075	.0	.0	.0	.0				
000076	258.	E6258.	E617.	E915.	E9			
000077	9.	E69.	E611.6	E611.6	E6			
000078	.25	.25	.26	.27		1500.	.0685	E-6
000079	.0	-8.4		+.0				
000080	.0					.1193	E63.11	
000081	.27	.27	1.4	1.4				
000082	376.	E6376.	E616.5	E916.5	E9			
000083	11.5	E611.5	E611.6	E611.6	E6			
000084	.175	.175	.02	.025				
000085	+.0129			+9.3				
000086	2.73	67.9						
000087	.3	.61	1.4	1.4				
000088	244.	E6825.	E616.5	E916.5	E9			
000089	11.5	E611.5	E611.5	E611.5	E6			
000090	.23	.1	.028	.028				
000091	+.0242			+4.98				
000092	4.85	58.5						
000093	.4	.55	1.4	1.4				
000094	15.75	E915.75	E916.5	E916.5	E9			
000095	11.5	E611.5	E611.5	E611.5	E6			
000096	.02	.02	.025	.032				
000097	+2.215		98.	E-6+2.95				
000098	51.2	145.5						
000099	.4	.4	.75	2.15				
000100	6.5	E96.5	E962.	E916.	E9			
000101	11.5	E611.5	E611.5	E611.5	E6			
000102	.09	.09	.017	.27				
000103	+.2			3.4				
000104	8.1	175.						
000105	.5	.5	1.5	1.5				
000106	6.5	E96.5	E912.	E986.	E9			
000107	11.5	E611.5	E611.5	E611.5	E6			
000108	.09	.09	.29	.04				
000109	+2.16		113.	E-6+3.0				
000110	46.	65.						
000111	.5	.5	.5	.5				
000112	6.5	E96.5	E910.3	E910.3	E9			
000113	11.5	E611.5	E611.5	E611.5	E6			
000114	.09	.09	.3	.3				
000115	+.2			+.3				
000116	9.8	40.						

000117	.3	.5	3.7	3.7	
000118	6.5	E96.5	E960.	E960.	E9
000119	11.5	E611.5	E611.5	E611.5	E6
000120	.09	.09	.07	.07	
000121	+2.16		138.	E-6+5.7	
000122	45.	100.			

```
@ ELT INT4,1,710422, 38771

      SUBROUTINE INT4(X,Y,XI,YO)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X,Y,XI,YO
      DIMENSION X(9),Y(9),XC(4),YC(4)
      EQUIVALENCE (XC(1),X1),(XC(2),X2),(XC(3),X3),(XC(4),X4),(YC(1),Y1)
     1,(YC(2),Y2),(YC(3),Y3),(YC(4),Y4)
   20 ASSIGN 30 TO NA
      J=2
      B=XI
   21 IF(X(J))26,22,26
   26 GO TO NA,(30,40)
   22 IF(Y(J))26,23,26
   23 IF(J-2)24,24,25
   24 YE=0.0
      GO TO 50
   25 ASSIGN 32 TO NB
      J=J-1
   27 X1=X(J)
      X2=X(J-1)
      X3=X(J-2)
      Y1=Y(J)
      Y2=Y(J-1)
      Y3=Y(J-2)
      GO TO NB,(32,42)
   30 IF(X(J)-B)29,37,37
   37 IF(J-2)31,31,28
   28 ASSIGN 40 TO NA
   29 J=J+1
      GO TO 21
   31 DO 60 J=1,3
      XC(J)=X(J)
   60 YC(J)=Y(J)
   32 D=X2-X1
      A1=B-X1
      A2=B-X2
      YE=A1*A2/2.0/D*((Y3-Y2)/(X3-X2)-(Y2-Y1)/D)-A2/D*Y1+A1/D*Y2
      GO TO 50
   40 ASSIGN 42 TO NB
      GO TO 27
   42 X4=X(J-3)
      Y4=Y(J-3)
      D=X3-X2
      A1=B-X2
      A2=B-X3
      XM12=(Y2-Y1)/(X2-X1)
      XM23=(Y3-Y2)/D
      XM34=(Y4-Y3)/(X4-X3)
      YE=A1*A2**2/2.0/D**2*(XM12-XM23)+A2*A1**2/2.0/D**2*(XM34-XM23)-A2*
     1Y2/D+A1*Y3/D
   50 YO=YE
      RETURN
      END
```

```
@ ELT MAIN,1,710420, 59942

      CMAIN                                                             13104 14
            IMPLICIT REAL*8 (A-H,O-Z)
      C                                                                 13104 15
      C     LATERAL VIBRATION ANALYSIS OF TWO ELASTICALLY COUPLED,UNDAMPED 13104 16
      C           LUMPED PARAMETER BEAMS     JOB  14043        J. F. BUSSIO 13104 17
      C                                                                 13104 18
            DIMENSION BLO(1)                                            13104 19
            COMMON /ARRAY/BLO/ARRAYZ/BHI                                13104 20
            DIMENSION TABP(26,10),TABK(26,10),NPIT(10),ISTA(10)         13104 21
            DIMENSION TITLE(12),IDP(4)                                  13104 22
            DIMENSION DL1(50),DL2(50),DL3(50),DL4(50),DEI1(50),DEI2(50), 13104 23
           1         DEI3(50),DEI4(50),DG1(50),DG2(50),DG3(50),DG4(50), 13104 24
           2         DC1(50),DC2(50),DC3(50),DC4(50),DIJ1(50),DX(50),   13104 25
           3         DGAMX(50),DIX2(50),DWN1(50),DWN2(50),DKN1(50),DKN2(50) 13104 26
            DIMENSION DETA(50),DBETA(50),DFLEX(50)                      13104 27
            DIMENSION E1MTRX(9,9),E2MTRX(9,9),AMATRX(9,9),BMATRX(9,9),  13104 28
           1          CMATRX(9,9),FMATRX(9,9),DLMTRX(9,1),SMATRX(9,1),  13104 29
           2          DUMMY(9),C(5),ID(5),X(4),KID(4),Q000FL(9,50) ,NREP(10) 13104 30
            DIMENSION DPN1(50),P0(50),DAN1(50),DBN1(50), P1(50),P2(50),P3(50) 13104 31
            COMMON  DL1     , DL2     , DL3     , DL4     , DEI1    , DEI2    , 13104 32
           1        DEI3    , DEI4    , DG1     , DG2     , DG3     , DG4     , 13104 33
           2        DC1     , DC2     , DC3     , DC4     , DIJ1    , DX      , 13104 34
           3        DGAMX   , DIX2    , DWN1    , DWN2    , DKN1    , DKN2    , 13104 35
           4        E1MTRX  , E2MTRX  , AMATRX  , BMATRX  , CMATRX  , FMATRX  , 13104 36
           5        KC      , KD      , KM      , KN      , KO      , KP      , 13104 37
           6        DETA    , DBETA   , DFLEX   , DPN1    , P0      , DAN1    , 13104 38
           7        DBN1    , IFLAG   , NTRIAL  , B       , DLMTRX  , SMATRX  , 13104 39
           8        DUMMY   , KK      , KA      , KB                           13104 40
            CALL EXPAND                                                 13104 41
            DO 999 II=1,10                                              13104 42
        999 NREP(II)=0                                                  13104 43
            LINE=6                                                      13104 44
       30   READ (5,3000,END=6000) TITLE, NSTA, (IDP(II), II=1,4)       13104 45
            READ ( 5,3001)NROOT,OMEGA,DOMGA,KK,KA,KB,KC,KD,KM,KN,KO,  KP,IFLAG13104 46
           1,NTRIAL                                                     13104 47
            DO 35 N=1,NSTA                                              13104 48
            CALL REPEAT(DL1(N-1),DL1(N),DL2(N-1),DL2(N),DL3(N-1),DL3(N),DL4(N-13104 49
           11),DL4(N),X(1),X(1),X(1),X(1),NREP(1))                      13104 50
            CALL REPEAT(DEI1(N-1),DEI1(N),DEI2(N-1),DEI2(N),DEI3(N-1),DEI3(N),13104 51
           1DEI4(N-1),DEI4(N),X(1),X(1),X(1),X(1),NREP(2))              13104 52
            CALL REPEAT(DG1(N-1),DG1(N),DG2(N-1),DG2(N),DG3(N-1),DG3(N),DG4(N-13104 53
           11),DG4(N),X(1),X(1),X(1),X(1),NREP(3))                      13104 54
            CALL REPEAT(DC1(N-1),DC1(N),DC2(N-1),DC2(N),DC3(N-1),DC3(N),DC4(N-13104 55
           11),DC4(N),DPN1(N-1),DPN1(N),DFLEX(N-1),DFLEX(N),NREP(4))    13104 56
            CALL REPEAT(DIJ1(N-1),DIJ1(N),DX(N-1),DX(N),DGAMX(N-1),DGAMX(N),DI13104 57
           1X2(N-1),DIX2(N),DETA(N-1),DETA(N),DBETA(N-1),DBETA(N),NREP(5))  13104 58
            CALL REPEAT(DWN1(N-1),DWN1(N),DWN2(N-1),DWN2(N),DKN1(N-1),DKN1(N),13104 59
           1DKN2(N-1),DKN2(N),DAN1(N-1),DAN1(N),DBN1(N-1),DBN1(N),NREP(6))  13104 60
       35   CONTINUE                                                    13104 61
            WRITE ( 6,4000)                                             13104 62
            WRITE ( 6,4001)TITLE,NSTA                                   13104 63
            WRITE ( 6,4021)NROOT,OMEGA,DOMGA,KA,KB,KC,KD,KM,KN,KO,   KP,IFLAG,13104 64
           1NTRIAL                                                      13104 65
            LINE=LINE+NSTA+4                                            13104 66
            IF(LINE-55)39,39,38                                         13104 67
         38 WRITE ( 6,4022)                                             13104 68
```

```
      LINE=1
   39 WRITE ( 6,4002)
      DO 40 N=1,NSTA
   40 WRITE ( 6,4008)DL1(N),DL2(N),DL3(N),DL4(N)
      LINE=LINE+NSTA+4
      IF(LINE-55)44,44,43
   43 WRITE ( 6,4022)
      LINE=1
   44 WRITE ( 6,4003)
      DO 45 N=1,NSTA
   45 WRITE ( 6,4008)DEI1(N),DEI2(N),DEI3(N),DEI4(N)
      LINE=LINE+NSTA+4
      IF(LINE-55)49,49,48
   48 WRITE ( 6,4022)
      LINE=1
   49 WRITE ( 6,4004)
      DO 50 N=1,NSTA
   50 WRITE ( 6,4008)DG1(N),DG2(N),DG3(N),DG4(N)
      LINE=LINE+NSTA+4
      IF(IFLAG)70,51,70
   70 IF(LINE-55)72,72,71
   71 WRITE ( 6,4022)
      LINE=1
   72 WRITE ( 6,4015)
      DO 73 N= 1,NSTA
   73 WRITE ( 6,4010)DC1(N),DC2(N),DC3(N),DC4(N),DPN1(N),      DFLEX(N)
      GO TO 56
   51 IF(LINE-55)54,54,53
   53 WRITE ( 6,4022)
      LINE=1
   54 WRITE ( 6,4005)
      DO 55 N=1,NSTA
   55 WRITE ( 6,4008)DC1(N),DC2(N),DC3(N),DC4(N)
   56 LINE=LINE+NSTA+4
      IF(LINE-55)59,59,58
   58 WRITE ( 6,4022)
      LINE=1
   59 WRITE ( 6,4006)
      DO 60 N=1,NSTA
   60 WRITE ( 6,4010)DIJ1(N),DX(N),DGAMX(N),DIX2(N) ,      DETA(N) , DB
     1ETA(N)
      LINE=LINE+NSTA+4
      IF(IFLAG)80,61,80
   80 IF(LINE-55)82,82,81
   81 WRITE ( 6,4022)
   82 WRITE ( 6,4017)
      DO 83 N= 1,NSTA
   83 WRITE ( 6,4010)DWN1(N),DWN2(N),DKN1(N),DKN2(N),      DAN1(N),
     1DBN1(N)
      GO TO 66
   61 IF(LINE-55)64,64,63
   63 WRITE ( 6,4022)
   64 WRITE ( 6,4007)
      DO 65 N=1,NSTA
   65 WRITE ( 6,4008)DWN1(N),DWN2(N),DKN1(N),DKN2(N)
   66 LINE=1
      WRITE ( 6,4022)
      IF(IFLAG)90,99,99
C**********************************************************************
C     FOR IFLAG = -1 P-K TABLES ARE INPUT (MAX. OF 10)
```

```
      C     TABP,TABK(3,M) SELECTS THE 3RD POINT OF THE MTH TABLE
      C     TABLES ARE SEQUENTIAL FOR INT4
      C*********************************************************************
         90 READ (5,2004) NTAB,(ISTA(II),II=1,NTAB )
       2004 FORMAT (I2,2X10I3)
            WRITE (6,1007)NTAB,(ISTA(II),II=1,NTAB )
      1007  FORMAT (1H1,8X18HNUMBER OF TABLES  I2,3X34H  INPUT AT THE FOLLOWIN
           1G STATIONS  ,2X10I3 )
            DO 213 M= 1,NTAB
            READ (5,3003)  TITLE,NPIT(M)
       3003 FORMAT (11A6,A4,I2)
            WRITE (6,3004) TITLE,NPIT(M)
       3004 FORMAT (1H0,11A6,A4,8X25HNUMBER OF POINTS IN TABLE,I3,6X
           1  12H(MAX IS 15.) )
            WRITE (6,3005)
       3005 FORMAT (1H0,24X1HP,19X1HK )
      C
            NPITM=NPIT(M)
            DO 91 J=1,NPITM
            READ (5,3002) TABP(J,M),TABK(J,M)
            WRITE (6,3006) TABP(J,M),TABK(J,M)
       3006 FORMAT (1H0,18X,E15.8,5X,E15.8)
         91 CONTINUE
      C
            TABP(NPITM+1,M)=0.0D0
            TABK(NPITM+1,M)=0.0D0
        213 CONTINUE
      C
      99    TMOD=OMEGA*6.283185307179586
            DMOD=DOMGA*6.283185307179586
      C
      C  INITILIZE E1,E2,AND F MATRICES
      C
        100 DO 110 I=1,9
            DO 110 J=1,9
            IF(I-J)105,106,105
       105  E1MTRX(I,J)=0.0D0
            E2MTRX(I,J)=0.0D0
            FMATRX(I,J)=0.0D0
            GO TO 110
       106  E1MTRX(I,J)=1.0D0
            E2MTRX(I,J)=1.0D0
            FMATRX(I,J)=1.0D0
       110  CONTINUE
            OMGW=TMOD-DMOD
            DOMG=DMOD
            DO 150 MMM=1,NROOT
            OMGW=OMGW+DOMG
      C*********************************************************************
            IF(IFLAG)4301,4300,4301
       4301 IF(MMM-4)4302,4302,434
       4302 GO TO (430,431,432,433),MMM
      C*********************************************************************
        430 DO 340 N= 1,NSTA
        340 P0(N)=DPN1(N)
            GO TO 4300
      C*********************************************************************
        431 DO 341 N= 1,NSTA
        341 P1(N)=.5D0*(P0(N)+DPN1(N))
            GO TO 4300
```

```
C*********************************************************************13104189
  432 DO 342 N= 1,NSTA                                                 13104190
  342 P2(N)=.5D0*(P0(N)+DPN1(N))                                       13104191
      GO TO 4300                                                       13104192
C*********************************************************************13104193
  433 DO 343 N= 1,NSTA                                                 13104194
      P3(N)=.5D0*(P0(N)+DPN1(N))                                       13104195
      P0(N)=3.D0*(P3(N)-P2(N))+P1(N)                                   13104196
  343 DPN1(N)=P0(N)                                                    13104197
      GO TO 4300                                                       13104198
C*********************************************************************13104199
  434 DO 344 N= 1,NSTA                                                 13104200
      P1(N)=P2(N)                                                      13104201
      P2(N)=P3(N)                                                      13104202
      P3(N)=.5D0*(P0(N)+DPN1(N))                                       13104203
      P0(N)=3.D0*(P3(N)-P2(N))+P1(N)                                   13104204
  344 DPN1(N)=P0(N)                                                    13104205
C     LOOP FOR BETTER K(X) S DURING EACH STATE VECTOR LOOP. 1ST PASS OK 13104206
 4300 DO 600 JERRY=  1,NTRIAL                                          13104207
C                                                                      13104208
C     FIND (DELTA)0                                                    13104
C                                                                      13104210
      DO 130 I=1,9                                                     13104211
      DO 130 J=1,9                                                     13104212
      IF(I-J)125,126,125                                               13104213
  125 CMATRX(I,J)=0.0D0                                                13104214
      GO TO 130                                                        13104215
  126 CMATRX(I,J)=1.0D0                                                13104216
  130 CONTINUE                                                         13104217
      M=1                                                              13104218
      DO 135 N=1,NSTA                                                  13104219
      IF(IFLAG)6,1,2                                                   13104220
    2 IF(DAN1(N))4,1,4                                                 13104221
    6 IF(DPN1(N))1,1,5                                                 13104222
    5 P0(N)=.5D0*(P0(N)+DPN1(N))                                       13104223
      CALL INT4(TABP(1,M),TABK(1,M),P0(N),DKN1(N))                     13104224
      M=M+1                                                            13104225
      DKN1(N)=DKN1(N)/(DFLEX(N)+1.0D0)                                 13104226
      GO TO 1                                                          13104227
    4 P0(N)=.5D0*(P0(N)+DPN1(N))                                       13104228
      DKN1(N)=1.0D0/(DFLEX(N)+1.0D0/(DAN1(N)*P0(N)**DBN1(N)))          13104229
    1 CALL SETUP(OMGW,N)                                               13104230
      CALL MATMPY(E1MTRX(1,1),CMATRX(1,1),AMATRX(1,1),9,9,9,9,9,9)     13104231
      CALL MATMPY(E2MTRX(1,1),FMATRX(1,1),BMATRX(1,1),9,9,9,9,9,9)     13104232
      CALL MATMPY(BMATRX(1,1),AMATRX(1,1),CMATRX(1,1),9,9,9,9,9,9)     13104233
  135 CONTINUE                                                         13104234
      OPRT=OMGW/6.283185307179586                                      13104235
  140 DO 1140 I=1,8                                                    13104236
 1140 DLMTRX(I,1)=0.0D0                                                13104237
      DLMTRX(9,1)=1.0D0                                                13104238
      ID(1)=1                                                          13104239
      ID(2)=2                                                          13104240
      ID(3)=3                                                          13104241
      ID(4)=4                                                          13104242
      ID(5)=-1                                                         13104243
      CALL SIMSZ
C                                                                      13104244
C     SOLVE FOR DELTA(M),DELTA(N),DELTA(O),AND DELTA(P)                13104245
C                                                                      13104246
      C(1)=CMATRX(KA,KM)                                               13104247
```

```
      C(2)=CMATRX(KA,KN)                                                13104248
      C(3)=CMATRX(KA,KO)                                                13104249
      C(4)=CMATRX(KA,KP)                                                13104250
      C(5)=-CMATRX(KA,9)                                                13104251
      CALL SIMST(C,ID,5,BLO,BHI)                                        13104252
      C(1)=CMATRX(KB,KM)                                                13104253
      C(2)=CMATRX(KB,KN)                                                13104254
      C(3)=CMATRX(KB,KO)                                                13104255
      C(4)=CMATRX(KB,KP)                                                13104256
      C(5)=-CMATRX(KB,9)                                                13104257
      CALL SIMST(C,ID,5,BLO,BHI)                                        13104258
      C(1)=CMATRX(KC,KM)                                                13104259
      C(2)=CMATRX(KC,KN)                                                13104260
      C(3)=CMATRX(KC,KO)                                                13104261
      C(4)=CMATRX(KC,KP)                                                13104262
      C(5)=-CMATRX(KC,9)                                                13104263
      CALL SIMST(C,ID,5,BLO,BHI)                                        13104264
      C(1)=CMATRX(KD,KM)                                                13104265
      C(2)=CMATRX(KD,KN)                                                13104266
      C(3)=CMATRX(KD,KO)                                                13104267
      C(4)=CMATRX(KD,KP)                                                13104268
      C(5)=-CMATRX(KD,9)                                                13104269
      CALL SIMST(C,ID,5,BLO,BHI)                                        13104270
      CALL SIMSD(X,KID,DXX,KERR,ITEQN)                                  13104271
      IF(KERR)310,145,310                                               13104272
  145 KSUB=KID(1)                                                       13104273
      DLMTRX(KM,1)=X(KSUB)                                              13104274
      KSUB=KID(2)                                                       13104275
      DLMTRX(KN,1)=X(KSUB)                                              13104276
      KSUB=KID(3)                                                       13104277
      DLMTRX(KO,1)=X(KSUB)                                              13104278
      KSUB=KID(4)                                                       13104279
      DLMTRX(KP,1)=X(KSUB)                                              13104280
      DO 147 I=1,9                                                      13104281
  147 DUMMY(I)=DLMTRX(I,1)                                              13104282
      IF(IFLAG)598,599,598                                              13104283
  598 WRITE ( 6,4018)                                                   13104284
  599 DO 149 N=1,NSTA                                                   13104285
      CALL SETUP(OMGW,N)                                                13104286
      CALL MATMPY(E2MTRX(1,1),FMATRX(1,1),AMATRX(1,1),9,9,9,9,9)        13104287
      CALL MATMPY(AMATRX(1,1),E1MTRX(1,1),BMATRX(1,1),9,9,9,9,9)        13104288
      CALL MATMPY(BMATRX(1,1),DLMTRX(1,1),SMATRX(1,1),9,9,9,9,1)        13104289
      DO 149 I=1,9                                                      13104290
      Q000FL(I,N)=SMATRX(I,1)                                           13104291
      DLMTRX(I,1)=SMATRX(I,1)                                           13104292
  149 CONTINUE                                                          13104293
      IF(IFLAG)605,146,605                                              13104294
  605 WRITE ( 6,4025)                                                   13104295
      DO 590 N= 1,NSTA                                                  13104296
      IF(DPN1(N))601,590,601                                            13104297
  601 DPN1(N)=DKN1(N)*DABS(Q000FL(4,N)-Q000FL(8,N))                     13104298
      WRITE ( 6,4019)N, DKN1(N), P0(N) , DPN1(N)                        13104299
  590 CONTINUE                                                          13104300
  600 CONTINUE                                                          13104301
  146 WRITE ( 6,4009)OPRT,DXX                                           13104302
      WRITE ( 6,4013)(DUMMY(K),K=1,4)                                   13104303
      DO 1149 I=1,NSTA                                                  13104304
 1149 WRITE ( 6,4008)(Q000FL(N,I),N=1,4)                                13104305
      LINE=NSTA+5                                                       13104306
      WRITE ( 6,4014)(DUMMY(K),K=5,8)                                   13104307
```

```
      DO 1150 I=1,NSTA                                                  13104308
      WRITE ( 6,4008)(Q00CFL(N,I),N=5,8)                                13104309
      LINE=LINE+1                                                       13104310
      IF(LINE-55)1150,1151,1151                                         13104311
 1151 WRITE ( 6,4023)                                                   13104312
      LINE=1                                                            13104313
 1150 CONTINUE                                                          13104314
      LINE=1                                                            13104315
  150 CONTINUE                                                          13104316
C                                                                       13104317
C  END OF CASE                                                          13104318
C                                                                       13104319
      WRITE ( 6,4011)                                                   13104320
      GO TO 30                                                          13104321
C                                                                       13104322
C     KERR FROM SIMSET                                                  13104323
C                                                                       13104324
  310 WRITE ( 6,5000)KERR,DXX                                           13104325
      NR=9                                                              13104326
      NC=9                                                              13104327
      CALL PRINTM(CMATRX(1,1),NR,NC,NR,12H   CMATRX    )                13104327
      GO TO 150                                                         13104329
 3000 FORMAT (11A6,A4,5I2)                                              13104330
 3001 FORMAT (I2,2E12.6,10I3,I4)                                        13104331
 3002 FORMAT (6E12.6)                                                   13104332
 4000 FORMAT (1H1,50X30HJOB  E13104 VIBRATION ANALYSIS///)              13104333
 4001 FORMAT (1H 11A6,A4,19HNUMBER OF STATIONS I2//)                    13104334
 4002 FORMAT (1H013X4HL(1),29X4HL(2),29X4HL(3),29X4HL(4)//)             13104335
 4003 FORMAT (1H012X5HEI(1),28X5HEI(2),28X5HEI(3),28X5HEI(4)//)         13104336
 4004 FORMAT (1H013X4HG(1),29X4HG(2),29X4HG(3),29X4HG(4)//)             13104337
 4005 FORMAT (1H013X4HC(1),29X4HC(2),29X4HC(3),29X4HC(4)//)             13104338
 4006 FORMAT (1H0,7X7HI SUBJ1,16X2HDX,16X8HGAMMA(X),14X8HI SUB J2,      13104339
     114X3HETA,16X4HBETA  // )                                          13104340
 4007 FORMAT (1H012X8HW SUB N1,25X8HW SUB N2,25X8HK SUB N1,25X8HK SUB N213104341
     1//)                                                               13104342
 4008 FORMAT (9XE15.8,3(18XE15.8))                                      13104343
 4009 FORMAT (1H1,26X8HOMEGA = E15.8,5X9HDETERM = E15.8)                13104344
 4010 FORMAT (6(6H      E15.8))                                         13104345
 4011 FORMAT (14H0 END OF CASE )                                        13104346
 4012 FORMAT (1H1,53X8HOMEGA = E15.8///)                                13104347
 4013 FORMAT (1H0,15X1HV,32X1HM,31X3HPHI,31X1HY///9XE15.8,3(18XE15.8))  13104348
 4014 FORMAT (1H011X7HV PRIME,26X7HM PRIME,25X9HPHI PRIME,25X7HY PRIME//13104349
     1/9XE15.8,3(18XE15.8))                                             13104350
 4015 FORMAT (1H0,7X7H C(1)   ,15X4HC(2),15X8H  C(3)   ,14X8H  C(4)   , 13104351
     1 12X7HP SUB X,15X4HFLEX  // )                                     13104352
 4016 FORMAT ( 5(6H      E15.8))                                        13104353
 4017 FORMAT (1H0,7X8HW SUB N1,13X8HW SUB N2,13X8HK SUB N1,14X8HK SUB N213104354
     1 , 12X7HA SUB X,13X7HB SUB X  // )                                13104355
 4018 FORMAT (1H0,32X5HSTA X,6X7HK SUB X,12X8HP SUB 0X,12X7HP SUB X )   13104356
 4019 FORMAT (1H ,33XI2,3(5X,E15.8) )                                   13104357
 4021 FORMAT (19H   NUMBER OF ROOTS I3,5X,13H       OMEGA F8.3,         13104358
     1   5X,13H DELTA OMEGA F8.3,5X,10I4,I7///)                         13104359
 4022 FORMAT (1H1)                                                      13104360
 4023 FORMAT (1H1,60X17H*** CONTINUED ***/12X7HV PRIME,26X7HM PRIME,    13104361
     1 25X9HPHI PRIME,25X7HY PRIME///9XE15.8,3(18XE15.8))               13104362
 4025 FORMAT (1H )                                                      13104363
 5000 FORMAT (1H ,32X66H*** THE EQUATIONS HAVE NOT BEEN SOLVED--AM GOING13104364
     1 TO NEXT ROOT. *** /// 54X7HKERR = I1,3X6HDXX = F8.3 )            13104365
 6000 STOP                                                              13104424
      END                                                               13104366
```

```
@ ELT MATMPY,1,710420, 59943

      SUBROUTINE MATMPY(A,B,C,K1,M1,K,M,N)                            13104472
      IMPLICIT REAL*8 (A-H,O-Z)                                       13104
      DIMENSION A(20),B(20),C(20)'                                    13104473
      DO 10 I=1,K                                                     13104474
      DO 10 J=1,N                                                     13104475
      II =(J-1)*K1+I                                                  13104476
      C(II) = 0.0D0                                                   13104477
      DO 10 L=1,M                                                     13104478
      JJ =(L-1)*K1+I                                                  13104479
      KK =(J-1)*M1+L                                                  13104480
   10 C(II) =C(II) +A(JJ)*B(KK)                                       13104481
      RETURN                                                          13104482
      END                                                             13104483
```

```
@ ELT PRINTM,1,710420, 59944

      SUBROUTINE PRINTM(A,NR,NC,MAXR,TITLE)                             13104445
      IMPLICIT REAL*8 (A-H,O-Z)
C                                                                       13104446
C                                                                       13104447
C SUBROUTINE TO PRINT ANY MATRIX WITH 2-WORD TITLE                      13104448
C     CALL PRINTM (CMATRX,8,8,8,12H    CMATRX    ) EXAMPLE CALL UP      13104449
C                                                                       13104450
      DIMENSION A(1),NHED(8),TITLE(2)                                   13104451
C                                                         MATRIX TITLE  13104452
      WRITE (6,22)TITLE                                                 13104453
   22 FORMAT (1H0,52X,2A6)                                              13104454
C                                                                       13104455
      DATA B /'   COL'/                                                 13104456
      DO 50 I=1,NC,8                                                    13104457
      II=NC-I+1                                                         13104458
      IF(II-8)20,20,10                                                  13104459
   10 II=8                                                              13104460
   20 DO 30 J=1,II                                                      13104461
   30 NHED(J)=I+J-1                                                     13104462
      WRITE (6,120)        (B,NHED(J),J=1,II)                           13104463
      DO 50 J=1,NR                                                      13104464
      KL=J+(I-1)*MAXR                                                   13104465
      KH=KL+(II-1)*MAXR                                                 13104466
   50 WRITE (6,130)        (J, A(K),K=KL,KH,MAXR)                       13104467
      RETURN                                                            13104468
  120 FORMAT (1H0,9X,10(A6,I4,4X))                                      13104469
  130 FORMAT (4H ROW,I3,5X,1P8E14.7)                                    13104470
      END                                                               13104471
```

```
@ ELT REPEAT,1,710420, 59945

      SUBROUTINE REPEAT(A,AA,B,BB,C,CC,D,DD,E,EE,F,FF,NR)                13104425
      IMPLICIT REAL*8 (A-H,O-Z)
C************************************************************************13104426
C     REPEAT READS IN A STATION CARD OR SIMULATES A REPEATED CARD BY     13104427
C     MOVING DATA.                                                       13104428
C     A,B,C,D,E,F   OLD         AA,BB,CC,DD,EE,FF   NEW                  13104429
C               NR = NUMBER OF REPEATS FOR A PARTICULAR CARD             13104430
C************************************************************************13104431
      IF(NR-1)400,100,100                                                13104432
  400 READ (5,3002)          AA,BB,CC,DD,EE,FF,NR                        13104433
 3002 FORMAT ( 6E12.6,I3)                                                13104434
      GO TO 700                                                          13104435
  100 AA=A                                                               13104436
      BB=B                                                               13104437
      CC=C                                                               13104438
      DD=D                                                               13104439
      EE=E                                                               13104440
      FF=F                                                               13104441
      NR=NR-1                                                            13104442
  700 RETURN                                                             13104443
      END                                                                13104444
```

@ ELT SETUP,1,710420, 59947

```
      SUBROUTINE SETUP(OMG,N)                                           13104367
      IMPLICIT REAL*8 (A-H,O-Z)                                         13104368
      DIMENSION DL1(50),DL2(50),DL3(50),DL4(50),DEI1(50),DEI2(50),      13104368
     1          DEI3(50),DEI4(50),DG1(50),DG2(50),DG3(50),DG4(50),      13104369
     2          DC1(50),DC2(50),DC3(50),DC4(50),DIJ1(50),DX(50),        13104370
     3          DGAMX(50),DIX2(50),DWN1(50),DWN2(50),DKN1(50),DKN2(50)  13104371
      DIMENSION DETA(50),DDETA(50),DFLEX(50)                            13104372
      DIMENSION E1MTRX(9,9),E2MTRX(9,9),AMATRX(9,9),BMATRX(9,9),        13104373
     1           CMATRX(9,9),FMATRX(9,9),DLMTRX(9,1),SMATRX(9,1),       13104374
     2           DUMMY(9),C(5),ID(5),X(4),KID(4),STORE(9,50)            13104375
      DIMENSION DPN1(50),P0(50),DAN1(50),DBN1(50), P1(50),P2(50),P3(50) 13104376
      COMMON  DL1     , DL2     , DL3     , DL4     , DEI1    , DEI2    , 13104377
     1        DEI3    , DEI4    , DG1     , DG2     , DG3     , DG4     , 13104378
     2        DC1     , DC2     , DC3     , DC4     , DIJ1    , DX      , 13104379
     3        DGAMX   , DIX2    , DWN1    , DWN2    , DKN1    , DKN2    , 13104380
     4        E1MTRX  , E2MTRX  , AMATRX  , BMATRX  , CMATRX  , FMATRX  , 13104381
     5        KC      , KD      , KM      , KN      , KO      , KP      , 13104382
     6        DETA    , DDETA   , DFLEX   , DPN1    , P0      , DAN1    , 13104383
     7        DBN1    , IFLAG   , NTRIAL  , B       , DLMTRX  , SMATRX  , 13104384
     8        DUMMY   , KK      , KA      , KB                            13104385
C                                                                       13104384
C       SETUP NON-ZERO,NON-UNIT ELEMENTS OF MATRICES E1,E2,AND F        13104385
C                                                                       13104386
    1 E1MTRX(2,1) = DL1(N)                                              13104 0A
      IF (DEI1(N).NE.0.D0) GO TO 2                                      13104 0B
      E1MTRX(4,2) = 0.D0                                                13104 0C
      E1MTRX(3,2) = 0.D0                                                13104 0D
      GO TO 3                                                           13104 0E
    2 E1MTRX(4,2) = DL1(N)**2*.5D0/DEI1(N)                              13104 0F
      E1MTRX(3,2) =-DL1(N)/DEI1(N)                                      13104 0G
    3 E1MTRX(3,1) =-E1MTRX(4,2)                                         13104 0H
      IF  ((DEI1(N).NE.0.D0.OR.DC1(N).NE.0.D0.AND.DL1(N).NE.0.D0)       13104 0I
     X     .AND.DG1(N).NE.0.D0)  GO TO 4                                13104 0J
      E1MTRX(4,1) = 0.D0                                                13104 0K
      GO TO 5                                                           13104 0L
    4 E1MTRX(4,1) = DL1(N)**3/6.D0/DEI1(N)-DC1(N)*DL1(N)/DG1(N)         13104 0M
    5 E1MTRX(4,3) =-E1MTRX(2,1)                                         13104 0N
      E1MTRX(6,5) = DL3(N)                                              13104 0P
      IF (DEI3(N).NE.0.D0) GO TO 6                                      13104 0Q
      E1MTRX(7,5) = 0.D0                                                13104 0R
      E1MTRX(7,6) = 0.D0                                                13104 0S
      E1MTRX(8,6) = 0.D0                                                13104 0T
      GO TO 7                                                           13104 0U
    6 E1MTRX(7,5) =-DL3(N)**2/2.D0/DEI3(N)                              13104 0V
      E1MTRX(7,6) =-DL3(N)/DEI3(N)                                      13104 0W
      E1MTRX(8,6) = DL3(N)**2/2.D0/DEI3(N)                              13104 0X
    7 IF  ((DEI3(N).NE.0.D0.OR.DC3(N).NE.0.D0.AND.DL3(N).NE.0.D0)       13104 0Y
     X     .AND.DG3(N).NE.0.D0)  GO TO 8                                13104 0Z
      E1MTRX(8,5) = 0.D0                                                13104 1A
      GO TO 9                                                           13104 1B
    8 E1MTRX(8,5) = DL3(N)**3/6.D0/DEI3(N)-DC3(N)*DL3(N)/DG3(N)         13104 1C
    9 E1MTRX(8,7) =-E1MTRX(6,5)                                         13104 1D
      E2MTRX(2,1) = DL2(N)                                              13104 1E
      IF (DEI2(N).NE.0.D0) GO TO 10                                     13104 1F
      E2MTRX(4,2) = 0.D0                                                13104 1G
      E2MTRX(3,2) = 0.D0                                                13104 1H
```

```
                   GO TO 11                                                   13104 1I
            10     E2MTRX(4,2) = DL2(N)**2*.5D0/DEI2(N)                       13104 1J
                   E2MTRX(3,2) =-DL2(N)/DEI2(N)                               13104 1K
            11     E2MTRX(3,1) =-E2MTRX(4,2)                                  13104 1L
                   IF  ((DEI2(N).NE.0.D0.OR.DC2(N).NE.0.D0.AND.DL2(N).NE.0.D0) 13104 1M
                  X    .AND.DG2(N).NE.0.D0)  GO TO 12                         13104 1N
                   E2MTRX(4,1) = 0.D0                                         13104 1O
                   GO TO 13                                                   13104 1P
            12     E2MTRX(4,1) = DL2(N)**3/6.D0/DEI2(N)-DC2(N)*DL2(N)/DG2(N)  13104 1Q
            13     E2MTRX(4,3) =-E2MTRX(2,1)                                  13104 1R
                   E2MTRX(6,5) = DL4(N)                                       13104 1S
                   IF (DEI4(N).NE.0.D0) GO TO 14                              13104 1T
                   E2MTRX(7,5) = 0.D0                                         13104 1U
                   E2MTRX(7,6) = 0.D0                                         13104 1V
                   E2MTRX(8,6) = 0.D0                                         13104 1W
                   GO TO 15                                                   13104 1X
            14     E2MTRX(7,5) =-DL4(N)**2/2.D0/DEI4(N)                       13104 1Y
                   E2MTRX(7,6) =-DL4(N)/DEI4(N)                               13104 1Z
                   E2MTRX(8,6) = DL4(N)**2/2.D0/DEI4(N)                       13104 2A
            15     IF  ((DEI4(N).NE.0.D0.OR.DC2(N).NE.0.D0.AND.DL4(N).NE.0.D0) 13104 2B
                  X    .AND.DG4(N).NE.0.D0)  GO TO 16                         13104 2C
                   E2MTRX(8,5) = 0.D0                                         13104 2E
                    GO TO 17                                                  13104 2F
            16     E2MTRX(8,5) = DL4(N)**3/6.D0/DEI4(N)-DC4(N)*DL4(N)/DG4(N)  13104 2G
            17     E2MTRX(8,7) =-E2MTRX(6,5)                                  13104 2H
                   FMATRX(1,4)=DWN1(N)*OMG/386.04D0*OMG-DKN1(N)               13104411
                   FMATRX(1,7)=DX(N)*DKN1(N)                                  13104412
                   FMATRX(1,8)=DKN1(N)                                        13104413
                   FMATRX(1,9)=DGAMX(N)*OMG**2+DETA(N)                        13104414
                   FMATRX(2,9)=DBETA(N)*OMG**2                                13104415
                   FMATRX(2,3)=-DIJ1(N)*OMG**2                                13104416
                   FMATRX(5,4)=FMATRX(1,8)                                    13104417
                   FMATRX(5,7)=-FMATRX(1,7)                                   13104418
                   FMATRX(5,8)=DWN2(N)*OMG/386.04D0*OMG-DKN1(N)-DKN2(N)       13104419
                   FMATRX(6,4)=FMATRX(1,7)                                    13104420
                   FMATRX(6,7)=DIX2(N)*OMG**2-DX(N)**2*DKN1(N)                13104421
                   FMATRX(6,8)=FMATRX(5,7)                                    13104422
                   RETURN                                                     13104423
                   END                                                        13104424
```

```
@ ELT SIMEQ,1,710422, 38770

      SUBROUTINE SIMEQ(A,B,NN,MM,NA ,ITEM,DD,NND,KERR)                  61210002
C*****                                                                  61210003
C     SOLVES MATRIX EQUATIONS - AX = B                                  61210004
C     GAUSS ELIMINATION WITH COMPLETE PIVOTING ON ABSOLUTE LARGEST      61210005
C     ELEMENT TO FORM TRIANGULAR MATRIX,WITH BACK SUBSTITUTION FOR      61210006
C     SOLUTION VECTORS.                                                 61210007
C******                                                                 61210008
C                                                                       61210009
C     CALL SIMEQ (A,B,NN,MM,NA ,ITEM,DD,NND,KERR )                      61210010
C     A          =    A(1,1) OF INPUT MATRIX                            61210011
C     B          =    INPUT VECTORS                                     61210012
C     NN         = NUMBER OF SIMULTANEOUS EQUATIONS.                    61210013
C     MM         = NUMBER OF B-VECTORS.                                 61210014
C     NA         = DIMENSION OF MATRIX A,THAT IS, A(NA,--)              61210015
C     ITEM       = TEMPORARY STORAGE (FOR PERMUTATION VECTOR)           61210016
C                  WITH DIMENSION - ITEM(NA)                            61210017
C     DD         = DETERMINANT                                          61210018
C     NND        = POWER OF TEN TO MULTIPLY DETERMINANT                 61210019
C     KERR       = ERROR CODE,  =K, SINGULAR RANK , =-1 SOLVED EQUATIONS 61210020
      DOUBLE PRECISION A(NA,NA),B(NA,1 ),PIVOT,XTEM,D,DD                61210021
      DIMENSION ITEM(2)                                                 61210022
C                                                                       61210023
      D          = 1.0D0                                                61210024
C     ND         = POWERS OF TENS FACTOR FOR DETERMINANT.               61210025
      ND         = 0                                                    61210026
      N=NN                                                              61210027
      M=MM                                                              61210028
C                                                                       61210029
C     SET-UP THE PERMUTATION VECTOR.                                    61210030
      DO 1       I=1,N                                                  61210031
    1 ITEM(I)    = I                                                    61210032
      N1         = N-1                                                  61210033
      DO 60      K=1,N                                                  61210034
C                                                                       61210035
C     SEARCH AND SET THE ABSOLUTE LARGEST ELEMENT AS THE PIVOT.         61210036
C                                                                       61210037
      PIVOT      = 0.D0                                                 61210038
      DO 10      I=K,N                                                  61210039
      DO 9       J=K,N                                                  61210040
      XTEM       = A(I,J)                                               61210041
      IF(DABS(XTEM) .LE. DABS(PIVOT))  GO TO 9                          61210042
      PIVOT      = XTEM                                                 61210043
      IS         = I                                                    61210044
      IT         = J                                                    61210045
    9 CONTINUE                                                          61210046
   10 CONTINUE                                                          61210047
C     COMPUTE DETERMINANT AND TEST FOR SINGULAR MATRIX.                 61210048
C                                                                       61210049
C                                                                       61210050
      D            = D*PIVOT                                            61210051
      IF(D.NE.0.D0)            GO TO 11                                 61210052
C     IF MATRIX IS SINGULAR,SET THE RANK OF MATRIX A IN KERR    AND EXIT61210053
      KERR       =    K-1                                               61210054
      GO TO 100                                                         61210055
   11 XTEM       =    DABS(D)                                           61210056
      IF(XTEM.LE.1.D0)         GO TO 13                                 61210057
```

```
      D         = D/10.D0
      ND        = ND+1
      GO TO 11
   13 IF(XTEM.GE.0.1D0)          GO TO 14
      D         = D*10.D0
      ND        = ND-1
      GO TO 11
   14 CONTINUE
      IF(K.EQ.IS)                GO TO 30
C
C     IF THE PIVOT IS NOT IN THE RIGHT ROW,INTERCHANGE ROWS.
C
      DO 20     J=1,N
      XTEM      = A(IS,J)
      A(IS,J)   = A(K,J)
      A(K ,J)   = XTEM
   20 CONTINUE
      DO 21     J=1,M
      XTEM      = B(IS,J)
      B(IS,J)   = B(K ,J)
      B(K ,J)   = XTEM
   21 CONTINUE
      D         = -D
   30 IF(K.EQ.IT)                GO TO 40
C
C     IF THE PIVOT IS NOT IN THE RIGHT COL.,EXCHANGE COLS AND RECORD
C     THIS IN THE PERMUTATION VECTOR.
C
      DO 31     I=1,N
      XTEM      = A(I,IT)
      A(I,IT)   = A(I,K )
      A(I,K )   = XTEM
   31 CONTINUE
      D         = -D
C
C     SET PERMUTATION VECTOR
C
      I         = ITEM(IT)
      ITEM(IT) = ITEM(K )
      ITEM(K ) = I
C
   40 CONTINUE
      K1        = K+1
      IF(K1.GT.N)                GO TO 60
C
C     MULTIPLY THE K-TH ROW BY -A(I,K)/PIVOT AND ADD TO THE I-TH ROW
      DO 50     I=K1,N
      DO 50     J=K1,N
      A(I,J)    = A(I,J) - A(K,J)/PIVOT * A(I,K)
   50 CONTINUE
      DO 51     I=K1,N
      DO 51     J=1,M
      B(I,J)    = B(I,J) - A(I,K)/PIVOT*B(K,J)
   51 CONTINUE
   60 CONTINUE
C
C     BACKSUBSTITUTION FOLLOWS.
C
      DO 70     J=1,M
      B(N,J)    = B(N,J)/A(N,N)
```

```
<****Δ****1****Δ****2****Δ****3****Δ****4****Δ****5****Δ****6****Δ****7****Δ****8****Δ****9****Δ****0****Δ****1****Δ****2****Δ****3*
*************************************ISD-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972******************************************
***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ
[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ
```

25 APR 72 P 14:42:31 IDENT=FYEE ACCOUNT=428999 CARDS IN= 9, OUT= 0

PAGES= 22, LINES= 919. TIME=00:00:06 (HMS)

* *

*** USER NOTICES - APRIL 20, 1972 ***

(1) ISD 1108 TERMINAL SERVICE IS SCHEDULED AS FOLLOWS

MON :	07:00 - 24:00
TUE - FRI :	00:00 - 04:00 ; 07:00 - 24:00
SAT :	00:00 - 22:00
SUN :	04:00 - 22:00

(2) LARGE-CORE (LCR) PRODUCTION JOBS ARE NOW BEING RUN ON AN OVERNIGHT BASIS STARTING AT 04:00 EACH DAY.

(3) ISD NOW HAS AVAILABLE REMOTE-BATCH JOB ENTRY VIA LOW-SPEED TELETYPE COMPATIBLE TERMINALS USING DIAL-UP COMMUNICATION LINES. THIS SERVICE HAS BEEN IN USE FOR OVER TWO MONTHS AND IS CALLED RON/I. THE DIAL-UP TELEPHONE NUMBERS AND TRANSMISSION RATES ARE LISTED BELOW.

10 CHAR/SEC 415-562-4035, 415-562-4036, 415-562-5186
30 CHAR/SEC 415-562-4716 ** EFFECTIVE 4/24/72 THIS NUMBER WILL BE CHANGED TO 415-562-4294 **

(4) ISD'S SECOND PUBLIC TERMINAL IN SAN FRANCISCO IS LOCATED AT # 1 CALIFORNIA ST., ROOM 2555.

(5) BEGINNING 4/24/72 AND AFFECTIVE MONDAY - FRIDAY TURNAROUND TIME SHOULD BE REDUCED BETWEEN THE HOURS OF 10:30 - 11:30 AND 14:00 - 16:00 FOR USERS SUBMITTING NON-TAPE JOBS WITH RUN TIMES ESTIMATED AT LESS THAN 6 MINUTES.

ADDITIONAL INFORMATION ON (2) & (3) IS NOW AVAILABLE TO ALL INTERESTED USERS BY CONTACTING YOUR SALESMAN AT 415-562-4204.

* *

```
 70   CONTINUE
      I         = N
      DO 73     K=2,N
      I1        = I
      I         = I-1
      PIVOT     = A(I,I)
      DO 72     IT=1,M
      XTEM      = 0.D0
      DO 71     J=I1,N
 71   XTEM      = A(I,J)*B(J,IT) + XTEM
 72   B(I,IT)   = (B(I,IT) - XTEM)/PIVOT
 73   CONTINUE
C
C     USE PERMUTATION VECTOR TO EXCHANGE ROWS OF B-MATRIX.
C
      DO 81     I=1,N
 79   IF(ITEM(I).EQ.I)          GO TO 81
      K         = ITEM(I)
      DO 80     J=1,M
      XTEM      = B(K,J)
      B(K,J)    = B(I,J)
      B(I,J)    = XTEM
 80   CONTINUE
      ITEM(I)   = ITEM(K)
      ITEM(K)   = K
      GO TO 79
 81   CONTINUE
      KERR=-1
      DD        =    D
      NND       =    ND
100   RETURN
      END
```

@ ELT SIMST,1,710420, 59949

```
      SUBROUTINE SIMST (C,K,M,A,B)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION C(5),K(5),X(5),ITEM(10),KI(4),A(10,1),XX(10)
      DATA IR/0/
      IR      = IR+1
      DO 10   I=1,10
   10 A(IR,I)  =0.D0
      DO 20     I=1,M
      J       = K(I)
      IF(J)         18,51,16
   16 A(IR,J)     = C(I)
      GO TO 20
   18 XX(IR)    = C(I)
   20 CONTINUE
 50   RETURN
   51 KER     = 4
      GO TO 50
      ENTRY SIMSD (X,KI,DET,KERR,IDUM)
      IF(KER-3)     53,70,53
   53 CONTINUE
      N       = IR
      IR    = 0
      DO 52  I=1,N
   52 KI(I)     = I
      D = 0.D0
      ND = 0
      CALL SIMEQ (A,XX,N,1,10,ITEM,D,ND,KER )
      DO 54   I=1,N
   54 X(I)    = XX(I)
      IF(D) 56,58,56
   56 CONTINUE
      DET     = D*10.D0**ND
   58 CONTINUE
      IF (KER) 70,65,65
   65 KER = 0
   70 KERR = KER + 1
   45 RETURN
      ENTRY SIMSZ
      IR    = 0
      RETURN
      END
```

4.   TRI X   14:42:30

END CUR

```
<***Δ****1****Δ****2****Δ****3****Δ****4****Δ****5****Δ****6****/****7****Δ****8****Δ****9****Δ****0****Δ****1****Δ****2****Δ****3*
***********************************ISD-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972*****************************************
***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ
[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ

25 APR 72 P 14:42:31    IDENT=FYEE    ACCOUNT=428999  CARDS IN=    9, OUT=    0

  PAGES=   22, LINES=   919. TIME=00:00:06 (HMS)

\\EOF\
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\RUN RECEIVED BY ISD AT 14:47:33 ON 25 APR 72:

\ @9 RUN WELLS,425465,5,200

\\EOF

APPENDIX G

PROGRAM E13112 LATERAL VIBRATION ANALYSIS OF A HARMONICALLY FORCED, UNDAMPED, LUMPED PARAMETER SINGLE BEAM SYSTEM SUPPORTED ON NON-LINEAR SPRINGS, USERS' MANUAL AND SAMPLE OF INPUT/OUTPUT

i

USERS' MANUAL FOR

JOB 14036

(E13112)

LATERAL VIBRATION ANALYSIS OF A HARMONICALLY FORCED, UNDAMPED, LUMPED PARAMETER BEAM SYSTEM SUPPORTED ON NON-LINEAR SPRINGS

September 10, 1965

Prepared by:

G.L. Goudreau

G.L. Goudreau

Design Engineer

Structural Analysis Program

Approved by:

L.K. Severud, Supervisor

Structural Analysis Program

J.D. McConnell, Manager

Structural Analysis Program

SUBJECT LATERAL VIBRATION ANALYSIS OF A HARMONICALLY FORCED, UNDAMPED, LUMPED PARAMETER BEAM SYSTEM SUPPORTED ON NON-LINEAR SPRINGS

DATE 9/9/65

WORK ORDER

BY G.L. Goudreau

CHK. BY

DATE

ABSTRACT

This computer program determines the steady state response of an undamped, lumped parameter beam system to harmonic forces and/or moments. The beam may be supported by non-linear lateral springs of the form $P = A\, y^B$. It is compatible with and supplements Job 14009 which determines the natural frequencies and mode shapes of such a beam on linear springs. It is four times faster than the double beam program 14043. This program has been extended to incorporate the exact load-deflection equations of angular contact ball bearings, permitting them to be represented exactly, including axial equilibrium and the effects of thrust on a rotating shaft. After convergence the program outputs at each station shears, moments, slopes, and deflections. For ball bearings, when present, a complete description of the equilibrium position of the bearing is determined, including axial, lateral, and rotational deformations, as well as load distribution and final contact angles of each ball.

TABLE OF CONTENTS

SUBJECT
LATERAL VIBRATION ANALYSIS OF A HARMONICALLY FORCED, UNDAMPED, LUMPED PARAMETER BEAM SYSTEM SUPPORTED BY NON*LINEAR SPRINGS

DATE 9/7/65

WORK ORDER

BY G.L. Goudreau

CHK. BY

DATE

INTRODUCTION

This computer program computes the amplitudes of the shears, moments, slopes, and deflections produced in a lumped parameter beam system by harmonic forces and/or moments. It constitutes an extension of Job 14009 developed by L.K. Severud, which computed the natural frequencies and associated modal information for a lumped parameter beam system supported by linear lateral springs. Job 14036 differs in the following respects: 1) It is a forced rather than a free vibration analysis; 2) It permits linear moment as well a lateral springs; 3) It permits non-linear lateral springs of the form $F = A\,y^{B}$; and 4) It includes the exact load deflection equations for angular contact ball bearings for specialized use in rotating machinery calculations. Input format is highly compatible between the two programs, facilitating determination of free and forced vibration information.

As in Job 14009, the beam system has a state vector of four variables (shear, moment, slope, and deflection), of which two at each end must equal zero. Ordinary beams, shear beams, beams on elastic foundations where the foundation modulus varies (thus, also axisymmetrical lateral vibrations of cylinders), and shafts of rotating machinery can be investigated with this program. In this last application, the lateral stiffness of roller bearings can be well represented by a power form of load deflection curve, and ball bearings can be accounted for exactly.

This program runs in about one fourth the time of Job 14043 which analyzes two elastically coupled beams, and is to be preferred where housing effects can be neglected.

SUBJECT: LATERAL VIBRATION ANALYSIS OF A HARMONICALLY FORCED, UNDAMPED, LUMPED PARAMETER BEAM SYSTEM SUPPORTED BY NON-LINEAR SPRINGS

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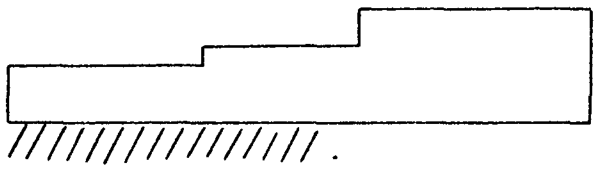
CHK. BY

DATE

THE LUMPED PARAMETER MODEL

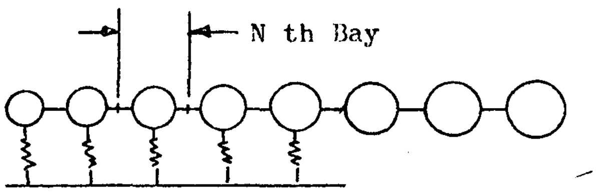
(The following three pages are taken from the Users' Manual for Job 14009 by L.K. Severud)

To describe the model, first consider the following beam on a discontinuous foundation:

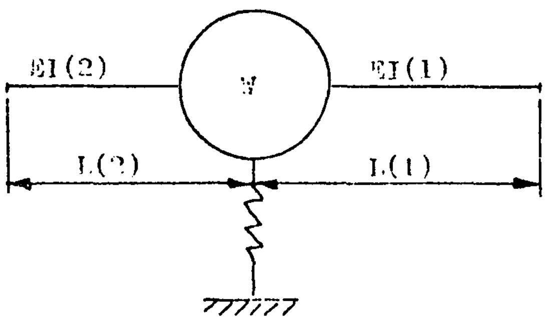


k (force/unit length)

We might represent this structure for the vibration analysis as:



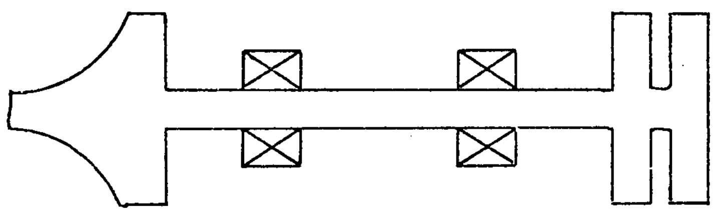
We see that a typical element, which is called the N th bay in the above sketch, is:



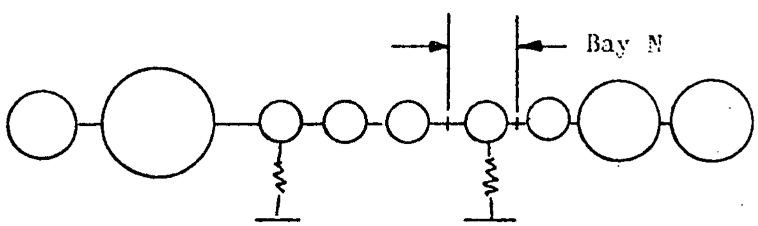
The weight of the bay, W , is lumped at point N. The beam lengths, which need not be equal, are designated L(1) and L(2). The associated bending rigidities are EI(1) and EI(2). To represent the elastic foundation acting on the bay, a spring constant K is utilized. Note that in this example K would equal $\left[L(1) + L(2)\right]$ k.

One can see that the physical beam rigidities are quite well represented in the analysis whereas the mass distribution and elastic foundation representation depends on the number of lumping stations used.

As a second example to describe the lumped parameter model, consider the following two bearing shaft with overhung rotors:

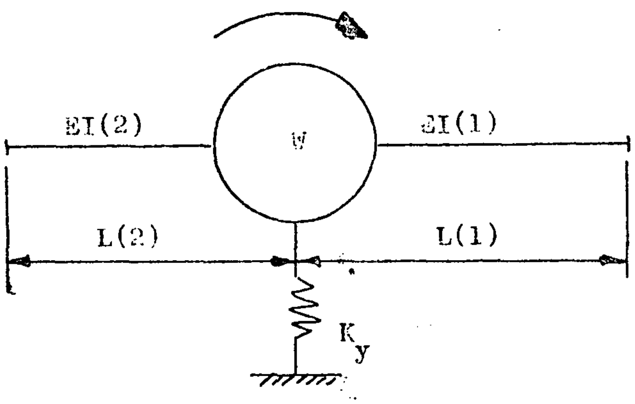


The lumped model is:



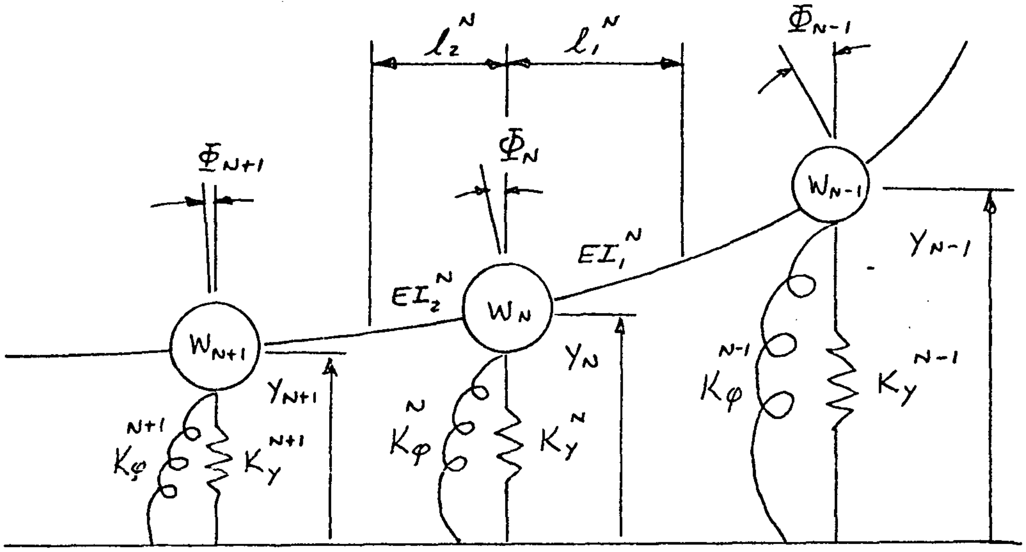
Enlarging the sketch of Bay N:

$$\Delta M = (I_j - I_x)\,\omega^2\,\Phi = -J\,\omega^2\,\Phi$$



The value ΔM accounts for the rotary inertia and gyroscopic effects of the mass (of prime importance for rotors). It is shown as a D'Alembert moment. Good discussions of these effects and derivation of the above formula can be found in References (2), (3), and (4). The parameter I_x is the mass moment of inertia of the mass about a diametral line and I_j is its mass polar moment of inertia. Understanding of the lumped parameter model is best obtained through a knowledge of the computational formulas used in the program. Therefore, the following section sets forth the theory and derivation of equations used in this vibration analysis.

1. LUMPED PARAMETER MODEL



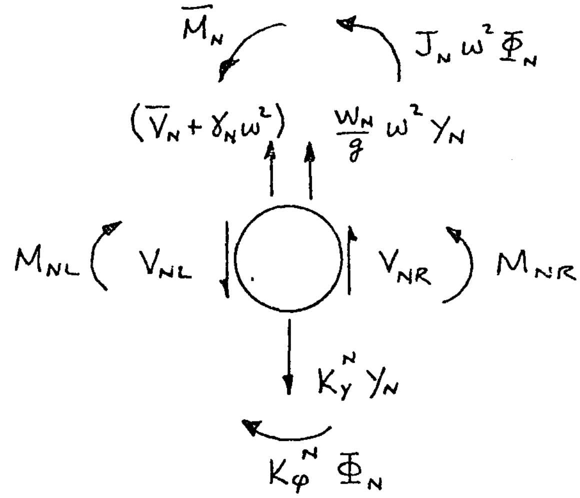
2. STATE VECTOR

$$\{\Delta_N\} = \begin{Bmatrix} V_N \\ M_N \\ \Phi_N \\ Y_N \\ 1 \end{Bmatrix}$$

The state vector Δ_N is defined as the column array of the shear, moment, slope, and deflection in the beam at the end of bay N. The fifth element of the state vector is the constant one which permits the inclusion of the load constants in the transfer matrices.

B

3. MASS TRANSFER MATRIX



$$V_{NL} = V_{NR} + \frac{W_N}{g} \omega^2 Y_N - K_Y^N Y_N + \overline{V}_N + \gamma_N \omega^2$$

$$M_{NL} = M_{NR} + J_N \omega^2 \Phi_N - K_\varphi^N \Phi_N + \overline{M}_N$$

$$\Phi_{NL} = \Phi_{NR} \quad ; \quad Y_{NL} = Y_{NR}$$

$$\begin{Bmatrix} V_{NL} \\ M_{NL} \\ \Phi_{NL} \\ Y_{NL} \\ \hline 1 \end{Bmatrix} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & \left(\frac{W_N}{g}\omega^2 - K_Y^N\right) & \overline{V}_N + \gamma_N \omega^2 \\ 0 & 1 & \left(J_N \omega^2 - K_\varphi^N\right) & 0 & \overline{M} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} V_{NR} \\ M_{NR} \\ \Phi_{NR} \\ Y_{NR} \\ \hline 1 \end{Bmatrix}$$

OR $\{\Delta_{NL}\} = [F_N] \{\Delta_{NR}\}$

SUBJECT: LATERAL VIBRATION ANALYSIS OF A HARMONICALLY FORCED, UNDAMPED, LUMPED PARAMETER BEAM SYSTEM SUPPORTED BY NON-LINEAR SPRINGS

DATE 5/8/65

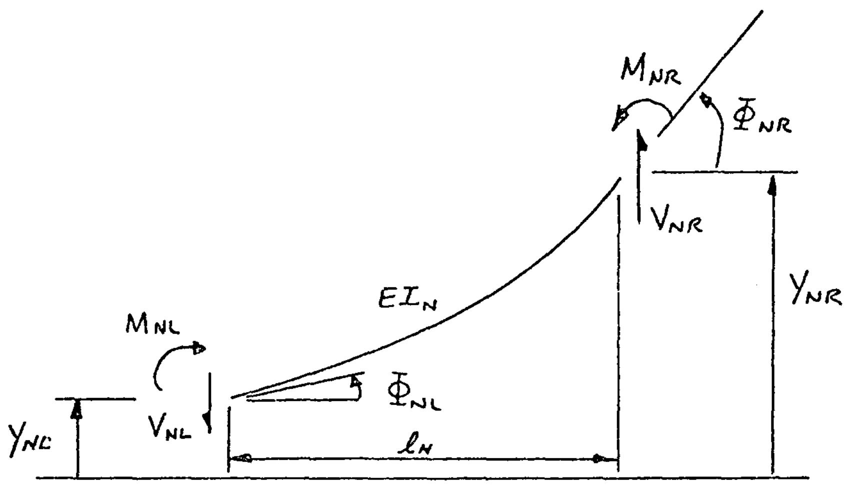
WORK ORDER

BY: G.L. Goudreau

CHK. BY

DATE

4. ELASTICITY TRANSFER MATRIX



$$\Phi_{NL} = \Phi_{NR} - \frac{\ell_N^2}{2EI_N} V_{NR} - \frac{\ell_N}{EI_N} M_{NR}$$

$$Y_{NL} = Y_{NR} - \Phi_{NR}\,\ell_N + \left(\frac{\ell_N^3}{6EI_N} - \frac{C_N \ell_N}{G_N}\right) V_{NR} + \frac{\ell_N^2}{2EI_N} M_{NR}$$

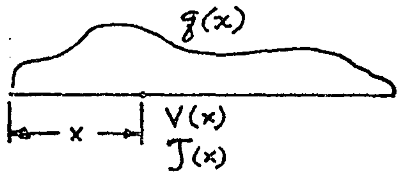
$$V_{NL} = V_{NR} \quad ; \quad M_{NL} = M_{NR} + V_{NR}\,\ell_{NR}$$

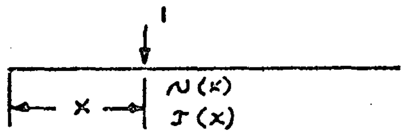
$$\begin{Bmatrix} V_{NL} \\ M_{NL} \\ \Phi_{NL} \\ Y_{NL} \\ \hline 1 \end{Bmatrix} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ \ell_N & 1 & 0 & 0 & 0 \\ -\frac{\ell_N^2}{2EI_N} & -\frac{\ell_N}{EI_N} & 1 & 0 & 0 \\ \left(\frac{\ell_N^3}{6EI_N} - \frac{C_N \ell_N}{G_N}\right) & \frac{\ell_N^2}{2EI_N} & -\ell_N & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} V_{NR} \\ M_{NR} \\ \Phi_{NR} \\ Y_{NR} \\ \hline 1 \end{Bmatrix}$$

$$\text{OR} \qquad \{\Delta_{NL}\} = [E_N]\,\{\Delta_{NR}\}$$

This transfer matrix symbolically represents both spans of bay N and for each the appropriate ℓ_N, EI_N, C_N, and G_N must be used.

SHEAR DEFLECTION COEFFICIENT FOR BEAMS



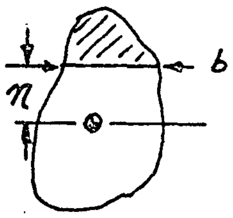


EQUATING WORK DONE –

$$\frac{1 \cdot \delta_s}{2} = \int_V \frac{1}{2}\,\mathcal{T}\left(\frac{\tau}{G}\right) dV$$

$$\therefore \quad \delta_s = \int_V \frac{\mathcal{T}\,\tau}{G}\, dV$$

GENERAL CROSS SECTION –



ASSUME $\mathcal{T} = \dfrac{VQ}{Ib}$, $\tau = \dfrac{\nu Q}{Ib}$

$$\delta_s = \int_0^L \frac{V\nu}{G}\left[\frac{1}{I^2}\int_A \frac{Q^2}{b^2}\, dA\right] dx$$

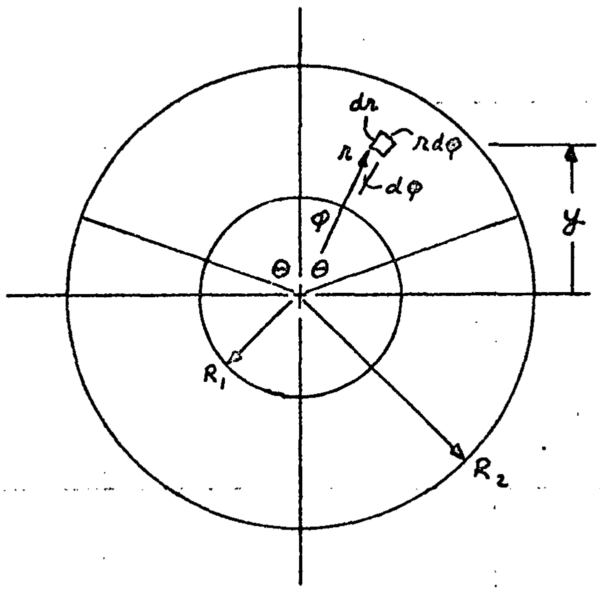
$$\therefore \quad \delta_s = \int_0^L k\,\frac{V\nu}{GA}\, dx$$

WHERE $k = \dfrac{A}{I^2}\displaystyle\int_A \frac{Q^2}{b^2}\, dA$

IF $dA = b\, d\eta$

$$\boxed{k = \frac{A}{I^2}\int_h \frac{Q^2\, d\eta}{b}}$$

HOLLOW CIRCULAR SECTION



$$k = \frac{A}{I^2} \int_A \frac{Q^2}{b^2}\, dA$$

$$R_1 = a R_2$$

$$A = \pi (R_2^2 - R_1^2) = \pi R_2^2 (1-a^2)$$

$$I = \frac{\pi}{4} (R_2^4 - R_1^4) = \frac{\pi R_2^4}{4} (1-a^4)$$

$$C = \frac{k}{A}$$

$$dA = r\, d\varphi\, dr \quad , \quad y = r \cos \varphi$$

$$Q(\theta) = \int_{-\theta}^{\theta} \int_{R_1}^{R_2} y\, dA$$

$$= \int_{-\theta}^{\theta} \int_{R_1}^{R_2} r \cos\varphi \; r\, d\varphi\, dr$$

$$Q(\theta) = \frac{2}{3} (R_2^3 - R_1^3) \sin\theta = \frac{2}{3} R_2^3 (1-a^3) \sin\theta$$

$$b(\theta) = 2(R_2 - R_1) = 2R_2 (1-a)$$

$$k = \frac{\pi R_2^2 (1-a^2) \; \frac{4}{9} R_2^6 (1-a^3)^2}{\frac{\pi^2}{16} R_2^8 (1-a^4)^2 \; 4R_2^2 (1-a)^2} \int_{-\pi}^{\pi} \int_{R_1}^{R_2} \sin^2\theta \; r\, d\theta\, dr$$

$$= \frac{16}{9\pi R_2^2} \frac{(1-a^2)(1-a^3)^2}{(1-a^4)^2 (1-a)^2} \; \frac{1}{2} (R_2^2 - R_1^2) \quad (\pi)$$

$$k = \frac{8}{9}\left[\frac{(1-a^2)(1-a^3)}{(1-a^4)(1-a)}\right]^2$$

$$k = \frac{8}{9}\left[\frac{(1+a+a^2)}{(1+a^2)}\right]^2$$

FOR THIN CIRCULAR SECTION $a \to 1$

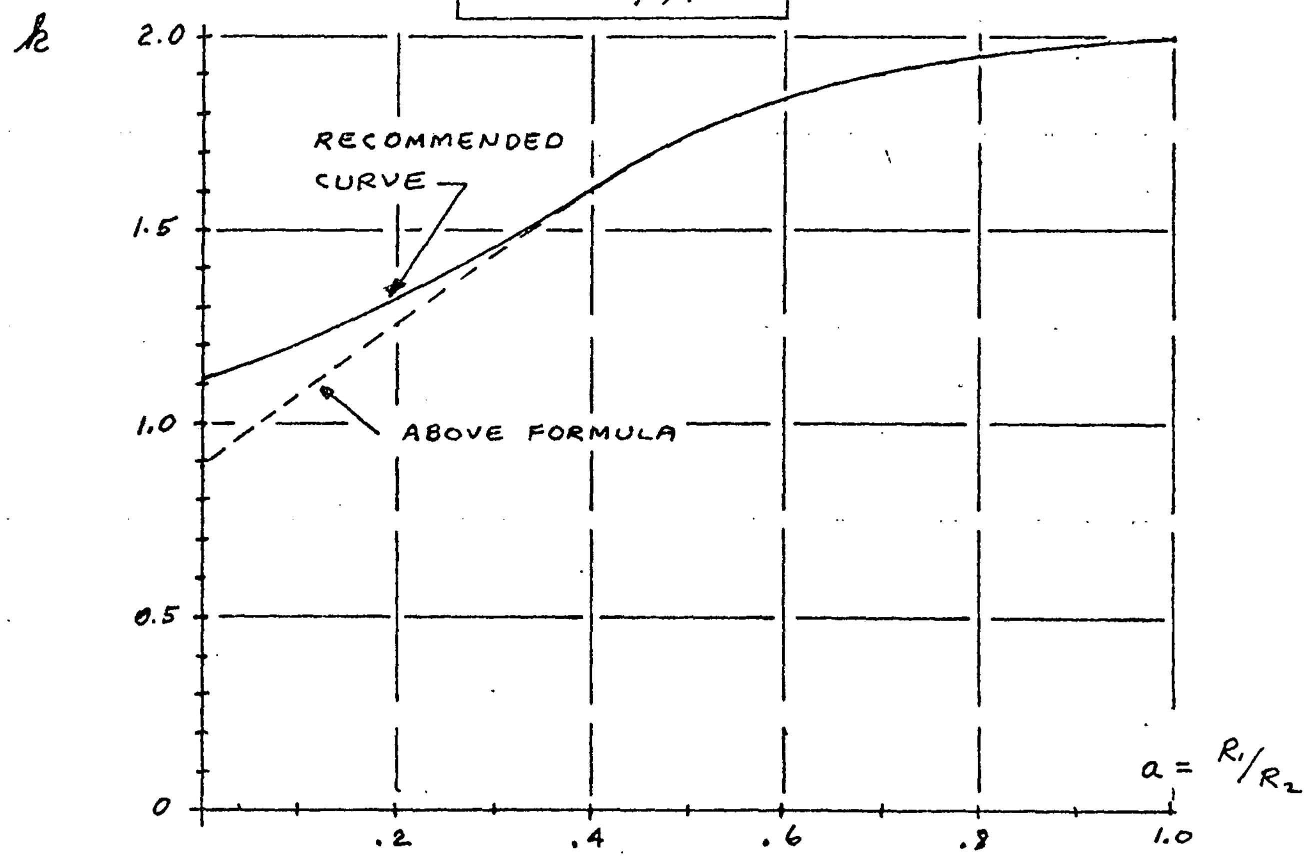
$$k = \frac{8}{9}\left(\frac{3}{2}\right)^2 = 2 \quad \text{OK}$$

FOR SOLID CIRCULAR SECTION $a \to 0$

$$k = \frac{8}{9} \neq \frac{10}{9} \quad \text{(NOT CORRECT)}$$

REF: ROARK, "FORMULAS FOR STRESS & STRAIN", p. 120.

$\boxed{C = k/A}$



5. SOLUTION PROCEDURE FOR A SET OF LINEAR SPRINGS

At the start, N = 0, thus, $\{\Delta_0\}$

Going across the first elasticity,

$$\{\Delta_1''\} = [\epsilon_1^1]\{\Delta_0\}$$

And across the first mass,

$$\{\Delta_1'\} = [F_1]\{\Delta_1''\} = [F_1][\epsilon_1^1]\{\Delta_0\}$$

Next across the second elasticity.

$$\{\Delta_1\} = [\epsilon_1^2]\{\Delta_1'\} = [\epsilon_1^2][F_1][\epsilon_1^1]\{\Delta_0\} = [C_1]\{\Delta_0\}$$

In like manner, transformations can be made across each bay, expressing each state vector in terms of the previous state vector, and thus in terms of the starting vector.

$$\{\Delta_{NSTA}\} = \prod_{N=1}^{NSTA} [C_N]\{\Delta_0\} = [D]\{\Delta_0\}$$

Expanding we get,

$$\begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ \hline 1 \end{Bmatrix}_{NSTA} = \left[\begin{array}{cccc|c} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{45} \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ \hline 1 \end{Bmatrix}_0$$

The fourth order nature of the governing beam equation requires the specification of two boundary conditions at each end. The program requires that two of the four variables of the state vector be zero at the beginning and at the end. Other homogeneous boundary conditions such as elastic restraints can be obtained by inputting a zero length so as to put the mass point at the boundary and then setting V and M equal to zero with appropriate lateral and/or moment springs. Likewise, concentrated end forces and moments can be inputted as $\overline{V}$ and $\overline{M}$ with the boundary condition that V = M = 0.

Let
$$\begin{Bmatrix} V \\ M \\ \Phi \\ Y \\ 1 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ 1 \end{Bmatrix}$$

Further, let

M = subscript of 1st zero variable at end of last bay

N = subscript of 2nd zero variable at end of last bay

R = subscript of 1st non-zero variable at start of 1st bay

S = subscript of 2nd non-zero variable at start of 1st bay

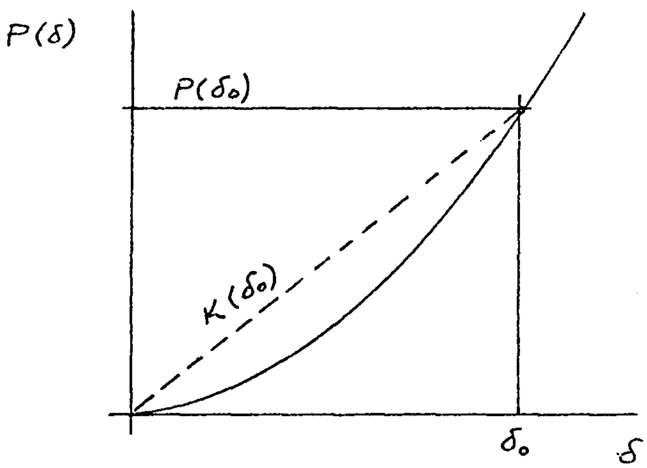
Considering the two equations associated with the zero variables at the end, and dropping those terms multiplied by the zero variables at the start:

$$\begin{Bmatrix} -d_{M5} \\ -d_{N5} \end{Bmatrix} = \begin{vmatrix} d_{MR} & d_{MS} \\ d_{NR} & d_{NS} \end{vmatrix} \begin{Bmatrix} Q_R \\ Q_S \end{Bmatrix}$$

These can be readily solved for Q_R and Q_S, the two unknowns at the start of the beam. Thus knowing the initial state vector, all succeeding state vectors can be found by "walking through" the system.

6. SOLUTION PROCEDURE FOR A SET OF NON-LINEAR SPRINGS

The previous section described the explicit procedure for determining the response of an elastically supported beam to a harmonic force or moment input of frequency ω. If the beam support (for example, bearings) has a non-linear load-deflection relation, the secant line intersecting the curve is not a constant, but is instead a function of the deformation of the beam.



The elastic analysis described in the previous section yields an elastic spring force which is simply the product of the spring constant times the deformation. If this elastic force equals the non-linear force for the same deformation (as determined from the non-linear force-deformation relation as typified above), then the linear spring used was the correct secant. Adjusting the choice of secants until this agreement is achieved for all the non-linear springs supporting the beam leads to the solution of the problem. After a given unsuccessful iteration, the initial and final secant values are averaged to obtain the trial spring for the next iteration.

The first trial spring for the first frequency investigated must be input to the program. When the response to a harmonic load of successive frequencies is desired, the converged secant value for the first frequency is used as the first trial spring for the second frequency. Likewise, the converged secant value for the second frequency is used as the first trial spring for the third frequency. After this, a parabola is fitted through the three previous converged frequency springs, and the first trial spring for the next frequency is extrapolated along this curve.

$$K_{\omega_i}^{(1)} = 3\left(K_{\omega_{i-1}}^{(F)} - K_{\omega_{i-2}}^{(F)}\right) + K_{\omega_{i-3}}^{(F)}$$

7. DETERMINATION OF THE NON-LINEAR FORCE

As described in the previous section, the forced vibration analysis for a set of linear springs yields deformations and associated elastic forces in the springs. In order to test for convergence, the non-linear force associated with that deformation must be determined. At present, the program permits two types of non-linear springs. The appropriate flag must be set in the input.

a) FLAG(N) = 1. Angular Contact Ball Bearing: The pertinent bearing data is input after all the station data, and the exact load-deflection equations for ball bearings are utilized, including the interaction of thrust with the lateral response.

b) FLAG(N) = 2. $P = A\,y^B$ where A and B are constants input to the program.

At present, roller bearing load-deflection curves are fitted by a form b). However, it is easily possible to introduce a third flag alternate and incorporate the exact load-deflection equations for roller bearings.

8. ANGULAR CONTACT BALL BEARINGS

The explicit analytical load-deflection relations for angular contact ball bearings have been derived and are extensively treated by A.B. Jones of New Departure in Reference (5).

The outer race of the bearing is assumed fixed in space. The inner race has three degrees of freedom with respect to the fixed outer race. It may move axially, laterally, and may rotate. It is also capable of transmitting three force resultants between shaft and bearing support (i.e., lateral force, axial force, and moment). Each of these force resultants can be expressed explicitly in terms of the three deformations. These functions are explicit, but non-linear. Their inverse cannot be explicitly stated, that is, the deformations cannot be expressed in terms of the three force resultants, nor can mixed functions of forces and deformations be expressed.

$$H = f_1 (\Delta_A , \Delta_R , \theta)$$

$$V = f_2 (\Delta_A , \Delta_R , \theta)$$

$$M = f_3 (\Delta_A , \Delta_R , \theta)$$

The significant parts of the derivation have been reproduced on the following pages. The value of "K" referred to on p 22 by Jones, and DKK in the program, is not computed internally by the program, though it could be, but is computed by IBM Job 773A. Since this number is a constant, it need be computed only once. Job 773A essentially programs Jones' equations in an iterative scheme to yield deformations as a function of input loads.

A. DERIVATION OF THE EQUATIONS (PER JONES)

I. Basic Geometric Relations.

The operating characteristics of a ball bearing depend to a great extent upon the internal fitup. Internal fitup is generally measured by the diametral clearance of the bearing.

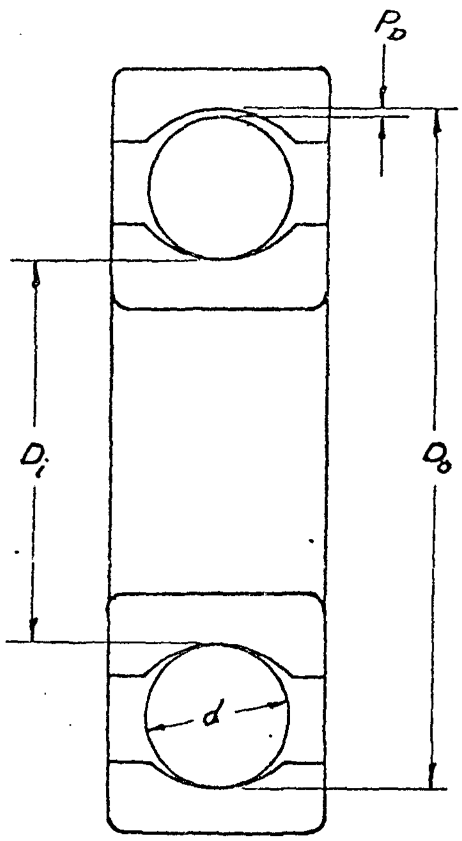


Fig. 1

Fig. 1 shows a cross section through a radial, single row bearing. Diametral clearance is denoted by P_D From Fig. 1:

$$P_D = D_o - D_i - 2d \qquad \text{Eq. 1}$$

Although diametral clearance is generally used in connection with single row, radial bearings, Eq. 1 is applicable to angular contact bearings as well since there is a definite relation between diametral clearance, race curvatures and free contact angle (See Eq. 8 p. ~~5~~ 13).

The value of P_D from Eq. 1 may be positive or negative. Loose bearings have positive diametral clearance. Tight bearings have negative values of P_D .

Diametral clearance in loose, single row, radial bearings is sometimes called radial clearance, radial play, radial shake, diametral play or diametral slackness.

For loose, single row, radial bearings diametral clearance may be defined as the maximum distance one race may move diametrally with respect to the other without the application of measureable force when both races lie in the same plane.

- -

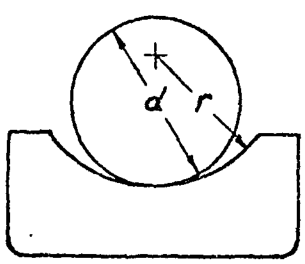


Fig. 2

Race curvature is a measure of the conformity of the race to the ball in a plane passing through the bearing axis and transverse to the raceway. It is expressed as a percentage or a decimal. Throughout this text decimal notation will be used.

The curvature of a race is defined as: (See Fig. 2)

$$f = \frac{r}{d}$$ Eq. 2

Thus, if the curvature and ball diameter are known, the radius of curvature is:

$$r = fd$$ Eq. 3

- -

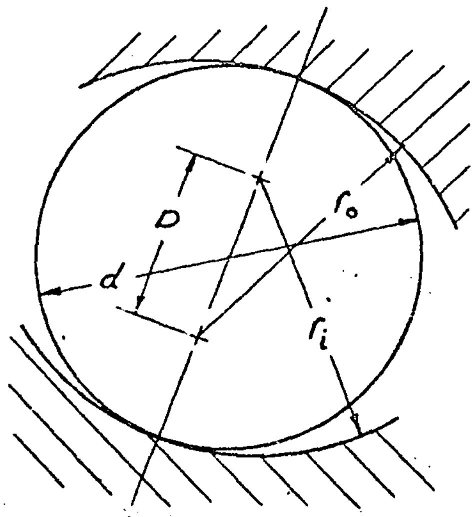


Fig. 3

The distance between the centers of curvatures of two races in line and line contact with a ball is of great importance. This distance is indicated by D in Fig. 3 and is a fixed quantity depending on race radii and ball diameter. Denoting quantities referred to the outer race by the subscript, $_o$, and quantities referred to the inner race by the subscript, $_i$, we have from Fig. 3:

$$D = r_o + r_i - d$$ Eq. 4

Since both r_o and r_i may be expressed in terms of outer and inner race curvatures, respectively, by Eq. 3, we have:

$$D = (f_o + f_i - 1)\,d$$ Eq. 5

Letting:

$$B = (f_o + f_i - 1)$$ Eq. 6

$$D = Bd$$ Eq. 7

The quantity B in Eq. 7 is known as the total curvature and is a measure of the conformity of both outer and inner races to the ball. Upon it depend all bearing deflection computations.

- -

Free contact angle is the angle made by a line passing through the points of contact of the ball and both raceways with a plane perpendicular to the axis of the bearing when both races are centered with respect to each other and one race is axially displaced with respect to the other without the application of measureable force.

The centers of curvature of both outer and inner races lie on the line defining the free contact angle. Free contact angle is denoted by β_o and is illustrated in Fig. 4.

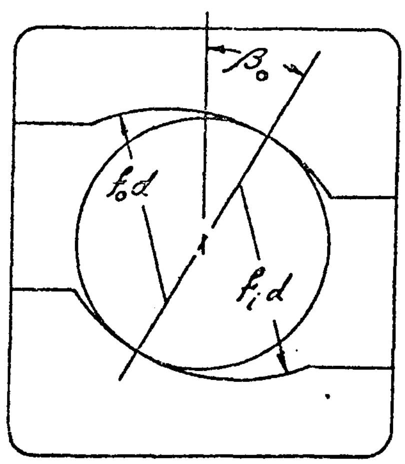


Fig. 4

Free contact angle is determined by diametral clearance, P_o, and total curvature, B, as:

$$\cos\beta_o = \frac{2Bd - P_o}{2Bd} \qquad \text{Eq. 8}$$

or:

$$P_o = 2Bd(1-\cos\beta_o) \qquad \text{Eq. 9}$$

In the case of radially tight bearings the value of P_o is negative and the value of $\cos\beta_o$ from Eq. 8 becomes greater than 1. Mathematically, this is an imaginary condition. However, the value of $\cos\beta_o$ for radially tight bearings obtained from Eq. 8 is of importance in certain deflection computations and has a definite physical significance.

Therefore, radially tight bearings may be considered as having an imaginary contact angle whose sine is zero and whose cosine is greater than 1 as defined by Eq. 8.

The effect of interference mounting fits on free contact angle is important. Due to the interference fit there is a change in diameter of the press fitted raceway and a corresponding reduction in diametral clearance. Hence the free contact angle is reduced by press fitting.

If ΔP_D is the total reduction in diametral clearance due to press fitting one or both race members, the initial mounted contact angle, β_o', is:

$$\cos \beta_o' = \frac{2Bd - P_D + \Delta P_D}{2Bd} \qquad \text{Eq. 10}$$

or:

$$\cos \beta_o' = \cos \beta_o + \frac{\Delta P_D}{2Bd} \qquad \text{Eq. 11}$$

For the effect of interference fits on ring dimensions see Chapter XVII p. 161.

- -

Free endplay is the maximum possible relative axial movement of inner race with respect to the outer, when both races are coaxially centered, without the application of measureable force. It is denoted by P_E

In practice, endplay is measured under a definite gauging load and is known as gauged endplay. Gauged endplay is always greater than free endplay because of the deflection of the bearing under the gauging load. See Chapter XV, p. 152 for the relation between gauged endplay and diametral clearance.

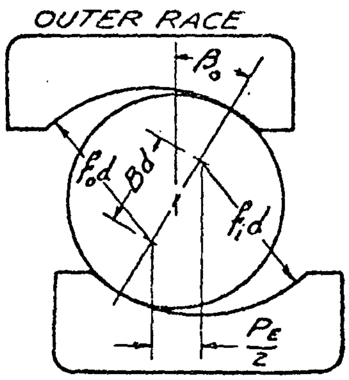


Fig. 5

Free endplay depends on total curvature and contact angle as shown in Fig. 5.

$$P_E = 2Bd \sin \beta_o \qquad \text{Eq. 12}$$

or:

$$\sin \beta_o = \frac{P_E}{2Bd} \qquad \text{Eq. 13}$$

- -

The relation between free endplay and diametral clearance is obtained by eliminating β_o between Eqs. 8 and 13.

$$P_D = 2Bd - \sqrt{(2Bd)^2 - P_E^2} \qquad \text{Eq. 14}$$

$$P_E = \sqrt{4Bd\,P_D - P_D^2} \qquad \text{Eq. 15}$$

- -

C

II. Solid Elastic Bodies In Contact.

When two, solid, elastic, curved bodies are pressed together under load a certain amount of flattening occurs in the neighborhood of the contact point. Due to the flattening there is produced an elliptical pressure area over which the total load is distributed. The relations governing the shape and size of the pressure area and the distribution of stress over the pressure area were mathematically investigated by Heinrich Hertz in 1881. These relations show good agreement with test results except where the dimensions of the projected pressure area are large in comparison to the principal radii of curvature of the contacting bodies. Good agreement is shown for conformities generally used in ball bearings.

Although Hertz's work was limited to an analysis of the distribution of stress at the pressure surface, more recent investigators have determined the nature and distribution of the stresses occurring beyond the pressure surface and have substantiated their results by photo-elastic tests.

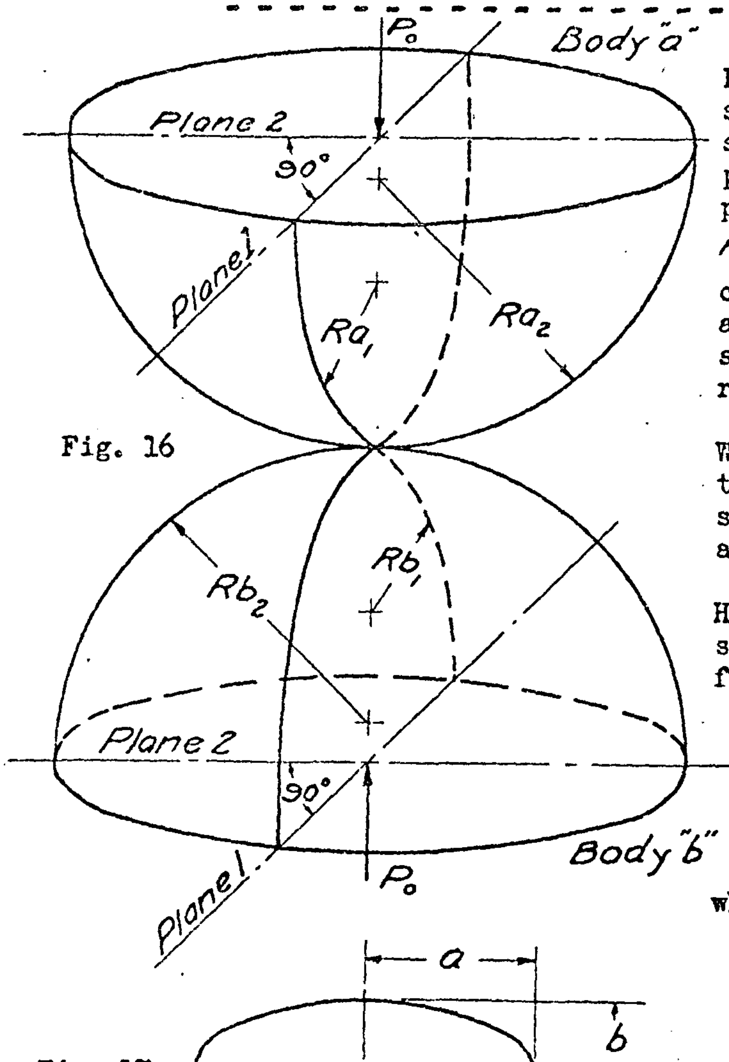


Fig. 16

Let the bodies be denoted by the subscripts "a" and "b", respectively, as shown in Fig. 16. Also, let the principal radii of curvature at the contact point be Ra_1 and Ra_2 for body "a" and Rb_1 and Rb_2 for body "b". The radii of curvature are measured in two planes, 1 and 2, at right angles to one another as shown in Fig. 16, the subscripts 1 and 2 referring to the respective planes.

When body "a" and body "b" are pressed together by the normal load, P_o, the resulting pressure area whose semi-axes are a and b is shown in Fig. 17.

Hertz gives the dimensions of the pressure area in terms of the transcendental functions μ and ν, as:

$$a = \mu g \qquad \text{Eq. 53}$$

$$b = \nu g \qquad \text{Eq. 54}$$

where:

$$g = \sqrt[3]{\frac{3P_o(\vartheta_a + \vartheta_b)}{8\left(\frac{1}{Ra_1} + \frac{1}{Ra_2} + \frac{1}{Rb_1} + \frac{1}{Rb_2}\right)}} \qquad \text{Eq. 55}$$

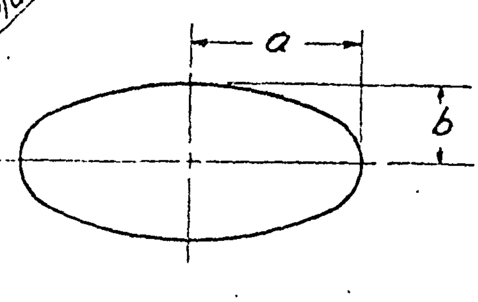


Fig. 17

$$\nu_b = \frac{4(1-\delta_b^2)}{E_b} \qquad \text{Eq. 57}$$

If both bodies are of steel with modulus of elasticity 29 x 10^6 #/sq. in. and with Poisson's ratio 1/4, the value of g from Eq. 55 is:

$$g = .0045944 \sqrt[3]{\frac{P_a}{\frac{1}{Ra_1} + \frac{1}{Ra_2} + \frac{1}{Rb_1} + \frac{1}{Rb_2}}} \qquad \text{Eq. 58}$$

The values of the principal radii of curvature, Ra_1, Ra_2, Rb_1 and Rb_2 are taken in accordance with Fig. 16.

The principal radii of curvature may be either positive or negative, depending on whether the centers of curvature lie within or without the body as shown in Fig. 18.

In addition, planes 1 and 2 should be so chosen that:

$$\frac{1}{Ra_1} + \frac{1}{Rb_1} > \frac{1}{Ra_2} + \frac{1}{Rb_2} \qquad \text{Eq. 59}$$

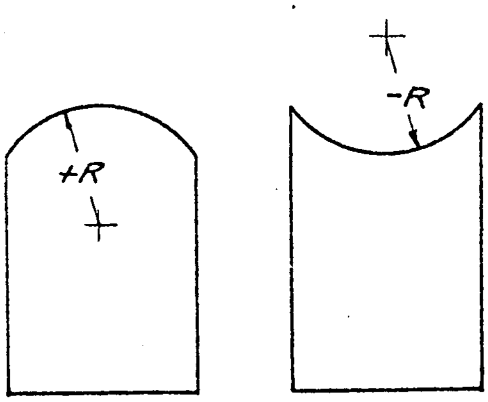


Fig. 18

Plane 1 then determines the direction of the semi-minor axis of the pressure area and plane 2 the direction of semi-major axis of the pressure area.

The values of the functions μ and ν for use in Eqs. 53 and 54 depend on the conformity of the contacting bodies in the vicinity of the pressure area as determined by the auxiliary angle, τ .

$$\cos \tau = \frac{\frac{1}{Ra_1} - \frac{1}{Ra_2} + \frac{1}{Rb_1} - \frac{1}{Rb_2}}{\frac{1}{Ra_1} + \frac{1}{Ra_2} + \frac{1}{Rb_1} + \frac{1}{Rb_2}} \qquad \text{Eq. 60}$$

Note that the denominator in the expression for cos τ is the same as that occurring under the radical in Eq. 55 and 58.

μ and ν are related by another auxiliary angle, ϵ , which depends on the shape of the pressure ellipse.

$$\cos \tau = 1 - \frac{2\left[K(\epsilon) - E(\epsilon)\right] \cot^2 \epsilon}{E(\epsilon)} \qquad \text{Eq. 61}$$

$$\nu = \sqrt[3]{\frac{2E(\epsilon)\cos \epsilon}{\pi}} \qquad \text{Eq. 62}$$

where:

$$\cos \epsilon = \frac{\nu}{\mu} = \frac{b}{a} \qquad \text{Eq. 63}$$

$K(\epsilon)$ and $E(\epsilon)$ are the complete elliptic integrals of the first and second order, having the modulus sin ϵ

$$K(\epsilon) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - \sin^2 \epsilon \sin^2 \varphi}} \qquad \text{Eq. 64}$$

$$E(\epsilon) = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2 \epsilon \sin^2 \varphi}\, d\varphi \qquad \text{Eq. 65}$$

Since accurate tables of $K(\epsilon)$ and $E(\epsilon)$ are not always available, values of $K(\epsilon)$ and $E(\epsilon)$ correct to ten decimal places are given on Charts 5 and 6. Four place tables may also be found in Jahnke and Emde's "Funktionentafeln" 1943 edition.

By assuming a series of values of the modulus, sin ϵ , corresponding values of cos τ , μ and ν may be calculated by Eqs. 61, 62 and 63.

Values of μ computed in this manner are plotted against corresponding values of cos τ in Charts 7 through 21. Values of ν are plotted against corresponding values of cos τ in Charts 22 through 31.

It must be emphasized that the semi-axes of the pressure ellipse, a and b, are the projected semi-axes and are not measured along the curvature of the pressure surface.

- -

IV. Load Distribution And Deflection In Ball Bearings - Generalized Solution.

A ball bearing derives its load carrying ability from the forces produced at the contact points of balls and races. These loads, called normal ball loads and designated by P_o, result from the elastic deformations of the contacting bodies.

Fig. 27

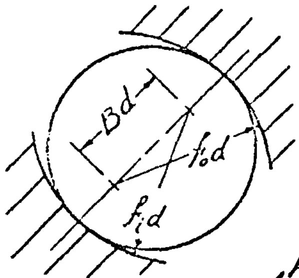


Fig. 28

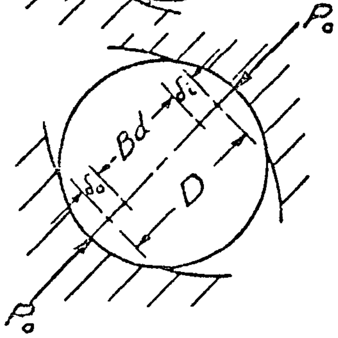


Fig. 27 shows a ball between two curved races. When the ball is in point (no load) contact with both races, the centers of curvature are separated by the distance Bd (see P.2) which depends on curvatures and ball diameter.

If the races are displaced with respect to each other so that the ball is compressed between them, the external force causing the compression is resisted by an elastic force (normal ball load), P_o, which acts along the line passing through the displaced centers of curvature of the two races as shown in Fig. 28.

The elastic deformations at the points of contact are δ_o and δ_i and the sum of these two equals the normal approach of the two races. Since the curvature centers are fixed with respect to their races and move with them, the original distance between race curvature centers, Bd, is increased by the normal approach of the two races. Calling the normal approach of the two races δ_N, <u>the distance between the displaced curvature centers</u> is:

$$D = Bd + \delta_N \qquad \text{Eq. 146}$$

or:

$$\delta_N = D - Bd \qquad \text{Eq. 147}$$

The relation between normal ball load and normal approach is:

$$P_o = K_N \delta_N^{3/2} \qquad \text{Eq. 148}$$

where the value of K_N is, from Eq. 143:

$$K_N = \frac{d^{1/2} \times 10^9}{\left[7.8107(C_{\delta_o} + C_{\delta_i})\right]^{3/2}} \qquad \text{Eq. 149}$$

C_{δ_o} and C_{δ_i} are obtained from Chart 56.

K_N may be more conveniently expressed in terms of the axial deflection constant, K, by the relation:

$$K_N = \frac{K d^{1/2}}{B^{3/2}} \qquad \text{Eq. 150}$$

Values of K may be obtained from Chart 57. See P. 49

In a complete ball bearing which involves a number of balls symmetrically disposed around a pitch circle, the normal load on any ball and the contact angle at which it acts may be completely determined and evaluated in terms of the following relative displacements of inner and outer races.

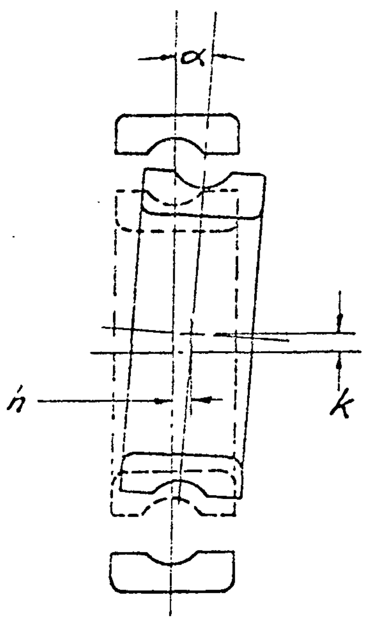


Fig. 29

1) A relative axial displacement, h, of inner and outer races.

2) A relative radial displacement, k, of inner and outer races.

3) A relative angular misalignment, α, of inner and outer races.

Fig. 29 shows these displacements. They are measured with reference to the relative position of inner and outer rings when all parts of the bearing are in symmetric, geometric contact under zero thrust load.

Some of the dimensions used in the following discussion are:

The radius of the locus of the center of curvature of inner race:

$$R_i = \frac{E}{2} + (f_i - .5) d \cos\beta_o \qquad \text{Eq. 151}$$

where: E = pitch circle diameter.

The radius of the locus of the center of curvature of the outer race:

$$R_o = R_i - Bd \cos \beta_o \qquad \text{Eq. 152}$$

and are also connected by the relations:

$$R_i - R_o = Bd \cos \beta_o \qquad \text{Eq. 153}$$

and:

$$R_i - R_o = Bd - \frac{P_D}{2} \qquad \text{Eq. 154}$$

where: P_D = Diametral Clearance

In order to express the normal ball loads and operating contact angles developed within the bearing in terms of the relative displacements of the inner race with respect to the outer, the following system is used.

The outer race is assumed to be fixed in space while the inner race is allowed to move with respect to the outer as shown in Fig. 29. The normal ball load and operating contact angle for a ball at any angle, φ, measured around the pitch circle from the heaviest loaded ball, are obtained by evaluating the change in distance between inner and outer race curvature centers in terms of the displacements shown in Fig. 29.

Fig. 30 shows the relative position of inner and outer race curvature center loci before displacement. The locus of the outer race curvature centers is a circle in space and is referred to a fixed, three dimensional coordinate system, X, Y, Z. The locus of the inner race curvature centers is also a circle in space and is referred to the movable, three dimensional coordinate system, X', Y', Z'.

Now, assume that the origin of the movable coordinate system is displaced the amounts h and k and misaligned the amount α as shown in Fig. 31. These displacements are those previously shown in Fig. 29.

In Fig. 31, the heaviest loaded ball lies in the X, Z plane. We are interested in the normal ball load, P_o, and operating contact angle, β_1, of a ball lying in the φ plane. This is determined by the relative positions of the intersection of the two race curvature loci with the plane.

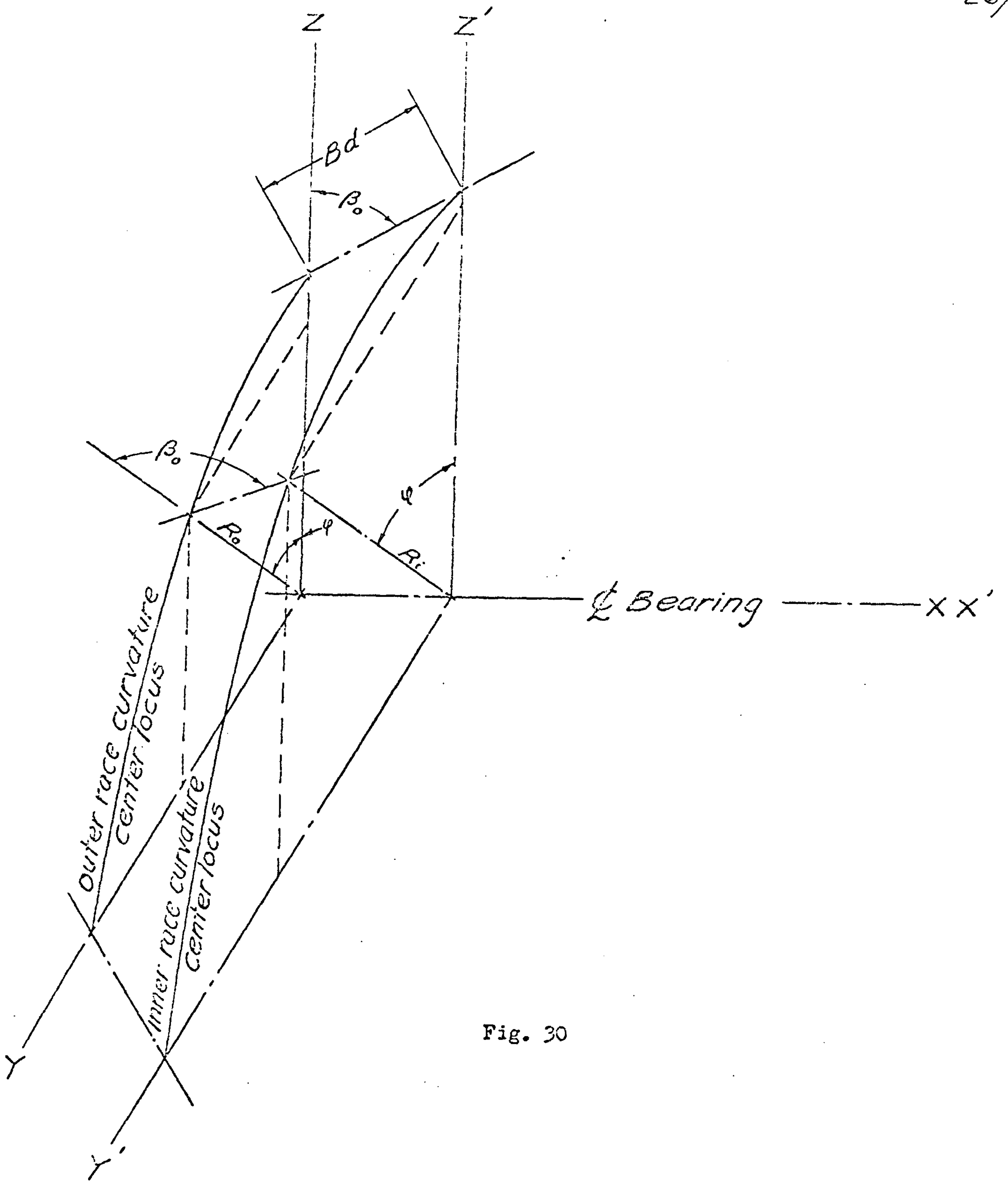

Z
Z'
Bd
β_o
β_o
φ
φ
R_o
R_i
℄ Bearing
X X'
outer race curvature center locus
inner race curvature center locus
Y
Y'

Fig. 30

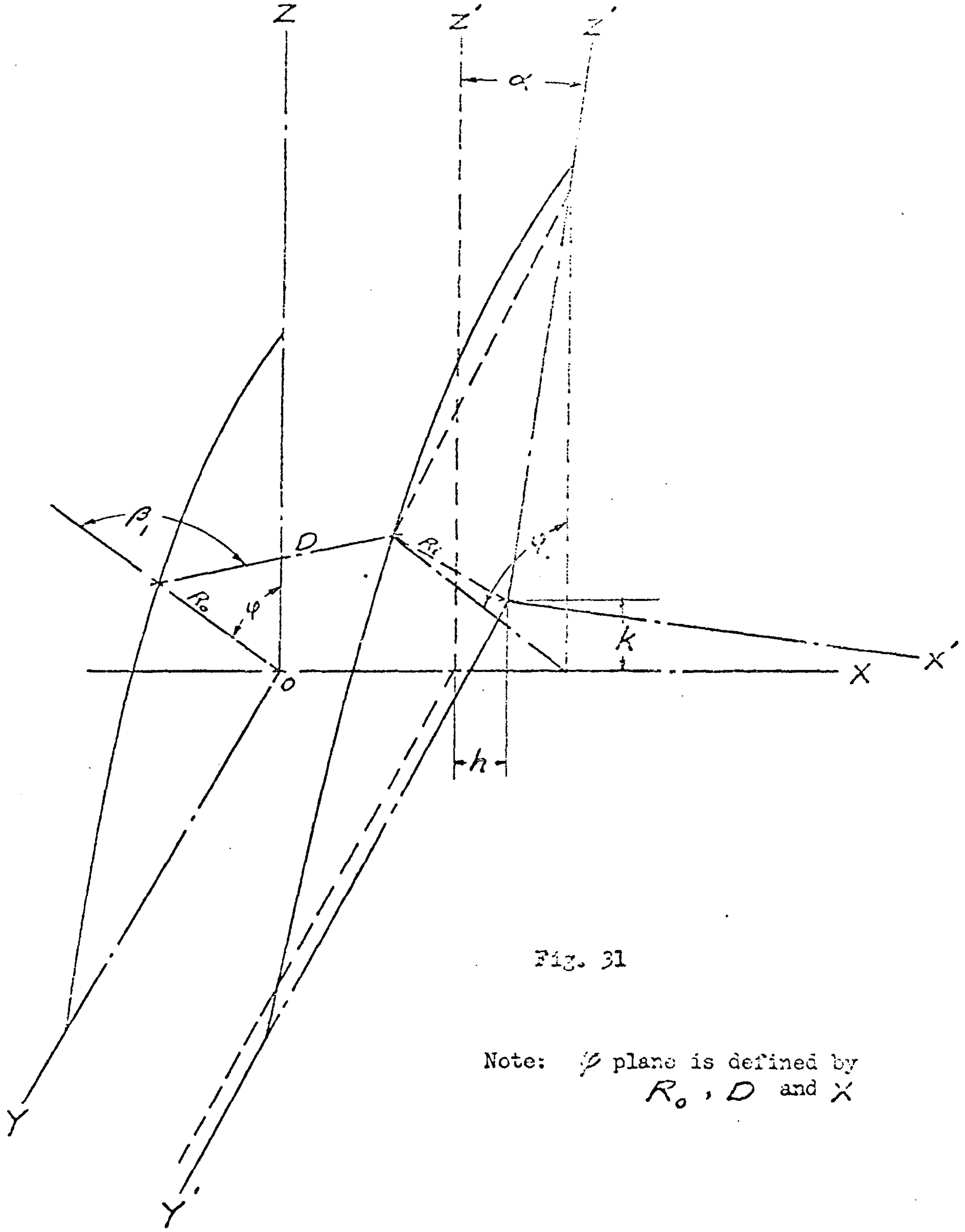


Fig. 31

Note: ϕ plane is defined by R_o, D and X

The distance, D, Fig. 31, between the centers of curvature of the inner and outer races after displacement and measured in the φ plane is:

$$D = Bd\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} \qquad \text{Eq. 155}$$

where:

$$h' = \frac{h}{Bd} \qquad \text{Eq. 156}$$

$$k' = \frac{k}{Bd} \qquad \text{Eq. 157}$$

$$\alpha' = \frac{\alpha}{Bd} \qquad \text{Eq. 158}$$

h, k and α being the three displacements of inner race with respect to the outer, Fig. 29. α is measured in radians. β_o is the free contact angle of the mounted bearing before load application.

The normal approach of the races, δ_N, is, from Eq. 147:

$$\delta_N = Bd\left[\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right] \qquad \text{Eq. 159}$$

The normal ball load, P_o, is, from Eq. 148:

$$P_o = K_N (Bd)^{3/2}\left[\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} \qquad \text{Eq. 160}$$

where K_N is the normal deflection constant from Eq. 149.

The normal ball load may be more conveniently expressed in terms of the axial deflection constant, K, as:

$$P_o = Kd^2\left[\sqrt{(\sin\beta_o + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_o + k'\cos\varphi)^2} - 1\right]^{3/2} \qquad \text{Eq. 161}$$

Values of K may be obtained from Chart 57.

The operating contact angle β_1 of a ball positioned in the φ plane is.

$$\sin\beta_1 = \frac{\sin\beta_0 + h' + \alpha' R_i \cos\varphi}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 162}$$

$$\text{or: } \cos\beta_1 = \frac{\cos\beta_0 + k' \cos\varphi}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 163}$$

If the normal ball load, P_0, which acts at the contact angle β_1 (along the line D in Fig. 31) is projected onto the X Z plane in Fig. 31, it may be resolved into two components. One is a thrust force, H, parallel to the X axis. The other is a vertical component, V, parallel to the Z axis.

The thrust component, H, is:

$$H = P_0 \sin\beta_1 \quad \text{Eq. 164}$$

or

$$H = \frac{Kd^2\left[\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2} - 1\right]^{3/2} (\sin\beta_0 + h' + \alpha' R_i \cos\varphi)}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 165}$$

The vertical component, V, is:

$$V = P_0 \cos\beta_1 \cos\varphi \quad \text{Eq. 166}$$

or

$$V = \frac{Kd^2\left[\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2} - 1\right]^{3/2} (\cos\beta_0 + k' \cos\varphi) \cos\varphi}{\sqrt{(\sin\beta_0 + h' + \alpha' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 167}$$

If it is assumed that the pitch circle radius does not appreciably change during the deformations, the moment of the thrust component about an axis through the center of the pitch circle and parallel to the Y axis in Fig. 31 is:

$$M = \frac{P_0 E}{2} \sin\beta_1 \cos\varphi \quad \text{Eq. 168}$$

where E is the pitch circle diameter.

or

$$M = \frac{EKd^2}{2} \frac{\left[\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2} - 1\right]^{3/2} (\sin\beta_0 + h' + d' R_i \cos\varphi) \cos\varphi}{\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 169}$$

In order that the bearing be in equilibrium after displacement, the following conditions must be satisfied:

$$\Sigma H = Kd^2 \sum \frac{\left[\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2} - 1\right]^{3/2} (\sin\beta_0 + h' + d' R_i \cos\varphi)}{\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 170}$$

$$\Sigma V = Kd^2 \sum \frac{\left[\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2} - 1\right]^{3/2} (\cos\beta_0 + k' \cos\varphi) \cos\varphi}{\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 171}$$

$$\Sigma M = \frac{EKd^2}{2} \sum \frac{\left[\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2} - 1\right]^{3/2} (\sin\beta_0 + h' + d' R_i \cos\varphi) \cos\varphi}{\sqrt{(\sin\beta_0 + h' + d' R_i \cos\varphi)^2 + (\cos\beta_0 + k' \cos\varphi)^2}} \quad \text{Eq. 172}$$

where ΣH and ΣV are respectively the thrust and radial components of the externally applied load and ΣM, the moment of the external load about the center of the pitch circle. The Σ in the right hand sides of the above equations indicates that the computations must be performed for each ball position in the bearing and the sum taken.

The equations of equilibrium, Eqs. 170, 171, and 172, above, are statically indeterminate; that is, a direct solution for the displacements in terms of the externally applied load is not possible without further reduction of the equations.

B. Axial Equilibrium and Thrust -

An angular contact ball bearing which is designed to carry lateral as well as thrust loads presents the problem of the interaction of the axial and lateral equilibrium and compatibility of the beam. The forced vibration analysis discussed in Section 6 obtained a lateral deflection and rotation at the bearing, based on the assumed lateral and moment springs used. However, in order to calculate the non-linear lateral force and moment, the axial deformation of the bearing must be known. Such bearings normally are mounted in pairs, face to face or back to back. The determination of axial deformation and axial forces on the bearings depends on axial equilibrium and compatibility.

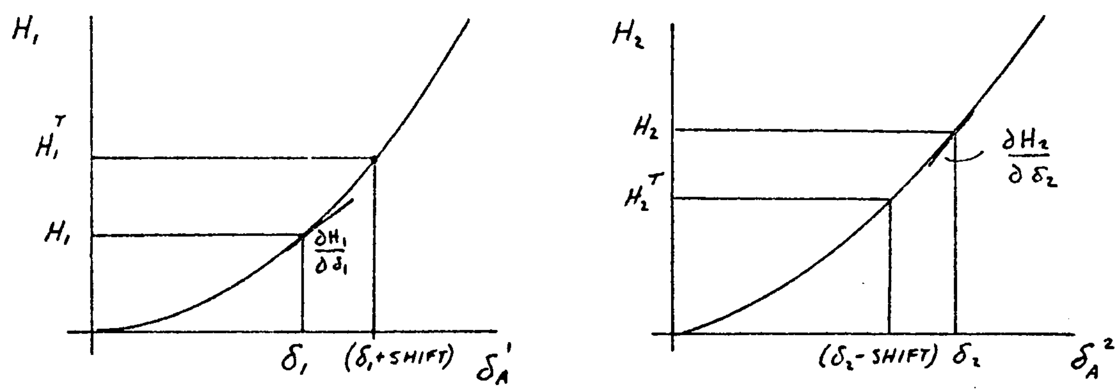
Preload deflection will be discussed in the next sub-section. Preload determines the relative axial position of the two angular contact ball bearings before any other load is imposed upon the system. The assumption of axial compatibility is that this relative position of the two inner races with respect to each other does not change during subsequent loading of the system. In other words, the shaft moves axially as a rigid body. The resulting axial position will be that such that the axial forces on the two ball bearings when combined with any thrust load on the shaft satisfies axial equilibrium. As the shaft whirls at a particular frequency, the shaft does not move axially, and so no axial inertia terms need be considered. The thrust must be a constant force with time, and not harmonic or any other time function in the steady state condition.

$$H_1 + H_2 + THRUST = 0.$$

For the first trial axial positions of the two ball bearings are assumed and input into the program. These would generally be $+\delta$ and $-\delta$ respectively, for the two balls, where δ is the preload deflection of the bearing. The non-linear forces and moment are then determined based on the assumed axial displacement, and the slope and lateral displacement computed by the forced linear vibration analysis described in Section 5. In general, the axial forces thus determined when combined with any thrust will not satisfy axial equilibrium. Thus, new values of axial deformation must be assumed for the second trial, along with the new lateral and moment springs discussed in Section 6.

$$\delta_A^{NEW} = \delta_A^{OLD} + SHIFT$$

The variable SHIFT is the estimated rigid body axial movement from the present axial position to the true axial position. For given values of rotation and lateral deflection for the two bearings, consider their respective axial load vs axial deformation curves:



$$H_1^T + H_2^T + THRUST = 0$$

It is expected that for small changes,

$$H_1^T \simeq H_1 + \frac{\partial H_1}{\partial \delta_1} \text{ SHIFT} = H_1 + DK1 * SHIFT$$

$$H_2^T \simeq H_2 + \frac{\partial H_2}{\partial \delta_2} \text{ SHIFT} = H_2 + DK2 * SHIFT$$

Substituting these expressions into the axial equilibrium equation and solving for SHIFT,

$$H_1 + DK1 * SHIFT + H_2 + DK2 * SHIFT + THRUST = 0$$

$$\therefore \quad SHIFT = -(H_1 + H_2 + THRUST)/(DK1 + DK2)$$

The problem then, is to determine the derivative of the axial force-deformation relation with respect to axial deformation, holding lateral deflection and rotation as constants.

Consider Eqs. 164 and 170 of Jones on pp 29 and 30:

$$H = \sum_{i=1}^{NBALL} P_i \sin \beta_{1i}$$

Per Eqs. 160 and 162,

$$P_i = DKK * d^2 \left[C_i - 1 \right]^{3/2}$$

where, $$C_i = \sqrt{A_i^2 + B_i^2}$$

$$A_i = \sin \beta_0 + DHP + DALP * DRI * \cos \varphi_i$$

$$B_i = \cos \beta_0 + DYP \cos \varphi_i$$

$$DHP = \delta_A / Bd$$

$$DYP = \delta_R / Bd$$

$$DALP = \theta / Bd$$

$$DRI = R_i$$

where β_o, R_i, and φ are defined by Jones on pages 18, 24, and 25 respectively.

Differentiating,

$$DK = \frac{\partial H}{\partial \delta_A} = \sum \left[P_i \frac{\partial}{\partial \delta_A} \sin \beta_{1i} + \sin\beta_{1i} \frac{\partial}{\partial \delta_A} P_i \right]$$

$$\frac{\partial P_i}{\partial \delta_A} = DKK\ d^2\ \frac{3}{2} \left[C_i - 1 \right]^{3/2} \frac{\partial C_i}{\partial \delta_A}$$

$$\frac{\partial C_i}{\partial \delta_A} = \frac{1}{2} \left[A_i^2 + B_i^2 \right]^{-1/2} \left[2A_i \frac{\partial A_i}{\partial \delta_A} + 2B_i \frac{\partial B_i}{\partial \delta_A} \right]$$

$$= \frac{1}{C_i} \left(A_i \frac{\partial A_i}{\partial \delta_A} + B_i \frac{\partial B_i}{\partial \delta_A} \right)$$

$$\frac{\partial A_i}{\partial \delta_A} = \frac{\partial}{\partial \delta_A} (DHP) = \frac{\partial}{\partial \delta_A} \left(\frac{\delta_A}{Bd} \right) = \frac{1}{(Bd)}$$

$$\frac{\partial B_i}{\partial \delta_A} = \frac{\partial}{\partial \delta_A} \left(\cos\beta_0 + DYP \cos\varphi_i \right) = 0$$

$$\frac{\partial C_i}{\partial \delta_A} = \frac{A_i}{C} \frac{1}{(Bd)} = \frac{\sin \beta_{1i}}{Bd}$$

$$\frac{\partial P_i}{\partial \delta_A} = DKK\ d^2 \left(\frac{3}{2} \right) \left[C_i - 1 \right]^{1/2} \frac{\sin\beta_{1i}}{Bd} = \frac{3}{2} \frac{P_i}{(C_i - 1)} \frac{\sin \beta_{1i}}{(Bd)}$$

$$\sin \beta_{1i} = A_i / C_i$$

$$\frac{\partial \sin \beta_{1i}}{\delta_A} = \frac{C_i \dfrac{\partial A_i}{\partial \delta_A} - A_i \dfrac{\partial C_i}{\partial \delta_A}}{C_i^{\,2}}$$

$$= \frac{1}{C_i} \frac{\partial A_i}{\partial \delta_A} - \frac{A_i}{C_i^{\,2}} \frac{\partial C_i}{\partial \delta_A}$$

$$\frac{\partial C_i}{\partial \delta_A} = \frac{\sin \beta_{1i}}{B d}$$

$$\frac{\partial A_i}{\partial \delta_A} = \frac{1}{Bd}$$

$$\frac{\partial \sin \beta_{1i}}{\partial_A} = \frac{1}{Bd\, C_i} - \frac{A_i}{C_i^{\,2}} \frac{\sin \beta_{1i}}{Bd}$$

$$= \left[1 - \frac{A_i \sin \beta_{1i}}{C_i} \right] / (C_i\, Bd)$$

$$= \left[1 - \sin^2 \beta_{1i} \right] / (C_i\, Bd)$$

$$= \cos^2 \beta_{1i} / (C_i\, Bd)$$

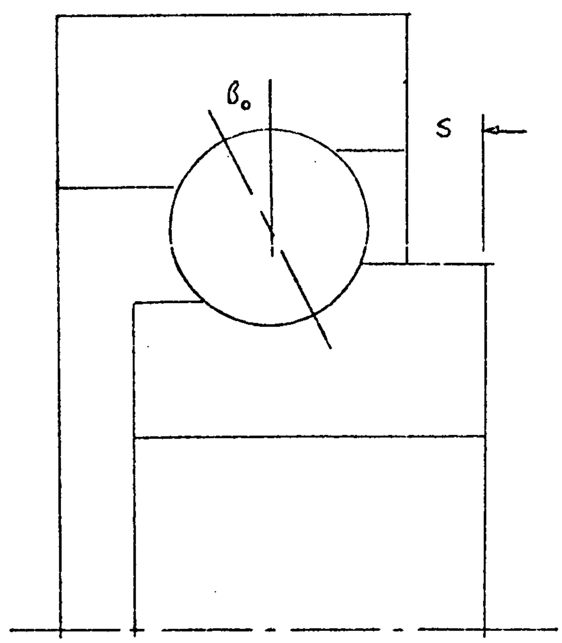
$$= B_i^{\,2} / (C_i^{\,3}\, Bd)$$

$$\therefore \quad DK = \sum P_i \left[\frac{B_i^{\,2}}{C_i^{\,3}} + \frac{3}{2} \frac{\sin^2 \beta_{1i}}{(C_i - 1)} \right] / (Bd)$$

$$DK = \sum P_i \left[B_i^{\,2} + \frac{3}{2} \frac{A_i^{\,2} C_i}{(C_i - 1)} \right] / (C_i^{\,3}\, Bd)$$

C. Preload Deflection

The faces of the inner and outer races of angular contact ball bearings are specified to be ground flush when under a certain axial load (for example, 50 lbs). Thus, when the bearings are assembled on the shaft and contact is first established between the races and balls under zero axial load, there is some stickout gap.



This stickout gap is closed by preloading the shaft, thus imparting to the bearing some initial axial displacement before any other loads are put on the shaft. Usually the preload force in the shaft far exceeds that required to close the stickout gap. Once it is closed, additional axial load is divided between the bearing (outer path) and the inner path. The final position of the preloaded bearing involves the simultaneous solution of linear and non-linear equations which are not too difficult, but must be done to determine the initial axial displacement at which the bearing awaits the lateral loads on the beam.

9. OTHER NON-LINEAR SPRINGS

Other non-linear springs (for example, roller bearings) can be included if the load-deflection curve can be expressed in the form,

$$P = A\, y^{B}$$

where A and B are constants to be inputted at the station at which the spring is to be located. The flag at that station must be set equal to 2.

Non-linear moment springs are not included, although to do so would not be difficult. A linear moment spring, however, may be input along with a non-linear lateral spring.

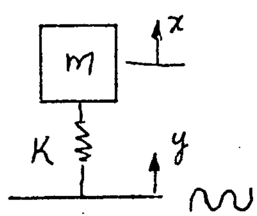
If the P vs y curve is plotted on log-log paper, the slope of the best fitting straight line through the points is the constant B. The constant A can then be found by inserting a value of P and y from the curve.

If the shaft is supported by two roller bearings, the shaft will whirl conically in contact with both bearings, taking up any radial clearance in the bearing, and so the clearance should not be included in the load-deflection curve. Such a canted position of the elastically undeflected shaft does, however, generally contribute an added unbalance at the large masses (especially overhung rotors). If the shaft is supported by two preloaded angular contact bearings, the undeflected position of the shaft will be the centerline of those bearings. If an additional roller bearing is present, any clearance there must be accounted for in its load-deflection relation. Such clearances should be added to the deflections in the load-deflection relation before fitting the power curve.

10. GROUND EXCITATION

The previous theory has been based on the assumption that the support to which the springs are connected is ground (i.e., that point has no deformation). In a shake test, however, the only load the beam sees is transmitted through the springs to the shaft by the harmonic occilations of the ground. This program permits the specification of the amplitude of the ground acceleration, and is assumed the same at all spring supports. It is assumed that the ground has no rotational acceleration (i.e., the moment springs are still attached to a face of zero rotation).

The derivation of the terms required to account for this phenomenon are illustrated by a single mass-spring model.



$$m\ddot{x} + K(x - y) = 0$$

LET $x = x_0 \cos \omega t$

$y = y_0 \cos \omega t$

$$\therefore \quad \ddot{x} = -\omega^2 x_0 \cos \omega t$$
$$\ddot{y} = -\omega^2 y_0 \cos \omega t$$

$$\therefore \quad (-m\omega^2 + K)\,x_0 - K\,y_0 = 0$$

OR $$(m\omega^2 - K)\,x_0 + K\,y_0 = 0$$

Now if the amplitude of the ground acceleration is specified as Ng (or N times the acceleration of gravity),

$$Ng = -\omega^2 y_0$$

OR $$y_0 = -\frac{Ng}{\omega^2}$$

Thus, at the station of each bearing,

$$\overline{V} = K y_o + \overline{V}$$

This is straightforward for a forced vibration analysis based on linear springs. However, for non-linear springs, whereever the non-linear force was a function of the lateral displacement δ_R, it must be considered a function of

$$\delta_R' = \delta_R - y_o$$

11. REFERENCES

(1) Users' Manual for Job 14009, "Lateral Vibration Analysis of a Free, Undamped, Lumped Parameter Beam System," by L.K. Severud, AGC, 3 June 1963.

(2) "Mechanical Vibrations," by J.P. Den Hartog, McGraw-Hill Book Co., Inc. 1956.

(3) "Theory of Mechanical Vibration," by Kin N. Tong, John Wiley and Sons, Inc., 1960.

(4) "Vibration Problems in Engineering," by S.P. Timoshenko, D. Van Nostrand Co., 3rd Edition, 1955.

(5) "New Departure - Analysis of Stresses and Deflections," Vol. 1 and 2, by A.B. Jones, New Departure Division, General Motors Corporation, 1946.

12. INPUT INFORMATION

I. Title card -

Col. 1 - 70 Title information

Col. 71 - 72 Number of stations (right adjusted)

II. Control card -

Col. 1 - 2 Number of frequencies (right adjusted)

Col. 3 - 14 Initial frequency (cps)

Col. 15 - 26 Increment in frequency (cps)

Col. 32 M = subscript of 1st zero variable at end of last bay

Col. 35 N = subscript of 2nd zero variable at end of last bay

Col. 38 R = subscript of 1st non-zero variable at start of 1st bay

Col. 41 S = subscript of 2nd non-zero variable at start of 1st bay

Col. 43 - 44 Maximum number of iterations allowed (right adjusted)

Col. 47 - 58 Accuracy (decimal)- if left blank, set = .05

Col. 59 - 70 Number of gravities acceleration of ground

III. Basic Station Data - Repeat sequence A, B, C for each station

A. Col. 1 - 12 L(1) inches

Col. 13 - 24 L(2) inches

Col. 25 - 36 EI(1) lb*inches^2

Col. 37 - 48 EI(2) lb*inches^2

Col. 49 - 60 G(1) lb/inch^2

Col. 61 - 72 G(2) lb/inch^2

Col. 74 - 75 Number of succeeding stations with the same data as Col. 1 - 72 (omit line A for those)

B. Col. 1 - 12 C(1) 1/inch^2

Col. 13 - 24 C(2) 1/inch^2

Col. 25 - 36 K(PHI) inch*lb/rad.

Col. 37 - 48 I(X) - I(J) lb*sec^2*inch

Col. 49 - 60 W lb

Col. 61 - 72 K(Y) lb/in

Col. 74 - 75 Number of succeeding stations with the same data as Col. 1 - 72. (omit line B for those)

C. Col. 1 - 12	DPBAR	lb	
Col. 13 - 24	DMBAR	in*lb	
Col. 25 - 36	A		
Col. 37 - 48	B		
Col. 49 - 60	GAMMA	$lb*sec^2$	
Col. 61 - 72	FLAG	1. Angular Contact Ball Bearing 2. Spring of form $P = A\ y^B$	
Col. 74 - 75	Number of succeeding stations with the same data as Col. 1 - 72. (omit line C for those)		

IV. Ball Bearing Input Data - Repeat A and B for each ball bearing.

A.	Col. 1 - 12	NBALL	Number of balls
	Col. 13 - 24	DIAM	Diameter of ball (inch)
	Col. 25 - 36	DFO	Ratio of radius of curvature of outer race to ball diameter
	Col. 37 - 48	DFI	Ratio of radius of curvature of inner race to ball diameter
	Col. 49 - 60	DE	Pitch circle diameter (in)
	Col. 61 - 72	DBETA	Initial unmounted contact angle (degree)
B.	Col. 1 - 12	DKK	Elastic coefficient computed by IBM Job 773A (Ref: pp 15 and 22)
	Col. 13 - 24	DHH	Initial axial deflection of bearing due to preload (in).

V. Thrust Input - Only if shaft supported by angular contact bearings.

Col. 1 - 12 TO lb $\quad THRUST = TO + DT\ \omega^2$

Col. 13 - 24 DT $lb*sec^2$

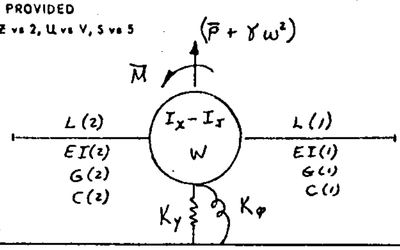
<u>CONVERSION TO FREE VIBRATION INPUT FOR JOB 14009</u>

1. Change NOMO to number of natural frequencies desired
2. In Col. 29 insert the number 1 for deflection and 2 for slope normalization in mode shape determination. (control card)
3. Omit cards III.C., IV., and V.
4. Run under Job 14009

CUSTOMER INSTRUCTIONS

1. ENTER DATA LEGIBLY WITHIN SPACES PROVIDED
2. DISTINGUISH BETWEEN I vs 1, Ø vs 0, Z vs 2, U vs V, S vs 5

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{Bmatrix} V \\ M \\ \Phi \\ Y \end{Bmatrix}$$



KEYPUNCH INSTRUCTIONS

- PUNCH 1 CARD PER HAND POSTED LINE ITEM
- PUNCH ALL * LINES WHETHER POSTED OR NOT. IF NECESSARY PROVIDE BLANK CARDS
- PUNCH ALL * LINES THAT ARE HAND POSTED
- PAS INCLUDING SPACES
- ALL SPACES MAY BE IGNORED
- ALL SPACES MAY BE IGNORED EXCEPT ON T CARD
- ALL SPACES MAY BE IGNORED EXCEPT (Specify cols.)
- ALL SIGNS ON KP LINES MUST BE PUNCHED
- DO NOT PUNCH PRE-PRINTED SIGNS SHOWN AFTER LAST HANDWRITTEN VALUE ENTRY

CUSTOMER	DATE
JOB NO. 11036	PROGRAMMER G.L. Goudreau
TITLE LATERAL VIBRATION ANALYSIS OF A HARMONICALLY FORCED, UNDAMPED, LUMPED PARAMETER BEAM	
FORM APPROVED (KEY PUNCH)	DATE

Card	Fields
I.	TITLE; NSTA
II.	NOMG; OMGO; DOMG; M; N; R; S; NITR; ACCUR; DNGS
III.A.	L(1); L(2); EI(1); EI(2); G(1); G(2); NR
B.	C(1); C(2); K(PHI); I(X); I(J); W; K(Y); NR
C.	DPBAR; DMBAR; A; B; GAMMA; FLAG; NR
IV.A.	NBALL; DIAM; DFO; DFI; DE; DBETA
B.	DRK; DHH
V.	TO; DT

43

AGC 2-410

PLEASE PRINT CLEARLY - USE BLACK PENCIL

13. OUTPUT INFORMATION

I. Input data - program dumps control and station data, ball bearing data, and thrust.

II. Spring data - At each station where there is a spring, the program prints out the station number, lateral spring value, elastic spring force, associated non-linear force if any, per cent difference if non-linear, moment spring value, elastic spring moment, associated non-linear moment if any, and per cent difference if non-linear.

III. Frequency and Determinant - For each iteration the program prints out the frequency in cps and rpm, the determinant of the set of two simultaneous equations solved in the forced vibration solution, and the value YO which is the lateral deformation of the ground, if any. A change in sign of the determinant through a frequency sequence indicates passing through a natural frequency or critical speed.

IV. State Vector - When the non-linear iteration has converged (trivially the first time if all springs linear), the program outputs the shear, moment, slope, and deflection at each station.

V. Ball bearing data - If angular contact ball bearings are encountered, then immediately following II. for each iteration the program outputs

TH1, TH2	- the axial force on the 1st and 2nd ball bearings
DH1, DH2	- the axial displacement of the 1st and 2nd "
DK1, DK2	- the axial derivative of the 1st and 2nd "
THRUST	- the value of the thrust load
PCT3	- the percent error in the axial equilibrium eq.
SHIFT	- the projected rigid body axial shift for the next iteration

When the non-linear iteration has converged, the program outputs a discription of the internal setup of the ball bearing, namely the circumferential position of each ball, its final contact angle, and its compressive force in lbs if any.

APPENDIX H

PROGRAM E13112 LISTING

i

```
59 RUN FYEE,428999,2,200   LIST E13112      25 APR 72   14:42:32.146

@ CTL UN=E13112                             25 APR 72   14:42:32.146

@R      ASG X=AN4153                        25 APR 72   14:42:32.217
        AN4153 ASSIGNED UNIT   3

@N      HDG                                 25 APR 72   14:42:32.227
```

2

XQT CUR 25 APR 72 14:42:32.229

```
1.      PEF X                                   14:42:32
2.      IN X                                    14:42:33
     END OF FILE -- UNIT X
3.      LIST 1                                  14:42:33
```

ELT DATA,1,710426, 41393

```
         ARES AXIAL FLOW TURBINE DESIGN B  11-24-65  40 MM BEARING          26
        30    600.0          20.0         1  2  3  4 10
          0.375         0.375         2.73E+06    2.73E+06    11.5E+06    11.5E+06
          1.55          1.55                 -    53.0E-06  0.259
                                                               0.620E-06
          0.425         0.425         7.45E+06    7.45E+06    11.5E+06    11.5E+06
          0.853         0.853                 - 1300.0E-06  0.898
                                                               1.970E-06
          0.440         0.440         7.45E+06    7.45E+06    11.5E+06    11.5E+06
          0.853         0.853                 - 1300.0E-06  0.910
                                                               1.805E-06
          0.315         0.315         6.56E+06    6.56E+06    11.5E+06    11.5E+06
          3.38          3.38                 -    78.4E-06  0.230
                                                               0.394E-06
          0.400         0.400         9.21E+06    9.21E+06    11.5E+06    11.5E+06
          2.56          2.56                 - 3930.0E-06  1.25
                                                               2.015E-06
          0.280         0.280        19.41E+06   19.41E+06    11.5E+06    11.5E+06
          0.502         0.502                 -   236.0E-06  0.435
                                                               0.627E-06
          0.160         0.160        16.30E+06   16.30E+06    11.5E+06    11.5E+06
          0.916         0.916                 -   138.0E-06  0.191
                                                               0.255E-06


                                     3.830E+07    1.45                       2.0
          0.575         0.575        16.30E+06   16.30E+06    11.5E+06    11.5E+06
          0.916         0.916                 -   315.0E-06  0.687
                                                               0.813E-06
          0.437         0.438        32.30E+06   32.30E+06    11.5E+06    11.5E+06
          0.852         0.852                 -   601.0E-06  0.558
                                                               .499E-06
          0.437         0.438        40.00E+06   40.00E+06    11.5E+06    11.5E+06
          0.720         0.720                 -   764.0E-06  0.654
                                                               0.442E-06
          0.565         0.565        48.50E+06   48.50E+06    11.5E+06    11.5E+06
          0.620         0.620                 - 1100.0E-06  0.968
                                                               0.415E-06
          0.375         0.375        48.50E+06   48.50E+06    11.5E+06    11.5E+06
          0.620         0.620                 - 6800.0E-06  1.873
                                                               0.356E-06
          0.390         0.390        48.50E+06   48.50E+06    11.5E+06    11.5E+06
          0.620         0.620                 - 6800.0E-06  1.932

          0.805         0.805        13.35E+06   13.35E+06    11.5E+06    11.5E+06
          1.04          1.04                 -    80.0E-06  0.782
                                                              -0.239E-06
          0.625         0.625        13.35E+06   13.35E+06    11.5E+06    11.5E+06
          1.04          1.04                 -   221.0E-06  0.640
                                                              -0.425E-06
          0.550         0.550        13.35E+06   13.35E+06    11.5E+06    11.5E+06
          1.04          1.04                 - 1400.0E-06  0.800
                                                              -0.625E-06
          0.400         0.400        13.35E+06   13.35E+06    11.5E+06    11.5E+06
          1.04          1.04                 -   223.0E-06  0.410
                                                              -0.489E-06
```

000057						
000058						
000059			3.830E+07	1.45		2.0
000060	0.150	0.150	13.35E+06	13.35E+05	11.5E+06	11.5E+06
000061	1.04	1.04	-	110.0E-05	0.195	
000062					-0.259E-06	
000063	0.615	0.615	14.70E+06	14.70E+05	11.5E+06	11.5E+06
000064	0.545	0.545	-	3200.0E-05	1.282	
000065					-1.950E-06	
000066	0.240	0.240	1.77E+06	1.77E+05	11.5E+06	11.5E+06
000067	2.26	2.26	-	29.5E-05	0.117	
000068					-0.204E-06	
000069	0.45	0.45	2.36E+06	2.36E+05	11.5E+06	11.5E+06
000070	2.21	2.21	-	537.0E-05	0.414	
000071					-0.787E-06	
000072	0.35	0.35	2.36E+06	2.36E+05	11.5E+06	11.5E+06
000073	2.21	2.21	-	525.0E-05	0.334	
000074					-0.703E-06	
000075	0.225	0.225	3.03E+06	3.03E+05	11.5E+06	11.5E+06
000076	1.45	1.45	-	39.7E-05	0.158	
000077					-0.355E-06	
000078	0.20	0.20	0.60E+06	0.60E+05	11.5E+06	11.5E+06
000079	2.97	2.97	-	7.1E-05	0.069	
000080					-0.163E-06	

5

@ ELT MAIN,1,710420, 62022

```
E13112
E13112
E13112
C     PROGRAM E13112 PLACED ON PRODUCTION APRIL 1,1970 BY F. YEE
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION TITLE(12)
C                           JOB 14036     VIBRATION ANALYSIS
C
C     DIMENSION DL1(50),DL2(50),DEI1(50),DEI2(50),DG1(50),DG2(50),DC1(50
C    1),DC2(50),DIJ(50),      DWN(50),DKN(50),E1MTRX(5,5),E2MTRX(5,5),
C    2AMATRX(5,5),BMATRX(5,5),CMATRX(5,5),FMATRX(5,5),DLMTRX(5,1),SHMTRX
C    3(5,1)   ,DGAMX(50)
      COMMON
     1    DL1 (50) ,DL2 (50) ,DEI1 (50),DEI2 (50),DG1 (50) ,DG2 (50) ,
     2    DC1 (50) ,DC2 (50) ,DIJ  (50),DGAMX(50),DWN (50) ,DKN (50) ,
     3    E1MTRX (5,5)       ,E2MTRX (5,5)       ,AMATRX (5,5)       ,
     4    BMATRX (5,5)       ,CMATRX (5,5)       ,FMATRX (5,5)       ,
     5    DLMTRX (5,1)       ,SHMTRX (5,1)       ,SUMG               ,
     6    OMGSO
C***********************************************************************
C      DIMENSION AND COMMON STATEMENTS ADDED FOR NON-LINEAR SPRINGS 1/7
C***********************************************************************
       DIMENSION NREP(3),DETA(50),DBETA(50),DAN1(50),DBN1(50),DPN1(50),
     1  IDP(3),P0(50),P1(50),P2(50),P3(50),Q000FL(4,50)
       COMMON DETA,DBETA,DAN1,DBN1,DPN1,IDP
C***********************************************************************
C PROGRAM STARTS HERE
C***********************************************************************
       DO 999  II=1,3
   999 NREP(II) = 0
C***********************************************************************
C INPUT HEADER AND NUMBER OF STATIONS
C***********************************************************************
    30 READ (5,111,END=444)TITLE,NSTA,(IDP(II),II=1,3)
   111 FORMAT (11A6,A4,5I2)
C***********************************************************************
C PRINT TITLE
C***********************************************************************
       WRITE  (6,13)
    13 FORMAT(74H1                                       JOB 14036
     1VIBRATION ANALYSIS///)
C***********************************************************************
C INPUT NUMBER OF ROOTS DESIRED, TRIAL ROOT, STEP SIZE, AND R.P.M.
C***********************************************************************
       READ  (  5 ,12)NOMODE,TROMGA,DELOMG,KK,KM,KN,KR,KS,NTRIAL,ACCUR
    12 FORMAT(I2,2E12.7,6I3,2X,E12.6)
       IF (KK)   114,113,114
   113 NTRIAL = 1
C***********************************************************************
C PRINT HEADER AND NUMBER OF STATIONS
C***********************************************************************
   114 WRITE (6,112) TITLE,NSTA
   112 FORMAT (1H1,10X,11A6,A4,10X,I2,2X8HSTATIONS )
       IF ( ACCUR )  116,115,116
   115 ACCUR  =  .05D0
```

```
  116 WRITE (  6   ,15)NOMODE,TROMGA,DELOMG,ACCUR
   15 FORMAT(23HONUMBER OF ITERATIONS =,I4,20H   STARTING OMEGA = ,
     1F12.4,17H   DELTA OMEGA = ,F12.4,5X7HACCUR = F12.4)
      WRITE (  6   ,18)KR,KS,KM,KN,NTRIAL
   18 FORMAT(34HOROW NUMBERS OF SELECTED SUBMATRIX,I4,4H AND,I4,
     139H   COLUMN NUMBERS OF SELECTED SUBMATRIX,I4,4H AND,I4,
     2   4X8HNTRIAL =  I2)
C*****************************************************************
C INPUT REMAINING DATA
C*****************************************************************
      DO 50 N=1,NSTA
      CALL REPEAT  ( DL1(N-1), DL1(N), DL2(N-1), DL2(N), DEI1(N-1),
     1     DEI1(N), DEI2(N-1), DEI2(N), DG1(N-1), DG1(N), DG2(N-1),
     2     DG2(N) , NREP(1) )
      CALL REPEAT  ( DC1(N-1),DC1(N),DC2(N-1),DC2(N),DIJ(N-1),DIJ(N),
     1     DGAMX(N-1),DGAMX(N),DWN(N-1),DWN(N),DKN(N-1),DKN(N),NREP(2) )
   50 CALL REPEAT  ( DETA(N-1),DETA(N),DBETA(N-1),DBETA(N),DAN1(N-1),
     1     DAN1(N),DBN1(N-1),DBN1(N),DPN1(N-1),DPN1(N),SUMG,SUMG,NREP(3))
C*****************************************************************
C PRINT STATION DATA
C*****************************************************************
      WRITE (  6   ,20)
   20 FORMAT(121H0             L(1)                        L(2)                          EI
     1(1)              EI(2)                     G(1)                          G(2)
     2)
      WRITE (  6   ,6)
    6 FORMAT(1H )
      LCTR = 0
      DO 57 N=1,NSTA
      LCTR = LCTR + 1
      WRITE (  6   ,22)DL1(N),DL2(N),DEI1(N),DEI2(N),DG1(N), DG2(N)
   22 FORMAT(6(6H      E15.8))
      IF (LCTR-5) 57,158,57
  158 LCTR = 0
      WRITE (  6   ,6)
   57 CONTINUE
      WRITE (  6   ,21)
   21 FORMAT(122H0             C(1)                        C(2)                          I(J
     1)-I(X)              GAMMA                     W SUB N                       K SUB
     2N)
      WRITE (  6   ,6)
      LCTR = 0
      DO 121 N=1,NSTA
      LCTR = LCTR + 1
      WRITE (  6   ,22)DC1(N),DC2(N),DIJ(N),DGAMX(N),DWN(N),DKN(N)
      IF (LCTR-5) 121,122,121
  122 LCTR = 0
      WRITE ( 6    ,6)
  121 CONTINUE
C*****************************************************************
C     PRINT ALL CARD NUMBER THREES.
C*****************************************************************
      WRITE (6,23)
   23 FORMAT (122H0             ETA                         BETA                          A
     1 SUB N              B SUB N                   P SUB N
     2          )
      WRITE (6,6)
      LCTR = 0
      DO 123 N = 1,NSTA
      LCTR = LCTR+1
```

```
      WRITE (6,22) DETA(N),DBETA(N),DAN1(N),DBN1(N),DPN1(N)
      IF (LCTR-5)  123,124,123
  124 LCTR = 0
      WRITE (6,6)
  123 CONTINUE
C
C*************************************************************************
      TMODE = TROMGA*6.2821854D0
      DELMOD = DELOMG*6.2821854D0
C*************************************************************************
C INITIALIZE E1, E2, AND F MATRICES
C*************************************************************************
      DO 51 I=1,5
      DO 51 J=1,5
      IF(I-J) 55,56,55
   56 E1MTRX(I,J)=1.0D0
      E2MTRX(I,J)=1.0D0
      FMATRX(I,J)=1.0D0
      GO TO 51
   55 E1MTRX(I,J)=0.0D0
      E2MTRX(I,J)=0.0D0
      FMATRX(I,J)=0.0D0
   51 CONTINUE
      OMGWRK = TMODE - DELMOD
      DELOMG = DELMOD
      DO 95 MM=1,NOMODE
      OMGWRK = OMGWRK + DELOMG
      OMGSQ = OMGWRK*OMGWRK
      SUMG = OMGSQ/386.04D0
      IF (  KK )     4301,4300,4301
 4301 IF ( MM-4 )    4302,4302,434
 4302 GO TO (430,431,432,433),MM
C*************************************************************************
  430 DO 340  N = 1,NSTA
  340 P0(N)  =  DPN1(N)
      GO TO  4300
C*************************************************************************
  431 DO 341  N = 1,NSTA
  341 P1(N) = .5D0 * (P0(N)+DPN1(N))
      GO TO  4300
C*************************************************************************
  432 DO 342  N = 1,NSTA
  342 P2(N) = .5D0 * (P0(N)+DPN1(N))
      GO TO  4300
C*************************************************************************
  433 DO 343  N = 1,NSTA
      P3(N) = .5D0 * (P0(N)+DPN1(N))
      P0(N) =3.0D0 * (P3(N)-P2  (N))+P1(N)
  343 DPN1(N)=  P0(N)
      GO TO  4300
C*************************************************************************
  434 DO  344  N= 1,NSTA
      P1(N)   =  P2(N)
      P2(N)   =  P3(N)
      P3(N) = .5D0 * (P0(N)+DPN1(N))
      P0(N) =3.0D0 * (P3(N)-P2  (N))+P1(N)
  344 DPN1(N) =  P0(N)
C*************************************************************************
C     LOOP FOR BETTER K(X) S DURING EACH STATE VECTOR LOOP. 1ST PASS OK
C*************************************************************************
```

```
 4300 NCF = 1
 4305 DO  600   JERRY  =  1,NTRIAL
      DO 58 I=1,5
      DO 58 J=1,5
      IF(I-J) 53,54,53
   54 CMATRX(I,J)=1.0D0
      GO TO 58
   53 CMATRX(I,J)=0.0D0
   58 CONTINUE
      DO 69 N=1,NSTA
      IF (  KK )  1,1,2
    2  IF (DPN1(N)) 4,1,4
    4 PO(N) = .5D0 * (PO(N)+DPN1(N))
      DKN(N)  =  DAN1(N)*PO(N)**DBN1(N)
    1 CALL MATELM(N)
      CALL MATMPY (E1MTRX(1,1),CMATRX(1,1),AMATRX(1,1),5,5,5,5,5)
      CALL MATMPY (E2MTRX(1,1),FMATRX(1,1),BMATRX(1,1),5,5,5,5,5)
      CALL MATMPY (BMATRX(1,1),AMATRX(1,1),CMATRX(1,1),5,5,5,5,5)
   69 CONTINUE
      DETNOW = CMATRX(KM,KR)*CMATRX(KN,KS) - CMATRX(KM,KS)*CMATRX(KN,KR)
      OMGPRT = OMGWRK / 6.28218540D0
      GO TO (71,73,71,71) ,NCF
   71 IF (JERRY-NTRIAL)   72,73,73
   72 IF (IDP(2)) 74,74,73
   73  WRITE ( 6    ,9)OMGPRT,DETNOW
    9 FORMAT(35H0                          OMEGA = E15.8,12H   DETERM =
     1E15.8)
   74  DLMTRX(1,1) = 0.0D0
       DLMTRX(2,1) = 0.0D0
       DLMTRX(3,1) = 0.0D0
       DLMTRX(4,1) = 0.0D0
       DLMTRX(5,1) = 1.0D0
       DLMTRX(KR,1) = (-CMATRX(KM,5)*CMATRX(KN,KS)+CMATRX(KM,KS)*
      1CMATRX(KN,5))/DETNOW
       DLMTRX(KS,1) = (-CMATRX(KM,KR)*CMATRX(KN,5)+CMATRX(KM,5)*
      1CMATRX(KN,KR))/DETNOW
       GO TO (75,77,75,75) ,NCF
   75 IF (JERRY-NTRIAL)   76,77,77
   76 IF (IDP(2) )  78,78,77
   77  WRITE ( 6    ,11)
   11 FORMAT(110H                     V                          M
     1                     PHI                         Y)
       WRITE ( 6    ,6)
       WRITE ( 6    ,10)DLMTRX(1,1),DLMTRX(2,1),D_MTRX(3,1),  DLMTRX(4,1)
   10 FORMAT(15H             E15.8,14H             E15.8,14H
     1    E15.8,14H            E15.8)
   78  WRITE ( 6    ,6)
       LCTR = 0
       DO 995 N=1,NSTA
       LCTR = LCTR + 1
       CALL MATELM(N)
       CALL MATMPY (E2MTRX(1,1),FMATRX(1,1),AMATRX(1,1),5,5,5,5,5)
       CALL MATMPY (AMATRX(1,1),E1MTRX(1,1),BMATRX(1,1),5,5,5,5,5)
       CALL MATMPY (BMATRX(1,1),DLMTRX(1,1),SHMTRX(1,1),5,5,5,5,1)
       GO TO (79,82,79,79) ,NCF
   79 IF (JERRY-NTRIAL)   80,82,82
   80 IF (IDP(2) )  83,83,82
   82  WRITE (  6    ,10)SHMTRX(1,1),SHMTRX(2,1),SHMTRX(3,1),  SHMTRX(4,1)
   83  DLMTRX(1,1) = SHMTRX(1,1)
       DLMTRX(2,1) = SHMTRX(2,1)
```

```
      DLMTRX(3,1) = SHMTRX(3,1)
      DLMTRX(4,1) = SHMTRX(4,1)
      Q000FL(4,N)=SHMTRX(4,1)
      IF(LCTR-5) 995,94,94
   94 LCTR = 0
      WRITE  (6,6)
  995 CONTINUE
      IF (IDP(2) )    85,85,84
   84 IDP(2) =  IDP(2)-1
   85 WRITE ( 6     ,6)
      IF (  KK )   598,95 ,598
  598 GO TO (201,202,201,201) ,NCF
  201 NCF = 2
      GO TO 599
  202 NCF = 4
  599 WRITE ( 6,4018)
 4018 FORMAT (1H0,32X5HSTA X,6X7HK SUB X,12X8HP SUB 0X,12X7HP SUB X ,
     1   18X3HPCT,6X3HNCF)
      DO 590  N = 1,NSTA
      IF ( DPN1(N) )   601,590,601
  601 DPN1(N) =  DKN (N)*DABS( Q000FL(4,N))
       PCTC =DABS(DABS(P0(N)/DPN1(N))-1.0D0)
      IF ( PCTC - ACCUR )      204,203,203
  203 NCF = 3
  204 PCTC = PCTC * 100.0D0
      WRITE ( 6,4019)N, DKN (N), P0(N) , DPN1(N ,PCTC,NCF
 4019 FORMAT (1H ,33XI2,3(5X,E15.8) , 9XF8.3,5XI1   )
  590 CONTINUE
      GO TO (600,4305,600,95) ,NCF
  600 CONTINUE
   95 CONTINUE
      WRITE ( 6     ,7)
    7 FORMAT(14H0 END OF CASE )
      GO TO 30
  444 STOP
      END
```

@ ELT MATELM,1,710420, 62024

```
      SUBROUTINE MATELM(N)
      IMPLICIT REAL*8 (A-H,O-Z)
C     DIMENSION DL1(50),DL2(50),DEI1(50),DEI2(50),DG1(50),DG2(50),DC1(50
C    1),DC2(50),DIJ(50),        DWN(50),DKN(50),E1MTRX(5,5),E2MTRX(5,5),
C    2AMATRX(5,5),BMATRX(5,5),CMATRX(5,5),FMATRX(5,5),DLMTRX(5,1),SHMTRX
C    3(5,1)   ,DGAMX(50)
      COMMON
     1    DL1 (50) ,DL2 (50) ,DEI1 (50),DEI2 (50),DG1 (50) ,DG2 (50) ,
     2    DC1 (50) ,DC2 (50) ,DIJ  (50),DGAMX(50),DWN (50) ,DKN (50) ,
     3    E1MTRX (5,5)       ,E2MTRX (5,5)        ,AMATRX (5,5)       ,
     4    BMATRX (5,5)       ,CMATRX (5,5)        ,FMATRX (5,5)       ,
     5    DLMTRX (5,1)       ,SHMTRX (5,1)        ,SUMG               ,
     6    OMGSQ
C*****************************************************************
C     DIMENSION AND COMMON STATEMENTS ADDED FOR NON-LINEAR SPRINGS 1/7
C*****************************************************************
      DIMENSION NREP(3),DETA(50),DBETA(50),DAN1(50),DBN1(50),DPN1(50),
     1 IDP(3),P0(50),P1(50),P2(50),P3(50),Q000EL(4,50)
      COMMON DETA,DBETA,DAN1,DBN1,DPN1,IDP
   61 E1MTRX(2,1) = DL1(N)
      E2MTRX(2,1) = DL2(N)
      IF (DEI1(N))62,29,62
   29 E1MTRX(4,2) =0.0D0
      GO TO 68
   62 E1MTRX(4,2) =0.5D0*DL1(N)*DL1(N)/DEI1(N)
   68 CONTINUE
      IF (DEI2(N))44,45,44
   45 E2MTRX(4,2) =0.0D0
      GO TO 444
   44 E2MTRX(4,2) =0.5D0*DL2(N)*DL2(N)/DEI2(N)
  444 CONTINUE
   63 E1MTRX(3,1) =    -E1MTRX(4,2)
      E2MTRX(3,1) =    -E2MTRX(4,2)
      IF (DEI1(N))64,43,64
   43 E1MTRX(3,2) =0.0D0
      GO TO 42
   64 E1MTRX(3,2) = -DL1(N)/DEI1(N)
   42 CONTINUE
      IF (DEI2(N))88,89,88
   89 E2MTRX(3,2) =0.0D0
      GO TO 888
   88 E2MTRX(3,2) = -DL2(N)/DEI2(N)
  888 CONTINUE
      IF(DEI1(N))26,27,26
   26 IF (DG1(N))65,27,65
   27 E1MTRX(4,1) =0.0D0
      GO TO 25
   65 E1MTRX(4,1) = (DL1(N)*((DL1(N)*DL1(N))/(6.0D0*DEI1(N))-DC1(N)/DG1
     1(N)))
   25 CONTINUE
      IF (DEI2(N)) 33,34,33
   33 IF (DG2(N))  35,34,35
   34 E2MTRX(4,1) =0.0D0
      GO TO335
   35 E2MTRX(4,1) = (DL2(N)*((DL2(N)*DL2(N))/(6.0D0*DEI2(N))-DC2(N)/DG2
     1(N)))
```

```
  335 CONTINUE
      E1MTRX(4,3) = -DL1(N)
      E2MTRX(4,3) = -DL2(N)
   66 FMATRX(1,4) = DWN(N)*SUMG - DKN(N)
      FMATRX(2,3) = -OMGSO*(DIJ(N))
      FMATRX(1,5)  =  DGAMX(N) * OMGSO + DETA(N)
      FMATRX(2,5)  =  DBETA(N) * OMGSO
C*****************************************************************************
C     OPTIONAL DUMP
C*****************************************************************************
      IF (IDP(1)) 1301,1301,1300
 1300 WRITE (6,130) N
  130 FORMAT (1H0,10X11HSTATION N = ,2XI2 )
      CALL PRINTM (E1MTRX(1,1),5,5,5,12H E1MTR(       )
      CALL PRINTM (E2MTRX(1,1),5,5,5,12H E2MTRX       )
      CALL PRINTM (FMATRX(1,1),5,5,5,12H FMATRX       )
      IDP(1) = IDP(1)-1
 1301 RETURN
      END
```

```
@ ELT PRINTM,1,710420, 62025

      SUBROUTINE PRINTM (A,NR,NC,MAXR,TITLE)
      IMPLICIT REAL*8 (A-H,O-Z)
C**********************************************************************
C FORTRAN IV PRINTM
C SUBROUTINE TO PRINT ANY MATRIX WITH 2-WORD TITLE
C     CALL PRINTM (CMATRX,8,8,8,12H  CMATRX     )   EXAMPLE CALL UP
C**********************************************************************
      DIMENSION A(1),NHED(8),TITLE(2)
C     CALL ATHRUZ (B,6H   COL  )
      DATA B/6H   COL   /
C                                                        MATRIX TITLE
      WRITE (6,22)TITLE
   22 FORMAT (1H0,52X,2A6)
C
      DO 50 I=1,NC,8
      II=NC-I+1
      IF (II-8) 20,20,10
   10 II=8
   20 DO 30 J=1,II
   30 NHED(J)=I+J-1
      WRITE    (6,120)          (B,NHED(J),J=1,II)
      DO 50 J=1,NR
      KL=J+(I-1)*MAXR
      KH=KL+(II-1)*MAXR
   50 WRITE    (6,130)          (J, A(K),K=KL,KH,MAXR)
      RETURN
  120 FORMAT(1H0,9X,10(A6,I4,4X))
  130 FORMAT (4H ROW,I3,5X,1P8E14.7)
      END
```

```
ELT REPEAT,1,710420, 62027

      SUBROUTINE REPEAT(A,AA,B,BB,C,CC,D,DD,E,EE,F,FF,NR)
      IMPLICIT REAL*8 (A-H,O-Z)
C*********************************************************************
C     REPEAT READS IN A STATION CARD OR SIMULATES A REPEATED CARD BY
C     MOVING DATA.
C     A,B,C,D,E,F   OLD         AA,BB,CC,DD,EE,FF   NEW
C                   NR = NUMBER OF REPEATS FOR A PARTICULAR CARD
C*********************************************************************
      IF (NR-1) 400,100,100
  400 READ    (5,3002)         AA,BB,CC,DD,EE,FF,NR
 3002 FORMAT ( 6E12.6,I3)
      GO TO 700
  100 AA=A
      BB=B
      CC=C
      DD=D
      EE=E
      FF=F
      NR=NR-1
  700 RETURN
      END

   4.      TRI X                                             14:42:35

   5.                                                        14:42:35

        END CUR
```

<***Δ****1****Δ****2****Δ****3****Δ****4****Δ****5****Δ****6****Δ****7****Δ****8****Δ****9****Δ****0****Δ****1****Δ****2****Δ****3*
***************************************ISD-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972***
Δ1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***[***Δ***1***Δ***2***Δ***3***Δ***4***Δ***5***Δ***6***Δ***7***Δ***0***Δ
[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ

25 APR 72 P 14:42:35 IDENT=FYEE ACCOUNT=428999 CARDS IN= 10, OUT= 0

PAGES= 13, LINES= 496, TIME=00:00:03 (HMS)

* *

*** USER NOTICES - APRIL 20, 1972 ***

(1) ISD 1108 TERMINAL SERVICE IS SCHEDULED AS FOLLOWS

MON : 07:00 - 24:00
TUE - FRI : 00:00 - 04:00 ; 07:00 - 24:00
SAT : 00:00 - 22:00
SUN : 04:00 - 22:00

(2) LARGE-CORE (LCR) PRODUCTION JOBS ARE NOW BEING RUN ON AN OVERNIGHT BASIS STARTING AT 04:00 EACH DAY.

(3) ISD NOW HAS AVAILABLE REMOTE-BATCH JOB ENTRY VIA LOW-SPEED TELETYPE COMPATIBLE TERMINALS USING DIAL-UP COMMUNICATION LINES. THIS SERVICE HAS BEEN IN USE FOR OVER TWO MONTHS AND IS CALLED RON/I. THE DIAL-UP TELEPHONE NUMBERS AND TRANSMISSION RATES ARE LISTED BELOW.

10 CHAR/SEC 415-562-4035, 415-562-4036, 415-562-5186
30 CHAR/SEC 415-562-4716 ** EFFECTIVE 4/24/72 THIS NUMBER WILL BE CHANGED TO 415-562-4294 **

(4) ISD'S SECOND PUBLIC TERMINAL IN SAN FRANCISCO IS LOCATED AT # 1 CALIFORNIA ST., ROOM 2555.

(5) BEGINNING 4/24/72 AND AFFECTIVE MONDAY - FRIDAY TURNAROUND TIME SHOULD BE REDUCED BETWEEN THE HOURS OF 10:30 - 11:30 AND 14:00 - 16:00 FOR USERS SUBMITTING NON-TAPE JOBS WITH RUN TIMES ESTIMATED AT LESS THAN 6 MINUTES.

ADDITIONAL INFORMATION ON (2) & (3) IS NOW AVAILABLE TO ALL INTERESTED USERS BY CONTACTING YOUR SALESMAN AT 415-562-4204.

* *

15

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ISO-27.16:INFORMATION-SYSTEMS-DESIGN:15-APR-1972
FGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ ABCDEFGHIJKLMNOPQRSTUVWXYZ)-+<=>&$*(%:?!,\0123456789';/.\ @[]#Δ

15 APR 72 P 14:42:35  IDENT=FYEE  ACCOUNT=428999  CARDS IN=  10, OUT=   0
 PAGES=   13, LINES=   496, TIME=00:00:03 (HMS)
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8/23/73

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CASE FILE

NMTC 105(3-72)