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A SIMPLE PERFORMANCE CALCULATION METHOD FOR LH_2/LOX ENGINES WITH DIFFERENT POWER CYCLES

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16. ABSTRACT

A simple method for the calculation of the specific impulse of an engine with a gas generator cycle is presented. The solution is obtained by a power balance between turbine and pump. Approximate equations for the performance of the combustion products of LH_2/LOX are derived. Performance results are compared with solutions of different engine types.

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DEFINITION OF SYMBOLS

Symbol	Definition
A	area
c^*	characteristic velocity
c_p	specific heat
C_F	thrust coefficient
E	energy
f	parameter
g	parameter
g_o	gravitational acceleration (9.81 m/sec^2)
h	enthalpy
h	parameter
I_{sp}	specific impulse
k	pressure drop factor
k_m	relative gas generator mass flow rate
m	mass flow
p	pressure
r	mixture ratio
T	temperature
γ	isentropic exponent
ϵ	expansion ratio

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
η	efficiency
ρ	density
a	ambient conditions
c	main compustion chamber
e	nozzle exit
fu	fuel
g	gas generator
ggc	gas generator cycle engine
i	pump inlet
min	minimum
ox	oxidizer
p	pump discharge
pb	preburner
scc	staged combustion cycle engine
t	throat
t	turbine
te	turbine exit
o	uncorrected
o	adopted

TECHNICAL MEMORANDUM X- 64749

A SIMPLE PERFORMANCE CALCULATION METHOD FOR LH_2/LOX ENGINES WITH DIFFERENT POWER CYCLES

SUMMARY

The selection of a feed system for an engine requires the estimation of its effects upon performance. For an engine with a gas generator cycle, the performance depends upon the amount of gas generator flow rate for the turbine drive. Based upon a power balance, the necessary mass flow and pressure formulations are derived. This leads to a simple equation for the performance, gas generator mass flow rate, and pump discharge pressures. The combination liquid hydrogen/liquid oxygen (LH_2/LOX) is selected as a typical propellant combination for vacuum space flight applications. Approximate equations for the performance of the combustion products are derived. The solution method has been applied to five different engine types leading to an optimization of the chamber pressure and turbine pressure ratio associated with maximum performance.

INTRODUCTION

The propulsion system has a dominant place in astronautics, since its performance essentially decides the feasibility and success of a mission. To increase the performance of an engine at sea level, the chamber pressure must be increased. This reduces the effects of dissociation, but much more important is the use of a larger area ratio.

For pure-vacuum space flight the situation is different, since the expansion ratio is limited only by the available geometric envelope of the engine. An engine using a gas generator cycle needs a certain amount of the whole mass flow to drive the turbopumps. Although the specific impulse of this type is lower than that of an engine with a staged combustion cycle, which uses the whole mass flow through the turbine in the main chamber, its use may lead to some advantages. The pump pressure for the gas generator system is lower and the development costs are smaller. The decision, which type of engine should be used, depends greatly on the amount of propellant flow necessary for the turbine drive. The performance of the engine with the gas generator cycle is very much influenced by the turbine and pump technology. A simple method for the calculation of the specific impulse, the necessary gas generator flow, and the pump, gas generator and chamber pressure permits an optimization of the principal design and a quick estimation of the effects of the various parameters on engine performance.

BASIC EQUATIONS FOR PERFORMANCE CALCULATION

Gas Generator Engine

A schematic representation of a rocket engine with a gas generator cycle and the assigned nomenclature is shown in Figure 1. The mass flow of oxidizer and fuel at pump pressure (p_p) into the gas generator is denoted by $\dot{m}_{ox_g}$ and $\dot{m}_{fu_g}$, respectively. After the combustion at pressure p_g the gases expand in the turbine to a pressure p_{te} and are injected into the main chamber nozzle. The injection occurs at a position where the main chamber nozzle's static wall pressure and the turbine exit pressure are almost the same. To simplify the calculations, no mixing and secondary chemical reaction of the turbine exhaust gases with the combustion chamber gases are assumed. The expansion of both mass flows is calculated independently.

Mass Flow Through the Gas Generator. For the calculation of the gas generator mass flow, an energy balance is performed. The required energy for the pressure increase of the propellants is

$$E_p = \dot{m}_{ox} \frac{p_{p_{ox}} - p_{i_{ox}}}{\rho_{ox}} \frac{1}{\eta_{p_{ox}}} + \dot{m}_{fu} \frac{p_{p_{fu}} - p_{i_{fu}}}{\rho_{fu}} \frac{1}{\eta_{p_{fu}}} . \qquad (1)$$

In this relation $\dot{m}_{ox}$ and $\dot{m}_{fu}$ denote the whole oxidizer and fuel mass flow, p_p and p_i the pump outlet and inlet pressures, ρ the propellant densities, and η_p the pump efficiencies. Assuming for convenience that both pump

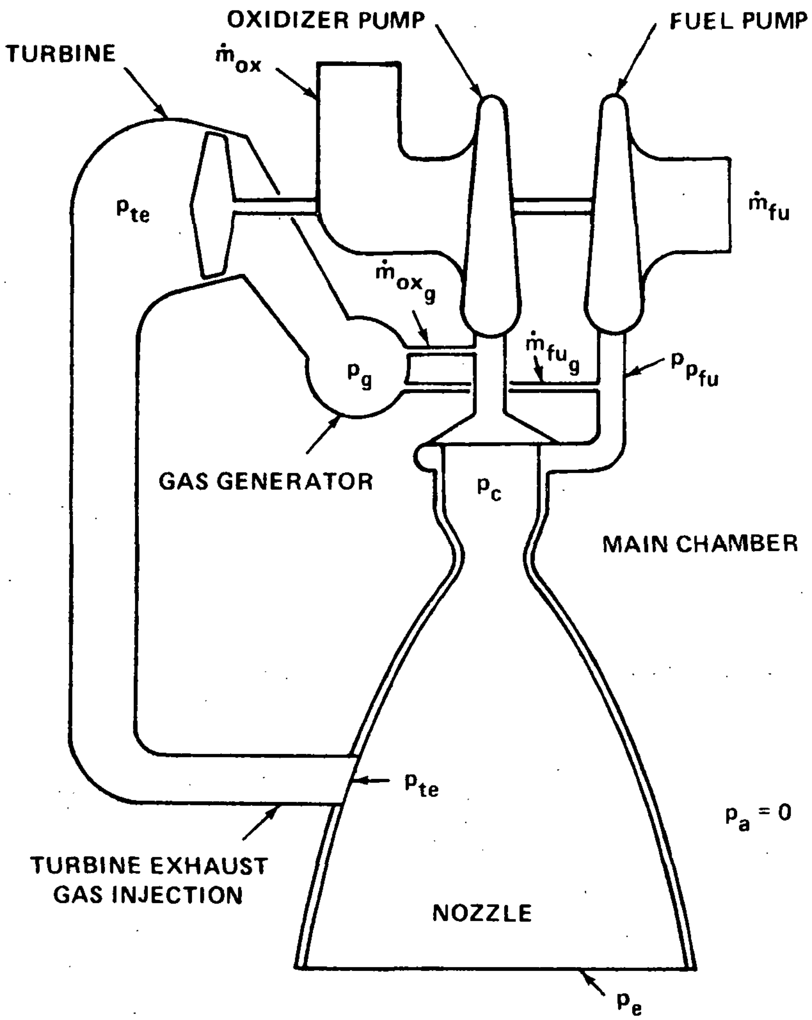

Figure 1. Gas generator cycle engine.

efficiencies are equal and that the pump inlet pressures are small compared with the discharge pressure, equation (1) can be rewritten as

$$E_p = \frac{1}{\eta_p} \left(\dot{m}_{ox} \frac{p_{p_{ox}}}{\rho_{ox}} + \dot{m}_{fu} \frac{p_{p_{fu}}}{\rho_{fu}} \right) . \tag{2}$$

Introduction of the overall mixture ratio r ,

$$r = \frac{\dot{m}_{ox}}{\dot{m}_{fu}} , \tag{3}$$

results in

$$E_p = \frac{\dot{m}}{\eta_{p(1 + 1/r)}} \left(\frac{p_{p_{ox}}}{\rho_{ox}} + \frac{p_{p_{fu}}}{\rho_{fu}} \frac{1}{r} \right) . \tag{4}$$

The turbine shaft energy (E_t) results from the enthalpy drop (Δh_t) between the gas generator and the turbine exit,

$$E_t = \dot{m}_g \Delta h_t \eta_t , \tag{5}$$

where $\dot{m}_g$ represents the gas generator mass flow and η_t the turbine efficiency. Instead of the enthalpy drop, the de St. Venant-Wantzel equation can be used* and one obtains

$$E_t = \dot{m}_g \eta_t (c_p T)_g \left[1 - \left(\frac{p_{te}}{p_g} \right)^{\frac{\gamma_g - 1}{\gamma_g}} \right] . \tag{6}$$

Here $(c_p T)_g$ describes the total enthalpy in the gas generator, p_g the gas generator pressure, which is equal to the turbine inlet pressure, and p_{te} the

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* Since the gas generator mixture ratio is normally very low, dissociation effects can be neglected and therefore equation (6) is a reasonable equation with constant γ_g .

turbine exhaust pressure. The isentropic exponent of the combustion gases in the turbine is denoted by γ_g . Under steady-state conditions both energies must be equal,

$$E_p = E_t \quad . \tag{7}$$

This results in a gas generator flow

$$\dot{m}_g = \frac{\dot{m}\left(\dfrac{p_{p_{ox}}}{\rho_{ox}} + \dfrac{1}{r}\,\dfrac{p_{p_{fu}}}{\rho_{fu}}\right)}{\left(1+\dfrac{1}{r}\right)\eta_t\,\eta_p\,(c_pT)_g\left[1-\left(\dfrac{p_{te}}{p_g}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]} \quad . \tag{8}$$

Replacing the absolute value ($\dot{m}_g$) by the ratio of the gas generator flow to the total flow,

$$k_m = \frac{\dot{m}_g}{\dot{m}} \quad , \tag{9}$$

the relative gas generator mass flow in terms of the overall mixture ratio is

$$k_m = \frac{\dfrac{p_{p_{ox}}}{\rho_{ox}} + \dfrac{1}{r}\,\dfrac{p_{p_{fu}}}{\rho_{fu}}}{\left(1+\dfrac{1}{r}\right)\eta_t\,\eta_p\,(c_pT)_g\left[1-\left(\dfrac{p_{te}}{p_g}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]} \quad . \tag{10}$$

Normally the gas generator mixture ratio (r_g) and the combustion chamber mixture ratio (r_c) are given and r can be expressed by these two. With

$$r = \frac{\dot{m}_{ox}}{\dot{m}_{fu}} = \frac{\dot{m}_{ox_c} + \dot{m}_{ox_g}}{\dot{m}_{fu_c} + \dot{m}_{fu_g}} , \tag{11}$$

one obtains after some transformations and the introduction of k_m

$$r = r_c \frac{1 + \dfrac{k_m}{1 - k_m} \dfrac{1 + 1/r_c}{1 + 1/r_g}}{1 + \dfrac{k_m}{1 - k_m} \dfrac{1 + r_c}{1 + r_g}} . \tag{12}$$

Due to pressure drop in the lines, injector, etc., the pump outlet pressure must be higher than the chamber pressure (p_c). A simple relation is formulated by

$$p_{p_{ox}} = p_c(1 + k_{ox}) \tag{13}$$

$$p_{p_{fu}} = p_c(1 + k_{fu}) , \tag{14}$$

where the k values are assumed to be constant within certain pressure limits. Since the results are not very sensitive to the pressure factors, this statement contains little error.

As gas generator pressure, the oxidizer pump discharge pressure, which is commonly lower than the fuel pressure, can be used following the previous formulation,

$$p_g = p_{p_{ox}} (1 - k_g) \tag{15}$$

$$= p_c (1 + k_{ox})(1 - k_g) \quad . \tag{16}$$

With the knowledge of the different constants for a given chamber pressure, the gas generator mass flow with turbine exit pressure as a parameter can be calculated.

From equation (10) a minimum gas generator flow results. The lowest turbine flow exit pressure for injection into the main nozzle is limited by the nozzle exit pressure (p_e). The minimum gas generator mass flow $\left(k_{m_{min}}\right)$ is determined by

$$k_{m_{min}} = \frac{\dfrac{p_{p_{ox}}}{\rho_{ox}} + \dfrac{1}{r_c} \dfrac{p_{p_{fu}}}{\rho_{fu}}}{\left(1 + \dfrac{1}{r_c}\right) \eta_t \, \eta_p \, (c_p T)_g \left[1 - \left(\dfrac{p_e}{p_g}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right]} \quad . \tag{17}$$

The numerical value results from a solution of equations (12) and (17). An approximate value can be obtained by using r_c instead of r in (17), since the gas generator mass flow is normally small compared with the total flow. An increase of the gas generator mass flow permits for the same turbine power generation the pressure ratio in the turbine to decrease.

<u>Engine Specific Impulse.</u> The engine specific impulse $\left(I_{sp_{ggc}}\right)$ resulting from the main chamber thrust and the turbine exhaust thrust can be expressed by

$$I_{sp_{ggc}} = \left[(1 - k_m) I_{sp_c} + k_m I_{sp_g} \right] \eta_{I_{sp}} \quad , \tag{18}$$

where I_{sp_c} and I_{sp_g} describe the main chamber and gas generator specific impulse, respectively. $\eta_{I_{sp}}$ considers the various losses such as divergence, friction, etc. For a given gas generator flow, the turbine pressure ratio can be obtained by solving equation (10). This results in

$$\frac{p_g}{p_{te}} = \left[1 - \frac{p_c \left(\dfrac{1 + k_{ox}}{\rho_{ox}} + \dfrac{1}{r} \dfrac{1 + k_{fu}}{\rho_{fu}} \right)}{\eta_p \eta_t \left(1 + \dfrac{1}{r} \right) k_m (c_p T)_g} \right]^{-\frac{\gamma_g - 1}{\gamma_g}} \tag{19}$$

The pressure ratio of the turbine gases for the expansion in the main nozzle is

$$\frac{p_{te}}{p_e} = \frac{p_{te}}{p_g} \frac{p_c}{p_e} \frac{p_g}{p_c} \tag{20}$$

$$= \frac{p_{te}}{p_g} \frac{p_c}{p_e} (1 + k_{ox})(1 - k_g) \quad , \tag{21}$$

where the main nozzle pressure ratio (p_c/p_e) is defined by the engine expansion ratio.

Staged Combustion Cycle Engine

In a staged combustion cycle engine, which is schematically represented in Figure 2, the turbine exhaust gases are fed into the main chamber through the injector plate. In this type of engine the expression preburner, instead of gas generator, is used. The calculation of the required turbine pressure ratio can be done with the previous equations. For a low pressure drop in the turbine, according to equation (19), the maximum possible flow rate in the preburner has to be used. This means that the whole fuel mass flow is injected

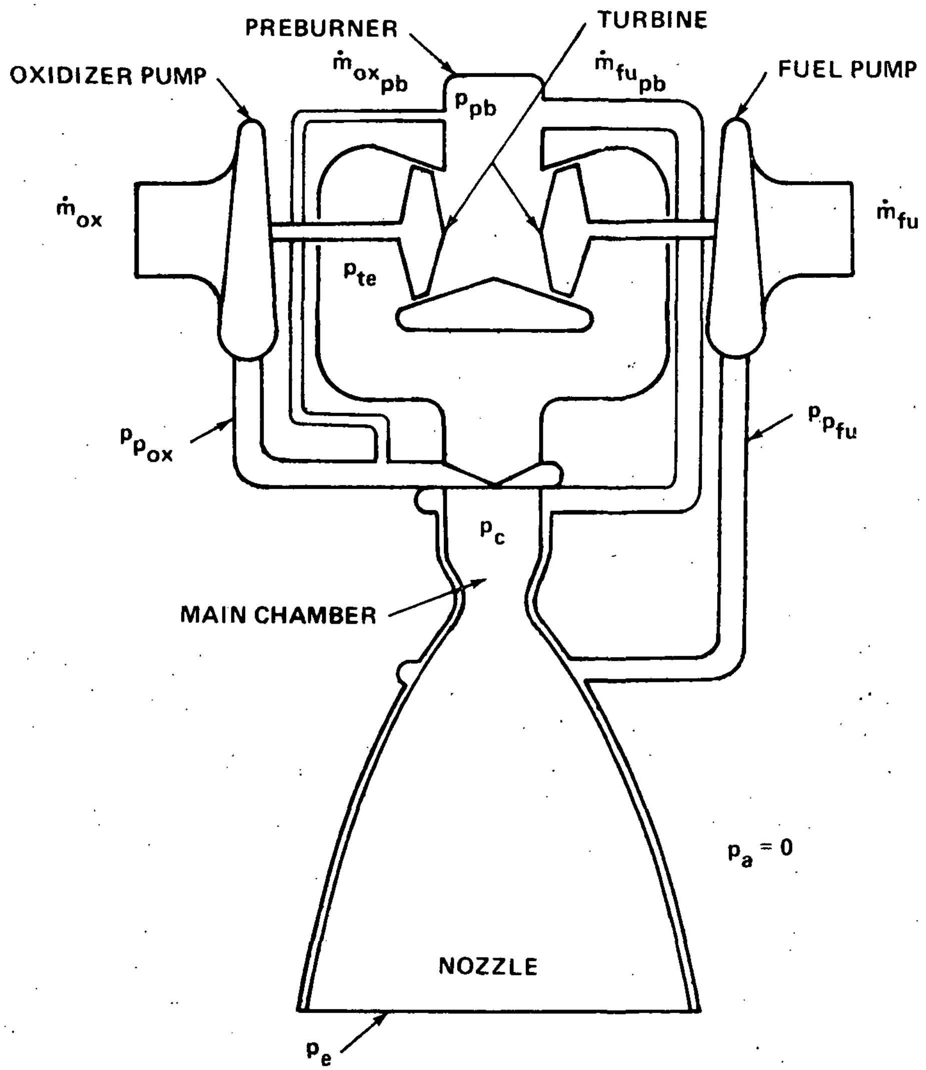

Figure 2. Staged combustion cycle engine.

into the preburner. With the subscript pb for the preburner, the mass flow is

$$\dot{m}_{fu_{pb}} = \dot{m}_{fu_c} \quad . \tag{22}$$

The preburner mixture ratio is defined by

$$r_{pb} = \frac{\dot{m}_{ox_{pb}}}{\dot{m}_{fu_{pb}}} \quad ,$$

and one obtains

$$\begin{aligned} \frac{\dot{m}_{pb}}{\dot{m}} &= \frac{1 + r_{pb}}{1 + r_c} \\ &= k_m \quad . \end{aligned} \tag{23}$$

Then with equation (19) the pressure drop in the turbine is calculated. Analogous to the linear relations for the pressure drops, a similar equation can be set up for the connection between turbine exit and main chamber pressure,

$$p_{te} = p_c(1 + k_{pb}) \quad . \tag{24}$$

The pressure drop in the turbine of a staged combustion cycle is normally much lower than that of a gas generator cycle. Therefore, equation (19) can be simplified by expanding into a power series and using only the linear term,

$$\frac{p_{pb}}{p_{te}} = 1 + \frac{\gamma_{pb} - 1}{\gamma_{pb}} \frac{p_{pb}\left(\frac{1 + k_{ox}}{\rho_{ox}} + \frac{1}{r_c}\frac{1 + k_{fu}}{\rho_{fu}}\right) r_c}{\eta_t \eta_p (1 + r_{pb})(c_p T)_{pb}} \quad . \tag{25}$$

In equation (25) the preburner pressure has to be used instead of p_c , since the pumps supply propellants to the preburner. Then together with equation (24), equation (25) can be rearranged and one obtains

$$\frac{p_{pb}}{p_{te}} = \frac{1}{1 - \frac{\gamma_{pb} - 1}{\gamma_{pb}} \frac{p_c(1 + k_{pb})\left(\frac{1 + k_{ox}}{\rho_{ox}} + \frac{1}{r_c}\frac{1 + k_{fu}}{\rho_{fu}}\right) r_c}{\eta_t \eta_p (1 + r_{pb})(c_p T)_{pb}}} \quad . \tag{26}$$

The pump discharge pressures are

$$p_{p_{ox}} = p_c(1 + k_{pb}) \frac{p_{pb}}{p_{te}} (1 + k_{ox}) \tag{27}$$

and

$$p_{p_{fu}} = p_c(1 + k_{pb}) \frac{p_{pb}}{p_{te}} (1 + k_{fu}) \quad . \tag{28}$$

In a staged combustion cycle the enthalpy drop in the turbine is recovered by the pump head. Therefore, the specific impulse can be calculated in the usual way with the tank enthalpies of the propellants. With the subscript scc for staged combustion cycle engine, the specific impulse is

$$I_{sp_{scc}} = I_{sp_c} \, \eta_{I_{sp}} \quad . \tag{29}$$

The various losses are comprised in $\eta_{I_{sp}}$.

Approximate Equations for the Thermodynamic Properties of the Combustion Products of O_2/H_2

For a simple treatment, a specific impulse relationship using only area ratio, chamber pressure, and mixture ratio is convenient. Since exact values for one-dimensional expansion can be obtained by Reference 1, it is practical to use "approximations to solutions" instead of "solutions of approximate equations" [2]. The de St. Venant-Wantzel equation belongs to the latter case and has the disadvantage that mixture ratio effects, etc., are omitted almost completely. This leads, especially in the near of the impulse optimum, to great discrepancies between these approximate values and the exact ones.

Therefore, simple equations should be used, which represent the exact one-dimensional values within small errors.

Combustion Chamber Gases. The vacuum specific impulse I_{sp_o} of an engine supplied with LOX/LH_2 at tank enthalpy level for different area ratios and chamber pressures is presented in Figure 3. Instead of a pure I_{sp_o} approximation, a combination of the characteristic velocity (c^*) and the thrust coefficient (C_F) is convenient. With C_{F_o} as the conventional thrust coefficient the usual relation yields

$$I_{sp_o} = \frac{c^*}{g_o} \left(C_{F_o} + \frac{A_e}{A_t} \frac{p_e}{p_c} \right) \tag{30}$$

The use of c^* , C_{F_o} , and p_c/p_e approximations has the advantage that individual effects can be analyzed.

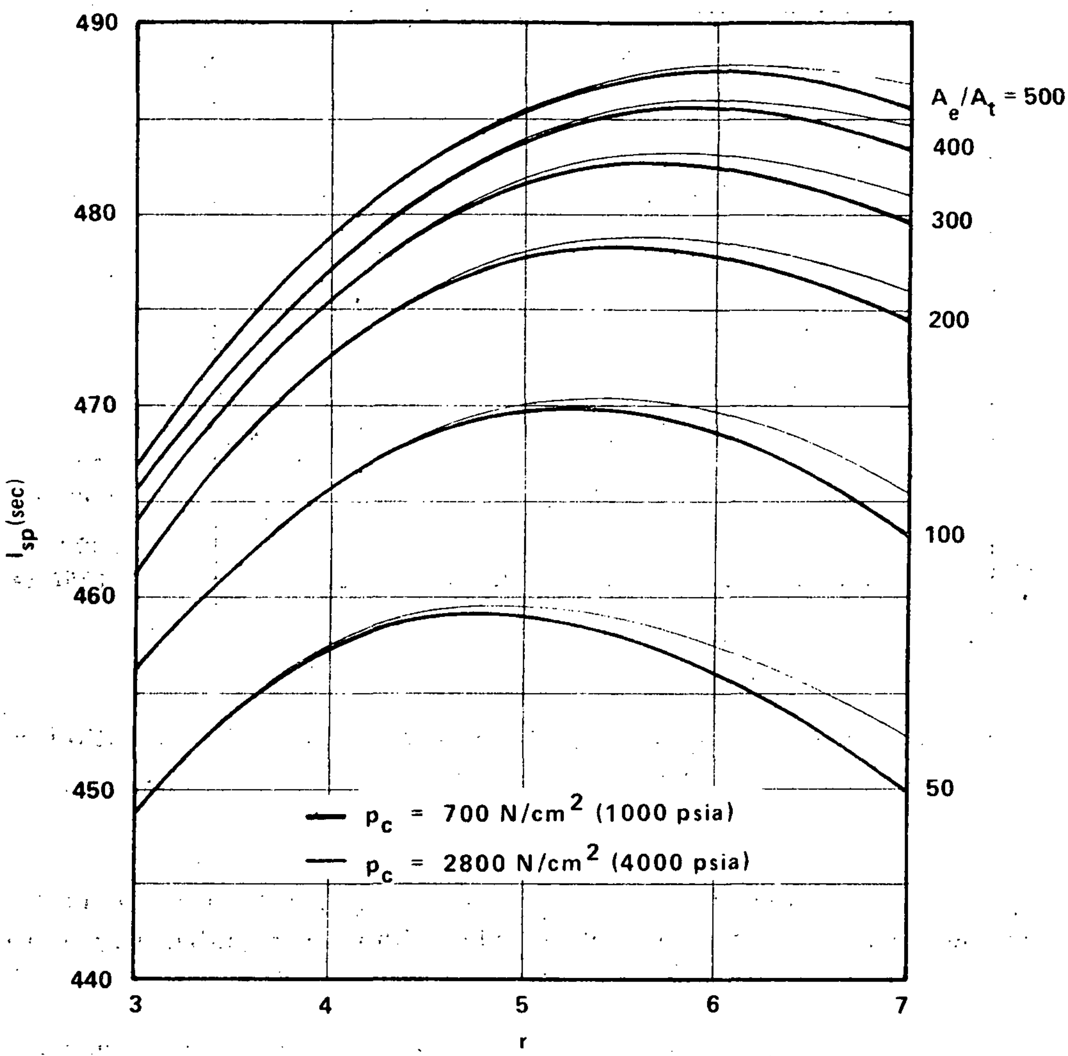

Figure 3. Vacuum specific impulse of LH_2/LOX versus mixture ratio for different area ratios and chamber pressures.

The characteristic velocity is shown in Figure 4. For the approximation of c^* the expression $c^*(1 + 1/r)$ can be used, which shows in the near of the optimum a fairly linear characteristic [3]. Selecting a power law formulation for the chamber pressure influence [4], the subsequent relation for the characteristic velocity within the limits $4 \leq r \leq 7$ results,

$$c^* = \frac{3660 - 160\, r \left(\frac{p_c}{70}\right)^{-0.022}}{1 + 1/r} \quad \text{(m/sec)} \quad . \tag{31}$$

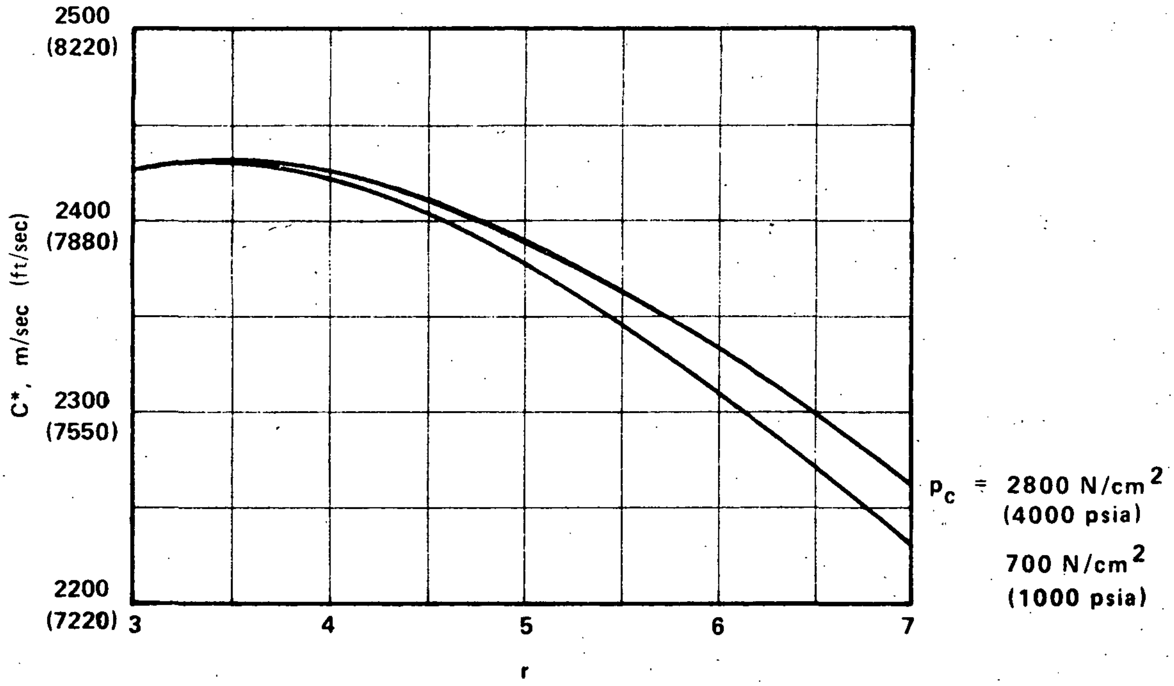

Figure 4. Characteristic velocity of LH_2/LOX versus mixture ratio for $p_c = 700\ N/cm^2$ (1000 psia) and $2800\ N/cm^2$ (4000 psia).

The equation is valid for chamber pressures between $500\ N/cm^2$ and $3000\ N/cm^2$; however, both limits may be extended without decreasing the accuracy significantly.

The thrust coefficient is shown in Figure 5. To simplify the approximation with the common equation for C_{F_o} a γ_{C_F} can be defined, which is easier to use,

$$C_{F_o} = \left\{ \gamma_{C_F} \left(\frac{2}{\gamma_{C_F} - 1} \right)^{\frac{\gamma_{C_F} + 1}{\gamma_{C_F} - 1}} \frac{2\gamma_{C_F}}{\gamma_{C_F} - 1} \left[1 - \left(\frac{p_e}{p_c} \right)^{\frac{\gamma_{C_F} - 1}{\gamma_{C_F}}} \right] \right\}^{0.5} . \quad (32)$$

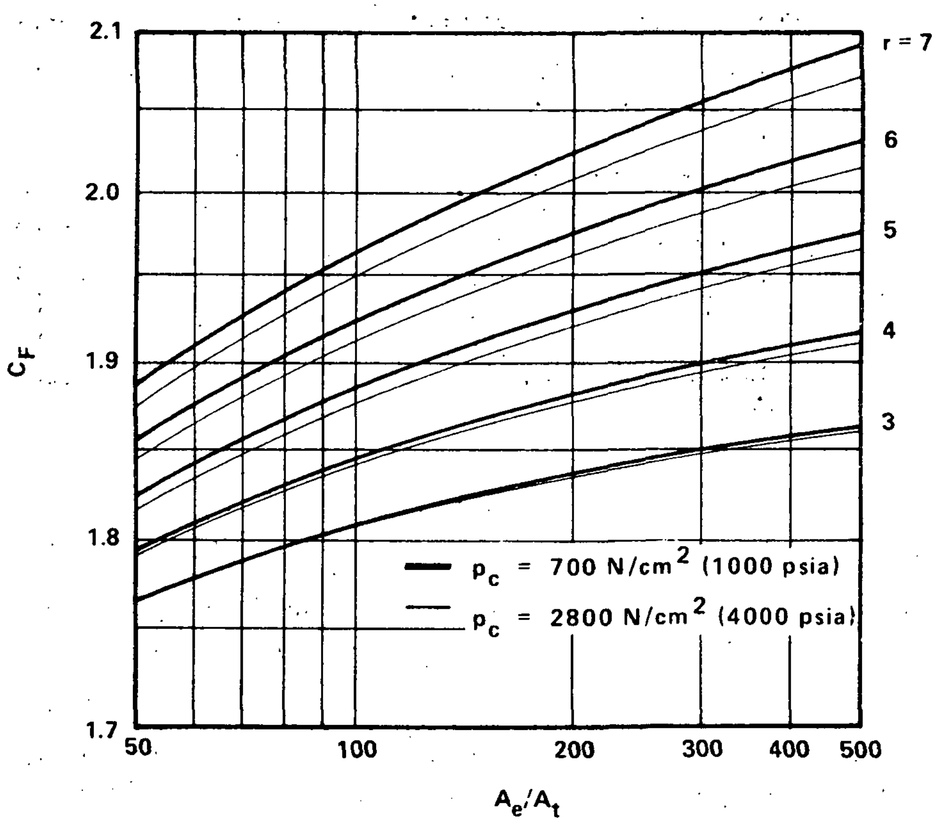

Figure 5. Thrust coefficient of LH_2/LOX versus area ratio for different mixture ratios and chamber pressures.

This γ_{C_F} is plotted in Figure 6. For space flight applications, the area ratio (A_e/A_t) is preferred over the pressure ratio (p_c/p_e), since the latter value is more characteristic for the engine size. The relation for γ_{C_F} is

$$\gamma_{C_F} = \exp\left[0.00534\ \ln\left(\frac{A_e}{A_t}\right) + 0.234 - 0.0311\ (r - 3)\left(\frac{p_c}{70}\right)^{-0.0555}\right] \quad . \tag{33}$$

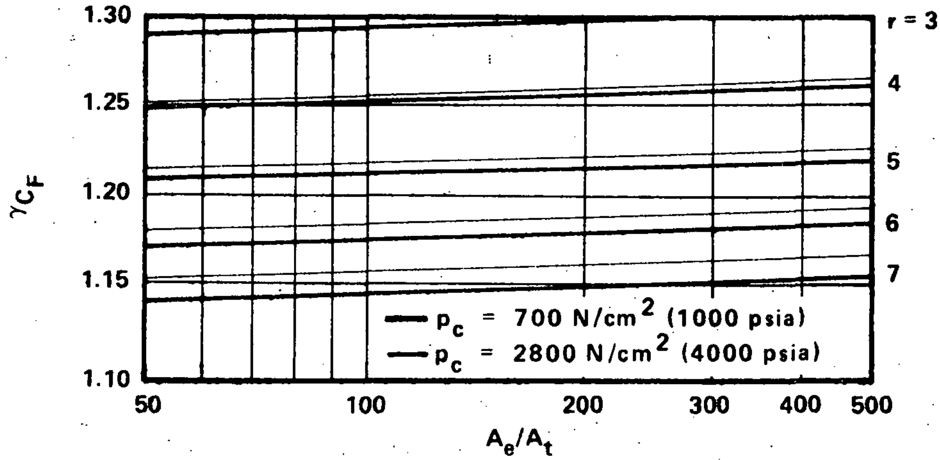

Figure 6. Specific heat ratio γ_{C_F} for the calculation of the thrust coefficient as a function of area ratio for different mixture ratios and chamber pressures.

The applicable pressure and mixture range are the same as for the characteristic velocity; in addition, the area ratio ranges from 50 to 500, where again the limits may be extended.

Finally the pressure ratio as a function of the area ratio is of interest. This correlation is shown in Figure 7. According to Reference 5, a power relation of the following type represents the characteristics of both quantities well:

$$\frac{p_c}{p_e} = \exp\left[(1.38 - 5.68\ 10^{-4}\ r)\ \ell n\left(\frac{A_e}{A_t}\right) + 1.58 - 0.1(r - 3)\left(\frac{p_c}{70}\right)^{-0.125}\right] \quad (34)$$