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STATUS REPORT ON CONTROL OPTIMIZATION, STABILIZATION AND COMPUTER ALGORITHMS FOR AIRCRAFT APPLICATIONS

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Fourteenth Status Report

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Fourteenth Status Report for Period 1 January 1973 to 1 May 1973

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FOURTEENTH STATUS REPORT

NASA NGL-22-009-124

1. GENERAL

This status report pertains to research progress under Research Grant No. NGL-22-009-124 during the period 1 January 1973 to 1 May 1973 by faculty members, research assistants and graduate students comprising the research staff assigned to this project. Progress was made in several areas as reported in the next section.

During the report period the following people contributed to this research: Professors S.K. Mitter, J.C. Willems and T.L. Johnson (all part time), and research assistant K. Glover. In addition, the following people contributed without requiring salary support from this grant: Instructor A. Eckberg, teaching assistant L. Platzman, and graduate students C-Y Chong, N. Sandel, A. Lopez-Toledo, and D. Willner. Professor Athans also contributed, but required no salary support from the grant during this time period.

2. TECHNICAL ACCOMPLISHMENTS

During this time period our research efforts have been directed toward several areas of practical and theoretical interest. There were six main areas in which advances were made. These are

1) Design Methods
2) Distributed Parameter Systems
3) Large Scale Systems
4) Structural Aspects of System Identification
5) Stochastic and Adaptive Control
6) A Practical Scheme for Adaptive Aircraft Flight Control Systems

A detailed qualitative description of each of the above topics is presented in the remainder of this section. Technical details are not given since they can be found in the cited references.

DESIGN METHODS

By design methods we mean computationally useful techniques that can aid the control engineer design good control systems for linear time-invariant dynamics. Our research in this area falls into two categories:

1. Suboptimal Control Methods
2. Saturating Controllers

Suboptimal Control Methods

Mr. L. Platzman and Prof. T.L. Johnson have been working in this area of design techniques.

In solving an optimal control problem, the prime considerations are usually: 1) The performance of the system, and 2) the cost of maintaining a high level of performance. For example, if the problem of guiding a rocket along a prescribed trajectory is considered, the controller might be designed to keep the rocket as close as possible to that trajectory without consuming an inordinate amount of fuel. The current research considers a third criterion: the cost of controller implementation. This 'cost' can be attributed to three components: The cost of designing the controller, the cost of the controller *per se*, and the reliability (or maintainance cost) of the controller. In other words, a complex controller is undesirable, because the computational procedures required to design the controller are expensive, the cost of building the controller is proportional to the number of components it contains, and a complex controller is more costly to maintain. A controller which is designed in such a way as to be less complex structurally than an optimal controller is called a suboptimal controller.

Previous research in suboptimal design has been limited primarily to linear-quadratic-gaussian applications [1,2]. For example, the optimal control of a linear plant with 15 states, 10 observations, and 5 actuator components requires 5 integrator and 140 summer components. A suboptimal controller can be designed which requires 2 integrators and 77 summers. In many cases, the performance might decrease by a factor of less than 10%. This compares favorably with the optimal design when one considers the fact that the number of components in the controller has decreased by nearly 50%.

The suboptimal design method becomes the only feasible method when the optimal design would in effect require an infinite number of components (e.g. distributed parameter systems). The ad hoc methods which are currently used to tackle such problems lack the rigor and methodology of the suboptimal design approach.

In addition to formulating a theory of suboptimal design, we were successful to obtain feasible computational algorithms for the following problems:

(a) Limited dimension control of a linear system with gaussian disturbances and quadratic criteria.

(b) Time-invariant controller design of time-varying systems [Optimal design methods often require the implementation of time-varying parameters. This might be accomplished by reading values off a tape - an expensive undertaking]

(c) Suboptimal control of markovian decision processes (finite-state finite-decision systems).

(d) Determination of optimal switching times for discrete decisions.

It is hoped that within the next year, the global theory will lead to feasible methods for optimal actuator location, optimal observer location, and other problems which occur in infinite dimensional systems analysis.

In the course of our investigation of suboptimal design methods, we discovered conditions for convergence of the Min-H Algorithm proposed by H.J. Kelley [3]. This algorithm is considered one of the fastest around when it works. In addition to proving that convergence does, indeed, occur under certain conditions, we developed a modified Min-H algorithm which circumvents these problems and is guaranteed to converge. Experience with the modified Min-H algorithm will be required to determine whether or not the efficiency of the original algorithm was preserved. We plan to carry out such experiments this summer.

The research will be more fully described in Platzman's S.M. Thesis [4].

References

[1] Levine, W.J., Johnson, T.L., and Athans, M. "Optimal Limited State Feedback Controllers for Linear Systems," IEEE Trans. on Aut. Control, Vol. AC-16, pp. 785-793, December, 1971.

[2] Kleinman, D.L., Fortmann, T., and Athans, M. "On the Design of Linear Systems with Piecewise-constant Feedback Gains," IEEE Trans. on Aut. Control, Vol AC-13, pp. 354-361, August 1968.

[3] Kelley, H.J., "Method of Gradients," in: Optimization Techniques (G. Lietman, Editor), Academic Press, New York (1962)

[4] Platzman, L. "On the Iterative Design of Optimal and Suboptimal Controllers", S.M. Thesis , M.I.T., May 1973 (in preparation).

Saturating Controllers

Prof. Athans has initiated a new research project that deals with fundamental understanding of handling multiple saturating controllers in a linear-quadratic-gaussian design framework. It turns out that if a control system has more than one control input, and due to disturbances one of the control inputs saturates, then the system may become temporarily unstable. We were successful in deriving conditions that will indicate if such an undesirable situation will occur, and techniques that prevent such a temporary instability are currently being developed.

This work will continue, and more details will be given in the next progress report.

DISTRIBUTED PARAMETER SYSTEMS

Control and estimation problems involving first-order distributed-parameter systems have been under continuous investigation by Prof. T.L. Johnson and outlined in previous status reports. In particular, we have in mind applications to problems such as control of electromagnetic field configurations, numerical weather prediction, control of Navier-Stokes equations, and boundary layer control problems for re-entry vehicles. The design of practical high-performance controllers for such systems, in our opinion, will grow very rapidly in the remainder of this century.

Several features of such control problems make them exceedingly difficult, both practically and theoretically; in fact, a successful design philosophy is almost certain to involve a very close interplay between mathematics and physical intuition. We cite the following examples:

1. Typically, physical laws lead to a set of coupled higher-order partial differential equations representing the static and/or dynamic behavior of such systems. Though one suspects that representing this higher-order system by a first-order one will yield conceptual advantages (just as in the case of choosing state variables for systems of higher-order ordinary differential equations), there is presently no theory to help us decide how to choose state variables. With partial differential equations, the problem goes far deeper than with finite dimensional systems, since solutions of the equations may fail to exist if the state variables are improperly chosen.

2. Such systems typically involve the interaction of diffusive properties (e.g., heat flow) with wave-type properties

(e.g., electromagnetic radiation), along with certain conservation conditions (e.g., energy conservation). Although these "classical" types of problems have received extensive study separately, our understanding of how to incorporate the strong interaction of such properties is limited, from a dynamical systems and control perspective. One central issue is how to determine proper boundary conditions. Another is how to append static constraints to the dynamic parts of the problem.

(3) Unfortunately, the natural analog of the finite-dimensional control problem with full-state feedback--i.e., distributed control, is of relatively little importance for realistic 1st-order problems. Rather, it is often desirable to control from the boundary. <u>We have derived necessary conditions for optimal boundary control in the simplest case, and the control laws suggest the use of weighted-averaging types of sensors</u>. The continuity properties of the controls and the relative effectiveness of boundary vs. distributed control are under investigation.

(4) Off-line and on-line computational requirements for optimal control of 1st-order systems are a crucial issue. <u>At least for wave-type systems, we have reason to believe that the on-line computational requirements for boundary control may be more modest than is conventionally assumed.</u> For example, a knowledge of wave speeds and directions plus the wave-shape at the boundary may suffice. It should be noted that conventional finite-element methods (e.g., discretization) may give slow

convergence (i.e., with increasing numbers of grid points) to the true solution of a problem, hence there is considerable impetus to find alternative computational schemes for the control problem.

To summarize, our research during recent months has addressed the problems of realizations (canonical forms) for first-order systems, and continuity properties of optimal boundary controls. In addition, an extension of the important geometric interpretation of the Maximum Principle of Pontryagin (due to Halkin) to distributed systems is under investigation. Technical discussion of results will appear in future publications.

LARGE SCALE SYSTEMS

Research in large scale system theory has progressed along several fronts. We outline below the specific areas of current research.

1. Linear Systems with Decentralized Control
2. The Value of Information Patterns
3. Periodic Decentralized Stochastic Control

Linear Systems with Decentralized Control

Prof. S.K. Mitter and Mr. A.E. Eckberg have continued to work on the decentralized control of linear systems.

Until fairly recently it has been traditional in system and control theory to consider a dynamical system as being controlled by a single control agent. In such a situation one assumes that all of the available information about the system and its state may be utilized by the single control agent in determining his control strategy. For rather obvious reasons this type of control philosophy is termed "centralized control".

Often, however, it is more reasonable to model the system as being controlled by two or more control agents, each of whom has access to an incomplete information set (i.e. the observations on the system and its state), and each of whom can influence only a portion of the system through his control strategies. Clearly, if there were a higher level control agent, or coordinator, which could receive information from, and send commands to, each of the other control agents, then the overall control of the system would be partially centralized. In the absence of such a coordinator we shall say that the control of the system is "decentralized".

Decentralized control systems fall into two classes: cooperative and noncooperative. In the former class the individual control agents all strive for the same goal, or at least similar goals; in the latter class the individual agents may be striving for conflicting goals. While there exist several examples of the latter class, e.g. differential games, we shall concern ourselves only with cooperative decentralized control systems.

Examples of cooperative decentralized control systems abound, especially when one considers large scale dynamical systems. A typical example where the control may be decentralized (at least partially) is the air traffic control system. Another example may be a flexible aircraft which is modelled as coupled beams where each beam is controlled separately.

Two fundamental results in the control of linear systems are the following:

(i) If a finite-dimensional linear time-invariant system is controllable ,then one can achieve arbitrary closed-loop pole configurations by means of linear constant state-variable feedback. If the complete state vector cannot be directly measured, then assuming the system is observable arbitrary pole-placement can be achieved by constructing a dynamic observer to estimate the "state" of the system and using linear feedback on the estimated state of the system. For a discussion of the above see, for example [1].

(ii) A deeper result, due to Rosenbrock [2] presents necessary and sufficient conditions for achieving arbitrary closed-loop Jordan Canonical Forms. This result is closely related to the work on invariants of linear control systems [3].

This work is concerned with generalizing the above results to the decentralized control of linear systems with decentralized memory less feedback and decentralized dynamic compensation. The details of this work may be found in the forthcoming thesis of A.E. Eckberg [4].

References:

1. J.C. Willems and S.K. Mitter: Controllability, Observability, Pole-Allocation and State Reconstruction, IEEE Transactions on Automatic Control, Vol. AC-16, December 1971, pp. 582-595.

2. H.H. Rosenbrock: State Space Theory of Multivariable Systems, John Wiley, 1970.

3. V.M. Popov: Invariant Description of Linear, Time-Invariant Controlable Systems, SIAM J. Control, 10 (1972) p. 252.

4. A.E. Eckberg: Algebraic System Theory with Applications to Decentralized Control. Doctoral Dissertation, Electrical Engineering Department, M.I.T., June 1973.

The Value of Information Patterns

A new project has been initiated during this time period by Mr. Nils Sandel and Prof. M. Athans, which deals with another issue that arises in stochastic control for large scale systems.

A fundamental problem in the design of large scale systems is the presence of conflict. Conflict can result from (1) the different goals of (human) controllers of the system or (2) different information available to the controllers. Our focus is presently directed at systems in which the second, but not the first, type of conflict is present.

We have in mind situations in which a large system is to be controlled by a number of computers. The computers' input channels receive data from other computers and various sensors. Each computer has its own distinct and finite storage, and its output channels carry signals to other computers and the various actuators which apply physical inputs to the system. The fundamental problems are (1) designing system interfaces and (2) designing the computer programs. At present, these problems are solved in a rather ad hoc fashion.

Of course, in order to develop an analytic treatment of these problems, it is necessary to make certain idealizations. Thus, each sensor-computer-actuator system becomes a controller which maps data into input signals, and the interfaces are specified by the information pattern. The reformulated problem is a (very general) stochastic control problem.

We are examining this problem from a very general viewpoint. Thus the problems of existance of solutions, various alternative descriptions of the uncertainty, sufficient statistics (data reduction), well-posedness of dynamic programming, etc., have received our attention. We have obtained encouraging preliminary results.

Periodic Decentralized Stochastic Control

Mr. C-Y Chong and Prof. Athans continued their research on specific strategies for the decentralized control of large scale systems.

Two aspects are very important in the control of large scale systems: computation and information. Even with large digital computers, it may not always be possible to compute the optimal controls and implement them in real time. Also, the different parts of the system may be physically apart such that perfect communication between different parts is technically or economically not feasible.

Most of the work dealing with the computational aspect of the control of large-scale systems is found in mathematical programming. Using the special structure of the problem, hierarchical decomposition schemes are suggested which give the optimal solution in an iterative manner. The drawback to this approach is its deterministic nature and the fact that the informational aspect of the problem has been neglected.

Another kind of work deals with decentralized information and is found under the name "team theory". It is a statistical decision problem when there are more than one decision agent present and each has different information about the state of the system. Optimal decision rules have been obtained for the case when the payoff function is quadratic and the probability densities are gaussian. The drawback to this approach is that the computation is centralized. Thus it is very complicated except for very simple problems. This in part explains why the extension team theory to dynamic teams has not been very successful.

Decentralization with respect to both information and computation has been considered in our research. For the static case, the stochastic optimization problem of several systems sharing a common source has been investigated. By the introduction of a coordinator who measures the resource available, the overall optimization problem has been decomposed into two levels, with the lower level consisting of uncoupled optimization problems. When the information patterns of the lower level decision agents and that of the coordinator satisfy certain conditions, the lower level problems depend only on certain messages sent out by the coordinator.

The dynamic problem also has been considered. In this case, a new kind of optimality is defined which reflects the fact that the system under consideration consists of subsystems coupled together. Again there exists a coordinator who does all the centralized computation and insures that the lower level controllers act in some "average optimal" manner. Two cases are considered: First, the off-line case when the coordinator chooses the coordinating parameters in an open-loop fashion; second, the coordinator makes on-line measurements periodically and updates the coordinating parameters. This decomposition of the off-line problem into two levels is done for a general nonlinear problem. Specialization to the linear-quadratic-gaussian case shows the optimal control strategies consists of an open-loop part depending on the coordinating parameters and a closed loop part depending on the measurements of each lower level controller. For the on-line case, it is found that the open-loop feedback optimal coordinating strategies consist of the coordinator updating his states estimates of the whole system periodically.

Further work will include the study of the loss of optimality of this scheme relative to the centralized information structure. Performance should improve as the coordinator has more information available to him. Necessary and sufficient conditions such that systems can be decomposed will also be studied.

This research will be summarized in the forthcoming doctoral dissertation of Chong [1].

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[1] C-Y Chong, "On the Decentralized Control of Large Scale Systems", Ph.D. Thesis, E.E. Dept., M.I.T., May 1973 (in preparation).

STRUCTURAL ASPECTS OF SYSTEM IDENTIFICATION

1. Summary and Introduction

There is presently great interest in system identification in many application areas, where better system models are needed in order to use the modern analytic techniques to design good controllers. The interest in identification for aerospace applications is evidenced by the great interest displayed in the recent NASA Symposium on Parameter Estimation Techniques and Applications in Aircraft Flight Testing, NASA Flight Research Center, Edwards, California, April 24-25, 1973.

In this research conducted by Mr. K. Glover, Prof. J. C. Willems, and Prof. S. K. Mitter properties of linear dynamical systems that effect system identification is considered. Firstly the use of canonical forms in identification is discussed, showing their shortcomings and possible modifications. Then it is assumed that a parametrization of the system matrices has been established from a priori knowledge of the system, and the question of when such unknown parameter can be identified from input/output observations is answered locally and globally.

The effect of feedback on identifiability is then considered. Further it is shown that system estimates being non-minimal can cause some algorithms to become ill-posed, and that such non-minimality is sometimes inevitable.

Finally identification of systems driven by white noise is studied when only the output is observed. Results on identifiability have been obtained in this case.

The technical details can be found in papers by Glover and Willems (1,2) and the forthcoming dissertation of Glover (3).

2. Canonical Forms for Linear Dynamical Systems

In this section the problem of parametrizing a linear dynamical system for identification, where there is little or no a priori system information, is described. A parametrization method is given which is both theoretically and practically attractive. This situation occurs in practice when a model is being determined not for its physical interpretation but for the subsequent design of a filter or controller.

Suppose a linear dynamical system is being identified from input/output observations, then it is well-known that there is not a unique state-space realization of the input/output response. Indeed there will be many equivalent realizations which will be related by a non-singular similarity transformation. It is therefore desired to put the system matrices in a so-called canonical form so that the identification problem has a unique solution. In a canonical form there is a unique realization of each transfer function of the appropriate dimension. For single input/ single output systems the standard observable or controllable forms are very useful canonical forms. However in the multivariable case, there does not exist a single parametrization of the system matrices that can represent all transfer functions of the appropriate dimension, while at the same time having a unique representation of each transfer function. Canonical forms for this situation have been produced by several authors in the literature (e.g., V. M. Popov, SIAM J. Control, Vol. 10, No. 2, May 1972), and rely on determining which columns (or rows) of the Hankel matrix are dependent and independent. Then a family of parametrizations is produced, one for each set of independent vectors.

The use of canonical forms of identification has two disadvantages. Firstly, a statistical test to show that a set of vectors are dependent is not well-posed. Secondly, the canonical representation of systems close to the boundary of one of the parametrizations becomes ill-posed with some elements of the matrices tending to zero while others tend to infinity. The first problem can lead to inaccuracies, and the second to severe numerical difficulties.

An alternative approach is proposed which overcomes both these problems as follows. Firstly, it is remarked that the uniqueness property of canonical forms is not essential in identification since when canonical forms are being used the state space does not have an immediate physical interpretation. Therefore, the fact that a particular system may be realized in a finite number of different ways is not a great concern. It is proposed to use a family of parametrizations, all of which can represent almost all systems of the appropriate dimension and together they can represent all such systems. Then when identifying these systems a particular parametrization is chosen on the results of a simple statistical test on a limited set of data. Then the resulting parametrization will be able to represent systems in a large neighborhood of the initial estimates in a well-posed and accurate manner.

3. Parameter Identifiability of Linear Dynamical Systems

In this section we discuss the parametrization of linear dynamical systems from a priori information, and give conditions when such parametrizations can be identified.

In many application areas, particularly in aerospace problems, state space equations can be written down from a priori knowledge of the physics of the system. The elements of the system matrices can then be written down as, either 1) zero

2) known physical constants

3) known functions of some unknown parameters.

In other words the system matrices can be expressed as a function of a, hopefully limited, set of parameters. This is in contrast to the situation discussed in section 2 where little was known a priori. A natural question is: can these unknown parameters be identified from some, possibly noisy, input/output observations? The answer to this question has two parts. Firstly, the input has to excite all the system modes and the observations must be sufficiently accurate. That is the transfer function must be identifiable from the observations. Then secondly given that the transfer function can be identified, the unknown parameters must be uniquely determined from the transfer function. The second problem has been studied here and it is clear that if too many parameters are unknown then they will not be identifiable (e.g., if the system matrices are completely unknown, any realization could be changed by a similarity transformation to give another equally good realization). It is shown that no more than $n(m+p)$ parameters can be identified from input/output observations (where, n = state dimension, m = number of inputs, p = number of outputs). Further, simple conditions for a parametrization to be identifiable near nominal values for the unknown parameters have been derived (i.e. local identifiability).

Local identifiability of a system parametrization is essentially a prerequisite for any identification routine to be well-posed and converge to a locally unique solution. A disadvantage of only having local identifiability is that the region where the parameters remain identifiable is difficult to compute. However, if a parametrization is locally identifiable for all values of the parameters, then an identification algorithm will converge to a solution, but may converge to one of several solutions depending on the initial parameter estimates and the particular data set received. The concept of global identifiability is thus considered and is in general a very difficult problem. However, if the parametrization is affine (i.e. linear plus a constant), then a good sufficient condition for global identifiability has been obtained.

Partial identifiability is defined as the identifiability of one set of parameters independently from another set of parameters. Conditions for local partial identifiability have been derived, and we determined if accurate estimates for some parameters can be found in the absence of good estimates for the other unknown parameters.

It is recommended that the identifiability conditions are checked before any algorithm is applied to ensure the parametrization will give no problems.

4. The Effect of Feedback on Identification

The presence of feedback can cause severe identification inaccuracies if the identification algorithm does not take the feedback into account and considers the input to be independent of the output. Feedback can enter explicitly in the form of some closed-loop automatic control gains, or more subtlely (as for example, a pilot's response). When the feedback is not known a priori, less information about the open loop can be identified, and conditions are given which determine whether a set of open loop parameters can be identified in the presence of a partially known feedback matrix.

5. Geometrical Properties or Minimal Systems

Most linear system identification algorithms assume that the McMillan degree of the system to be identified is known or has already been determined. Then the algorithm produces a sequence of parameter estimates that converge to the true parameter values. Now certain algorithms require that each estimate represent a minimal system, so that for example a matrix inverse can be computed. However, the assumption that the actual system has degree n will not necessarily imply that the estimates will have degree n. Furthermore, that an estimate is non-minimal cannot be blamed on "bad luck" but in certain circumstances is an inevitable consequence of the initial estimate, as illustrated by the following result due to R.W. Brockett (1972) on scalar transfer functions. He proved that the parameter space is divided into several regions which are separated from each other by non-minimal systems (i.e. not controllable or not observable systems). The implication is, therefore, that if the initial system estimate is in the wrong region, then the algorithm can only converge to the correct solution by passing through a system estimate of lower degree. This can also present problems in some adaptive control algorithms, if for example minimality is required to obtain a solution to the control problem.

The multivariable case was studied and shown not to exhibit the above behavior in general. This is because the non-minimal systems form a lower dimensional surface in the multivariable case, and hence cannot disconnect the parameter space.

It is recommended that because of the above result on single input/single output systems that on-line algorithms that require the systems estimates to be minimal are not used until reasonable parameter estimates have been found by an alternate method.

6. Identifiability From Output Correlation

In this section we have considered the identification of linear dynamical systems driven by white noise, when only the output is observed. The situation occurs when there is process noise present (e.g., wind turbulence) as against observation noise, or when it is desired to model a correlated noise process as the output of a linear system driven by white noise.

In the steady state the most information that can be obtained is the power spectral density of the output. The realization problem then reduces to a spectral factorization problem, and relationships have been derived between all the equivalent solutions, (i.e., systems with the same output spectral density). The above relationships then lead to conditions for local identifiability from the output spectral density. In this situation, except in the single input/single output case, fewer parameters can be identified than when input observations are present.

7. References

K. Glover and J.C. Willems, "On the Identifiability of Linear Dynamical Systems", Third IFAC Symposium on Identification and System Parameter Estimation, The Hague, The Netherlands, June 12-15, 1973.

K. Glover and J.C. Willems, "Parameter Identifiability of Linear Dynamical Systems", NASA Symposium on Parameter Estimation Techniques and Applications in Aircraft Flight Testing, NASA Flight Research Center, April 24-25, 1973.

K. Glover, "Structural Aspects of System Identification", Ph.D. dissertation, Electronic Systems Laboratory, M.I.T., September 1973.

STOCHASTIC AND ADAPTIVE CONTROL

Once more several important issues have been under consideration during this time period. The areas are:

1. Simultaneous Identification and Control of Linear Systems
2. Adaptive Control

Simultaneous Identification and Control of Linear Systems

In applying system-theoretic techniques to problems arising from present day technology it is often found that a model of the plant can not be constructed without including the presence of uncertain parameters. It is clear that, for a given performance measure, the optimal performance of the control system will be degraded by the uncertainty in the characterization of the plant. This uncertainty can be decreased by the use of standard identification algorithms which find updated estimates of the plant parameters. The efficiency of the identification process -- and therefore the reduction of the uncertainty -- is critically dependent on the nature of the driving input signals applied to the plant. At the same time, of course, the input signals -- as in any control problem -- will have to be selected so as to result in a desired behavior of the state trayectory. In the combined identification and control problem, therefore, the input signals will have a dual purpose: affecting the information gathering process on the plant parameters and affecting the behavior of the state trayectory. It is not difficult to observe that these dual objectives are often conflicting. Consider, for instance, the class of systems driven by white noise and whose output is observed through additive white noise: on the one hand it is desired the output (or the state) to follow a specified trayectory (which may be zero) and on the other hand, the output should be "high" in order to improve the signal to noise ratio and obtain maximum information about the unknown parameters.

Mr. A. Lopez-Toledo and Professor M. Athans have been working to gain further insight into this problem for a particular class of linear systems driven by white Gaussian noise and with perfect observations of the system output.

The identification aspects of the problem have been analyzed. For real-time identification of random parameters, we have determined how to make use of the a-priori knowledge about the parameters and initial conditions to obtain optimal driving inputs. The value of a given input signal with regard to the identification objective is measured in terms of the amount of information about the unknown parameters contained in the corresponding output signal. This information can be expressed as the expected value of a quadratic function of the output gradient. The input sequence that optimizes this criterion has been shown to be the difference between the eigenvector corresponding to the maximum eigenvalue of certain matrix depending on the a-priori distribution and a vector depending on the initial conditions. This problem bears some resemblance with a class of problems in communication theory and statistical decision theory. These relationships are currently being investigated and future research will involve the use of the estimates of the parameters and the accuracy in the estimation for control purposes. The results will be extended to multiple input-multiple output linear systems.

A similar problem is also being investigated by Mr. A. Sarris and Professor M. Athans. We considered system models characterized by their structure and parameters within the structure. We assume that the structure remains fixed, i.e., the equations that describe the inherent dynamics of the system do not change. However, it is very likely that some of the parameters in the equations might change with time. As an example,aging parts of a machine or an airplane could give rise to the behavior described above.

The research objective is twofold. At first detection of the change must take place and secondly a corrective control must be applied to optimize the changing plant. Before a method for identifying the stochastically changing parameter is found we must try to pose a model according to which the parameters change. This model should be general enough to describe a large number of possible changes. One which we have examined assumes the first time differences of the parameters to be stationary. This description, with the assumption that the measurements of the system state are exact, renders the problem solvable via Kalman estimation techniques. The control in certain cases can be found relatively easily because separation holds, namely the controlling procedure depends only on the conditional means and variances of the parameters.

There are two main directions for our future research. The first one has to do with the model that is used to describe the parameter changes. The one we used, and which almost all other researchers in this area have used, does not respond very fast to sudden parameter variations. We have in mind another model based on second time differences of the parameters, that has been found in other contexts (e.g. statistics), to respond much faster to time changes. We want to devise and implement algorithms for this model and test its response speed.

The second direction of our research is related to the control question. In the previously mentioned context separation holds only if the criterion is to optimize performance for a single period ahead. At first we would like to examine if the new model for parameter change that we have in mind gives rise to separation for more realistic (or equivalently more complicated) cost criteria. We would like to investigate the extent to which separation holds for various identification techniques. Finally it is our intuitive feeling that the optimal control should not depend very much on the estimation technique, so we would like to investigate control laws that are more or less invariant under different parameter identification schemes.

Adaptive Control

Mr. Dieter Willner and Professor M. Athans are continuing their research on adaptive control and its applications with special emphasis on the problem of control for partially unknown systems.

A system is considered to be partially unknown if it can be described by one of N systems (which have known dynamics), e.g., it could be system $i = 1, \ldots, N$. Each of these systems is represented by a linear stochastic difference equation with stochastic measurements:

$$x(k+1) = A_i(k)\, x(k) + B_i(k)\, u(k) + \xi_i(k)$$

$$z(k) = C_i(k)\, x(k) + \theta_i(k)$$

for $i = 1, \ldots, N$. The problem is to find the optimal control $u^*(t)$ which minimizes a standard quadratic criterion of the form

$$J(u) = E\left\{\sum_{i=t}^{T} x_i(i)'\, Qx_i(i) + u'(i)Ru(i)\right\}$$

Note that for $N \to \infty$ we have the control problem with continuously unknown parameters. Most problems can be formulated such that $N \leq N_o \leq \infty$ with sufficient accuracy since some knowledge of the parameter variations is usually available and parameter quantization can be used. The results are presented in two cases:

Case I: Exact Measurements $z(i) = x(k)$

Case II: Stochastic Measurements $z(k) = C_i(k)\, X(k) + \theta_i(k)$

The controllers are very similar in both cases, yet Case I is much easier to derive and to compare to currently available results.

Optimal Solution: It has been shown that the optimal control u* can be computed in an analytically closed form only for $t = T-1$. For $t < T-1$ the expressions are nonlinear and no closed form solutions are feasible. A solution procedure is outlined for arbitrary t to compute (near) optimal controls of sufficient computer memory and time is available.

Suboptimal Solutions: Because of the impracticality of the optimal solution, two philosophies for generating suboptimal solutions were considered:

Philosophy 1: (Pick a reasonable cost criterion and compute the resulting control) This leads to formulating (and/or bounding) a number of open loop cost criteria to compute the true cost or its bound and the corresponding control. Several algorithms were considered and their performance was evaluated from both theoretical and simulation studies.

Philosophy 2: Start with a desirable control (i.e., one which converges to the optimal control as T increases, and is easy to implement). Such a control is $u(t) = \sum_{i=1}^{N} p_i(t)\, u^*_i(t)$, where $p_i(t)$ = probability of observing system i at time t, and $u_i^*(t)$ = optimal control for system i. For a simple example it has been shown that this control is a first order approximation to the optimal control. An attempt has been made (using both analysis and simulation results) to show that this is true in general.

The above general results will be fully reported in the forthcoming Ph.D. thesis of Willner [1]. However, some of the results based upon philosophy #2 above were obtained also for continuous time problems, and we feel that they are of extreme importance in practical applications. [2]

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[1] D. Willner, "Observation and Control of Partially Unknown Stochastic Systems", Ph.D. Thesis, E.E. Dept., M.I.T., May, 1973 (in preparation).

[2] M. Athans and D. Willner, "A Practical Scheme for Adaptive Aircraft Flight Control Systems", Proc. Symposium on Parameter Estimators Techniques and Applications in Aircraft Flight Testing, NASA Flight Research Center, Edwards AFB, California, April, 1973.

A PRACTICAL SCHEME FOR ADAPTIVE AIRCRAFT FLIGHT CONTROL SYSTEMS

This research was conducted by Prof. M. Athans and D. Willner and was motivated by practical considerations in the design of stability augmentation systems for high performance aircraft and STOL aircraft whose dynamic characteristics are changing. Such changes in the aerodynamic derivatives have been the primary motivation behind numerous techniques for parameter identification of aerodynamic parameters. Although such procedures are useful for off-line parameter estimation, it is not quite clear how they should be used for on-line closed loop automatic control.

There seems to be general agreement that on-line computational requirements of least-squares, maximum likelihood, and extended Kalman filter methods are far too severe for on-board implementation. If one also considers the additional computational requirements for the determination of appropriate autopilot gains (say using the linear-quadratic-gaussian methods of modern control theory), then there is little doubt that simultaneous identification and control is not a feasible practical method for aircraft control system design, at least in the near future.

This research was directed toward both analytical and simulation results associated with a method for approximate identification of aircraft characteristics and subsequent generation for control signals that appears practical for on-board implementation.

The method presupposes that off-line calculation of aerodynamic derivatives has been made at N distinctly different flight conditions. For each flight condition one has a linear time-invariant state variable model which represents approximately the aircraft dynamics.

The aircraft changing dynamic characteristics may not however coincide always with one of the N dynamical models. Hence, a switching type autopilot may not be sufficiently good, especially if the total number N of models available is small.

The identification and control technique that was investigated, is a type of smooth transitional autopilot, in contradistinction to a switching one. It consists of:

(a) a bank of N linear time-invariant steady state Kalman-Bucy continuous time filters, each matched to the N available dynamical models, followed by:

(b) linear-quadratic time invariant optimal controllers designed on the basis of the N models available.

It turns out that by nonlinear operations upon the residuals (innovation processes) of each Kalman Bucy filter one can calculate on-line a time varying probability that the actual aircraft dynamics corresponds to each one of the N models (this involves hypothesis testing). More important, from a practical point of view, the calculation of these probabilities can be accomplished using analog and time-invariant implementation of a set of first order nonlinear differential equations.

These time varying probability signals are then used to weigh on-line the control signals generated by each linear time invariant dynamic compensator (the combination of each Kalman filter and linear quadratic controller) to produce the actual values of the control variables applied to the aircraft.

The practicality of the above "analog" scheme, hinges upon the fact that integrated circuits can be used to construct high reliability, low volume, and low weight constant coefficient dynamical systems. Thus, no actual digital computer hardware is needed.

In addition to the theoretical results we have simulation results based on numerical values for STOL aircraft longitudinal dynamics (seven state variables).

Details on both the theory and the STOL aircraft simulations can be found in the paper by Athans and Willner. [1].

[1] M. Athans and D. Willner "A Practical Scheme for Adaptive Aircraft Flight Control Systems" Proc. Symposium on Parameter Estimation Techniques and Applications in Aircraft Flight Testing, NASA Flight Research Center, Edwards AFB, Cal., April 1973.

3. BUDGET STATUS

As of April 30, 1973, approximately 70% of the grant funds had been expended in approximately 66% of the current grant period. No effect has been given to step funding in these figures.

4. PUBLICATIONS

The publications below describe research reported in this or previous progress reports and supported all or in part by NASA grant NGL-22-009-124.

Copies have been transmitted to the NASA Technical Information Center, and to Mr. Brian F. Doolin at NASA/Ames Research Center.

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1. L. K. Platzman, "Computation of Optimal Output Feedback Compensators for Linear Time Invariant Systems" Report ESL-R-488, 1972.

2. R. Ku and M. Athans, "Open Loop Feedback Optimal Adaptive Stochastic Control of Linear Systems" Accepted for 3rd IFAC Symposium on Identification and System Parameter Estimation, Netherlands, June 1973.

3. H. P. Geering and M. Athans, "The Infimum Principle" Report ESL-P-492, 1973.

4. M. Athans and D. Willner, "A Practical Scheme for Adaptive Aircraft Flight Control Systems" Proc. Symposium on Parameter Estimation Techniques and Applications in Aircraft Flight Testing, NASA Flight Research Center, Edwards AFB, April 1973.

5. M. Athans and H. P. Geering, "Necessary and Sufficient Conditions for Differentiable Nonscalar Valued Functions to Attain Extrema" IEEE Trans. Auto. Control, Vol. AC-18, April 1973, pp. 132-139.

6. K. Glover and J. C. Willems, "Parameter Identifiability of Linear Dynamical Systems" Proc. Symposium on Parameter Estimation Techniques and Applications in Aircraft Flight Testing, NASA Flight Research Center, Edwards AFB, April 1973.

7. K. Glover and J. C. Willems, "On the Identifiability of Linear Dynamical Systems" Accepted for 3rd IFAC Symposium on Identification and System Parameter Estimation, Netherlands, June 1973.

8. T. L. Johnson and M. Athans, "A Minimum Principle for Smooth First Order Distributed Systems" submitted to IEEE Trans. on Auto. Control.

9. N. Sandell and M. Athans, "On Type-L Multivariable Linear Systems" Automatica, Vol. 9, January 1973, pp. 131-136.