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GLOBAL CORRELATIONS

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Introduction

Recent interest in the geophysical exploration of the oceans and the unsurveyed parts of the land areas, and the recent advances in space techniques which make it possible to collect geophysical data by using the artificial earth satellites in multiple ways, have led to vast amounts of geophysical and geodetic data which are global in character. Since various forms of geophysical information are all indicators of the physical properties of the planet which we persue to study in any geophysical investigation, it is frequently useful to study the interrelationships of these various geophysical parameters in an attempt to determine whether they are generated by the same mechanism and if not, to assess the association of the various generating mechanisms. In this paper we summarize the formulas for studying such correlations.

Theory

Let $F(\phi, \lambda)$ and $f(\phi, \lambda)$ be the scalar potential functions representing the appropriate planetary force fields. Then, their cross-correlation function, normalized to the product of the root mean square values of the two functions (see, for examples, Lee 1960 or Khan 1971a) is

$$R(F, f) = \frac{\int_S F(\phi, \lambda)\, f(\phi, \lambda)\, dS}{\left[\int_S F^2(\phi, \lambda)\, dS \int_S f^2(\phi, \lambda)\, dS\right]^{1/2}} \tag{1}$$

Let $F(\phi, \lambda)$ and $f(\phi, \lambda)$ be rewritten (see, for example, Hobson 1931) as

$$\begin{matrix} F(\phi, \lambda) \\ \\ f(\phi, \lambda) \end{matrix} = \sum_n \sum_m \begin{bmatrix} A_{nm} & & B_{nm} & \\ & \cos m\lambda + & & \sin m\lambda \\ a_{nm} & & b_{nm} & \end{bmatrix} P_{nm}(\sin \phi) \tag{2}$$

where

$P_{nm}(\sin\phi)$ = associate Legendre function

A_{nm}, B_{nm} constant coefficients in the spherical harmonic

a_{nm}, b_{nm} = representation of functions $F(\phi, \lambda)$, $f(\phi, \lambda)$

Then, using the orthogonal properties of the spherical harmonic functions $P_{nm}(\sin\phi)\cos m\lambda$ and $P_{nm}(\sin\phi)\sin m\lambda$, Equation (1) can be rewritten as

$$R(F, f) = \frac{\sum_n \sum_m \sigma_{nm}(F, f)}{\left[\sum_n \sum_m \sigma_{nm}(F) \sum_n \sum_m \sigma_{nm}(f)\right]^{1/2}} \tag{3}$$

where

$$\sigma_{nm}(F, f) = A_{nm} a_{nm} + B_{nm} b_{nm}$$

$$\sigma_{nm}(F) = A_{nm}^2 + B_{nm}^2$$

$$\sigma_{nm}(f) = a_{nm}^2 + b_{nm}^2$$

The correlation between $F(\phi, \lambda)$ and $f(\phi, \lambda)$ as a function of frequency is generally studied by the degree correlation function R_n which is obtained from Equation (3) by dropping the summation $\sum_n$. See Khan (1971a, 1971b)

Rotated Correlations

Sometimes a planetary force field is shifting its position in a planet-fixed reference system and it is useful to bring it in phase with another stationary force field of the same planet before the correlation of the two can be usefully studied (Hide & Malin, 1970; Khan 1971c, Khan 1971d). This can be done by substituting for the coefficients a_{nm}, b_{nm} the coefficients α_{nm}, β_{nm} in Equation (3) such that

$$\alpha = a\, r(m\bar{\lambda})$$

where

$$\alpha = (\alpha_{nm} \beta_{nm}) \tag{4}$$

$$a = (a_{nm} b_{nm})$$

and $r(m\lambda)$ is the usual rotation matrix in which $\bar{\lambda}$ indicates an appropriate rotation along the latitude circle. The resulting formula then becomes

$$R(F, f) = \frac{\sum_n \sum_m \bar{\sigma}_{nm}(F, f)}{\left[\sum_n \sum_m \sigma_{nm}(F) \sum_n \sum_m \sigma_{nm}(f)\right]^{1/2}} \tag{5}$$

where

$$\bar{\sigma}_{nm}(F, f) = A_{nm}\alpha_{nm} + B_{nm}\beta_{nm}$$

and other symbols are the same as in Equation (3).

The statistical significance of the rotated correlations must be scrutinized rather carefully with the help of the tests available in the theory of circular distributions (Khan 1971d) as sometimes rotation may cause high but spurious correlations because of the structural properties of the rotated correlation function.

Correlations of Identical Frequencies

The degree correlation coefficient R_n compares the amplitudes of the two functions contained in each degree n and thus, it does not correlate essentially similar wave patterns, since, for each n, it contains $\sum_{m=0}^{n}$. Khan (1971b) has developed formulas for correlating the identical frequencies of functions $F(\phi, \lambda)$ and $f(\phi, \lambda)$ by means of a correlation parameter called an 'identical frequency correlation function' which must be split into longitudinal and latitudinal correlations as below:

Longitudinal Correlation

The identical frequency correlation function R_m^λ, compares the frequencies along the parallels of latitude. It is given by

$$R_m^\lambda = \frac{\sum_{n=m} \sigma_{nm}(F, f)}{\left[\sum_{n=m} \sigma_{nm}(F) \sum_{n=m} \sigma_{nm}(f)\right]^{1/2}} \tag{6}$$

The usual correlation coefficient R can be obtained from the above by introducing the summation $\sum\limits_{m}$ to the left of each summation $\sum\limits_{n=m}$ in the above equation.

Latitudinal Correlation

The correlation coefficient R_p^{ϕ} ($p = n - m$) correlates frequencies along the meridian circles. It is given by

$$R_p^{\phi} = \frac{\sum\limits_{n=p} \sigma_{n(n-p)}(F, f)}{\left[\sum\limits_{n=p} \sigma_{n(n-p)}(F) \sum\limits_{n=p} \sigma_{n(n-p)}(f)\right]^{1/2}} \tag{7}$$

The correlation coefficient R can again be obtained from the above by introducing the summation $\sum\limits_{p}$ to the left of each summation $\sum\limits_{n=p}$ in the above equation.

Rotated Identical Frequency Correlations

The rotated counterparts of Equations (6) and (7) can be obtained with the help of the matrix conversions given in Equation (4). These are

$$R_m^{\lambda} = \frac{\sum\limits_{n=m} \overline{\sigma}_{nm}(F, f)}{\left[\sum\limits_{n=m} \sigma_{nm}(F) \sum\limits_{n=m} \sigma_{nm}(f)\right]^{1/2}} \tag{8}$$

and

$$R_p^{\phi} = \frac{\sum\limits_{n=p} \overline{\sigma}_{n(n-p)}(F, f)}{\left[\sum\limits_{n=p} \sigma_{n(n-p)}(F) \sum\limits_{n=p} \sigma_{n(n-p)}(f)\right]^{1/2}} \tag{9}$$

where

$$\overline{\sigma}_{n(n-p)}(F, f) = \sigma_{n(n-p)}(F, f) \cos(n - p)\,\overline{\lambda} - (B_{n(n-p)} a_{n(n-p)} - A_{n(n-p)} b_{n(n-p)})$$
$$\sin(n - p)\,\overline{\lambda}$$

with corresponding formulas for $\overline{\sigma}_{nm}(F, f)$.

As in the case of Equation (5), the statistical significance of the quantities obtained from equations (8) and (9) must be carefully investigated for the same reasons stated therein.

Results

The results given here are based on Equations (3) and (5) only. The identical frequency correlations are not reported here as they are still under investigation.

Global Topography and Gravity

The degree correlation function R_n between typical models of global topography and free air and isostatic gravity fields is listed in Table 1. These correlations are computed on the basis of Lee and Kaula's (1967) topographic model, Smithsonian standard earth II gravity model and Khan's (1972) isostatic reduction potential model, but the correlations remain substantially the same irrespective of which global gravity and topography models are used.

Table 1
Correlation of Gravity* and Topography**

Degree n	Free air gravity	Isostatic gravity
2	-0.53	-0.59
3	0.03	-0.08
4	0.49	0.31
5	-0.58	-0.72
6	0.58	0.39
7	0.31	0.13
8	0.27	-0.12
9	0.57	0.30
10	0.29	-0.02
11	0.02	-0.25
12	-0.17	-0.56
13	0.33	-0.02
14	0.48	0.01
15	0.26	-0.05
16	0.43	0.18

* Global topography - Lee and Kaula (1967)
** Free air gravity model - Smithsonian Standard Earth II. Isostatic gravity model Khan (1972)

Geogravity and Geomagnetic Fields

Khan and Woollard (1970) investigated the correlations between the geogravity field and the geomagnetic field and its secular variations, using a wide spectrum of the mathematical models of geogravity potential and geomagnetic potential and its secular variations which are documented in Khan (1970). Typical values for these correlations which remain more or less the same for all the models investigated are given in Table 2. Hide and Malin (1970) and Khan (1970; 1971c) have also investigated correlations between these two fields after introducing a phase lag between the two. They find that for phase lag angles lying between 160° and 170°, the geogravity and geomagnetic fields have a high correlation of between 0.8 and 0.9, particularly in the lower range of frequencies. The correlations remain substantially the same for a large number of geogravity and geomagnetic models investigated by them. A typical representation of this correlation is shown in Figure 1. In this diagram, the correlation coefficient R^N is equivalent to R , the superscript N indicating the harmonic degree to which the functions $F(\phi, \lambda)$ and $f(\phi, \lambda)$ are considered in computing the correlations. A much more detailed analysis and discussion of these correlations, including those of the geogravity field with the drifting and non-drifting parts of the geomagnetic field and its secular variations, is given in Khan (1971c).

Table 2
Correlation of Geogravity and Geomagnetic Fields

Degree n	Geomagnetic Field*	Geomagnetic Secular Variations*
2	-0.86	-0.68
3	0.40	0.77
4	0.63	0.49
5	-0.15	-0.12
6	0.19	0.26
7	0.27	0.30
8	-0.12	0.33
9	-0.07	
10	0.27	
11	0.15	
12	0.33	

* 1. Zonal parts of gravity and magnetic potenials excluded
2. Geogravity model - Smithsonian Standard Earth II
Geomagnetic model - Hurwitz (1970)

The geophysical interpretation of these results is not regarded within the purview of the National Geodetic Satellite Program Handbook.

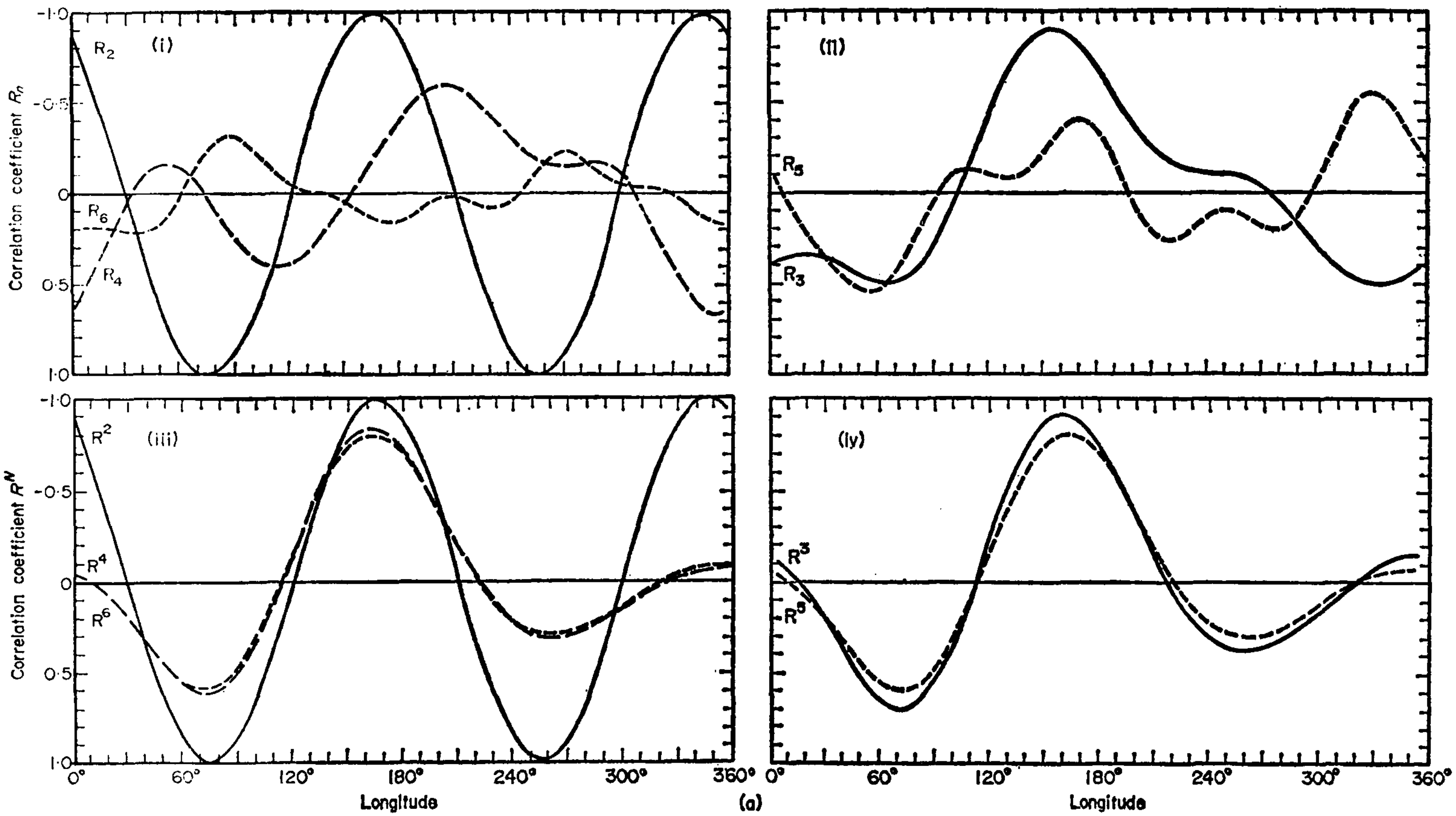

Figure 1(a). Degree correlation R_n, and correlation coefficient R^N, as functions of longitude, for the geogravity and geomagnetic fields. (i) Degree correlation R_n, $n = 2, 4, 6$; (ii) Degree correlation R_n, $n = 3, 5$; (iii) Correlation coefficient R^N, $N = 2, 4, 6$; (iv) Correlation coefficient R^N, $N = 3, 5$.

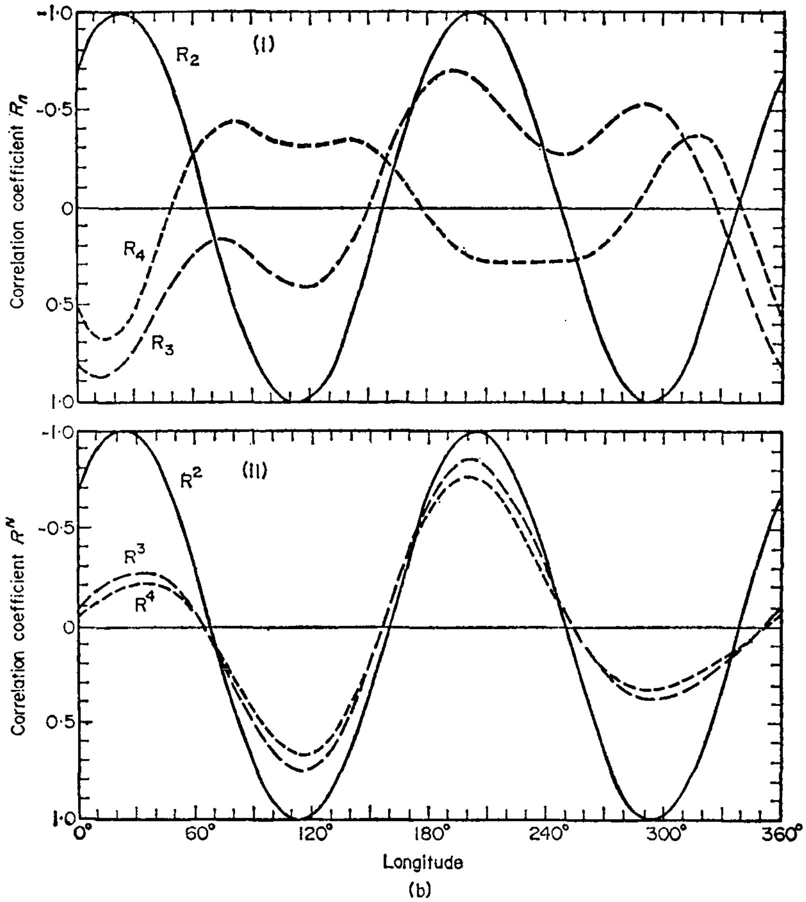

Figure 1(b). Degree correlation R_n, and correlation coefficient R^N, as functions of longitude, for the geogravity and geomagnetic secular variation fields. (i) Degree correlation R_n, n = 2, 3, 4; (ii) Correlation coefficient R^N, N =2, 3, 4.

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