Wave-mean flow interactions in the

upper atmosphere

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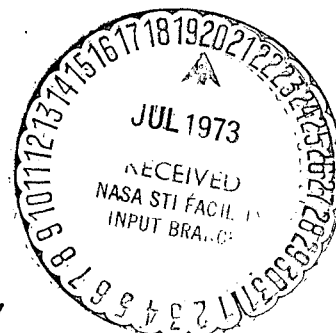
Abstract.

The nature of internal gravity waves is described with special emphasis on their ability to transport energy and momentum. The conditions under which these fluxes interact with the mean state of the atmosphere are described and the results are applied to various problems of the upper atmosphere, including the quasi-biennial oscillation, the heat budget of the thermosphere, the general circulation of the mesosphere, turbulence in the mesosphere, and the "4-day" circulation of the Venusian atmosphere.

Wave-mean flow interactions in the upper atmosphere

1. Introduction.

The question of wave-mean flow interactions assumes major importance in the upper atmosphere where wave amplitudes tend to be much larger than those found in the troposphere. Amplitudes in horizontal velocity of 10-100 m/s are common. The reason for the magnitude of waves in the upper atmosphere is primarily that wave amplitudes grow as $e^{x/2}$, where x = height in scale heights, and this factor can account for a factor of 10^3 between the ground and 100 km.

I shall in section 2 review the relevant theorems and concepts involved in the interaction of internal gravity waves with the mean state of the atmosphere[1]. This section will undoubtedly repeat matters dealt

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[1] Though I shall not emphasize the point, it should be remarked that the interaction of internal Rossby waves with the mean flow in many ways parallels what I shall describe for gravity waves (Dickinson, 1969; Matsuno, 1971).

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with in other lectures, but will, at the least set the terminology for the remainder of this lecture. The remaining sections (3-6) will deal with explicit applications to the quasibiennial oscillation of the tropical

stratosphere, the heating of the thermosphere, the temperature structure of the mesopause, and the circulation of the ozonosphere. Peripherally, I will also discuss the Venusian 4-day circulation. In the concluding section I will discuss the validity of the above applications. No attempt will be made at completeness or rigor. My intention, instead, is merely to suggest the wide range of possibilities.

2. Theoretical review.

The theory of internal gravity waves is amply described in the literature (Hines, 1961; Bretherton, 1971; Lindzen, 1971). Internal gravity waves are atmospheric oscillations whose restoring force arises from the buoyancy (implying the presence of gravity) of displaced fluid elements in a stably stratified atmosphere. Such waves can propagate vertically if their doppler shifted frequency is less than the Brunt-Vaisala frequency, ω_B, given by

$$\omega_B^2 = \frac{g}{T_o} \left(\frac{\partial T_o}{\partial z} + \frac{g}{c_p} \right) \tag{1}$$

where   g = acceleration of gravity

T_o = undisturbed temperature

c_p = heat capacity at constant pressure

and   z = altitude,

and greater than the Coriolis parameter, f, where

$$f = 2\Omega \sin\varphi$$

Ω = rotation rate of earth

φ = latitude   (2

In an isothermal atmosphere without mean motion internal gravity waves approximately satisfy the following dispersion relation

$$n^2 = \frac{\omega_B^2 \left(1 - \left(\frac{\omega}{\omega_B}\right)^2\right)}{\omega^2 - f^2} k^2 - \frac{1}{4H^2} \qquad (3$$

where   n = vertical wavenumber

K = horizontal wavenumber

ω = wave frequency

H = atmospheric scale height

and, for plane waves, the various oscillatory fields (u, the horizontal velocity, w, the vertical velocity, δT, the temperature perturbation, p'/p_o, the fractional pressure perturbation, and ρ'/ρ_o, the fractional density perturbation) behave as follows in the absence of damping:

$$e^{i(\kappa x + nz + \omega t)} e^{\frac{z}{2H}} \qquad (4$$

where   x = horizontal distance

t = time

The above relations assume f is constant. What happens when its variation with latitude is accounted for is dealt with in Lindzen (1967,

1971a, 1972).

Our primary concern here is what happens to the above waves in inhomogeneous atmospheres where, for example, there are mean flows. This matter has been studied in some detail by Eliassen and Palm (1961), Hines and Reddy (1967), Booker and Bretherton (1967), and Benny and Bergeron (1969).

Eliassen and Palm (1961) developed two important theorems for linearized plane, internal gravity waves in the absence of rotation. First, they crudely associated the vertical flux of energy with the quantity $\overline{p'w'}$ (where the overbar refers to the average over one cycle of the wave) and the vertical flux of horizontal momentum with $\rho_o \overline{u'w'}$. They then found

i) $\qquad \overline{p'w'} = -\rho_o (U-c) \overline{u'w'}$ $\qquad\qquad$ (5

where $\qquad$ U = mean flow

$\qquad\quad \rho_o$ = basic density

$\qquad\quad c = -\omega/\kappa$ = horizontal phase speed

and

ii) $\qquad \dfrac{d}{dz} \left(\rho_o \overline{u'w'} \right) = 0$ $\qquad\qquad$ (6

(i) implies that an upward moving wave ($\overline{p'w'} > 0$) carries easterly momentum if its phase speed is easterly relative to the mean flow and westerly momentum if its phase speed is westerly relative to the mean flow. (ii) implies that none of the mean momentum carried by the

wave is deposited in the mean flow.[1]

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[1]Jones (1967) and Bretherton (1969) have extended (i) and (ii) to a plane

rotating atmosphere. The only modification required is that the vertical

flux of horizontal momentum be identified with $\rho_0 (\overline{u'w'} - \Omega \overline{\eta'w'})$

(where $\eta' = \dfrac{i v'}{\hat{\omega}}$ = meridional displacement; $\hat{\omega}$ = doppler

shifted frequency) rather than $\rho_0 \overline{u'w'}$. Lindzen (1972) showed that for

normal modes on an equatorial β-plane bars should be replaced by $\langle \overline{\quad} \rangle$

where $\langle \quad \rangle$ indicate an integration over all latitudes.

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The validity of both (i) and (ii) depends on the absence of damping, while

the validity of (ii) depends additionally on $U \neq c$ and on the local absence

of thermal excitation for the wave (as might be found in atmospheric tides

or in flows over urban "heat islands"). All the above conditions are,

at least sometimes, violated in the real atmosphere. Before discussing

what then happens, a criticism due to Hines and Reddy (1967) ought to

be mentioned. They pointed out that (ii) implies that the wave is really

not interacting with mean flow while (i) implies that $\dfrac{d}{dz} \overline{p'w'} \neq 0$ when

$\dfrac{dU}{dz} \neq 0$. The resolution of this paradox consisted in noting that $\overline{p'w'}$

is not the total flux of energy, F_E. Another term representing the

advection by wave fields of the kinetic energy of the mean flow is needed

leading to

$$F_E = \overline{p'w'} - \rho_0 \overline{u'w'} \, U \qquad\qquad (7$$

Now $\frac{d F_E}{d z} = 0$. However, now F_E is arbitrary to the extent of a Galilean transformation and it is no longer possible to associate F_E with the direction of wave propagation. I shall return to this point later. First I wish to describe what happens when the conditions for the validity of (ii) are violated.

What happens when a critical level is reached (i.e., when $U = c$) has been studied by Booker and Bretherton (1967) and in a more implicit manner by Benney and Bergeron (1969).[1] The basic problem is that

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[1] It is probably premature to suggest that only people whose surnames begin with "B" work in this area.

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within the context of linearized, inviscid theory for plane waves, the behavior at a critical level is not completely determined. There are two approaches to this problem. First we might note that as a wave approaches a critical level its vertical group velocity approaches zero while its vertical wavelength also approaches zero. Thus, if any dissipation is present, it will become important sufficiently near the critical level, and this remains true in the limit of vanishing dissipation. It is this limit which Booker and Bretherton investigated.[2] They found that the

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[2] Hazel (1967) more explicitly studied the effect of finite viscosity.

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momentum flux, $F_M = \rho_o \overline{u' w'}$, suffered a discontinuity at the critical level leading essentially to the absorption of the wave at the critical level

(assuming the Richardson number $\gtrsim 1$). Such absorption would lead to an acceleration of the mean flow in a manner described by Lindzen and Holton (1968) and Jones and Houghton (   ). As an alternative to the above approach one might note that in the absence of damping, the amplitudes of oscillations in horizontal velocity become infinite as a wave approaches a critical level. Thus, for any initial amplitude there will be some neighborhood of the critical level where nonlinear effects should become important, and this remains true in the limit of vanishing amplitude. This limit is what Benney and Bergeron investigated, and they found that $\rho_o \overline{u'w'}$ must be continuous across a critical level. Coupled with the Eliassen and Palm theorems this would imply total reflection at a critical level. What really happens must depend on whether dissipative damping occurs before amplitudes have grown to the point where nonlinear effects become important in which case Booker and Bretherton's results should be applicable and this appears to be the case for the upper atmosphere.

The next thing I would like to describe are some new results I have obtained for the situation when gravity waves are thermally forced as is the case for tides in the atmosphere. Let us consider a plane, non rotating two-dimensional situation where the waves are forced by a heat source of the form

$$J(z) \, e^{i(\kappa x - \kappa c t)}$$

where J is the heating per unit time per unit mass. Within the region where $J \neq 0$, (6) is no longer true. Instead

$$\frac{d}{dz} \, \rho_0 \, \overline{u'w'} = -\frac{\kappa \rho_0}{(U-c)} \left\{ \frac{1}{H} \, \overline{\zeta J} + \frac{1}{P_0} \, \overline{P'J} \right\} \tag{8}$$

where $\quad \zeta = \frac{W'}{i\omega}$, $\kappa = \frac{\gamma-1}{\gamma}$, $\gamma = c_P/c_v$.

It is difficult to say much about the right hand side of (8) except that averaged over the entire heating region, it must represent a net easterly acceleration of the mean flow if $c - U > 0$ and a net westerly acceleration if $c - U < 0$. The reason for this is that outside the region of excitation (5) and (6) are still true and the momentum flux must come from someplace. Since thermal forcing is not associated with any momentum, the momentum must come from the mean flow in the forcing region. However, this applies to the average acceleration. Locally (8) may lead to either westerly or easterly acceleration. Stephen Fels, at the University of Chicago, has made explicit calculations of this effect for atmospheric tides on earth and on Venus and finds quantitatively significant effects which I will discuss later.

I turn now to the question of what happens to internal waves when finite but small damping is present. If the damping is small then the inviscid, adiabatic solutions are a reasonable first approximation with (4) modified by a factor

$$e^{-\int \mu \, dz} \tag{9}$$

This in turn implies that

$$F_M = \rho_0 \overline{u'w'} \propto e^{-2\int\limits_{0}^{z} \mu \, dz}$$

(10

and

$$F_E = \overline{p'w'} - \rho_0 \overline{u'w'} \, U \propto e^{-2\int\limits_{0}^{z} \mu \, dz}$$

(11

That is to say wave damping leads to the deposition by the waves of

energy and zonal momentum in the mean flow. In the tropical stratosphere

there are large internal waves with periods on the order of a week; there

is also damping due to infrared cooling with a time scale of about a week.

The resulting depositions of momentum appear to account for the quasi-

biennial oscillation in zonal wind. This will be described further in

section 3.

The presence of damping which perturbs undamped waves only

slightly provides a framework for reconsidering the arbitrariness of the

energy flux in Hines and Reddy's formulation -- at least in a simple

situation where $\dfrac{dU}{dz} = 0.$ Presumably the small convergence of

F_M will accelerate the mean flow, while the convergence of F_E will

both heat and increase the kinetic energy of the mean flow. We then

have (ignoring time variations in ρ_0 -- which is all right for a Boussinesq

fluid at least)

$$\rho_0 \frac{\partial U}{\partial t} = - \frac{d}{dz} \rho_0 \overline{u'w'} \qquad (12$$

and

$$\rho_0 \frac{\partial}{\partial t} \left(\tfrac{1}{2} U^2 + Q \right) = - \frac{d}{dz} \left(\overline{p'w'} - \rho_0 \overline{u'w'} \, U \right) \qquad (13$$

where Q is a measure of the fluid's heat. Multiplying (12) by U we get

$$\rho_0 \frac{\partial}{\partial t} \left(\tfrac{1}{2} U^2 \right) = - U \frac{d}{dz} \rho_0 \overline{u'w'} \qquad (14$$

and subtracting (14) from (13) we get

$$\rho_0 \frac{\partial}{\partial t} Q = - \frac{d}{dz} \overline{p'w'} \qquad (15$$

which suggests that in the presence of damping the convergence of $\overline{p'w'}$ goes into heat while the convergence of $-\rho_0 \overline{u'w'} \, U$ goes into kinetic energy. Since we do not wish the absorption of a wave to cool a fluid, it appears that the direction of $\overline{p'w'}$ should be associated with the direction of wave propagation -- as was done by Eliassen and Palm.

As we have seen, internal waves interact with the mean flow in

regions of thermal excitation and whenever the waves are being

damped -- the most extreme example being at a critical level. Wave

damping may also, in some instances, be associated with the unstable

breakdown of an internal gravity wave (assuming this leads to motions

which are dissipated locally). Such a possibility was noted by Hines

(1963) and Lindzen (1967) and more explicitly described by Hodges (1967)

and Lindzen (1968). The possibility is based on the following line of

argument: Within a Boussinesq fluid a linear, plane internal gravity

wave is also an exact solution of the nonlinear equations (because the

wave's flow field, u', is orthogonal to the wave vector). Thus any

wave amplitude is a permissible solution. Now, for sufficiently large

amplitudes potential temperature surfaces can become so deformed

as to produce local regions of static instability. Even if this doesn't

occur, the wave flow may produce shears so large that the local

Richardson number, if not negative, will at least be less than 0.25. In

such cases we would expect unstable breakdown of the wave provided

that the time scale for the development of the instability is less than

the wave period/2π. The requisite amplitudes for such instability

develop in the upper atmosphere because of the $e^{x/2}$ growth cited

at the beginning of this lecture. Some conjectural work on the con-

sequences of such instabilities has been done by Lindzen (1968) and

Orlanski and Bryan (1971) among others.

The above completes my personal overview of the relevant

mechanisms of wave-mean flow interaction of importance in the upper

atmosphere. I now turn to specific examples.

3. Quasi-biennial cycle.

In figure 1 I show a time height cross-section of averaged (monthly mean) zonal wind over Canton Island (   ); a similar picture would be obtained at any other near-equatorial station. What we see is a system of winds with a period of about 26 mos, and an amplitude of about 20 m/s between 20 and 30 km moving down at the rate of about 1 km/mo. This wind system is essentially confined to altitudes between 18 km and 30 km and to between $\pm 10^{\circ}$ latitude. Above 30 km the tropical zonal winds are dominated by a semiannual oscillation whose origin I will speculate on later. It now appears that the quasibiennial cycle results from an interaction of internal waves generated in the upper troposphere with the semiannual wind oscillation aloft. The basic facts associated with this analysis are the following:

i) For internal gravity waves to propagate vertically ω^2 must be greater than f^2. However f^2 decreases toward zero as one approaches the equator. Thus, disturbances with meteorological time scales (O (1 week)) which are trapped at middle latitudes, can in the tropics propagate vertically as modified internal gravity waves (Lindzen 1967). Two such waves have been clearly identified in the tropical stratosphere: an easterly wave (Maruyama, 1967; Lindzen and Matsuno, 1968) and a westerly wave (Kousky and Wallace, 1971   and Holton and Lindzen, 1968

), both with phase speeds of O(20 m/s).

ii) Lindzen and Holton (1968) hypothesized a continuous distribution of waves of the above type with phase speeds between ±20 m/s whose source region was assumed to be at about 17 km. They were able to parameterize the critical level absorption of such a distribution of waves and show that the resulting momentum exchanges would lead to a quasi-biennial oscillation. The main difficulty of this mechanism *is* that only isolated easterly and westerly waves are observed -- not a continuous distribution in phase speed.

iii) Lindzen (1971) noted that existing infrared cooling significantly damps observed equatorial waves leading to a continuous exchange of momentum with the mean flow -- without the need for a continuous distribution of phase speeds. Holton and Lindzen (1972) parameterized this mechanism and were again able to simulate a quasi-biennial oscillation. The simulation is shown in figure 2. Basically we began with both an easterly and a westerly wave generated at 18 km. The wave for which the doppler shifted frequency is least is absorbed more rapidly. If this wave is the westerly wave, its absorption will cause a regime of westerly zonal winds to descend. However, the absorption of the westerly wave will leave greater altitudes unaffected by the westerly wave and the easterly wave will have an opportunity to accelerate these regions in the easterly direction. The result is a descending sequence of alternating wind regimes whose period is determined by the intensity of the shorter period waves.

4. Mean heating of the thermosphere.

Above 110 km the temperature of the atmosphere increases from about $280^{\circ}K$ to about $800^{\circ}K$ (averaged over the day) at about 180 km. Above this height the temperature is approximately independent of height at what is called the exosphere temperature. The exosphere temperature has a daily variation of about $\pm 150^{\circ}K$. Over long periods of time, it is believed that the major heat source for the thermosphere is extreme ultraviolet solar radiation absorbed above 10 km; the major loss is believed to be the molecular conduction of heat downward into the lower thermosphere and mesosphere (Craig, 1965). However, data casts considerable doubt on this picture. The most recent measurements of EUV flux and estimates of quantum efficiency (Hinterreger, 1970, Hays, 1970) suggest that this source is incapable of maintaining an exosphere temperature much in excess of $500^{\circ}K$. On the other hand the daily variation in the observed EUV flux is sufficient to explain the observed daily variation in temperature (Lindzen 1971b, c). Where then does the additional mean heating come from? It was suggested by Hines (1965) that such heating could come from the dissipation of internal gravity waves. The effectiveness of such heating depends on both the magnitude of the waves and the heights at which they are dissipated (primarily by molecular viscosity and conductivity). The greater the height at which dissipation occurs, the greater the exospheric temperature produced. The height at which waves are dissipated increases as their vertical wavelength increases and as their period decreases. Lindzen and Blake

(1970) were able to show that the main solar semidiurnal tidal mode can,
by virtue of its very long vertical wavelength, penetrate above 130 km and
produce a mean exospheric temperature of about $700^{\circ}K$ even without EUV
heating.

The absorption of atmospheric tides in the thermosphere must also
have consequences for the momentum budget of that region, but these
have not been investigated as yet.

5. Turbulence in the upper mesosphere.

As mentioned in section 2, internal gravity waves can, in
principle, reach such large amplitudes as to render the waves, themselves,
unstable to other disturbances which might in turn lead to turbulence.
This situation is especially likely in the upper atmosphere where wave
amplitudes are increasing with height roughly as $1/\rho_o^{1/2}$. For the main
propagating diurnal tidal mode in the atmosphere (viz. Chapman and
Lindzen, 1970, for a detailed description), theoretical calculations,
which agree with observations up to 60 km (above which height there
is too little data for detailed comparisons) predict that this mode becomes
unstable at 90 km over the tropics (Lindzen and Blake, 1970). Rather
crude rocket observations (Theon, et al 1967) tend to confirm this picture.
In figure 3 I show day-night temperature differences as a function of
height over the tropics; presumably these day-night differences are due
to the diurnal tide. This view is supported by the fact that the temperature
difference keeps changing sign with a wavelength $\sim$ 20 km which is the

vertical wavelength associated with the main propagating diurnal mode. The observations show a rough $1/\rho_o^{1/2}$ growth up to 90 km where temperature gradients which seem to be unstable appear.

A similar situation is observed at high latitudes in winter where temperature soundings are severely distorted by what appear to be internal gravity waves (viz. Fig.4) (in summer temperature soundings are smooth replicas of "standard" atmospheres). Here again there appears to be $1/\rho_o^{1/2}$ growth until static instability is reached, above which no further growth of amplitude is observed. Presumably the growth of amplitude is limited by the generation of turbulence. Lindzen (1971) has made some attempt at estimating eddy coefficients due to such turbulence.

6. Latitude gradient of temperature at the mesopause.

The mesosphere is primarily heated by ultraviolet radiation absorbed by ozone. At the stratopause (50 km) where this heating (as measured in degrees/day) is greatest, the heating is proportional to the length of the day. Hence the greatest heating occurs over the summer pole rather than over the equator, and, indeed, the observed temperature at 50 km increases from the winter pole to the summer. Ozone heating virtually disappears by 80 km at which point a temperature minimum known as the mesopause is found. The latitude variation of temperature at the mesopause is characterized by the summer polar mesopause being the coldest point in the atmosphere with temperature increasing all the way to the winter polar mesopause. Various reasons

have been suggested for this peculiar behavior but one, due to Leovy

(1964), seems particularly attractive. Leovy showed that in the likely

event that

i) the motion of the mesosphere is primarily forced by ozone heating, and

ii) the zonal motion of the mesosphere is in geostrophic balance, then

iii) if mechanical friction is present, it will cause the vertical shear of

the zonal wind to reverse from what it was below the heating just above

the heating region, and

iv) if geostrophy is to be maintained the north-south temperature gradient

must reverse as well.

The question is where does sufficient friction come from. At

least in winter it is clear that the unstable breakdown and subsequent

dissipation of internal gravity waves could provide the answer. While it

is unclear what the precise origin of the observed waves is, it appears

plausible that surface orography is a primary contender in which case

the phase speeds of the observed waves (relative to the ground) will be

centered around zero. Assuming a symmetric distribution of phase

speeds, I have been able to show that the dissipation of such waves leads

to Rayleigh friction if the range of phase speeds exceeds the mean winds

in the upper mesosphere. In any case, however, the dissipation of

these waves will tend to drag the mean flow to zero. In addition to

providing friction, the dissipation of these waves will provide an additional

source of heating for the winter mesopause. This last possibility has

been discussed by Hines (1965).

7. The general circulation of the tropical mesosphere.

Leovy (1964) has shown that an axially symmetric, thermally driven circulation will not produce mean zonal winds at the equator. On the other hand mean winds are obviously observed there and they must, therefore, be forced by asymmetric motions or eddies, of which internal waves are examples. One such case was described in section 3. In this section I wish to describe another mechanism which produces equatorial flows by redistributing momentum. It was mentioned in section 2 that thermally excited waves will exchange momentum with the mean flow in the region of thermal forcing. Now atmospheric tides are excited by insolation absorbed by water vapor and ozone. The heating region extends from the ground to almost 80 km. The semidiurnal modes, for a variety of reasons, produce negligible divergences of Reynolds stress. However, the situation is markedly different for the diurnal modes. In figure 5 I show the results of a calculation by Fels for the divergence of the Reynolds stress due to the generation of the main propagating diurnal tidal mode (viz. Chapman and Lindzen, 1970) by ozone heating. Several features of the result ought to be noted:

i) The acceleration near 50 km of 15 cm/sec/day could, in six months, produce winds comparable with observed mean winds (Reed, 1965). This presumes relatively little friction, but evidence from the quasi-biennial cycle suggests that this presumption is not unrealistic.

ii) Although a net westerly acceleration is produced (consistent with the discussion in section 2), the net acceleration is much smaller than

the actual acceleration which is alternately westerly and easterly

depending on altitude. Much of the acceleration is due to a redistribution

of momentum within the heating region rather than to an export of

easterly momentum out of the heating region.

Ozone is not the only source of excitation; water vapor in the

troposphere is even more important for the main propagating diurnal

mode (Lindzen, 1967a). What happens when we have both sources of

excitation is seen in figure 5. The magnitude of the accelerations

within the ozone heating region are doubled (accelerations in the troposphere

are small due to the high density of the lower atmosphere) and their

directions are reversed! What is happening in terms of equ. (8) is that

wave fields excited by water vapor are interacting with the ozone forcing

and catalyzing (so to speak) an exchange of momentum within the mean

flow.

The ability of a wave excited at a low level to grow as it propagates

upward and catalyze an exchange of momentum in an upper level region

of thermal excitation opens up several additional possibilities. For

example most diurnal thermal excitation goes into a vertically trapped

mode which by itself is associated with no fluxes of energy or momentum.

However, it has been shown that the interaction of a propagating diurnal

mode excited by water vapor with a trapped diurnal mode excited in the

ozone layer can catalyze a redistribution of zonal momentum in both

the meridional and vertical directions.[1] Although the detailed calculation

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[1] Details of these mechanisms will be described in a forthcoming

paper by myself and Stephen Fels.

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of this effect is not yet completed, it appears possible that the mean

flows generated may be able to explain both the mean flow of the tropical

mesosphere and its seasonal variations -- in a manner similar to that

suggested by Lindzen (1967b).

8. The four day circulation of the Venusian atmosphere.

Much of the work cited in section 7 was in fact stimulated by a

desire to explain the observed four day circulation on Venus. Venus is

covered by cloud whose top is about 50 km above the surface. It is

observed that identifiable cloud features move around the planet every

four days ($\sim$ 100 m/s at the equator) in a direction opposite to the motion

of the sun as seen from the surface (about 4 m/s). Early attempts to

understand this behavior (Schubert and Whitehead, 1969) interpreted it

in terms of the moving flame experiments of Fultz (1956) who showed that

moving a flame around a fluid contained within a stationary annulus

induced mean surface currents. It was found by Stern (1959) that such

currents required the existence of viscosity. The reason is simply that

in a contained inviscid fluid there cannot be Reynolds stresses induced

by heating because there is no place for the momentum flux to go. When

viscosity is present, the viscous boundary layers can provide suitable

sinks. In the context of this paper we see that attempting to model an

atmosphere by a contained fluid is highly inappropriate. In a real

semi-infinite atmosphere a flux of momentum can always go "away" (viz. the radiation condition) and hence there is no pressing need for viscosity to be important. Indeed, if we assume that solar radiation is absorbed in a thin region near the top of Venus cloud layer then equ. (8) suggests that a flow opposite the sun's apparent motion will be induced in this layer due to the forcing of an internal gravity wave. The situation is complicated by the fact that the period of the wave will be long and infrared damping will be important. Thus the wave generated at the center of the heating region will be absorbed as it propagates away and this will give rise to an easterly acceleration (let us adopt the convention that the sun is moving in an easterly direction). When the fluid velocity outside the center of the heating region is accelerated to something close to 4 m/s, critical level absorption will block the wave. Further wave generation within the heating region will cause the critical level to move closer to the center of the heating region (from both above and below) in a manner similar to that involved in the quasi-biennial mechanism of section 3. The overall process is subject to two limitations:

i) The westerly momentum generated within the heating region must equal the easterly momentum generated outside since the fundamental mechanism does not generate net momentum; it merely redistributes momentum.

ii) The approach of the critical level(s) to the center of the heating region must be limited by shear instability. When the Richardson number goes below 0.25, we may expect that further forcing will simply produce

turbulent energy while the Richardson number remains close to 0.25.

The detailed calculation of the above procedure is still in its preliminary stages, and the difficulties include uncertainty as to even the specification of basic physical mechanisms. However, certain aspects of the above procedure can be used to gain quantitative insights into the mechanism. A schematic of the situation is given in fig. 6. For simplicity, it is assumed that the region between z_1 and z_2 has all been brought to 4 m/s (I will return to this point later.). What happens above z_3 is not too important since the mass of this region is relatively small. Limitations (i) and (ii) then imply

$$\frac{1}{2}(\rho_2 - \rho_3)U_2 = -(\rho_1 - \rho_2)U_1$$

$$\approx -\rho_1 U_1 \tag{16}$$

and

$$Ri = \frac{\omega_B^2}{\left(\dfrac{U_2}{\frac{1}{2}(z_3 - z_2)}\right)^2} \cong 0.25 \tag{17}$$

where

$$U_2 = 100 \text{ m/s}$$

$$U_1 = -4 \text{ m/s.}$$

From (17) we get

$$U_2 \approx \omega_b \Delta z$$

where $\Delta z = z_3 - z_2$

If $\omega_b \approx \dfrac{2\pi}{300 \text{ sec}}$

then $\Delta z \approx 5 \text{ km}.$ (18

From (18) it may be shown that

$$P_3 \approx \frac{1}{2} P_2 \tag{19}$$

and from (19) and (16) we then get

$$\frac{P_1}{P_2} \approx 6 \tag{20}$$

which is to say that the wave generated in the heating region must penetrate at least two scale heights down. Our schematic description of the region between z_1 and z_2 is almost certainly wrong. According to Gierasch (1970) the time scale for infrared cooling increases downward and is fairly long below z_2. Thus we would expect waves excited in the heating region to penetrate quite far down giving rise to accelerations which will be greatest near the heating region. A critical level will eventually form near z_2 preventing the wave from penetrating further. The region below z_2 will have speeds less than 4 m/s but will extend much deeper than z_1. The overall effect, however, should not be

significantly different from that envisaged in fig. *6*.

9. Concluding remarks

As shown in section 2 internal gravity waves carry mean momentum and energy and can exchange these quantities with the mean flow in the presence of damping, critical levels, and local thermal forcing. In addition the unstable breakdown of internal waves must also lead to the deposition of wave fluxes in the mean flow. In the remaining sections I have shown how these processes can lead to important observed features of the mean structure and circulation of the upper atmosphere (even on Venus). These examples utilized the simplest (and most common) expressions for the fluxes of energy and momentum -- evaluating these quadratic fluxes with wave solutions obtained from linear theory. This seemingly straightforward procedure is, however, still without a completely rigorous foundation (Jones, 1971, Bretherton, 1971). Especially in rotating systems where mean temperature and wind fields are coupled through the thermal wind relation and where mean meridional circulations are generated to maintain this balance (Eliassen, 1950), care must be taken to include the effects of these meridional circulations in evaluating the response of the atmosphere to wave fluxes. Such a procedure is implicit in the discussion in section 6. For some of the other examples it was assumed that such considerations are not of paramount importance at the equator, and, in the case of the quasi-biennial oscillation observations (Kousky and Wallace, 1971) seem to support this view.

Still, as long as one accepts the basic premises that internal waves can carry momentum and energy, that the demise of these waves must lead to the deposition of these fluxes, and that in the case of thermally excited waves the momentum flux must come from someplace, then the remainder of the discussion is mere detail -- albeit important detail. The examples presented can then be viewed, as I stated in the introduction, as indicative of possibilities. It would, however, be dishonest for me to deny my belief that the examples are also explicitly relevant.

Acknowledgments

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Figure Legends

Fig. 1. Time-height cross-section of monthly mean zonal winds over

Canton Island. After Reed and Rogers, 1962.

Fig. 2. Theoretically calculated time-height cross-section of zonal wind

over the equator. After Holton and Lindzen, 1972.

Fig. 3. Daytime temperature minus nighttime temperature (ΔT) vs.

altitude over the equator. After Theon, et al, 1967.

Fig. 4. Winter and summer temperature profiles for the years 1962

through 1965 at Churchill (59 N) measured with the rocket grenade

technique. After Theon, et al, 1967.

Fig. 5. Mean zonal acceleration due to thermal excitation of propagating
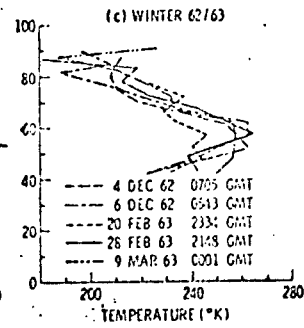
diurnal tidal mode.

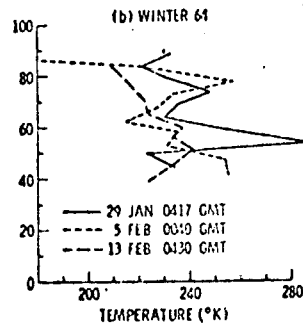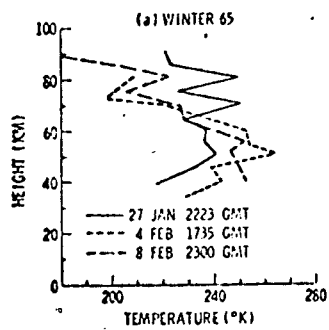Fig. 6. Schematic of Venusian 4-day circulation.

CANTON ISLAND

HEIGHT (KM)

TIME (YEARS)

PITOT TUBE EQUATOR
1248 LST—0126 LST
9 MARCH, 1965

O_3 ONLY

$O_3 + H_2O$

Acceleration (m/sec/10days)