THE UNIVERSITY OF ALABAMA

COLLEGE OF ENGINEERING
BUREAU OF ENGINEERING RESEARCH

INTERIM REPORT

on

Contract NAS8-28019

ACTIVE CONTROL OF PRIMARY MIRROR OF AN ORBITING TELESCOPE WITH THERMAL EXCITATION

by

James L. Hill and John N. Youngblood
Co-Principal Investigators

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center

May, 1973

BER Report No. 153-09

UA ... UNIVERSITY, ALABAMA 35486

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I. INTRODUCTION

In recent years tremendous advances in the practice of astronomy have become possible by escaping the atmospheric shield surrounding earth-based astronomical observatories. Earth-orbiting telescopes are unaffected by the atmosphere and thus capable of achieving resolving power that is completely beyond the reach of terrestrial instruments. Refraction anomalies and background glow, heretofore inherent limiting factors in celestial observations, are problem areas that have been successfully eliminated. They have been replaced by problems associated with maintaining extremely low optical tolerances in equipment exposed to the harsh environment of rocket launch and orbit. Nonetheless, the problem of maintaining small tolerances in construction and alignment is a solvable problem, whereas the problems of accuracy in land based telescopes can only be solved up to the limit imposed by irregular atmospheric refraction.

As stated, the most critical problem of orbiting telescopes is the maintenance of accuracy in construction and alignment. This problem is compounded by the uncertainty of the disturbances which affect the telescope during launch and in orbit. For this reason the critical parts of the instruments must be designed to be as insensitive as possible to external disturbances. There are a number of ways this "passive control" of optical surfaces may be provided.

In addition to the passive shielding method, it is necessary to consider slightly more elaborate techniques to actively control some of the most critical optical surfaces. The object of these "active" methods is not to replace the passive shielding, but to nullify the effects of those disturbances that cannot be predicted and adequately compensated by passive means.

The most critical surface on an orbiting telescope is the primary mirror.

The limits on the resolving power of the instruments are imposed by the size and precision of the surface. For this reason, a number of studies have considered the "active" suppression of surface irregularities (1, 2). Such an approach to the problem has proven to be entirely feasible and these studies are continuing. The active mirror studies which have been performed thus far have considered the use of either external forces on the back of the mirror or segmented mirrors to control the shape of the mirror surface (3). It has been shown that an overall improvement in the surface figure may be achieved in this manner and that such a figure compensation can be performed automatically in a servo control mode (4).

Having determined that active control of the mirror surface is feasible and desirable, attention is directed to the determination of the best way to mechanize the controlled flexing of the surface. To this end such things as cost, weight, reliability and range of operation must be considered in the selection of the best means of implementation.

The results of a study of the feasibility of an active method of surface error control using thermal elements are presented in this report. It is shown that the control effort of the thermal elements is sufficient for the purpose, and that such benefits as low cost, low weight and high reliability may be achieved in conjunction with a significant reduction in the mirror surface error figure.

II. ANALYTICAL APPROACH

Introduction

To accomplish the objectives of this research it was necessary to formulate a thermoelastic response model of the primary mirror which could be used to simulate the response of the structure to disturbances and to controlled thermal inputs. In addition the control strategies for the active system were developed through the use of the response model.

The features that were included in the response model were as follows:

(a) steady state thermal response

(b) transient thermal response

(c) structural response corresponding to a and b

(d) generation of the influence matrix (thermal input - surface deflection output) for use with the control program.

The features that were included in the control program were:

(a) arbitrary control and output node selection

(b) free thermal expansion removal

(c) optimal mean-squared thermal control computation

(d) surface error computation.

Concepts of Discretization

Due to the axisymmetric geometry of the unperturbed mirror it was decided, after preliminary investigation, to use modal decomposition in one dimension (angular) and to use a two dimensional finite element discretization in the radial and longitudinal coordinates. Such a system had been shown to give highly reliable results in work done on solid rocket grains by Wilson (5) and was familiar to one of the authors. It was felt that such a method would be superior in many respects to a complete three dimension finite element discretization if the number of angular modes arising from unsymmetrical disturbance and thermal control excitation were small.

The angular span of the thermal patches were on the order of 10°, which results in a significant number of harmonics necessary to characterize their input effect. (Early in the work we were prepared to use 100 modes in the analysis). However, the inverse of the thermal coefficient matrix C which is obtained after the discretization is completed contains elements that are proportional to n^{-2}, where n is the order of the mode. Therefore, the temperature of the mirror is affected by any mode of a patch in proportion to the inverse square of the order of the mode. This has the effect of reducing the total number of modes used for each patch, since the higher order harmonics are attenuated to such an extent that their presence is inconsequential. It has been shown that a truncation after ten modes for a 10° patch angle results in a temperature error less than .1% at any point in the mirror.

Due to the ability of this program to function with a very small number of radial modes, it is estimated that a savings of 90% in total number of computations and 99% in storage locations is made over a full three dimensional finite element model.

One disadvantage of the discretization scheme is the necessity for a large storage to compute a true transient response in time. For example, to compute the deformation of the structure at time t_1, it is necessary to compute the nodal temperatures of the two dimensional finite element models corresponding to all angular modes, sum the temperatures, and compute the deformation. If the process is to be repeated at a later time t_2, then all of the nodal temperatures for every mode must be computed and stored at time t_1.

This amount of storage was not available to us, so we programmed the transient loop to compute the structural response at times t_1, t_2, t_3, ... etc., starting each computation at $t = 0$. This replaces storaged needed by increased run time.

Thermal Response Formulation

The first stage of a thermoelastic response is the determination of the temperature field, $T(r,\Theta,z,t)$ that exists in the mirror. The temperature is governed by the partial differential equation

$$\nabla^2 T = \frac{\rho c}{k} T_t \qquad \text{(II-1)}$$

with boundary conditions

$$q = -kT_n \text{ on the bottom}$$

$$q = -kT_n = 0 \text{ on the sides}$$

and

$$q = -kT_n = -e(T^4 - T_0^4) \text{ on the front}$$

for the geometry shown in Figure II-1.

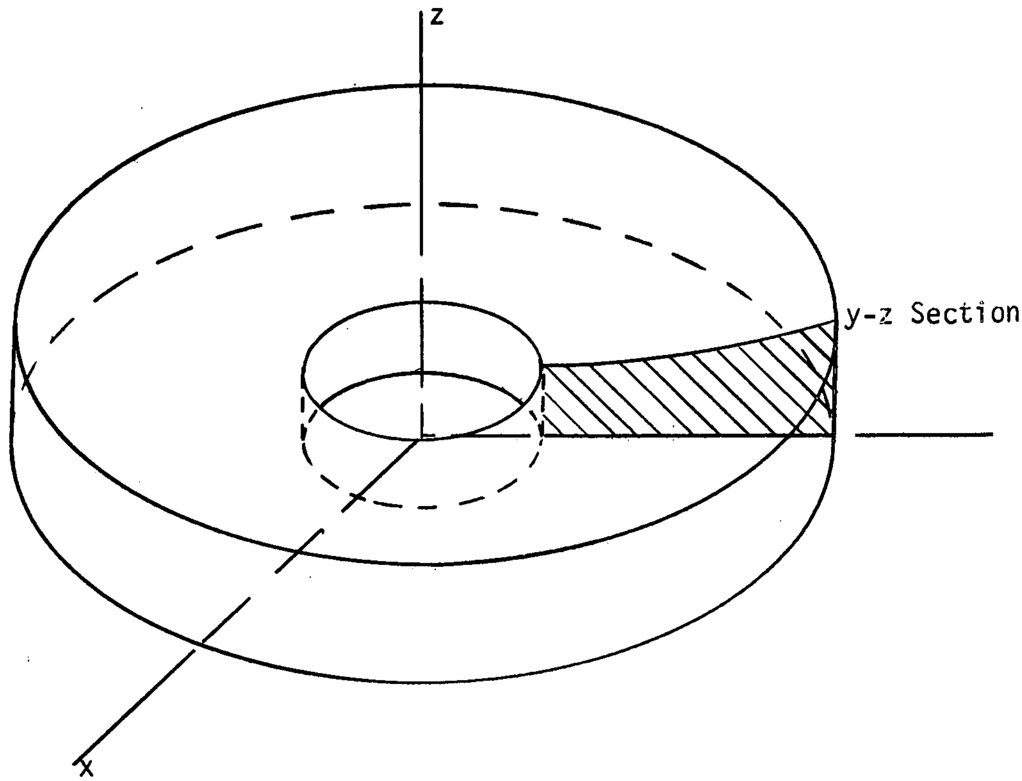


Figure II-1

This problem may be reformulated as a variational problem where the variation is

$$\delta J = \int_V [k\nabla T \cdot \Delta\delta T + c\rho T_t \,\delta T]dv + \int_S \bar{q} \cdot \bar{n}\, \delta T dS = 0 \qquad \text{(II-2)}$$

If the modal decomposition of u and q

$$T(r,\Theta,z,t) = \sum_{n=0}^{\infty} T_n(r,z,t) \cos n\Theta + \sum_{n=1}^{\infty} S_n(r,z,t) \sin n\Theta$$

$$\bar{q} \cdot \bar{n} = \sum_{n=0}^{\infty} p_n \cos n\Theta + \sum_{n=1}^{\infty} q_n \sin n\Theta$$

are substituted into II-2 and reduced the result is

$$\delta_n \int_A \left[k\left(\frac{\partial T_n}{\partial r}\frac{\partial \delta T_n}{\partial r} + \frac{n^2}{r^2} T_n \delta T_n + \frac{\partial T_n}{\partial z}\frac{\partial \delta T_n}{\partial z}\right) + \rho c \frac{\partial T_n}{\partial t}\delta T_n\right] rdA$$

$$+ \int_C p_n \delta T_n rdS = 0, \quad n = 0,1,2, \ldots \qquad \text{(II-3a)}$$

where $\delta_n = 1$ for $n \neq 0$

$\delta_n = 2, \quad n = 0$

and

$$\int_A \left[k\left(\frac{\partial S_n}{\partial r}\frac{\partial}{\partial r}\delta S_n + \frac{n^2}{r^2} S_n \delta S_n + \frac{\partial S_n}{\partial z}\frac{\partial \delta S_n}{\partial z}\right) + \rho c \frac{\partial S_n}{\partial t}\delta S_n\right] rdA$$

$$+ \int_C q_n \delta S_n rdS = 0, \quad n = 1,2, \ldots \qquad \text{(II-3b)}$$

The finite element discretization is obtained initially by designating nodes in the r, z plane and identifying triangular areas between node sets. The equations II-3 hold over any area in the r-z plane and in particular over the triangular finite element areas. Within each triangular area it is assumed that the variation of $T_n(S_n)$ is a linear function of r and z. The discretization will be carried out for the cosine modes only ($T_n(r,z,t)$) the sine modes are very similar.

For the purposes of this feasibility study the initial temperature was assumed uniform throughout the mirror, the sink for front surface radiation was uniform and the patches were heated one at a time. Within these restrictions the temperature distribution was symmetric with respect to a diametral plane through the center of the patch. Because of this the temperature and displacement fields were calculated with the patch at the correct radial position but centered about the x-axis. Then the temperature and displacement fields were rotated about the

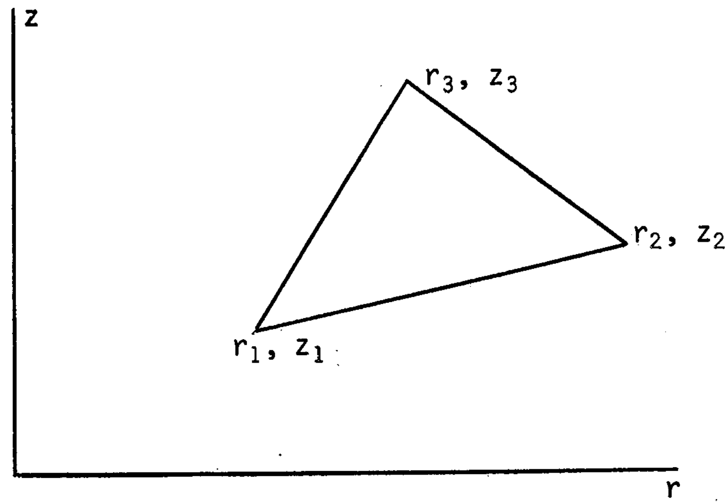


Figure II-2

z-axis until the patch was in the correct angular position. Then the displacement field was given a rigid body displacement to make the axial displacements zero at the supports. Because of this procedure it was only necessary to retain the cosine terms in the temperature field and the terms that are indicated later in the three displacement fields.

$$T(r,z,t) = [1 \quad r \quad z] \begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \\ \alpha_3(t) \end{bmatrix} = \Phi^t \alpha(t) \tag{II-4}$$

The value of T may be computed at the $L^{\underline{th}}$ corner of the triangle

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 1 & r_1 & z_1 \\ 1 & r_2 & z_2 \\ 1 & r_3 & z_3 \end{bmatrix} \alpha$$

or

$$T_n = A^{-1}\alpha_n$$

$$\alpha n = AT_n$$

$$T = \Phi^t AT_n \qquad \text{(II-5)}$$

When II-5 is substituted into II-3a one has

$$0 = \delta T_n A^t \int_A k[\Phi_1\Phi_1{}^t + \frac{n^2}{r^2}\Phi\Phi^t + \Phi_2\Phi_2{}^t]rdA\ AT_n$$

$$+ \delta T_n{}^t A^t \int_A \rho c\Phi\Phi^t rdA\ A\dot{T}_n$$

$$+ \delta T_n{}^t A^t \int_C \Phi p_n rdS \qquad \text{(II-6)}$$

where $\Phi_1{}^t = (0\ 1\ 0)$

$\Phi_2{}^t = (0\ 0\ 1)$

Identifying the integrals in II-6 as C, D and q we have

$$C_n T_n + D_n \dot{T}_n + q_n = 0. \qquad \text{(II-7)}$$

The triangular elements are grouped by fours to form quadrilaterals as shown in Figure II-3.

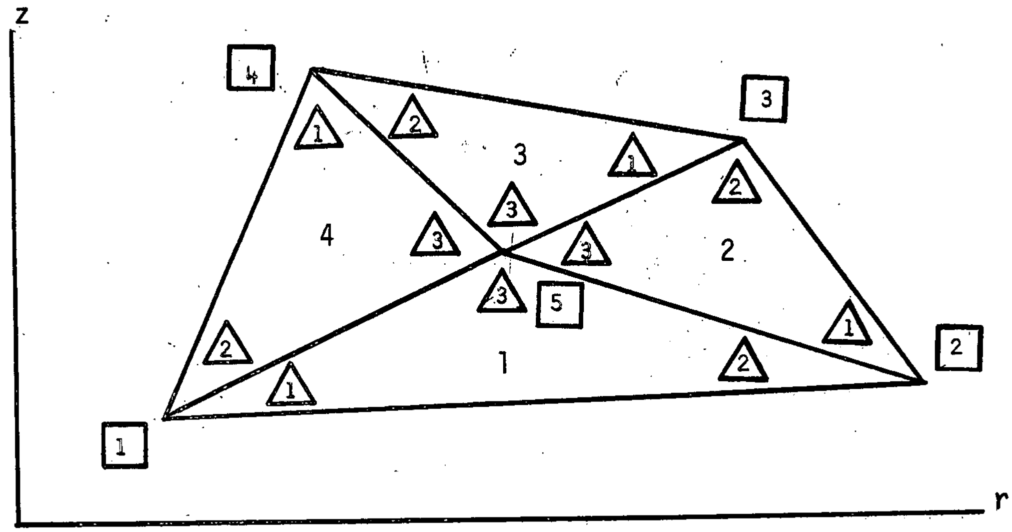


Figure II-3

Here the numbers in triangles represent triangle corners, numbers in squares represent quadrilateral indicies, and the other numbers represent elements.

The assembly of the finite elements are based upon two ideas:

1. Global indexing of nodal point values related to local element nodal point values (Renaming) and

2. Summing the element nodal heat flux to equal the applied heat flux (Equilibrium).

The renaming is done by observation as follows

$$T_1 = T_1^1 = T_2^4$$

$$T_2 = T_2^1 = T_1^2$$

etc.

The equilibrium requires

$$q_1 = q_1^1 + q_2^4$$

$$q_2 = q_2^1 + q_1^2$$

etc.

The reassembly is done via the renaming and equilibrium schemes operating on the sets of element equations II-7.

After the quadrilateral elements have been formed from the triangular elements, the index 5 (middle index) is eliminated from the quadrilateral representation. This is a straightforward elimination procedure and is not outlined here.

The four-index quadrilateral elements are arranged to fill the r-z section of the mirror as indicated in Figure II-4.

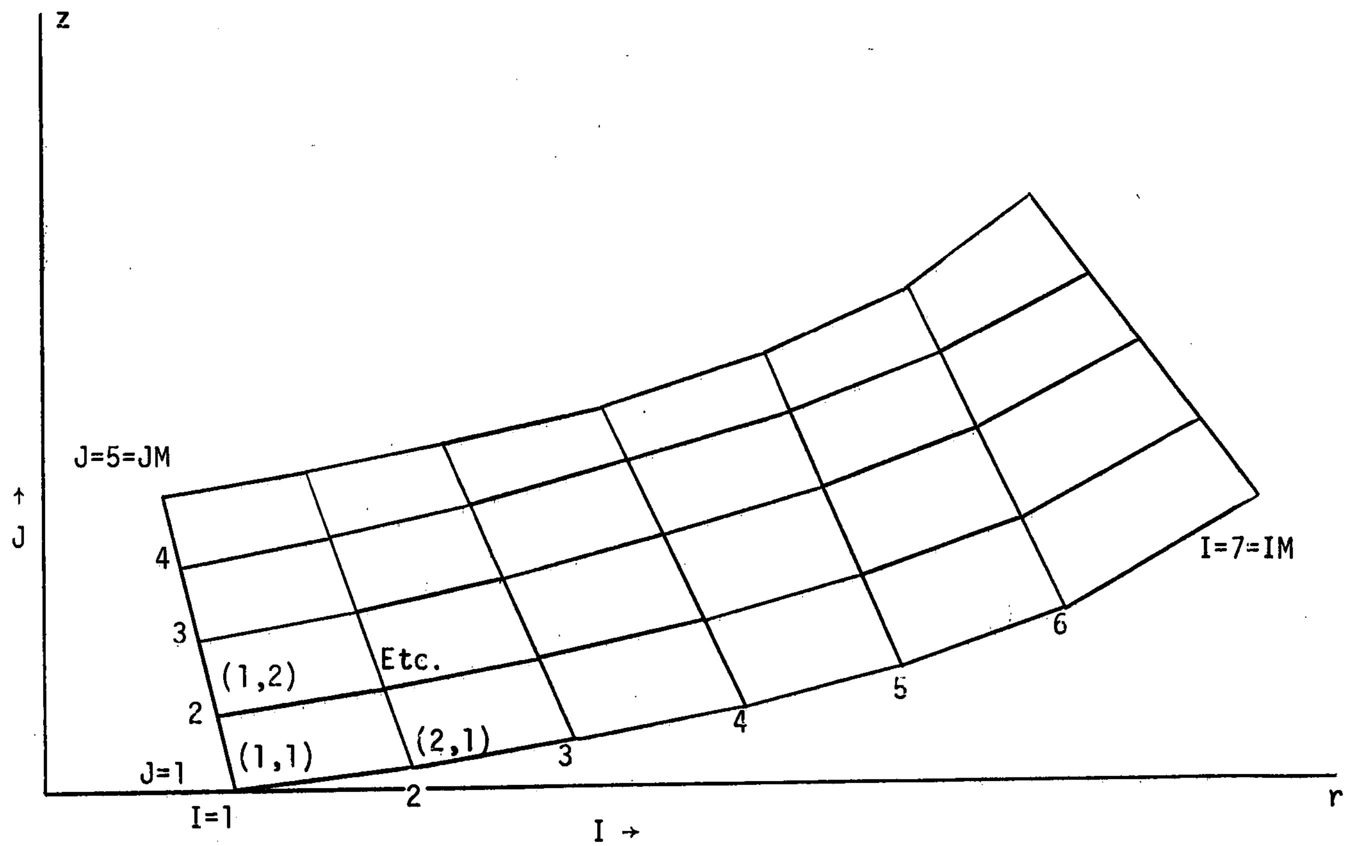


Figure II-4

Thus a node is located by I, J. In general the situation is as shown in Figure II-5.

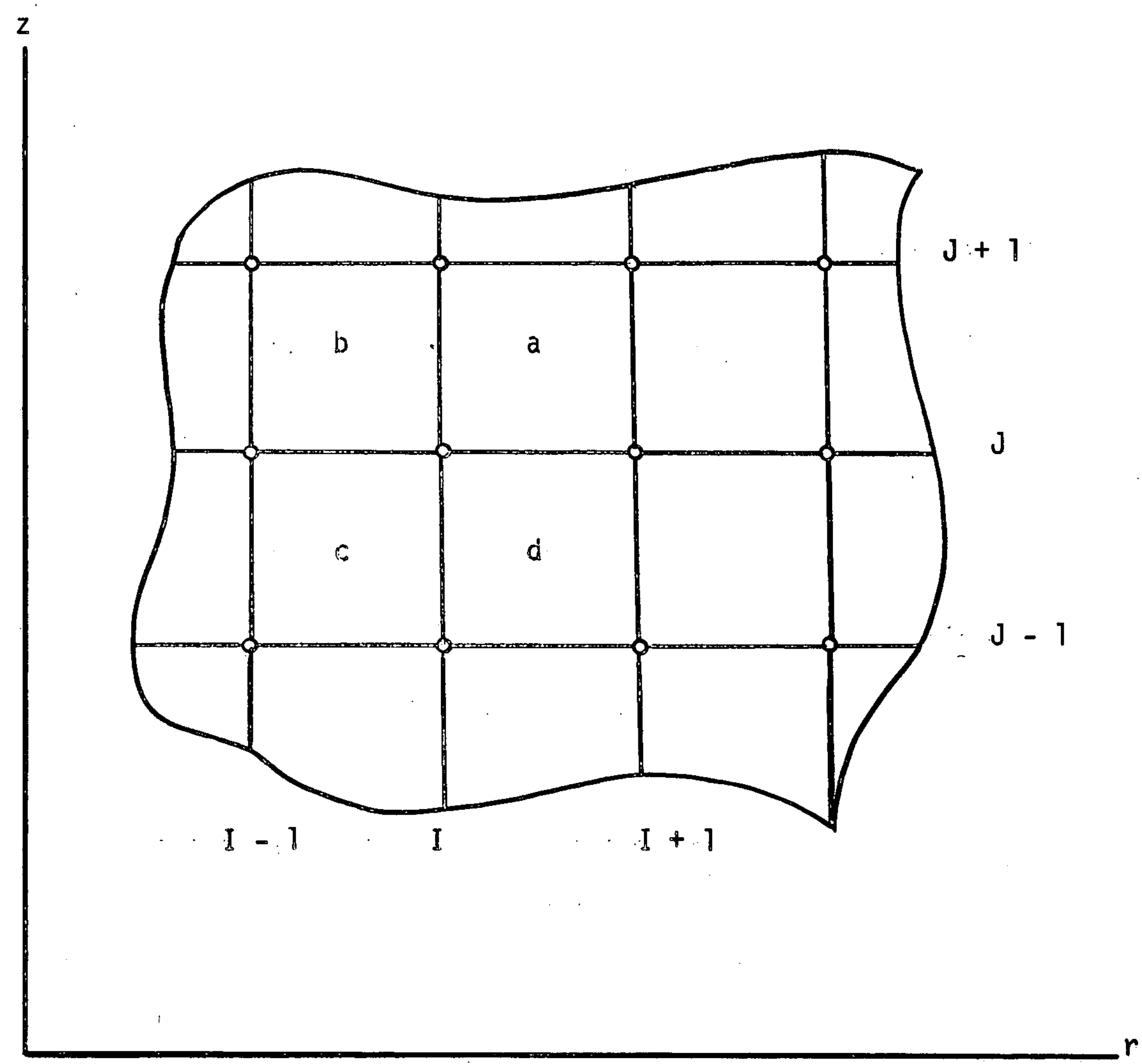


Figure II-5

Here

$$
\begin{aligned}
q_{IJ} &= q_3^c + q_4^d + q_1^a + q_2^b \\
&= C_{31}^c T_{I-1,\,J-1} + C_{32}^c T_{I,\,J-1} + C_{33}^c T_{I,\,J} + C_{34}^c T_{I-1,\,J} \\
&+ C_{41}^d T_{I,\,J-1} + C_{42}^d T_{I+1,\,J-1} + C_{43}^d T_{I+1,\,J} + C_{44}^d T_{I,\,J} \\
&+ C_{11}^a T_{I,\,J} + C_{12}^a T_{I+1,\,J} + C_{13}^a T_{I+1,\,J+1} + C_{14}^a T_{I,\,J+1} \\
&+ C_{21}^b T_{I-1,\,J} + C_{22}^b T_{I,\,J} + C_{23}^b T_{I,\,J+1} + C_{24}^b T_{I-1,\,J+1} \\
&+ \text{like } D\dot{T} \text{ terms.}
\end{aligned}
$$

In this way the (I-1) x (J-1) quadrilateral element equations of dimension 4 x 4 are corresponding to each mode are compacted into one equation which is IM x JM in dimension. The coefficient matrices are banded matrices with a small number of elements on the band.

The resulting equation

$$C_n T_n + D_n \dot{T}_n + q_n = 0 \qquad \text{(II-8)}$$

is integrated to determine the model temperature T_n.

Thermoelastic Formulation

The temperature field $T(r,\Theta,z,t)$ determined from the previous thermal model as modal temperatures $T_n(r,z,t)$ are the inputs to the thermoelastic response. The thermoelastic response is the displacement field $u_i(x_1, x_2, x_3,t)$. The inertial effects are ignored with the time dependence coming only from the time dependence of the temperature field. The governing equations will be expressed in Cartesian tensor form although they are implemented in cylindrical coordinates. The displacements u_i must satisfy the Cauchy-Navier equations

$$\lambda u_{k,ki} + \mu(u_{i,jj} + u_{j,ij}) - \beta T_{,i} = 0 \tag{II-9}$$

within the body and satisfy the boundary conditions

$$[\lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i})]n_j = \beta T n_i \tag{II-10}$$

on the surface of the body. Where λ and μ are Lame' elastic constants and β is related to the linear coefficient of thermal expansion α by

$$\beta = \alpha(3\lambda + 2\mu)$$

Equations (II-9 and 10) are equivalent to the variational principle that potential energy must be stationary for equilibibirum. This is given as

$$-\delta PE = \int_S \beta T n_i \delta u_i \, dS - \int_V \beta T_{,i} \delta u_i \, dV \tag{II-11}$$

$$-\int_V [\lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i})]\delta u_{i,j} dV = 0$$

The variational principle was expressed in cylindrical coordinates and components of the displacement field u_r, u_Θ, u_z. This is not presented due to its length. The displacement fields were assumed as

$$u_r(r,\Theta,z) = \sum_{n=0}^{\infty} U_n(r,z) \cos n\Theta$$

$$u_\Theta(r,\Theta,z) = \sum_{n=1}^{\infty} V_n(r,z) \sin n\Theta$$

$$u_z(r,\Theta,z) = \sum_{n=0}^{\infty} W_n(r,z) \cos n\Theta$$

The sine terms for u_r and u_z and the cosine terms for u_Θ were not retained as previously explained.

These displacement forms were inserted into the variational principle of Equation (II-11) to obtain individual variational principles for each mode (n). In addition the modal temperature and displacement fields are assumed to be linear functions of r and z inside each finite element triangle. Then,

$$U_n = \begin{bmatrix} U \\ V \\ W \end{bmatrix} = CX_n$$

where $X^T = [U_1\ V_1\ W_1 \quad U_2\ V_2\ W_2 \quad U_3\ V_3\ W_3]$

and

$$C = [C_1 I \quad C_2 I \quad C_3 I]$$

$$C_i = a_i + b_i r + d_i z$$

Under these assumptions the variational principles become

$$-\delta PE_n = 0 = \delta X_n^t(K_n X_n + P_n T_n) \qquad n = 0,1,.... \qquad \text{(II-13)}$$

for an interior triangle, and

$$-\delta PE_n = 0 = \delta X_n^t(K_n X_n + P_n T_n - R_n T_n) \qquad n = 0,1,.... \qquad \text{(II-14)}$$

for a triangle having an edge on the surface. Where the coefficient matrices K_n, P_n, and R_n can be identified from the integrals in the variational statement.

The thermoelastic finite element equations for a single triangle are similar to the thermal equations. They are of dimension nine due to the three displacements of the three corners, however, there is no rate term. The assembly of triangles to form quadrilaterals and the systematic assembly of the quadrilateral elements proceeds exactly as in the case of the thermal program. For that reason it is not repeated here. The assembled equations are of the form

$$K_n X_n + P_n T_n = 0 \qquad \text{(II-15)}$$

Let M = IM x JM then K_n is a square matrix of order 3M, X_n is a row vector of order 3M, P_n is 3M x M and T_n is a row vector of order M.

Response Computation

The thermal and deflection programs whose modal-finite element discretization have been discussed in the two previous sections yield the following vector matrix equations

$$CT + D\dot{T} = q \qquad \text{for each mode}$$

and

$$KX + PT = 0 \qquad \text{for each mode}$$

where

q is the vector of nodal heat inputs

T is the vector of nodal temperature

X is the vector of nodal displacements.

To compute the displacement of any node at time t the thermal equation is integrated to yield the temperature and the displacement is computed from the displacement equation where the nodal heat input rates q must be inputed.

The integration algorithm that was used is as follows

$$\dot{T}_i = \frac{T_i - T_{i-1}}{h} - \dot{T}_{i-1}$$

where h is the integration delta time. The thermal equation via the integration algorithm yields

$$(C + \frac{2D}{h})(\frac{T_{i+1} + T_i}{2}) = \frac{2}{h} DT_i + \frac{1}{2}(q_i + q_{i+1})$$

$$T_{i+1} = 2(\frac{T_{i+1} + T_i}{2}) - T_i$$

In using this procedure to iterate the nodal temperatures, no problems in matrix inversion were encountered. The anticipation q_{i+1} was removed by assuming

$$q_i = \frac{1}{2}(q_i + q_{i+1}),$$

a reasonable assumption for slowly changing heat inputs and high iteration rates.

The displacement vector of the body is obtained from the nodal temperature vector by matrix inversion and multiplication in the thermo-elastic equation. No problems with matrix inversion were encountered.

When the deflection associated with each successive mode is computed it is added to the preceeding modes. The vector displacement of each node is established.

The Influence Matrix Computation

The influence matrix computation is associated with the complete steady-state response of the nodes of the body excited by a designated pattern of constant heat inputs. A subset of the nodes on the front surface of the mirror is assigned the role of output nodes. A pattern of patches is specified on the rear of the mirror. A unit heat rate is supplied to the patches one at a time and the vector of longitudinal steady state deflection is recorded. These deflection vectors are the columns of the influence matrix A, where

$$w = Aq$$

w is the longitudinal output deflection vector and q is the heat input vector whose components represent the heat input at each patch.

This influence matrix may be used in control simulation for any output node set and patch pattern set which are subsets of the set for which the influence matrix is computed.

Removal of Free Thermal Expansion

In heating the mirror for control purposes a significant amount of heat goes into the free thermal expansion of the mirror. This deflection does not contribute to local error smoothing and may be compensated by refocussing of the mirror. Moreover, if retained as part of the control, it significantly reduces the sensitivity of the control function. For these reasons the free thermal expansion terms are measured and their effect is removed from the influence matrix.

Consider the mirror geometry shown in Figure II-6.

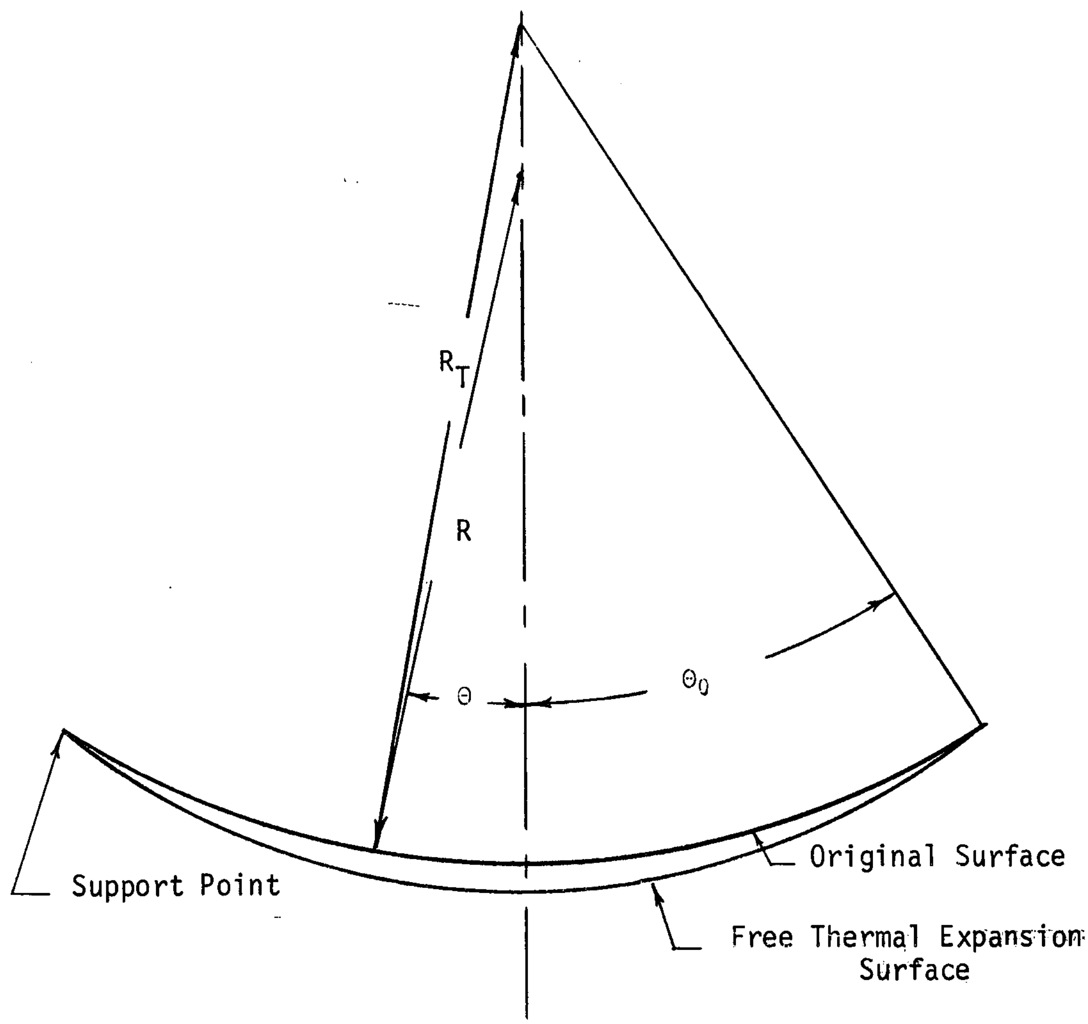


Figure II-6

The difference in the two reference spheres is

$$y = (R_T - R)\left[\frac{\cos\Theta}{\cos\Theta_o} - 1\right]$$

The best fit sphere is found by minimizing the functional

$$J = \sum_{i}\sum_{j} (w_{ij} - y_i)^2$$

where w_{ij} is the displacement of the $ij^{\underline{th}}$ surface nodal point due to heat input on the rear.

The minimization yields

$$\Delta = R_T - R = \frac{\sum_{i}\sum_{j} w_{ij}\left[\frac{\cos\Theta_i}{\cos\Theta_o} - 1\right]}{j\sum_{i}\left[\frac{\cos\Theta_i}{\cos\Theta_o} - 1\right]^2}$$

which represents the adjustment of the reference sphere.

Error Computation and Control

The deflection of the front of the mirror is composed of a disturbance w_D and control induced deflection

$$w_C = Aq$$

so

$$w = w_D + Aq$$

The front surface error figure is

$$J = w^tQw \ ,$$

where Q is a symmetric weighting matrix. Since one may wish to minimize the use of the control a control effort cost term

$$C = q^tRq$$

is also introduced, where in most cases R is a diagonal matrix, whose elements reflect the priority associated with control cost versus surface error reduction.

The minimization of the index

$$w^tQw + q^tRq$$

under the constraint

$$w = w_DAq$$

yields the optimal control vector

$$\hat{q} = -[A^tQA + R]^{-1} A^tQw_D$$

and associated best index

$$\hat{J} + \hat{C} = w_D{}^t[Q - QA[A^tQ + R]^{-1} A^tQ]w_D \quad .$$

III. DISCRIPTION OF FORTRAN PROGRAMS

A. RESPONSE

Purpose

This program computes the thermoelastic deflection of a set of nodes specified in an axisymmetric body. Heat is inputed on one surface of the body and radiates at the other.

Options

This program provides the following options.

(1) The temperature and deflection of the node set may be computed at any time.

(2) The steady state temperature and deflection may be computed.

(3) The influence matrix of surface deflection for a designated heat input pattern may be computed.

Input Parameter Definition

Parameter	Definition
NUMPAT	The total number data cases to be run.
IM	Number of node locations in the radial direction. Cannot exceed 15.
JM	Number of node locations in the logitudinal direction. Cannot exceed 5.
CK	Thermal conductivity of the material.
CP	Specific heat of the material (0. value indicates steady-state solution).
CF	Radiation coefficient of the front surface.
TO	Temperature of radiating reference medium.

Parameter	Definition
DELT	Integration time.
NTS	Number of integration steps.
IPRINT	Number of integration steps between printings.
TI	Initial mirror temperature.
ALF	Thermal expansion coefficient.
E	Young's modulus.
V	Poission's ratio.
NM	Highest harmonic (angular) component.
IS	Location of radial node of support ring.
KM	Number of angular divisions. Must not exceed 12.
S2	Angular position of second support (first one is at 0).
S3	Angular position of third support.
INFLU	Switch to choose simulation of response or influence matrix computation. If INFLU = 0 Program calculates the response to heating one patch. If INFLU > 0 Program calculates the thermoelastic influence coefficient matrix. This matrix is written in a data file through I-0 unit 4.
IX	Index of the radial ring on the back at which the modal boundary conditions are applied.
IP	Radial node of single patch input.
KP	Angular position of single patch input.
PA	Patch angle for single patch.

Parameter	Definition
PH	Heat input rate for single patch. (Negative is input positive is output).
PAN(I)	Patch angle for i-th radial node in full pattern generation.
DO	Outside diameter of mirror.
DI	Inside diameter of mirror.
H	Thickness of mirror.
FNO	F-number of the mirror = Focal Length/Diameter of Mirror.

Input Data Card Listing

Card No.	Parameter	Data Field	Format
1	NUMPAT	1-5	I5
2	IM	1-5	I5
	JM	6-10	I5
	CK	11-20	F10.5
	CP	21-30	F10.5
	CF	31-40	F10.5
	TO	41-50	F10.5
	DELT	51-60	F10.5
	NTS	61-65	I5
	IPRINT	66-70	I5
	TI	71-80	F10.5
3	ALF	1-10	F10.5
	E	11-20	F10.5
	V	21-30	F10.5
4	NM	1-5	I5
	IS	6-10	I5
	KM	11-15	I5
	S2	16-20	I5
	S3	21-25	I5
	INFLU	26-30	I5
	IX	31-35	I5

Card No.	Parameter	Data Field	Format
5*	IP	1-5	I5
	KP	6-10	I5
	PA	11-20	F10
	PH	21-30	F10
5**	PAN(1)...PAN(7)	1-70	7F10.5
.			
.			
5+N	PAN(IM-7)...PAN(IM-1)	1-70	7F10.5
6+N	DO	1-10	F10.5
	DI	11-20	F10.5
	H	21-30	F10.5
	FNO	31-40	F10.5

Cards 2 through 6+N constitute one data set and there should be NUMPAT data sets.

---

* Used only if INFLU equals zero.

** Used only if INFLU is greater than zero.

Output of Program

A. Option INFLU = 0

1. Repeated input data.
2. Table of temperatures and deflection of the node (I=IM, J=JM) for each mode for a check of convergence.
3. Table of displacements and temperatures of the nodes on the front surface before the support constraints are added.
4. Table of displacements of the nodes on the front surface after supports are added.

B. Option INFLU = 1

1. Repeated input data.
2. Patch angles of each patch ring.
3. The displacements of the nodal points on the front surface of the mirror corresponding to each patch location are read into a file.

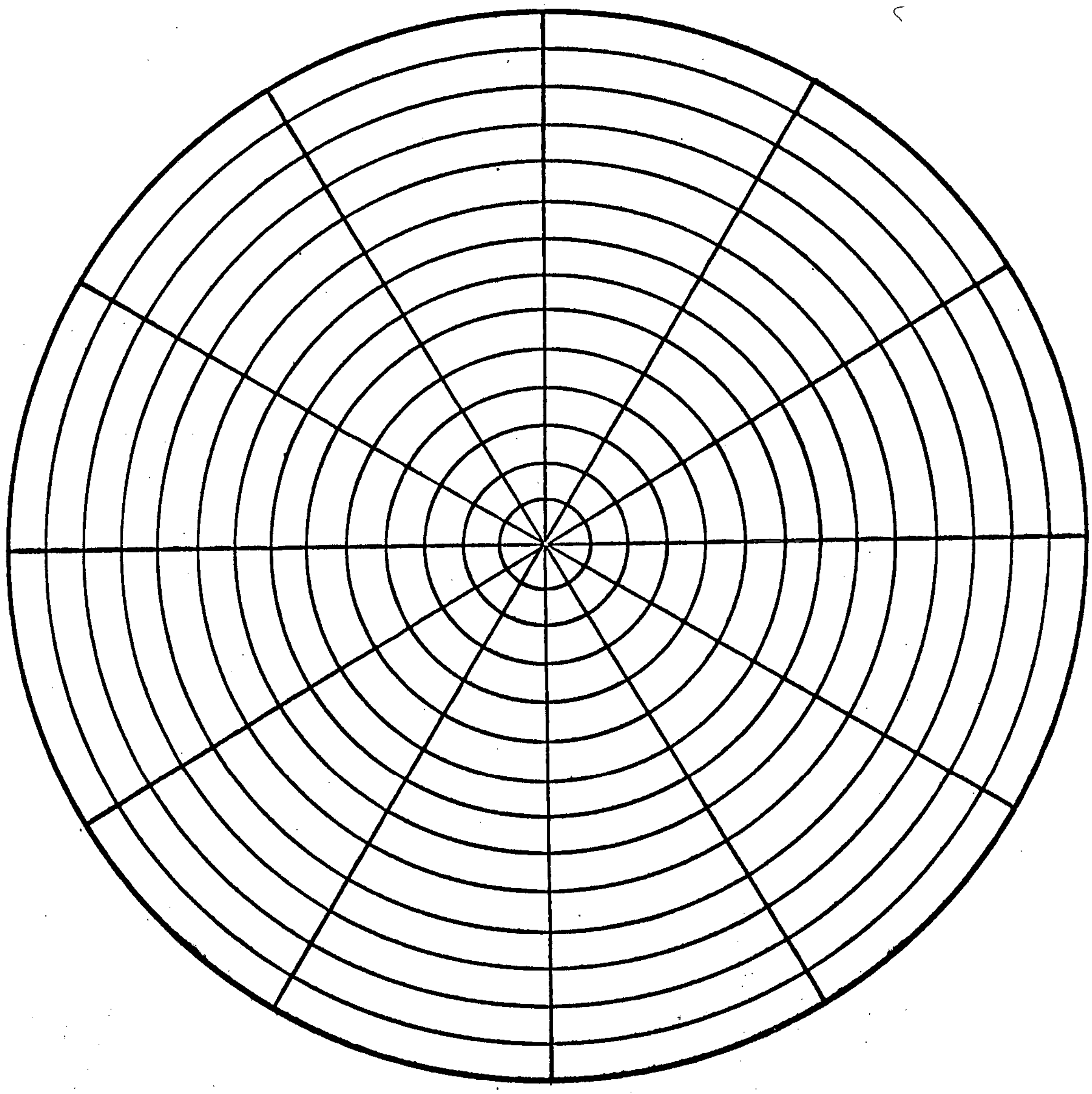
Heat Input and Deflection Measurement Points.

The node distribution for both the front and the rear of the mirror is shown in Figure III-1. The surface has IM concentric nodal rings and KM nodal rays. The nodes occur at intersections of rings and rays. The nodes are selected by the program after mirror dimensions, KM, and IM are given.

The nodes for measurement of surface deflection are chosen in the control program. Any subset of the full set of nodes may be chosen.

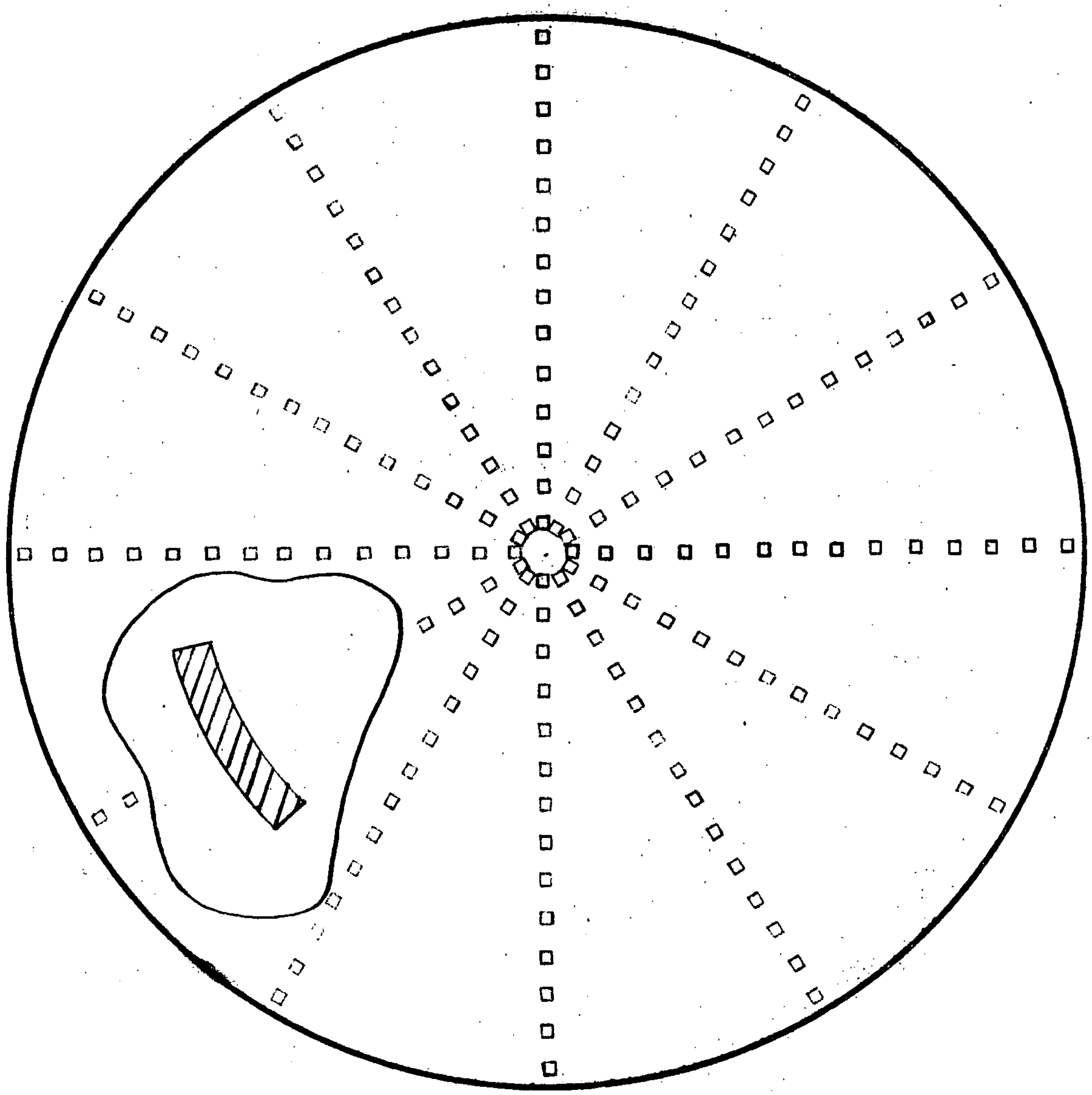
The heat is applied to the rear by patches. The patches extend the entire distance between rings and make an angle PAN centered about a ray. Patch location and patch angle may be inputed in the control program.

Several typical patch location patterns are shown in Figure III-2 through III-5.



Sample Points on Front Surface

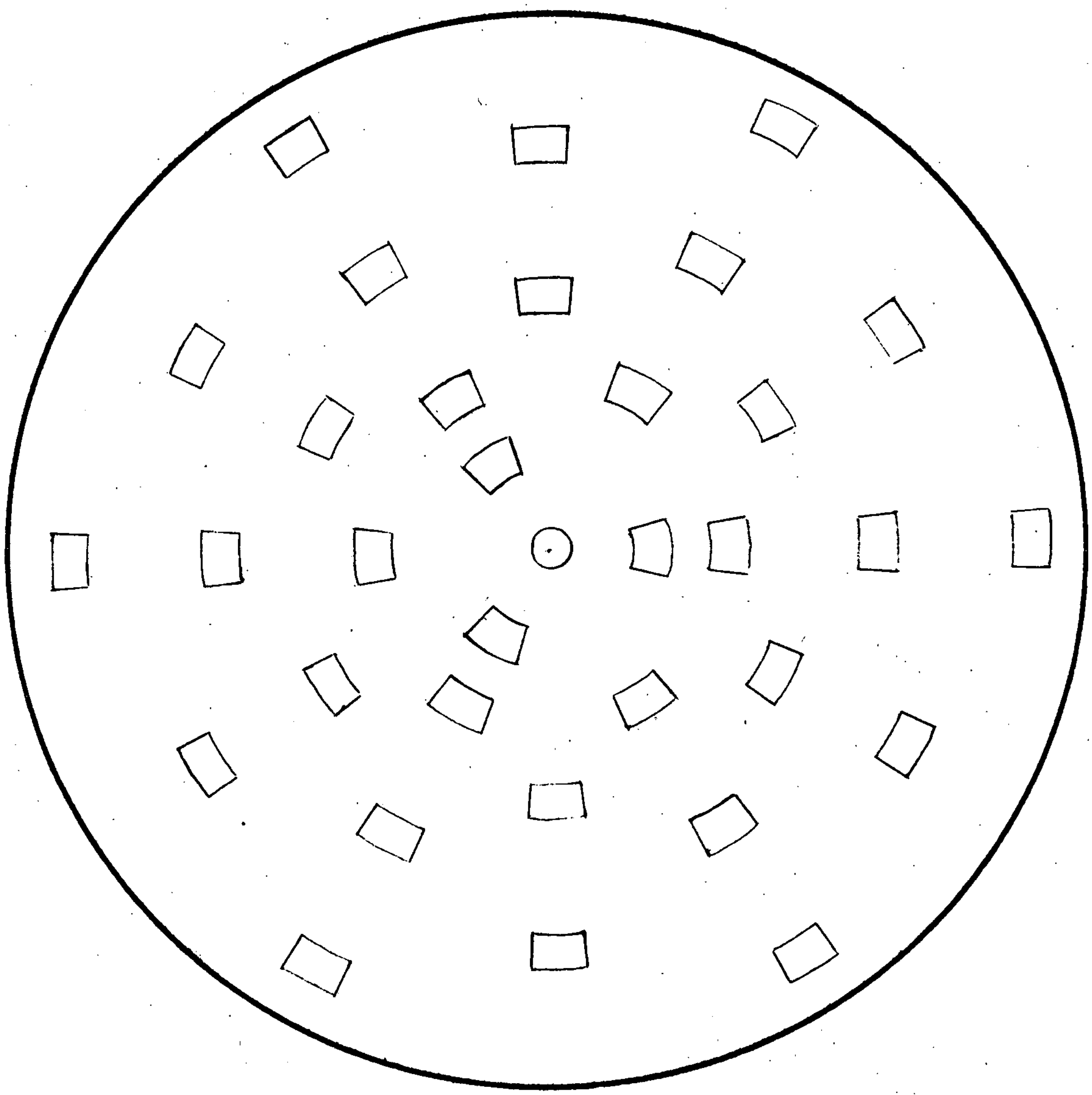
Figure III-1



Heat Patch Locations

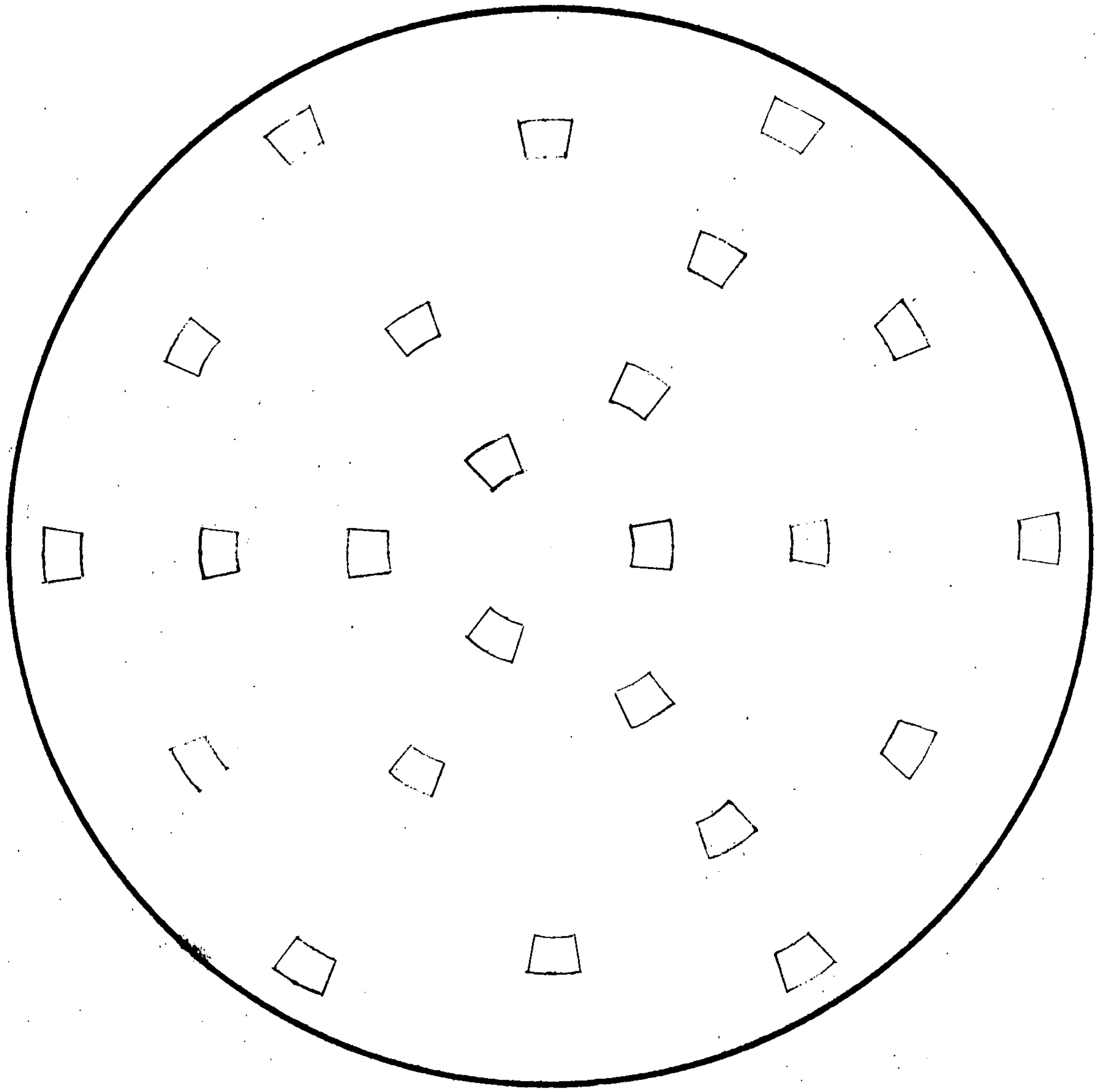
-Typical Patch-

Figure III-2



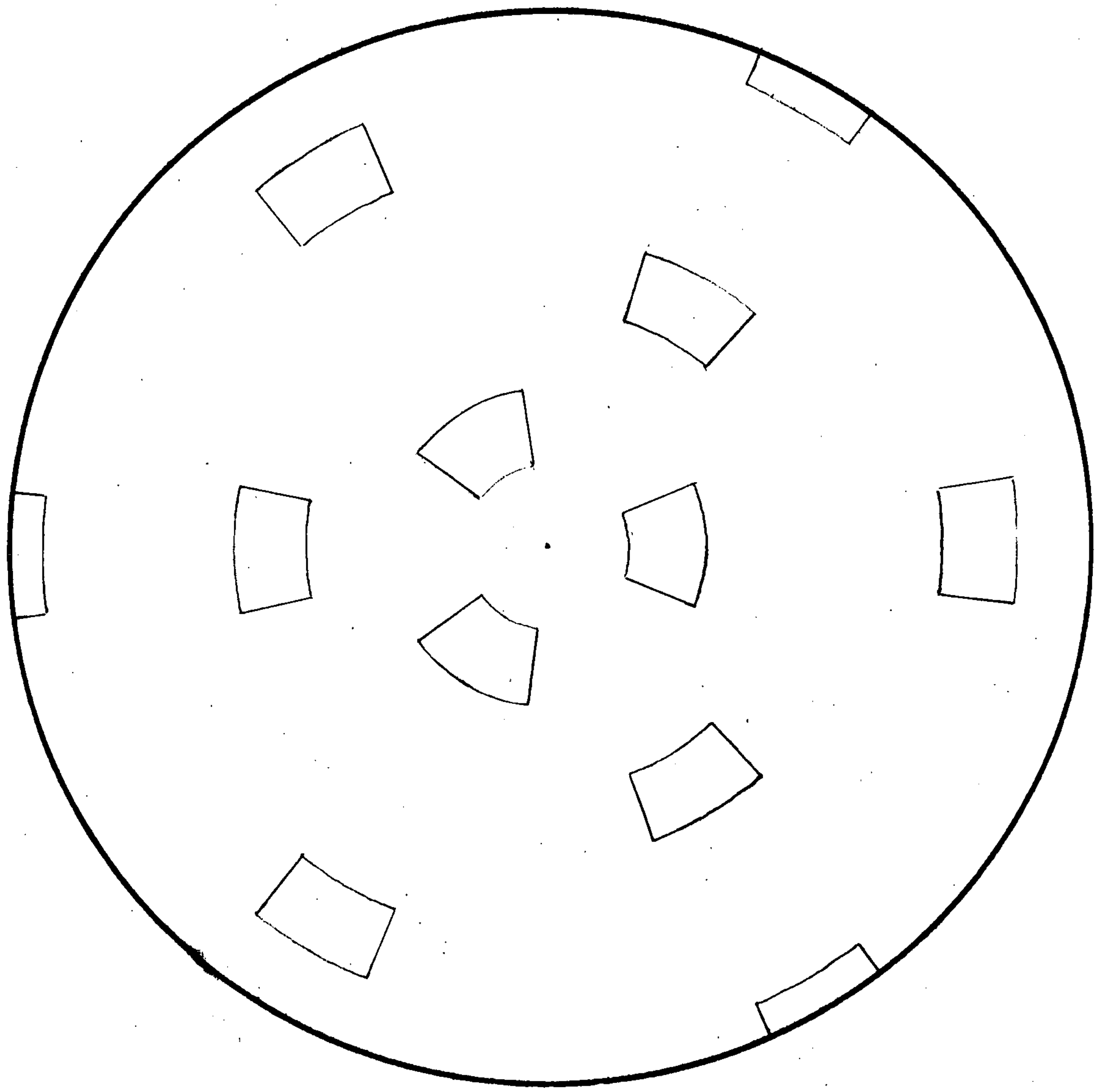
34 Patch Distribution

Figure III-3



24 Patch Distribution

Figure III-4



12 Patch Distribution

Figure III-5

B. CONTROLS

Purpose

This program computes the patch heats necessary to minimize the weighted surface error of the mirror taken at designated sample points, using designated heater locations. In addition it computes the surface error both before and after application of the thermal input. The disturbance error is computed internally.

Input Parameter Definition

Parameter	Definition
IM	Number of nodes in raidal direction
KM	Number of angular divisions
DI	Inner Diameter
DO	Outer Diameter
FNO	Focal Length/Diameter
NHP	Number of heater locations
NSP	Number of sample points
IHP	Individual heater location
ISP	Individual sample point
A(I,J)	Coefficients of the influence matrix computed in RESPONSE (read from file)

Input Data Card Listing

Card No.	Parameter	Data Field	Format
1	IM	1-5	I5
1	KM	6-10	I5
1	DI	11-20	F10.5
1	DO	21-30	F10.5
1	FNO	31-40	F10.5
2	NHP	1-5	I5
2	NSP	6-10	I5
3	IHP(I)	1-80	16I5
4	ISP(I)	1-80	16I5

Output of Program

1. Repeated heat patch points.
2. Repeated sample points.
3. Coefficients of the reduced influence matrix corresponding to the heat patches and sample points selected.
4. The surface error of the sample points before control.
5. The performance index before control.
6. The performance index after control.

IV. FEASIBILITY RESULTS

The results presented here were obtained for a small fused-silica mirror whose properties and dimensions are given in Table IV-1. The deflection patterns are shown in Figures IV-1 through IV-10 for various one patch heater locations. The mirror is supported at 120° intervals as shown in the figures. Figures IV-1 through IV-6 show the steady state deflection patterns for heat inputs of 3 watts and various patch locations. Figures IV-8 and IV-9 shows a transient deflection pattern for the same heat rate. Figures IV-10 and IV-11 show the transient deflection of selected points on the front surface as a function of time. These points are shown in Figure IV-7.

SMALL FUSED SILICA MIRROR

•INNER RADIUS	0.
•OUTER RADIUS	20. IN
•THICKNESS	.667 IN

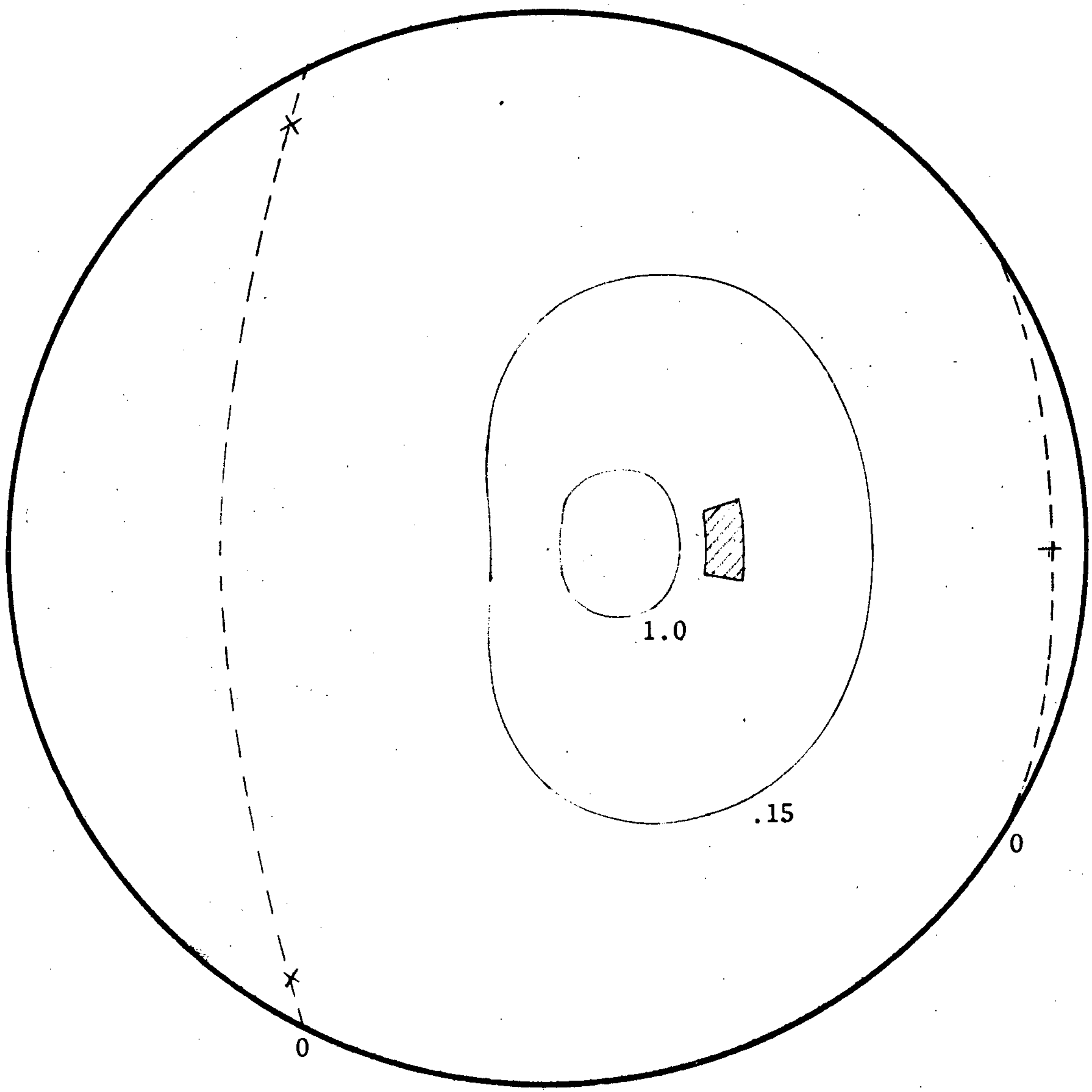
Structural Properties

•SPECIFIC HEAT	.015 BTU/IN^3 DEG F
•CONDUCTIVITY	.05 BTU/HR•IN•DEG F
•EMISSIVITY	.04
•POISSON'S RATIO	.17
•YOUNG'S MODULUS	106 10^8 LB/IN^2
•COEFFICIENT OF THERMAL EXPANSION	$.311 \bullet 10^{-6}$/DEG F.

Response Properties

•NOMINAL HEATING POWER	3 WATTS
•THERMAL TIME CONSTANT	19 HOURS

Table IV-1

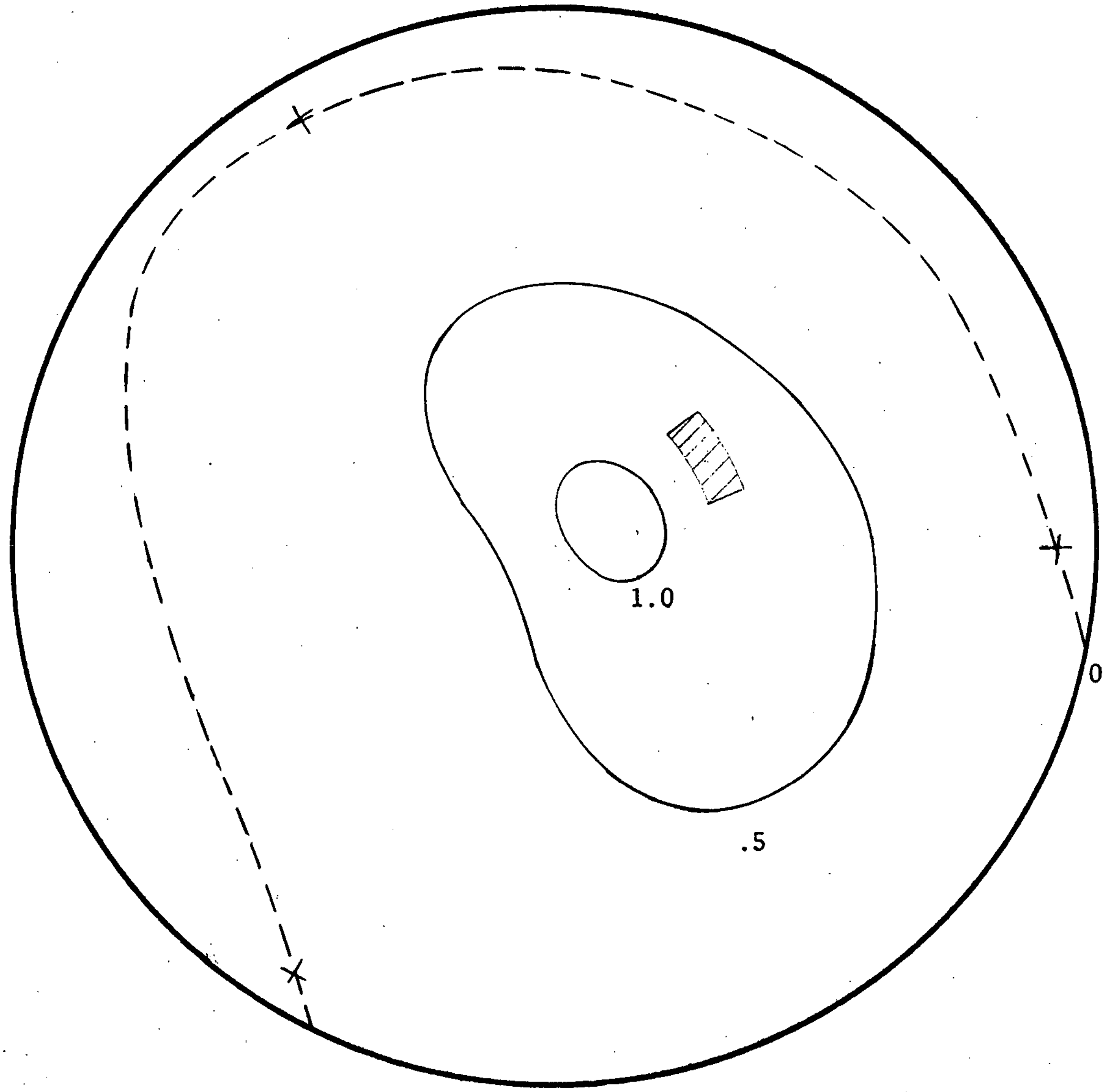


Heat Input 10^{-3} BTU/HR•IN^2

Patch Location IP = 5, KP = 0

STEADY STATE DEFLECTION (MICRO-INCH)

Figure IV-1

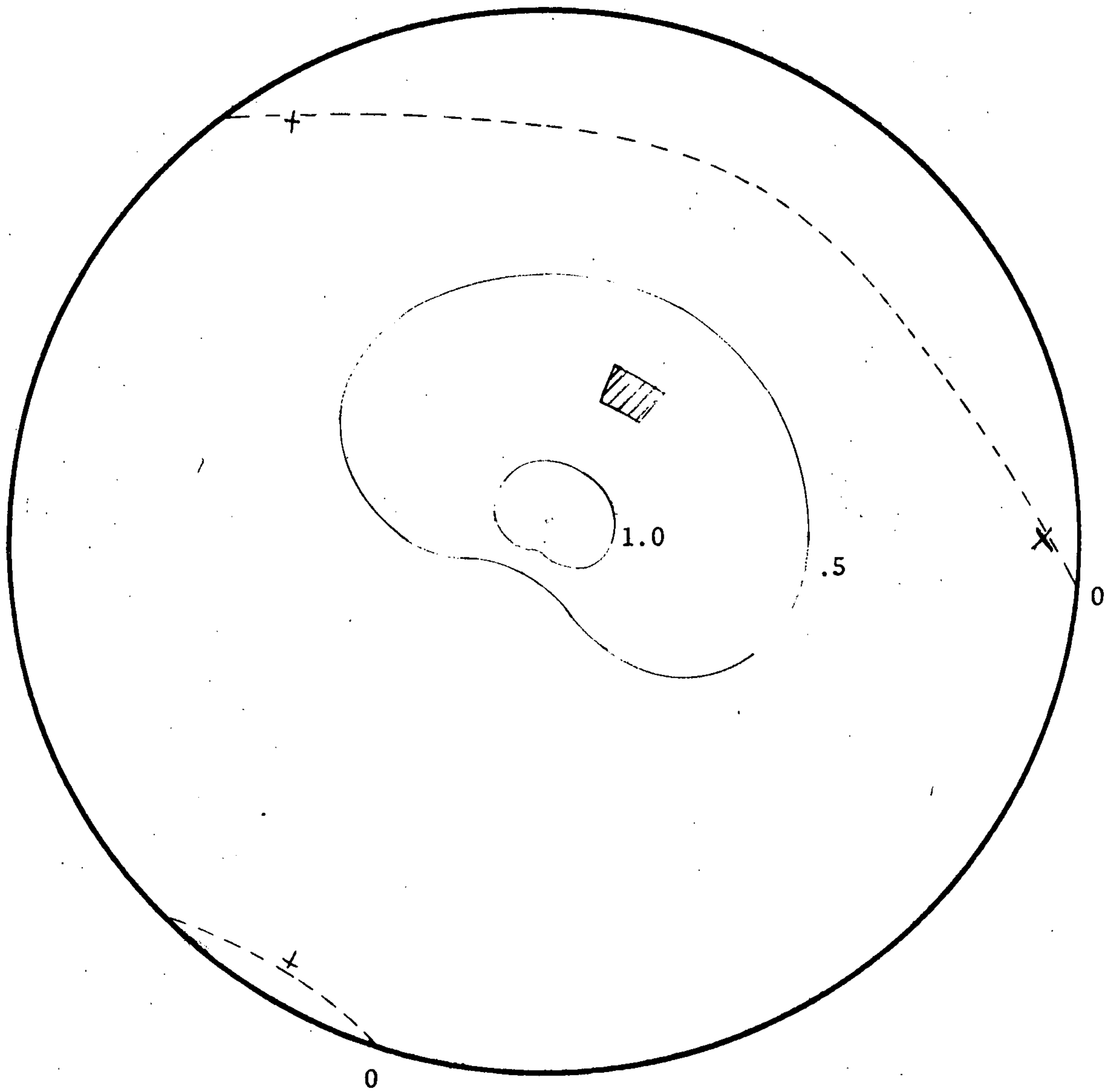


Heat Input: 10^{-3} BTU/HR•IN^2

Patch Location IP = 5, KP = 2

STEADY STATE DEFLECTION (MICRO-INCHES)

Figure IV-2

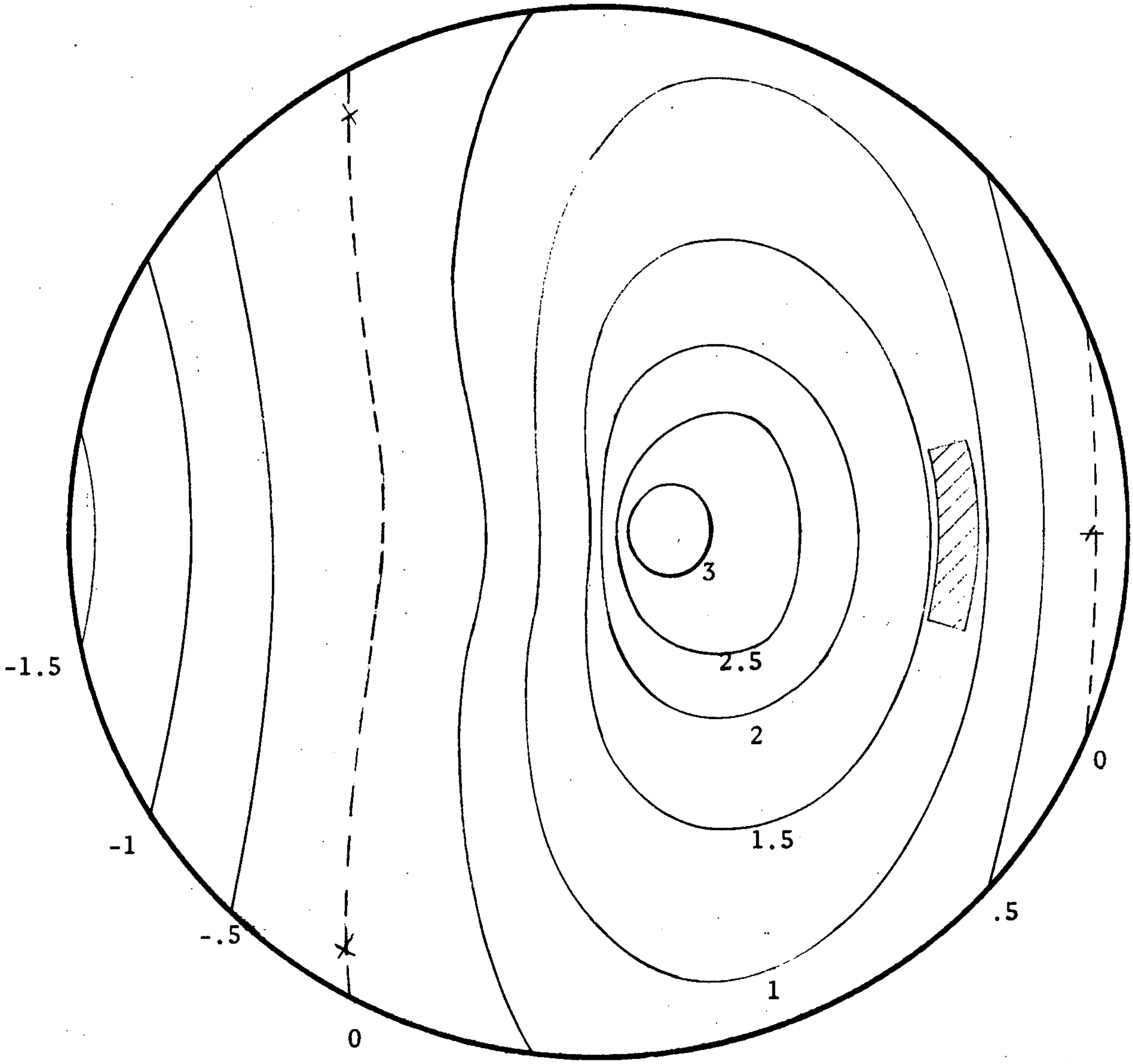


Heat Input: 10^{-3} BTU/HR•IN^2

Patch Location: IP = 5, KP = 3

STEADY STATE DEFLECTION (MICRO-INCHES)

Figure IV-3

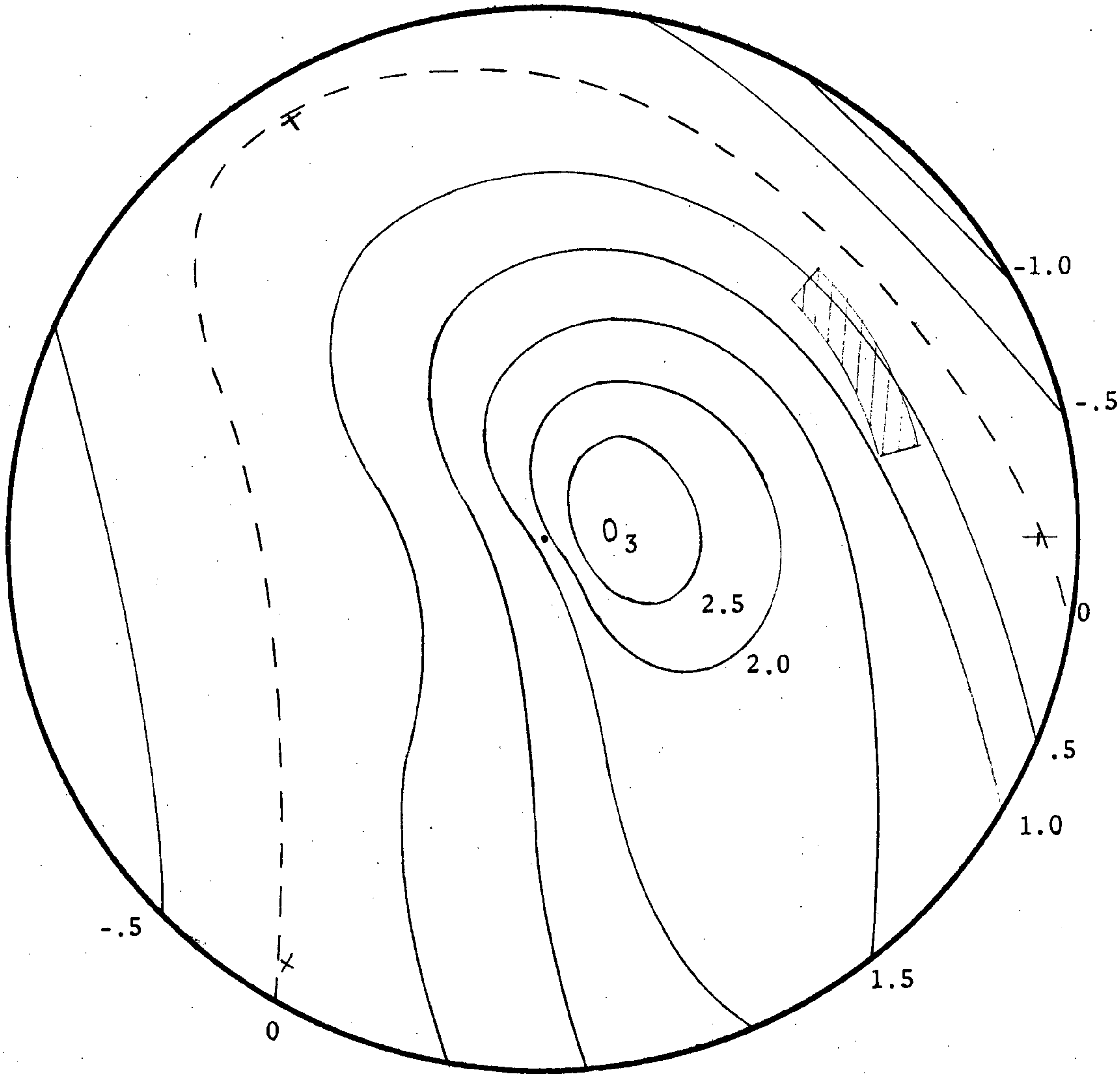


Heat Input 10^{-3} BTU/HR·IN^2

Patch Location IP = 10, KP = 1

STEADY STATE DEFLECTION (MICRO-INCHES)

Figure IV-4

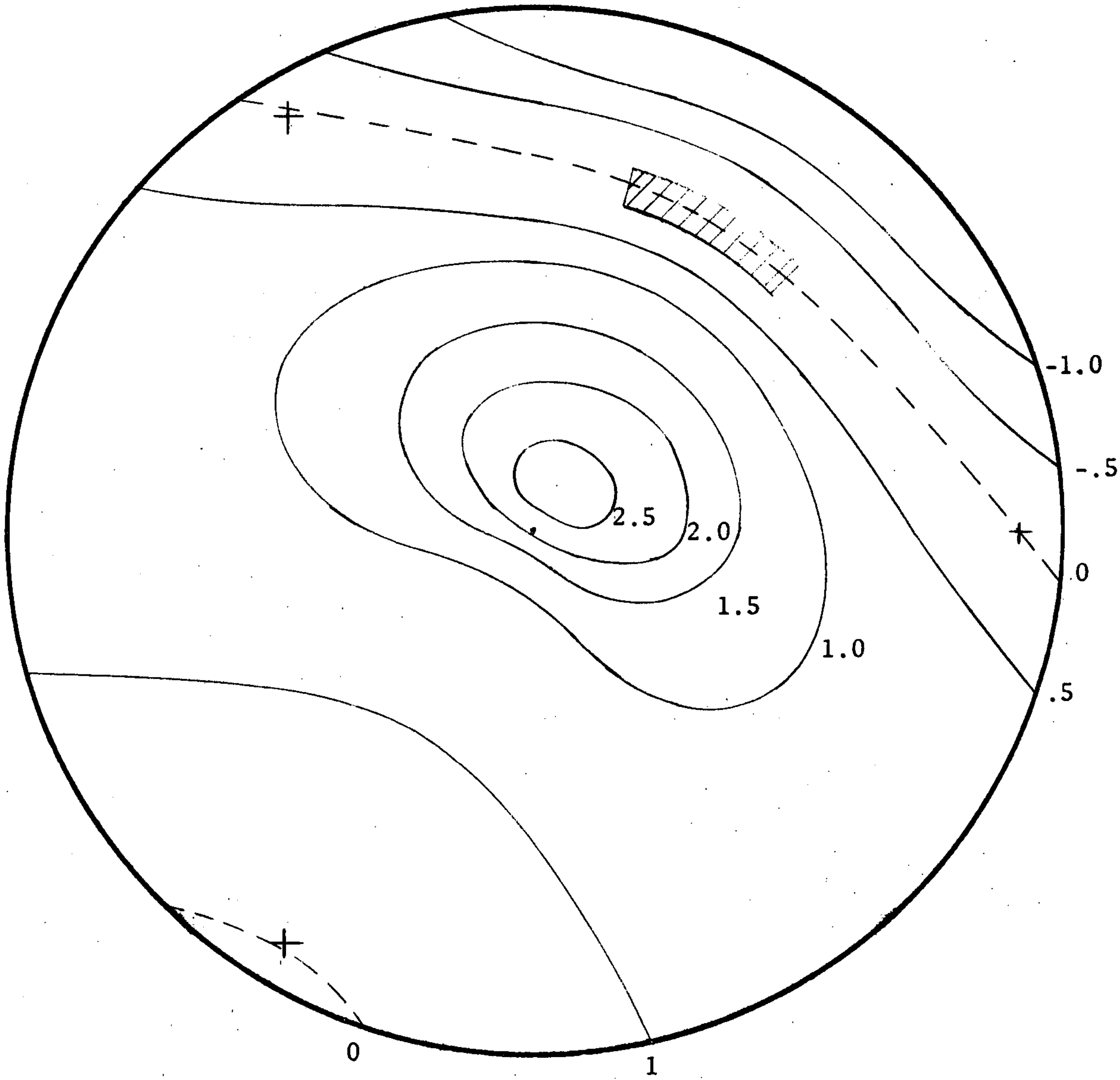


Heat Input 10^{-3} BTU/HR•IN^2

Patch Location IP = 10, KP = 2

STEADY STATE DEFLECTION (MICRO-INCHES)

Figure IV-5

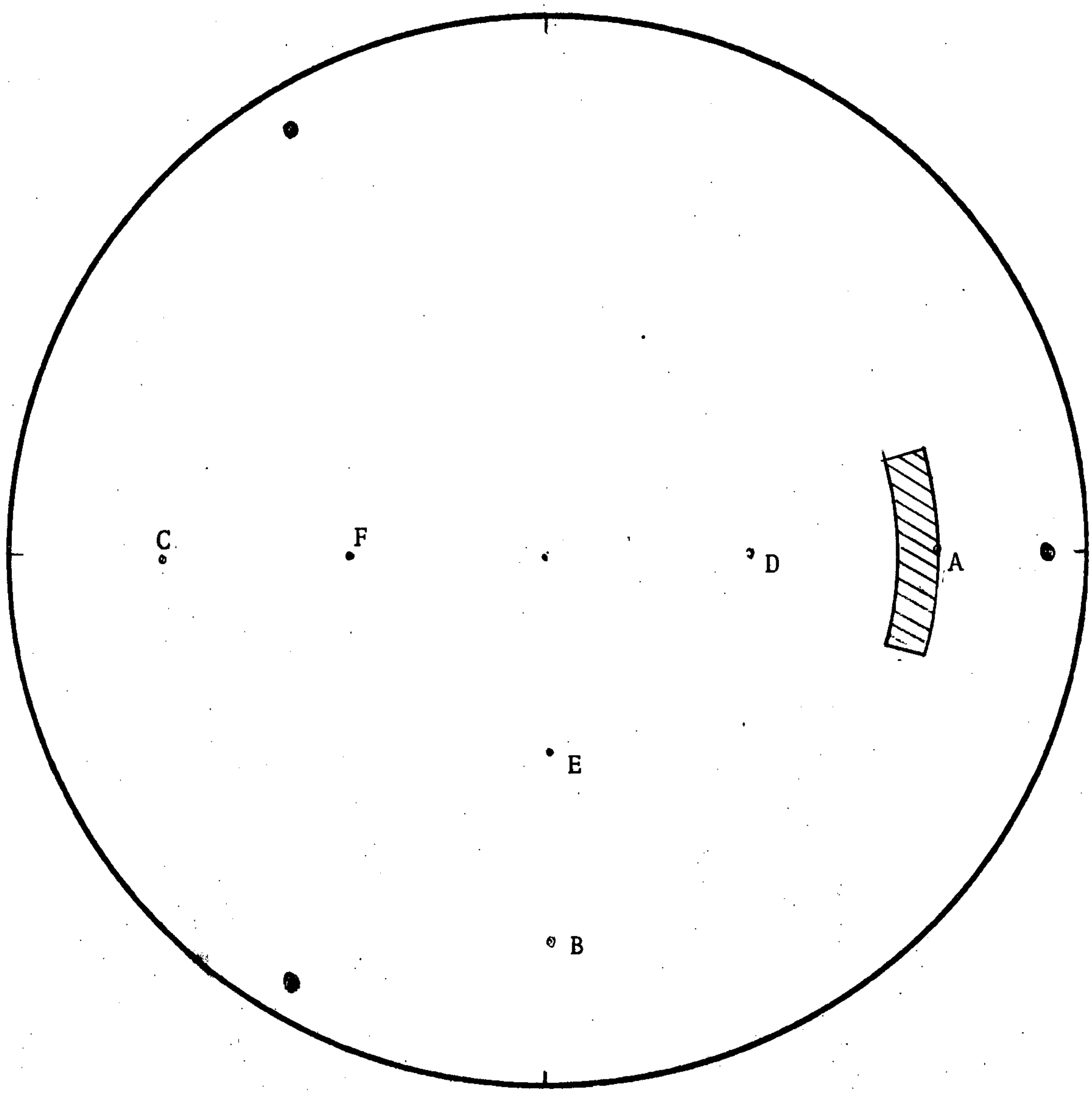


Heat Input 10^{-3} BTU/HR•IN²

Patch Location IP = 10, KP = 3

STEADY STATE DEFLECTION (MICRO-INCHES)

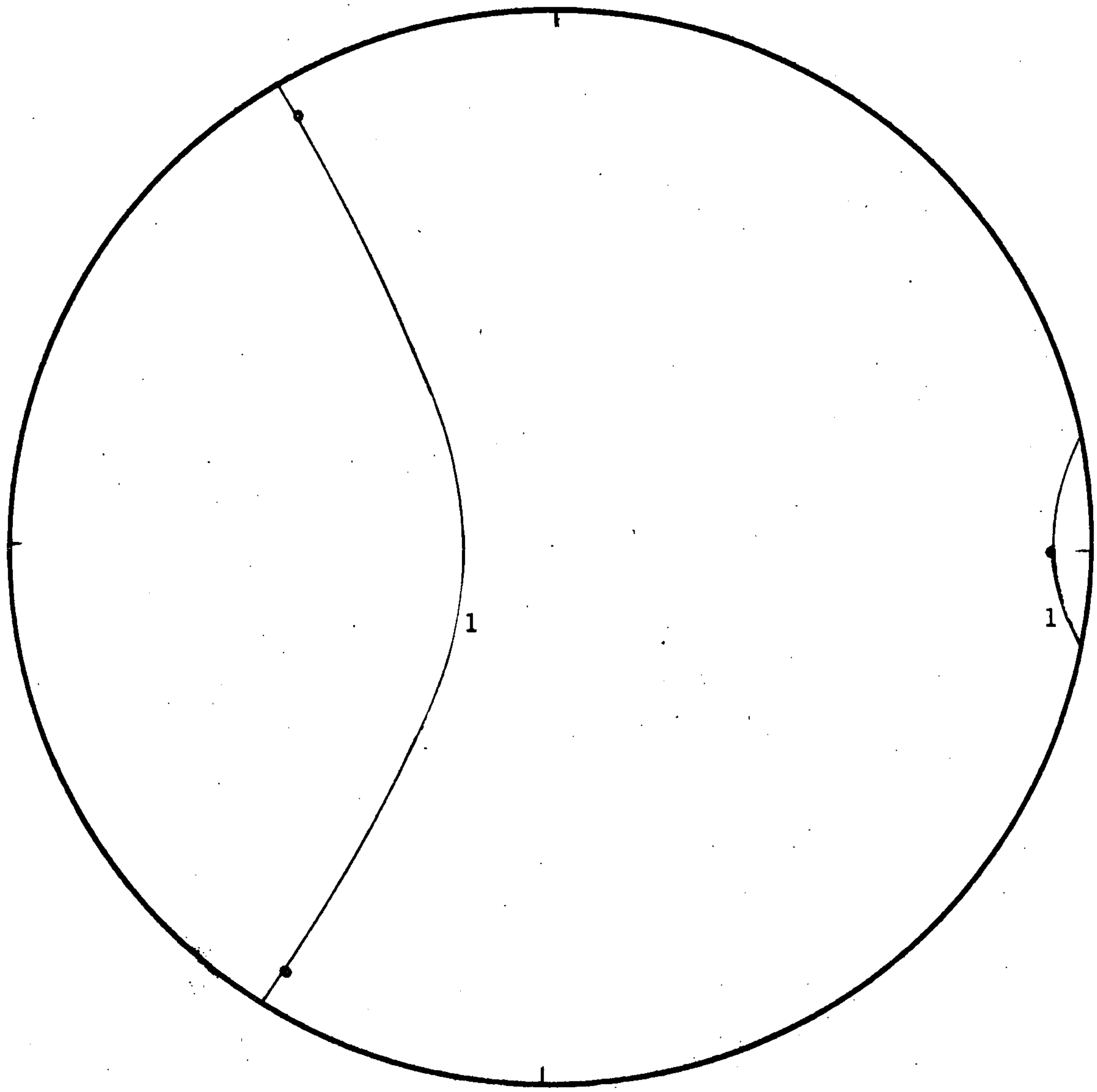
Figure IV-6



Transient Response

Heat Input 10^{-3} BTU/HR•IN^2

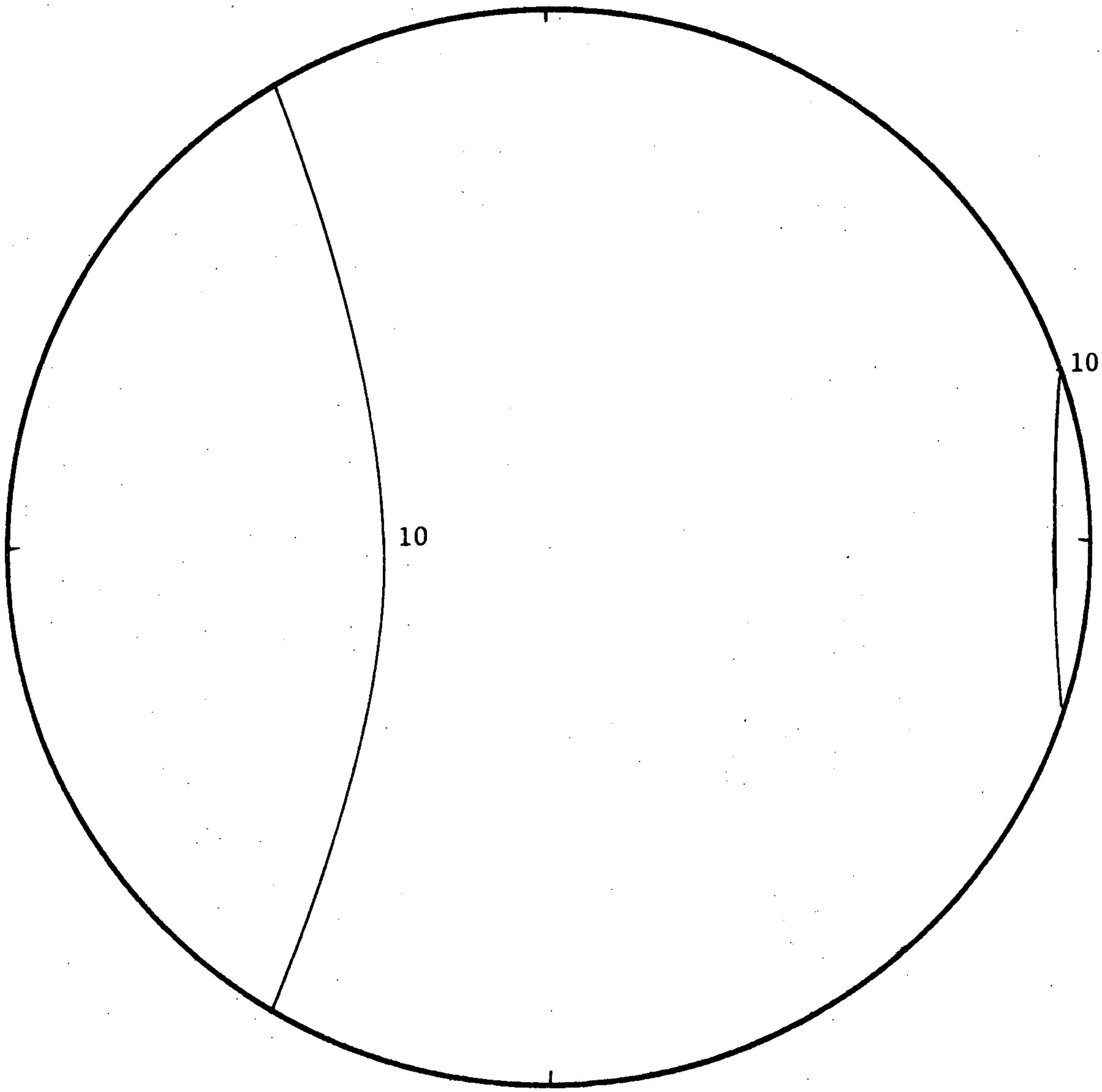
Figure IV-7



ZERO DEFLECTION CONTOUR

ONE HOUR

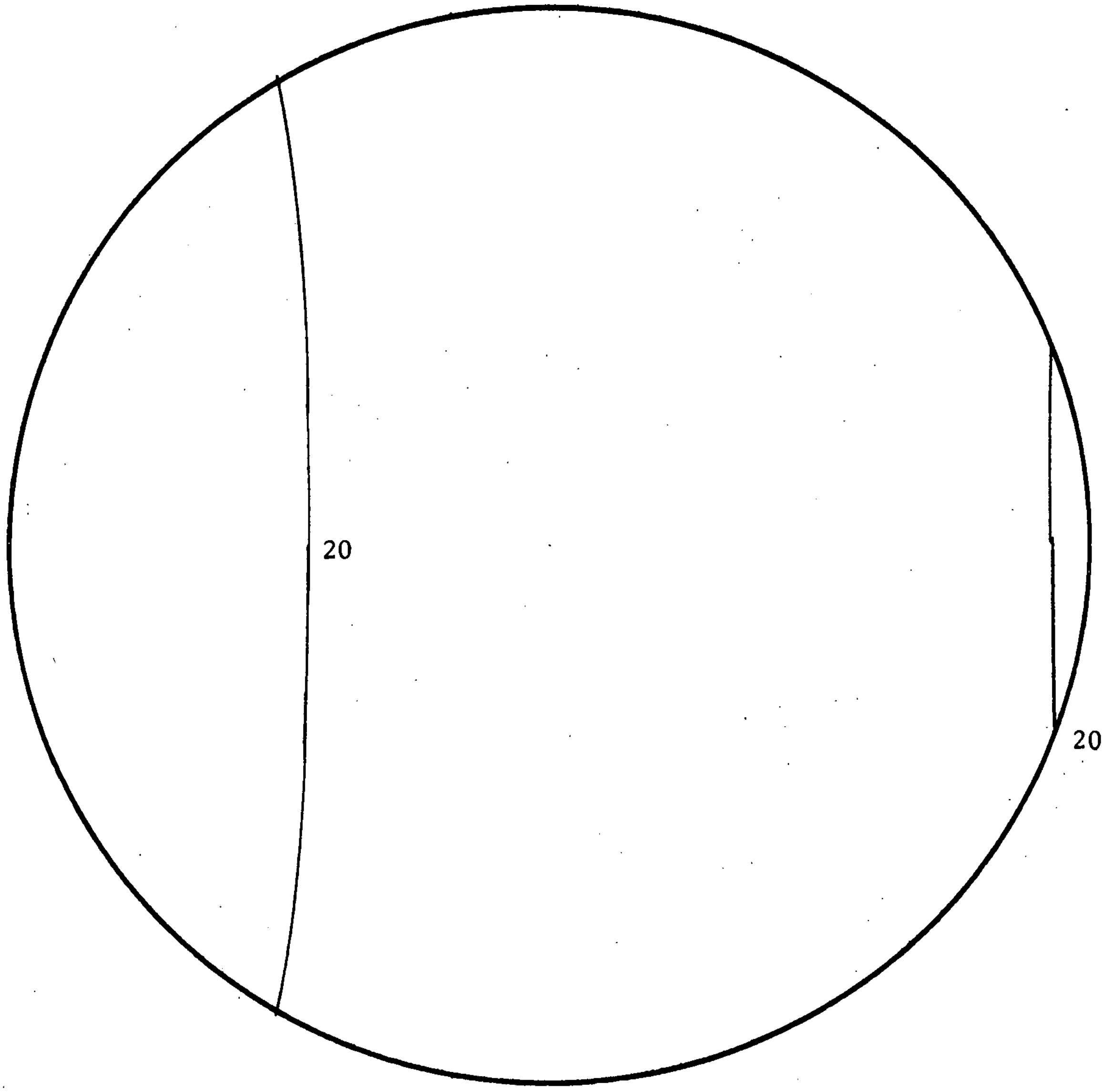
Figure IV-8(a)



ZERO DEFLECTION CONTOUR

TEN HOURS

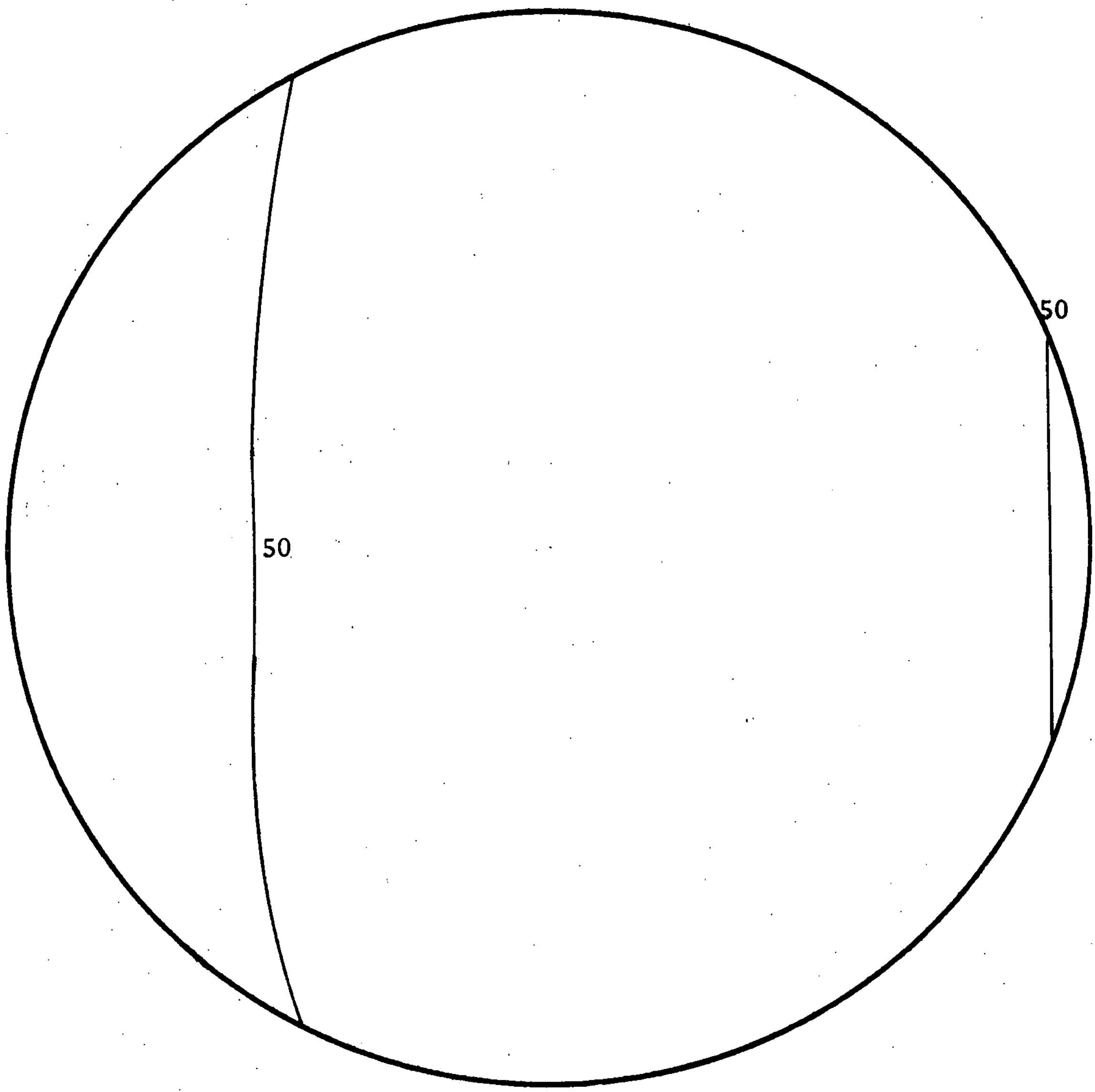
Figure IV-8(b)



ZERO DEFLECTION CONTOUR

TWENTY HOURS

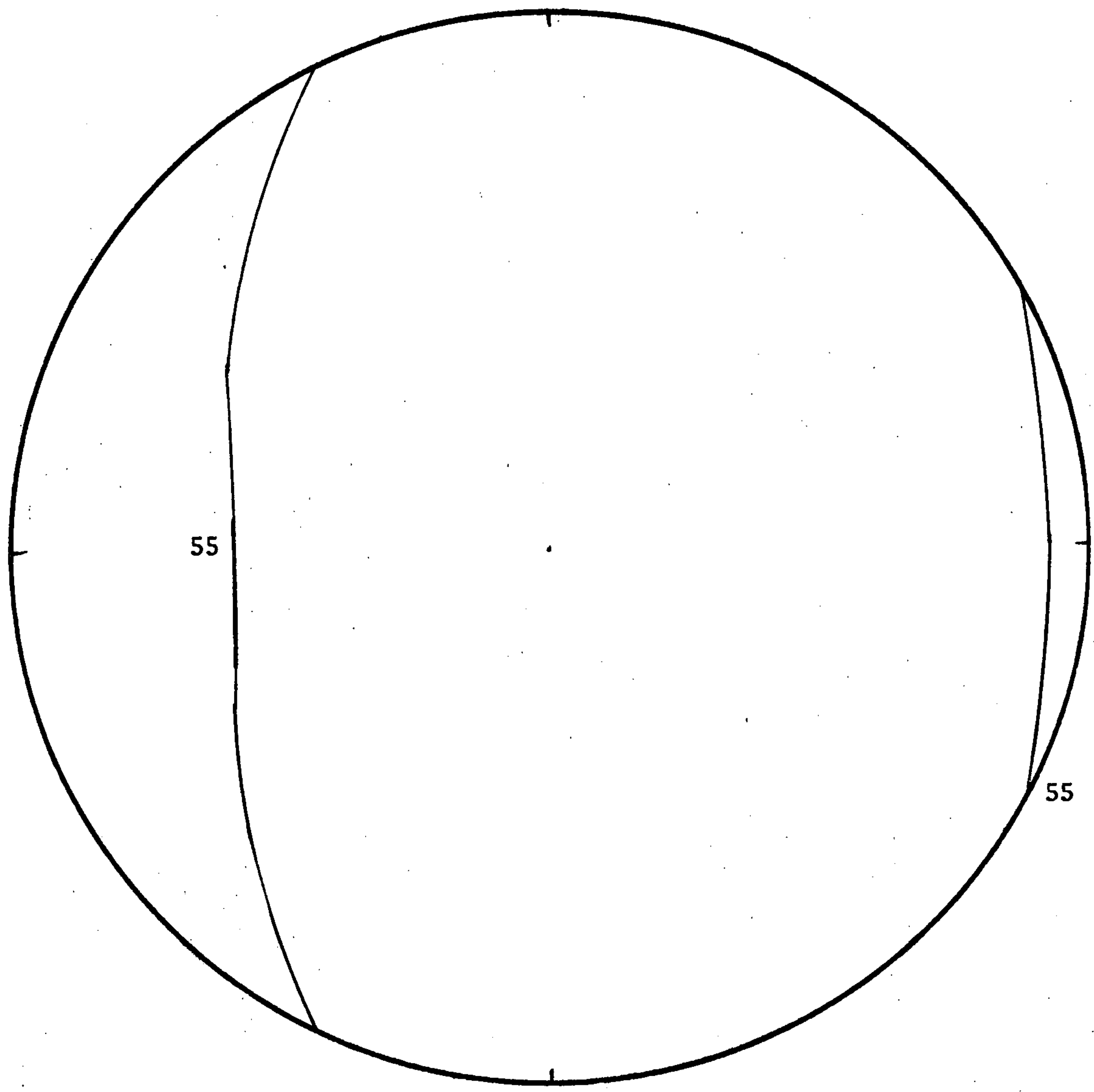
Figure IV-8(c)



ZERO DEFLECTION CONTOUR

FIFTY HOURS

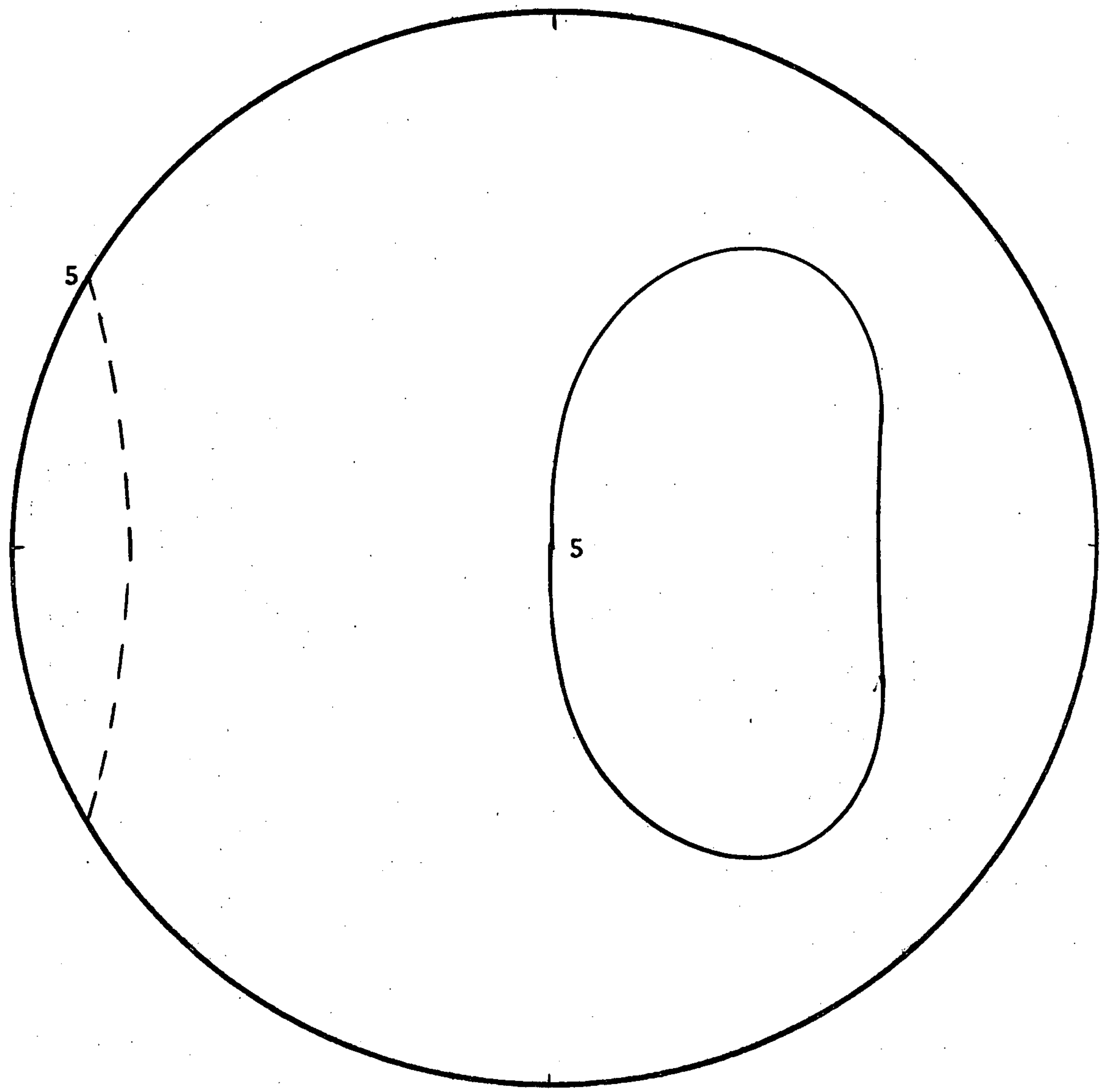
Figure IV-8(d)



ZERO DEFLECTION CONTOUR

FIFTY-FIVE HOURS

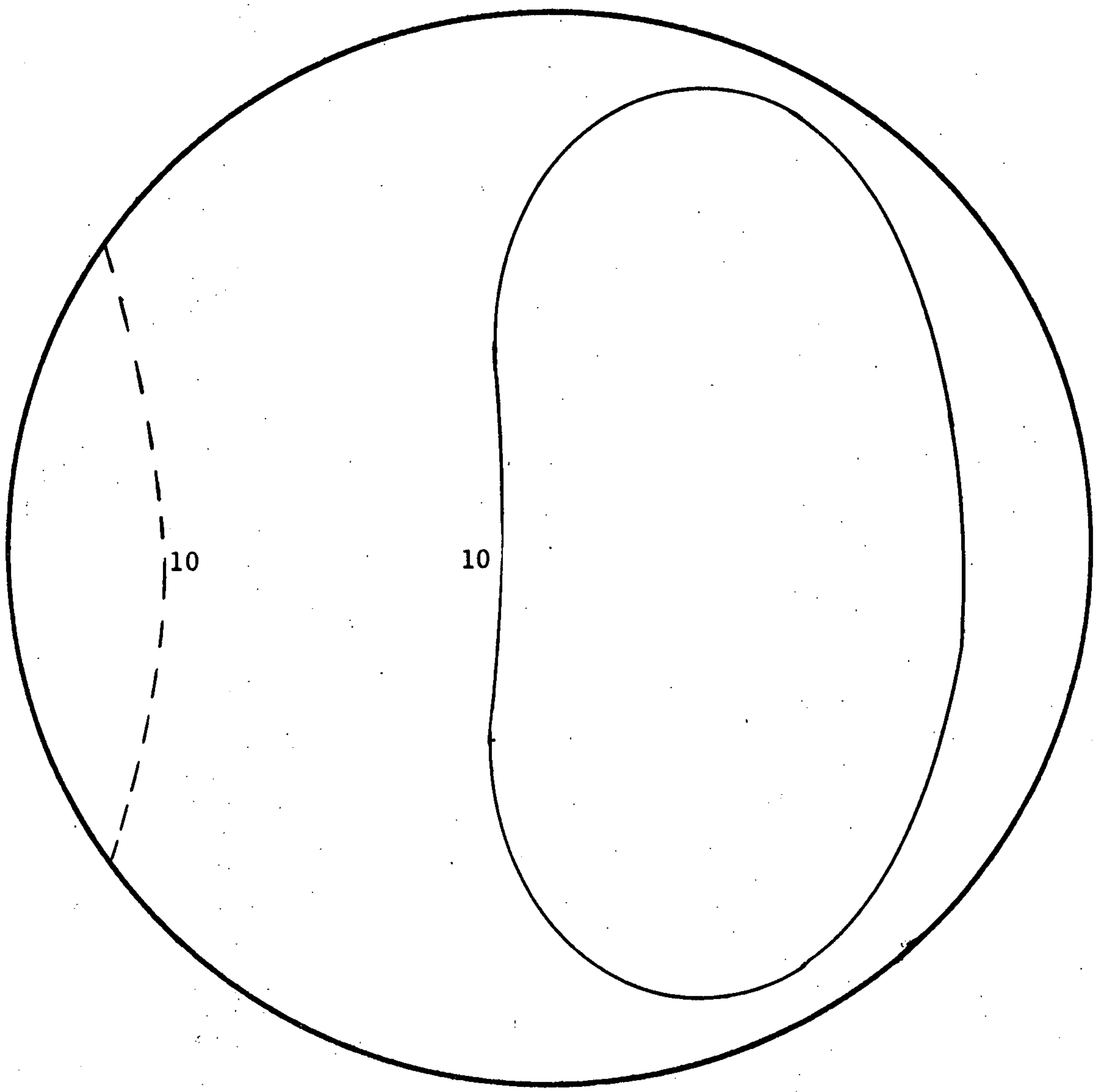
Figure IV-8(e)



1 μ MICRO-INCH CONTOUR

FIVE HOURS

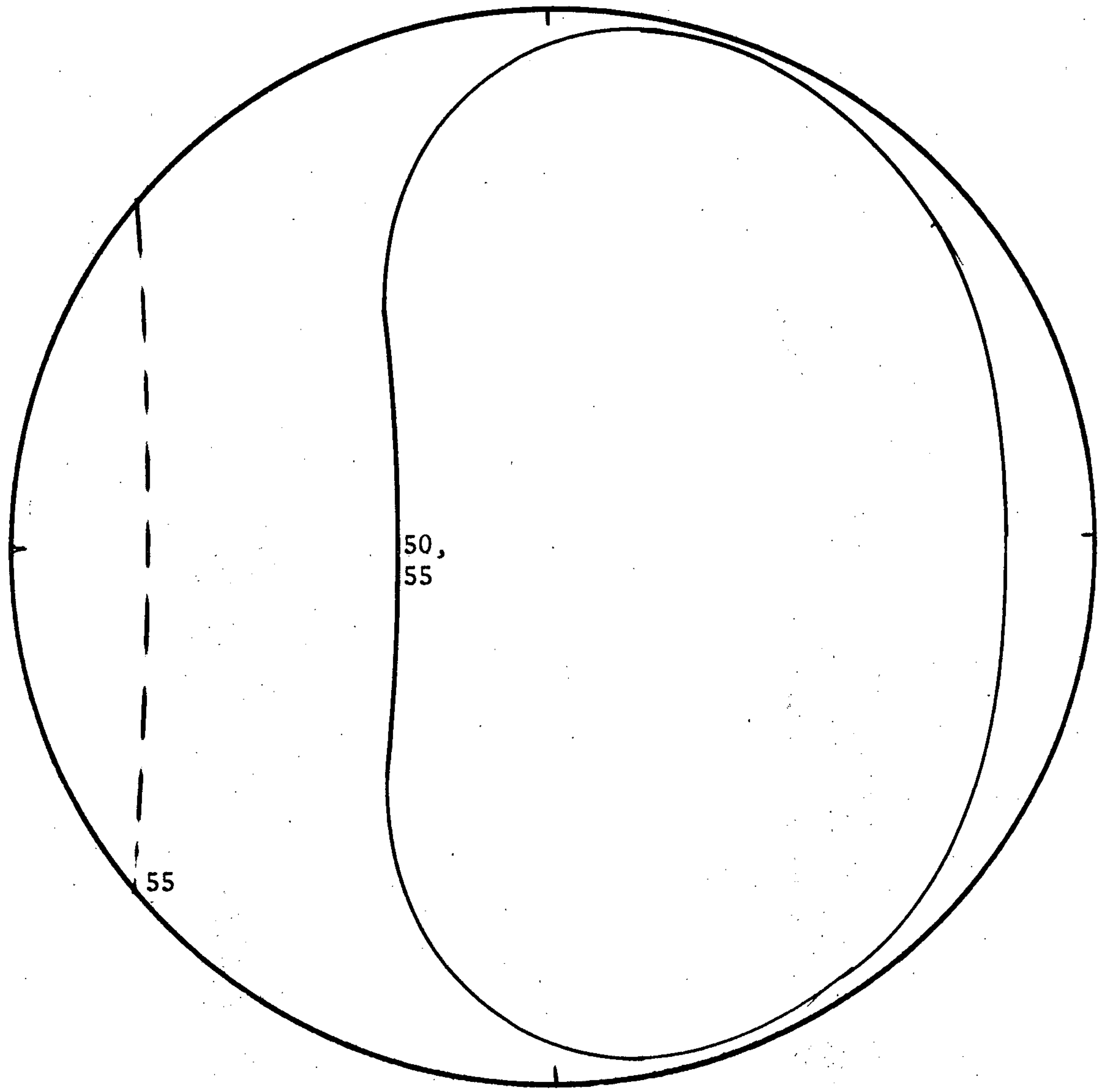
Figure IV-9(a)



1 μ INCH CONTOUR

TEN HOURS

Figure IV-9(b)



1 μ INCH CONTOUR

FIFTY AND FIFTY-FIVE HOURS

Figure IV-9(c)

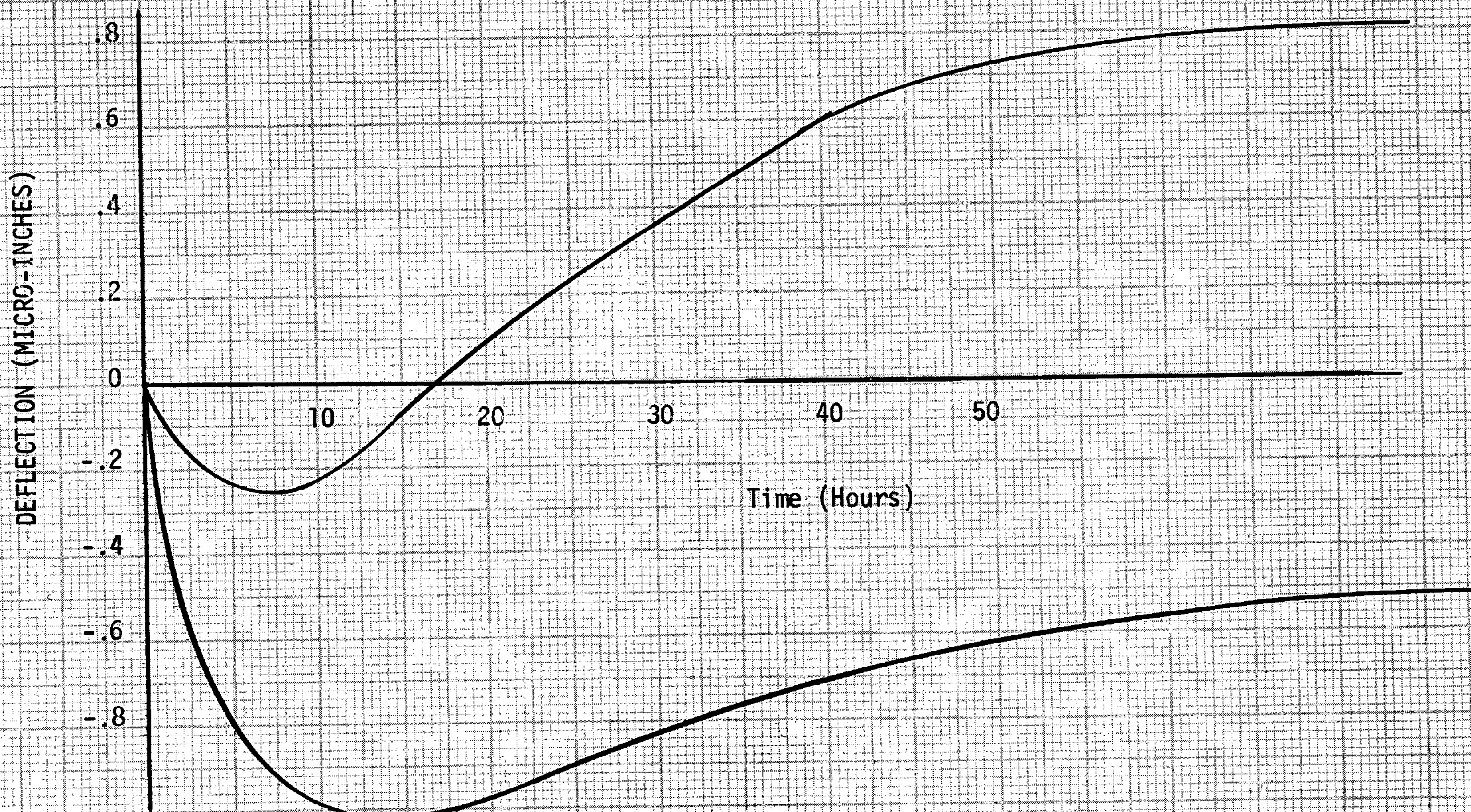


Figure IV-10

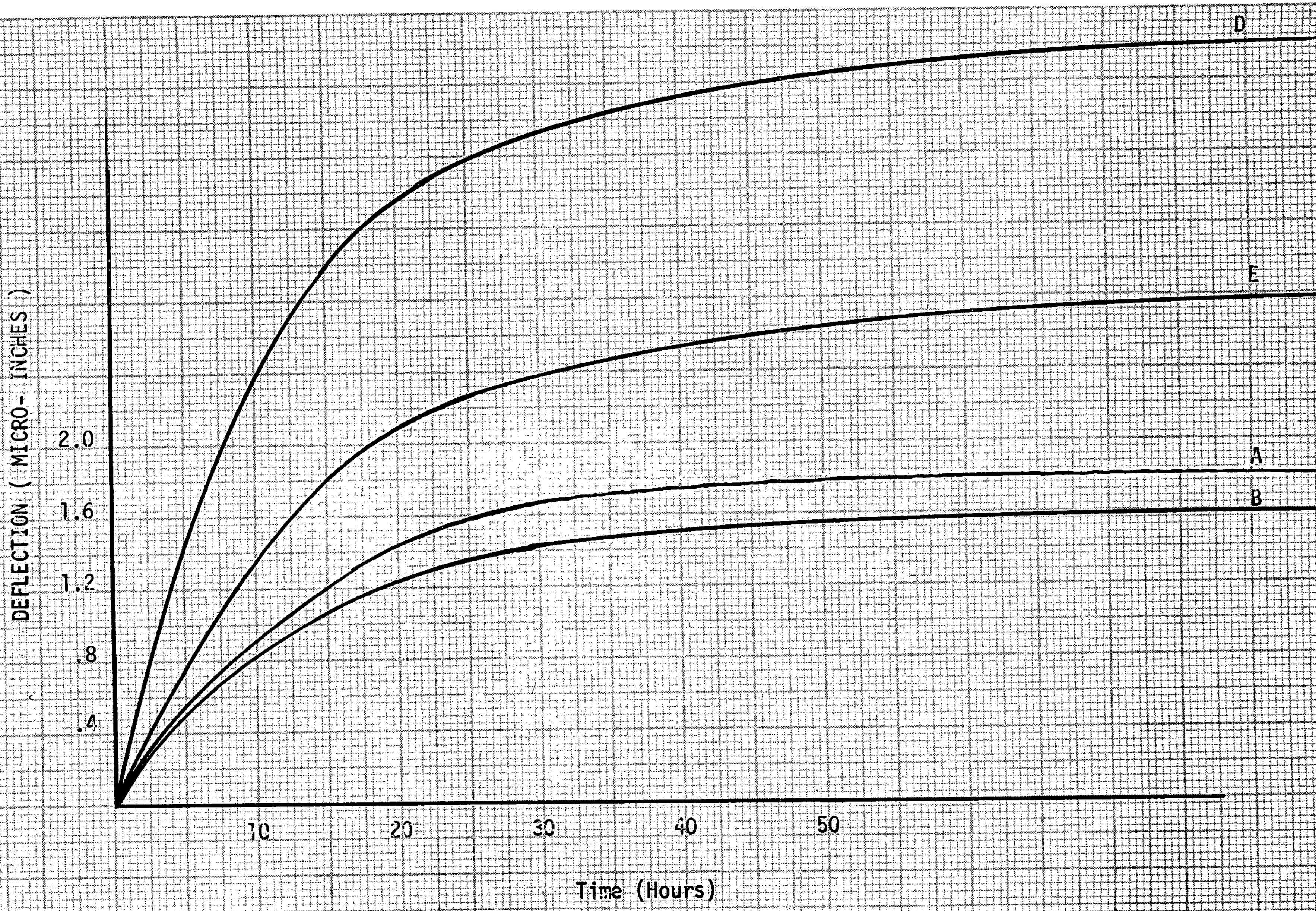

D
E
A
B
DEFLECTION (MICRO- INCHES)
2.0
1.6
1.2
.8
.4
10
20
30
40
50
Time (Hours)

Figure IV- 11

V. REFERENCES

1. Robertson, H.J., etal, "Active Optical System for Spaceborne Telescopes", Perkin Elmer Corporation, Norwalk, Conn., NASA CR 66297, Oct. 14, 1966.

2. ____________________, "Active Optical System for Spaceborne Telescopes", Perkin Elmer Corporation, Norwalk, Conn., NASA CR 66489, Dec. 7, 1967.

3. Crane, Robert, "An Experimental 20 Inch Segmented Active Mirror", IEEE Transactions on Aerospace and Electronic Systems, Vol. AES 5, March, 1969.

4. MacKinnon, D., etal., "Optical Mirror Figure Control", C.S. Draper Lab Report, R-665, Dec., 1970.

5. Wilson, Edward L., "Structural Analysis of Axisymmetric Solids", AIAA Journal, Vol. 3, No. 12, December 1965, pp. 2269-2275.

APPENDIX

PROGRAM LISTING

```
  MAIN
C       PROGRAM TO DETERMINE TEMPERATURE DISTRIBUTION AND
C       THERMAL DISTORTION OF A MIRROR
C
C       PARAMETERS
C       CP=SPECIFIC HEAT
C       CK=THERMAL CONDUCTIVITY
C       CF=RADIATION FACTOR FOR FRONT FACE
C       TO=REFERENCE TEMPERATURE
C       I,J=ROW AND COL INDICES OF GRID PT
C       R(I,J),Z(I,J)=COORDINATES OF GRID POINT
C       IM,JM=MAX VALUES OF I AND J
C       NC=NUMBER OF THE HARMONIC COMPONENT
C       TC(I)=TEMPERATURE COMPONENT
C       DELT=INTEGRATION TIME STEP
C       NTS=NUMBER OF TIME INTERVALS
C       ALF=LINEAR COEFFICIENT OF THERMAL EXPANSION
C       E=YOUNGS MODULUS OF ELASTICITY
C       V= POISSONS RATIO
C       DO=OUTSIDE DIAMETER OF MIRROR
C       DI=INSIDE DIAMETER OF MIRROR
C       H=THICKNESS OF MIRROR
C       FNO=F-NUMBER OF THE MIRROR (FOCAL LENGTH/DO)
        DIMENSION AS(15,15),DK(15,15),DKK(15,15),DKKK(15,15),IPIVOT(15),
       1U(15,15),INDEX(15,2),ZZ(15,1),SPP(15),SPPP(15),Q2(15,15),
       2SAP(15),SS(15),SSS(15),SP(15,15),P1(15,15,15)
        DIMENSION D(75),R(15,5),Z(15,5),C(75,75,2),QR(15),QB(15,14),
       1TC(75),X(75),A(75,75),Q(75),Q1(75),P(14),F(15,15)
        DIMENSION WF(15,12),WB(12),TT(15,12)
        DIMENSION HEAT(100)
        DIMENSION PAN(14)
        INTEGER S1,S2,S3
        COMMON R,Z
        COMMON/STIF/KD,KS,PD,PS,PL,A
        COMMON /CND/C,D,QR,QB
        DIMENSION KD(15,15,15),KS(15,15,15),PD(15,5,15),PS(15,5,14),
       3PL(15,5,14),KL(15,15,15)
        REAL KD,KS,KL
C
C     LOOP TO INPUT ALL THE PATCHES
C
        READ (5,3100)NUMPAT
  3100  FORMAT (I5)
        DO 72 NU=1,NUMPAT
        WRITE(6,6500)
  6500  FORMAT(1H1)
        READ(5,100)IM,JM,CK,CP,CF,TO,DELT,NTS,IPRINT,TI
   100  FORMAT(2I5,5F10.5,2I5,F10.5)
        READ(5,101)ALF,E,V
   101  FORMAT(3F10.5)
        WRITE (6,102)IM,JM,CK,CP,CF,TO,DELT,NTS
        WRITE(6,107) ALF,E,V
        READ(5,2600)NM,IS,KM,S2,S3,INFLU,IX
  2600  FORMAT(7I5)
        NMP=NM+1
        WRITE(6,2800)TI
  2800  FORMAT(27H INITIAL TEMPERATURE, TI = ,F10.5/////)
```

```
      WRITE(6,2601)NMP,KM,IS,S2,S3,IX
 2601 FORMAT(33H NUMBER OF HARMONIC COMPONENTS = ,I5/
     137H NUMBER OF ANGULAR PATCH POSITIONS = ,I5/
     231H RADIAL NODE OF SUPPORT RING = ,I5/
     348H ANGULAR POSITIONS OF SUPPORTS ARE,K =  1,  K = ,I2,2X,
     410H AND, K = ,I2/
     56H IX = ,I5/)
      N1=3
      MM=15
      LL=15
      MAT=IM
      FKM=KM
      NTP=NTS+1
      IM1=IM-1
      M=IM*JM
      FJM1=JM-1
      FIM1=IM1
      IF(INFLU.GT.0)GO TO 2500
      READ(5,2602)IP,KP,PA,PH
 2602 FORMAT(2I5,2F10.5)
      WRITE(6,2603)IP,KP,PA,PH
 2603 FORMAT(52H THERMOELASTIC RESPONSE OF THE MIRROR IS CALCULACTED/
     124H RADIAL NODE OF PATCH = ,I5/
     229H ANGULAR POSITION OF PATCH = ,I5/
     315H PATCH ANGLE = ,F10.5/
     414H PATCH HEAT = ,F10.5)
      GO TO 2501

 2500 READ(5,2604)(PAN(I),I=1,IM1)
 2604 FORMAT(7F10.5)
      WRITE(6,2605)(I,PAN(I),I=1,IM1)
 2605 FORMAT(72H THERMOELASTIC INFLUENCE COEFFICIENTS ARE CALCULATED AND
     1 WRITTEN ON FILE/
     24X,1HI,5X,11HPATCH ANGLE/
     3(I5,5X,F10.5))
 2501 CONTINUE
      PI=3.1415927
C
C     GRID GENERATION FOR SPHERICAL MIRROR
C
      READ(5,112)DO,DI,H,FNO
      WRITE (6,113) DO,DI,H,FNO
      IF(FNO.GT.99)    GO TO 201
      RF=2.*DO*FNO
      RB=RF+H
      DR=H/FJM1
      SI=.5*DI/RF
      CI=SQRT(1.-SI*SI)
      SO=.5*DO/RF
      CO=SQRT(1.-SO*SO)
      THETI=ATAN(SI/CI)
      THETO=ATAN(SO/CO)
      DTHET=(THETO-THETI)/FIM1
      EM=THETI
      DO 39 I=1,IM
      RR=RB
      DO 38 J=1,JM
```

```
      R(I,J)=RR*SIN(EM)
      Z(I,J)=RB-RR*COS(EM)
   38 RR=RR-DR
   39 EM=EM+DTHET
      GO TO 202
  201 DH=.5*(DO-DI)/FIM1
      DZ=H/FJM1
      DO 200 I=1,IM
      FI=I-1
      DO 200 J=1,JM
      FJ=J-1
      R(I,J)=0.5*DI+FI*DH
  200 Z(I,J)=FJ*DZ
 202  CONTINUE
C
C     CALL SUBROUTINE TO CALCULATE C,D,KD,KS,PD,PS,PL
C
      CALL CAND (IM,JM,M,CK,CP,CF)
C
COMPONENTS OF PATCH HEAT
C
      IF(INFLU.LE.0)GO TO 2502
      IP=1
 2505 PA=PAN(IP)
      IP1=IP+1
      AP=PA*3.14159265/360.*(R(IP1,1)**2-R(IP,1)**2)
      PH=-1./AP
 2502 CONTINUE
      DO 2000 N=1,NM
      FN=N
      HEATO=PH*PA/360.
 2000 HEAT(N)=(2.*PH*SIN(FN*PA*PI/360.))/(FN*PI)
C
C  INITIALIZE WF AND WB
C
      DO 2001 K=1,KM
      WB(K)=0.0
      DO 2001 I=1,IM
      TT(I,K)=0.0
 2001 WF(I,K)=0.0
C
C    START MODE LOOP
C
      IF(INFLU.GT.0) GO TO 6000
      WRITE(6,500)
  500 FORMAT(/////31H CONVERGENCE OF MODAL RESPONSES//
     11X,4HMODE,11X,10HHEAT INPUT,4X,1HI,4X,1HJ,10X,10HDEFLECTION,
     29X,11HTEMPERATURE)
 6000 DO 2002 NT=1,NMP
      N1=3
      NC=NT-1
      IF(NC.EQ.0)N1=2
      JM3=N1*JM
      LCV=JM3
      IJM=IM*JM3
      CALL STIFF(E,V,ALF,IM,JM,NC,CK,CF)
```

```
C
C       REARRANGE KD AND KS AND FORM KL FOR POTTER
C
      DO 50 I=1,IM
      DO 50 L1=1,JM3
      DO 50 L2=1,JM3
   50 KL(L1,L2,I)=KD(L1,L2,I)
      DO 51 I=1,IM
      DO 51 L1=1,JM3
      DO 51 L2=1,JM3
   51 KD(I,L1,L2)=KL(L1,L2,I)
      DO 52 I=1,IM
      DO 52 L1=1,JM3
      DO 52 L2=1,JM3
   52 KL(L1,L2,I)=KS(L1,L2,I)
      DO 53 I=1,IM
      DO 53 L1=1,JM3
      DO 53 L2=1,JM3
   53 KS(I,L1,L2)=KL(L1,L2,I)
      DO 54 I=2,IM
      DO 54 L1=1,JM3
      DO 54 L2=1,JM3
   54 KL(I,L1,L2)=KS(I-1,L2,L1)
C       INITIAL CONDITIONS ON TEMPERATURE
C
      IF(NC.NE.0) GO TO 131
      DO 37 I=1,M
   37 TC(I)=TI
      GO TO 132
  131 DO 133 I=1,M
  133 TC(I)=0.0
  132 CONTINUE
C
C       INITIALIZE DISPLACEMENTS
C
      DO 24 L1=1,IM
      DO 24 L2=1,JM3
   24 U(L1,L2)=0.0
C
C       RADIATION FROM FRONT
C
      FC=NC
      DO 20 L1=1,M
      X(L1)=0.0
   20 Q1(L1)=0.0
      IF(NC.NE.0) GO TO 22
      DO 21 L1=1,IM
      LR=L1*JM
   21   Q1(LR)=Q1(LR)+   TO*QR(L1)
   22 CONTINUE
C
C       SET UP AND INVERT EFFECTIVE CONDUCTIVE MATRIX IN A
C
      DO 2 L1=1,M
      DO 2 L2=1,M
      A(L1,L2)=C(L1,L2,1)+FC*FC*C(L1,L2,2)
```

```
      IF(L1.EQ.L2)A(L1,L2)=A(L1,L2)+2.0*D(L1)/DELT
    2 CONTINUE
      CALL INVERT(A,M,75)
C
C     INITIALIZE SOLUTION AND HEAT INPUT
C

      DO 99 I=1,IM1
   99 P(I)=0.0

C
C     STEP BY STEP SOLUTION
C
      IPR=0
      IF(CP.GT.0.0001) GO TO 73
      NTP=2
      IPRINT=1
 73   DO 6 II=1,NTP
      DO 45 K1=1,M
 45   X(K1)=0.0
      FI=II-1
       TIME=FI*DELT
      IF(II.EQ.1)GO TO 40
      IPR=IPR+1
C
C     HEAT INPUT ON BACK
C
      IF(NC .NE. 0) P(IP)=HEAT(NC)
      IF(NC .EQ. 0) P(IP)= HEATO
      DO 28 L=2,IM1
      IJ=(L-1)*JM+1
   28 Q1(IJ)=QB(L,L-1)*P(L-1)+QB(L,L)*P(L)
      Q1(1)=QB(1,1)*P(1)
      I2=M+1-JM
      Q1(I2)=QB(IM,IM1)*P(IM1)
C
C     SET UP THE COMBINE HEAT INPUT VECTOR
C
      DO 3 L1=1,M
    3 Q(L1)=2.0*D(L1)*TC(L1)/DELT +Q1(L1)
C
C     CALCULATE THE NEW TEMPERATURE
C
 5002 DO 10 L1=1,M
      DO 10 L2=1,M
 10   X(L1)=X(L1)+A(L1,L2)*Q(L2)
      IF(CP.LT.0.0001) GO TO 70
      DO 4 L1=1,M
    4 TC(L1)=-TC(L1)+2.*X(L1)
      IF(IPR.NE.IPRINT)GO TO 6
      IPR=0
 5001 CONTINUE
      IF(CP.GT.0.0001)GO TO 5000
 70   DO 62 L1=1,M
   62 TC(L1)=X(L1)

C
```

```
C     SET UP THE THERMAL FORCE VECTOR
C
 5000 IF(NC.NE.0) GO TO 71
      DO 55 L1=1,M
   55 TC(L1)=TC(L1)-T1
   71 DO 41 L1=1,IM
      DO 41 L2=1,JM3
   41 F(L1,L2)=0.0
      DO 43 I1=1,IM
      DO 43 L2=1,JM
      DO 43 J=1,JM
      DO 42 K=1,N1
      L1=N1*(J-1)+K
      L3=JM*(I1-1)+L2
      L4=L3+JM
      L5=L3-JM
      F(I1,L1)=-PD(L1,L2,I1)*TC(L3)+F(I1,L1)
      IF(I1.NE.IM)F(I1,L1)=F(I1,L1)-PS(L1,L2,I1)*TC(L4)
      IF(I1.EQ.1)GO TO 42
      F(I1,L1)=F(I1,L1)-PL(L1,L2,I1-1)*TC(L5)
   42 CONTINUE
  43  CONTINUE
C
C     IMPOSE BOUNDARY CONDITIONS
C
      N=NC
      IF(N.NE.0)GO TO 49
      DO 44 L1=1,JM3
      IF(IX.NE.1)KS(IX-1,L1,2)=0.
      IF(IX.NE.IM)KL(IX+1,L1,2)=0.
      KD(IX,L1,2)=0.
      KS(IX,2,L1)=0.0
      IF(IX.NE.1)KL(IX,2,L1)=0.0
   44 KD(IX,2,L1)=0.0
      KD(IX,2,2)=1.0
      F(IX,2)=0.0
      GO TO 48
   49 IF(N.NE.1)GO TO 48
      DO 47 L2=1,3
      DO 46 L1=1,JM3
      KS(IX,L2,L1)=0.0
      IF(IX.NE.1)KL(IX,L2,L1)=0.0
   46 KD(IX,L2,L1)=0.0
      KD(IX,L2,L2)=1.0
   47 F(IX,L2)=0.0
   48 CONTINUE
      DO 9000 L1=1,JM3
      DO 9000 L2=1,JM3
 9000 AS(L1,L2)=KL(2,L1,L2)
      WRITE(6,9002)((F(L1,L2),L2=1,15),L1=1,15)
 9002 FORMAT(15E8.3)
      CALL MATINV(AS,JM3,SS,0,DETM,IPIVOT,INDEX,MM,ISCALE)
      WRITE(6,6001)N,DETM
 6001 FORMAT(5H N = ,I2,5X,7HDETM = ,E20.8)
C     CALCULATE DISPLACEMENTS
C
      CALL POTTER (KL,KD,KS,F,U,MAT,LCV,AS,DK,DKK,DKKK,IPIVOT,INDEX,ZZ,
```

```
     4SPP,SPPP,Q2,P1,SAP,SS,SP,SSS,MM,LL)
 40    CONTINUE
       WRITE(6,9003)((U(L1,L2),L2=1,15),L1=1,15)
 9003 FORMAT(7H U(I,J)/(15E8.3))
    6 CONTINUE
C
COMPUTE WB AND WF
C
      IF(NC.NE.0)GO TO 4000
      DO 4001 L1=1,M
 4001 TC(L1)=TC(L1)+TI
 4000 CONTINUE
      DO 2003 K=1,KM
      FN=NC
      FK=K
      WB(K)=WB(K)+U(IS,N1)*COS(FN*(FK-1.)*PI*2./FKM)
      DO 2003 I=1,IM
      IT=I*JM
      TT(I,K)=TT(I,K)+TC(IT)*COS(FN*(FK-1.)*PI*2./FKM)
 2003 WF(I,K)=WF(I,K)+U(I ,JM3)*COS(FN*(FK-1.)*PI*2./FKM)
C
C     OUTPUT RESULTS OF A MODE
C
      IF (INFLU.GT.0)GO TO 2002
      I=1
      J=JM
      JJ=J*N1
      JJJ=JM*(I-1)+J
      WRITE(6,501) NC,P(IP),I,J,U(I,JJ),TC(JJJ)
  501 FORMAT(I5,E20.8,2I5,2E20.8)
 2702 CONTINUE
 2002 CONTINUE
      DTHD=360./FKM
      IF(INFLU.GT.0)GO TO 3000
      WRITE (6,1501)
      DO 1006 K=1,KM
      WRITE(6,1505)
      FK=K-1
      THD=FK*DTHD
      DO 1006 I=1,IM
 1006 WRITE(6,1502)I,K,R(I,JM),THD,WF(I,K),TT(I,K)
      WRITE(6,1503)
      DO 1007 K=1,KM
      FK=K-1
      THD=FK*DTHD
 1007 WRITE(6,1504)K,RS,THD,WB(K)
C
C     ROTATION OF OUTPUT FOR SUPPORTS
C
 3000 CONTINUE
      DIMENSION WBR(12),WFR(15,12)
      S1=1
      IF(INFLU.LE.0)GO TO 2503
      KP=1
 2503 CONTINUE
      FKM=KM
      FS2=S2
```

```
      FS3=S3
      DTHET=2.*PI/FKM
      DTHD=360./FKM

      KR=KP-1
      DO 1001 K=1,KM
      KR=KR+1
      IF (KR.EQ. KM+1) KR=1
      DO 1002 I=1,IM
 1002 WFR(I,KR)=WF(I,K)
 1001 WBR(KR)=WB(K)
 1000 CONTINUE
      W1=WBR(S1)
      W2=WBR(S2)
      W3=WBR(S3)
      RS=R(IS,1)
      THET2=(FS2-1.)*DTHET
      THET3=(FS3-1.)*DTHET
      X1=RS
      X2=RS*COS(THET2)
      X3=RS*COS(THET3)
      Y2=RS*SIN(THET2)
      Y3=RS*SIN(THET3)
      DET=Y3*(X2-X1)+Y2*(X1-X3)
      IF(DET.GT.0.0001)GO TO 1003
      WRITE(6,1510)
 1510 FORMAT(23H SUPPORTS LOCATED WRONG)
      GO TO 72
 1003 WO=((X2*Y3-X3*Y2)*W1-X1*Y3*W2+X1*Y2*W3)/DET
      THX=((X3-X2)*W1+(X1-X3)*W2+(X2-X1)*W3)/DET
      THY=((Y3-Y2)*W1-Y3*W2+Y2*W3)/DET
      IF (INFLU.GT.0) GO TO 3001
      WRITE(6,1500) WO,THX,THY
 3001 CONTINUE
 1500 FORMAT(//35H RIGID BODY DISPLACEMENT PARAMETERS/17H Z-TRANSLATION
     1= ,E15.8,5X,13HX-ROTATION = ,E15.8,5X,13HY-ROTATION = ,E15.8//)
      DO 1004 K=1,KM
      FK=K
      DO 1005 I=1,IM
      RI=R(I,JM)
      TK=(FK-1.0)*DTHET
      XIK=RI*COS(TK)
      YIK=RI*SIN(TK)
 1005 WFR(I,K)=WFR(I,K)-WO-YIK*THX+XIK*THY
      XIS=RS*COS(TK)
      YIS=RS*SIN(TK)
 1004 WBR(K)=WBR(K)-WO-YIS*THX+XIS*THY
      IF(INFLU.GT.0)GO TO 3002
      WRITE (6,1501)
      DO 1506 K=1,KM
      WRITE(6,1505)
      FK=K-1
      THD=FK*DTHD
      DO 1506 I=1,IM
 1506 WRITE(6,1502)I,K,R(I,JM),THD,WFR(I,K),TT(I,K)
 3002 CONTINUE
      IF(INFLU.LE.0)GO TO 72
```

```
      WRITE(6,5099) IP,KP
5099  FORMAT(6H IP = ,I5,5X,5HKP = ,I5)
      WRITE(4,1600)((WFR(I,K),I=1,IM),K=1,KM)
1600  FORMAT(6E13.8)

      WRITE(4,7006)((WFR(I,K),I=1,IM),K=1,KM)
7006  FORMAT(6E13.8)
7005  CONTINUE
      IF (INFLU.GT.0) GO TO 3003
      WRITE (6,1503)
      DO 1507 K=1,KM

      FK=K-1
      THD=FK*DTHD
1507  WRITE(6,1504)K,RS,THD,WBR(K)
1501  FORMAT (29H Z-DISPLACEMENTS ON THE FRONT //4X,1HI,4X,1HK,10X,
     11HR,14X,5HTHETA,12X,1HW,16X,11HTEMPERATURE)
1502  FORMAT(2I5,2E15.5,2E20.8)
1505  FORMAT (//)
1503  FORMAT(//50H Z-DISPLACEMENTS AT THE SUPPORT RADIUS ON THE BACK//
     14X,1HK,14X,2HRS,13X,5HTHETA,10X,1HW)
1504  FORMAT(I5,3E20.8)
3003  CONTINUE
 102  FORMAT(////39H TEMPERATURES AND THERMAL DISPLACEMENTS,///
     119H NO. OF GRID ROWS = ,I5,5X,20H NO. OF GRID COLS. = ,I5/
     2/18H MIRROR PROPERTIES/
     320H CONDUCTIVITY, CK = ,E15.6/21H SPECIFIC HEAT, CP = ,E15.6/
     424H RADIATION FACTOR, CF = ,E15.6/29H REFERENCE TEMPERATURE, TO =
     5,E15.6//24H SOLUTION TIMER CONTROLS/32H INTEGRATION TIME STEP, DELT
     6TA = ,E15.6,5X,28HNUMBER OF TIME STEPS, NTS = ,I5//)
 107  FORMAT(48H LINEAR COEFFICIENT OF THERMAL EXPANSION, ALF = ,E15.6/
     135H YOUNGS MODULUS OF ELASTICITY, E = ,E15.6/
     321H POISSONS RATIO, V = ,E15.6/////)

 112  FORMAT(4F10.5)
 113  FORMAT(////16H MIRROR GEOMETRY//20H OUTSIDE DIAMETER = ,F10.5/
     119H INSIDE DIAMETER = ,F10.5/13H THICKNESS = ,F10.5/12H F-NUMBER =
     2 ,F10.5///)
      IF(INFLU.LE.0)GO TO 72
      KP=KP+1
      IF(KP.GT.KM)GO TO 2504
      GO TO 2503
2504  IP=IP+1
      IF(IP.LE.IM1)GO TO 2505
 72   CONTINUE
      STOP
      END
```

CAND

```
      SUBROUTINE CAND(IM,JM,M,CK,CP,CF)
      COMMON R,Z
      COMMON /CND/C,D,QR,QB
      DIMENSION C(75,75,2),D(75),R(15,5),Z(15,5)
      DIMENSION CT(3,3,2),CQ(5,5,2),GT(3,3,2),DQ(4),A(3,3),F1(5,5,2),
     2F2(5,5,2),G(5,5,2),D1(5),D2(5),GQ(3),QR(15),QB(15,14)
      DIMENSION RZ(3),ZR(3),X(10)
      IM1=IM-1
      JM1=JM-1
      DO 20 L1=1,M
      D(L1)=0.0
      DO 20 L2=1,M
      DO 20 L3=1,2
   20 C(L1,L2,L3)=0.0
      DO 47 L=1,IM
   47 QR(L)=0.0
      DO 60 M1=1,IM
      DO 60 M2=1,IM1
60    QB(M1,M2)=0.0
      DO 25 L1=1,JM
      D1(L1)=0.0
      D2(L1)=0.0
      DO 25 L2=1,JM
      DO 25 L3=1,2
      F1(L1,L2,L3)=0.0
      F2(L1,L2,L3)=0.0
   25 G(L1,L2,L3)=0.0
      DO 24 I=1,IM
      DO 46 L=1,JM
      D2(L)=0.0
      DO 46 LL=1,2
      DO 46 K=1,JM
      G(L,K,LL)=0.0
   46 F2(L,K,LL)=0.0
      DO 21 J=1,JM1
      IF (I.EQ.IM)GO TO 21
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      R3=R(I+1,J+1)
      Z3=Z(I+1,J+1)
      R4=R(I,J+1)
      Z4=Z(I,J+1)
      R21=R2-R1
      R32=R3-R2
      R41=R4-R1
      R34=R3-R4
      Z21=Z2-Z1
      Z32=Z3-Z2
      Z41=Z4-Z1
      Z34=Z3-Z4
      AREA=.5*(R41*Z41-R21*Z21-R32*Z32+R34*Z34)-R32*Z21+R34*Z41
      AR=.5*(R41*Z41)*(R1+2.*R41/3.0)-.5*R21*Z21*(R1+2.*R21/3.)-R32*Z21*
     3(R2+.5*R32)-.5*R32*Z32*(R2+2.*R32/3.)+.5*R34*Z34*(R4+2.*R34/3.)+R3
     44*Z41*(R4+.5*R34)
```

```
      AZ=.5*R41*Z41*(Z1+Z41/3.)-.5*R21*Z21*(Z1+Z21/3.)-R32*Z21*(Z1+.5*Z2
     51)-.5*R32*Z32*(Z2+Z32/3.)+.5*R34*Z34*(Z4+Z34/3.)+R34*Z41*(Z1+.5*Z4
     61)
      RC=AR/AREA
      ZC=AZ/AREA
      DO 16 L1=1,5
      DO 16 L2=1,5
      DO 16 L3=1,2
   16 CQ(L1,L2,L3)=0.0
      DO 17 L1=1,4
   17 DQ(L1)=0.0
      DO 15 K=1,4
      IF(K.NE.1)GO TO 1
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      GO TO 4
    1 IF(K.NE.2)GO TO 2
      R1=R(I+1,J)
      Z1=Z(I+1,J)
      R2=R(I+1,J+1)
      Z2=Z(I+1,J+1)
      GO TO 4
    2 IF(K.NE.3)GO TO 3
      R1=R(I+1,J+1)
      Z1=Z(I+1,J+1)
      R2=R(I,J+1)
      Z2=Z(I,J+1)
      GO TO 4
    3 IF(K.NE.4)GO TO 4
      R1=R(I,J+1)
      Z1=Z(I,J+1)
      R2=R(I,J)
      Z2=Z(I,J)
    4 R3=RC
      Z3=ZC
      AT=0.5*(R2*Z3-R3*Z2+R3*Z1-R1*Z3+R1*Z2-R2*Z1)
      A(1,1)=.5*(R2*Z3-R3*Z2)/AT
      A(2,1)=.5*(Z2-Z3)/AT
      A(3,1)=.5*(R3-R2)/AT
      A(1,2)=.5*(R3*Z1-R1*Z3)/AT
      A(2,2)=.5*(Z3-Z1)/AT
      A(3,2)=.5*(R1-R3)/AT
      A(1,3)=.5*(R1*Z2-R2*Z1)/AT
      A(2,3)=.5*(Z1-Z2)/AT
      A(3,3)=.5*(R2-R1)/AT
      DO 5 L1=1,3
      GQ(L1)=0.0
      DO 5 L2=1,3
      DO 5 L3=1,2
      GT(L1,L2,L3)=0.0
    5 CT(L1,L2,L3)=0.0
      IF(ABS(R1-R2).LT.0.000001)GO TO 6
      ALF12=(Z1-Z2)/(R1-R2)
      BET12=(Z2*R1-Z1*R2)/(R1-R2)
      GO TO 7
```

```
 6 ALF12=0.0
   BET12=0.0
 7 IF(ABS(R2-R3).LT.0.000001)GO TO 8
   ALF23=(Z2-Z3)/(R2-R3)
   BET23=(Z3*R2-Z2*R3)/(R2-R3)
   GO TO 9
 8 ALF23=0.
   BET23=0.
 9 IF(ABS(R3-R1).LT.0.000001)GO TO 10
   ALF31=(Z3-Z1)/(R3-R1)
   BET31=(Z1*R3-Z3*R1)/(R3-R1)
   GO TO 11
10 ALF31=0.0
   BET31=0.0
11 CONTINUE
   RZ(1)=R1
   RZ(2)=R2
   RZ(3)=R3
   ZR(1)=Z1
   ZR(2)=Z2
   ZR(3)=Z3
   CALL INTGRL(RZ,ZR,X)
   GT(2,2,1)=X(3)
   GT(3,3,1)=X(3)
   GT(1,1,2)=X(1)
   GT(1,2,2)=X(2)
   GT(1,3,2)=X(6)
   GT(2,1,2)=X(2)
   GT(2,2,2)=X(3)
   GT(2,3,2)=X(7)
   GT(3,1,2)=X(6)
   GT(3,2,2)=X(7)
   GT(3,3,2)=X(10)

   DO 50 L1=1,3
   DO 50 L2=1,3
   DO 50 L3=1,2
50 GT(L1,L2,L3)=CK*GT(L1,L2,L3)
   IF(J.NE.JM1)GO TO 51
   IF(K.NE.3)GO TO 51
   SA=SQRT(1.+ALF12**2)
   N=1
   GT(1,1,N)=GT(1,1,N)+CF*SA*(R1**2-R2**2)/2.
   GT(1,2,N)=GT(1,2,N) +CF*SA*(R1**3-R2**3)/3.0
   GT(1,3,N)=GT(1,3,N)+CF*SA*(ALF12*(R1**3-R2**3)/3.0+0.5
  1*BET12*(R1**2-R2**2))
   GT(2,1,N)=GT(1,2,N)
   GT(2,2,N)=GT(2,2,N)+CF*SA*0.25*(R1**4-R2**4)
   GT(2,3,N)=GT(2,3,N)+CF*SA*(0.25*ALF12*(R1**4-R2**4)
  1 +BET12*(R1**3-R2**3)/3.0)
   GT(3,1,N)=GT(1,3,N)
   GT(3,2,N)=GT(2,3,N)
   GT(3,3,N)=GT(3,3,N)+CF*SA*(0.25*ALF12**2*(R1**4-R2**4)+2.0*ALF12
  1 *BET12*(R1**3-R2**3)/3.0+0.5*BET12**2*(R1**2-R2**2))

   GQ(1)=CF*SA*0.5*(R1**2-R2**2)
   GQ(2)=CF*SA*(R1**3-R2**3)/3.0
```

```
      GQ(3)=CF*SA*(ALF12*(R1**3-R2**3)/3.0+0.5*BET12*(R1**2-R2**2))
   51 CONTINUE
      DO 48 L1=1,3
      QR(I)=QR(I)+A(L1,2)*GQ(L1)
      IF(I.EQ.IM)GO TO 48
      QR(I+1)=QR(I+1)+A(L1,1)*GQ(L1)
   48 CONTINUE
      DO 12 L1=1,3
      DO 12 L2=1,3
      DO 12 L3=1,3
      DO 12 L4=1,3
      DO 12 L5=1,2
   12 CT(L1,L2,L5)=CT(L1,L2,L5)+A(L3,L1)*GT(L3,L4,L5)*A(L4,L2)
      DQ(K)=DQ(K)+.5*GT(2,2,2)*CP/CK
      IF(K.EQ.4)GO TO 31
      DQ(K+1)=DQ(K+1)+.5*GT(2,2,2)*CP/CK
      GO TO 32
   31 DQ(1)=DQ(1)+.5*GT(2,2,2)*CP/CK
  32  CONTINUE
      DO 14 L1=1,2
      CQ(K,K,L1)=CQ(K,K,L1)+CT(1,1,L1)
      CQ(K,5,L1)=CQ(K,5,L1)+CT(1,3,L1)
      IF(K.EQ.4)GO TO 30
      CQ(K,K+1,L1)=CT(1,2,L1)
      CQ(K+1,K+1,L1)=CQ(K+1,K+1,L1)+CT(2,2,L1)
      CQ(K+1,5,L1)=CQ(K+1,5,L1)+CT(2,3,L1)
      GO TO 13
  30  CQ(1,1,L1)=CQ(1,1,L1)+CT(2,2,L1)
      CQ(1,4,L1)=CT(1,2,L1)
      CQ(1,5,L1)=CQ(1,5,L1)+CT(2,3,L1)
   13 CQ(5,5,L1)=CQ(5,5,L1)+CT(3,3,L1)
   14 CONTINUE
      IF(J.NE.1)GO TO 15
      IF(K.NE.1)GO TO 15
      IF(I.EQ.IM)GO TO 15
      SAR=SQRT(1.+ALF12**2)/(R1-R2)
      QB(I,I)= SAR*(R2**2+R1*R2-2.0*R1**2)/6.0
      QB(I+1,I)= SAR*(2.0*R2**2-R1*R2-R1**2)/6.0
   15 CONTINUE
      DO 19 L1=1,2
      DO 18 L2=2,5
      L4=L2-1
       DO 18 L3=1,L4
   18 CQ(L2,L3,L1)=CQ(L3,L2,L1)
      DO 19 L2=1,4
      DO 19 L3=1,4
   19 CQ(L2,L3,L1)=CQ(L2,L3,L1)-CQ(L2,5,L1)*CQ(L3,5,L1)/CQ(5,5,L1)
      DO 40  L3=1,2
      F1(J,J,L3)=F1(J,J,L3)+CQ(1,1,L3)
      F1(J,J+1,L3)=F1(J,J+1,L3)+CQ(1,4,L3)
      F1(J+1,J+1,L3)=F1(J+1,J+1,L3)+CQ(4,4,L3)
      F2(J,J,L3)=F2(J,J,L3)+CQ(2,2,L3)
      F2(J,J+1,L3)=F2(J,J+1,L3)+CQ(2,3,L3)
      F2(J+1,J+1,L3)=F2(J+1,J+1,L3)+CQ(3,3,L3)
      G(J,J,L3)=G(J,J,L3)+CQ(1,2,L3)
      G(J,J+1,L3)=G(J,J+1,L3)+CQ(1,3,L3)
      G(J+1,J,L3)=G(J+1,J,L3)+CQ(4,2,L3)
```

```
      G(J+1,J+1,L3)=G(J+1,J+1,L3)+CQ(4,3,L3)
   40 CONTINUE
      D1(J)=D1(J)+DQ(1)
      D1(J+1)=D1(J+1)+DQ(4)
      D2(J)=D2(J)+DQ(2)
      D2(J+1)=D2(J+1)+DQ(3)
   21 CONTINUE
      DO 22 L3=1,2
      DO 22 L1=1,JM
      DO 22 L2=L1,JM
      IR=JM*(I-1)
      LR=IR+L1
      LC=IR+L2
      C(LR,LC,L3)=F1(L1,L2,L3)
   22 F1(L1,L2,L3)=F2(L1,L2,L3)
      DO 26 L1=1,JM
      IR=JM*(I-1)
      LR=IR+L1
      D(LR)=D1(L1)
   26 D1(L1)=D2(L1)
      IF(I.EQ.IM)GO TO 24
      DO 23 L3=1,2
      DO 23 L1=1,JM
      DO 23 L2=1,JM
      IR=JM*(I-1)
      LR=IR+L1
      LC=IR+JM+L2
   23 C(LR,LC,L3)=G(L1,L2,L3)
   24 CONTINUE
      DO 27 L3=1,2
      DO 27 L1=2,M
      L1M=L1-1
      DO 27 L2=1,L1M
   27 C(L1,L2,L3)=C(L2,L1,L3)
      RETURN
      END
```

```
STIFF
      SUBROUTINE STIFF (E,V,ALF,IM,JM,N,CK,CF)
      COMMON R,Z
      COMMON/STIF/KD,KS,PD,PS,PL,D
      REAL LAM,MU,KD,KT,KQ,KS
      DIMENSION R(15,5),Z(15,5),KD(15,15,15),KS(15,15,15),PD(15,5,15),
     1PS(15,5,14),PL(15,5,14),D(75,75)
      DIMENSION A(3,3),NN(10),RT(9,3),KT(9,9),KQ(15,15),PT(9,3),
     2PQ(15,5),GT(3,3,2),CQ(5,5,2),CP(5,5),DQ(4),GQ(3),CT(3,3,2)
      DIMENSION RZ(3),ZR(3),X(10)
      FN=N
      IM1=IM-1
      JM1=JM-1
      LAM=V*E/((1.+V)*(1.-2.*V))
      MU=0.5*E/(1.+V)
      BET=ALF*(3.*LAM+2.*MU)
      E1=LAM+2.*MU
      NN(1)=1
      NN(2)=3
      NN(3)=4
      NN(4)=6
      NN(5)=7
      NN(6)=9
      NN(7)=10
      NN(8)=12
      NN(9)=13
      NN(10)=15

C
C     INITIALIZE
C
      DO 101 L1=1,15
      DO 101 L2=1,15
      DO 101 L3=1,15
      KS(L1,L2,L3)=0.0
  101 KD(L1,L2,L3)=0.0
      DO 102 L1=1,15
      DO 102 L2=1,5
      DO 102 L3=1,15
  102 PD(L1,L2,L3)=0.0
      DO 222 L1=1,15
      DO 222 L2=1,5
      DO 222 L3=1,14
      PL(L1,L2,L3)=0.0
  222 PS(L1,L2,L3)=0.0
C
C     OUTER LOOP ON I BEGINS HERE
C
      DO 402 I=1,IM1
C
C     INNER LOOP ON J BEGINS HERE
C
      DO 21 J=1,JM1
C     INITIALIZE FOR I,J QUAD
      DO 104 L1=1,15
      DO 104 L2=1,15
  104 KQ(L1,L2)=0.0
```

```
      DO 105 L1=1,15
      DO 105 L2=1,5
  105 PQ(L1,L2)=0.0
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      R3=R(I+1,J+1)
      Z3=Z(I+1,J+1)
      R4=R(I,J+1)
      Z4=Z(I,J+1)
      R21=R2-R1
      R32=R3-R2
      R41=R4-R1
      R34=R3-R4
      Z21=Z2-Z1
      Z32=Z3-Z2
      Z41=Z4-Z1
      Z34=Z3-Z4
      AREA=.5*(R41*Z41-R21*Z21-R32*Z32+R34*Z34)-R32*Z21+R34*Z41
      AR=.5*(R41*Z41)*(R1+2.*R41/3.0)-.5*R21*Z21*(R1+2.*R21/3.)-R32*Z21*
     3(R2+.5*R32)-.5*R32*Z32*(R2+2.*R32/3.)+.5*R34*Z34*(R4+2.*R34/3.)+R3
     44*Z41*(R4+.5*R34)
      AZ=.5*R41*Z41*(Z1+Z41/3.)-.5*R21*Z21*(Z1+Z21/3.)-R32*Z21*(Z1+.5*Z2
     51)-.5*R32*Z32*(Z2+Z32/3.)+.5*R34*Z34*(Z4+Z34/3.)+R34*Z41*(Z1+.5*Z4
     61)
      RC=AR/AREA
      ZC=AZ/AREA
      DO 16 L1=1,5
      DO 16 L2=1,5
      DO 16 L3=1,2
   16 CQ(L1,L2,L3)=0.0
      DO 17 L1=1,4
   17 DQ(L1)=0.0
      DO 15 K=1,4
      DO 601 L1=1,9
      DO 601 L2=1,9
601   KT(L1,L2)=0.0
      DO 602 L1=1,9
      DO 602 L2=1,3
602   PT(L1,L2)=0.0
      IF(K.NE.1)GO TO 1
      R1=R(I,J)
      Z1=Z(I,J)
      R2=R(I+1,J)
      Z2=Z(I+1,J)
      GO TO 4
    1 IF(K.NE.2)GO TO 2
      R1=R(I+1,J)
      Z1=Z(I+1,J)
      R2=R(I+1,J+1)
      Z2=Z(I+1,J+1)
      GO TO 4
    2 IF(K.NE.3)GO TO 3
      R1=R(I+1,J+1)
      Z1=Z(I+1,J+1)
      R2=R(I,J+1)
```

```
      Z2=Z(I,J+1)
      GO TO 4
    3 IF(K.NE.4)GO TO 4
      R1=R(I,J+1)
      Z1=Z(I,J+1)
      R2=R(I,J)
      Z2=Z(I,J)
    4 R3=RC
      Z3=ZC
      AT=0.5*(R2*Z3-R3*Z2+R3*Z1-R1*Z3+R1*Z2-R2*Z1)
      A(1,1)=.5*(R2*Z3-R3*Z2)/AT
      A(2,1)=.5*(Z2-Z3)/AT
      A(3,1)=.5*(R3-R2)/AT
      A(1,2)=.5*(R3*Z1-R1*Z3)/AT
      A(2,2)=.5*(Z3-Z1)/AT
      A(3,2)=.5*(R1-R3)/AT
      A(1,3)=.5*(R1*Z2-R2*Z1)/AT
      A(2,3)=.5*(Z1-Z2)/AT
      A(3,3)=.5*(R2-R1)/AT
      DO 5 L1=1,3
      GQ(L1)=0.0
      DO 5 L2=1,3
      DO 5 L3=1,2
      GT(L1,L2,L3)=0.0
    5 CT(L1,L2,L3)=0.0
      IF(ABS(R1-R2).LT.0.000001)GO TO 6
      ALF12=(Z1-Z2)/(R1-R2)
      BET12=(Z2*R1-Z1*R2)/(R1-R2)
      GO TO 7
    6 ALF12=0.0
      BET12=0.0
    7 IF(ABS(R2-R3).LT.0.000001)GO TO 8
      ALF23=(Z2-Z3)/(R2-R3)
      BET23=(Z3*R2-Z2*R3)/(R2-R3)
      GO TO 9
    8 ALF23=0.
      BET23=0.
    9 IF(ABS(R3-R1).LT.0.000001)GO TO 10
      ALF31=(Z3-Z1)/(R3-R1)
      BET31=(Z1*R3-Z3*R1)/(R3-R1)
      GO TO 11
   10 ALF31=0.0
      BET31=0.0
   11 CONTINUE
      RZ(1)=R1
      RZ(2)=R2
      RZ(3)=R3
      ZR(1)=Z1
      ZR(2)=Z2
      ZR(3)=Z3
      CALL INTGRL(RZ,ZR,X)
      GT(2,2,1)=X(3)
      GT(3,3,1)=X(3)
      GT(1,1,2)=X(1)
      GT(1,2,2)=X(2)
      GT(1,3,2)=X(6)
      GT(2,1,2)=X(2)
```

```
      GT(2,2,2)=X(3)
      GT(2,3,2)=X(7)
      GT(3,1,2)=X(6)
      GT(3,2,2)=X(7)
      GT(3,3,2)=X(10)
      R314=R3**4-R1**4
      R313=R3**3-R1**3
      R234=R2**4-R3**4
      R233=R2**3-R3**3
      R312=R3**2-R1**2
      R232=R2**2-R3**2
      GD1=0.25*(ALF31-ALF12)*R314+(BET31-BET12)*R313/3.0+0.25*(ALF23-
     4ALF12)*R234+(BET23-BET12)*R233/3.0
      GD2=0.125*(ALF31**2-ALF12**2)*R314+(ALF31*BET31-ALF12*BET12)*    R
     5313/3.0+.25*(BET31**2-BET12**2)*R312+0.125*(ALF23**2-ALF12**2)*
     6R234+(ALF23*BET23-ALF12*BET12)*R233/3.0+.25*(BET23**2-BET12**2)*
     2R232
      GD3=(ALF31**3-ALF12**3)*R314/12.0+(ALF31**2*BET31-ALF12**2*BET12
     6)*R313/3.0+0.5*(ALF31*BET31**2-ALF12*BET12**2)*R312+(BET31**3-  BE
     7T12**3)*(R3-R1)/3.0+(ALF23**3-ALF12**3)*R234/12.0+(ALF23**2*BET23-
     8ALF12**2*BET12)*R233/3.0+0.5*(ALF23*BET23**2-ALF12*BET12**2)*R232+
     9(BET23** 3-BET12**3)*(R2-R3)/3.0

C     FORM KT AND PT FOR TRIANGLE
C     INSERT A
      Y1=1.
      Y2=1.
      IF(N.EQ.0)Y1=2.
      IF(N.EQ.0)Y2=0.
      DO 209 I1=1,3
      DO 209 J1=1,3
      AI=A(1,I1)
      BI=A(2,I1)
      DI=A(3,I1)
      AJ=A(1,J1)
      BJ=A(2,J1)
      DJ=A(3,J1)
      GG1=GT(2,2,1)
      GG2=AJ*GT(1,2,2)+BJ*GG1+DJ*GT(2,3,2)
      GG3=AI*GT(1,2,2)+BI*GG1+DI*GT(2,3,2)
      GG4=AI*AJ*GT(1,1,2)+(AI*BJ+AJ*BI)*GT(1,2,2)+(AI*DJ+AJ*DI)*GT(
     11,3,2)+BI*BJ*GG1+(BI*DJ+BJ*DI)*GT(2,3,2)+DI*DJ*GT(3,3,2)
      GG5=AI*GG1+BI*GD1+DI*GD2
      GG6=AI*AJ*GT(1,2,2)+(AI*BJ+AJ*BI)*GG1+(AI*DJ+AJ*DI)*GT(2,3,2)+
     1BI*BJ*GD1+(BI*DJ+BJ*DI)*GD2+DI*DJ*GD3
      GG7=AI*GG1+BI*GD1+DI*GD2
      DO 208 L1=1,3
      DO 208 L2=1,3
      LR=(I1-1)*3+L1
      LC=(J1-1)*3+L2
      IF(L1.NE.1)GO TO 202
      IF(L2.NE.1)GO TO 200
      PT(LR,J1)=BET*Y1*GG5*BJ
      KT(LR,LC)=Y1*(E1*BI*BJ*GG1+LAM*(BI*GG2+BJ*GG3)+E1*GG4)+Y2*FN*FN
     2*GG4*MU+Y1*MU*DI*DJ*GG1
      GO TO 208
  200 IF(L2.NE.2)GO TO 201
```

```
      KT(LR,LC)=Y1*FN*(LAM*BI*GG2+E1*GG4)+Y2*MU*(GG4-BJ*GG3)*FN
      GO TO 208
  201 KT(LR,LC)=Y1*(LAM*DJ*(BI*GG1+GG3)+MU*DI*BJ*GG1)
      GO TO 208
  202 IF(L1.NE.2)GO TO 205
      IF(L2.NE.1)GO TO 203
      PT(LR,J1)=-BET*FN*Y2*GG6
      KT(LR,LC)=Y1*FN*(LAM*BJ*GG3+E1*GG4)+Y2*MU*FN*(GG4-BI*GG2)
      GO TO 208
  203 IF(L2.NE.2)GO TO 204
      KT(LR,LC)=Y1*FN*FN*E1*GG4+Y2*MU*(BI*BJ*GG1-BI*GG2-BJ*GG3+GG4+DI
     3*DJ*GG1)
      GO TO 208
  204 KT(LR,LC)=Y1*LAM*FN*DJ*GG3-Y2*MU*FN*DI*GG2
      GO TO 208
  205 IF(L2.NE.1)GO TO 206
      PT(LR,J1)=BET*Y1*DJ*GG7
      KT(LR,LC)=Y1*(LAM*(DI*BJ*GG1+DI*GG2)+MU*DJ*BI*GG1)
      GO TO 208
  206 IF(L2.NE.2)GO TO 207
      KT(LR,LC)=Y1*LAM*FN*DI*GG2-Y2*MU*FN*DJ*GG3
      GO TO 208
  207 KT(LR,LC)=Y1*(E1*DI*DJ+MU*BI*BJ)*GG1+Y2*MU*FN*FN*GG4
  208 CONTINUE
  209 CONTINUE
C
C     SUBROUTINE TO CALCULATE RT
C
      DO 210 I1=1,9
      DO 210 J1=1,3
  210 RT(I1,J1)=0.0
      Y3=1.
      IF(J.EQ.1.AND.K.EQ.1)GO TO 211
      IF(J.EQ.JM1.AND.K.EQ.3) GO TO 211
      GO TO 212
  211 CONTINUE
      GR1=R2**4/12.-R2**2*R1**2/2.+2.*R2*R1**3/3.-R1**4/4.
      GR2=R1**3*R2/6.-R1**4/12.-R1*R2**3/6.+R2**4/12.
      GR3=R2**4/4.-2.*R1*R2**3/3.+R1**2*R2**2/2.-R1**4/12.
      BBB=BET*Y1*Y3/(R2-R1)**3
      RT(1,1)=BBB*(Z2-Z1)*GR1
      RT(3,1)=-1.*BBB*(R2-R1)*GR1
      RT(4,1)=BBB*(Z2-Z1)*GR2
      RT(6,1)=-1.*BBB*(R2-R1)*GR2
      RT(1,2)=RT(4,1)
      RT(3,2)=RT(6,1)
      RT(4,2)=BBB*(Z2-Z1)*GR3
      RT(6,2)=-1.*BBB*(R2-R1)*GR3
      DO 213 I1=1,9
      DO 213 J1=1,3
  213 PT(I1,J1)=PT(I1,J1)-RT(I1,J1)
  212 CONTINUE
      IF(I.EQ.1.AND.K.EQ.4) GO TO 221
      IF(I.EQ.IM1.AND.K.EQ.2)GO TO 221
      GO TO 225
  221 CONTINUE
      GR4=Z2**4/12.-Z2**2*Z1**2/2.+2.*Z2*Z1**3/3.-Z1**4/4.
```

```
      GR5=Z1**3*Z2/6.-Z1**4/12.-Z1*Z2**3/6.+Z2**4/12.
      GR6=Z2**4/4.-2.*Z1*Z2**3/3.+Z1**2*Z2**2/2.-Z1**4/12.
      GR7=Z2**3/3.-Z2**2*Z1+Z2*Z1**2-Z1**3/3.
      GR8=Z1**2*Z2/2.-Z1**3/6.-Z1*Z2**2/2.+Z2**3/6.
      GR9=Z1**2*Z2-Z1**3/3.-Z1*Z2**2+Z2**3/3.
      ALP=(R2-R1)/(Z2-Z1)
      BEP=(R1*Z2-R2*Z1)/(Z2-Z1)
      BBB=BET*Y1*Y3/(Z2-Z1)**3
      RT(1,1)=BBB*(Z2-Z1)*(ALP*GR4+BEP*GR7)
      RT(3,1)=BBB*(R2-R1)*(ALP*GR4+BEP*GR7)*(-1.0)
      RT(4,1)=BBB*(Z2-Z1)*(ALP*GR5+BEP*GR8)
      RT(6,1)=BBB*(R2-R1)*(ALP*GR5+BEP*GR8)*(-1.0)
      RT(1,2)=RT(4,1)
      RT(3,2)=RT(6,1)
      RT(4,2)=BBB*(Z2-Z1)*(ALP*GR6+BEP*GR9)
      RT(6,2)=BBB*(R2-R1)*(ALP*GR6+BEP*GR9)*(-1.0)
      DO 223 I1=1,9
      DO 223 J1=1,3
  223 PT(I1,J1)=PT(I1,J1)-RT(I1,J1)
  225 CONTINUE
C
C     FORM CT
C
      DO 50 L1=1,3
      DO 50 L2=1,3
      DO 50 L3=1,2
   50 GT(L1,L2,L3)=CK*GT(L1,L2,L3)
      IF(J.NE.JM1)GO TO 51
      IF(K.NE.3)GO TO 51
      SA=SQRT(1.+ALF12**2)
      GT(1,1,1)=GT(1,1,1)+CF*SA*(R1**2-R2**2)/2.
      GT(1,2,1)=GT(1,2,1)+CF*SA*(R1**3-R2**3)/3.
      GT(1,3,1)=GT(1,3,1)+CF*SA*(ALF12*(R1**3-R2**3)/3.+.5*BET12*(R1**2
     3-R2**2))
      GT(2,1,1)=GT(1,2,1)
      GT(2,2,1)=GT(2,2,1)+CF*SA* .25*(R1**4-R2**4)
      GT(2,3,1)=GT(2,3,1)+CF*SA*(0.25*ALF12*(R1**4-R2**4)+BET12*(R1**3
     1 -R2**3)/3.0)
      GT(3,1,1)=GT(1,3,1)
      GT(3,2,1)=GT(2,3,1)
      GT(3,3,1)=GT(3,3,1)+CF*SA*(0.25*ALF12**2*(R1**4-R2**4)+2.0*ALF12
     1 *BET12*(R1**3-R2**3)/3.0+0.5*BET12**2*(R1**2-R2**2))
   51 CONTINUE
      DO 12 L1=1,3
      DO 12 L2=1,3
      DO 12 L3=1,3
      DO 12 L4=1,3
      DO 12 L5=1,2
   12 CT(L1,L2,L5)=CT(L1,L2,L5)+A(L3,L1)*GT(L3,L4,L5)*A(L4,L2)

C     NOW FOR THE QUADRILATERAL
C
      DO 300 K1=1,3
      DO 300 K2=1,3
      KR=3*(K-1)+K1
      KC=3*(K-1)+K2
      KR5=12+K1
```

```
      KC5=12+K2
      KRR=3*K+K1
      KCC=3*K+K2
      KQ(KR,KC)=KQ(KR,KC)+KT(K1,K2)
      KQ(KR,KC5)=KQ(KR,KC5)+KT(K1,K2+6)
      KQ(KR5,KC)=KQ(KR5,KC)+KT(K1+6,K2)
      IF(K.EQ.4) GO TO 301
      KQ(KR,KCC)=KQ(KR,KCC)+KT(K1,K2+3)
      KQ(KRR,KC)=KQ(KRR,KC)+KT(K1+3,K2)
      KQ(KRR,KCC)=KQ(KRR,KCC)+KT(K1+3,K2+3)
      KQ(KRR,KC5)=KQ(KRR,KC5)+KT(K1+3,K2+6)
      KQ(KR5,KCC)=KQ(KR5,KCC)+KT(K1+6,K2+3)
      GO TO 302
  301 KQ(K1,K2)=KQ(K1,K2)+KT(K1+3,K2+3)
      KQ(K1,K2+9)=KQ(K1,K2+9)+KT(K1+3,K2)
      KQ(K1+9,K2)=KQ(K1+9,K2)+KT(K1,K2+3)
      KQ(KRR,K2)=KQ(KRR,K2)+KT(K1+6,K2+3)
      KQ(K1,KCC)=KQ(K1,KCC)+KT(K1+3,K2+6)
  302 KQ(KR5,KC5)=KQ(KR5,KC5)+KT(K1+6,K2+6)
  300 CONTINUE
      DO 303 K1=1,3
      KR=3*(K-1)+K1
      PQ(KR,K)=PQ(KR,K)+PT(K1,1)
      PQ(KR,5)=PQ(KR,5)+PT(K1,3)
      PQ(K1+12,K)=PQ(K1+12,K)+PT(K1+6,1)
      IF(K.EQ.4) GO TO 304
      PQ(KR,K+1)=PQ(KR,K+1)+PT(K1,2)
      PQ(KR+3,K)=PQ(KR+3,K)+PT(K1+3,1)
      PQ(KR+3,K+1)=PQ(KR+3,K+1)+PT(K1+3,2)
      PQ(KR+3,5)=PQ(KR+3,5)+PT(K1+3,3)
      PQ(K1+12,K+1)=PQ(K1+12,K+1)+PT(K1+6,2)
      GO TO 305
  304 PQ(K1,4)=PQ(K1,4)+PT(K1+3,1)
      PQ(K1,5)=PQ(K1,5)+PT(K1+3,3)
      PQ(K1+9,1)=PQ(K1+9,1)+PT(K1,2)
      PQ(K1,1)=PQ(K1,1)+PT(K1+3,2)
      PQ(K1+12,1)=PQ(K1+12,1)+PT(K1+6,2)
  305 PQ(K1+12,5)=PQ(K1+12,5)+PT(K1+6,3)
  303 CONTINUE
      DO 14 L1=1,2
      CQ(K,K,L1)=CQ(K,K,L1)+CT(1,1,L1)
      CQ(K,5,L1)=CQ(K,5,L1)+CT(1,3,L1)
      IF(K.EQ.4)GO TO 30
      CQ(K,K+1,L1)=CT(1,2,L1)
      CQ(K+1,K+1,L1)=CQ(K+1,K+1,L1)+CT(2,2,L1)
      CQ(K+1,5,L1)=CQ(K+1,5,L1)+CT(2,3,L1)
      GO TO 13
   30 CQ(1,1,L1)=CQ(1,1,L1)+CT(2,2,L1)
      CQ(1,4,L1)=CT(1,2,L1)
      CQ(1,5,L1)=CQ(1,5,L1)+CT(2,3,L1)
   13 CQ(5,5,L1)=CQ(5,5,L1)+CT(3,3,L1)
   14 CONTINUE
      DO 18 L1=1,2
      DO 19 L2=2,5
      L4=L2-1
      DO 19 L3=1,L4
   19 CQ(L2,L3,L1)=CQ(L3,L2,L1)
```

```
   18 CONTINUE
   15 CONTINUE

      DO 500 L1=1,5
      DO 500 L2=1,5
  500 CP(L1,L2)=CQ(L1,L2,1)+FN*FN*CQ(L1,L2,2)
C
C     NEED TO ELIMINATE MIDDLE NODE
C
C     INSERT A
      N1=3
      IF(N.NE.0)GO TO 801
      N1=2
      DO 800 M1=1,10
      M3=NN(M1)
      DO 802 M5=1,5
  802 PQ(M1,M5)=PQ(M3,M5)
      DO 800 M2=1,10
      M4=NN(M2)
  800 KQ(M1,M2)=KQ(M3,M4)

  801 CONTINUE
      N2=2*N1
      N3=3*N1
      N4=4*N1
      DO 306 K1=1,N1
      DO 306 K2=1,N1
  306 D(K1,K2)=KQ(K1+N4,K2+N4)

      CALL INVERT(D,N1,75)

      DO 307 K1=1,N4
      DO 307 K2=1,N4
      DO 307 K3=1,N1
      DO 307 K4=1,N1
      L3=K3+N4
      L4=K4+N4
  307 KQ(K1,K2)=KQ(K1,K2)-KQ(K1,L3)*D(K3,K4)*KQ(L4,K2)

      DO 309 L1=1,N4
      DO 309 L2=1,4
  309 PQ(L1,L2)=PQ(L1,L2)-PQ(L1,5)*CP(5,L2)/CP(5,5)
      DO 308 L1=1,N1
      DO 308 L2=1,4
  308 PQ(N4+L1,L2)=PQ(N4+L1,L2)-PQ(N4+L1,5)*CP(5,L2)/CP(5,5)
      DO 310 L1=1,N4
      DO 310 L2=1,4
      DO 310 K1=1,N1
      DO 310 K2=1,N1
  310 PQ(L1,L2)=PQ(L1,L2)-KQ(L1,K1+N4)*D(K1,K2)*PQ(N4+K2,L2)
C
C     ASSEMBLE THE ROW MATRICES KD,KS,PD,PS
C
      DO 400 K1=1,N1
      DO 400 K2=1,N1
      KR=N1*(J-1)+K1
      KC=N1*(J-1)+K2
```

```
      KD(KR,KC,I)=KD(KR,KC,I)+KQ(K1,K2)
      KD(KR,KC+N1,I)=KD(KR,KC+N1,I)+KQ(K1,K2+N3)
      KD(KR+N1,KC,I)=KD(KR+N1,KC,I)+KQ(K1+N3,K2)
      KD(KR+N1,KC+N1,I)=KD(KR+N1,KC+N1,I)+KQ(K1+N3,K2+N3)
      KD(KR,KC,I+1)=KD(KR,KC,I+1)+KQ(K1+N1,K2+N1)
      KD(KR,KC+N1,I+1)=KD(KR,KC+N1,I+1)+KQ(K1+N1,K2+N2)
      KD(KR+N1,KC,I+1)=KD(KR+N1,KC,I+1)+KQ(K1+N2,K2+N1)
      KD(KR+N1,KC+N1,I+1)=KD(KR+N1,KC+N1,I+1)+KQ(K1+N2,K2+N2)
      KS(KR,KC,I)=KS(KR,KC,I)+KQ(K1,K2+N1)
      KS(KR,KC+N1,I)=KS(KR,KC+N1,I)+KQ(K1,K2+N2)
      KS(KR+N1,KC,I)=KS(KR+N1,KC,I)+KQ(K1+N3,K2+N1)
      KS(KR+N1,KC+N1,I)=KS(KR+N1,KC+N1,I)+KQ(K1+N3,K2+N2)
  400 CONTINUE
      DO 401 K1=1,N1
      KR=N1*(J-1)+K1
      PD(KR,J,I)=PD(KR,J,I)+PQ(K1,1)
      PD(KR,J+1,I)=PD(KR,J+1,I)+PQ(K1,4)
      PD(KR+N1,J,I)=PD(KR+N1,J,I)+PQ(K1+N3,1)
      PD(KR+N1,J+1,I)=PD(KR+N1,J+1,I)+PQ(K1+N3,4)
      PD(KR,J,I+1)=PD(KR,J,I+1)+PQ(K1+N1,2)
      PD(KR,J+1,I+1)=PD(KR,J+1,I+1)+PQ(K1+N1,3)
      PD(KR+N1,J,I+1)=PD(KR+N1,J,I+1)+PQ(K1+N2,2)
      PD(KR+N1,J+1,I+1)=PD(KR+N1,J+1,I+1)+PQ(K1+N2,3)
      PS(KR,J,I)=PS(KR,J,I)+PQ(K1,2)
      PS(KR,J+1,I)=PS(KR,J+1,I)+PQ(K1,3)
      PS(KR+N1,J,I)=PS(KR+N1,J,I)+PQ(K1+N3,2)
      PS(KR+N1,J+1,I)=PS(KR+N1,J+1,I)+PQ(K1+N3,3)
      PL(KR,J,I)=PL(KR,J,I)+PQ(K1+N1,1)
      PL(KR,J+1,I)=PL(KR,J+1,I)+PQ(K1+N1,4)
      PL(KR+N1,J,I)=PL(KR+N1,J,I)+PQ(K1+N2,1)
      PL(KR+N1,J+1,I)=PL(KR+N1,J+1,I)+PQ(K1+N2,4)
  401 CONTINUE
   21 CONTINUE
  402 CONTINUE
      RETURN
      END
```

INTGRL

```
      SUBROUTINE INTGRL(R,Z,X)
      REAL ICON,ICONP,IZ,IZP,IZ2,IZ2P
      DIMENSION R(3),Z(3),X(10),XI(10),AI(10)
      RI=R(1)
      RJ=R(2)
      RK=R(3)
      DATA(XI(I),AI(I),I=1,10)/-.97390653,.066671344,-.86506337,.1494513
     15,-.67940957,.21908636,-.43339539,.26926672,-.14887434,.29552422,
     2.14887434,.29552422,.43339539,.26926672,.67940957,.21908636,
     3.86506337,.14945135,.97390653,.066671344/
      DO 2001 N1=1,10
 2001 X(N1)=0.
C****  CALCULATION OF INTEGRALS BY GAUSSIAN QUADRATURE
   70 RMIN=AMIN1(RI,RJ,RK)
      RMAX=AMAX1(RI,RJ,RK)
      DO 7 N1=1,3
    7 IF(ABS(R(N1)-RMIN).LE.0.00001)I1=N1
      DO 8 N1=1,3
    8 IF(ABS(R(N1)-RMAX).LE.0.00001)I3=N1
      DO 9 N1=1,3
    9 IF(N1.NE.I1.AND.N1.NE.I3) I2=N1
       R1=R(I1)
      R2=R(I2)
      R3=R(I3)
      Z1=Z(I1)
      Z2=Z(I2)
      Z3=Z(I3)
      FAC=1.0
      DR12=ABS(R1-R2)
      DR13=ABS(R1-R3)
      IF(R1.GT.0.0001)GO TO 100
      IF(DR12.LT.0.0001.OR.DR13.LT.0.0001)FAC=1000.0
  100 CONTINUE
      S12=(Z2-Z1)/(R2-R1)
      S13=(Z3-Z1)/(R3-R1)
      S23=(Z3-Z2)/(R3-R2)
      DR=R2-R1
      DRP=R3-R2
      DO 12 N1=1,10
      RR=R1+DR*(XI(N1)+1.)/2.
      RRP=R2+DRP*(XI(N1)+1.)/2.
      ZZ1=S13*(RR-R1)+Z1
      ZZ1P=S13*(RRP-R1)+Z1
      ZZ2=S12*(RR-R1)+Z1
      ZZ3=S23*(RRP-R2)+Z2
       ICON=ABS(ZZ2-ZZ1)
       ICONP=ABS(ZZ3-ZZ1P)
      IZ=(ZZ1**2-ZZ2**2)/2.
      IF(ZZ1.LT.ZZ2) IZ= -IZ
      IZP=(ZZ1P**2-ZZ3**2)/2.
      IF(ZZ1P.LT.ZZ3) IZP= -IZP
      IZ2=ABS(ZZ2**3-ZZ1**3)/3.
      IZ2P=ABS(ZZ3**3-ZZ1P**3)/3.
      DO 10 N2=1,5
      X(N2)=X(N2)+AI(N1)*ICONP*RRP**(N2-2)*DRP
   10 IF(ABS(RR).GT.0.0000001)
```

```
     1X(N2)=X(N2)+AI(N1)*(ICON*RR**(N2-2)*DR)
      DO 11 N2=6,9
      X(N2)=X(N2)+AI(N1)*IZP*RRP**(N2-7)*DRP
   11 IF(ABS(RR).GT.0.0000001)
     1X(N2)=X(N2)+AI(N1)*(IZ*RR**(N2-7)*DR)
   12 X(10)=X(10)+AI(N1)*(IZ2/RR*DR+IZ2P/RRP*DRP)
      DO 13 N1=1,10
   13 X(N1)=X(N1)/2.
      X(1)=FAC*X(1)
      X(6)=FAC*X(6)
      X(10)=FAC*X(10)
      RETURN
      END
```

```
  POTTER
       SUBROUTINE POTTER(A,B,C,DL,Z,MAT,LCV,AS,DK,DKK,DKKK,IPIVOT,INDEX,
     1ZZ,SPP,SPPP,Q,P,SAP,SS,SP,SSS,MM,LL)
CHANGED  5 MARCH 71  FOR DIMENSION STATEMENT CONTINUITY
C*  VARIABLES  MM  AND  LL  ADDED TO ARGUMENT LIST
       DIMENSION AS(LL,LL),DK(LL,LL),DKK(LL,LL),DKKK(LL,LL),
     1  ZZ(LL,1),SPP(LL),SPPP(LL),Q(MM,LL),Z(MM,LL),P(MM,LL,LL),
     2  SAP(LL),SS(LL),SP(LL,LL),SSS(LL),A(MM,LL,LL),B(MM,LL,LL),
     3  C(MM,LL,LL),DL(MM,LL)
       DOUBLE PRECISION DETERM
       DIMENSION IPIVOT(LL),INDEX(LL,2)
       M=MAT
       N=M-1
       NMAX=LL
       DO 1 I=1,LCV
       DO 1 J=1,LCV
     1 C(M,I,J)=0.
C
C LOGIC TO STATEMENT 22 CALCULATES P1 AND Q2 MATRICES
C
       DO 4 I=1,LCV
       DO 4 J=1,LCV
     4 AS(I,J)=A(2,I,J)
       CALL MATINV(AS,LCV,ZZ,0,DETERM,IPIVOT,INDEX,NMAX,ISCALE)

       DO 15 I=1,LCV
       DO 15 J=1,LCV
       DK(I,J)=.0
       DO 15 K=1,LCV
    15 DK(I,J)=DK(I,J)+B(1,I,K)*AS(K,J)
       DO 16 I=1,LCV
       DO 16 J=1,LCV
       DKK(I,J)=0.
       DO 16 K=1,LCV
    16 DKK(I,J)=DK(I,K)*B(2,K,J) +DKK(I,J)
       DO 17 I=1,LCV
       DO 17 J=1,LCV
    17 DKKK(I,J)=DKK(I,J)-C(1,I,J)
       CALL MATINV (DKKK,LCV,ZZ,0,DETERM,IPIVOT,INDEX,NMAX,ISCALE)

       DO 18 I=1,LCV
       DO 18 J=1,LCV
       SP(I,J)=0.
       DO 18 K=1,LCV
    18 SP(I,J)=SP(I,J)+DK(I,K)*C(2,K,J)
       DO 19 I=1,LCV
       DO 19 J=1,LCV
       P(2,I,J)=0.
       DO 19 K=1,LCV
    19 P(2,I,J)=P(2,I,J)+DKKK(I,K)*SP(K,J)
       DO 20 I=1,LCV
       SPP(I)=0.
       DO 20 K=1,LCV
    20 SPP(I)=SPP(I)+DK(I,K)*DL(2,K)
       DO 21 I=1,LCV
    21 SPPP(I)=SPP(I)-DL(1,I)
       DO 22 I=1,LCV
```

```
      Q(2,I)=0.
      DO 22 K=1,LCV
   22 Q(2,I)=Q(2,I)+DKKK(I,K)*SPPP(K)
C
C LOGIC TO STATEMENT 23 CALCULATES,IN A LOOP, THE Q3....QM AND THE
C    P2 TO PM MATRICES
C
      DO 23 L=3,M
      DO 24 I=1,LCV
      DO 24 J=1,LCV
      DK(I,J)=0.
      DO 25 K=1,LCV
   25 DK(I,J)=DK(I,J)+A(L,I,K)*P(L-1,K,J)
   24 DK(I,J)=-DK(I,J)+B(L,I,J)
      CALL MATINV (DK,LCV,ZZ,0,DETERM,IPIVOT,INDEX,NMAX,ISCALE)

      DO 26 I=1,LCV
      DO 26 J=1,LCV
      P(L,I,J)=0.
      DO 26 K=1,LCV
   26 P(L,I,J)=P(L,I,J)+DK(I,K)*C(L,K,J)
      DO 27 I=1,LCV
      SAP(I)=0.
      DO 28 K=1,LCV
   28 SAP(I)=SAP(I)+A(L,I,K)*Q(L-1,K)
   27 SAP(I)=-SAP(I)+DL(L,I)
      DO 29 I=1,LCV
      Q(L,I)=0.
      DO 29 J=1,LCV
   29 Q(L,I)=Q(L,I)+DK(I,J)*SAP(J)
      IF(L.EQ.M) GO TO 50
      GO TO 23
   50 DO 30 I=1,LCV
   30 Z(L,I)=Q(L,I)
   23 CONTINUE
C
C  LOGIC TO STATEMENT  32 CALCULATES, IN A LOOP, THE Z3 TO Z(M-1) MATRICES
C
      M=N-2
      DO 32 L=1,M
      K=M-L+3
      DO 32 I=1,LCV
      Z(K,I)=0.
      DO 31 J=1,LCV
   31 Z(K,I)=Z(K,I)+P(K,I,J)*Z(K+1,J)
   32 Z(K,I)=-Z(K,I)+Q(K,I)
C
C REMAINING LOGIC CALCULATES Z1 AND Z2
C
      DO 33 I=1,LCV
      SAP(I)=.0
      DO 33 J=1,LCV
   33 SAP(I)=SAP(I)-SP(I,J)*Z(3,J)
      DO 34 I=1,LCV
   34 SAP(I)=SAP(I)+SPPP(I)
      DO 35 I=1,LCV
      Z(2,I)=.0
```

```
      DO 35 J=1,LCV
35    Z(2,I)=Z(2,I)+DKKK(I,J)*SAP(J)
      DO 36 I=1,LCV
      SS(I)=.0
      DO 36 J=1,LCV
   36 SS(I)=SS(I)+B(2,I,J)*Z(2,J)
      DO 37 I=1,LCV
      SSS(I)=.0
      DO 37 J=1,LCV
   37 SSS(I)=SSS(I)+C(2,I,J)*Z(3,J)
      DO 38 I=1,LCV
   38 SSS(I)=-SSS(I)-SS(I)+DL(2,I)
      DO 40 I=1,LCV
      Z(1,I)=.0
      DO 40 J=1,LCV
   40 Z(1,I)=Z(1,I)+AS(I,J)*SSS(J)
       RETURN
      END
```

INVERT

```
      SUBROUTINE INVERT(D,ACT,DIM)
C     INVERSION OF SYMMETRIC MATRIX
      INTEGER ACT,DIM
      DIMENSION D(DIM,DIM),LOC(76)
      DOUBLE PRECISION DP
      DP=1.D0
      DO 1 N=1,ACT
    1 LOC(N)=N
      DO 6 N1=1,ACT
      M=0
      PIVOT=0.
      DO 2 N2=N1,ACT
      NN=LOC(N2)
      IF (ABS(D(NN,NN)).LE.ABS(PIVOT)) GO TO 2
      M=N2
      PIVOT=D(NN,NN)
    2 CONTINUE
      IF (M.EQ.0) GO TO 8
      N=LOC(M)
      LOC(M)=LOC(N1)
      LOC(N1)=N
      D(N,N)=-1.
      DO 3 J=1,ACT
    3 D(N,J)=D(N,J)/PIVOT
      DO 5 I1=1,ACT
      I=LOC(I1)
      IF (N.EQ.I.OR.D(I,N).EQ.0.) GO TO 5
      DO 4 J1=I1,ACT
      J=LOC(J1)
      IF (N.EQ.J) GO TO 4
      D(I,J)=D(I,J)-D(I,N)*D(N,J)*DP
      D(J,I)=D(I,J)
    4 CONTINUE
    5 CONTINUE
      DO 6 I=1,ACT
    6 D(I,N)=D(N,I)
      DO 7 I=1,ACT
      DO 7 J=1,ACT
    7 D(I,J)=-D(I,J)
      RETURN
    8 WRITE(6,9)
    9 FORMAT (42H0MATRIX IS SINGULAR - EXECUTION TERMINATED   )
      STOP
      END
```

```
 MATINV
      SUBROUTINE MATINV(A,N,B,M,DETERM,IPIVOT,INDEX,NMAX,ISCALE)
C
C     MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
C
      DIMENSION A(NMAX,N),B(NMAX,M),IPIVOT(N),INDEX(NMAX,2)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)
C
C     INITIALIZATION
C
    5 ISCALE=0
    6 R1=10.0**18
    7 R2=1.0/R1
   10 DETERM=1.0
   15 DO 20 J=1,N
   20  IPIVOT(J)=0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVOT(J)-1) 60, 105, 60
   60 DO 100 K=1,N
   70 IF (IPIVOT(K)-1) 80, 100, 740
   80 IF (ABS(AMAX)-ABS(A(J,K)))85,100,100
   85 IROW=J
   90 ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IF (AMAX) 110,106,110
  106 DETERM=0.0
      ISCALE=0
      GO TO 740
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM) 140, 260, 140
  140 DETERM=-DETERM
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  205 IF(M) 260, 260, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
  310 PIVOT=A(ICOLUM,ICOLUM)
C
C     SCALE THE DETERMINANT
C
 1000 PIVOTI=PIVOT
```

```
1005 IF(ABS(DETERM)-R1)1030,1010,1010
1010 DETERM=DETERM/R1
     ISCALE=ISCALE+1
     IF(ABS(DETERM)-R1)1060,1020,1020
1020 DETERM=DETERM/R1
     ISCALE=ISCALE+1
     GO TO 1060
1030 IF(ABS(DETERM)-R2)1040,1040,1060
1040 DETERM=DETERM*R1
     ISCALE=ISCALE-1
     IF(ABS(DETERM)-R2)1050,1050,1060
1050 DETERM=DETERM*R1
     ISCALE=ISCALE-1
1060 IF(ABS(PIVOTI)-R1)1090,1070,1070
1070 PIVOTI=PIVOTI/R1
     ISCALE=ISCALE+1
     IF(ABS(PIVOTI)-R1)320,1080,1080
1080 PIVOTI=PIVOTI/R1
1    ISCALE=ISCALE+1
     GO TO 320
1090 IF(ABS(PIVOTI)-R2)2000,2000,320
2000 PIVOTI=PIVOTI*R1
     ISCALE=ISCALE-1
     IF(ABS(PIVOTI)-R2)2010,2010,320
2010 PIVOTI=PIVOTI*R1
     ISCALE=ISCALE-1
 320 DETERM=DETERM*PIVOTI
C
C    DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
 330 A(ICOLUM,ICOLUM)=1.0
 340 DO 350 L=1,N
 350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
 355 IF(M) 380, 380, 360
 360 DO 370 L=1,M
 370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT
C
C    REDUCE NON-PIVOT ROWS
C
 380 DO 550 L1=1,N
 390 IF(L1-ICOLUM) 400, 550, 400
 400 T=A(L1,ICOLUM)
 420 A(L1,ICOLUM)=0.0
 430 DO 450 L=1,N
 450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
 455 IF(M) 550, 550, 460
 460 DO 500 L=1,M
 500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
 550 CONTINUE
C
C    INTERCHANGE COLUMNS
C
 600 DO 710 I=1,N
 610 L=N+1-I
 620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
 630 JROW=INDEX(L,1)
 640 JCOLUM=INDEX(L,2)
```

```
650 DO 705 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLUM)
700 A(K,JCOLUM)=SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
    END
```

```
MAIN
C      PROGRAM TO COMPUTE OPTIMAL CONTROL
C
C
      DIMENSION IHP(50),ISP(180),A(180,169),B(50,50),C(50,180),D(50,50),
     1  WD(180),WW(180)
      BN=1.0
C     INITIALIZATION
      DO 12 I=1,180
      WD(I)=0.0
      WW(I)=0.0
      ISP(I)=0
      DO 11 J=1,50
   11 C(J,I)=0.0
      DO 12 J=1,168
   12 A(I,J)=0.0
      DO 20 I=1,50
      IHP(I)=0.0
      DO 20 J=1,50
   20 B(I,J)=0.0
      READ(5,50) IM,KM,DI,DO,FNO
   50 FORMAT(2I5,3F10.5)
      RI=DI/2.0
      RO=DO/2.0
C
C     INPUT SAMPLE POINT AND HEATER LOCATIONS
C
      READ(5,100) NHP,NSP
  100 FORMAT(2I5)
      READ(5,110) (IHP(I),I=1,NHP)
  110 FORMAT(16I5)
      READ(5,120) (ISP(I),I=1,NSP)
  120 FORMAT(16I5)
      WRITE(6,259)
  259 FORMAT(29H INDICES OF HEAT PATCH POINTS//4X,1HI,2X,3HIHP)
      DO 260 L1=1,NHP
  260 WRITE(6,261) L1,IHP(L1)
  261 FORMAT(2I5)
      WRITE(6,262)
  262 FORMAT(25H INDICES OF SAMPLE POINTS//4X,1HI,2X,3HISP)
      DO 263 L1=1,NSP
  263 WRITE(6,261) L1,ISP(L1)
C
C     INPUT MATRIX A AND COMPUTE REDUCED MATRIX
C
      READ(4,130)((A(I,J),I=1,180),J=1,169)
  130 FORMAT(6E13.8)
C
C     SUBTRACT OUT SPHERICAL PART OF A(I,J)
C
      DIMENSION THET(15),BE(15),Y(15)
      FIM1=IM-1
      FKM=KM
      RF=2.0*DO*FNO
      SI=.5*DI/RF
      CI=SQRT(1.0-SI*SI)
      SO=.5*DO/RF
```

```
      CO=SQRT(1.0-SO*SO)
      THETI=ATAN(SI/CI)
      THETO=ATAN(SO/CO)
      DTHET=(THETO-THETI)/FIM1
      THETO=THETO-DTHET
      B2=0.0
      DO 39 I=1,IM
      THET(I)=THETI+(I-1)*DTHET
   39 BE(I)=COS(THET(I))/COS(THETO)-1.0
      DO 63 KH=1,NHP
      JH=IHP(KH)
      WB=0.0
      B2=0.0
      DO 61 IS=1,NSP
      I=ISP(IS)/IM
      I=ISP(IS)-I*IM
      JS=ISP(IS)
      WB=WB+A(JS,JH)*BE(I)
   61 B2=B2+BE(I)**2
      DO 62 I=1,IM
   62 Y(I)=WB/B2*BE(I)
      DO 63 IS=1,NSP
      I=ISP(IS)/IM
      I=ISP(IS)-I*IM
      JS=ISP(IS)
   63 A(JS,JH)=A(JS,JH)-Y(I)
      DO 150 I=1,NSP
      DO 150 J=1,NHP
      IS=ISP(I)
      JH=IHP(J)
  150 A(I,J)=A(IS,JH)
C
C     TO COMPUTE   I-A(1/ATA)AT
C
      DO 200 I=1,NHP
      DO 200 J=1,NHP
      DO 200 K=1,NSP
  200 B(I,J)=B(I,J)+A(K,I)*A(K,J)*BN
      DO 198 L1=1,NHP
      DO 198 L2=1,NHP
  198 D(L1,L2)=B(L1,L2)/BN
      CALL INVERT (B,NHP,50)
      DO 199 L1=1,NHP
      DO 199 L2=1,NHP
  199 B(L1,L2)=  BN *B(L1,L2)
      DO 197 I=1,NHP
      DO 197 J=1,NHP
      DO 197 K=1,NHP
  197 C(I,J)=C(I,J)+B(I,K)*D(K,J)
      DO 195 I=1,NHP
      DO 195 J=1,NHP
  195 C(I,J)=0.0
      DO 250 I=1,NHP
      DO 250 J=1,NSP
      DO 250 K=1,NHP
  250 C(I,J)=C(I,J)+B(I,K)*A(J,K)
C
```

```
C       COMPUTE WD
C
        RS=RI+(RO-RI)*(IM-2)/(IM-1)
        DO 500 I=1,IM
        DO 500 J=1,KM
        K=IM*(J-1)+I
        R=RI+(RO-RI)*(I-1)/(IM-1)
        TH=0.5235988*(J-1)
        ARG=3.1415927*R/RS
  500   WD(K)=SIN(ARG)**2
C
C       SUBTRACT OUT SPHERICAL PART FROM WD
C
        WB=0.0
        DO 300 IS=1,NSP
        JS=ISP(IS)
        I=ISP(IS)/IM
        I=ISP(IS)-I*IM
  300   WB=WB+WD(JS)*BE(I)
        DO 301 I=1,IM
  301   Y(I)=BE(I)*WB/B2
        DO 302 IS=1,NSP
        JS=ISP(IS)
        I=ISP(IS)/IM
        I=ISP(IS)-I*IM
  302   WD(JS)=WD(JS)-Y(I)
        IKM=IM*KM
        WRITE(6,264)
  264   FORMAT(12H DISTURBANCE)
        WRITE(6,265)(WD(K),K=1,IKM)
  265   FORMAT(15E8.3)
        DO 550 K=1,NSP
        IS=ISP(K)
  550   WD(K)=WD(IS)
C
C       COMPUTE PERFORMANCE INDEX
C
        AJ1=0.0
        DO 600 I=1,NSP
  600   AJ1=AJ1+WD(I)**2
        DO 700 I=1,NSP
        DIJ=0.0
        DO 700 J=1,NSP
        IF(I.EQ.J) DIJ=1.0
        DO 701 K=1,NHP
  701   DIJ=DIJ-A(I,K)*C(K,J)
  700   WW(I)=WW(I)+DIJ*WD(J)
        AJ2=0.0
        DO 750 I=1,NSP
  750   AJ2=AJ2+WD(I)*WW(I)
        WRITE(6,1000) AJ1,AJ2
 1000   FORMAT(//46H THE PERFORMANCE INDEX BEFORE COMPENSATION IS ,E10.5,
       1//45H THE PERFORMANCE INDEX AFTER COMPENSATION IS ,E10.5)
        BJ1=SQRT(AJ1/NSP)
        BJ2=SQRT(AJ2/NSP)
        WRITE(6,1050) BJ1,BJ2
 1050   FORMAT(20H RMS ERROR BEFORE = ,E20.8/
```