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# PRAIRIE VIEW A. AND M. COLLEGE

PRAIRIE VIEW, TEXAS 77445

# SCHOOL OF ENGINEERING

# UNDERGRADUATE RESEARCH REPORT

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Submitted by

H. Y. YEH

FEBRUARY 1973

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Part I

# STATISTICAL METHOD OF DETERMINING YIELD STRENGTH IN MILD STEEL RODS

By

Thomas Hadnot

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# CHAPTER I

# INTRODUCTION AND OBJECT

# Preface

Two students' investigative papers are including in this report. One paper is on "STATISTICAL METHOD OF DETERMINING YIELD STRENGTH IN MILD STEEL RODS" by Mr. Thomas E. Hadnot, a major in Civil Engineering. This paper discusses the probability distribution of yield strength for mild steel. For the purpose of obtaining sufficient data for statistical analysis, tensil experiment was conducted for 50 cold rolled and 38 hot rolled mild steel rods. The yield strength distribution of mild steel is assumed to follow normal or lognormal laws. Based on 95 percent condidence level, chi-square, kolmogro-Smirnov and goodness-of-fit methods are used to test the normality or lognormality of experiment results. The other paper is on "VIBRATION ANALYSIS OF MECHANICAL SYSTEMS" by Mr. Sanders Marshall, a Mechanical Engineering major. The emphasis of this paper is on the application of numerical techniques to solve mechanical vibration problems. As an example, a non-linear dash-pot-speing system subjected to seismic excitation are analyzed by using fourth-order Runge-Kutta integrated method and the aid of IBM 360-65 Computer.

This undergraduate research was supported by NASA under contract NGR-044-033-002, and was conducted at School of Engineering of which Mr. A. E. Greaux is the Dean. Thanks are due to Miss Josephine Chisom for her excellent typing.

## Introduction

It is common practice today in structural design offices to pick up a value for the strength properties of materials from specification. The materials have been tested and the strength property value published is a true representation of the population of the materials tested. However, it cannot be said that this value is a true representation of the same material that will be made in the future even though it is made by the same manufacturer.

Statistical research provides a means for determining, with a certain level of confidence, the strength properties as well as other vital information that is of paramount importance to a design engineer. For instance, an engineer designs a structure with a factor of safety of 2 and existing strength properties of the material. Does he know, in fact, that this will be a safe structure? No. He has to perform certain test on the material himself. Then, by using the value that he gets from his experiments, and the value that he receives from the manufacturer, he can make a comparison and decide whether or not the manufacturer is supplying him with good or bad material. Then and only then will he know that it is safe to go ahead and build the structure.

This problem of testing material involves a very important function of statistics---statistical inference.[1] There is no way to measure the average yield strength of a group of structural steel rods unless they were tested individually. Of course, if that were done, the average yield strenght of that group of rods could be obtained but the rods would be useless. They certainly could not be used after they have been ruptured. This would, of course, require another group of rods, but they could not be used because the average yield strength would not be known. Through experimentation and statistical inferences, however, a decision can be made about the population of steel rods from the experimental results of the sample tested.

As one can see this is not a new type of experiment. Experiments of this type are conducted almost daily on a small scale. It is not limited to any one type of material. For instance, there has been statistical research done to determine the crushing strength of concrete cubes, the breaking strength of steel mesh wire, and of course, the yield force of different types of steel bars, and they all have been effective.

## Object

The object of this investigative paper is concerned with the yield strength---commonly referred to as the

[1]Samuel B. Richmond, Statistical Analysis (New York: The Ronald Press Company, 1964), p. 4.

measure of the elastic strength of materials---of mild strutural steel. By using statistical methods a study of the variation of the yield strength of each individual rod, and uncertainty of the value of the yield strength as well as other variations and uncertainties can be made.

The yield strength of cold rolled steel rods and hot rolled steel rods are studied in order to determine the probability model that best suit the distribution.

# CHAPTER II

## THEORY

Sampling

In order to make inferences about the population, a sample from that population has to be taken and examined. Often times it is too costly, too time consuming, or impossible to examine the complete population. Also, the inspection process may be destructive, in which case sampling inspection is the only possible technique.[2] As is the case with this investigation, the inspection process would be destructive because each individual rod would have to be stretched to its yield and there would be none left for use.

Therefore it is generally recognized that the reason for taking samples is one of the following: (1) Due to limitations of time, money, or personnel, it is impossible to study every item in the population; (2) the population, as defined, may not physically exist; (3) to examine an item may require that the item be destroyed.[3]

In statistical research it is important to know whether a sample or a complete population is being investigated. A population can consist of many samples; however, a sample

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[2] Ibid., p. 324.

[3] Bernard Ostle, Statistics in Research (Ames, Iowa: The Iowa State University Press, 1963). p. 44.

cannot be a population. The concept of a sample as opposed to a population is very important.[4] A sample is defined as a part of a population selected by some rule or plan and the important things to know are: (1) that we are dealing with a sample and (2) which population has been sampled.[5]

On the other hand a population is defined as the totality of all possible values (measurements or counts) of a particular characteristic for a specified group of objects.[6] It is not general in engineering that a complete population is investigated.

A sample can be taken from different population in various ways. Samples selected according to some chance mechanism are known as probability samples if every item in the population has a known probability of being in the sample.[7] The accuracy of the results obtained from a specified sample cannot really be judged.[8] That is to say that it is not known how accurate an estimate is rather the precision of the estimating technique.

By use of statistical inference, decisions can be made about the population and be right some known proportion of

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[4] _Ibid_., p. 45.

[5] _Ibid_., p. 45.

[6] _Ibid_., p. 44.

[7] _Ibid_., p. 45.

[8] Samuel B. Richmond, _Statistical Analysis_ (New York: The Ronald Press Company, 1964), p. 325.

the time, and on the other hand be wrong some proportion of the time. For instance, with a 5 percent level of significance a decision can be made that would be right in 95 percent of the time in estimating population boundaries. By the same token 5 percent of the decisions would be wrong. The 5 percent error is unaviodable because there has to be some degree of error. In the final analysis the goodness or badness of the sample is determined by the way that the sample was obtained.

Statistical inference and the formulas for standard error that will be used in this experiment is based upon simple random sampling in which each item of the population or subpopulation has an equal chance of being in the sample.

After the sample has been selected, tested, and the data calculated, the distribution curve for the mild structural steel rods in this experiment was assumed.

Normal Distribution

Computation of probabilities are based on probabilistic models which have been proposed to describe the probabilistic behavior of certain physical variables.[9] The engineer uses his sense of practicality from observations or studies theory to propose a probability model usch as the normal, or lognormal distribution which are the distributions proposed for this experiment. In another situation, for the sake of convenience, a distribution is selected because it is

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[9]Wilson, H. C. Tang, Notes on Statistical Inferences (Urbana, Illinois: University of Illinois, 1970), p. 1.

believed to be a satisfactory description of the phenomenon.[10] In both cases the parameters of the distribution has to be evaluated to find out whether the sample is representative of the characteristics of the population from which the sample has been taken; or when the characteristics are unknown, through observations and collection of data, the parameters can be estimated and a decision can be made as to the goodness or badness of the research made.

The normal distribution as proposed in this experiment is generally used for populations whose members are measured for some characteristic such as height or yield.[11] The variable flows without a break from one member to the next; it is continuous with no limit to the number of members with different measurements.[12] It should be made clear, however, that the variables could be distributed in other ways too.

The normal distribution is one of the oldest of statistical inference probability models. Its equation was published as early as 1733 by DeMoivre.

The equation of the distribution is

$$p = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)/2\sigma^2}$$

Where: p is the probability
x is the random variable
$\mu$ is the means given by the equation

$$\mu = \frac{\Sigma f_x}{\Sigma f}$$

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[10] Ibid., p. 1.

[11] George W. Snedecor, Statistical Methods (Ames, Iowa: The Iowa State University Press, 1956), p. 35.

[12] Ibid., p. 35

Where: f is the frequency
x is the random variable

$\sigma$ is the standard deviation given by the equation

$$\sigma^2 = \frac{\Sigma f (x-\mu)^2}{\Sigma f}$$

The normal distribution curve is shown in figure 2.1.

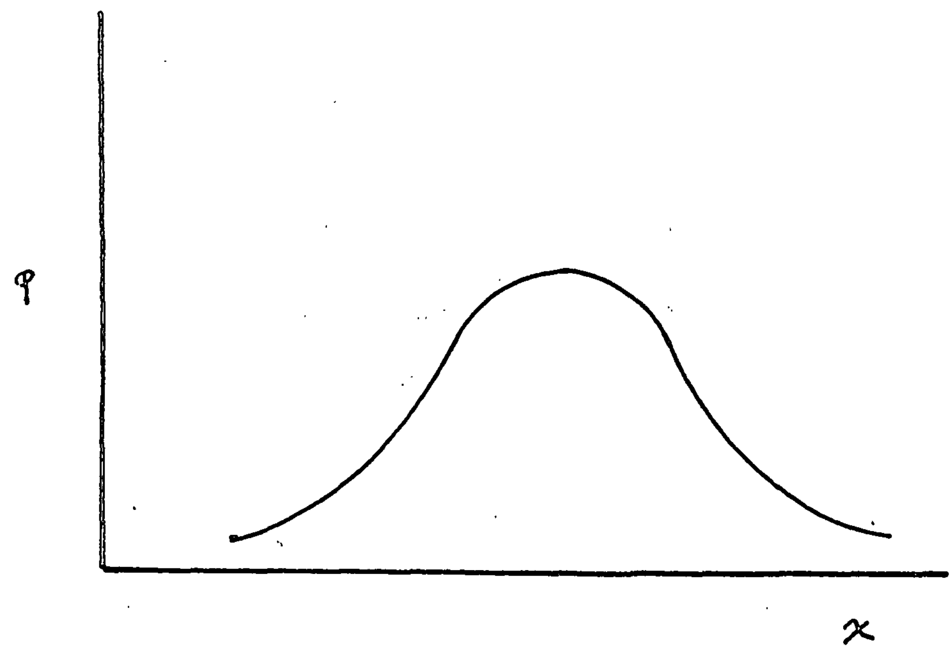


Fig. 2.1 Normal distribution curve

## The Logarithmic Normal Law

A random variable x has a logarithmic normal probability distribution if ln x (the natural logarithm of x) has a normal probability distribution.[13] In this case, the density function of x is,

$$P_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2} \frac{(\ln x - \bar{x})^2}{\sigma}\right]$$

Where $\bar{x} = E(\ln x)$ and $\sigma = \sqrt{(\ln x)}$ are respectively, the mean and standard deviation of ln x.

Examples of lognormal density function are shown in figure 2.2

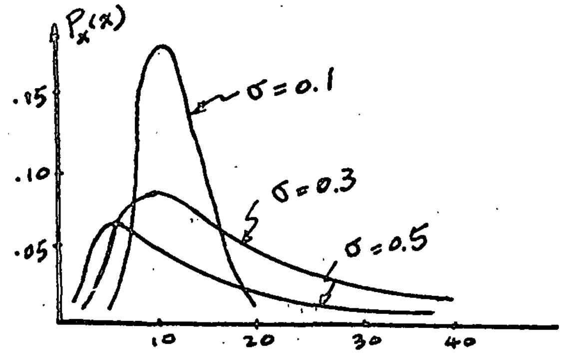


Fig. 2.2 Lognormal Density Functions

In view of the facility for the calculation of probabilities of lognormal random variables, and also because the values of the random variable is limited on the left, the logarithmic normal distribution is useful in many practical applications.[14]

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[13] Alfredo, H. S. Ang, and Mohammed Amin, <u>Probabilistic Structural Mechanics and Engineering</u> (Urbana, Ill.: University of Illinois, 1970), p. 77.

[14] Ibid., p. 79.

Chi-Square Test

"A problem that arises frequently in statistics is the testing of the compatibility of a set of observed and theoretical frequencies. If a theoretical distribution function has been fitted to an empirical distribution the question whether the fit is satisfactory naturally arises. . . . It is usually assumed that the data from a normal population and that the fitted normal curve is an approximation of the population distribution. Thus, the question whether the fit is satisfactory in this case can be answered only if one knows what sort of histograms will be obtained in random samples from a normal population."[15]

The chi-square test is one of the methods of testing the goodness of fit for distribution. It involves the comparison of observed frequencies with expected frequencies. Using the chi-square test, the test of different kinds of hypotheses about frequencies can be made. A comparison of observed frequencies with theoretical or expected frequencies, and a comparison of two or more sets of observed frequencies can be made with the chi-square test in order to determine whether the difference among the two sets of frequencies are significant; that is, whether the observed differences are too great to be attributable to chance.[16]

The general procedure for chi-square test is:

(1) The expected or theoretical frequency is calculated.

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[15]Wilson, H. C. Tang, Notes on Statistical Inferences (Urbana, Illinois: University of Illinois, 1970), p. 50.

[16]Richmond, Samuel B., Statistical Analysis (New York: The Ronald Press Company, 1964), p. 280.

(2) The sample observed frequency is obtained.

(3) The frequencies are compared by computing chi-square which depends upon the differences between the corresponding frequencies.

(4) The value is compared with the known theoretical distribution of chi-square to determine whether the value of chi-square is significantly different from 0.

The computed value for chi-square is designated by the Greek symbol $\chi^2 = \Sigma(\frac{o_i - e_i}{e_i})$

Where: $O_i$ is the observed frequency

$e_i$ is the computed (expected or theoretical) frequency

Kolmogorov-Smirnov Test

Another test designed to test the goodness-of-fit for proposed distribution function is the kolmogorov-smirnov test. It was named for the two Russian mathematicians whose names are attached to it. It is said to be a more powerful test than the chi-square test and its uses is encouraged.[17] It proceeds as follows:

(1) Let F(x) be the completely specified theoretical cumulative distribution function under the null hypothesis.

(2) Let $S_n(x)$ be the sample c.d.f. basen on n observations. For any observed x, $S_n(x) = \frac{k}{n}$ where k is the number of observations less than or equal to x

[17]Bernard, Ostle, Statistics in Research (Ames, Iowa: The Iowa State University Press, 1963), p. 471.

(3) Determine the maximum deviation, D, defined by

$$D = \max \left[ F(x) - S_n(x) \right]$$

(4) If, for the chosen significance level, the observed value of D is greater than or equal to the critical value taken from the kolmogorov-smirnov goodness-of-fit table, the hypothesis will be rejected.

# CHAPTER III

# PROCEDURE AND EXPERIMENTAL RESULTS

## Procedure

With the aforementioned principles and theories of statistics in mind the experiment was conducted in the following manner:

(1) A sample of 50 cold rolled steel rods was selected at random; a sample of 38 hot rolled steel rods was selected at random.

(2) These two samples were tested, at different times, using the universal testing machine for the yield strength. With the aid of an extensometer used in conjunction with the aforementioned equipment and the Baldwin Microformer autographic stress-strain recorder, a copy of the stress-strain diagram was achieved for each speciman tested. The extensometer was attached directly to the speciman which were round steel rods with a .505 inch diameter. The gage length of the extensometer is 2 inches and the measuring range is 0.040 inches.

(3) The yield strength of .2 percent offset was then calculated for each individual speciman.

(4) The data collected was then grouped, charted and graphed as applicable.

(5) A normal distribution probability model was proposed, calculated, and charted.

(6) A lognormal distribution was graphed and the theoretical frequency calculated.

(7) The chi-square test and the Kolmogrov-Smirnov Test were applied to determine the goodness-of-fit of the normal and lognormal distribution curves.

(8) Inferences and conclusions were drawn from the applied theories of statistics.

## Experimental Results

DATA

TABLE 3.1 - YIELD STRENGTH (PSI)
(COLD ROLLED STEEL RODS)

| | | |
|---|---|---|
| 62406 | 58912 | 64903 |
| 62906 | 62906 | 57913 |
| 62406 | 61158 | 65903 |
| 61907 | 62656 | 65903 |
| 60909 | 64903 | 62406 |
| 59910 | 62656 | 59910 |
| 64903 | 62406 | 62906 |
| 59910 | 62356 | 62906 |
| 64903 | 63656 | 59910 |
| 61408 | 58912 | 59910 |
| 60909 | 57913 | 64903 |
| 62906 | 64403 | 59910 |
| 61250 | 62656 | 58912 |
| 63405 | 62906 | 62406 |
| 63405 | 60909 | 64903 |
| 62906 | 62906 | 57913 |
| 64903 | 58412 | |

DATA

TABLE 3.2 - TALLY SHEET FOR DATA OF TABLE 3.1
(COLD ROLLED STEEL RODS)

| YIELD STRENGTH | TALLY | FREQUENCY |
|---|---|---|
| 57913 | 111 | 3 |
| 58412 | 1 | 1 |
| 58912 | 11 | 2 |
| 59910 | ~~1111~~ 1 | 6 |
| 60909 | 111 | 3 |
| 61158 | 11 | 2 |
| 61408 | 1 | 1 |
| 61907 | 1 | 1 |
| 62356 | 1 | 1 |
| 62406 | ~~1111~~ | 5 |
| 62656 | 111 | 3 |
| 62906 | ~~1111~~ 111 | 8 |
| 63405 | 11 | 2 |
| 63656 | 1 | 1 |
| 64403 | 1 | 1 |
| 64903 | ~~1111~~ 1111 | 9 |

TABLE 3.3 - FREQUENCY DISTRIBUTION AND CALCULATION OF $\overline{x}$, $s^2$, and s

(Cold Rolled Steel Rods)

| AT LEAST | BUT LESS THAN | f | r.f | x | fx | x - x | $(x - \overline{x})^2$ | $F(x - x)^2$ |
|---|---|---|---|---|---|---|---|---|
| 57.0 | 58.0 | 3 | .06 | 57.5 | 172.5 | -4.28 | 18.32 | 54.96 |
| 58.0 | 59.0 | 4 | .08 | 58.5 | 234.0 | -3.28 | .76 | 43.03 |
| 59.0 | 60.0 | 6 | .12 | 59.5 | 357.0 | -2.28 | 5.20 | 31.19 |
| 60.0 | 61.0 | 3 | .06 | 60.5 | 181.5 | -1.28 | 1.64 | 4.92 |
| 61.0 | 62.0 | 4 | .08 | 61.5 | 246.0 | -0.28 | 0.08 | 0.31 |
| 62.0 | 63.0 | 17 | .34 | 62.5 | 1062.5 | 0.72 | 0.5184 | 8.81 |
| 63.0 | 64.0 | 3 | .06 | 63.5 | 190.5 | 1.72 | 2.96 | 8.87 |
| 64.0 | 65.0 | 10 | .20 | 64.5 | 645.0 | 2.72 | 7.40 | 73.98 |
| 665.0 | 66.0 | 0 | 0 | 65.5 | 0 | - | - | 0 |
| $\Sigma$ | | 50 | 1.00 | | 3089. | | | 225.16 |

$\bar{x} = \frac{\Sigma f x}{\Sigma f} = \frac{3089}{50} = 61.78$ $\quad S^2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f} = \frac{225.16}{50} = 4.51$ $\quad S = \sqrt{4.51} = 2.126$

TABLE 3.4 - CLASS INTERVALS AND POINTS
FOR NORMAL DISTRIBUTION CURVE

(Cold Rolled Steel Rods)

| CLASS INTERVAL | NORMAL DISTRIBUTION | *LOGNORMAL f |
|---|---|---|
| 57.5 | 0.0246 | 0.08 |
| 58.5 | 0.0569 | 0.05 |
| 59.5 | 0.1054 | 0.06 |
| 60.5 | 0.1565 | 0.10 |
| 61.5 | 0.1860 | 0.15 |
| 62.5 | 0.1772 | 0.20 |
| 63.5 | 0.1352 | 0.36 |
| 64.5 | 0.0826 | 0 |

Normal distribution equation

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

*Lognormal values taken from lognormal curve. Fig. 3.2

(Cold Rolled Steel Rods)

Calculations for chi-square test for goodness-of-fit

$$\chi^2 = \sum\left(\frac{o_i - e_i}{e_i}\right)$$

Where: $o_i$ = observed frequency

$e_i$ = expected or theoretical frequency

(1). Normal

$$\chi^2 = \frac{(.06 - .0246)^2}{.0246} + \frac{(.08 - .0569)^2}{.0569} + \frac{(.12 - .1054)^2}{.1054}$$

$$+\frac{(.06 - .1565)^2}{.1565} + \frac{(.08 - .1860)^2}{.1860} + \frac{(.34 - .1772)^2}{.1772}$$

$$+\frac{(.06 - .1352)^2}{.1352} + \frac{(.20 - .0826)^2}{.0826}$$

$$\chi^2 = .5405\ (50) = 27.02$$

(2). Lognormal

$$\chi^2 = \frac{(.08 - .0246)^2}{.0246} + \frac{(.05 - .0569)^2}{.0569} + \frac{(.06 - .1054)^2}{.1054}$$

$$+\frac{(.10 - .1565)^2}{.1565} + \frac{(.15 - .1860)^2}{.1860} + \frac{(.20 - .1771)^2}{.1771}$$

$$+\frac{(.36 - .1352)^2}{.1352} + \frac{(0 - .0826)^2}{.0826}$$

$$\chi^2 = .6320\ (50) = 31.60$$

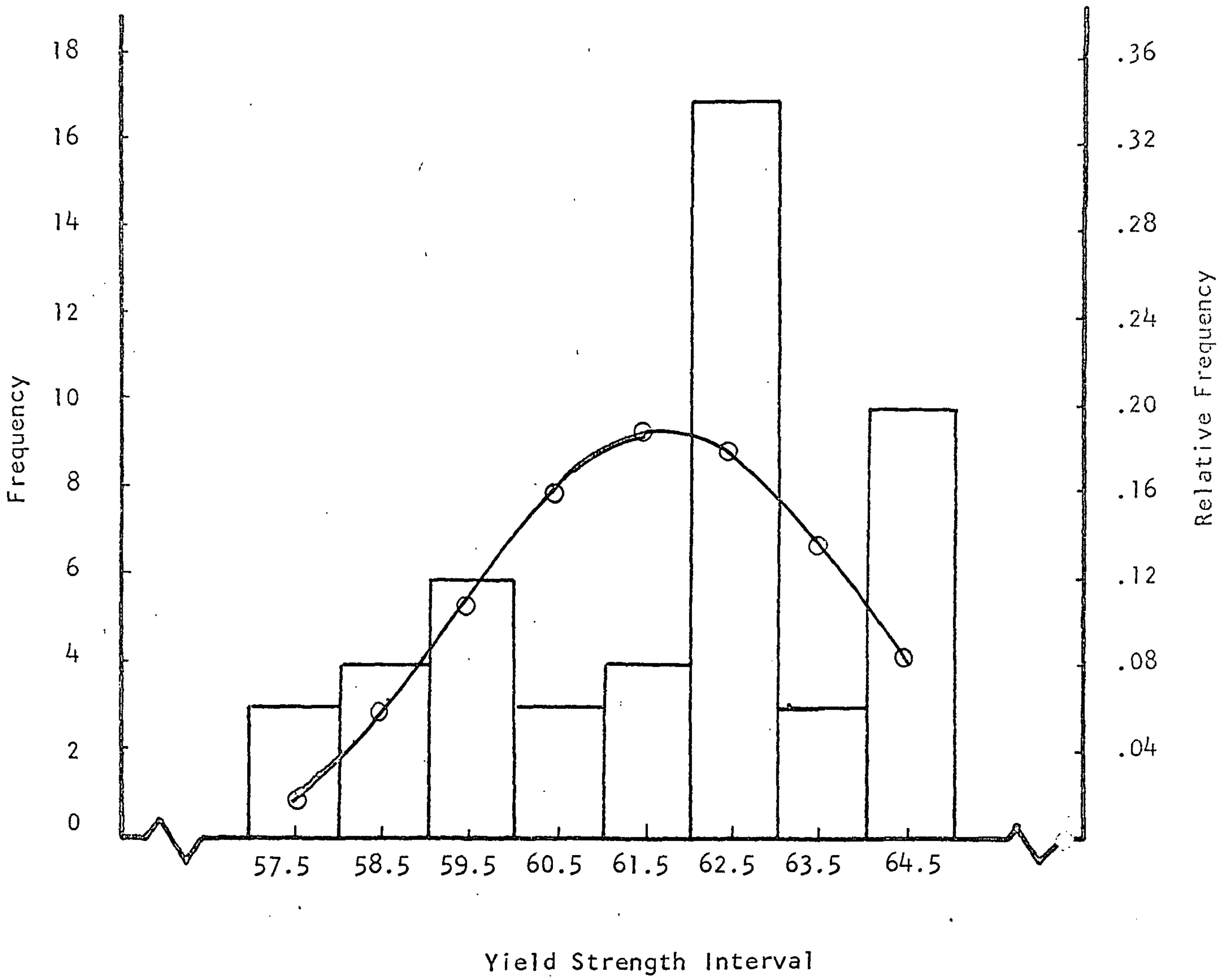



Figure 3.1 Frequency histogram and normal distribution curve plotted from Table 3.3.

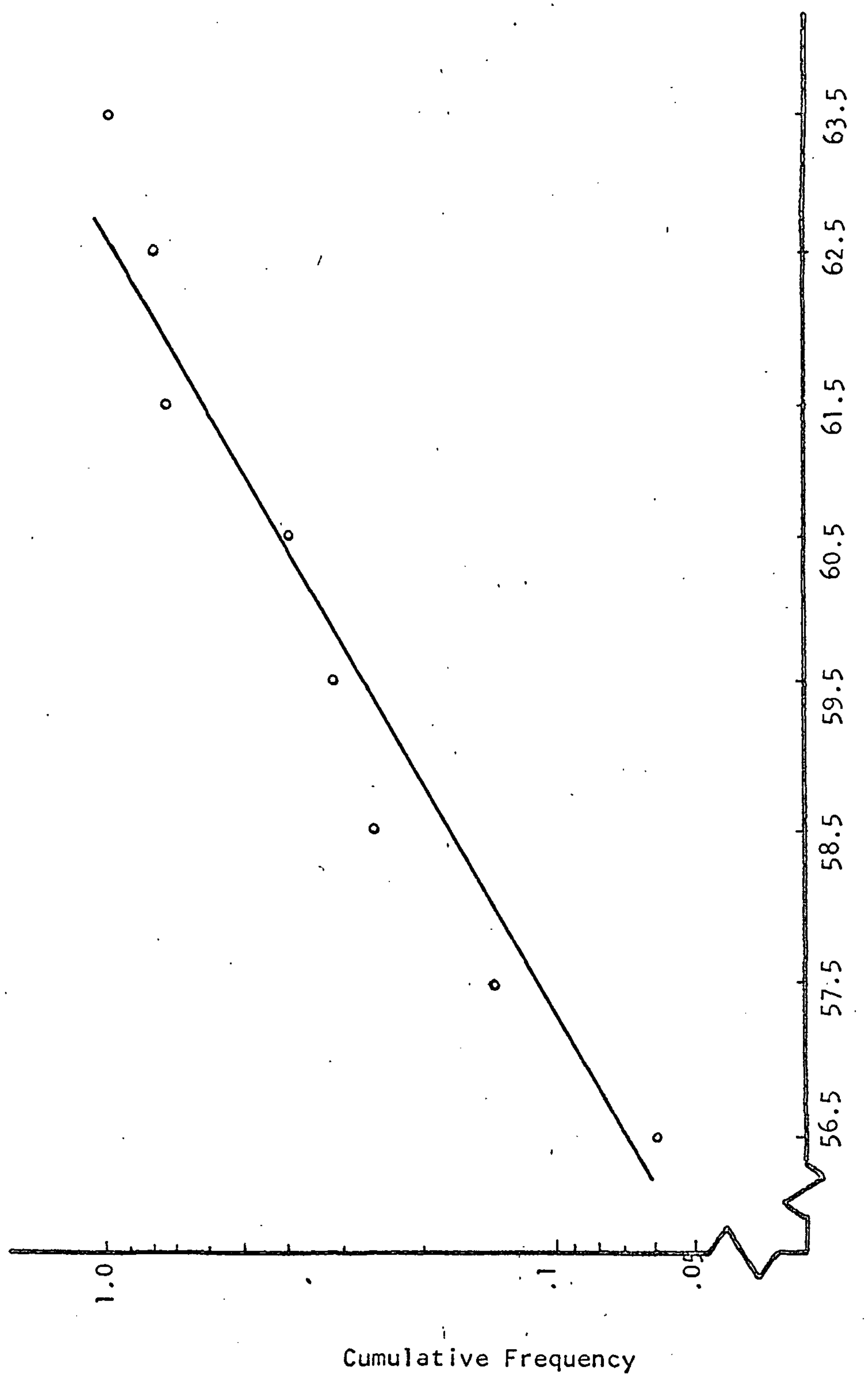
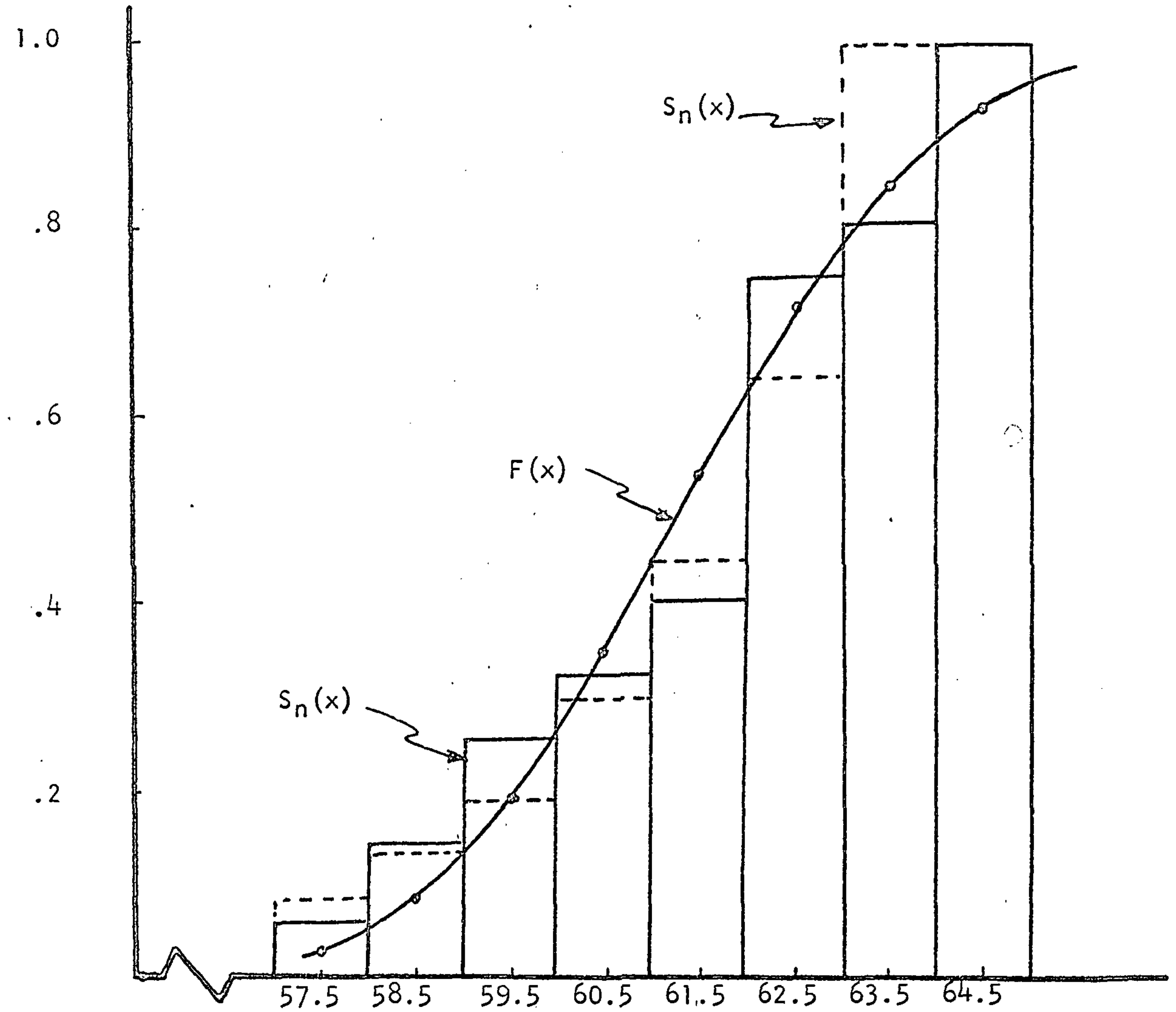



Figure 3.2 Lognormal distribution curve




Yield Strength Intervals

Figure 3.3 Sample and Theoretical Cumulative Distribution Functions

DATA

TABLE 3.1a - YIELD STRENGTH (PSI)
(Hot Rolled Steel Rods)

| | | |
|---|---|---|
| 39940 | 39940 | 39940 |
| 42436 | 41187 | 41438 |
| 41937 | 39940 | 39441 |
| 39940 | 41937 | 42187 |
| 40439 | 39940 | 37943 |
| 41937 | 39940 | 42187 |
| 42436 | 41937 | 36994 |
| 39940 | 41687 | 37444 |
| 42436 | 42436 | 39940 |
| 42436 | 42936 | 37444 |
| 42436 | 42686 | 37444 |
| 40939 | 42436 | 37444 |
| 39940 | 42187 | ----- |

DATA

TABLE 3.2a - TALLY SHEET FOR DATA OF TABLE 3.1a
(Hot Rolled Steel Rods)

| Yield STRENGTH | TALLY | FREQUENCY |
|---|---|---|
| 36994 | 1 | 1 |
| 37444 | 1111 | 4 |
| 37943 | 1 | 1 |
| 39441 | 1 | 1 |
| 39940 | ~~1111~~ ~~1111~~ | 10 |
| 40439 | 1 | 1 |
| 40939 | 1 | 1 |
| 41187 | 1 | 1 |
| 41438 | 1 | 1 |
| 41687 | 1 | 1 |
| 41937 | 1111 | 4 |
| 42187 | 111 | 3 |
| 42436 | ~~1111~~ 11 | 7 |
| 42686 | 1 | 1 |
| 42936 | 1 | 1 |

TABLE 3.3a - FREQUENCY DISTRIBUTION AND
CALCULATION OF $\bar{x}$, $s^2$, AND s

(Hot Rolled Steel Rods)

| AT LEAST | BUT LESS THAN | f | r.f. | x | fx | x - x | $(x - x)^2$ | $f(x - x)^2$ |
|---|---|---|---|---|---|---|---|---|
| 36 | 37 | 1 | .03 | 36.5 | 36.5 | -4.03 | 16.24 | 16.24 |
| 37 | 38 | 5 | .13 | 37.5 | 187.5 | -3.03 | 9.18 | 45.90 |
| 38 | 39 | 0 | 0 | 38.5 | 0 | -2.03 | 4.12 | 0 |
| 39 | 40 | 11 | .29 | 39.5 | 434.5 | -1.03 | 1.06 | 11.66 |
| 40 | 41 | 2 | .05 | 40.5 | 81.0 | -0.03 | .001 | .002 |
| 41 | 42 | 7 | .18 | 41.5 | 290.5 | .97 | .94 | 6.58 |
| 42 | 43 | 12 | .32 | 42.5 | 510.0 | 1.97 | 3.88 | 46.56 |
| $\Sigma$ | | 38 | 1.00 | | 1540.0 | | | 126.94 |

$$\bar{X} = \frac{\Sigma f x}{\Sigma f} = \frac{1540}{38} = 40.53 \qquad S^2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f} = \frac{126.94}{38} = 3.34 \qquad S = \sqrt{3.34} = 1.83$$

DATA

TABLE 3.4a - CLASS INTERVALS
AND POINTS FOR NORMAL DISTRIBUTION CURVE

(Hot Rolled Steel Rods)

| Class Interval | Normal Distribution | *Lognormal f |
|---|---|---|
| 36.5 | 0.0193 | 0.04 |
| 37.5 | 0.0554 | 0.04 |
| 38.5 | 0.1178 | 0.07 |
| 39.5 | 0.1861 | 0.14 |
| 40.5 | 0.2180 | 0.23 |
| 41.5 | 0.1894 | 0.48 |
| 42.5 | 0.1221 | |

Normal Distribution Equation

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/2\sigma^2}$$

*Lognormal values taken from lognormal curve. Fig. 3.2a

Calculations for chi-square test for goodness-of-fit

$$\chi^2 = \sum\left(\frac{o_i - e_i}{e_i}\right)$$

Where: $o_i$ is the observed frequency

$e_i$ is the expected or theoretical frequency

(1). Normal

$$\chi^2 = \frac{(.03 - .0193)^2}{.0193} + \frac{(.13 - .0554)^2}{.0554} + \frac{(0 - .1178)^2}{.1178}$$

$$+ \frac{(.29 - .1861)^2}{.1861} + \frac{(.05 - .2180)^2}{.2180} + \frac{(.18 - 1894)^2}{.1894}$$

$$+ (.32 - .1221)^2 \qquad \chi^2 = .7330 \times (38) = 27.85$$

(2). Lognormal

$$\chi^2 = \frac{(.04 - .0193)^2}{.0193} + \frac{(.04 - .0554)^2}{.0554} + \frac{(.07 - .1178)^2}{.1778}$$

$$+ \frac{(.14 - .1861)^2}{.1861} + \frac{(.23 - .2180)^2}{.2180} + \frac{(.48 - .1894)^2}{.1894}$$

$$+ \frac{(0 - .1221)^2}{.1221} \qquad \chi^2 = .6260 \times (38) = 23.79$$

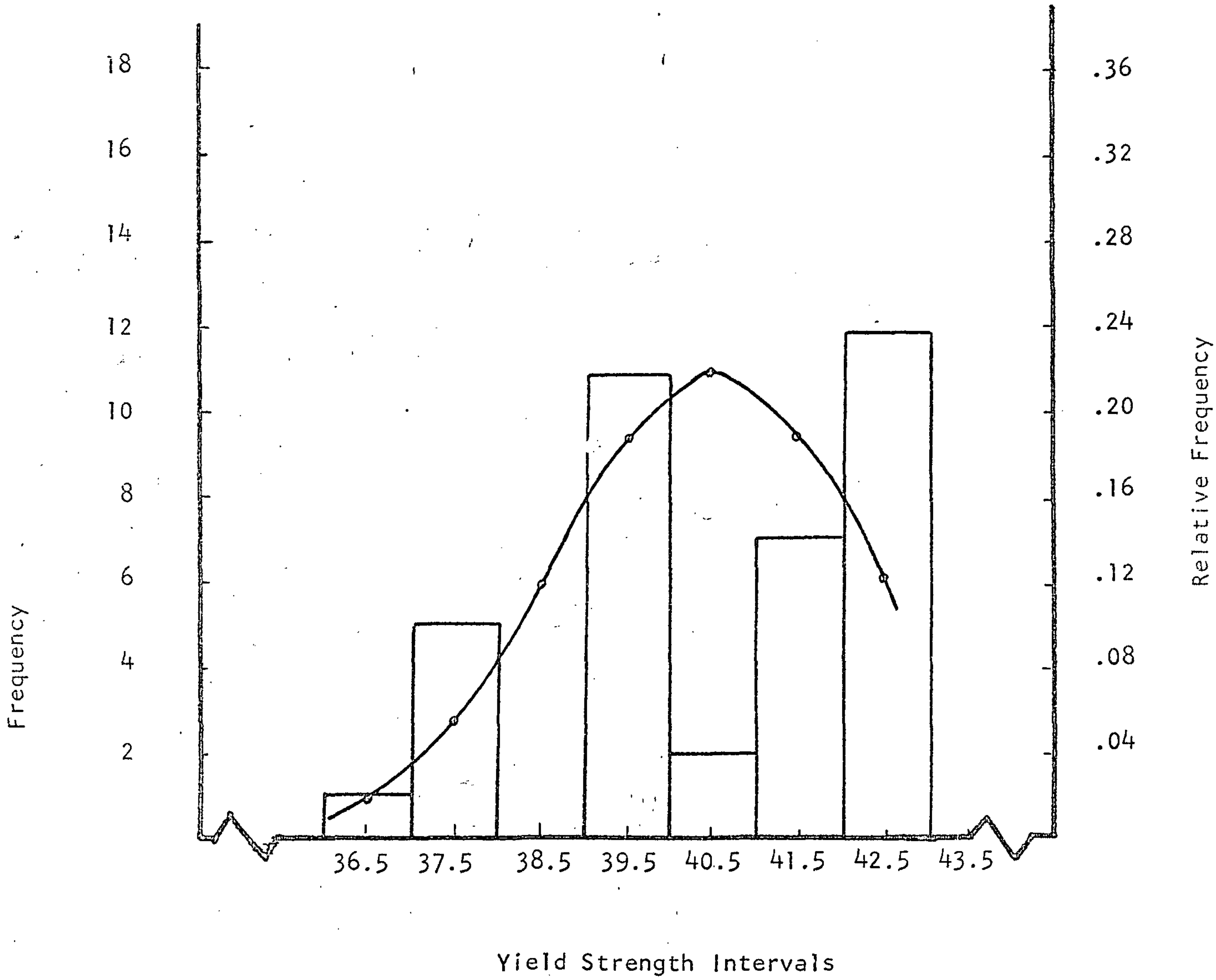



Figure 3.1a Frequency histogram and normal distribution curve plotted from Table 3.3a

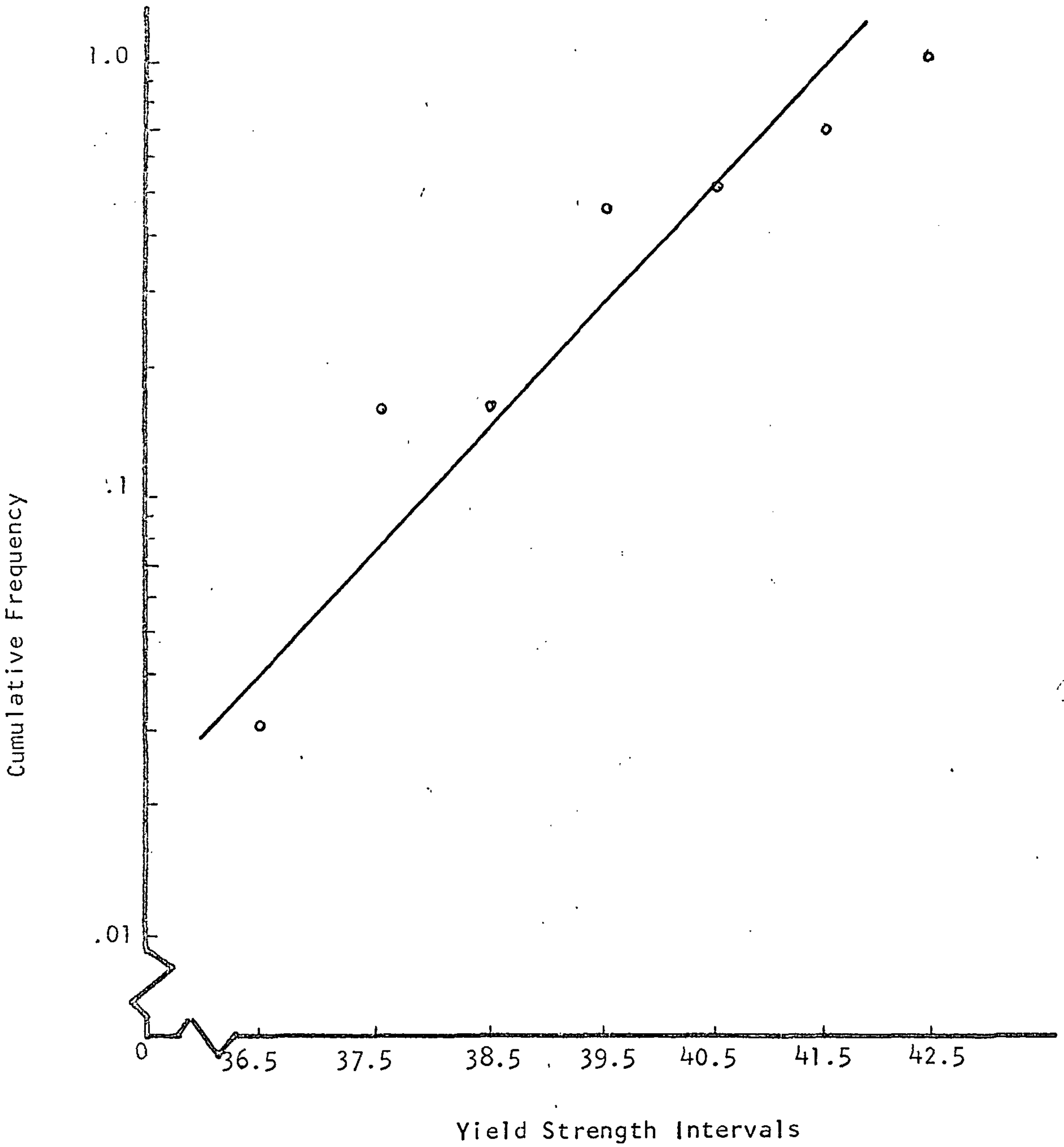



Figure 3.2a. Lognormal distribution curve

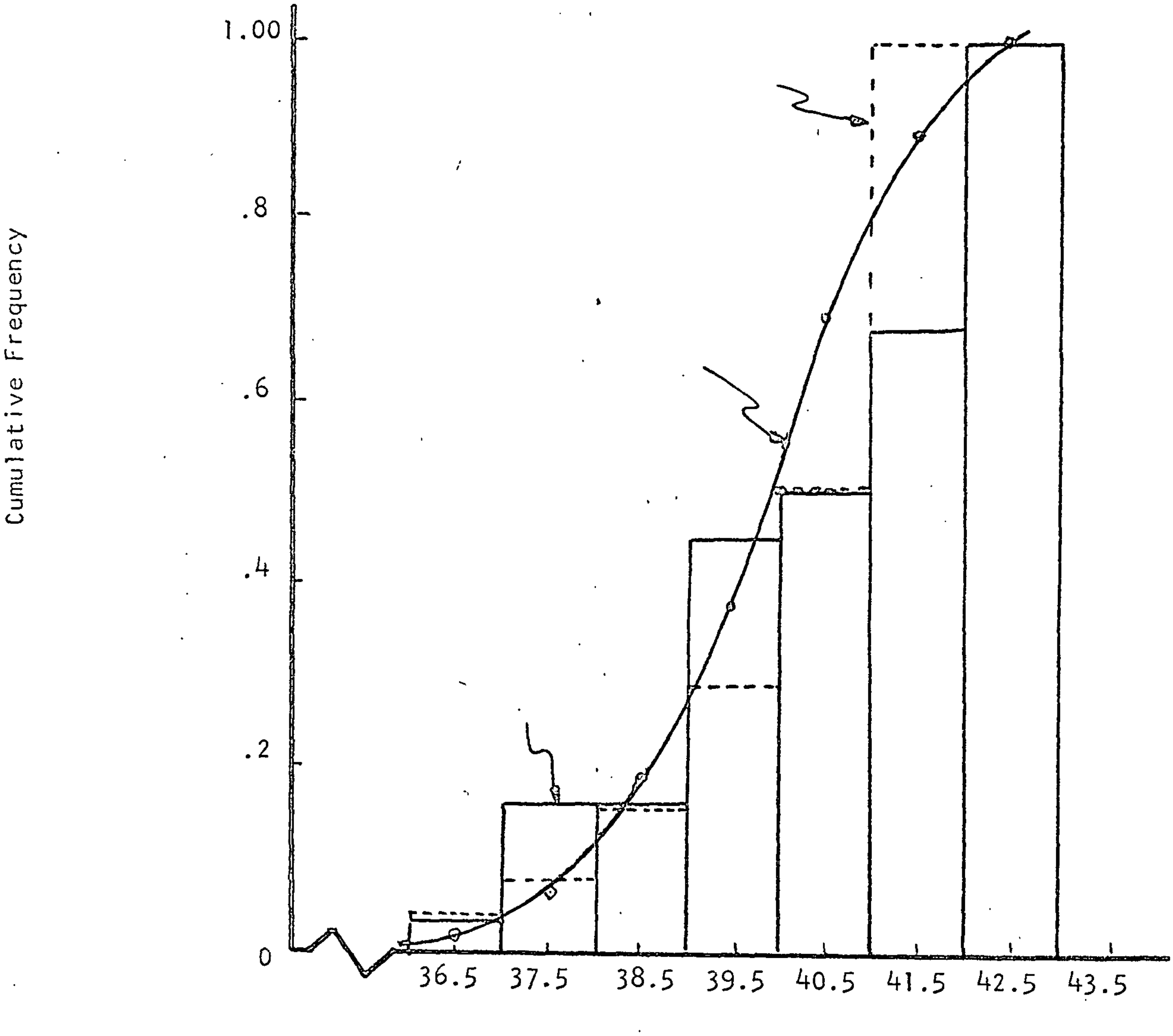



Figure 3.3a. Sample and Theoretical Cumulative Distribution Functions

Analyzing the data

The hypothesis is that the distribution of the samples tested follows the normal or lognormal probability model. This hypothesis will be accepted with a 95 percent level of confidence if the area under the normal curve between $\mu - 1.96\sigma$ and $\mu + 1.96\sigma$ consists of 95 percent of the sample tested.

The 95 percent level of confidence means that if other samples of the population were tested one could be sure that 95 percent of the time the mean of the sample would lie in the interval $\mu \pm 1.96\sigma$ and that only 5 percent of the samples tested would lie outside that range.

After the normal distribution curve has been fitted to the histogram of frequencies, a test as to the goodness-of-fit of the normal distribution or lognormal distribution, as is the case here, will be performed. This test will enable the analyst to decide whether the sample may be regarded as a random sample from a population with a normal or lognormal distribution. The two tests used to test the goodness-of-fit or the proposed distribution are: (1) the Chi-Square test, and (2) the Kolmogorov-Smirnov test.

# CHAPTER IV

# DISCUSSION AND CONCLUSION

Discussion

This experiment was designed very carefully as to the kind of sampling to be used, the hypothesised distribution, and the kinds of tests that were to be used to test the validity of the hypothesis.

A statistical hypothesis is a statement made about the population and it is tested to give the facts collected about the sample a chance to discredit it. If the sample data does, in fact, discredit the hypothesis, the hypothesis will be rejected and considered as false. On the other hand, if the sample data does not discredit the hypothesis, the hypothesis will be accepted and considered as true.

The statistical inferences and results of this experiment are based upon the simple random sampling theory. The 50 cold rolled steel rods and the 38 hot rolled steel rods used in this experiment were assumed to be picked up according to that theory. It is believed that this kind of sampling would best represent the way that the manufacturer or engineer would select his material as he would begin to sell or build a structure.

Since the normal and lognormal distributions are generally used to describe the distribution of the yield strength of cast iron, the distribution of the data collected from this experiment for the yield strength of cold rolled and hot rolled steel will be assumed to follow these two probability laws.

To test the assumption of normality and lognormality the chi-square test and the kolmogorov-smirnov test for goodness-of-fit based on a 95 percent level of confidence were used.

## Conclusion

The hypothesis that the distributions of yield strength of cold rolled and hot rolled steel rods are a normal and lognormal distribution must be rejected based on the following reasons.

1). The normal curve with $\mu = 61.78$ and $\sigma = 2.126$ as shown in Figure 3.1, and the normal curve with $\sigma = 1.83$ and $\mu = 40.53$ as shown in Figure 3.1a were examined by use of the chi-square test. The lognormal curve in Figure 3.2 and 3.2a were also examined by the chi-square test.

The calculated values for chi-square by using the observed and theoretical frequencies were calculated to be 27.02 for the normal distribution and 31.60 for the lognormal distribution for cold rolled steel rods and 27.85 for the normal distribution and 23.79 for the lognormal distribution for hot rolled steel rods respectively. All of these values for chi-square are above the acceptable values of $X^2_{.05} = 11.07$ and $X^2_{.05} = 9.49$ for a 5 percent level of significance or a 95 percent level of confidence for cold rolled and hot rolled steel rods respectively. Therefore, according to the chi-square test for goodness-of-fit of the hypothesis has to be rejected.

2). According to the theory of the Kolmogorov-Smirnov test for goodness-of-fit, the hypothesis should be rejected if the maximum vertical distance between the cumulative distribution of the yield strength of the sample and that of the proposed distribution exceeds the value taken from the table of critical values for $D_n^{\alpha}$ in the Kolmogorov-Smirnov test for $\alpha$ percent levle of significance.

From Figure 3.3 it can be found that the maximum vertical distance between the cumulative distribution and the proposed distribution for the cold rolled steel sample is .22 for the normal and .20 for the lognormal. Also from Figure 3.3a the maximum vertical distance between the cumulative distribution and the proposed distribution for the hot rolled steel sample is .27 for the normal and .27 for the lognormal distributions.

The critical values are $D_{50}^{.05} = .19$ for cold rolled steel rods and $D_{38}^{.05} = .22$ for hot rolled steel rods.

Since the observed D values are greater than the critical values, the hypothesis has to be rejected.

From all the data collected in this experiment it can be concluded that the yield strength of cold rolled and hot rolled steel rods does not follow the normal or the lognormal probability models.

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Part II

# VIBRATION ANALYSIS
# OF MECHANICAL SYSTEMS

By

Sanders Marshall

TABLE OF CONTENTS

# CHAPTER I

## INTRODUCTION

The study of vibrations treats the oscillatory motion of mechanical systems and the dynamic condition related thereto. It deals with the behavior of bodies under the influence of oscillatory forces. The vibration motion may be of regular form and repeated continuously, or it may be irregular of a random nature.

Vibrations are accompanied by, or are produced by forces which vary in an oscillatory manner. Such forces are frequently produced by unbalance in rotating machines or by the motion of the body itself. Most machines and engineering structure experience vibration in differing degrees, and their design generally requires consideration for their oscillatory behavior.

Although the term "vibration" usually implies a mechanical oscillation, similar conditions prevail in other areas, such as for alternating electric circuit, electromagnetic waves, and acoustics. The condition may be related in some manner, in different fields; for example, a mechanical vibration may produce an electric oscillation or vice versa. The basic principles, analyses, mathematical formulations, and terminology for oscillatory phenomena are similar in the various fields.

Oscillatory systems can be broadly characterized as linear or non-linear. For linear systems the principle of superposition holds, and the mathematical techniques available for their study are well-developed. In contrast, techniques for the analysis of nonlinear systems are less well known and diffecult to apply. Some knowledge of nonlinear systems is desirable since all systems tend to become nonlinear with increasing emplidute of oscillation. Many excellent books (1 to 6)* are available for study on this subject.

*Numbers within the parenthesis refers to the references given in Bibliography.

## CHAPTER II

## LINEAR VIBRATIONS

Vibration of linear systems fall into two general classes, free and forced. Free vibration takes place when an elastic system vibrates under the action of forces inherent in the system itself. The system under free vibration will vibrate at one or more of its natural frequencies, which are properties of the elastic system.

Vibration that takes place under the excitation of external forces is called forced vibration. Forced vibration takes place at the frequency of the exciting force, which is an arbitrary quantity independent of the natural frequencies of the system.

Perhaps the simplest of a free linear vibration problem is furnished by a mechanical system consisting of a mass attached to a spring which exerts a force (called the restoring or spring force) proportional to the displacement x of the mass (see Figure 2.1). If the addition, the mass is considered to move in a medium which exerts a resistance proportional to the velocity (a viscous damping force), the equation of motion is

$$m\ddot{x} + c\,\dot{x} + k\,x = o \qquad (2.1)$$

Where m, c and k are the mass damper and spring constant of the mechanical system respectively.

Equation (2.1), being a homogeneous second-order differential equation, can be solved by assuming a solution of the form

$$x = e^{st} \tag{2.2}$$

where s is a constant to be determined. Upon substitution of equation (2.2) into equation (2.1), we obtain the following equation

$$\left(s^2 + \frac{c}{m}s + \frac{K}{m}\right) e^{st} = o \tag{2.3}$$

Equation (2.3) is satisfied for all values of t if and only if

$$s^2 + \frac{c}{m}s + \frac{k}{m} = o \tag{2.4}$$

Equation (2.4), which is known as the characteristic equation, has two roots

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \tag{2.5}$$

and hence the general solution for the damped free vibration as described by equation (2.1) is

$$X = Ae^{s1t} + Be^{s2t} \tag{2.6}$$

Where A and B are arbitrary constants depending on how the motion is started. The behavior of the damped system of Figure (2.1) depends on the numerical value of the radical of equation (2.5).

Consider now the motion which results when an external force F (t) depending only on the time is applied to the previous discuss free vibration system (Figure 2.2). The force F(t) may be harmonic, nonharmonic, or random. The equation of motion is then the nonhomogeneous linear differential equation

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{2.7}$$

For most practical purposes F (t) is a periodic harmonic function.

Let's assume F (t) to be a simple harmonic function given by

$$F(t) = P \sin \omega t \tag{2.8}$$

in which P is amplitude and $\omega$ is the circular frequency. The solution of equation (2.7) is

$$X = X_a + X_b \tag{2.9}$$

where

$$X_a = \text{the complementary solution} = Ae^{s_1 t} + Be^{s_2 t}$$

$$X_b = \text{The particular solution} \quad \frac{P \sin (\omega t - \theta)}{(k - m\omega^2) + (c\omega)^2}$$

The complementary solution is the free-vibration component, and the particular solution represents the forced-vibration part of the motion. The complete motion consists of the sum of these two parts.

# CHAPTER III

## NONLINEAR VIBRATIONS

Linear system analysis serves to explain much of the behavior of oscillatory systems. However, there are a number of oscillatory phenomena which cannot be predicted or explained by the linear theory. In the linear system, cause and effect are related linearly. In a nonlinear system, this relationship between cause and effect is no longer proportional. For example, the center of an oil can may move proportionally to the force for small loads, but at a certain critical load it will snap over to a large displacement. The same phenomenom is also encountered in the buckling of columns, electrical oscillations of circuits containing inductance with an iron core, and vibration of mechanical systems with nonlinear restoring forces.

For the single-degree-of-freedom nonlinear system (Figure 3.1), the general form of the equation is

$$m\ddot{x} + f(\dot{x}, x, t) = F(t) \tag{3.1}$$

Such equations are distinguished from linear equations in that the principle of superposition does not hold for their solution.

The general method for the exact solution of nonlinear differential equations is a yet known. Exact solutions which are known are relatively few, and a large part of the progress in the knowledge of nonlinear systems comes from approximate and graphical solutions, and from studies made on machine computers.

Efforts in the search for exact solutions of nonlinear equations have led to a number of analytical techniques yielding approximate solutions. Some analytical techniques include the perturbation method, and the jump phenomenon. In particular, iteration and perturbation can be applied to obtain directly the solutions of differential equations. These methods can also be applied more indirectly as a means of determining the coefficients of the Fourier series developments of the solutions.

Before the advent of the electronic computer, nearly all non-linear differential equation were solved analytically. This usaully required analytic simplification to the point where the answers had only a remote connection with the original problem. However, today with the use of the computer, many numerical methods have been developed to solve nonlinear vibration problems. These numerical methods include Euler's, Euler's modified, Runge-Kutta, Milne's, and Hamming's methods. The methods will vary in complexity. The following chapter will compare the numerical solution of a viration system with the true solution.

# CHAPTER IV

## COMPUTER TECHNIQUES IN VIBRATION ANALYSIS

Modern computer technology has provided a number of powerful tools for the vibration analyst. The tools that now permit not only the rapid and convenient solution to vibration problems but also the analysis of highly complex vibratory systems may be grouped in three broad categories: circuits constructed from electrical analogies, and analog computer, and the digital computer.

The analogous behavior for electric circuits and mass-elastic systems has been recognized for many years. The vibratory behavior of complex mechanical system may be analyzed by series and parallel combinations of resistors, capacitors, and inductors. Systems imputs and responses in the form of voltages or currents can be easily obtained and analyzed. For example, a simple spring-mass-damper system may be represented by a series resistance, inductance, and capacitance circuit, where the force excitation is represented by an imput voltage, and the velocity of the mass is observed by monitoring the current. This system of elements, where inductance is analogous to mass, resistance is analogous to viscous damping, and capacitance is analogous to the inverse of stiffness, is called a force-voltage analogy. It is also possible to utilize a parallel electric circuit, in this case, it is termed a force-current analogy. After some experience in dealing with analogous quantities, it is possible to construct extremely complex electric networks to stimulate such mechanical systems as gear train automobile suspensions, structures, and almost any system defined by linear differential equations.

There is, however, one fundamental drawback to the utilization of analogies for the solution of vibration problems. The limitation is primarily that an analogy provides a very special computer which will solve only the given physical case at hand. If one wishes to add springs or change the number of masses, expand the system into more degrees of freedom, or make any other modification in the configuration of the system, it is necessary to construct a new analogous circuit.

The general-purpose analog computer is a device that is naturally suited for the study of the dynamic behavior of any vibratory system. This computer can be described as a machine consisting of elements which, when properly coupled together, may be used to solve differential equations or sets of differential equations. All variable are represented by voltages, as well as system outputs or responses. The behavior of a system may be observed and the data recorded by using oscilloscopes and electromechanical recorders. The accuracy by the precision of the components which make up the computer and the ability to measure voltages accurately. Most modern commercial computers are, however, capable of providing result sufficiently accurate for engineering analysis and synthesis.

While the analog computer is extremely useful for analyzing most vibratory systems, it has particular value in the study of nonlinear systems. The outstanding flexibility of analog equipment is a result of modern technology and the development of simple-to-use nonlinear function-generation components. Linear systems are defined by linear differential equations, and many classic solutions are available to the analyst. The principal of superposition for linear analysis provides a degree of organized general solution that is not possible in nonlinear-problem analysis. Thus, an analog simulation of a nonlinear problem may be the only practical engineering approach.

The digital computer is also very useful in vibration analysis. It may be used simply as a means for evaluating the response of a system for a wide variety of system parameters. In some instances an engineer may wish to know the effects of changing certain design parameters on the behavior of a system, which necessitates the solving of the problem many times with different set of data.

Many equations in engineering problems, even though they can be solved analytically in closed form, require a great deal of tiresome and time-consuming work which can be eliminated by programming the equations to a digital computer. Other equations cannot be solved analytically, and, although their approximate solution may be obtained by various numerical methods, these often involve large number of calculations which are time-consuming when performed manually. A digital computer can be employed to perform the large number of calculations require, and, since they are executed at tremendous speeds, solutions are obtained quickly as well as accurately.

## CHAPTER V

## NUMERICAL TECHNIQUE

It is frequently necessary to solve sets of simultaneous first-order differential equations in analyzing engineering systems. Such equations occur in obtaining solutions of higher-order differential equations which are transformed to sets of the solution process. Runge-Kutta methods are well-suited for the solution of higher-order differential equations.

An nth-order differential equation can be solved by transforming the equation to a set of N simultaneous first-order differential equations and applying N Rung-Kutta formulas.

Consider the second-order differential equation

$$\frac{d^2x}{dt^2} = f\left(t,\ x,\ \frac{dx}{dt}\right) \qquad (5\text{-}1)$$

Letting $v = \frac{dx}{dt}$, equation (5-1) can be transformed to the 2 first order differential equations

$$\frac{dv}{dt} = f(t,x,v)$$

$$\frac{dx}{dt} = v \qquad (5\text{-}2)$$

The following 2 fourth-order Runge-Kutta formulas could be used to solve equation (5-2)

Where

$$V_i{+}1 = V_i + 1/6\ (K_1 + 2K_2 + 2K_3 + K_4)$$

$$K_1 = (\Delta t)\ f\ (t_i,\ x_i,\ v_i) \tag{5-3}$$

$$K_2 = (\Delta t)\ f\ (t_i + \frac{\Delta t}{2},\ x_i + \frac{q1}{2},\ v_i + \frac{ki}{2})$$

$$K_3 = (\Delta t)\ f\ (t_i + \frac{\Delta t}{2,}\ x_i + \frac{q1}{2,}\ V_i + \frac{K2}{2}) \tag{5-4}$$

$$K4 = (\Delta t)\ f\ (t_i + \Delta t,\ x_i + q_3,\ v_i + K_3)$$

and

$$X_{i+1} = x_i\ 1/6\ (q_i + 2q_2 + 3q_3 + q4) \tag{5-5}$$

where

$$q_1 = (\Delta t)\ v_i$$

$$q2 = (\Delta t)\ (v_i + \frac{k1}{2})$$

$$q3 = (\Delta t)\ (v_i + \frac{k2}{2})$$

$$q4 = (\Delta t)\ (v_i + k_3) \tag{5-6}$$

We may obtain a more compact set of equations by substitution the expression for the q's into both the expressions for the k's and the recurrence formula of equation (5-5). Performing these substitutions yeilds

$$X_i{+}1 = x_i + (\Delta t)\ v_i + \frac{\Delta t}{6}\ (k_1 + K_2 + K_3)$$

$$V_i{+}1 = v_i + 1/6\ (K_1 + 2K_2 + 2K_3 + K4) \tag{5-7}$$

where

$$K_1 = (\Delta t)\ f\ (t_i,\ x_i,\ v_i)$$

$$K2 = (\Delta t)\ f\ (t_i + \frac{\Delta t}{2},\ x_i + \frac{\Delta t}{2}v_i\ ;\ v_i + \frac{k1}{2}) \tag{5-8}$$

$$K_3 = (\Delta t)\ f\ (t_i + \frac{\Delta t}{2}, x_i + \frac{t}{2}v_i + \frac{\Delta t k_1}{4},\ v_i + \underline{k2})$$

$$K4 = (\Delta t)\ f\ (t_i + \Delta t,\ x_i + (\Delta t)\ v_i + \frac{\Delta t}{2}k2,\ v_i + K_3)$$

The Runge-Kutta methods are very useful in the solution of nonlinear vibration systems. A series of computer programs were ran on the digital computer IBM/1401 at Prairie View A. and M. College and the IBM/360 at Texas A. and M. applying the Runge-Kutta method to the solution of the following nonlinear system:

$$\ddot{X} + 2\mu\omega_n \dot{x} - \frac{2\omega_n 2_D}{3.14} \tan\left(\frac{3.14x}{2D}\right) = F(t) \qquad (5\text{-}9)$$

where

$\mu$ = damping coefficeint

$\omega_n$ = natural frequency

D = displacement

F (t) = 1940 El Centro Earthquake NS Component Excitation

Equation (5-9) can be solved by the use of the computer program in the Appendix.

The nonlinear vibration system was analyze with $\mu$ =0.01, D = 15 inches and $\omega$ varing from 2.0 to 10.0 rad/sec. The data obtained from the numerical solution are plotted as shown in (Figure 5.4 to 5.8). It was observed that as the natural frequency of the spring-mass system was increased the period of oscillation decreased. Also the amplitude of the vibration tends to decease since the stiffness of the spring is a function of the natural frequency. When $\omega_n$ = 6.0, the amplitude of the oscillation increased slightly. This phenomena may be due to the fact that the natural frequency of the system corresponded to the frequency of the excitation force F(t).

A similar computer program was ran on the IBM/1401 to determine the accuracy of the Runge-Kutta method. The Runge-Kutta method was applied to a linear vibration system shown in Figure 5.1 A computer program is furnished in the Appendix. The data obtained from the numerical solution are plotted in graphical from in Figure (5.2) and Figure (5.3). A comparison of the Numerical solution and true solution is given in Table 1 and Table 2. For small step size ($\Delta t$), the solution computed by the Runge-Kutta fourth-order method is extremely accurate. The truncation error increases with increasing step size.

# CHAPTER IV

## CONCLUSION

It has been shown that solution of a vibratory system can be obtained through the application of numerical methods to digital computers. Today, application of computer techniques in the field of vibration analysis is becoming more widespread. Engineering methods and techniques have changed considerable during the past decade, as a result of the extensive use of highspeed computers in the solution of vibration problems. Therefore, it is essential for the modern vibration analysis to be familiar with the numerical methods used in programming problems on the computers, as well as the mathematical analysis involved.

APPENDIX - COMPUTER PROGRAMS

```
                                   FORTRAN   RUN
FORTRAN COMPILATION  VER 2 MOD 2
$NO DICTIONARY
$OBJECT MACHINE SIZE = 15999
          WRITE(3,1)
        1 FORMAT(1H1,26H   TIME   DISPL   VELOCITY/)
          READ(1,2) X,XD,DELT,DTPR,TMAX
        2 FORMAT(5F10.0)
          T=0.
          TPR=DTPR
        3 AK1=DELT*F(X,XD)
          AK2=DELT*F(X+DELT/2.*XD,XD+AK1/2.)
          AK3=DELT*F(X+DELT/2.*(XD+AK1/2.),XD+AK2/2.)
          AK4=DELT*F(X+DELT*(XD+AK2/2.),XD+AK3)
          X=X+DELT*(XD+(AK1+AK2+AK3)/6.)
          XD=XD+(AK1+2.*AK2+2.*AK3+AK4)/6.
          T=T+DELT
          IF(T .LT. TPR) GO TO 3
        4 WRITE(3,5) T, X, XD
        5 FORMAT(1H , F6.2,3X,F10.7,4X,F10.7)
          TPR=TPR+DTPR
          IF(T .EQ. TMAX) GO TO 6
          GO TO 3
        6 STOP
          END
```

```
                                   FORTRAN   RUN
FORTRAN COMPILATION  VER 2 MOD 2
$NO DICTIONARY
$OBJECT MACHINE SIZE = 15999
          FUNCTION F(X,XD)
          F=-55.794*XD-85.837*X
          RETURN
          END
```

```
//SWATFIVW JOB (10808,PVAM-,*25,001,59),'MARSHALL  THESIS 4/1'  2
      DIMENSION TT(5), AC(5), ACCEL(300)
      WRITE(6,1)
    1 FORMAT (1H1, 26H  TIME    DISPL     VELOCITY/)
      READ(5,2) COEFF,X,XD,DELT,DTPR,TMAX,OMEGA
    2 FORMAT (7F10.0)
      I1=1
      I2= 4
      TS=0
      TT(5)=0.
      TPR=0.
  100 READ(5,9) (TT(I), AC(I), I=I1, I2)
    9 FORMAT (4(F6.2, F12.7))
      T=0.
      DEL=0.01
      J1=2
      IR=0
      I2=I2-1
      DO 110 I=1, I2
      DI=(TT(I+1)-TT(I))/DEL
      ID=DI
      CHECK=ID
      IF(CHECK-DI) 15, 16, 16
   15 ID=ID+1
   16 IF(TT(I) .NE. 0.) GO TO 10
      IF(I .EQ. 3) GO TO 11
      GO TO 12
   10 IF(I .EQ. 4) GO TO 11
   12 IR=IR+ID
      GO TO 14
   11 IR=IR+ID+1
   14 ACCEL(J1-1)=AC(I)*0.3864
      DO 120 J=J1, IR
  120 ACCEL(J)=ACCEL(J-1)+(AC(I+1)-AC(I))*0.3864/DI
      J1=ID+J1
  110 CONTINUE
      IF(TT(5) .NE. 0.) GO TO 90
      TT(5)=TT(4)
      AC(5)=AC(4)
   90 TT(1)=TT(5)
      AC(1)=AC(5)
      I1=2
      I2=5
    3 N1=T/DEL+1.0
      N2=(T+DELT/2.)/DEL+1.0
      N3=(T+DELT)/DEL+1.0
      AK1=DELT*F(X, XD, COEFF, OMEGA, ACCEL(N1))
      AK2=DELT*F(X+DELT/2.*XD, XD+AK1/2., COEFF, OMEGA, ACCEL(N2))
      AK3=DELT*F(X+DELT/2.*(XD+AK1/2.),XD+AK2/2.,COEFF,OMEGA,ACCEL(N2))
      AK4=DELT*F(X+DELT*(XD+AK2/2.), XD+AK3, COEFF, OMEGA, ACCEL(N3))
      X=X+DELT*(XD+(AK1+AK2+AK3)/6.)
      XD=XD+(AK1+2.*AK2+2.*AK3+AK4)/6.
      IF(TS .LT. TPR) GO TO 4
      TPR=TPR+DTPR
      WRITE(6,5) TS, X, XD
    5 FORMAT(1H , E16.7, 3X, E16.7, 4X, E16.7)
    4 TS=TS+DELT
      T=T+DELT
      IF(TS .GE. TT(5)) GO TO 100
      IF(TS .GE. TMAX) GO TO 6
```

```
      GO TO 3
    6 STOP
      END

      FUNCTION F(X, XD, COEFF, OMEGA, ACCEL)
      F=-2.0*COEFF*OMEGA*XD-2.0*OMEGA**2*14.99/3.14*SIN(3.14*X/29.98)/
     1COS(3.14*X/29.98)+ACCEL
      RETURN
      END
```

//$DATA

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TABLE I

COMPARISON OF NUMERICAL AND TURE

SOLUTION OF A VIBRATION SYSTEM

| Time sec | Numerical Solution inch Displacement | True Solution inch Displacement | Error |
|---|---|---|---|
| 0.5 | 1.8668234 | 1.8792000 | .0123766 |
| 1.0 | 0.8458096 | 0.8571000 | .0112904 |
| 1.5 | 0.3832145 | 0.3910000 | .0077855 |
| 2.0 | 0.1736246 | 0.1783000 | .0046754 |
| 2.5 | 0.0786648 | 0.0813000 | .0026352 |
| 3.0 | 0.0356410 | 0.0371000 | .0014590 |
| 3.5 | 0.0161480 | 0.0169000 | .0007520 |
| 4.0 | 0.0073163 | 0.0077000 | .0003837 |
| 4.5 | 0.0033148 | 0.0035000 | .0001852 |
| 5.0 | 0.0015019 | 0.0016000 | .0009810 |

TABLE 2

COMPARISON OF NUMERICAL AND TRUE

SOLUTION OF A VIBRATION SYSTEM

| Time<br><br>sec | Numerical Solution<br><br>inch/sec Velocity | True Solution<br><br>inch/sec Velocity | Error |
|---|---|---|---|
| 0.5 | -2.9559260 | -2.9556557 | .0002703 |
| 1.0 | -1.3392538 | -1.3481328 | .0088790 |
| 1.5 | -0.6067814 | -0.6149099 | .0081285 |
| 2.0 | -0.2749170 | -0.2804725 | .0055555 |
| 2.5 | -0.1245578 | -0.1279290 | .0033712 |
| 3.0 | -0.0564339 | -0.0583509 | .0019170 |
| 3.5 | -0.0255688 | -0.0266150 | .0010462 |
| 4.0 | -0.0115845 | -0.0121396 | .0005551 |
| 4.5 | -0.0052487 | -0.0055371 | .0002884 |
| 5.0 | -0.0023780 | -0.0025256 | .0001476 |

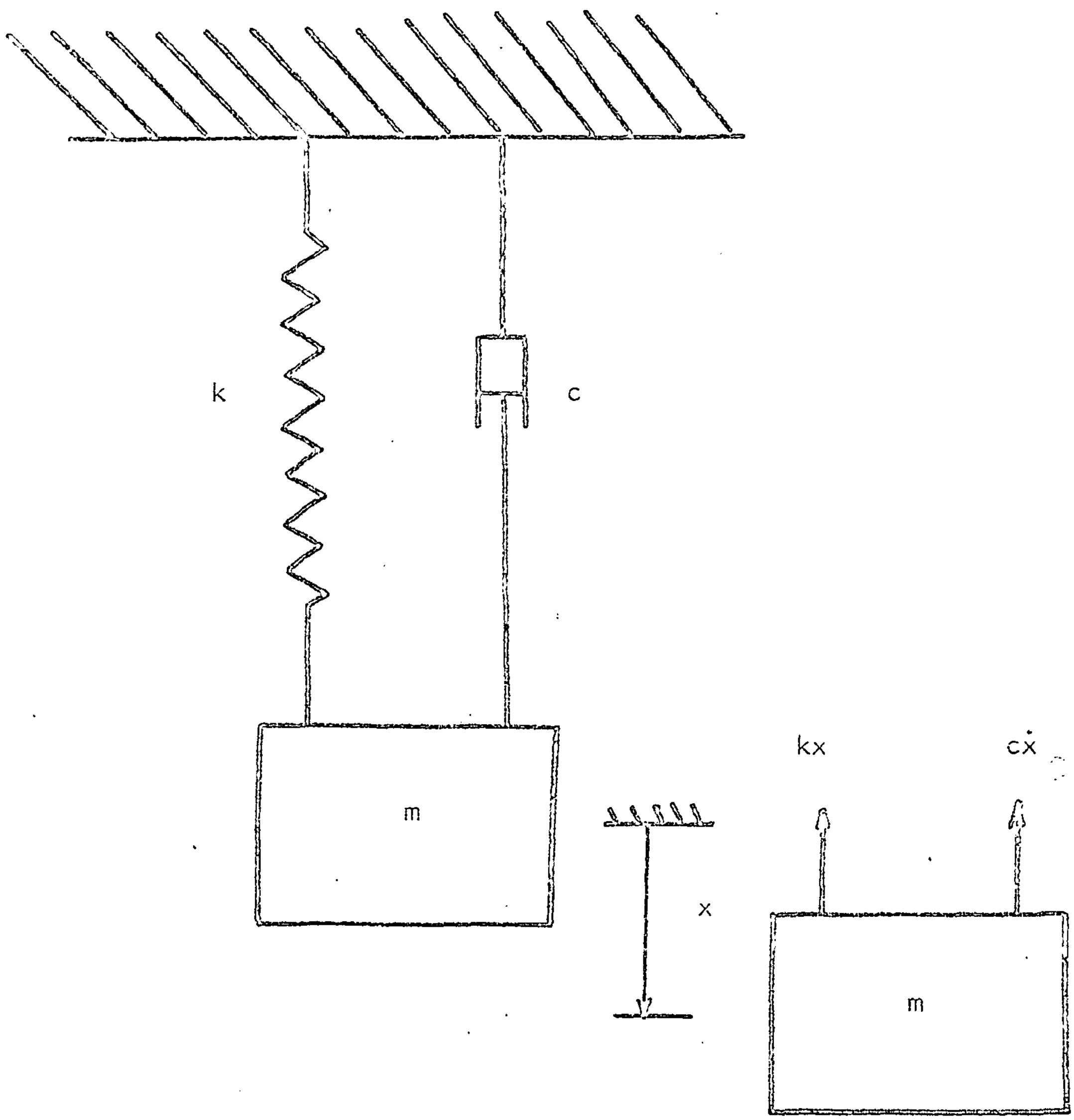


Fig. 2.1. Free vibration with viscous damping.

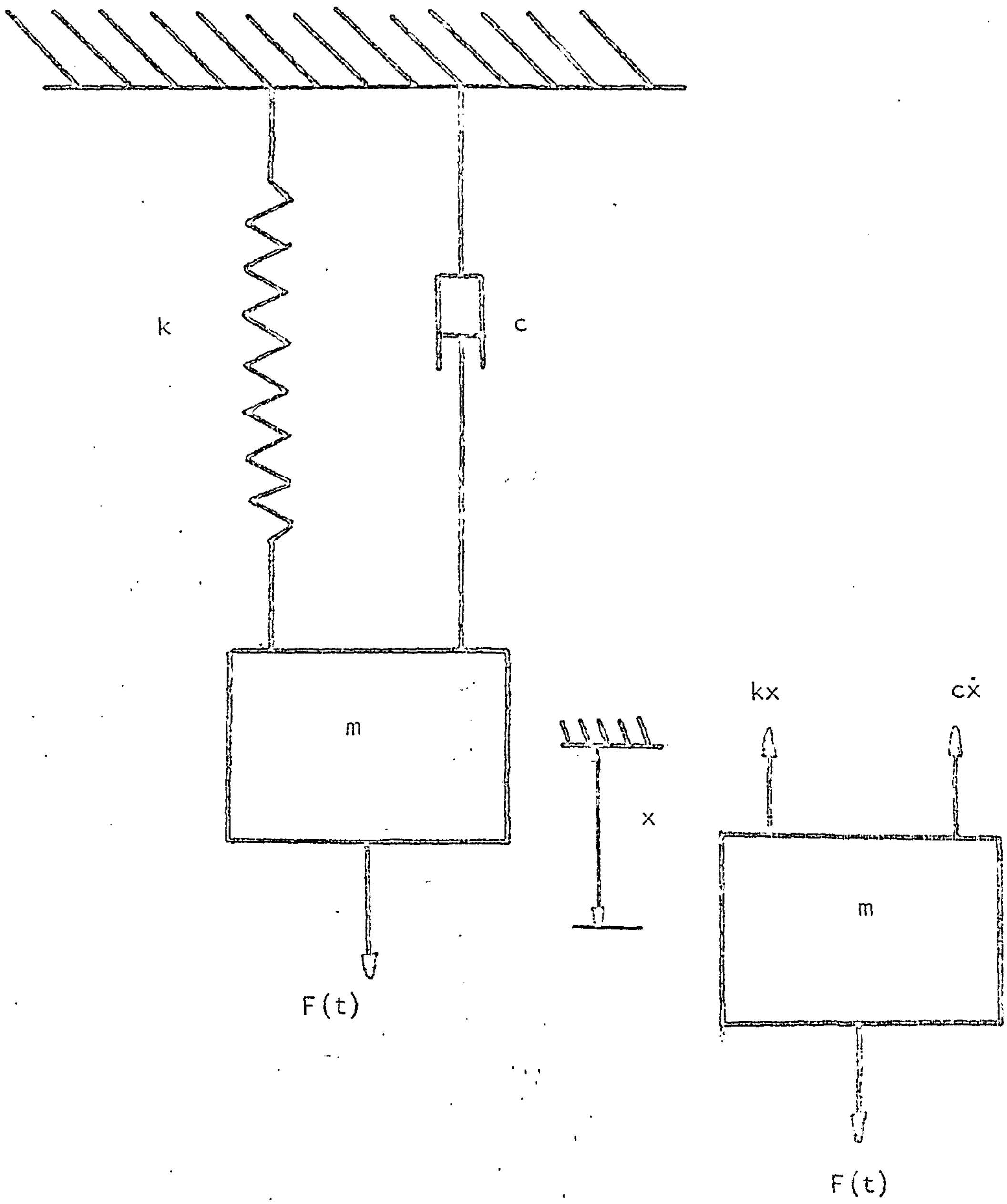


Fig. 2.2. Forced vibration with viscous damping.

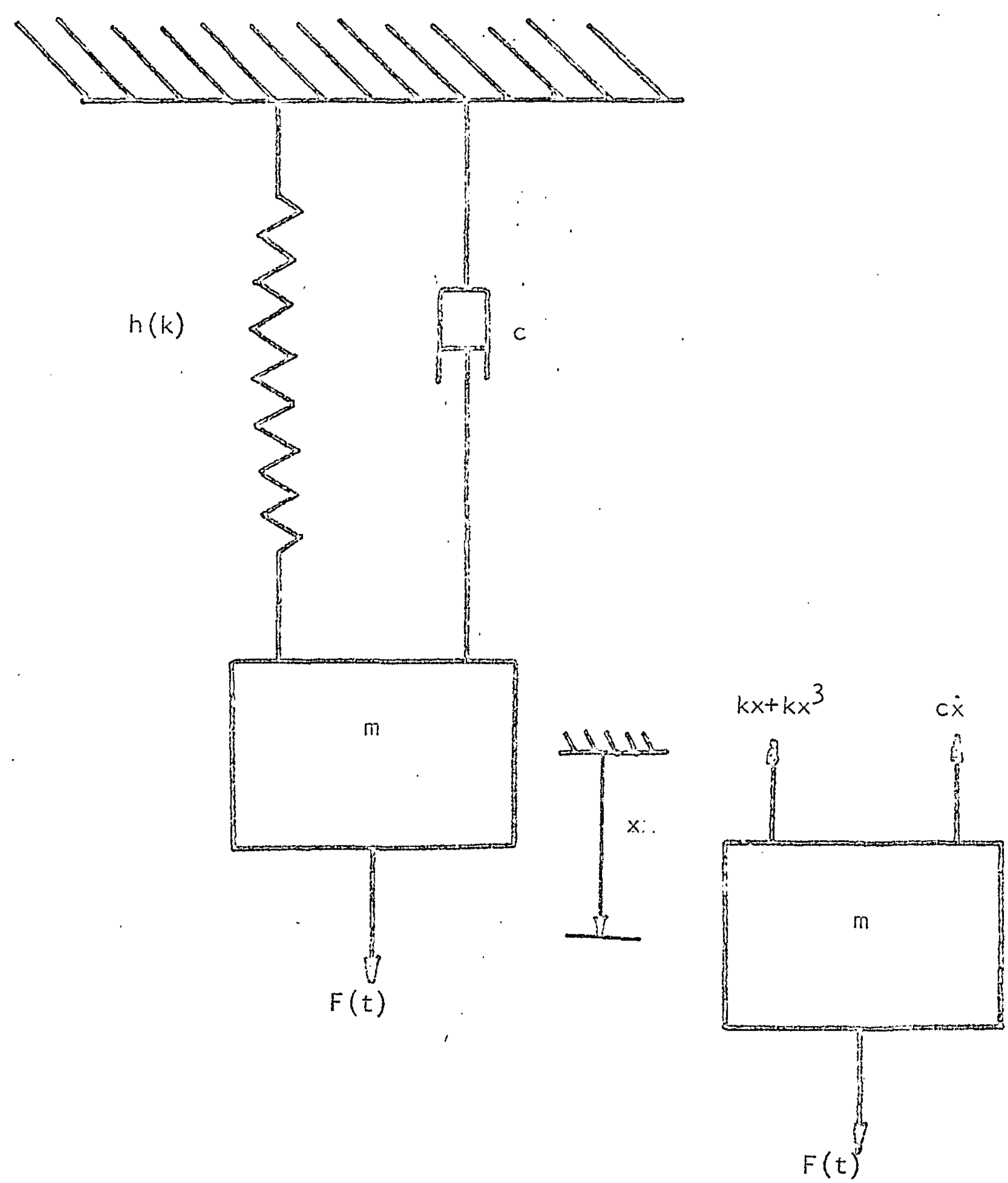


Fig. 3.1. Forced vibration with viscous damping and a non-linear restoring force.

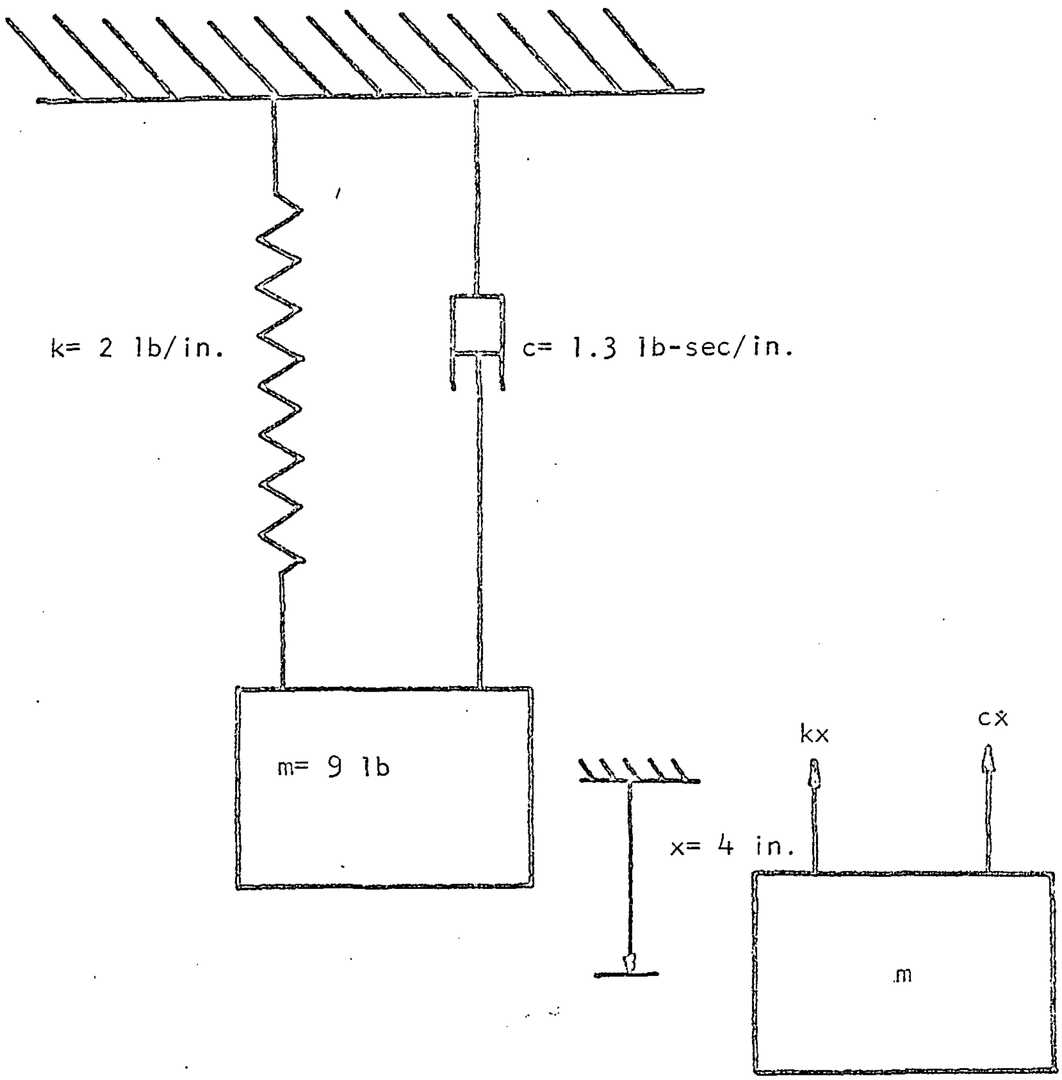


Fig. 5.1 Free vibration with viscous damping.

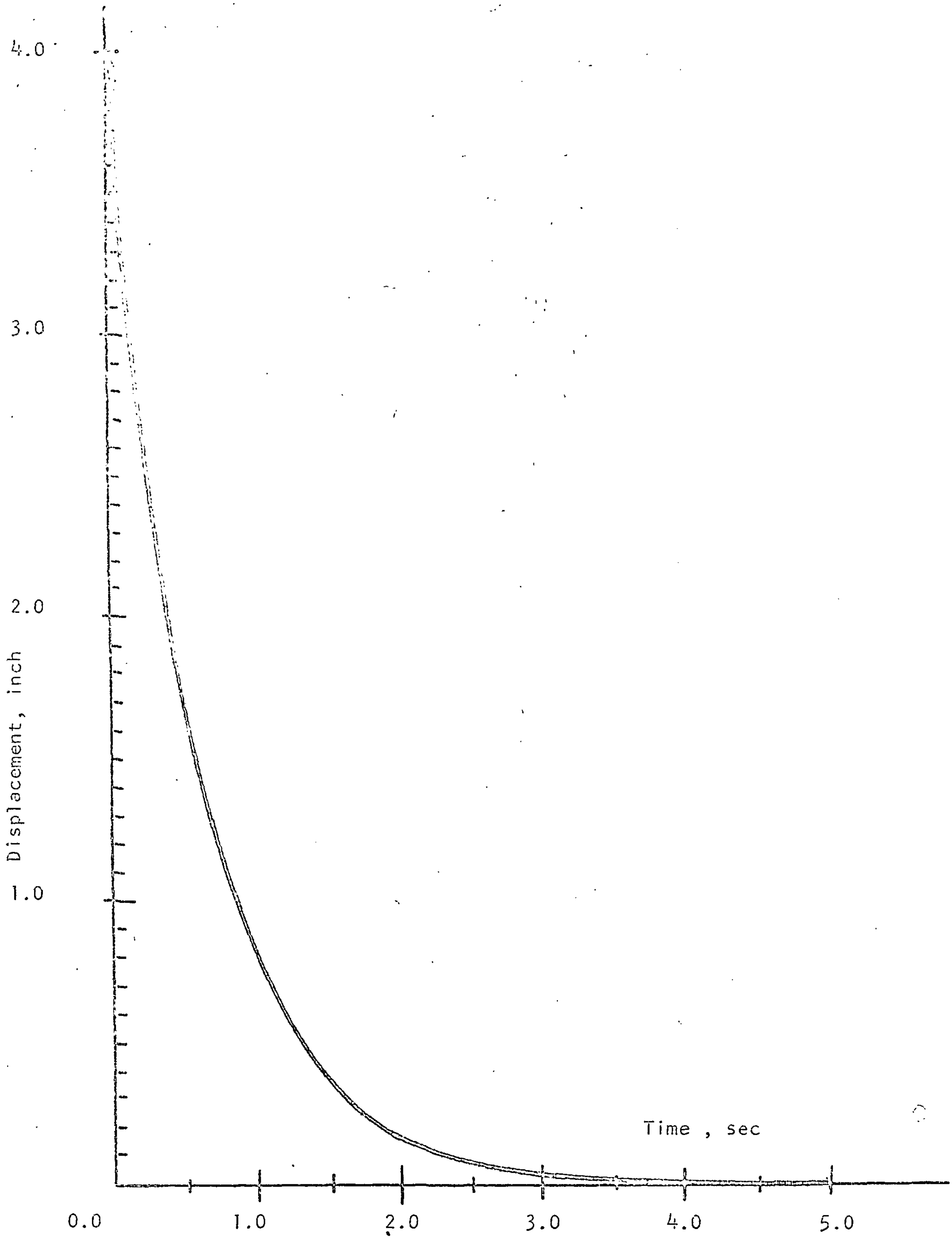


Fig. 5.2. Displacement-time data.

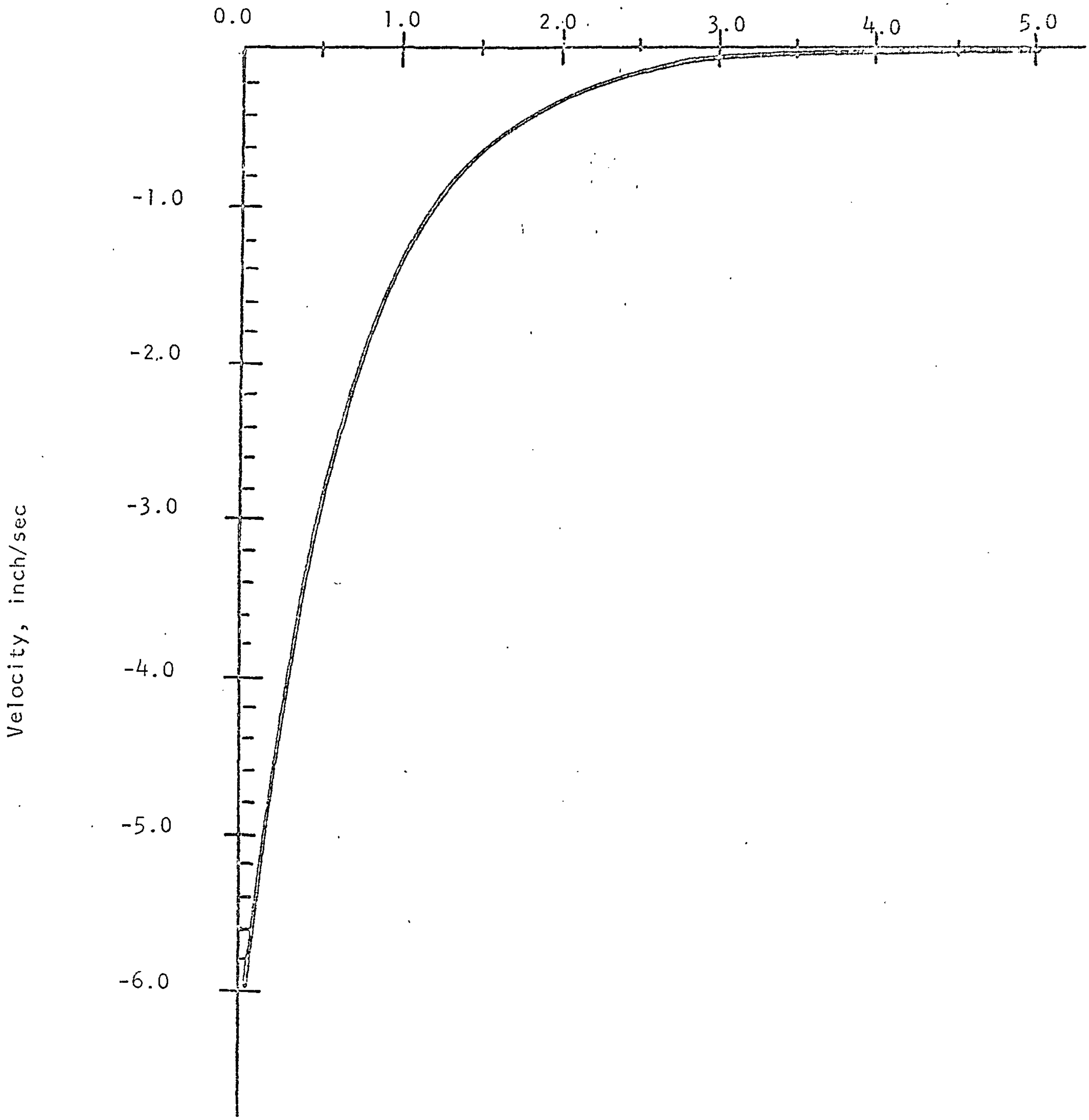


Fig. 5.3. Velocity-time data.

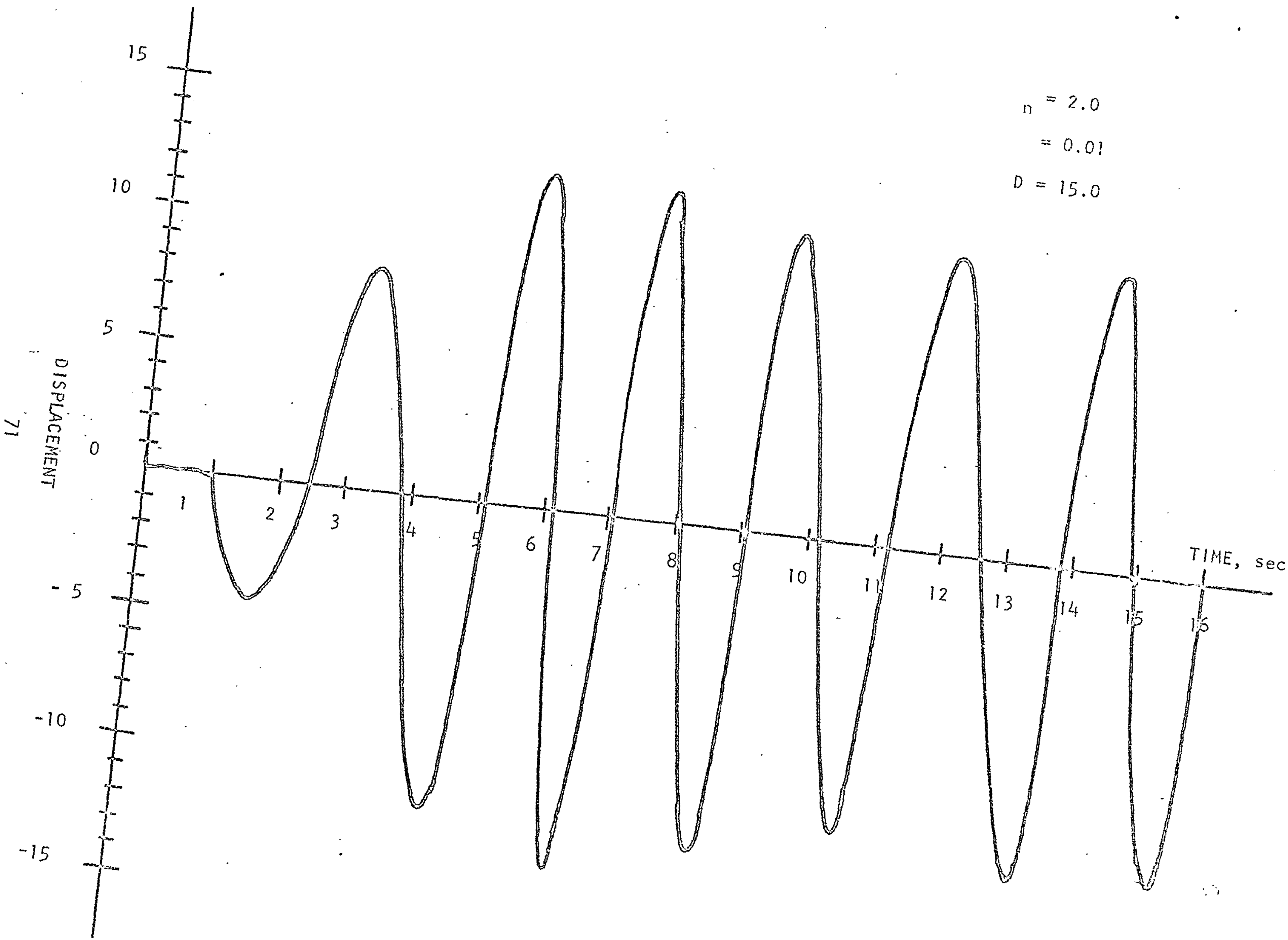


Fig. 5.4. Displacement-time curve

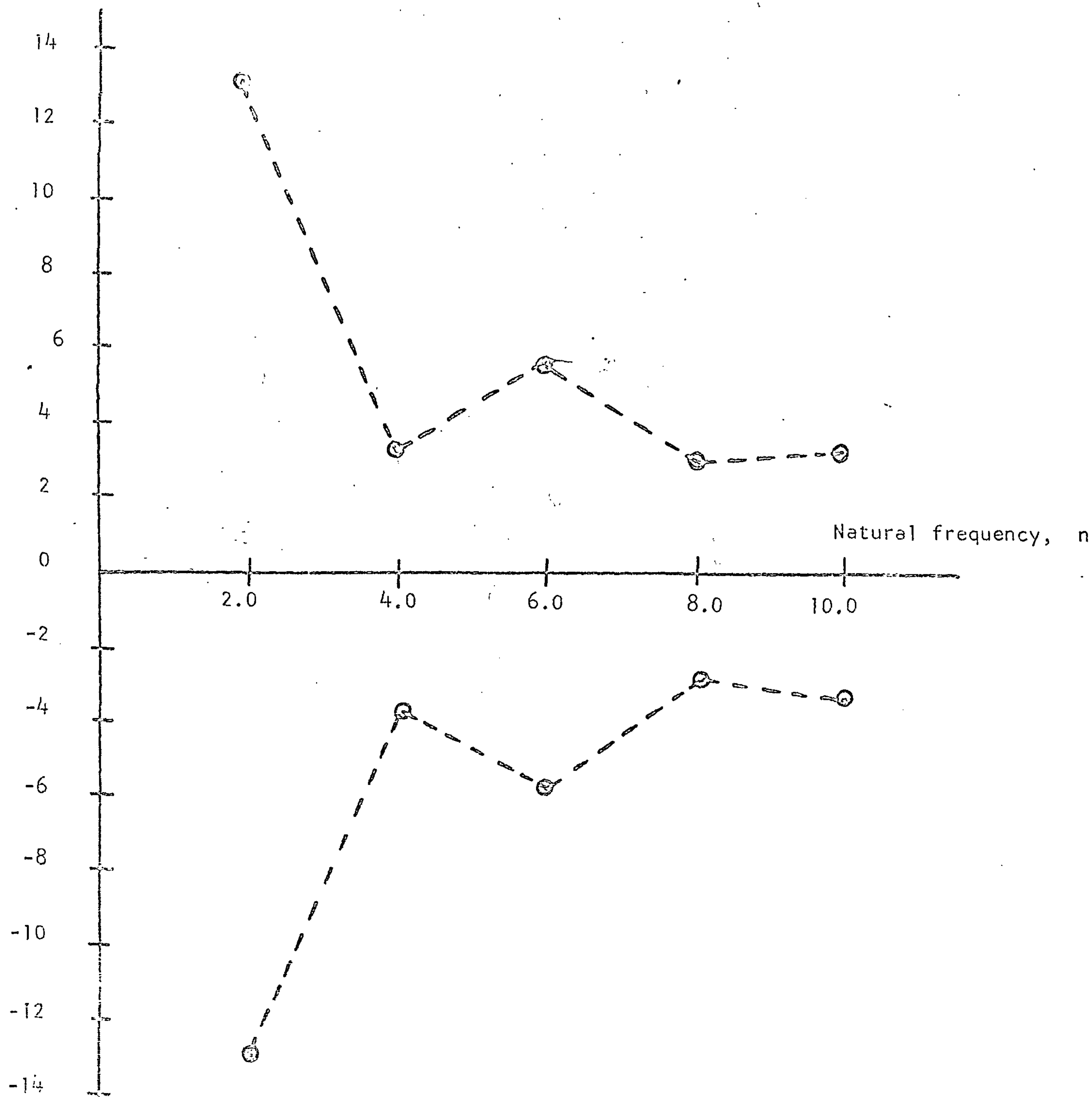


Fig. 5.5. Minimum and maximum displacement vs. natural frequency.