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ACOUSTIC IMPEDANCE OF CURVED MULTILAYERED DUCT LINERS

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16. Abstract

This paper investigates the effect of curvature of annular duct liners on the liner acoustic impedance. Exact equations are derived for the impedance of point-reacting liners which are made from an arbitrary number of thin cylindrical layers of porous material separated by small radially oriented cells. Equations are given for liners with convex curvature (inner annulus wall) and for liners with concave curvature (outer annulus wall). For ducts with small curvature, it is shown that these equations reduce to the equations for a flat liner. It is shown, by analytical and numerical examples, that the effect of liner curvature is significant in practical noise reduction problems.

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ACOUSTIC IMPEDANCE OF CURVED MULTILAYERED DUCT LINERS

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SUMMARY

This paper investigates the effect of curvature of annular duct liners on the liner acoustic impedance. Exact equations are derived for the impedance of point-reacting liners which are made from an arbitrary number of thin cylindrical layers of porous material separated by small radially oriented cells. Equations are given for liners with convex curvature (inner annulus wall) and for liners with concave curvature (outer annulus wall). For ducts with small curvature, it is shown that these equations reduce to the equations for a flat liner. It is shown, by analytical and numerical examples, that the effect of liner curvature is significant in practical noise reduction problems.

INTRODUCTION

Layers of sound-absorbing material are frequently installed on the walls of air ducts to reduce transmitted sound. In aircraft noise reduction applications, duct liners are used in the inlet and bypass ducts for this purpose. These duct liners, which are described in reference 1, are often made from a thin layer of porous material or perforated plate which is placed over a compartmented or honeycomb backing cavity. In the analysis of the acoustic properties of these liners, it is usually assumed that the liner is flat, although there is obvious curvature in cylindrical and annular ducts such as the inlet of an aircraft turbofan engine. It is shown that this assumption of flatness may lead to significant practical errors.

In this paper, the material layers are assumed to be infinitesimally thin and acoustically linear. In liners which are made of thick layers of material, waves may propagate in the material and it is necessary to consider the coupling between the acoustic waves in the material and in the duct. These thick-material coupling effects are not considered in this paper.

The purpose of this brief paper is to set down the appropriate equations for the acoustic impedance of multilayered duct liners with a single curvature. These equations are derived by using one-dimensional cylindrical waves to represent the acoustic field between the porous layers of acoustic material. Numerical results are presented which show the effects of curvature in practical liner designs.

SYMBOLS

b^*	duct radius, cm
c	ambient speed of sound, cm/sec
d_k^*	distance between material layers k and $k - 1$, cm
$d_k = \frac{2\pi d_k^*}{\lambda}$	
f	frequency, Hz
i	unit imaginary number
J_0, Y_0	Bessel functions of the first and second kind, respectively, of order zero
p^*	acoustic pressure, dynes/cm^2
$p = \frac{p^*}{\rho c^2}$	
P_0, Q_0, R_0, S_0	Bessel function cross products of order zero
r^*	radius, cm
$r = \frac{2\pi r^*}{\lambda}$	
R	resistance ratio of material
t	time, sec
v_r^*	radial acoustic velocity, cm/sec
$v_r = \frac{v_r^*}{c}$	
X	reactance ratio of material
Z^*	acoustic impedance, dyne-sec/cm^3
$Z = \frac{Z^*}{\rho c} = (R - iX)$	

β acoustic admittance of duct liner, $1/Z$

λ wavelength, cm

ρ ambient density, g/cm^3

A prime indicates differentiation with respect to the argument.

GENERAL ANALYSIS

The acoustic properties of curved duct liners are determined from the acoustic behavior of the material and from the properties of the radial cylindrical waves in the spaces between the material. Figure 1 shows two layers of a thin acoustic material in a typical curved duct liner. The layers of material are separated by a space which is divided into small cells (cell dimensions transverse to the radius are small compared to wavelength) such that only radial cylindrical waves may exist in the liner. The acoustic impedance may then be defined as the complex ratio of the pressure to the radial velocity (a time factor $\exp(-2\pi ift)$ is understood)

$$Z = \frac{p}{v_r} \tag{1}$$

The acoustic impedance is a function of the radius. At a layer of material, which is assumed to be infinitesimally thin and acoustically linear, there is an impedance discontinuity which is represented by

$$Z_j^- - Z_j^+ = R_j - X_j \tag{2}$$

The acoustic field between any two layers of material located at r_j and r_k ($k = j + 1$ in fig. 1) is governed by the equation

$$\frac{d^2p}{dr^2} + \frac{1}{r}\frac{dp}{dr} + p = 0 \tag{3}$$

and the radial velocity is proportional to the pressure gradient

$$v_r = -i\frac{dp}{dr} \tag{4}$$

The general solutions for the pressure and radial velocity are therefore

$$p(r) = AJ_0(r) + BY_0(r) \tag{5}$$

$$v_r = -i\left[AJ_0'(r) + BY_0'(r)\right] \tag{6}$$

Combining equations (1), (5), and (6) at two radial locations, $r = r_j + \epsilon$ and $r = r_k - \epsilon$ $(\epsilon << 1)$, and eliminating the coefficients A and B give a general relation between the impedances Z_j^+ and Z_k^-

$$P_0(r_j,r_k) + iZ_k^- Q_0(r_j,r_k) + iZ_j^+ R_0(r_j,r_k) - Z_j^+ Z_k^- S_0(r_j,r_k) = 0 \tag{7}$$

where the Bessel function cross products are given by the equations

$$P_0(r_j,r_k) = J_0(r_j)Y_0(r_k) - J_0(r_k)Y_0(r_j) \tag{8a}$$

$$Q_0(r_j,r_k) = J_0(r_j)Y_0'(r_k) - J_0'(r_k)Y_0(r_j) \tag{8b}$$

$$R_0(r_j,r_k) = J_0'(r_j)Y_0(r_k) - J_0(r_k)Y_0'(r_j) \tag{8c}$$

and

$$S_0(r_j,r_k) = J_0'(r_j)Y_0'(r_k) - J_0'(r_k)Y_0'(r_j) \tag{8d}$$

CONVEX DUCT LINERS

The inner liner of an annular duct is convex in the outward radial direction. In order to find the impedance of this liner at the duct wall, a recurrence formula is developed by using equations (2) and (7). It is assumed that the impedance of the outer surface of the innermost layer Z_0^+ is known so that the recurrence formula may be used to find the impedance at the surfaces of succeeding layers. Since the impedance on the inner wall of the duct is usually referred to the outward velocity from the duct $-v_r$, a sign change in the impedance of equations (2) and (7) is introduced in order to conform to this convention. The resulting recurrence formula is

$$Z_n^+ = \left(R_n - iX_n\right) - \left[\frac{iP_0(r_{n-1},r_n) + Z_{n-1}^+ R_0(r_{n-1},r_n)}{Q_0(r_{n-1},r_n) - iZ_{n-1}^+ S_0(r_{n-1},r_n)}\right] \tag{9}$$

where $n - 1$ and n correspond to j and k, respectively, in figure 1.

When the liner curvature is small, equation (9) must reduce to a recurrence formula for a flat liner. The curvature is the inverse of the radius; therefore, the asymptotic expressions for the cross products for large r_n, with finite values of $(r_n - r_{n-1})$, give the flat-liner recurrence formula. The asymptotic formula for the cross products may be obtained directly from the asymptotic formulas for the Bessel functions, which are

$$J_m(r) \sim \sqrt{\frac{2}{\pi r}} \cos\left(r - m\frac{\pi}{2} - \frac{\pi}{4}\right) \tag{10a}$$

$$Y_m(r) \sim \sqrt{\frac{2}{\pi r}} \sin\left(r - m\frac{\pi}{2} - \frac{\pi}{4}\right) \tag{10b}$$

The asymptotic formulas for the cross products are

$$P_0 \sim S_0 \sim \frac{2}{\pi r} \sin d_k \tag{11a}$$

$$Q_0 \sim -R_0 \sim \frac{2}{\pi r} \cos d_k \tag{11b}$$

where $r_{k-1} \leqq r \leqq r_k$ and $d_k = r_k - r_{k-1}$.

With formulas (11), equation (9) becomes

$$Z_n^+ \sim \left(R_n - iX_n\right) + \left(\frac{Z_{n-1}^+ \cos d_n - i \sin d_n}{\cos d_n - iZ_{n-1}^+ \sin d_n}\right) \tag{12}$$

Equation (12) may be derived also by considering the plane waves between sheets of material in a flat liner.

In the special case of a single-layer flat liner with a rigid backing, such that $Z_0^+ \rightarrow \infty$, equation (12) reduces to

$$Z_1^+ = R_1 - i\left(X_1 - \cot d_1\right) \tag{13}$$

Equation (13) shows the well-known $\cot d$ reactance contributed by the backing cavity.

When the liner is a hollow cylinder with a single outer layer of material, equation (9) reduces to

$$Z_1^+ = R_1 - i\left[X_1 + \frac{J_0\left(r_1\right)}{J_0'\left(r_1\right)}\right] \tag{14}$$

Equations (13) and (14) may be compared to see the effect of large curvature on backing-cavity reactance. The tuning frequency is sometimes defined as the lowest frequency where the liner reactance is zero. If X_1 is small, this tuning frequency occurs when $d_1 = \pi/2$ for the flat liner whereas it is near the lowest zero of $J_0(r)$ or about $r = 2.4$ for the cylinder liner. Thus, the tuning frequency of the cylinder would be about 1.5 times the tuning frequency of the flat liner. Although this is an extreme case, it shows that curvature can be an important effect in the properties of duct liners.

CONCAVE DUCT LINERS

The layers of the concave liners which are used on the outer wall of an annular duct are numbered starting with $n = 0$ at the outermost layer and working inward to the duct wall where $n = N$. In equation (7), the outward direction is the $+v_r$ direction; therefore, since $r_j < r_k$, let $r_j \rightarrow r_n$ and $r_k \rightarrow r_{n-1}$. The recurrence formula for this case is

$$Z_n^- = \left(R_n - iX_n\right) + \left[\frac{iP_0\left(r_n, r_{n-1}\right) - Z_{n-1}^- Q_0\left(r_n, r_{n-1}\right)}{R_0\left(r_n, r_{n-1}\right) + iZ_{n-1}^- S_0\left(r_n, r_{n-1}\right)}\right] \tag{15}$$

Equation (15) also reduces to equation (12) when r_n is large.

EFFECT OF CURVATURE ON SINGLE-LAYER LINERS

The effect of duct curvature on the properties of single-layer liners is shown in figure 2. Since the curvature affects only the backing reactance, this reactance parameter is shown as a function of the dimensionless backing depth for several values of the ratio d_1^*/b^* where b^* is the duct radius. The negative value of d_1^*/b^* is for a concave (outer cylinder wall) liner whereas the positive values of d_1^*/b^* are for convex liners. The curve for $d_1^*/b^* = 0$ is the well-known cotangent function for the flat liner. These curves show that a concave liner will "tune" at lower frequency than a flat liner of the same depth and that convex liners will "tune" at higher frequencies.

The curvature effects shown in figure 2 are larger than will occur in many practical problems; however, there can be a significant effect on the liner's acoustic properties even when the curvature is small. Figure 3 shows the acoustic admittance β (reciprocal of impedance) of a concave cylindrical liner where the cylinder radius is 19 cm and the backing depth is 3 cm. The ratio d_1^*/b^* for this liner is only 0.16; however, as the curves for the flat and curved liners show, both the real and imaginary parts of the admittance may differ by as much as 0.5 in some frequency ranges.

CONCLUDING REMARKS

General recurrence formulas have been derived for the impedance of point-reacting cylindrical duct liners which are made from an arbitrary number of material layers separated by spaces.

In practical duct-liner designs, liner curvature effects can be significant. These effects have been substantiated by the computational results of this paper.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., April 26, 1973.

REFERENCE

1. Anon.: Proceedings of the Aircraft Noise Symposium – Acoustical Duct Treatments for Aircraft. J. Acoust. Soc. Amer., vol. 48, no. 3, pt. 3, Sept. 1970, pp. 779-842.

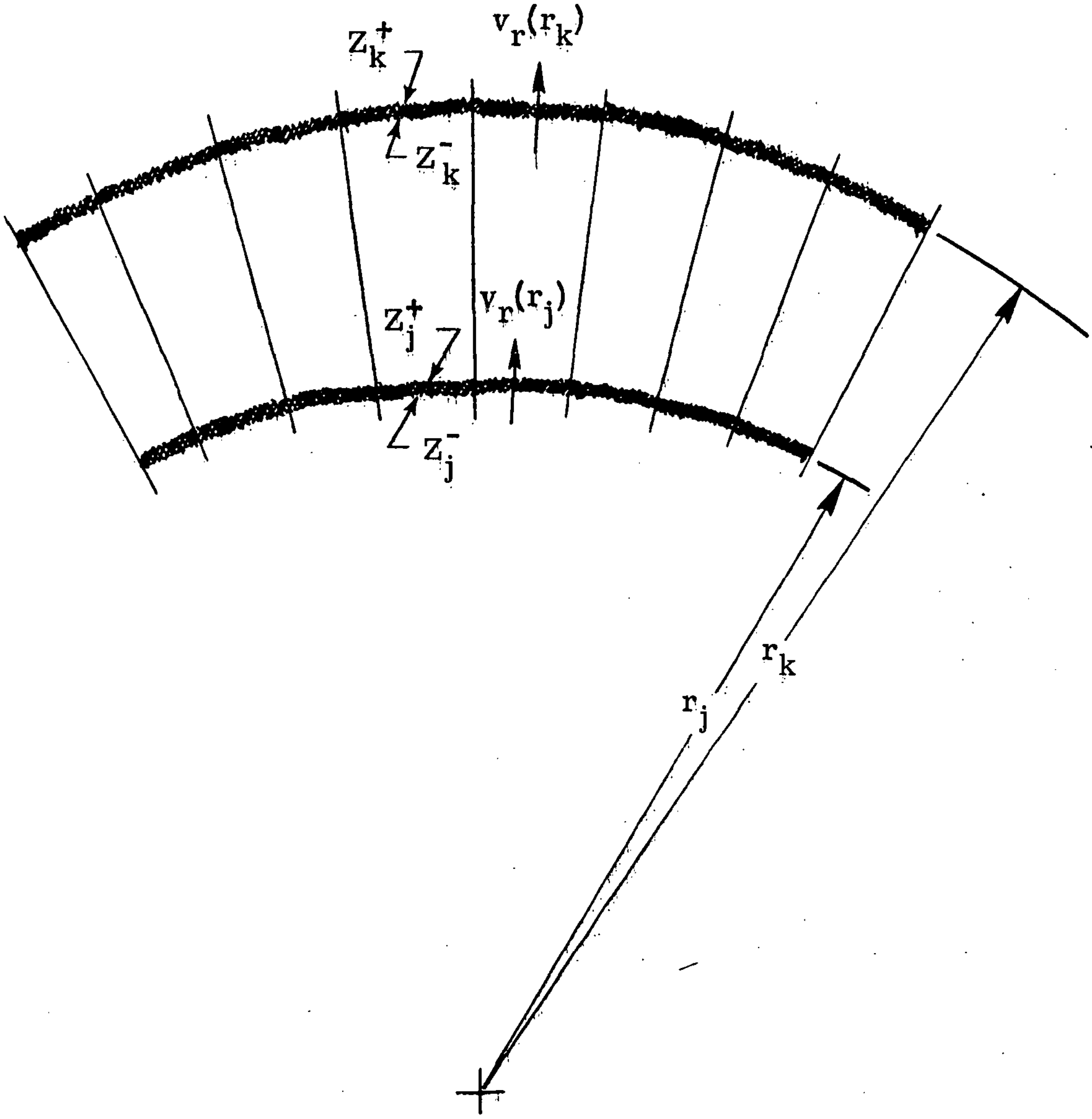

Figure 1.- Typical layers in curved liner.

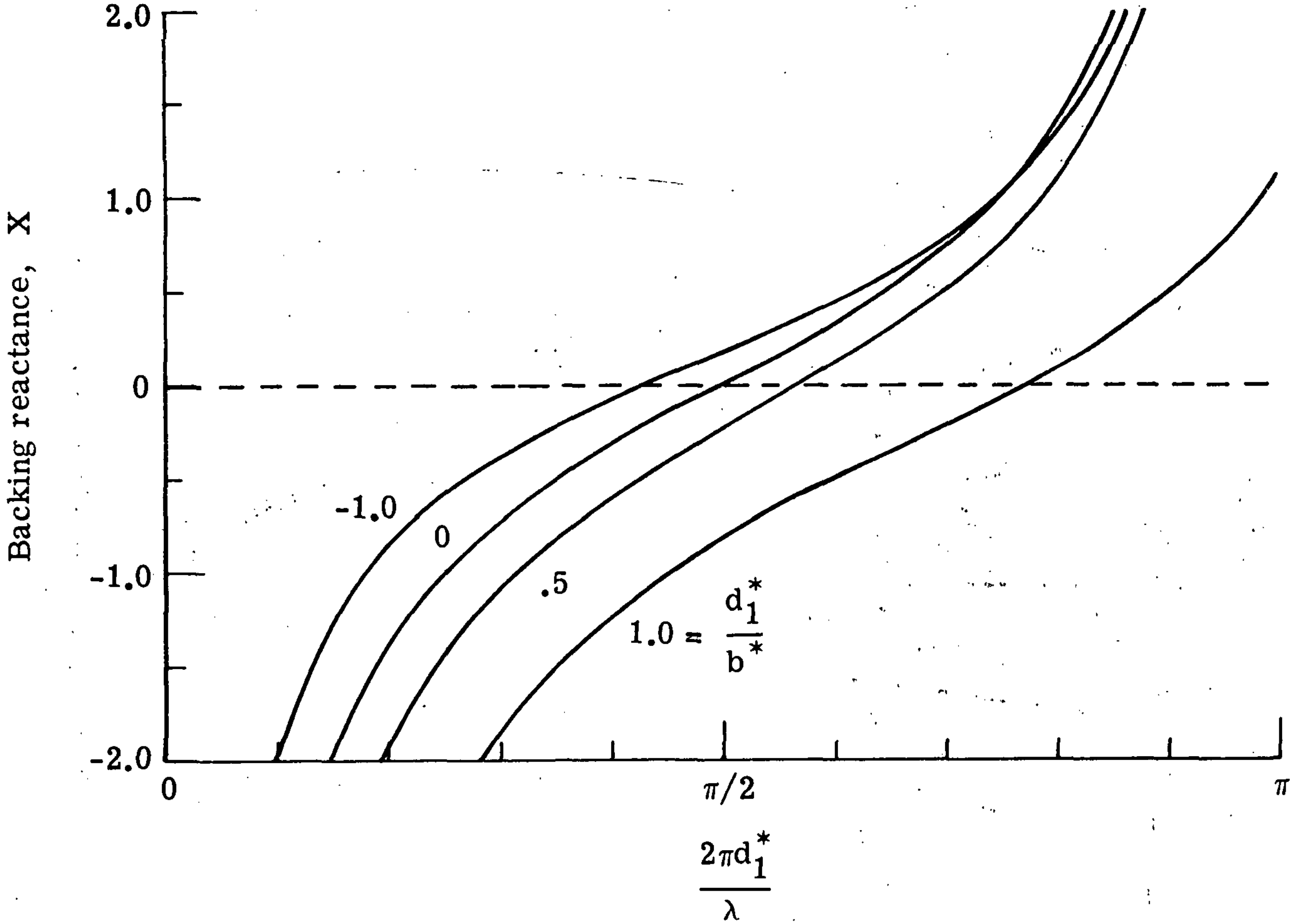

Figure 2.- Effect of curvature on backing reactance (b^* = Duct radius).

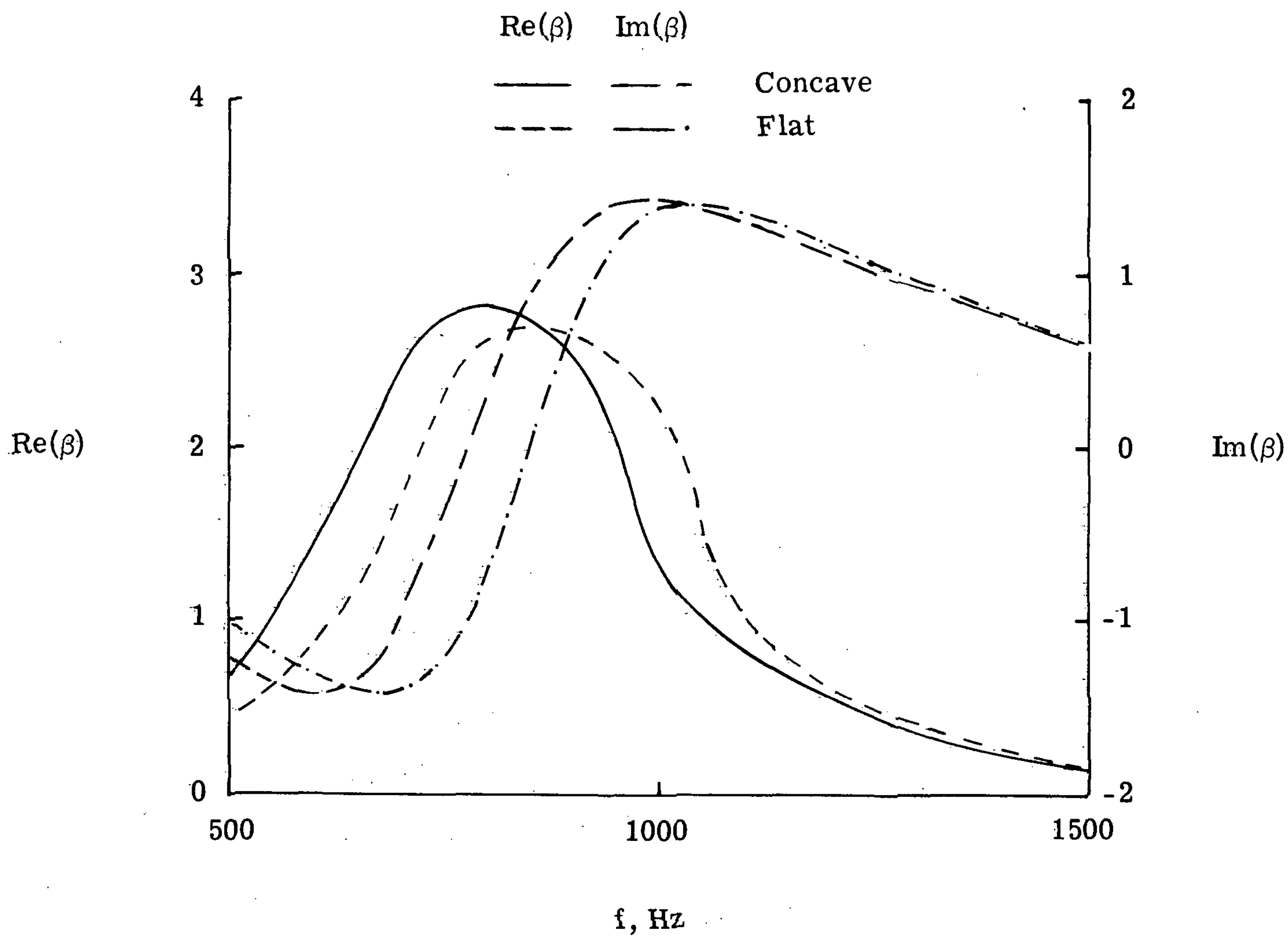

Figure 3.- Admittances of single-layer liners. $Z_1 = 0.7 - i(f \times 4 \times 10^{-4}\ \text{sec})$; $d_1 = 9.0$ cm; $b^* = 19$ cm.