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SCHOOL OF ENGINEERING

RESEARCH REPORT

THE EFFECT OF YIELD STRENGTH AND DUCTILITY TO FATIGUE DAMAGE

by

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TECHNICAL REPORT CE-3

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ABSTRACT

The cumulative damage of aluminium alloys with different yield strength and various ductility due to seismic loads has been studies. The responses of an idealized beam with a centered mass at one end and fixed at the other end to El Centro's and Taft's earthquakes are computed by assuming that the alloys are perfectly elastoplastic materials and by using numerical technique. Consequently, the corresponding residual plastic strain can be obtained from the stress-strain relationship. The revised Palmgren-Miner cumulative damage theorem is utilized to calculate the fatigue damage. The numerical results show that in certain cases, the high ductility materials are more resistant to seismic loads than the high yield strength materials. The results also show that if a structure collapse during the earthquake, the collapse always occurs in the very early stage.

TABLE OF CONTENTS

INTRODUCTION

There are two different approaches to structural design. One is based on the concept of allowable stress and elastic behavior of materials and the other on the concept of ultimate load and inelastic behavior. Since the structure designed by using plastic theory is more economic than those by applying elastic theory, plastic design concept has been gradually accepted by most engineers and has been widely applied in constructions.

If the external forces are large enough, the plastic-design oriented structures, undoubtedly, will undergo plastic deformation and create residual plastic strain. The residual plastic strain will gradually accummulate if the structure is subjected to dynamic loads such as wind force, seismic loads, etc. If the cumulative plastic strain reaches the critical values, the structures will collaspe. This process is usually termed low cycle fatigue failure. Hence the study of low cycle fatigue failure is of importance to the structures designed by using the concept of plastic theory.

There are many papers dealing with inelastic behavior of materials (2, 9, 10, 12, 13)* and the fatigue damage of structures under the action of seismic loads. (4, 5, 6, 10). Few of them have stressed the importance of the comparison of different fatigue life among the same materials with different yield strength and ductility.

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*The numerals in parenthese refer to the list of references.

Since low cycle fatigue damage can be expressed in terms of plastic strain, the fatigue life of structural systems can always be improved by choosing the materials with high ductility. However, high ductile materials generally have low yield strength which will cause the larger deformation than the materials with high yield strength. It becomes a complicated problem to choose material for structural members to resist dynamic loading. Based on the given random behavior of plastic deformation, a method for evaluation of alternative materials to be used in structures subjected to random White Noise excitation was presented (15). Since there is not any white noise excitation in reality, it is desirable to study the fatigue damage of materials subjected to earthquake laod in more details.

The purpose of this investigation is to study the fatigue damage of materials with different yeild strength and various ductility due to seismic loads. The revised Palmgren-Miner cumulative damage theorem expressed in terms of plastic strain is used to compute the fatigue damage. As an example, aluminium alloys (5052 and 3030 groups) are considered on an ideal beam, subjected to 1948 El Centro's and 1951 Taft's earthquakes.

LOW CYCLE FATIGUE DAMAGE MODEL

In computing the fatigue damage, the Palmgren-Miner criterion (8), because of its simplicity, has been most widely used during the past decades. This theorem can be mathematically expressed as follows:

$$\sum_{i=1}^{N} d_i = D \qquad \text{<1>}$$

Where d_i is the fatigue damage during its cycle, and D is the total cumulative damage under N cycles. The theorem states that the fatigue failure will occur if the total damage D reaches some critical level, which depends on material properties. At present, this study is concentrated on the low cycle fatigue life which usually is referred to the case with life less than 10^5 cycles. If a structure is supposed to collaspe within such a limited cycle, the magnitude of cyclic loading may be large enough to cause the plastic strain in the materials. Hence the Palmgren-Miner criterion for low cycle fatigue life was developed to associate with plastic strain. Manson and Gross et. al. (3, 7) proposed the following low cycle fatigue life criterion by introducing plastic strain for predicting the total number of cycles of reversed-strain to cause fatigue failure:

$$N^m (\Delta e_t) = C \qquad \text{<2>}$$

Where Δe_t is the plastic tensil strain, m is a constant depending on material properties. N is the fatigue life and C is some constant. Later Yao and Munse (14) suggested that, for uniaxially loaded metal, the cumulative damage can be expressed in terms of material ductility and plastic strains as follows:

$$\sum_{i=1}^{N} \left\langle \left(\frac{\Delta e_t}{e_f}\right)^{1/m} \right\rangle = 1 \qquad \langle 3 \rangle$$

Where e_f is the ultimate plastic strain and is constant for a given material, Δe_t is the plastic strain at tensil cycle and m is material constant dependent on ambient and loading rate. Usually m is defined as function of plastic strain in tensile and compressive cycles, denoted by e_c and e_t, respectively.

$$1/m = 1 - 0.086 \left(\frac{\Delta e_t}{\Delta e_c}\right) \qquad \langle 4 \rangle$$

The cyclic history of plastic strains in tensile and compressive cycles is illustrated in Figure 1. Equation <3> and <4> will be considered as the fatigue damage criterion through this investigation.

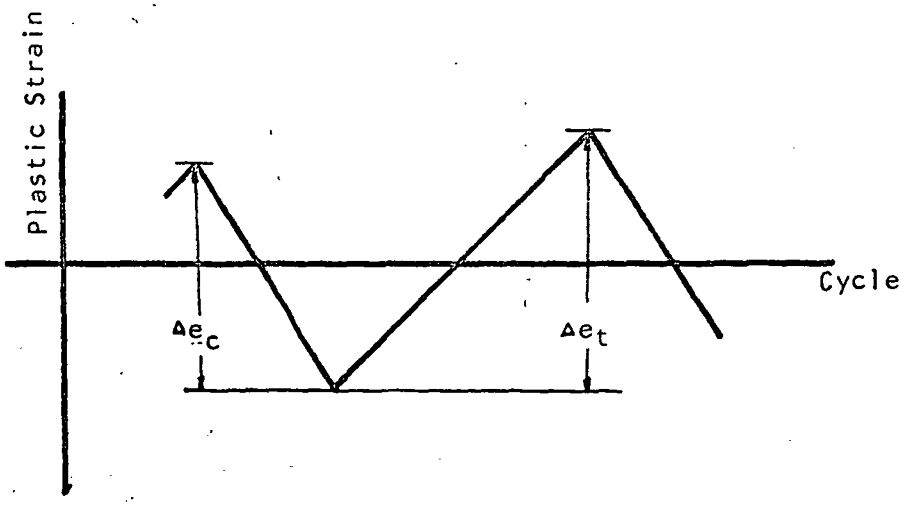

Figure 1. Cyclic History of Plastic Strain

MECHANICAL MODEL

Consider a beam with a centered mass at one end and fixed at the other end subjected to a fluctuating excitation as shown in Figure 2.

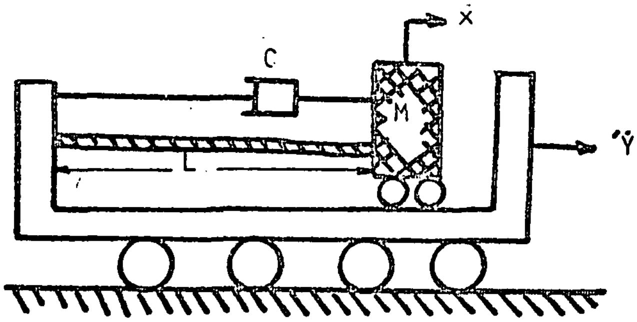

Figure 2. Mechanical Model

The equation of motion of this spring-dash-pot system is well known.

$$M \ddot{x} + C \dot{x} + G(x) = M \ddot{Y} \qquad \langle 5 \rangle$$

Where M is the concentrated mass of the beam.

C is the damping of the mechanical system.

x is the deformation of the beam.

$\ddot{y}$ is the accelerogram of seismic loads.

G(x) us tge firced-deformation function dependent on the stress-strain curve.

The force-deformation function, G(x), can be be derived from the stress-strain relationship of a given material. For simplifying the computation, the aluminium alloys considered here are assumed to be elasto-plastic materials. The stress--strain diagram of such a material is shown in Figure 3.

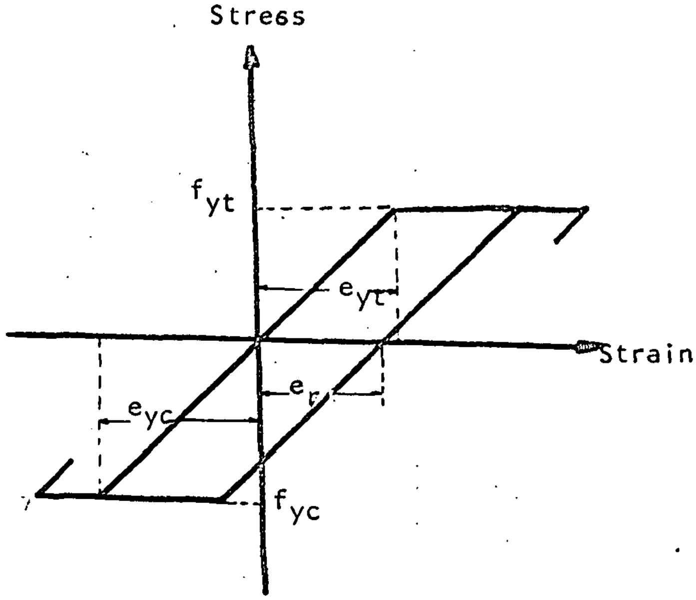

Figure 3. Elasto-plastic stress-strain diagram

Let A be the cross-sectional area of the beam.

E be the modulus of elasticity.

L be the length of beam.

e_r be the residual strain.

e_y be the yielding strain.

f_{yt} be the yielding tensil stress.

f_{yc} be the compressive yielding stress.

If $f_{yt} = f_{yc} = f_y$, the force-deformation function can be formulated as follows:

$$
\begin{aligned}
G(x) &= EA \left(\frac{X}{L} - e_r \right) && \text{If } \left| \frac{X}{L} - e_r \right| < | e_y | \\
G(x) &= A f_y && \text{If } \left| \frac{X}{L} - e_r \right| > | e_y |
\end{aligned}
\qquad \langle 6 \rangle
$$

Since the analytical solution of equation (5) is not available, Ruge-Kutta Numerical intergretion was used to obtain the deformation, x. From the known deformation and stress-strain curve, the cyclic history of plastic strain can be found, then the fatigue damage can be calculated from equation <3> .

NUMERICAL EXAMPLES

In order to illustrate the low cycle fatigue damage of structures subjected to dynamic loads, several aluminium alloys are studied. The mechanical properties of these materials are tabulated in Table (1).

Alloy	Yield strength ksi	Ductility %	Modulus of Elasticity 10^3 ksi
Alclad 3003 - H12	18	20	10.0
Alclad 3003 - H14	21	16	10.0
Alclad 3003 - H16	25	14	10.0
Alclad 3003 - H18	27	10	10.0
5052 - H32	28	18	10.0
5052 - H34	31	14	10.0
5052 - H36	35	10	10.0
5052 - H38	37	8	10.0

Table 1. Mechanical Properties of Aluminium Alloys

Since the natural frequency can not be well defined if the structural systems allow to undergo plastic deformation, the terminology of "artificial frequency" is adopted. Let p_a be the artificail frequency which is defined as $\sqrt{(AE) / (ML)}$. If both sides of equation <5> are divided by the mass M, it becomes

$$\ddot{x} + 2\, c\, p_a\, \dot{x} + g(x) = \ddot{y} \qquad \langle 7 \rangle$$

Where c is the damping ratio, p_a is the artificial frequency and g(x) is G(x)/M. It is to be noted that g(x) can be reformulated in terms of artificial frequency.

In order to make the comparision, two seismic loads are utilized in this study. One is 1948 El Centro's earthquake NS component with maximum peak in the order of 0.312g, the other is 1951 Taft's earthquake S69E component with a peak value in the order of 0.157g, where g denotes the gravitational force. Because of the high peak value of El Centro's earthquake, the artificial frequency of the mechanical system varies from 50 rad/sec to 70 rad/sec, and for the case of Taft's earthquake, the frequency varies from 35 rad/sec to 45 rad/sec. With the length of beam, L, being 10 inches and the information shown in Table 1, the fatigue damages of aluminium alloys subjected to El Centro's and Taft's earthquakes are computed by the aid of 360/65 IBM computer. The computer program is listed in Appendix. The results are tabulated in Tables 2, 3, and 4, and also plotted in Figures 4, 5, and 6.

Cumulative damage / Alloy / Frequency	5052 - H32	5052 - H34	5052 - H36	5052 - H38
50	2.9920149	3.1137094	2.5031681	3.9078865
52.5	1.5593367	1.5319510	1.7257175	1.8590279
55	0.97939056	1.1105976	1.4277601	1.2899484
57.5	0.85163534	0.96612877	1.1536770	0.82214361
60	0.62915808	0.66661555	0.44669420	0.58149391
62.5	0.32020706	0.32498884	0.4687804	0.31088388
65	0.32253897	0.32985848	0.14389163	0.1470437
67.5	0.25728476	0.17950523	0.13773221	0.13794059
70	0.12210464	0.090874553	0.06572252	0.06312008

Table 2 Fatigue Damages of 5052 Group Alloys Subjected to

El Centro's Earthquake

Cumulative damage / Frequency — Alloy	Alclad 3003 - H12	Alclad 3003 - H14	Alclad 3003 - H16	Alclad 3003 - H18
52.5	3.9831467	4.1246414	4.5256672	4.8264341
55	2.9735933	3.6988564	1.9604654	2.4367723
57.5	2.9497690	2.8602915	1.5088053	2.1295109
60	2.5558205	1.7559586	1.2991552	1.6246128
62.5	1.9535789	1.6068087	1.0175724	0.68900013
65	1.8724232	0.9148501	0.67170912	0.51194036
67.5	1.2662811	0.7284845	o.52865228	0.48792696

Table 3 Fatigue Damages of 3003 Group Alloys Subjected to

El Centro's Earthquake

Cumulative damage / Alloy	5052-H32	5052-H34	5052-H36	5052-H38
Frequency	18	14	10	8
35	1.9330864	2.2903147	2.4820929	2.7911043
37.5	1.6505632	1.3604631	1.7209997	1.5659914
40	1.0455103	0.99359804	0.77765310	0.68719399
42.5	0.63108575	0.43125600	0.32834977	0.31635483
45	0.31469911	0.18018502	0.14462858	0.14523304

Yable 4 Fatigue Damages of 5052 Group Alloys Subjected to

Taft's Earthquake

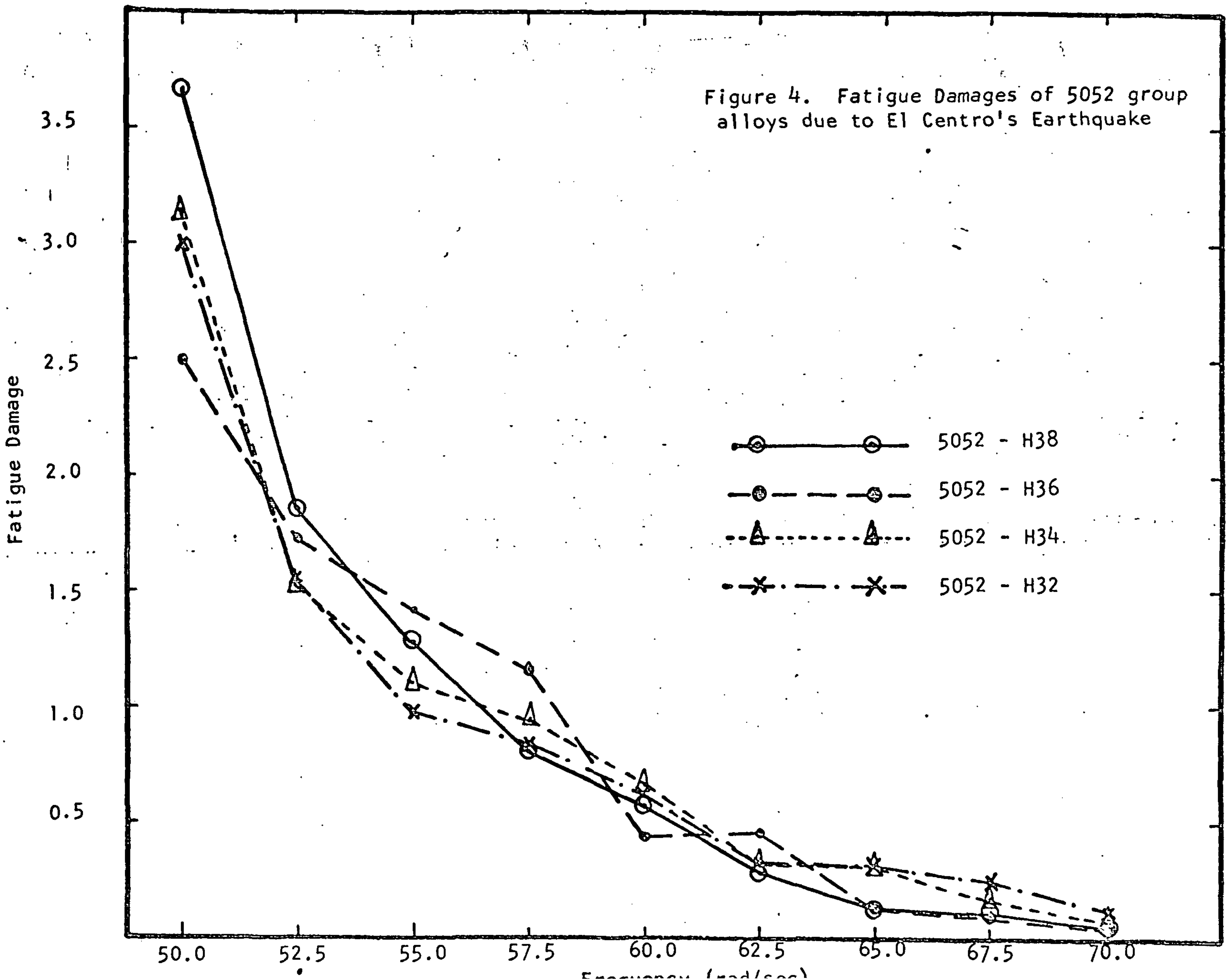

Figure 4. Fatigue Damages of 5052 group alloys due to El Centro's Earthquake

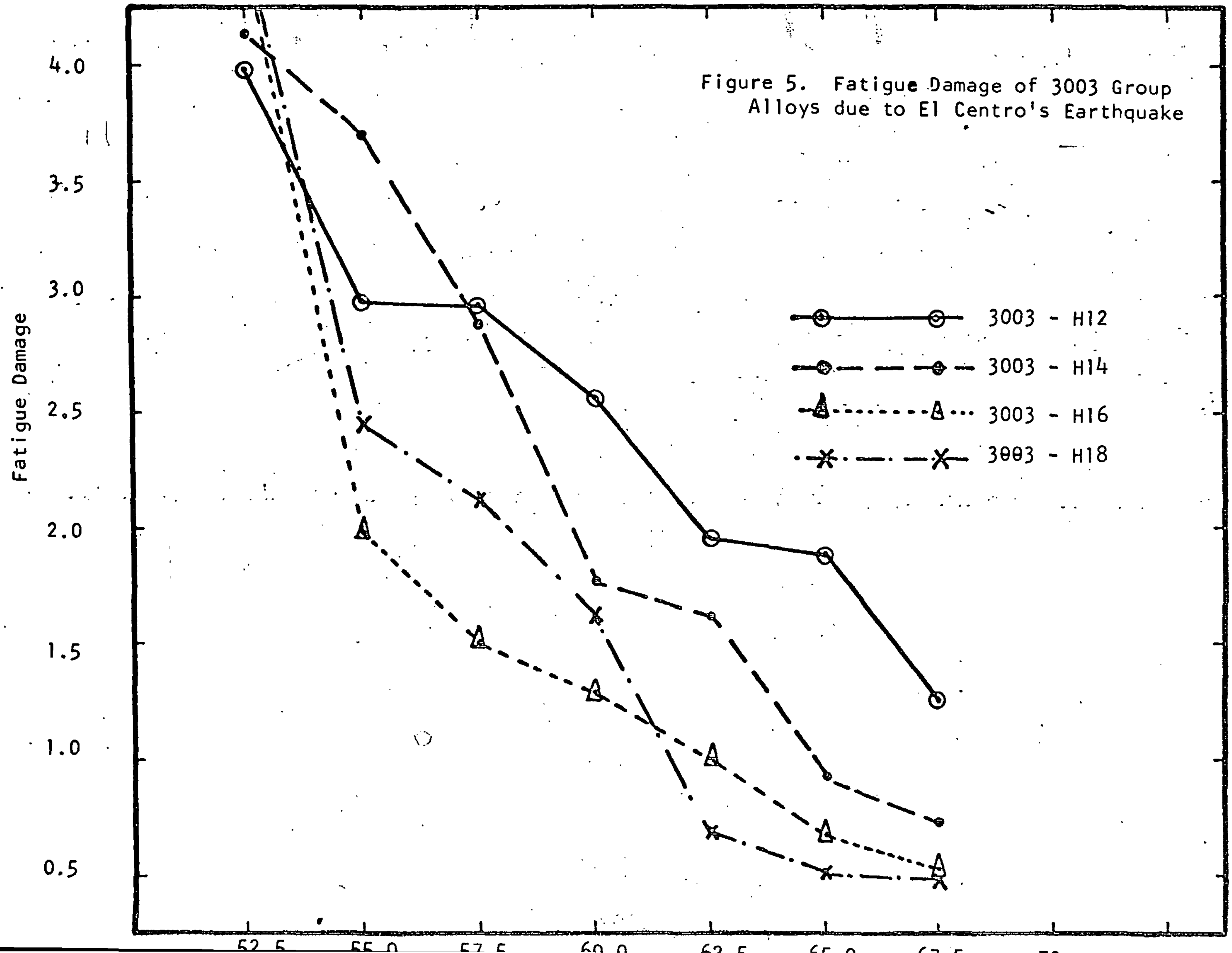

Figure 5. Fatigue Damage of 3003 Group Alloys due to El Centro's Earthquake

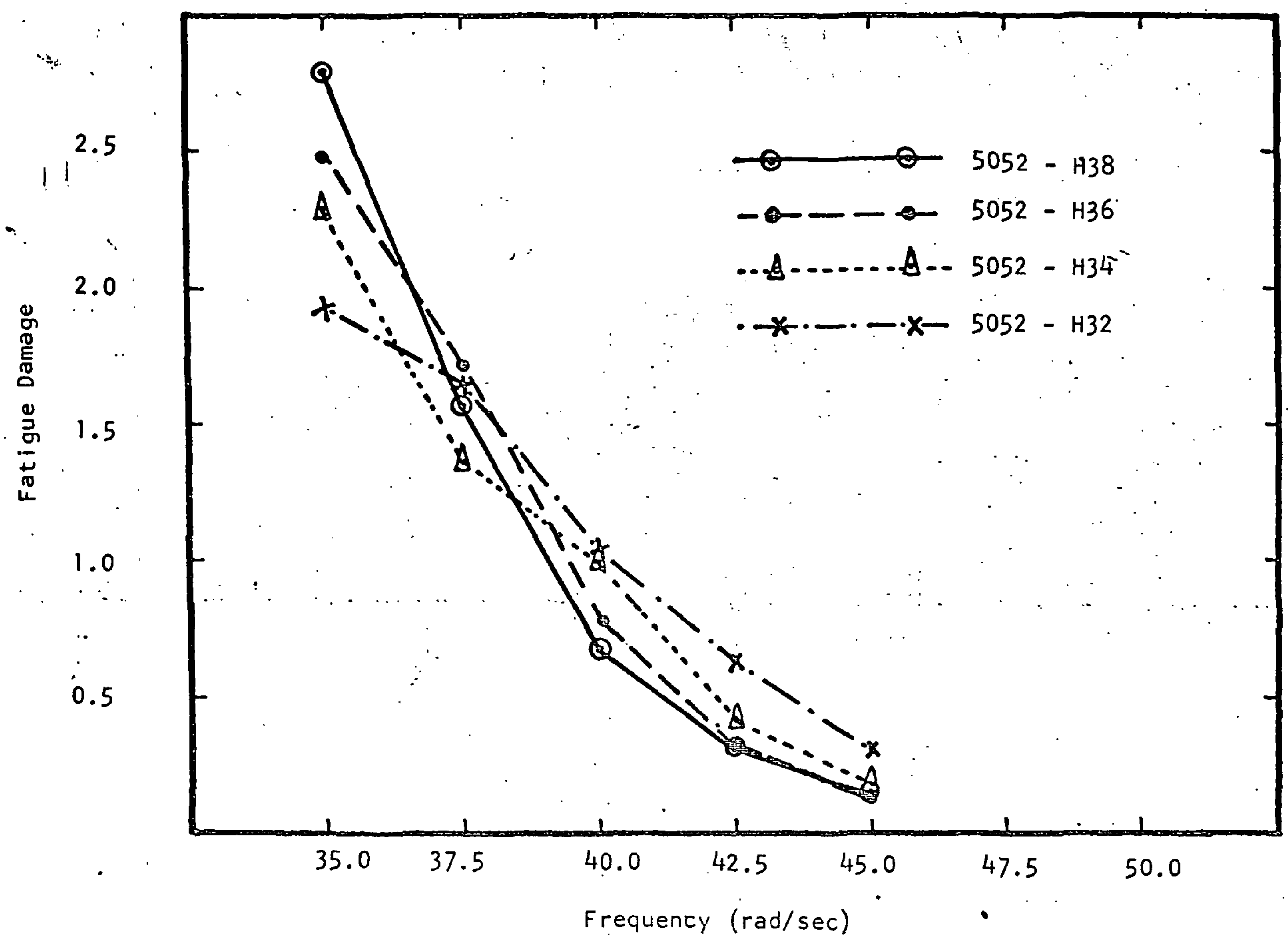

Figure 6. Fatigue Damage of 5052 Group Alloys due to Taff's Earthquake.

DISCUSSION AND CONCLUSION

For Table 2, 3, and 4 or Figures 4, 5, 6, they show that in general, the fatigue damage will increase as the stiffness of the beam decreases. However, in some cases, the fatigue damages of higher stiffness member are smaller than that of lower stiffness member as it can be seen in frequencies 62.5 rad/sec and 65.0 rad/sec for alloys 5052-H32, and 5052-H36. This phenomenon is dur to the fact that the frequency of the beam coincide with the frequency of seismic load.

Since a structural system is usually defined as "collapse" if the cumulative fatigue damage reaches one, the discussion here will concentrate on those materials whose cumulative damage smaller than one. Under Taft's earthquake, the cumulative damages increase as the yield strength of alloys 5052 group decreases regardless the ductility of materials. But it is not the case for the same materials subjected to El Centro's earthquake. The results show that in certain frequency domain, the material with high ductility is stronger to resist seismic load than the material with high yield strength. For easy comparision, the cumulative damage historigrams are plotted versus time as shown in Figure 7 and 8 for alloys 5052 group in the frequencies of 60.0 rad/sec and 8 for alloys 5052 group in the frequencies of 60.0 rad/sec and 62.5 rad/sec.

The damage historigrams are also plotted for alloys 5052 subjected to Taft's earthquake in some frequency domain. It is very interesting to note that the cumulative damages pile up very fast in the early stage of earthquake action as shown in Figures 7, 8, 9, and 10. In other words, fatigue damage is always created during the first 5 - second period for the 30-second duration El Centro's earthquake, and the first 12-second period for 60-second duration Taft's earthquake. It can be concluded that if a structure can survive during the first quarter period of earthquake action, it surely will not collaspe and under this earthquake. On the contrary, it can be said that if a structure collaspses during the earthquake, the failure always occurs in the very early stage.

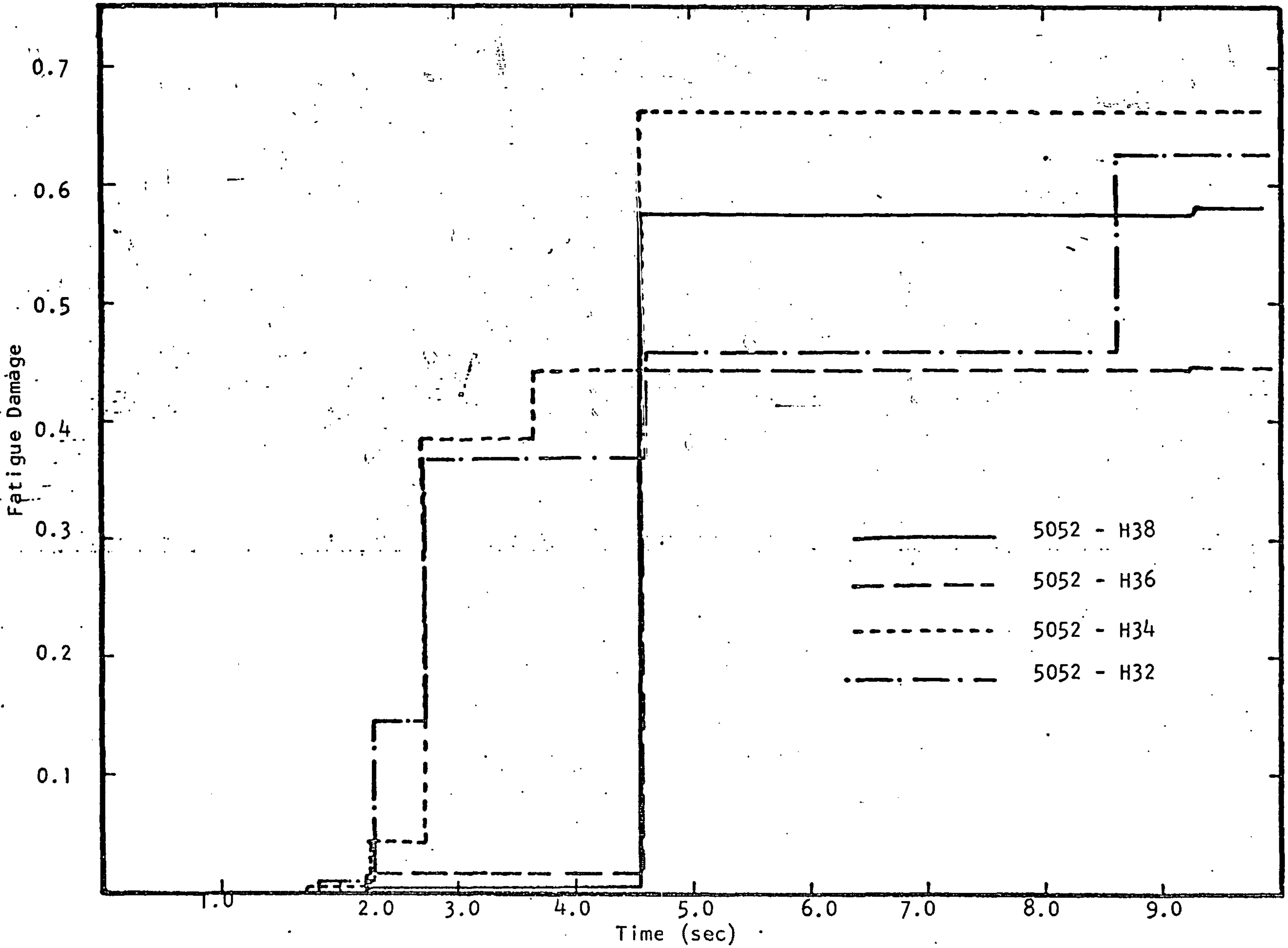

Figure 7. Fatigue Damage Historigram for 5052 Group Alloy's in the Frequency of 60 rad/sec Due to El Centro's Earthquake.

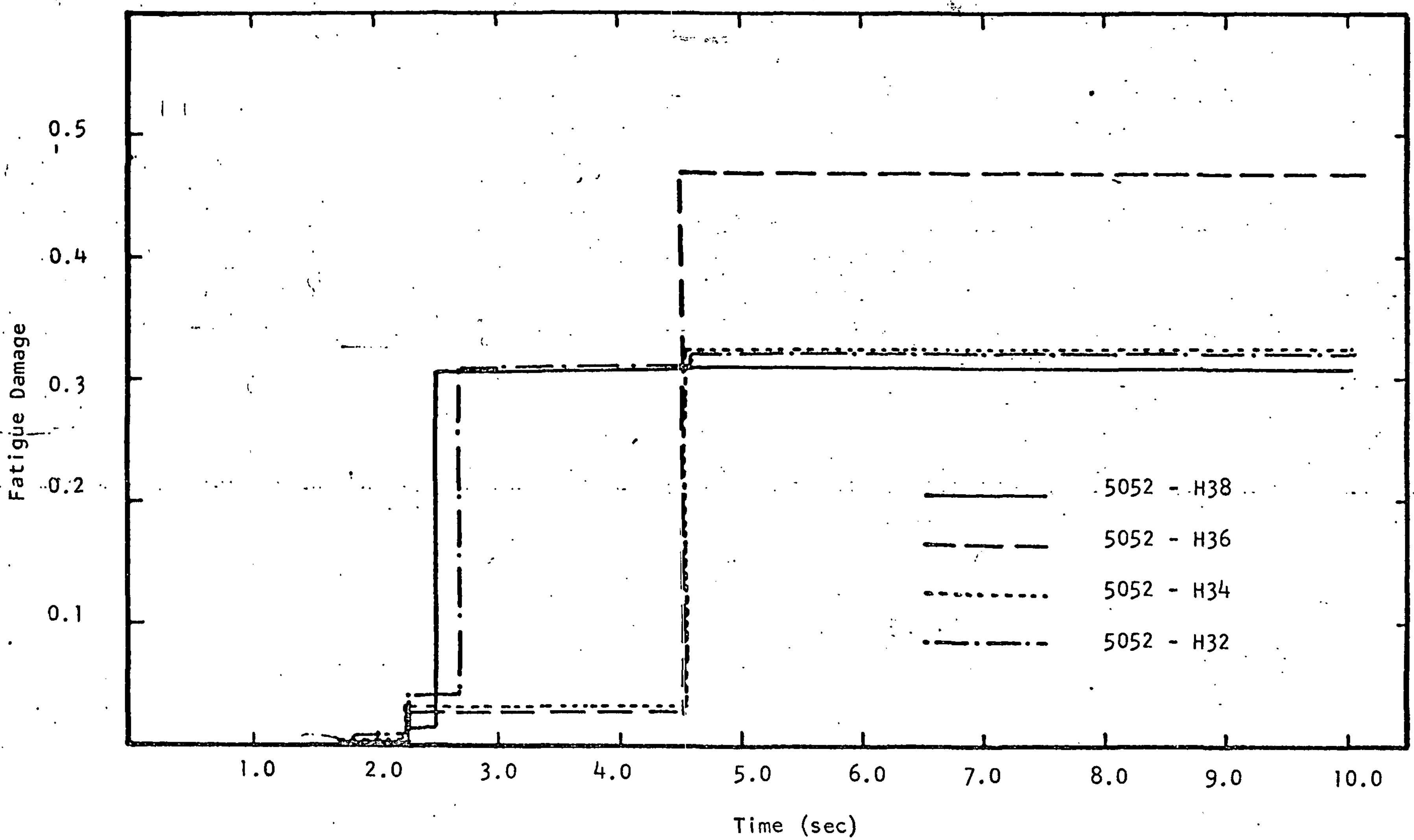

Figure 8. Fatigue Damage Historigram for 5052 Group Alloys in the Frequency of 62.5 rad/sec Due to El Centro's Earthquake.

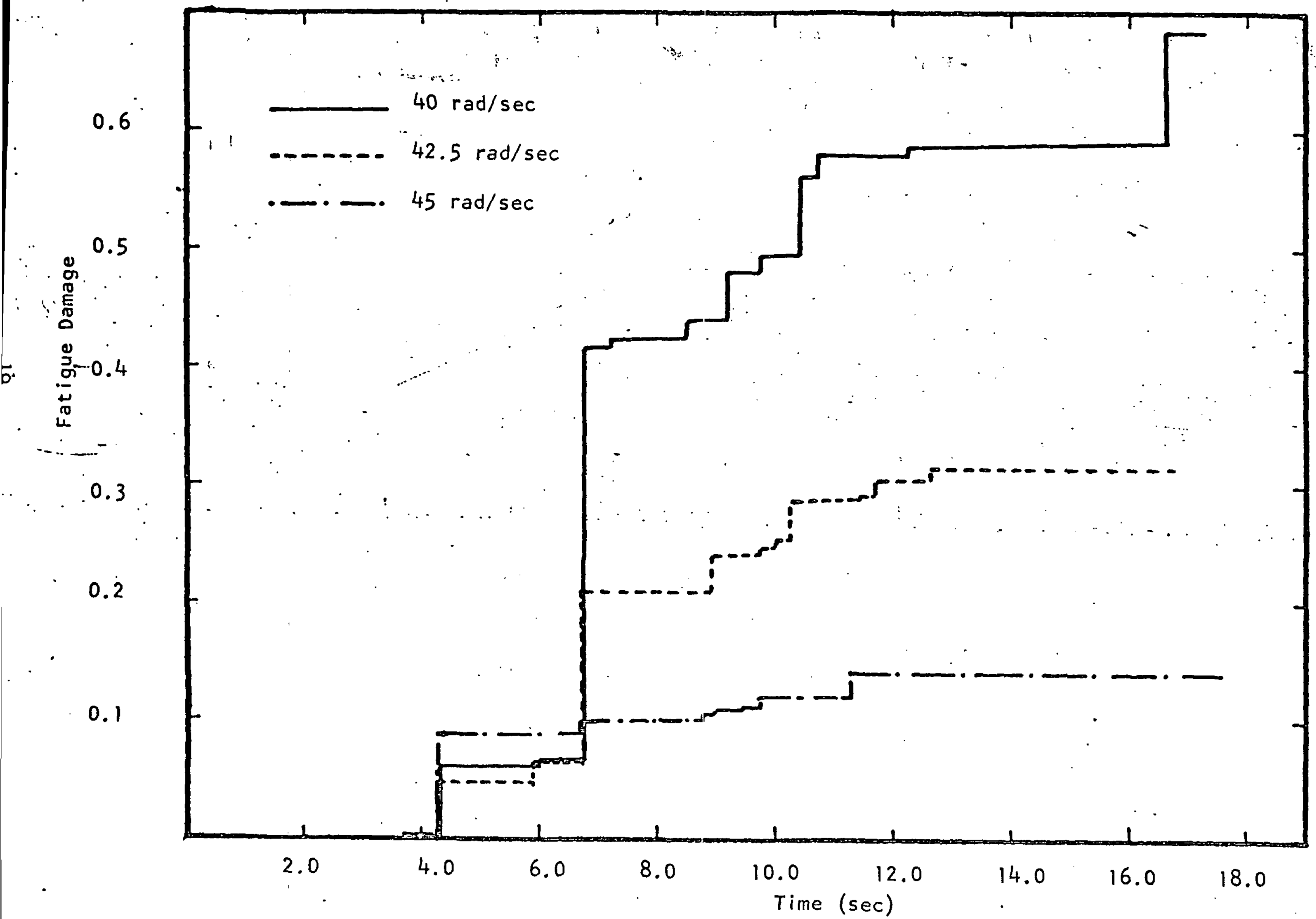

Figure 9. Fatigue Damage Historigram for 5052 - H38 Alloy Due to Taft's Earthquake.

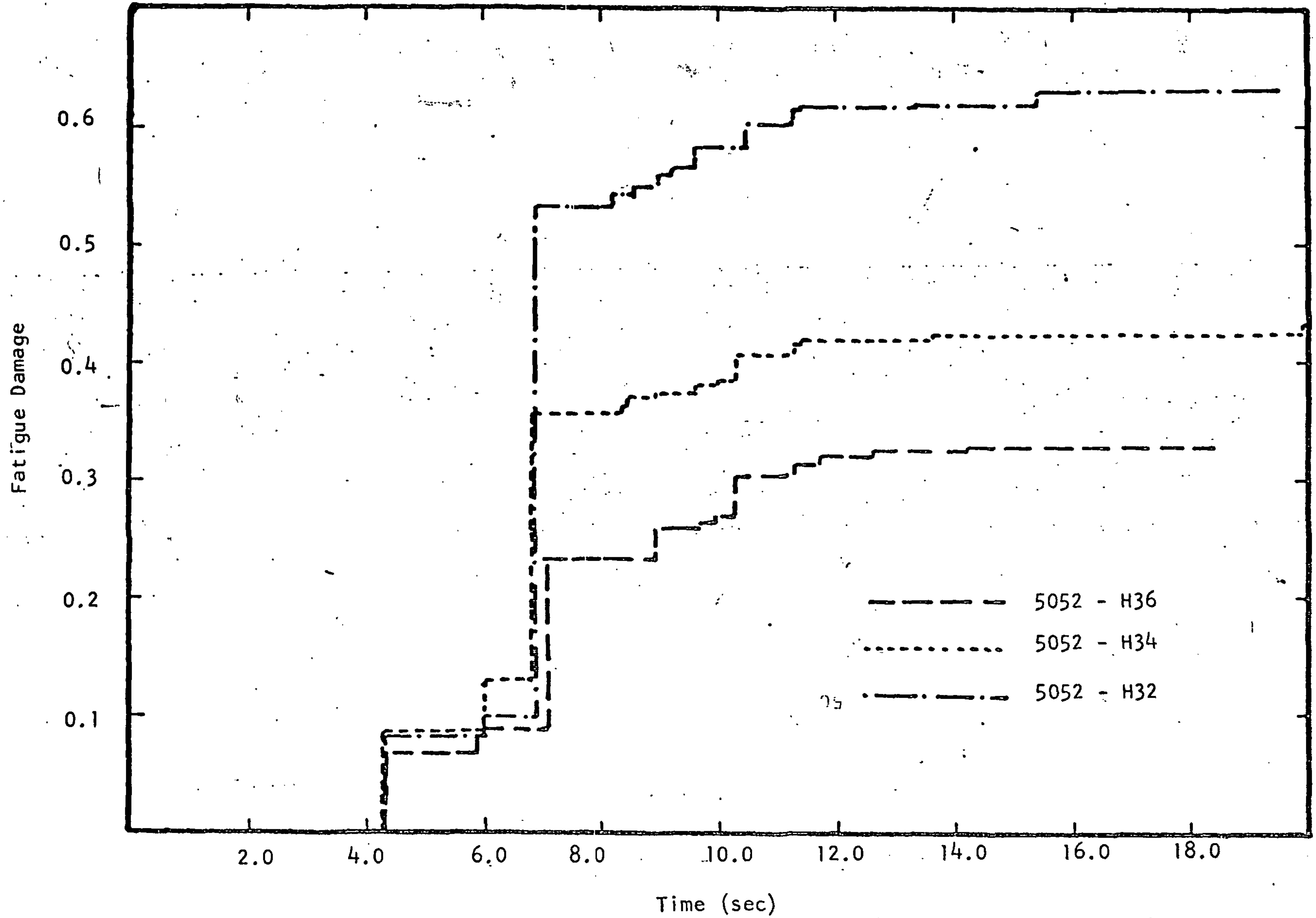

Figure 10. Fatigue Damage Historigram For 5052 Group Alloys in the Frequency of 42.5 rad/sec due to Taft's Earthquake.

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C
C      FATIGU DAMAGE DUE TO EARTHQUAKE EXCIATATION (MECHANICAL MODEL)
C
C
       DIMENSION PRMT(5),Y(2 ),DERY(2 ),AUX(8,2 ),YDIS1(3000)
      1,S(500), AC(500) ,ACCEL(3000),XT(3000),YDIS2(3000)
      2,OMGA(2),ETA(2),DULTY(2),EYELD(2),CON(2),Q(2),YDMIN(2),YDMAX(2),
      3OME(2),SK1(2),SK2(2),QMAX(2),QMIN(2),YMAX(2),YMIN(2),YELD(2),TL(2)
      4,DAMGC(2),DAMGV(2),K(2),KT1(3000),KT2(3C00)
       COMMON/VI/ACCEL,OMGA,ETA
       COMMON/VP/YMAX,YMIN,YELD,QMAX,QMIN,Q,SK1,SK2,QME
       COMMON/VD/DULTY,TL,NL
       COMMON/VO/YDIS1,YDIS2,YDMIN,YDMAX,NT,XT,DMU,TI,APM,BPM,CPM
       EXTERNAL FCT,OUTP
       READ(5,905) IEN,DEL
 905   FORMAT(I5,F10.5)
       READ(5,906) (PRMT(I),I=1,5)
 906   FORMAT(5F10.5)
       READ(5,903) (S(I),AC(I),I=1,IEN)
 903   FORMAT(4(F6.2,F12.7))
       J1=2
       IR=0
       IEN1=IEN+1
       S(IEN1)=PRMT(2)
       AC(IEN1)=0.
       DO 110 I=1,IEN
       DI=(S(I+1)-S(I))/DEL
       ID=DI
       IID=DI+0.5
       IF (ID-IID) 80,90,100
 80    ID=ID+1
       GO TO 90
 100   WRITE (6,904) I
 904   FORMAT(2X,12HERROR IN AC ,I3)
       ID=ID-1
 90    IR=IR+ID
       ACCEL(J1-1)=AC(I)*0.3864
       DO 120 J=J1,IR
 120   ACCEL(J)=ACCEL(J-1)+(AC(I+1)-AC(I))*0.3864/DI
       J1=ID+J1
 110   CONTINUE
       READ(5,907) APM,BPM,CPM
 907   FORMAT( 3F10.6)
       READ(5,900) N
       NDIM=N*2
 900   FORMAT(I5)
       WRITE(6,999)
 999   FORMAT(1H1)
```

```
31     READ(5,902) (TL(I),I=1,N)
902    FORMAT(2F10.5)
       READ(5,908) TLMU,LCHECK
908    FORMAT(F10.5,I5)
       DO 20 I=1,N
       READ(5,901) OMGA(I),ETA(I),DULTY(I),EYELD(I),CON(I)
901    FORMAT(5F10.5)
       YELD(I)=EYELD(I)*TL(I)
       Q(I)=0.
       YDMIN(I)=0.
       YDMAX(I)=0.
       QME(I)=OMGA(I)**2*YELD(I)
       SK1(I)=OMGA(I)**2
       SK2(I)=SK1(I)*CON(I)
       QMAX(I)=QME(I)
       QMIN(I)=-QME(I)
       YMAX(I)=YELD(I)
       YMIN(I)=-YELD(I)
       WRITE(6,923) OMGA(I),ETA(I),DULTY(I),YELD(I),CON(I),TL(I)
923    FORMAT(2X,10HFREQUENCY F10.5,17H  DAMPING RATIO  F10.5,11H DUCTILI
      1TY F10.5,11H  YIELDING F10.5,13H STIFFNESS 2 F10.5,3H L F10.5//)
20     CONTINUE
       DIM=NDIM
       DO 10 I=1,NDIM
       Y(I)=0.
       DERY(I)=1./DIM
10     CONTINUE
       DMU=DEL*TLMU
       TT=0.
       NT=1
       CALL RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
       NL=NT
       CALL DAMAGE(YDIS1,YELD(1),DAMGC(1),DAMGV(1),K(1),KTIM1,TL(1),
      1DULTY(1),NL,KT1,1)
       CALL DAMAGE(YDIS2,YELD(2),DAMGC(2),DAMGV(2),K(2),KTIM2,TL(2),
      1DULTY(2),NL,KT2,2)
       TEND1=XT(KTIM1)
       TEND2=XT(KTIM2)
       WRITE(6,800) TEND1,TEND2
800    FORMAT (2X,3HT1=1PE16.7,10X,3HT2=1PE16.7)
       DO 30  I=1,N
       WRITE(6,921) N, YDMAX(I),YDMIN(I)
30     WRITE (6,920) N,DAMGC(I),DAMGV(I),K(I)
920    FORMAT(10X,I3,6H STORY,16H  DAMAGE FACTOR 1P2E16.7,20H   NUMBER OF
      1 CYCLES  I7//)
921    FORMAT(10X,I3,6H STORY,19H  MAX DISPLACEMENT 1PE16.7,19H  MIN DISP
      1LACEMENT 1PE16.7//)
       K1=K(1)
```

```
      K2=K(2)
      DO 140 I=1,K1
      J=KT1(I)
  140 WRITE(6,940) I,XT(J)
      DO 150 I=1,K2
      J=KT2(I)
  150 WRITE(6,950) I,XT(J)
  940 FORMAT(2X,12HNO OF CYCLES I5,2X,5HTIME=1PE16.7,2X,2H01)
  950 FORMAT(2X,12HNO OF CYCLES I5,2X,5HTIME=1PE16.7,2X,2H02)
      IF(LCHECK) 31,32,31
   32 CONTINUE
      STOP
      END
```

```
C
      SUBROUTINE RKGS(PRMT,Y,DERY,NDIM,IHLF,FCT,OUTP,AUX)
C
C
      DIMENSION Y(2),DERY(2),AUX(8,2),A(4),B(4),C(4),PRMT(5)
      DO 1 I=1,NDIM
    1 AUX(8,I)=.06666667*DERY(I)
      X=PRMT(1)
      XEND=PRMT(2)
      H=PRMT(3)
      PRMT(5)=0.
      CALL FCT(X,Y,DERY)
C
C     ERROR TEST
      IF(H*(XEND-X))38,37,2
C
C     PREPARATIONS FOR RUNGE-KUTTA METHOD
    2 A(1)=.5
      A(2)=.2928932
      A(3)=1.707107
      A(4)=.1666667
      B(1)=2.
      B(2)=1.
      B(3)=1.
      B(4)=2.
      C(1)=.5
      C(2)=.2928932
      C(3)=1.707107
      C(4)=.5
C
C     PREPARATIONS OF FIRST RUNGE-KUTTA STEP
      DO 3 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=0.
    3 AUX(6,I)=0.
      IREC=0
      H=H+H
      IHLF=-1
      ISTEP=0
      IEND=0
C
C
C     START OF A RUNGE-KUTTA STEP
    4 IF((X+H-XEND)*H)7,6,5
    5 H=XEND-X
    6 IEND=1
C
```

```
C     RECORDING OF INITIAL VALUES OF THIS STEP
    7 CALL OUTP(X,Y,DERY,IREC,NDIM,PRMT)
      IF(PRMT(5))40,8,40
    8 ITEST=0
    9 ISTEP=ISTEP+1
C
C
C     START OF INNERMOST RUNGE-KUTTA LOOP
      J=1
   10 AJ=A(J)
      BJ=B(J)
      CJ=C(J)
      DO 11 I=1,NDIM
      R1=H*DERY(I)
      R2=AJ*(R1-BJ*AUX(6,I))
      Y(I)=Y(I)+R2
      R2=R2+R2+R2
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1
      IF(J-4)12,15,15
   12 J=J+1
      IF(J-3)13,14,13
   13 X=X+.5*H
   14 CALL FCT(X,Y,DERY)
      GOTO 10
C     END OF INNERMOST RUNGE-KUTTA LOOP
C
C
C     TEST OF ACCURACY
   15 IF(ITEST)16,16,20
C
C     IN CASE ITEST=0 THERE IS NO POSSIBILITY FOR TESTING OF ACCURACY
   16 DO 17 I=1,NDIM
   17 AUX(4,I)=Y(I)
      ITEST=1
      ISTEP=ISTEP+ISTEP-2
   18 IHLF=IHLF+1
      X=X-H
      H=.5*H
      DO 19 I=1,NDIM
      Y(I)=AUX(1,I)
      DERY(I)=AUX(2,I)
   19 AUX(6,I)=AUX(3,I)
      GOTO 9
C
C     IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
   20 IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)21,23,21
   21 CALL FCT(X,Y,DERY)
```

```
      DO 22 I=1,NDIM
      AUX(5,I)=Y(I)
   22 AUX(7,I)=DERY(I)
      GOTO 9
C
C     COMPUTATION OF TEST VALUE DELT
   23 DELT=0.
      DO 24 I=1,NDIM
   24 DELT=DELT+AUX(8,I)*ABS(AUX(4,I)-Y(I))
      IF(DELT-PRMT(4))28,28,25
C
C     ERROR IS TOO GREAT
   25 IF(IHLF-10)26,36,36
   26 DO 27 I=1,NDIM
   27 AUX(4,I)=AUX(5,I)
      ISTEP=ISTEP+ISTEP-4
      X=X-H
      IEND=0
      GOTO 18
C
C     RESULT VALUES ARE GOOD
   28 CALL FCT(X,Y,DERY)
      DO 29 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=AUX(6,I)
      Y(I)=AUX(5,I)
   29 DERY(I)=AUX(7,I)
      CALL OUTP(X-H,Y,DERY,IHLF,NDIM,PRMT)
      IF(PRMT(5))40,30,40
   30 DO 31 I=1,NDIM
      Y(I)=AUX(1,I)
   31 DERY(I)=AUX(2,I)
      IREC=IHLF
      IF(IEND)32,32,39
C
C     INCREMENT GETS DOUBLED
   32 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      IF(IHLF)4,33,33
   33 IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)4,34,4
   34 IF(DELT-.02*PRMT(4))35,35,4
   35 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      GOTO 4
```

```
C
C
C      RETURNS TO CALLING PROGRAM
   36 IHLF=11
      CALL FCT(X,Y,DERY)
      GOTO 39
   37 IHLF=12
      GOTO 39
   38 IHLF=13
   39 CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
   40 RETURN
      END
```

```
C
      SUBROUTINE FCT(X,Y,DERY)
      DIMENSION Y(4 ), DERY(4 ), ACCEL(3000),
     1OMGA(2),ETA(2),YMAX(2),YMIN(2),YELD(2),QMAX(2),QMIN(2),Q(2),SK1(2)
     2,SK2(2),QME(2)
      COMMON/VP/YMAX,YMIN,YELD,QMAX,QMIN,Q,SK1,SK2,QME
      COMMON/VI/ACCEL,OMGA,ETA
      XN=X/0.01+1.
      NX=XN
      TEMP=NX
      CALL PLAS(QF1,Y(1),YMAX(1),YMIN(1),YELD(1),QMAX(1),QMIN(1),Q(1),
     1SK1(1),SK2(1),QME(1))
      CALL PLAS(QF2,Y(2),YMAX(2),YMIN(2),YELD(2),QMAX(2),QMIN(2),Q(2),
     1SK1(2),SK2(2),QME(2))
      DERY(1)=Y(2)
      DERY(2)=-2.*ETA(1)*OMGA(1)*Y(2)-QF1-ACCEL(NX)-(ACCEL(NX+1)-
     1ACCEL(NX))*(XN-TEMP)+2.*ETA(2)*OMGA(2)*Y(4)+QF2
      DERY(3)=Y(4)
      DERY(4)=-4.*ETA(2)*OMGA(2)*Y(4)-2.*QF2+2.*ETA(1)*OMGA(1)*Y(2)+QF1
      RETURN
      END
```

```
C
      SUBROUTINE DAMAGE(Y,YELD,DAMGC,DAMGV,K,KTIME,TL,DULTY,NL,KT,IDX)
      DIMENSION Y(3000),DPP(3000),DPN(3000),KT(3000)
      KTIME=0
      DO 401 I=1,NL
      DPP(I)=0.
  401 DPN(I)=0.
      SK=1./TL
      YMAX=0.
      YMIN=0.
      EPP=0.
      EPN=0.
      DAMGV=0.
      DAMGC=0.
      N=1
      J=1
  402 IF (ABS(Y(N))-YELD) 403,403,404
  403 N=N+1
      IF (N-NL) 402,402,420
  404 IF (Y(N)) 419,405,405
  405 YYE=Y(N)-YELD
  406 EPP=YYE*SK
      YMAX=Y(N)
      DPP(J)=EPP-EPN
      KT(J)=N
  407 N=N+1
      IF (N-NL) 408,408,420
  408 IF (Y(N)-YMAX) 410,409,409
  409 YYE=YYE+Y(N)-YMAX
      GO TO 406
  410 YYN=Y(N)-(YMAX-2.*YELD)
      IF (YYN) 411,407,407
  411 EPN=YYN*SK
      YMIN=Y(N)
      IF (DPP(J)) 412,414,413
  412 WRITE(6,999)
  999 FORMAT (10X,16H ERROR IN DPP(J)//)
      RETURN
  413 J=J+1
  414 DPN(J)=EPN-EPP
  415 N=N+1
      IF (N-NL) 416,416,420
  416 IF (Y(N)-YMIN) 417,417,418
  417 YYN=YYN+Y(N)-YMIN
      EPN=YYN*SK
      YMIN=Y(N)
      GO TO 414
  418 YYE=Y(N)-YMIN-2.*YELD
```

```
      IF (YYE) 415,415,406
419   YYN=Y(N)+YELD
      GO TO 411
420   K=J
      DO 422 J=1,K
      IF (DPP(J)) 421,422,421
421   COES=1.-0.86*DPN(J)/DPP(J)
      DAMGV=DAMGV+(DPP(J)/DULTY)**COES
      IF (KTIME.NE.0) GO TO 423
      IF (DAMGV.GE.1.) KTIME=KT(J)
423   DAMGC=DAMGC+(DPP(J)/DULTY)**1.86
      WRITE(6,998) DAMGC,DAMGV,J,IDX
998   FORMAT (2X,6HDAMGC=1PE16.7,2X,6HDAMGV=1PE16.7,2X,12HNO OF CYCLES
     1I5,I5,5HSTORY)
422   CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE PLAS(QM,Y,YMAX,YMIN,YELD,QMAX,QMIN,Q,SK1,SK2,QME)
      IF (Q) 212,201,208
 201  IF(ABS(Y)-YELD) 202,202,203
 202  QM=SK1*Y
      GO TO 216
 203  IF(Y) 204,206,206
 204  QM=QMIN+(Y-YMIN)*SK2
 205  Q=-1.
      YMIN=Y
      QMIN=QM
      GO TO 216
 206  QM=QMAX+(Y-YMAX)*SK2
 207  Q=1.
      YMAX=Y
      QMAX=QM
      GO TO 216
 208  IF (Y-YMAX) 209,206,206
 209  YMA2=YMAX-Y-2.*YELD
      IF (YMA2) 210,211,211
 210  QM=QMAX+(Y-YMAX)*SK1
      GO TO 216
 211  QM=QMAX-2.*QME-YMA2*SK2
      GO TO 205
 212  IF (Y-YMIN) 204,204,213
 213  YMI2=Y-YMIN-2.*YELD
      IF (YMI2) 214,214,215
 214  QM=QMIN+(Y-YMIN)*SK1
      GO TO 216
 215  QM=QMIN+2.*QME+YMI2*SK2
      GO TO 207
 216  RETURN
      END
```

```
      SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)
      DIMENSION Y(4 ),DERY(4 ),PRMT(5),YDIS1(3000),YDIS2(3000),XT(3000)
     1,YDMAX(2),YDMIN(2)
      COMMON/VO/YDIS1,YDIS2,YDMIN,YDMAX,NT,XT,DMU,TT,APM,BPM,CPM
      N=NDIM/2
      DO 212 I=1,N
      J=2*I-1
      IF (Y(J)) 211,212,213
211   IF(YDMIN(I)-Y(J)) 212,212,214
214   YDMIN(I)=Y(J)
      GO TO 212
213   IF (YDMAX(I)-Y(J)) 215,212,212
215   YDMAX(I)=Y(J)
212   CONTINUE
      IF (IHLF-10) 216,216,217
217   WRITE(6,910) X,IHLF
910   FORMAT (10X,2HX=,F10.5,10X,5HIHLF=,I3)
      RETURN
216   IF(X-TT) 227,228,228
228   XT(NT)=X
      YDIS1(NT)=Y(1)
      YDIS2(NT)=Y(3)
      NT=NT+1
      TT=TT+DMU
227   IF (NT-3000) 225,226,226
225   CHEK=ABS(Y(1))
      DO 219 I=2,NDIM
      IF (CHEK-ABS(Y(I))) 219,219,218
218   CHEK=ABS(Y(I))
219   CONTINUE
220   IF(CHEK.EQ.0.) RETURN
      IF (CHEK-1.0E-03) 221,221,222
221   PRMT(4)=APM
      RETURN
222   ICHEK=CHEK
      IF (ICHEK) 223,224,223
224   CHEK=CHEK*10.
      GO TO 222
223   CHEKI=ICHEK
      PRMT(4)=CHEK/CHEKI*BPM
      RETURN
226   WRITE (6,911) X
911   FORMAT (10X,8HNT=3000 F10.5)
      PRMT(5)=1.
      RETURN
```