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**ELASTOHYDRODYNAMIC FILM THICKNESS MODEL
FOR HEAVILY LOADED CONTACTS**

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ABSTRACT

An empirical elastohydrodynamic (EHD) film thickness formula for predicting the minimum film thickness occurring within heavily loaded contacts (maximum Hertz stresses above $1.04 \times 10^9 \text{ N/m}^2$ (150 000 psi)) was developed. The formula was based upon X-ray film thickness measurements made with synthetic paraffinic, fluorocarbon, Type II ester and polyphenyl ether fluids covering a wide range of test conditions. Comparisons were made between predictions from an isothermal EHD theory and the test data. The deduced relationship was found to adequately reflect the high-load dependence exhibited by the measured data. The effects of contact geometry, material and lubricant properties on the form of the empirical model are also discussed.

NOMENCLATURE

a minor semi-axis of Hertzian contact, m (in.)

B constant, eq. (7)

b major semi-axis of Hertzian contact, m (in.)

$C_{i,j}$ coefficient, eq. (1)

E_1, E_2 modulus of elasticity of elements 1 and 2, N/m^2 (psi)

E' $\left(\frac{1 - \nu_1^2}{\pi E_1} + \frac{1 - \nu_2^2}{\pi E_2} \right)^{-1} \text{ N/m}^2$ (psi)

$f(p_{\text{Hz}})_j$ film thickness-stress function, eq. (2)

\bar{H}_{min} nondimensional minimum film thickness, $\frac{h_{\text{min}}}{R'}$

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h_c	film thickness in Cheng's theory, see eq. (11), m (in.)
h_{min}	minimum film thickness, m (in.)
i	experimental maximum Hertz stress subscript
j	test lubricant subscript
K_j, K_j^*, k_j	lubricant coefficients in empirical film thickness formula
$n_{i,j}$	nondimensional speed-viscosity parameter exponent, eq. (1)
\bar{n}_j	mean value of $n_{i,j}$, eq. (2)
\bar{P}_{Hz}	nondimensional stress parameter, p_{Hz}/E'
p_{Hz}	maximum Hertz stress, N/m^2 (psi)
R_1, R_2	radius of elements 1 and 2 in rolling direction, m (in.)
R'	equivalent radius, $\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$, m (in.)
T_o	disk temperature, K ($^{\circ}F$)
\bar{U}	nondimensional speed-viscosity parameter, $\mu_o u/E'R'$
u	mean surface velocity, $1/2(u_1 + u_2)$, m/sec (in./sec)
u_1, u_2	surface velocities of elements 1 and 2, m/sec (in./sec)
α	pressure-viscosity coefficient, m^2/N (psi^{-1})
μ_o	inlet absolute viscosity, $N\text{-sec}/m^2$, (lb-sec/in. 2)
$\nu_{1,2}$	Poisson's ratio of elements 1 and 2
ϕ_s	high contact stress factor, eqs. (8), (9), and (10)
$\psi_{i,j}, \psi_s$	film thickness-stress function, eqs. (4), (5)

INTRODUCTION

The importance of maintaining a sufficient elastohydrodynamic (EHD) film thickness between dynamically contacting machine elements has in recent years been more fully appreciated. The prediction of EHD film thickness has been the focal point of many theoretical and experimental investigations, and has been summarized well in [1,2].¹

¹Numbers in brackets designate References at end of paper.

The ratio of EHD minimum film thickness to composite surface roughness of the mating contact surfaces has become an acceptable indicator of the effectiveness of the lubricant film within the rolling-element contact zone. It has been experimentally shown that this ratio influences the fatigue life of rolling-element bearings [3,4]. Predetermination of this lubricant parameter with an accurate prediction of minimum film thickness will be of value to the designer in obtaining more realistic estimates of rolling-element fatigue life [5].

The bulk of the experimental work conducted in elastohydrodynamic lubrication has been confined to conditions of moderate speeds; that is, up to 25.4 meters per second (1000 in./sec), and moderate loads; that is, maximum Hertz stresses to $1.24 \times 10^9 \text{ N/m}^2$ (180,000 psi) [6 to 9]. The research of [10,11] has extended the EHD film thickness measurements to maximum Hertz stresses of $2.42 \times 10^9 \text{ N/m}^2$ (350,000 psi) which include the design operating range of most machine components such as bearings and gears. This data was obtained on a rolling-disk machine using an X-ray transmission technique to measure minimum film thickness. The film thickness measurements showed good qualitative agreement with full scale bearing test results [12]. That is, very low film thicknesses were measured at conditions similar to those where the bearings suffered surface damage.

In contrast to the results obtained by previous investigators which showed reasonably good correlation at moderate speeds and loads between elastohydrodynamic theory and film thickness measurement, the data of [10,11] showed a marked deviation between predicted and experimental values of film thickness. In particular, at high contact stresses; that is, maximum Hertz stresses greater than $1.38 \times 10^9 \text{ N/m}^2$ (200,000 psi), the

sensitivity of the film thickness to load as determined experimentally is far greater than that predicted by classical EHD theory of [13,14].

Several attempts have been made to resolve the apparent discrepancy between theory and experiment. A critical examination of the X-ray technique itself was made [15] for possible load dependent experimental errors. However, no experimental factors were uncovered which could seriously alter the accuracy of the X-ray measurements. On the theoretical side, the influence of several possible rheological factors has been investigated, such as the effects of a non-Newtonian lubricant of the Ree-Eyring form [16], the effects of heating at the inlet of the contact region [17] and the effects of a reduced lubricant viscosity-pressure dependence using a composite exponential model [18] and using a power-law model [19].

While each of the above modifications to elastohydrodynamic theory has succeeded somewhat in improving the agreement between theory and experimental data within the heavy load regime, the resulting predicted values of film thickness differed little in magnitude from those computed using classical EHD theory. Furthermore, the modified theories do not sufficiently account for the high film thickness-load dependence to allow accurate predictions of film thickness under realistic operating conditions.

The experimental data of [20,21] also show a film thickness sensitivity to stress greater than theoretical for maximum Hertz stresses greater than about $1.04 \times 10^9 \text{ N/m}^2$ (150,000 psi). These data, obtained by an optical interferometry technique with sliding point contacts tend to confirm the measurements obtained by the X-ray technique of [10,11].

It was the objective of the work reported herein to develop an empirical elastohydrodynamic film thickness model based on an analysis of the experimental data of [10,11] and to compare the empirical relation derived with that of conventional elastohydrodynamic theory.

EHD FILM THICKNESS EXPERIMENTAL DATA

The empirical film thickness model which is presented herein was developed from film thickness data [10,11] obtained in an X-ray rolling-disk machine. The four lubricants studied were a Type II ester, a fluorocarbon, a polyphenyl ether, and a synthetic paraffinic oil. Properties of these lubricants are shown in Table 1.

The method of measuring film thickness with the X-ray technique comprises projecting X-rays between the surfaces of the two contacting disks, and detecting the rate of X-ray transmission through the contact. Since the greatest constriction occurs at the trailing edge of the contact, the X-ray count thus becomes a measure of the lubricant's minimum film thickness.

The range of test conditions include disk temperatures from 339 to 589 K (150° to 600° F), surface speeds from 9.4 to 37.6 meters per second (370 to 1480 in./sec) corresponding to disk rolling speeds from 5000 to 20,000 rpm, and maximum Hertz stresses from 1.04×10^9 to 2.42×10^9 N/m² (150,000 to 350,000 psi). Two crowned-cone AISI M-50 steel disks each with a rolling radius of 1.83 centimeters (0.72 in.) and a surface finish of 2.5×10^{-6} to 5.0×10^{-6} centimeters (1 to 2 μ in.) rms were used as the test specimens.

Both crowned disks and crowned-cone disks (with a cone angle of 10°) were tested and no significant differences were reported between the two

sets of film thickness data [10,11]. All the data reported herein were generated with the crowned-cone test disks appearing in Fig. 1.

FORMULATION OF AN EMPIRICAL FILM THICKNESS EXPRESSION

To formulate a generalized film thickness expression for the four test lubricants over the wide range of experimental operating conditions it is most convenient to reflect the effects of the several test variables on the value of measured minimum film thickness within the confines of a single plot. This has been accomplished in Fig. 2 which shows the sensitivity of minimum film thickness h_{\min} to mean surface speed u and lubricant viscosity μ_o at the various experimental maximum Hertzian stress levels p_{Hz} for each of the four test fluids. In these and later plots the nondimensional groupings common to EHD theory, viz. the dimensionless film thickness parameter $\bar{H}_{\min} = (h_{\min}/R')$, the dimensionless speed-viscosity parameter $\bar{U} = (\mu_o u/E'R')$ and the dimensionless stress parameter $\bar{p}_{Hz} = (p_{Hz}/E')$, have been introduced to facilitate the handling of the experimental data.

The measured data on these log-log plots can be satisfactorily fitted by straight lines for all of the test fluids (see [22,23] for details) and therefore can be represented by the following simple power relationship

$$(\bar{H}_{\min})_{i,j} = C_{i,j} \bar{U}^{n_{i,j}} \quad (1)$$

where subscript $i = 1 \rightarrow 5$ designates one of the five experimental maximum Hertz stress levels and subscript $j = 1 \rightarrow 4$ designates one of the four test fluids and where \bar{U} ranges from the lowest to highest experimental value.

What distinguishes one lubricant from the next in these plots is the effect of contact stress on the sensitivity of minimum film thickness to variations in \bar{U} . In view of conventional elastohydrodynamic theory, it would not be expected that the stress level would appreciably influence the relation of \bar{H}_{\min} to \bar{U} . However, both the synthetic paraffinic and polyphenyl ether film thickness data show a somewhat enhanced sensitivity to \bar{U} with increasing contact stress, as evidenced by the change in slope of the lines appearing in Figs. 2(a) and (d). The magnitude of this variation, that is, the variation of exponent $n_{i,j}$ with increasing contact pressure, is tabulated in Table 2 for the four test lubricants.

It is apparent from this table that exponents $n_{i,2}$ and $n_{i,3}$ for the fluorocarbon and Type II ester fluids are essentially unaffected by variations in contact stress. Their mean values, designated as \hat{n}_2 and \hat{n}_3 , were determined to be 0.61 and 0.60, respectively. With regard to the synthetic paraffinic and polyphenyl ether test fluids, one notices an appreciable variation in the value of $n_{i,j}$ over the operating stress range. For purposes of developing a generalized film thickness formula of an approximate nature, the complications of a pressure dependent speed-viscosity parameter exponent can be avoided without introducing serious inaccuracies by selecting mean values of exponents $n_{i,1}$ and $n_{i,4}$. Table 3 lists the values of exponent \hat{n}_j which have been determined for each of the test lubricants from the X-ray test data.

Equation (1) can now be written in the following form.

$$(\bar{H}_{\min})_j = \bar{U}^{\hat{n}_j} f(p_{Hz})_j \quad (2)$$

where the presently unknown continuous function $f(p_{Hz})_j$ has been introduced to describe the dependence of minimum film thickness upon the

applied contact load for each lubricant. The main objective in this approach is to separate the effects of maximum Hertz stress on the value of film thickness from those effects contributed by surface speed and lubricant viscosity. Having isolated the effects of contact pressure, there remains the task of representing the influence of contact pressure on film thickness by a single mathematical expression for all test fluids.

By combining equations (1) and (2), an expression can be written for $f(p_{Hz})_j$ where

$$f(p_{Hz})_j = C_{i,j} \bar{U}^{(n_{i,j} - \hat{n}_j)} \quad (3)$$

and where the discrete values of $f(p_{Hz})_j$ must equal the experimentally deduced values of the expression on the right side of the above equation for any \bar{U} .

It is evident from equation (3) and the previous discussion that $f(p_{Hz})_j$ is unavoidably an explicit function of \bar{U} . In the case of the fluorocarbon and Type II ester fluids, exponent $n_{i,j}$ is essentially constant and equal to \hat{n}_j for all p_{Hz} so that the exponent of \bar{U} in equation (3) will be essentially zero. Thus the effect of \bar{U} on the $f(p_{Hz})_j$ term in equation (2) would be negligible in accordance with EHD theory. With regard to the synthetic paraffinic and polyphenyl ether test fluids, the sensitivity of minimum film thickness to maximum Hertz stress does vary slightly with changes in operating speed and disk temperature as evidenced by Figs. 2(a) and (d). However, this variation due to \bar{U} is not severe [22,23] and the inaccuracies incurred by evaluating the expression in equation (2) at some mean experimental value of \bar{U} are tolerable.

If $\log f(p_{Hz})_{i,j}$, evaluated at the mean experimental value of \bar{U} , is plotted against the log of the dimensionless stress parameter \bar{P}_{Hz} for each of the four test fluids, the curves on Fig. 3 representing the film thickness - stress function $f(p_{Hz})_j$ result. It can be seen that there is great similarity in the shape of the curves appearing in this figure. That is to say, the effect of maximum Hertz stress on the ratio of $(\bar{h}_{min})_j$ to $\bar{U}^{\hat{n}_j}$ is nearly the same, apart from some constant multiplier, say k_j , for all the test fluids [23]. Defining factor $\psi_{i,j}$ such that

$$\psi_{i,j} = \frac{f(p_{Hz})_{i,j}}{k_j} \quad (4)$$

where constant k_j is some lubricant parameter used to normalize the value $f(p_{Hz})_{i,j}$ at $\bar{P}_{Hz} = 3.09 \times 10^{-3}$. Table 4 lists constant k_j and parameter $\psi_{i,j}$ as a function of \bar{P}_{Hz} for each of the four test fluids. It is apparent from inspecting Table 4 that the value of $\psi_{i,j}$ at a given contact stress level does not differ appreciably from lubricant to lubricant. Thus taking the mean values of $\psi_{i,j}$ at each \bar{P}_{Hz} , this variation can be represented for all of the test fluids by a single generalized function ψ_s . Equation (4) may be rewritten as follows.

$$\psi_s = \frac{f(p_{Hz})_j}{k_j} \quad (5)$$

Substituting equation (5) into equation (2) yields

$$(\bar{h}_{min})_j = k_j \bar{U}^{\hat{n}_j} \psi_s \quad (6)$$

where function ψ_s satisfactorily describes the effect of p_{Hz} on h_{min} for all four test lubricants.

The above expression is, in itself, a film thickness correlation

which can be used to satisfactorily forecast minimum film thickness at high contact stress levels. However, as a matter of convenience, equation (6) will be altered slightly to a somewhat more familiar form.

DEVELOPMENT OF EHD HIGH-CONTACT-STRESS FACTOR

As previously discussed, film thickness values forecasted by currently accepted EHD theory has shown reasonably good agreement with experimental data for maximum Hertz pressures less than approximately $1.04 \times 10^9 \text{ N/m}^2$ (150,000 psi). Above this stress level, conventional theory seriously overestimates the extent of the film generated by the lubricant as evidenced by test data [10,11]. Thus, it is most desirable to introduce some factor to adjust current film thickness formulae for the deviation between theory and experiment at high applied loads.

It is generally recognized that film thickness is only moderately dependent upon contact stress at the lower stress levels. Typically, for line contact, film thickness is proportional to maximum Hertz stress to the -0.22 power, i.e., $\bar{H} \propto (\bar{P}_{\text{Hz}})^{-0.22}$, where the stress parameter exponent selected here comes from the isothermal theory of Cheng [24]. This proportionality can be introduced into the empirical film thickness relationship shown in equation (6) by simply defining a factor ϕ_s , such that

$$\phi_s = B \frac{\psi_s}{(\bar{P}_{\text{Hz}})^{-0.22}} \quad (7)$$

where constant B has been arbitrarily chosen to equal $(3.09 \times 10^{-3})^{-0.22}$ to make $\phi_s = \psi_s = 1$, at $\bar{P}_{\text{Hz}} = 3.09 \times 10^{-3}$. By incorporating equation (7) into equation (6), yields

$$\bar{H}_{\min} = K_j^* \bar{U}^{\hat{n}_j} \bar{P}_{Hz}^{-0.22} \phi_s \quad (8)$$

where coefficient $K_j^* = k_j/B$ and exponent \hat{n}_j are lubricant parameters listed in Table 3, and parameter ϕ_s is a reduction factor to account for the much higher sensitivity of film thickness to load than normally predicted.

Fig. 4 shows the effect of contact stress on factor ϕ_s together with the following polynomial expression which closely fits this curve,

$$\phi_s = \bar{P}_{Hz} (150. - 27.5 \times 10^3 \bar{P}_{Hz}) + 0.806 \quad (9)$$

It is important to emphasize that the accuracy of the empirical relationship shown in equation (6) has not been affected by the introduction of the $(\bar{P}_{Hz})^{-0.22}$ term in equation (8). Either of these expressions may be utilized to forecast minimum film thickness quite satisfactorily but the latter is perhaps more convenient to use.

In viewing the variation of the \bar{U} exponent \hat{n}_j , appearing in Table 3, it is apparent that the value of \hat{n}_j for the first three test fluids are nearly equal, averaging approximately 0.62. However, the value of \hat{n}_j for the polyphenyl ether fluid is significantly higher than the rest. In the optical film thickness experiments conducted by Westlake and Cameron [25], the speed-viscosity parameter exponent of a similar polyphenyl ether fluid, at a value of 0.82, was found to be somewhat larger than the exponent of any other fluid tested including that for a fluorocarbon and a synthetic paraffinic lubricant. In contrast to the results of the present work, no variation of \hat{n}_j with P_{Hz} for the minimum film thickness case was observed for either the polyphenyl ether or synthetic paraffinic oils. However, due to differences in the experi-

mental apparatus, both the operational shear rate and contact stress levels are significantly higher here than in the work reported in [25].

It was determined from numerical comparisons between the X-ray test data and predictions from equation (8) that only a small loss of accuracy would result by setting \hat{n}_j at a nominal value of 0.62 for all four test fluids. Taking advantage of this last simplification, equation (8) can be written in the following final form,

$$\bar{H}_{\min} = K_j \bar{U}^{0.62} \bar{P}_{Hz}^{-0.22} \phi_s \quad (10)$$

where the lubrication parameter K_j^* has been adjusted to K_j to reflect the change in the exponent of \bar{U} . Table 3 lists the appropriate value of K_j for equation (10).

DISCUSSION OF RESULTS

Comparison with Test Data and Conventional Theory

The deviations in magnitude between film thicknesses forecasted by current isothermal EHD film thickness formulae and those experimentally observed are partially attributable to the uncertainties encountered in selecting appropriate values of the pressure-viscosity coefficient α for computational use [26]. Furthermore, part of this magnitude difference between isothermally predicted and measured film thicknesses at high rolling speeds is undoubtedly linked to inlet shear heating effects of the lubricant. Under the appropriate dynamic circumstances, inlet shear heating effects are known to cause appreciable film thinning [18,27]. What has not yet been satisfactorily resolved from a theoretical standpoint is the increasing sensitivity of the minimum film thickness to contact stress with increasing applied load. This anomaly is illustrated in

Fig. 5 which is a plot of nondimensional minimum film thickness as a function of maximum Hertz pressure. This figure compares the X-ray test data and isothermal theory for a synthetic paraffinic oil at temperatures of 339 K (150° F) and 422 K (300° F) and disk speeds of 5000 and 15 000 rpm which correspond to surface speeds of 9.4 and 28.2 meters per second (370 and 1110 in./sec), respectively. Comparisons using the other test lubricants showing similar results could be made.

The EHD film thickness formula used for the above comparison comes from the isothermal theory of Cheng [24]. Chen's formula is considered representative of those EHD formulas which predict nominal film thickness for bodies in line contact. This equation can be written

$$\frac{h_c}{R'} = 1.47 \left(\frac{\alpha_o u}{R'} \right)^{0.74} \left(\frac{p_{Hz}}{E'} \right)^{-0.22} \quad (11)$$

In keeping with Cheng [18], a correction factor of 0.8 has been applied to the above equation to adjust Cheng's center film thickness to minimum film thickness. The limitations of this type of an adjustment are recognized. The necessity for it underscores the uncertainties attendant with simplified film thickness formulas as applied to minimum film thickness calculations.

The values of α utilized for this computation are based upon the numerical integration of the reciprocal viscosity-pressure isotherm as published in [20]. They appear in Table 5 as a function of temperature.

Film thickness correlation. - The results of the present analysis are compared with the X-ray test data at selected temperatures in Fig. 6. In this figure, nondimensional minimum film thickness is plotted as a function of maximum Hertz pressure at several rolling speeds. The numer-

ical results of this comparison are summarized in Table 6. It is evident that the present film thickness formulation (eq. (10)), although reduced to a very simple form, is still in reasonably good agreement with the measured data for all four test fluids over the full range of test conditions.

The application of the empirical film thickness formula to systems where somewhat different lubrication conditions prevail will be considered next.

Effect of Lubricant on Film Thickness Correlation

The empirical factors utilized in formulating the present correlation have been developed from the experience gained with four test fluids. At present no meaningful generalizations can be made regarding the extension of the present deduced relationship to systems employing different lubricant types or formulations without the benefit of additional experimental information. On the other hand, application of the film thickness correlation to systems utilizing the lubricants under study over similar conditions can be made with reasonable confidence.

Lubrication coefficient K_j . - It may be apparent that the pressure-viscosity coefficient α which is customarily used to characterize the film forming capabilities of a lubricant, apart from the effects of absolute viscosity, is conspicuously absent from the present formulation. In the present model, the role formerly played by α has been fulfilled in part by the lubrication coefficient K_j . That is, a lubricant's film forming capabilities can be ascertained by knowing its K_j and its absolute viscosity μ_0 at a given operating condition. An important distinction between α and K_j , is that α is temperature dependent where K_j

as presently defined is not. Careful examination of test data revealed that the effects of temperature on minimum film thickness are adequately reflected by the variation in absolute viscosity and that the added complication of an additional temperature dependent variable could thus be avoided. Further, the availability of pertinent pressure-viscosity data at elevated temperature under the appropriate shear rate and pressure conditions for film thickness calculation purposes have been generally limited. There has been, however, increasingly more attention directed at obtaining these pressure-viscosity data in recent years [21,26,28,29]. In view of the aforementioned, it is advantageous to dispense with α for the present film thickness model.

Effects of Contact Geometry and Material

Contact geometry. - The present correlation is based exclusively upon measurements made with a single disk geometry chosen to simulate the ball-inner race contact of a 120-mm bore angular-contact ball bearing [30]. The contact between the test disks approaches the condition of line contact with an ellipticity ratio b/a of 5.9 where b and a are the major and minor semi-axes of the contact ellipse, respectively. The equivalent radius of curvature in the direction of rolling R' for the test disks is 0.915 centimeter (0.36 in.).

It is difficult to extend the results presented herein to different contact geometries with complete assurance without further experimental verification. However, from a practical standpoint, it is speculated that the overall effect of contact geometry on the value of film thickness is minimal. Cheng [24] has theoretically shown that the proportions of the contact ellipse with b/a varying from 1 to 5 have a relatively

mild effect on film thickness. That is, the dependence of film thickness upon mean surface speed u , absolute viscosity μ_0 , and contact stress p_{Hz} changes little as the shape of the contact ellipse varies from point to line contact. Similarly, Archard and Cowking [31] have shown that there is great similarity between the EHD lubrication of point and line contacts. A second factor is that the contacts between the races and the balls or rollers in rolling-element bearings and the contacts between gear teeth normally approximate the line contact case. In view of these considerations, the empirical minimum film thickness formula presented can be used with reasonable certainty for most practical applications without further modifications for small differences in contact geometry.

With regard to the size of the contacting elements, elastohydrodynamic theory indicates that film thickness is moderately dependent upon the contacting elements' equivalent radius of curvature in the rolling direction R' [1]. The current film thickness formula implies that film thickness is a function of R' to the 0.38 power at a given contact stress level. This value is in approximate accord with conventional EHD theory. (See eq. (11) where $h_{min} \propto R'^{0.26}$ at equal p_{Hz} .) However, it is recognized that the sensitivity of h_{min} to R' in the heavy load regime remains to be established experimentally. Until such time, minimum film thicknesses forecasted by equation (10) will be most successful for those systems in which the R' of the contact approximates that of the system under study.

Material. - The effects of material properties in terms of Young's modulus upon film thickness at high contact pressures (up to 3.45×10^9 N/m² (5×10^5 psi) maximum Hertz stress) has been demonstrated by experi-

ment [32] to be minimal. These tests which confirm theoretical expectations were conducted by Cohar [32] utilizing interferometry to measure the film generated between a rolling steel ball and a flat glass plate. It is anticipated that the choice of materials other than steel will not appreciably alter the form of equation (10). However, caution must once again be exercised if the elastic properties of the material of interest are markedly different from those of steel.

CONCLUDING REMARKS

The film thickness relationship developed herein represents an initial attempt at empirically modeling the effects of high contact stress (above $1.04 \times 10^9 \text{ N/m}^2$ (150,000 psi)) on minimum film thickness in an elastohydrodynamic contact. Understandably, this expression, stemming from a data base originating from a single source, will require additional refinements to become more universally attractive. However if utilized judiciously, the present film thickness formula will aid the designer in obtaining a more realistic appraisal of the extent of oil film separating his contacting machine elements.

SUMMARY

An empirical elastohydrodynamic (EHD) film thickness formula was developed for heavily loaded contacts based upon X-ray film thickness measurements made with synthetic paraffinic, fluorocarbon, Type II ester and polyphenyl ether test fluids. The film thickness test data covered a wide and practical range of operating conditions.

Maximum Hertz stresses ranged from 1.04×10^9 to $2.42 \times 10^9 \text{ N/m}^2$ (150,000 to 350,000 psi), disk temperatures from 339 to 505 K (150° to 450° F), and mean surface speeds from 9.4 to 37.6 meters per second

(370 to 1480 in./sec). Predicted values of minimum film thickness were compared to X-ray film thickness measurements and contrasted against the results from a well known isothermal EHD analysis. The effects of contact geometry, material and lubricant properties upon predicted film thickness were considered. The following results were obtained:

1. In contrast to commonly accepted elastohydrodynamic theory, the present film thickness formula reflects the high sensitivity of minimum film thickness to contact stress exhibited by the test data under heavy loads. Good agreement with the X-ray test data existed over the full range of test conditions.

2. The measured minimum film thickness data in the case of the synthetic paraffinic and polyphenyl ether fluids was observed to display an enhanced sensitivity to mean surface speed and lubricant absolute viscosity with increasing contact stress.

3. It was judged that the empirical film thickness formula can be used to forecast minimum film thickness under heavy loads with reasonable certainty for rolling-element bearing and gear systems employing the lubricants studied herein, and whose contact geometry approximates that upon which the model is based.

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TABLE 1. - VISCOSITY PROPERTIES OF TEST LUBRICANTS

Lubricant	Kinematic viscosity cs (or $10^{-6} \text{ m}^2/\text{sec}$)		Specific gravity at 478 K (400° F)
	At 311 K (100° F)	At 372 K (210° F)	
Synthetic paraffinic	443	39.7	0.74
Fluorocarbon	298	29.8	1.59
Type II ester	29	5.4	0.85
Polyphenyl ether	358	13.0	1.05

TABLE 2. - EMPIRICAL MINIMUM FILM THICKNESS COEFFICIENTS FOR EQUATION (1)

\bar{P}_{Hz}	$\frac{P_{Hz}}{N/m^2}$ (psi)	Synthetic paraffinic		Fluorocarbon		Type II ester		Polyphenyl ether	
		C_{i1}	N_{i1}	C_{i2}	N_{i2}	C_{i3}	N_{i3}	C_{i4}	N_{i4}
3.09×10^{-3}	1.04×10^9 (1.5×10^5)	28.1	0.58	90.6	0.60	35.8	0.60	58.1	0.60
4.12×10^{-3}	1.38×10^9 (2.0×10^5)	44.6	.61	109.	.61	34.2	.60	7.50×10^3	.81
5.14×10^{-3}	1.72×10^9 (2.5×10^5)	93.5	.65	127.	.62	36.8	.61	5.25×10^4	.90
6.17×10^{-3}	2.07×10^9 (3.0×10^5)	231.	.70	64.6	.61	28.1	.61	5.83×10^5	1.0
6.69×10^{-3}	2.24×10^9 (3.25×10^5)	-----	-----	38.8	.59	-----	-----	-----	-----
7.20×10^{-3}	2.4×10^9 (3.5×10^5)	655.	.76	-----	-----	12.6	.59	-----	-----

TABLE 3. - MEAN VALUES OF THE DIMENSIONLESS SPEED VISCOSITY

PARAMETER EXPONENT \hat{n}_j AND LUBRICANT
 PARAMETERS K_j^* AND K_j

	Synthetic paraffinic	Fluoro-carbon	Type II ester	Polyphenyl ether
\hat{n}_j	0.66	0.61	0.60	0.83
K_j^*	43.8	30.6	10.7	3940.
K_j	18.2	44.8	18.2	24.9

TABLE 4. - VARIATION OF $\psi_{i,j}$ WITH \bar{P}_{Hz} FOR THE FOUR TEST FLUIDS

	k_j	$\psi_{i,j}$				
		$\bar{P}_{Hz} = 3.09 \times 10^{-3}$	$\bar{P}_{Hz} = 4.17 \times 10^{-3}$	$\bar{P}_{Hz} = 5.15 \times 10^{-3}$	$\bar{P}_{Hz} = 6.17 \times 10^{-3}$	$\bar{P}_{Hz} = 7.2 \times 10^{-3}$
Synthetic paraffinic	156	1.0	0.92	0.78	0.61	0.43
Fluorocarbon	109	1.0	.93	.78	.58	----
Type II ester	38	1.0	.92	.79	.62	.44
Polyphenyl ether	1.4×10^4	1.0	.85	.71	.52	----
ψ_s		1.0	0.91	0.77	0.58	0.48

TABLE 5. - PRESSURE VISCOSITY COEFFICIENT α

FOR A SYNTHETIC PARAFFINIC OIL

	$T_0 = 339 \text{ K}$ (150° F)	$T_0 = 422 \text{ K}$ (300° F)
α	$17.1 \times 10^{-9} \text{ M}^2/\text{N}$ ($11.8 \times 10^{-5} \text{ psi}^{-1}$)	$10.8 \times 10^{-9} \text{ M}^2/\text{N}$ ($7.5 \times 10^{-5} \text{ psi}^{-1}$)

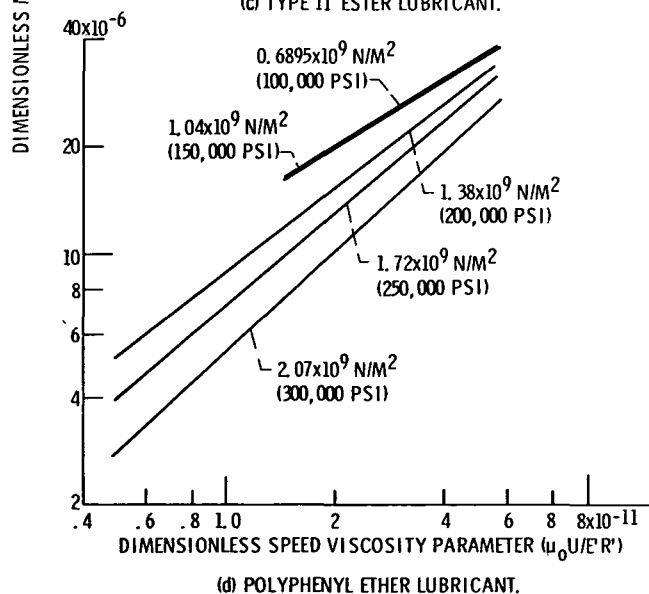
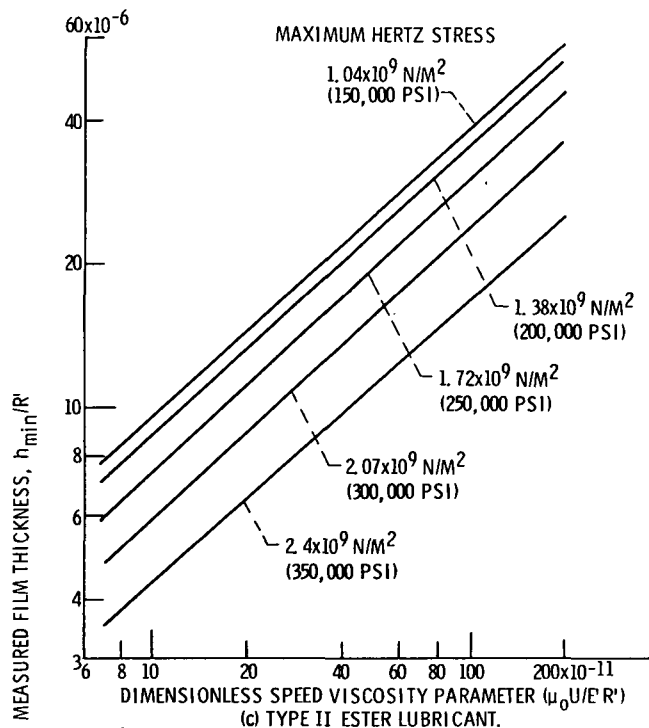
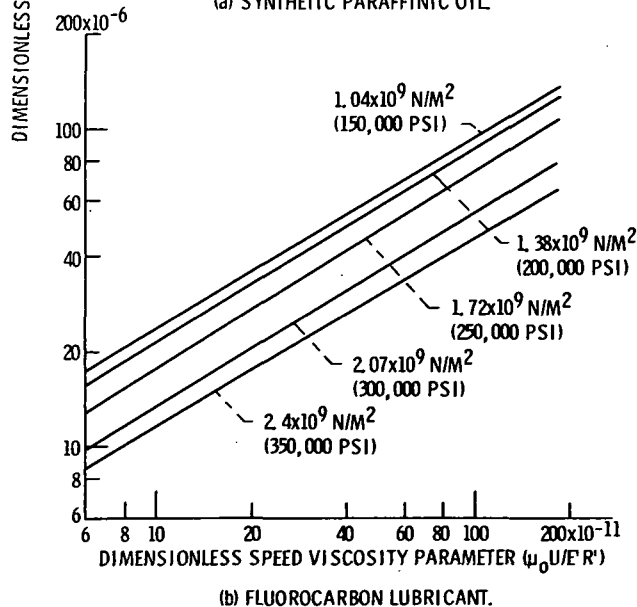
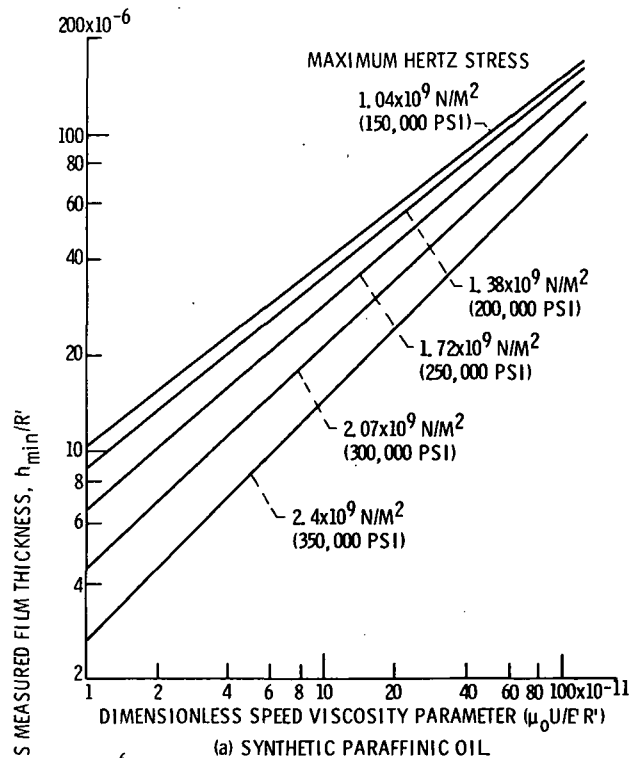


Figure 2 - Effect of maximum hertz stress on the sensitivity of minimum film thickness to changes in speed and viscosity.

Figure 2 - Concluded.

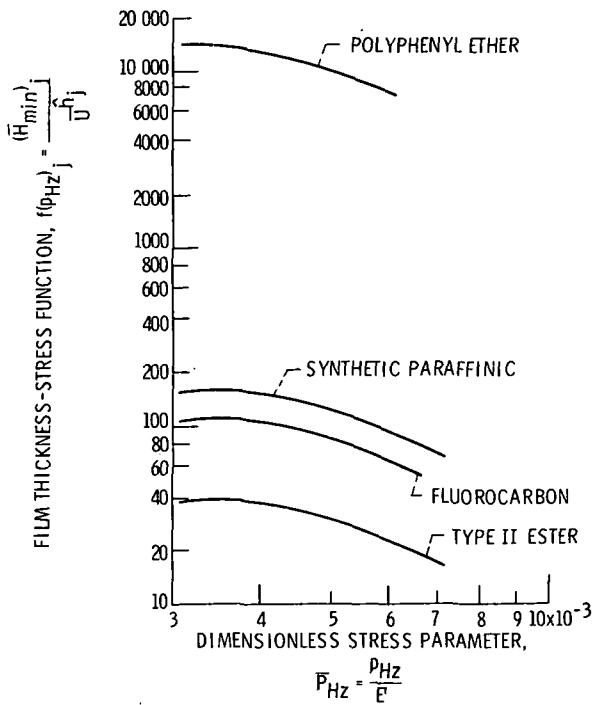


Figure 3. - Effect of maximum hertz stress upon measured minimum film thickness.

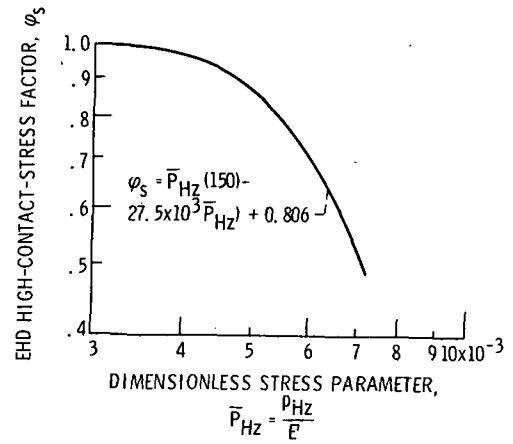


Figure 4. - Variation of EHD high-contact-stress factor with operating contact stress.

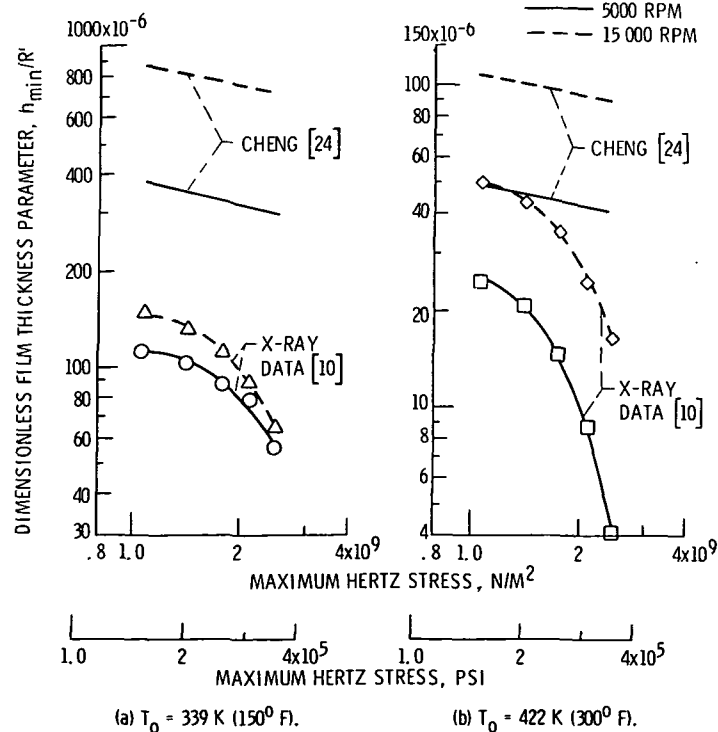


Figure 5. - Comparison of X-ray measured minimum film thickness with isothermal theory for a synthetic paraffinic oil.

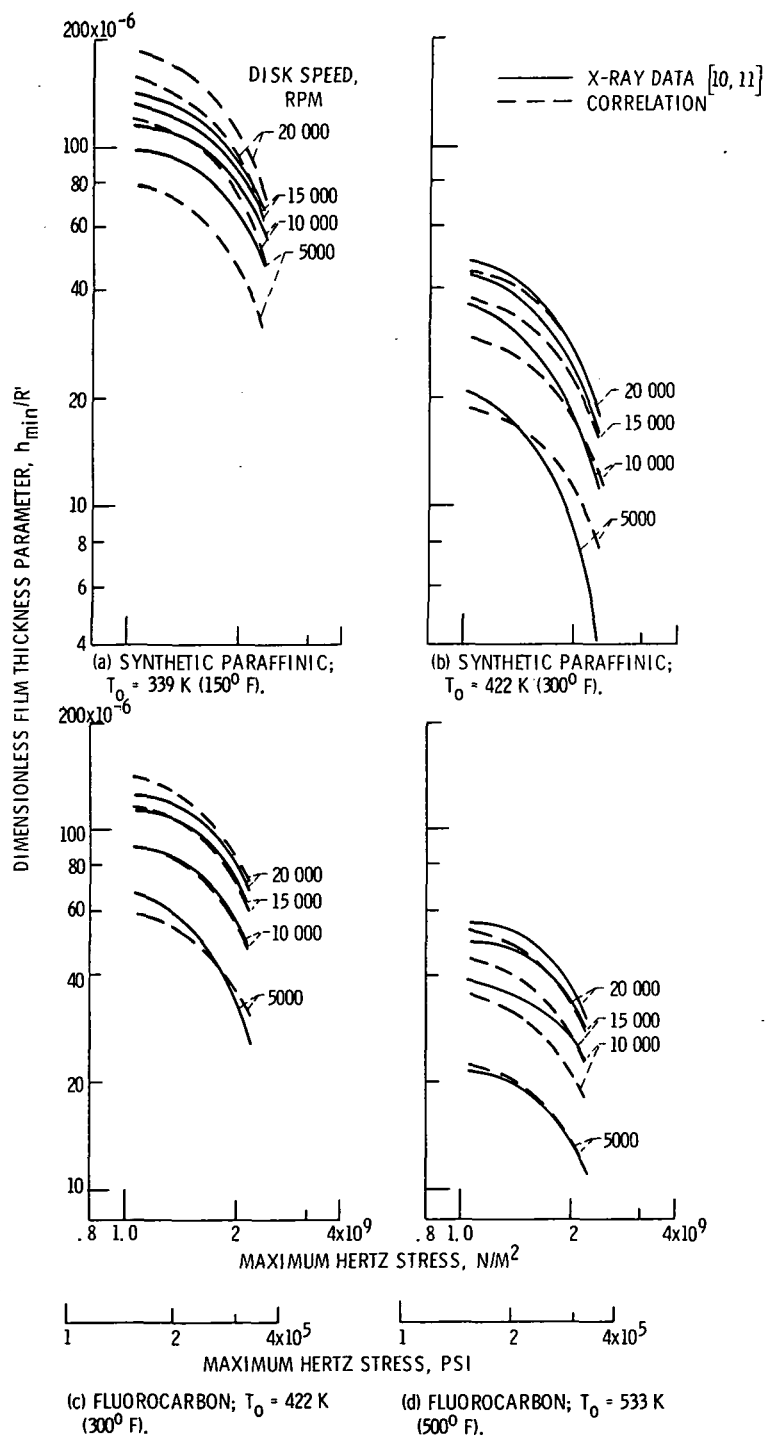


Figure 6. - Comparison between predicted minimum film thickness and X-ray test data [10, 11].

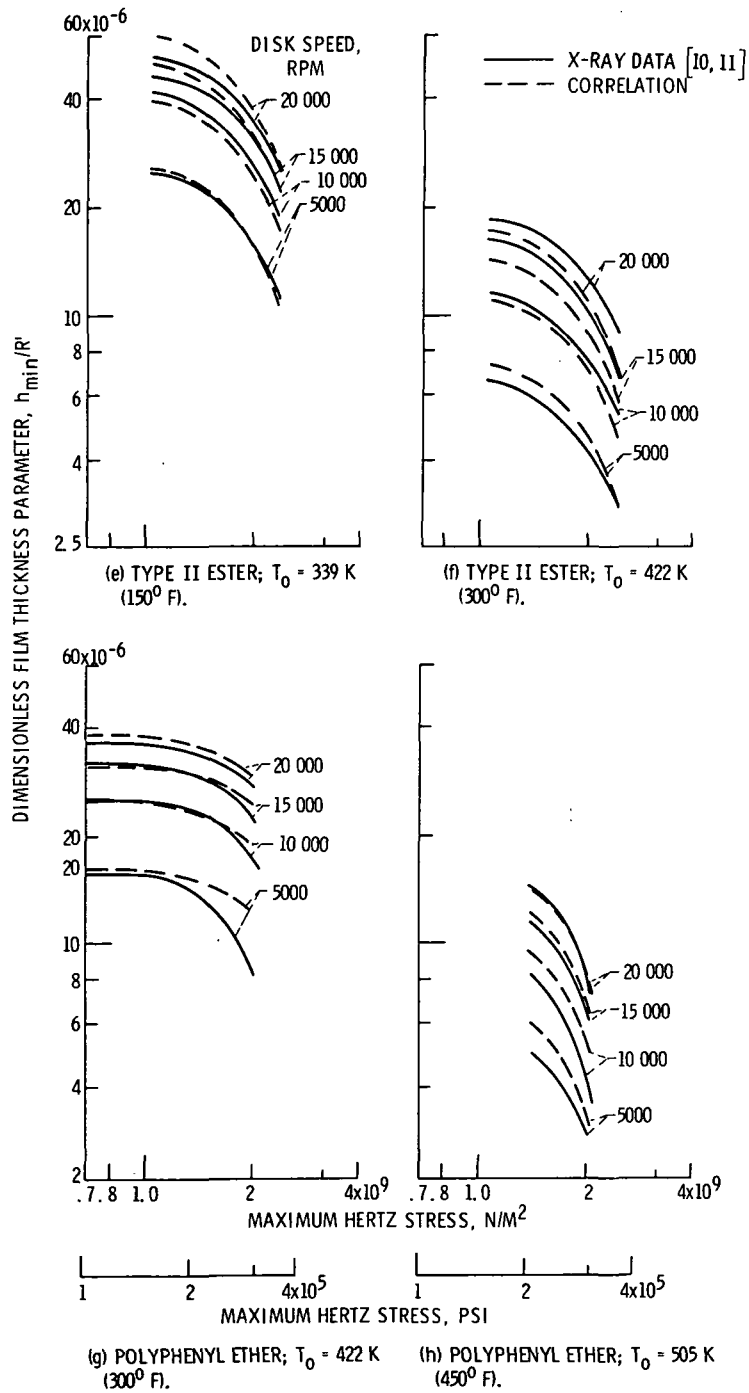


Figure 6. - Concluded.