

A KINEMATIC EDDY VISCOSITY MODEL INCLUDING THE INFLUENCE OF DENSITY VARIATIONS AND PRETURBULENCE

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SUMMARY

A model for the kinematic eddy viscosity has been developed which accounts for the turbulence produced as a result of jet interactions between adjacent streams as well as the turbulence initially present in the streams. In order to describe the turbulence contribution from jet interaction, the eddy viscosity suggested by Prandtl has been adopted, and a modification has been introduced to account for the effect of density variation through the mixing layer. The form of the modification was ascertained from a study of the compressible turbulent boundary layer on a flat plate. A kinematic eddy viscosity relation which corresponds to the initial turbulence contribution has been derived by employing arguments used by Prandtl in his mixing length hypothesis. The resulting expression for self-preserving flow is similar to that which describes the mixing of a submerged jet.

Application of the model has led to analytical predictions which are in good agreement with available turbulent mixing experimental data.

INTRODUCTION

In the analytical treatment of turbulent shear flows, the local shear stress may be expressed as the product of an eddy viscosity and the local velocity gradient by analogy with the laminar flow representation. However, while the molecular viscosity for laminar flow depends only on the fluid properties, the eddy viscosity is related to the structure of the turbulence in the shear flow. At present, turbulent flow phenomena are not well understood, so that empirical hypotheses are utilized to create a mathematical basis for the investigation of turbulent motion. These phenomenological theories lead to a formulation of the kinematic eddy viscosity (eddy viscosity divided by local density) which may be used with the equations of motion and a suitable equation of state to determine the local time-average conditions throughout a flow field.

The constant exchange coefficient hypothesis for the kinematic eddy viscosity suggested by Prandtl is widely used in analytical studies of the mixing layer formed at the boundary between adjacent fluid streams. In this hypothesis the kinematic eddy viscosity is taken to be proportional to the product of the mixing-layer width and the difference between the velocities at the edges of the mixing layer. Application of this formulation

to the investigation of the mixing of incompressible streams of the same fluid, for example, the classical analysis conducted by Görtler (ref. 1), has yielded results which have been verified experimentally. Recent studies of compressible jet mixing (refs. 2 and 3), however, have shown that Prandtl's relationship is not valid when the fluid density varies through the mixing layer. Moreover, the prediction that turbulent transport will cease when there is no velocity gradient in the flow is inconsistent with the experimental evidence (refs. 4 and 5). In this case, it appears that the initial turbulence of the fluid streams plays an important role in jet mixing. As a result of these findings several new formulations of the kinematic eddy viscosity or equivalent mixing parameter have been recommended (refs. 5 to 9). Unfortunately, the general validity of these new expressions has not been satisfactorily demonstrated.

The present investigation was performed to resolve the discrepancies regarding the effect of density on the kinematic eddy viscosity and the influence of initial jet turbulence. Prandtl's hypothesis is adopted to describe the mixing of isothermal, incompressible streams of the same fluid which results from the turbulence produced by interactions between the streams. The hypothesis is modified to account for density variations through the mixing layer by making use of flat-plate, compressible turbulent boundary-layer information. The contribution of initial stream turbulence to the kinematic eddy viscosity is investigated for a constant turbulent intensity and an intensity which decays in the flow direction. In the latter case, use is made of the initial period decay law which characterizes turbulent flow downstream of grids.

SYMBOLS

a	constant
b	transverse extent of mixing layer, m (ft)
C_1, C_2, C_3	constants
C_D	drag coefficient
D	diameter, m (ft)
d	characteristic dimension of jet nozzle, m (ft)
e	exponent for two-dimensional ($e = 0$) or axisymmetric ($e = 1$) flow
g_c	conversion factor, 32.2 lbf-ft/lbf-sec ²

I	intensity of turbulence defined by equation (33)
ℓ	mixing length, m (ft) (see eq. (2))
ℓ'	fluctuating mean free path of fluid particles, m (ft)
M	Mach number
m	ratio of external stream velocity to centerline velocity
m_1	velocity ratio fixed by turbulence level
N	characteristic dimension of grid, m (ft)
n	ratio of external stream density to centerline density
n_1	density ratio fixed by turbulence level
R_θ	Reynolds number
r	distance from tube centerline, m (ft) (except where indicated)
$r_{1/2}$	half thickness of jet, m (ft)
T	temperature, K (degree R)
U	velocity of stream approaching grid, m/sec (ft/sec)
u	velocity in longitudinal direction, m/sec (ft/sec)
u'	longitudinal velocity fluctuation, m/sec (ft/sec)
u_τ	friction velocity, m/sec (ft/sec) (see eq. (7))
\bar{u}	average velocity defined in equation (24), m/sec (ft/sec)
v'	transverse velocity fluctuation, m/sec (ft/sec)
x	axial distance, m (ft)

x_c	length of core region, m (ft)
x_1	axial distance at which $m = m_1$, m (ft)
y	transverse distance, m (ft) (except where indicated)
\bar{y}	transverse distance from centerline to inner boundary of mixing layer, m (ft)
z	distance from wall, m (ft) (except where indicated)
α_1, α_2	constants in equations (15) and (16)
β	constant in equation (41)
γ	ratio of specific heats
δ	boundary-layer thickness, m (ft)
ϵ	kinematic eddy viscosity, m^2/sec (ft^2/sec)
η_j	mass fraction of species j
θ	centerline decay exponent
κ	constant in equation (3)
ξ	parameter defined by equation (13)
ρ	density, kg/m^3 (lbm/ft^3)
σ	spreading parameter (see eq. (22))
τ	shear stress, N/m^2 (lbf/ft^2)

Subscripts:

\dagger	centerline
e	external stream

j	jet stream
o	denotes value for incompressible submerged jet
r	reference
τ	total condition
w	wall
∞	asymptotic value

KINEMATIC EDDY VISCOSITY MODEL

When a jet discharges into a quiescent or flowing external stream, a distinctive flow field develops which may be divided into two principal regions. (See fig. 1.) In the initial region or "core" region, a mixing layer of finite thickness with a continuous distribution of velocity, temperature, and species concentration forms at the boundary between the two streams. For the idealized system shown in figure 1, the velocities of the jet and external streams u_j and u_e are uniform, the jet nozzle with characteristic dimension d has infinitesimally thin walls, and the pressure is constant throughout the flow. The mixing layer gradually broadens in the direction of flow and ultimately extends to the centerline of the jet at $x = x_c$, which marks the end of the initial region. In the developed region, the velocity on the centerline u_c decreases while the width of the layer continues to increase.

The equations of motion, modified according to the usual boundary-layer assumptions (ref. 10), are used in the analytical treatment of jet mixing. For turbulent jets, the local shear stress is related to a kinematic eddy viscosity ϵ according to the Boussinesq hypothesis

$$\tau_c = \rho \epsilon \frac{\partial u}{\partial y} \quad (1)$$

Thus, once the kinematic eddy viscosity is specified, all local conditions throughout the flow field may be determined for selected boundary conditions.

Prandtl proposed two formulations to characterize the rate of mixing resulting from jet-induced turbulence. In the earlier mixing-length hypothesis, Prandtl suggested the relation

$$\epsilon = \ell^2 \left| \frac{\partial u}{\partial y} \right| \quad (2)$$

in which the mixing length ℓ may be thought of as the transverse extent of an identifiable lump of fluid (i.e., an eddy). In Prandtl's new theory, referred to as the constant exchange coefficient hypothesis, it is assumed that the kinematic eddy viscosity varies in the axial direction only. From considerations of flow similarity, for example, that the ratio of the mixing length to the transverse extent of the mixing layer b is constant throughout the flow, it can be shown that

$$\epsilon = \epsilon(x) = \kappa b |u_t - u_e| \quad (3)$$

where κ is a universal constant in each region of the jet for a given geometry. Analytical studies conducted by Tollmien (ref. 11) with equation (2) and Görtler (ref. 1) with equation (3) produced the velocity distributions through a mixing layer presented in figure 2. As very little difference between the profiles can be discerned, the formulations of equations (2) and (3) can be considered to be equivalent for practical purposes, and hence the ratio of ℓ/b may be considered to be a constant in an incompressible free jet mixing layer. Equation (3) is simpler to use, however, and for this reason it is adopted as the basic relation in the present model.

In any general formulation of the kinematic eddy viscosity it is necessary to consider the turbulence initially present in the streams, that is, the "preturbulence," as well as the turbulence produced as a result of the interactions between the streams. When the velocity ratio $m \equiv \frac{u_e}{u_t}$ differs significantly from unity, the growth of the mixing layer is controlled by jet interaction since shearing stresses of large magnitude occur, which induce high-intensity turbulent activity. As m approaches unity, however, the preturbulence contribution may become the dominant factor. Thus, in a jet mixing situation where $m_{x=0} \ll 1$, the initial spread of the mixing layer depends on jet-induced turbulence, while far downstream, after appreciable decay of the centerline velocity, the effect of preturbulence may become important. It should be noted, however, that when mixing aids such as vortex generators or mixing "fingers" are present in the nozzle supplying the fluids or when the fluids to be mixed are introduced into the main flow through angled injectors, the preturbulence contribution to the eddy diffusivity might predominate even for $m \ll 1$. Two classes of mixing phenomena may therefore be considered to stem from the two sources of turbulence identified above: (1) The turbulence level is initially set by the jet turbulence and then gradually decays to the background or "preturbulence" level; (2) the jet turbulence level is below the background level at $x = 0$ and the "preturbulence" is the controlling parameter over the full extent of the mixing region.

Effect of Density Variation

As noted in the introduction, the results presented in references 2 and 3 indicate that equation (3) is not suitable when there is a significant density variation in the mixing

layer. Unfortunately, the available compressible mixing layer data are not sufficiently extensive or reliable to indicate how equation (3) should be modified for this effect. However, since the free-shear mixing layer and the wake or outer region of the turbulent boundary layer are described by the same system of equations, it is suggested that the influence of compressibility for zero pressure gradient can be ascertained by examining the behavior of the compressible boundary layer on a flat plate for which comprehensive information exists. In order to implement this approach, an expression must be obtained which relates ϵ to parameters which characterize the boundary layer. Thus, if equations (1) and (2) are combined to eliminate the velocity gradient, the result is

$$\epsilon = \ell \left(\frac{g_c \tau}{\rho} \right)^{1/2} \quad (4)$$

which may be written in the expanded form

$$\epsilon = \delta \left(\frac{\ell}{\delta} \right) \left(\frac{\tau}{\tau_w} \right)^{1/2} \left(\frac{g_c \tau_w}{\rho_w} \right)^{1/2} \left(\frac{\rho_w}{\rho} \right)^{1/2} \quad (5)$$

In equation (5) δ is the boundary-layer thickness and the subscript w denotes conditions at the plate surface. It was shown in reference 12 that neither the ratio of the local shear stress to wall shear stress nor the parameter ℓ/δ displays a great sensitivity to either the Mach number or the Reynolds number. Therefore, it may be concluded that

$$\frac{\epsilon}{\delta u_\tau (\rho_w/\rho)^{1/2}} = f(y/\delta) \quad (6)$$

where the friction velocity u_τ is given by

$$u_\tau = \left(\frac{g_c \tau_w}{\rho_w} \right)^{1/2} \quad (7)$$

Equation (6) implies that the normalization of the kinematic eddy viscosity with the product $\delta u_\tau (\rho_w/\rho)^{1/2}$ results in a universal parameter, which is a function of only the dimensionless transverse distance through the boundary layer. This has been verified by using the calculation procedure described in reference 12, with the exception that a value for the exponent on the density ratio ρ_w/ρ of 0.4 fits the data better than the derived value of 0.50. Results of the calculation are given in figure 3 for Mach numbers of 0, 2.0, and 5.0.

By applying the analogy between the wake region of the turbulent boundary layer and the turbulent mixing layer, it is assumed that equation (6) may be rewritten for the mixing layer in the following form:

$$\frac{\epsilon}{bu_{\tau,r}(\rho_r/\rho)^{0.4}} = f_1(y/b)$$

where ρ_r and $u_{\tau,r}$ are jet reference conditions and b is the extent of the mixing layer in the direction normal to the jet axis. Furthermore, since the kinematic eddy viscosity is independent of y for an incompressible jet, or shear layer, the form of f_1 is then known and the relation may be written as

$$\frac{\epsilon}{bu_{\tau,r}(\rho_r/\rho)^{0.4}} = \text{Constant} \quad (8)$$

for both compressible and incompressible mixing layers. Assuming the existence of velocity and temperature profile similarity, it may be deduced that

$$u_{\tau,r}^2 = \frac{g_c \tau_r}{\rho_r} = \epsilon \left. \frac{\partial u}{\partial y} \right|_r \sim \epsilon \frac{|u_{\xi} - u_e|}{b} \quad (9)$$

Thus,

$$\epsilon = \kappa \left(\frac{\rho_r}{\rho} \right)^{0.8} b |u_{\xi} - u_e| \quad (10)$$

which generalizes equation (3) to include density variations. It still remains, however, to specify the ratio ρ_r/ρ . It should be noted that this ratio must reduce to unity when there is no density variation through the mixing layer and that it must reflect the experimentally observed decrease in the mixing rate when the Mach number of a supersonic, submerged jet is increased at constant static temperature (refs. 13 and 14). A possible representation is

$$\frac{\rho_r}{\rho} = f^* \frac{\rho_{\xi} + \rho_e}{2\rho_{\xi}} \quad (11)$$

where f^* is an empirical parameter equal to unity for incompressible jet mixing, which may vary with Mach number for example (ref. 15). Introducing equation (11) into equation (10) leads to

$$\epsilon = \kappa \left(f^* \frac{\rho_{\xi} + \rho_e}{2\rho_{\xi}} \right)^{0.8} b |u_{\xi} - u_e| \quad (12)$$

Comparison of Modified Formulation With Experimental Jet Spreading Data

The spreading parameter σ is usually reported in the results of jet mixing studies. This parameter may be thought of as a scaling factor in the transformation from the x-y physical coordinate system into a system with the single independent variable ξ defined as

$$\xi \equiv \sigma \frac{y - \bar{y}}{x} \quad (13)$$

Under conditions of profile similarity, there is a value of σ that will allow all velocity profiles

$$\frac{u - u_e}{u_{\xi} - u_e} = F(\xi) \quad (14)$$

to be collapsed to a single curve.

Another consequence of profile similarity is the existence of a unique relationship between the spreading parameter σ and the extent of the mixing region b for a given flow situation. The extent of the mixing region may be defined as the transverse distance between the points at which

$$u - u_e = \alpha_1(u_{\xi} - u_e) = u_1 - u_e \quad (15)$$

and

$$u - u_e = \alpha_2(u_{\xi} - u_e) = u_2 - u_e \quad (16)$$

where α_1 and α_2 are arbitrary but universal constants and U_1 and U_2 are the velocities at the extrema of the mixing region located at y_1 and y_2 , respectively.

From equation (14) (the similarity law) it follows that

$$\alpha_1 = F(\xi_1) = F\left(\sigma \frac{y_1 - \bar{y}}{x}\right) \quad (17)$$

and

$$\alpha_2 = F(\xi_2) = F\left(\sigma \frac{y_2 - \bar{y}}{x}\right) \quad (18)$$

From the definition of the extent of the mixing region,

$$b = y_1 - y_2 = (y_1 - \bar{y}) - (y_2 - \bar{y}) \quad (19)$$

and hence

$$\frac{\sigma b}{x} = \sigma \frac{y_1 - \bar{y}}{x} - \sigma \frac{y_2 - \bar{y}}{x} = \xi_1 - \xi_2 \quad (20)$$

Therefore, for any arbitrary nondimensional profile function F , it is seen that

$$\frac{\sigma b}{x} = F^{-1}(\alpha_1) - F^{-1}(\alpha_2) = \text{Constant} \quad (21)$$

where F^{-1} is the inverse of the profile function of equation (14) and the constant depends only on the form of the profile function once the values of α_1 and α_2 have been chosen. Hence, it is seen that the spreading parameter is proportional to the cotangent of the spreading angle of the mixing region, and if σ is a constant, then b must vary linearly with x .

Investigations of incompressible jet mixing systems have shown that the lateral extent of the mixing region b does vary linearly with x , for example, in the core region, or in a fully developed region, where the external stream is quiescent. It may be further shown that in this case the kinematic eddy viscosity is related to the spreading parameter by the equation

$$\sigma = \sqrt{\frac{2\bar{u}x}{4(1-e)\epsilon}} \quad (22)$$

and hence the spreading parameter is proportional to the square root of the turbulent Reynolds number of the jet, which is also a constant. In equation (22) $e = 0$ for a two-dimensional jet, $e = 1$ for an axisymmetric jet, and \bar{u} is the characteristic velocity given as

$$\bar{u} = \frac{u_\epsilon + u_e}{2} \quad (23)$$

For the present consideration of density variation through the mixing layer, it is reasonable to extend the validity of equation (22), at least for moderate variations in density, by employing a suitable definition of the characteristic velocity, and the kinematic eddy viscosity determined in equation (12). The following expression for the characteristic velocity proposed by Yakovlevskiy (ref. 16) for the range of $0.3 \leq \frac{\rho_e}{\rho_\epsilon} \leq 2$ will be adopted:

$$\bar{u} = \frac{\rho_e u_e + \rho_\epsilon u_\epsilon}{\rho_e + \rho_\epsilon} \quad (24)$$

Then, introducing equations (12) and (24) into equation (22) and defining $m \equiv \frac{u_e}{u_t}$ and $n \equiv \frac{\rho_e}{\rho_t}$, result in

$$\sigma = \left[\frac{(1 + mn) \left(\frac{1 + n}{2} \right)^{-1.8}}{4(1-e)_{kf}^{*0.8} \left(\frac{b}{x} \right) |1 - m|} \right]^{1/2} \quad (25)$$

which pertains to compressible jet mixing flows in which b/x is constant.

Typical experimental velocity profiles for incompressible and compressible jets mixing with quiescent and moving external stream are plotted from references 13, 17, and 18 in figure 4. The values of σ used in figure 4 were determined by matching the slopes of the experimental velocity profiles with the slope of the profile for the incompressible submerged jet at $\frac{u - u_e}{u_t - u_e} = 0.5$. As can be seen, the choice of a suitable value of σ for each mixing flow system results in essentially exact coincidence of all velocity profiles, thus lending validity to the similarity assumption of equation (14) upon which the validity of equation (25) rests. Equation (25) may be put into a more useful form by forming the ratio σ/σ_0 , where σ_0 is the spreading parameter for an incompressible, submerged jet (i.e., $m = 0$, $n = 1$) having a value of approximately 11.0 in the core region (ref. 17). This results in the expression

$$\frac{\sigma}{\sigma_0} = \frac{(1 + mn)}{\left(\frac{1 + n}{2} \right)^{1.8} |1 - m| f^{*0.8}} \quad (26)$$

For an incompressible jet discharging into a moving external stream, $n = 1$ and $f^* = 1$, and hence equation (26) reduces to

$$\frac{\sigma}{\sigma_0} = \frac{1 + m}{|1 - m|} \quad (27)$$

which is in general agreement with experimental measurements taken from references 16, 17, 19, and 20. (See fig. 5.) For compressible submerged jets (i.e., $m = 0$),

$$\frac{\sigma}{\sigma_0} = \left(\frac{1 + n}{2} \right)^{-1.8} (f^*)^{-0.8} \quad (28)$$

which may be written as

$$\frac{\sigma}{\sigma_0} = (f^*)^{-0.8} \left(\frac{1 + \frac{\gamma_{\phi} - 1}{2} M_{\phi}^2}{1 + \frac{\gamma_{\phi} - 1}{4} M_{\phi}^2} \right)^{1.8} \quad (29)$$

for constant static pressure and molecular weight throughout the mixing layer. Equation (29) is compared with data from references 11, 13, 14, 17, and 21 to 27 in figure 6 for various assumed variations of f^* with jet Mach number. A value of unity for f^* over the Mach number range 0 to 3.0 appears to result in fair agreement with most of the data points. In particular, the choice of $f^* = 1$ is predictive of the comprehensive experimental studies of Maydew and Reed (ref. 13) and Olson and Miller (ref. 14). Accordingly, the parameter f^* will be taken as unity in the remainder of this work.

Experimental data from two-stream mixing studies in which there exists a density variation through the mixing layer provide the most important test of equation (26). Such data, obtained from references 28 and 29, are compared with equation (26) in figure 7. The agreement between measured and calculated values is very satisfactory except for velocity ratios greater than about 0.50. This discrepancy will be investigated in some detail in subsequent sections. A listing of the test conditions and jet widths from references 29 and 30 as well as the calculated results from equation (26) is also given in table I.

Effect of Initial Turbulence

As a result of recent experimental studies, it has been suggested (ref. 5) that initial turbulence becomes the controlling factor in jet mixing at values of the velocity ratio m near unity. This breakdown of the jet interaction mechanism, exemplified by equation (12), is also apparent from some of the data presented in figures 5 and 7 for $m \geq 0.4$, which deviate from the analytical result (eq. (26)). When preturbulence controls, it appears that the spreading parameter becomes independent of m , at least for $n = 1.0$, so that equation (26) is no longer valid.

For the purpose of developing a formulation of the kinematic eddy viscosity for the preturbulence mechanism, it is convenient to begin with the basic expression (ref. 30)

$$\epsilon = -\overline{v'l'} \quad (30)$$

Equation (30) relates the kinematic eddy viscosity to a parameter associated with the eddy size of the turbulence field l' and the transverse velocity fluctuation v' . In the spirit

of Prandtl's mixing length hypothesis (ref. 20), it is assumed that the mean of the product of fluctuating quantities is proportional to the product of the means of the absolute values of these quantities, that is,

$$\epsilon = -a \overline{|\mathbf{v}'| \cdot |\ell'|} \quad (31)$$

with $0 < a \leq 1$ ($a \neq 0$). Although nothing is known about the numerical parameter a , it may be inferred that it is related to a correlation factor which is descriptive of the turbulent field. As shown in reference 30, $|\overline{\ell'}|$ may be taken to be proportional to the mixing length, which for similar flows is also proportional to the width of the mixing region. Hence,

$$e \sim bI\bar{u} \quad (32)$$

in which the turbulent intensity I is defined as

$$I \equiv \frac{|\overline{\mathbf{v}'|}}{\bar{u}} \quad (33)$$

which for situations involving isotropic turbulence is identical with the conventional definition of intensity and where \bar{u} is given by equation (23). Noting that the longitudinal gradient of the mixing layer width varies directly with the turbulent intensity (ref. 20), as in the case of jet-induced turbulence, then,

$$b = b_i + C_1 \int_{x_i}^x I dx \quad (34)$$

It follows from equation (32) that

$$\epsilon = C_2 I \bar{u} \left(b_i + C_1 \int_{x_i}^x I dx \right) \quad (35)$$

where C_2 is a constant.

In general, the turbulence will decay from some initial value starting a small distance downstream from $x = 0$. The decay will continue with distance in the flow direction until a value of intensity commensurate with the background turbulence level is attained.

An interesting special case of equation (35) results if it is assumed that the mean absolute value of the transverse velocity fluctuation always varies in direct proportion to the average flow velocity. In this event, that is,

$$|\overline{v'}| \sim \overline{u}$$

it follows that

$$I = I_{x=0}$$

and

$$b \sim x$$

for $b_i = 0$ at $x = 0$. Introduction of this assumption implies that the shear flow induced by the preturbulence is self-preserving, in that the distribution of the nondimensional turbulent shear stress across the shear region is similar at any cross section. The turbulent (Reynolds) stress is given by

$$\frac{\rho_c \tau}{\rho \overline{u}^2} = -\frac{\overline{u'v'}}{\overline{u}^2} \sim \frac{|\overline{u'}| \cdot |\overline{v'}|}{\overline{u}^2}$$

but

$$|\overline{u'}| \sim |\overline{v'}|$$

following Prandtl (ref. 31). Therefore,

$$\frac{\rho_c \tau}{\rho \overline{u}^2} \sim I_{x=0}^2$$

as required. Verification of this behavior for plane jets in most of the developed region is provided by measurements presented in reference 32.

The spreading parameter for a constant turbulent intensity is obtained by taking $I = I_{x=0}$ and combining equations (22) and (35) with the result

$$\sigma = \left\{ (0.5)^4 (1-e) C_2 I_{x=0} \left[\frac{b_i}{x} + C_1 I_{x=0} \left(1 - \frac{x_i}{x} \right) \right] \right\}^{-1/2} \quad (36)$$

which is independent of the local velocity ratio. For a free jet where the mixing is initially controlled by jet-induced turbulence, there must exist some distance $x = x_1$, at which the velocity ratio attains a value m_1 and the spreading parameters given by equations (25) and (36) must coincide. Thus,

$$C_2 I_{x=0} = \kappa \frac{|1 - m_1|(1 + n_1) \left(\frac{1 + n_1}{2}\right)^{0.8}}{1 + m_1 n_1} \quad (37)$$

so that the appropriate value of the velocity ratio m_1 at which the mixing becomes controlled by the preturbulence mechanism may be obtained, in principle, from the main-stream turbulence level. The mixing downstream of the position x_1 is controlled by the preturbulence mechanism, and the appropriate kinematic eddy viscosity relation is obtained by combining equations (35) and (37), that is,

$$\epsilon = \kappa \left(\frac{1 + n_1}{2}\right)^{0.8} b |1 - m_1| u_{\text{t}} \left[\left(\frac{1 + n_1}{1 + n}\right) \left(\frac{1 + mn}{1 + m_1 n_1}\right) \right] \quad (38)$$

It is interesting to note that equation (38) corresponds to the expression describing the mixing of an incompressible submerged jet since the factor in the brackets is only a slowly varying function of m and n .

Far downstream from the origin of jet mixing, m and n approach unity and the asymptotic kinematic eddy viscosity for the turbulent jet becomes

$$\epsilon_{n=m=1} = \epsilon_{\infty} = \kappa b |1 - m_1| u_{\text{t}} \left(\frac{1 + n_1}{1 + m_1 n_1}\right) \left(\frac{1 + n_1}{2}\right)^{0.8} \quad (39)$$

This expression may be compared with the asymptotic relation given in reference 2, that is,

$$\epsilon_{\infty} = 0.04 r_{1/2} u_{\text{t}} \quad (40)$$

which was used to correlate data from a study involving the mixing of coflowing hydrogen and air at nearly equal stream velocities. In equation (40) $r_{1/2}$ is the distance between the jet centerline and the transverse position at which $u - u_e = 0.5(u_{\text{t}} - u_e)$. Since $b \approx 2r_{1/2}$, equation (40) may be rewritten as

$$\epsilon_{\infty} \approx 0.02 b u_{\text{t}}$$

which is identical in form to equation (39).

Initial turbulence levels are generally low in jets which are produced by expanding a gas through a nozzle. However, if there are blockages in the flow as in the case of a ducted fan engine or if mixing aids such as vortex generators are deliberately introduced into the flow, high levels of turbulence can result. In these cases, the initial turbulence

level cannot be sustained in the jet flow, and it is expected that $|\bar{v}'|$ will decay with distance downstream. Although the nature of this decay is not known, it is of interest to apply the initial period decay law found for isotropic turbulence downstream of grids to the present problem. The decay law

$$\frac{U^2}{v'^2} = \frac{\beta}{C_D} \left(\frac{x}{N} - \frac{x_0}{N} \right) \quad (41)$$

has been verified by a number of authors including Batchelor and Townsend (ref. 33) for grid Reynolds numbers from 640 to 5600, Webb (ref. 34) for grid Reynolds numbers from 2000 to 12 000 at various pressures with argon, helium, and air, and Kistler (ref. 35) for high Reynolds numbers. In equation (41) β is an absolute constant, $\frac{1}{2} \rho U^2 C_D$ is the drag of unit cross-sectional area of the grid, and N is an effective unit of length, which depends on the spacing of the grid elements. The ratio x_0/N , which has a value between 5 and 15, corresponds to the station at which the decay begins.

In order to adapt the decay law for use in equation (35), it is convenient to use the following modified form of equation (41):

$$\frac{1}{v'^2} - \frac{1}{v'^2_{x=0}} = \frac{\beta x}{U^2 C_D N} \quad (42)$$

or

$$\frac{v'^2}{v'^2_{x=0}} = \left(1 + \frac{\beta x v'^2_{x=0}}{U^2 C_D N} \right)^{-1} \quad (43)$$

Furthermore, by assuming that

$$\frac{v'^2}{v'^2_{x=0}} \sim \frac{|\bar{v}'|^2}{|\bar{v}'|^2_{x=0}} \quad (44)$$

the eddy diffusivity for the preturbulence mechanism following the decay law of equation (42) takes the form

$$\epsilon = C_4 \left(\frac{N U^2 C_D}{\beta} \right) \frac{\int_0^\theta \frac{d\theta}{\bar{u}(1+\theta)^{1/2}}}{(1+\theta)^{1/2}} \quad (45)$$

where b_i has been taken to be equal to zero, and

$$\theta \equiv \frac{\beta x v'_{x=0}{}^2}{U^2 C_D N} \quad (46)$$

The integral in equation (45) may be evaluated immediately to yield

$$\epsilon = 2C_4 \left(\frac{NU^2 C_D}{\beta \bar{u}} \right) \left[1 - (1 + \theta)^{-1/2} \right] \quad (47)$$

for the core and asymptotic regions, as \bar{u} is constant for these cases. Moreover, since $\beta \approx 100$ and $C_D \approx 1$ (ref. 33) and expected turbulence levels are such that $0.05 \leq \frac{v'^2}{U^2} \leq 1.0$, θ becomes large relative to unity a small number of grid spacings downstream of the initial station. In this event,

$$\epsilon = 2C_3 \left(\frac{NU^2 C_D}{\beta \bar{u}} \right) = \text{Constant} \quad (48)$$

in the core region, while far downstream of the initial station

$$\epsilon \rightarrow \epsilon_\infty = 2C_3 \left(\frac{NU^2 C_D}{\beta u_\epsilon} \right) \quad (49)$$

The kinematic eddy viscosity formulations derived above cannot be verified at this time since pertinent experimental information does not exist. It is interesting to note with respect to equation (48), however, that a constant kinematic eddy viscosity often successfully correlates experimental mixing data (ref. 4).

Application of the Model

The principal results of the preceding analysis are embodied in the three expressions for the kinematic eddy viscosity, equations (12), (38), and (47). The choice of which form to use in a particular mixing study depends on the expected value of m_1 and whether the turbulent intensity in the jet streams decays or remains approximately constant with distance downstream. While a value of m_1 close to unity may be obtained, in theory, in a very carefully designed experiment, values of m_1 between 0.4 and 0.5 are found to be representative of a major portion of the existing mixing data (ref. 20, also figs. 5 and 7).

Some caution should be exercised in the application of the model to flows in which the ratio of external to centerline density is very large compared with unity, for example, the mixing of a central hydrogen jet with an outer air stream when the static temperatures of the two streams are not too different. As the density ratio increases and/or the velocity ratio increases above unity, the rate of mixing increases and ultimately leads to the generation of a significant positive pressure gradient in the initial mixing region and flow reversal of the inner jet (refs. 36 and 37). According to the data presented in reference 38, a back-flow vortex is formed at a momentum flux ratio ρu^2 of 169. Furthermore, characteristics of wakelike flow, for example, centerline velocity initially decreasing, then increasing, are observed for momentum flux ratios greater than about 4. Use of the models for momentum flux ratios much in excess of 4, therefore, is not recommended.

In summary, when the ratio of velocities at the edges of the mixing layer is less than m_1 , equation (12) is used; otherwise, equation (38) or (47) is used. It is recommended that equation (47) be used only in studies in which artificially high levels of turbulence are present because of the introduction of mixing aids into the flow field.

DISCUSSION OF RESULTS – COMPARISON OF THEORY WITH EXPERIMENTAL DATA

The kinematic eddy viscosity model developed in the preceding sections was used with the United Aircraft Research Laboratories mixing-combustion computer program (ref. 38) to generate flow-field information which could be compared with available measurements. A value of m_1 , treated as a program input, indicated that the kinematic eddy viscosity was to be calculated from equation (12) for $m \leq m_1$ and from equation (38) for $m > m_1$. The value of n_1 was calculated in the program at the longitudinal station where $m = m_1$. If the initial velocity ratio exceeded m_1 , n_1 was taken as the ratio of the external stream density to the initial jet density. For the results discussed below, m_1 was taken to be equal to 0.40. The transverse extent of the mixing zone b was calculated according to the method discussed earlier (eqs. (15) and (16)) with $\alpha_1 = 0.95$ and $\alpha_2 = 0.05$ in the core region. Values of κ which were employed are given in table II (refs. 39 and 40).

The Two-Dimensional Shear Layer

Computed velocity profiles at two axial stations for $m = 0.01$ and $m = 0.10$ are compared with Görtler's theoretical profile (ref. 1) in figure 8. The excellent agreement obtained is an indication that the computer program is operating properly.

Mixing of Coaxial Incompressible Jets

Landis (ref. 28) investigated the mixing of a 0.64-cm-diameter (0.25-in.) heated air jet with a coflowing annular air stream at room temperature. In certain tests, small amounts of helium or carbon dioxide were also metered into the central jet. The velocity of the central jet was about 60 m/sec (200 ft/sec) while the velocity of the external stream was varied from 15 to 41 m/sec (50 to 135 ft/sec) to change m . The largest temperature difference between the streams was 180 K (325° R).

Measurements of the longitudinal variation of the centerline velocity ratio for various tests are compared with calculated results in figures 9 and 10. For the data presented in figures 9(a) and (b), the initial velocity ratio is 0.25 and the eddy diffusivity is calculated from equation (12) until $m \geq 0.4$. In figure 9(c) and figures 10(a) and (b), the initial velocity ratio exceeds m_1 , and equation (38) of the model is employed throughout. Although the initial values of m and n are identical for both parts of figure 10, the nonunity value of n for figure 10(a) resulted from a temperature difference of 58 K (105° R) between the jet and external streams, while that in figure 10(b) was caused principally by the addition of helium to the jet. Thus, the use of a density correction to account for temperature and/or concentration variations through the mixing layer appears to be justified.

Subsonic Mixing in a 53-cm-Diameter (21-in.) Tube

In the experiments of reference 41, a Mach 0.3 jet at 733 K (1320° R) was brought into contact with a cold, Mach 0.1 external stream in a duct. The measure of agreement between calculated and experimental velocity and temperature profiles at two axial stations is shown in figures 11 and 12. Both magnitudes and trends are seen to be reproduced accurately.

Mixing of a Submerged Supersonic Free Jet

Eggers (ref. 42) conducted an analytical and experimental study of the mixing of a Mach 2.22 air jet with quiescent air. The axisymmetric jet which issued from a 2.56-cm-diameter (1.007-in.) nozzle was probed at seven axial stations in the core region and 23 axial stations in the developed region.

Predicted and experimental profiles at three stations are shown in figure 13. In order to treat this problem with the existing mixing analysis, it was necessary to assume that the external stream had some velocity. The chosen value of $u_e = 30$ m/sec (100 ft/sec) is thought to be sufficiently small relative to the jet velocity so as not to invalidate the comparison. The predicted mixing region is seen to spread somewhat more rapidly than is indicated by the measurements but the agreement is still considered to be good.

Ducted Mixing of a Supersonic Jet With an Annular Subsonic Jet

Isoenergetic mixing of a central Mach 2.6 air jet with a low-velocity external air stream was investigated in reference 43. Mach number profiles from the cited reference are shown with the computed results in figures 14 and 15. The effect of a shock system on the profile at $x/D = 2.5$ should be noted. Good agreement is obtained at the four axial locations shown over most of the duct cross section. The lower predicted values in the vicinity of the pipe centerline in the downstream profiles are to be expected inasmuch as the wall boundary layer was not accounted for. The analytical results and the data show the interesting phenomenon of the acceleration of the subsonic external stream to supersonic Mach numbers.

Other Examples

The models presented in this paper have been applied extensively so that numerous other comparisons with data are available in the literature. Groves (ref. 44) and Cohen and Guile (ref. 45) have utilized the recommended kinematic eddy viscosity formulations in a treatment of the mixing and combustion of a supersonic central hydrogen jet with an outer supersonic vitiated air stream. The momentum flux ratio of the jet mixing system studied was approximately 1.5. Eggers (ref. 46) studied supersonic hydrogen-air mixing ($nm^2 \approx 2.2$ and 7.7) and found that a kinematic eddy viscosity of the form given by equation (12) satisfactorily correlated his data.

CORRELATION OF EXPERIMENTAL CENTERLINE DECAY RATES

The decay of centerline concentration is generally presented in the form

$$\eta_{j,c} = \left(\frac{x}{x_c} \right)^{-\theta}$$

where x_c is the core length. A correlation of existing data (including that from refs. 47, 48, and 49) emerges within the framework of the ideas presented in this paper, when the observed values of θ are plotted against the initial jet velocity ratio $m_{x=0}$. It is found from figure 16 that θ is approximately unity over the range of velocity ratios where preturbulence predominates independent of density ratio. Over the range of velocity ratios where jet interactions constitute the dominant turbulence producing mechanism, θ is larger than unity and depends on both m and n . These findings are consistent with the jet spreading data of reference 50.

CONCLUSIONS

On the basis of the analysis presented herein and comparison between the analysis and existing data, it may be stated that

1. The proposed model for the kinematic eddy viscosity, involving an extension of Prandtl's constant exchange coefficient hypothesis to account for the effect of density variation through the mixing layer, yields good agreement with measured jet spreading parameters.

2. Transport of heat and mass can occur when the velocities of the jet and external streams are equal as a result of initial turbulence.

3. Additional mixing data are required to provide verification of the kinematic eddy viscosity model. In particular, information concerning the rate of mixing of jets at several different initial turbulence levels would be of value.

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TABLE I.- COMPARISON OF EXPERIMENTAL AND PREDICTED
SPREADING PARAMETERS

m	n	^a (b/x) _{experimental}	(σ/σ_0) _{experimental}	(σ/σ_0) _{calculated}
Landis (ref. 28)				
0.25	1.300	0.185	1.372	1.37
.25	1.087	.165	1.540	1.57
.46	1.087	.095	2.680	2.57
.50	1.300	.106	2.40	2.57
Willis and Glassman (ref. 29)				
0	0.915	0.216	1.18	1.08
.382	.927	.1298	1.96	2.34
.678	.952	.0759	3.35	5.30
0	.842	.1968	1.29	1.16
.279	.852	.1421	1.79	1.97
.496	.876	.0986	2.57	3.20
.319	1.324	.1403	1.81	1.60
.567	1.361	.0836	3.04	3.04
.234	1.218	.1655	1.53	1.394
.415	1.251	.1266	2.00	2.10

^a (b/x)₀ determined to be 0.254 from data. Value suggested by Abramovich (ref. 20) is 0.270.

TABLE II.- SPREADING PARAMETERS FOR CONSTANT DENSITY JET MIXING

	σ	a_{κ}
Core region (e = 0)	11.0 (ref. 17)	0.00764
Developed region:		
Axisymmetric (e = 1.0)	22.6 (ref. 39)	0.0089
Two-dimensional (e = 0)	9.1 (ref. 40)	0.0136

^a Calculated from equations (3) and (22) with $(b/x)_0 = 0.27$ in the core region and $(b/x)_0 = 0.22$ in the developed region as suggested in reference 20.

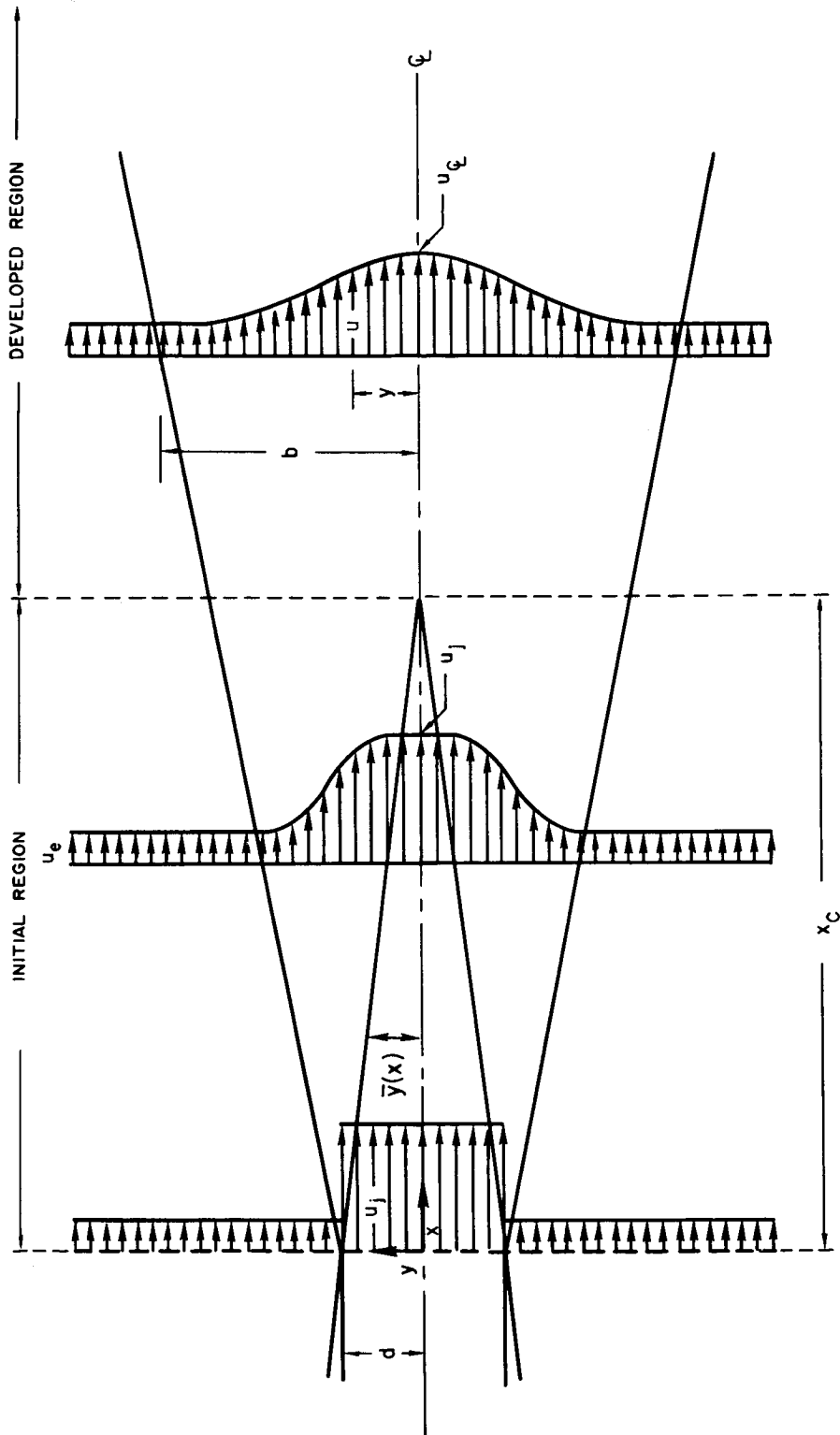


Figure 1.- Schematic of mixing of coflowing streams.

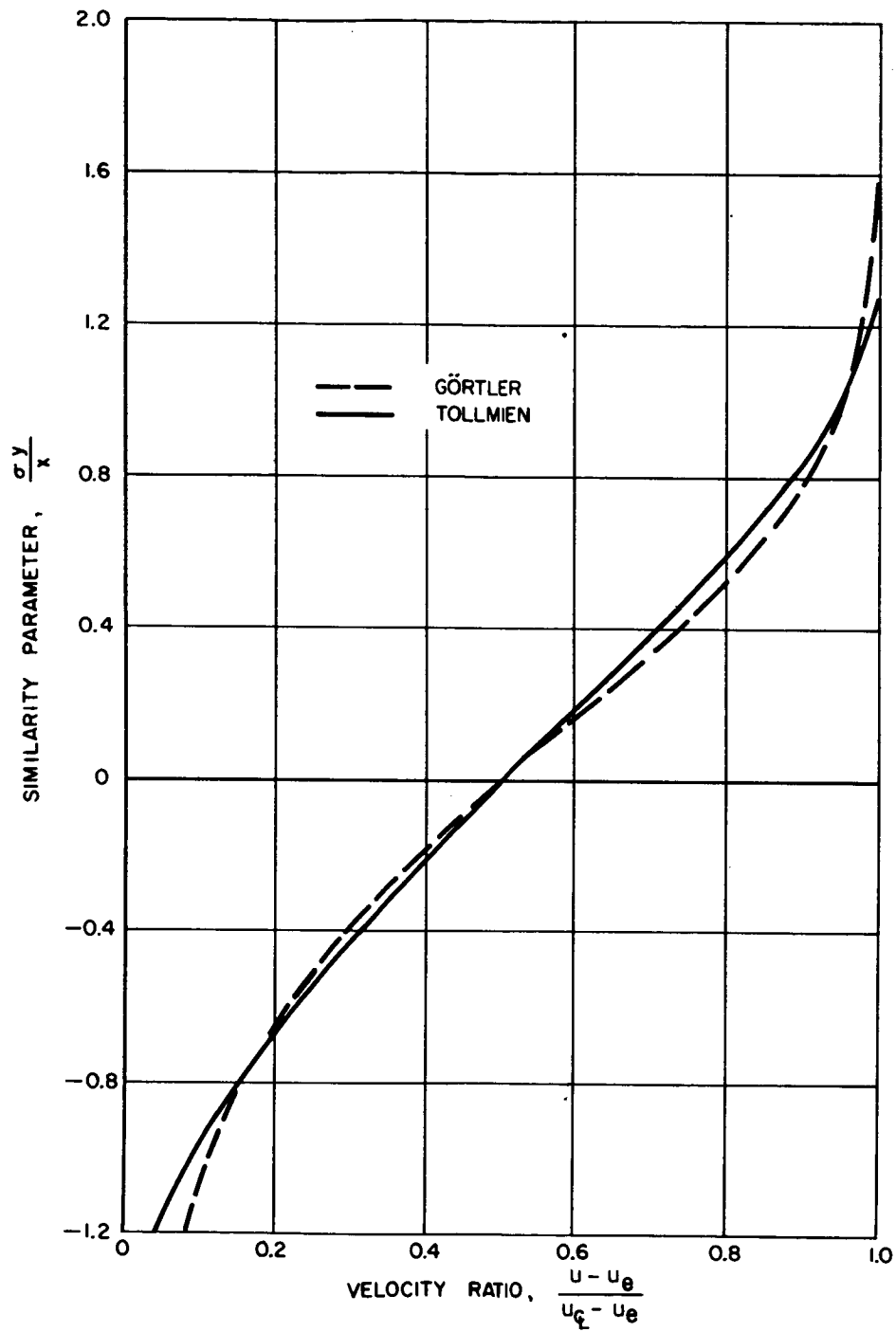


Figure 2.- Theoretical velocity profiles.

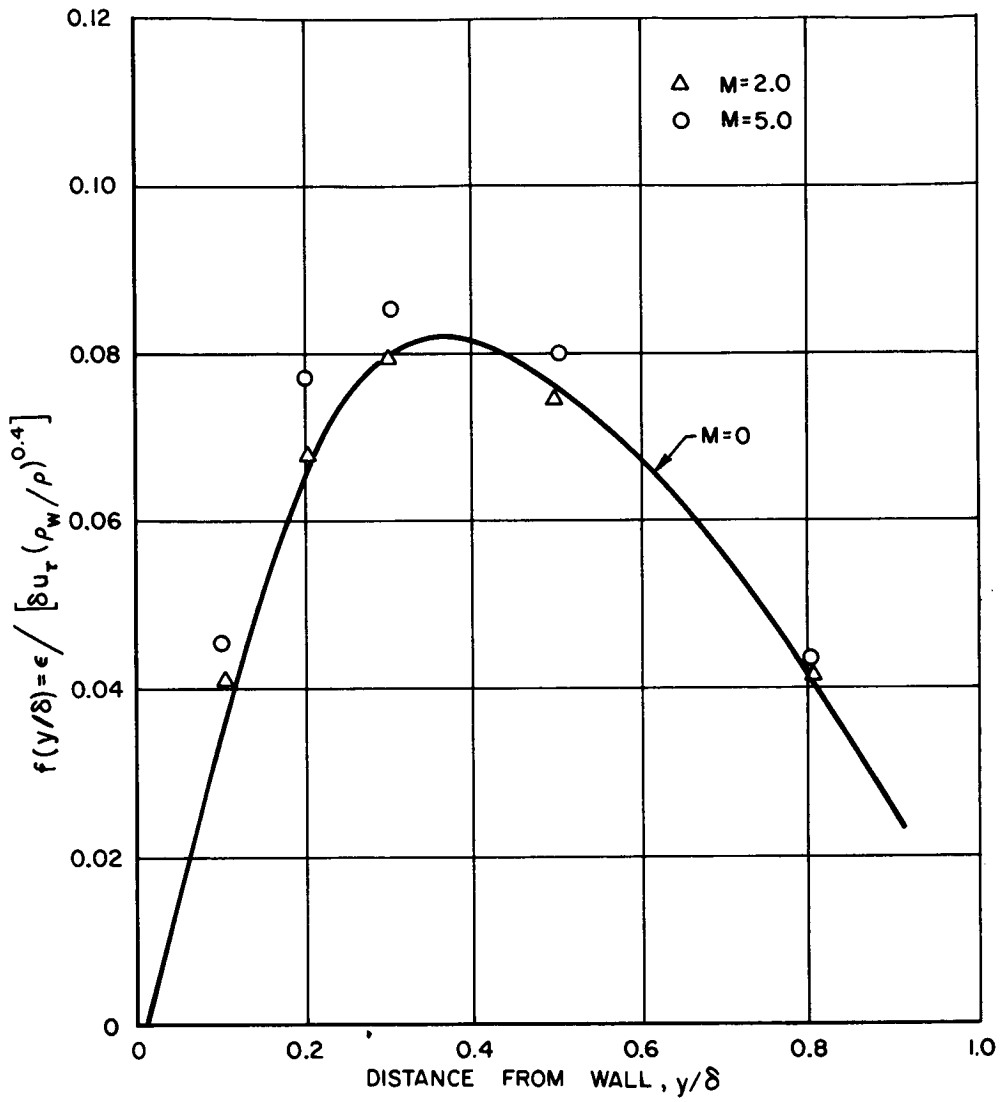


Figure 3.- Correlation of kinematic eddy viscosity for flat-plate, turbulent, compressible, boundary layers. $R_\theta = 10^5$.

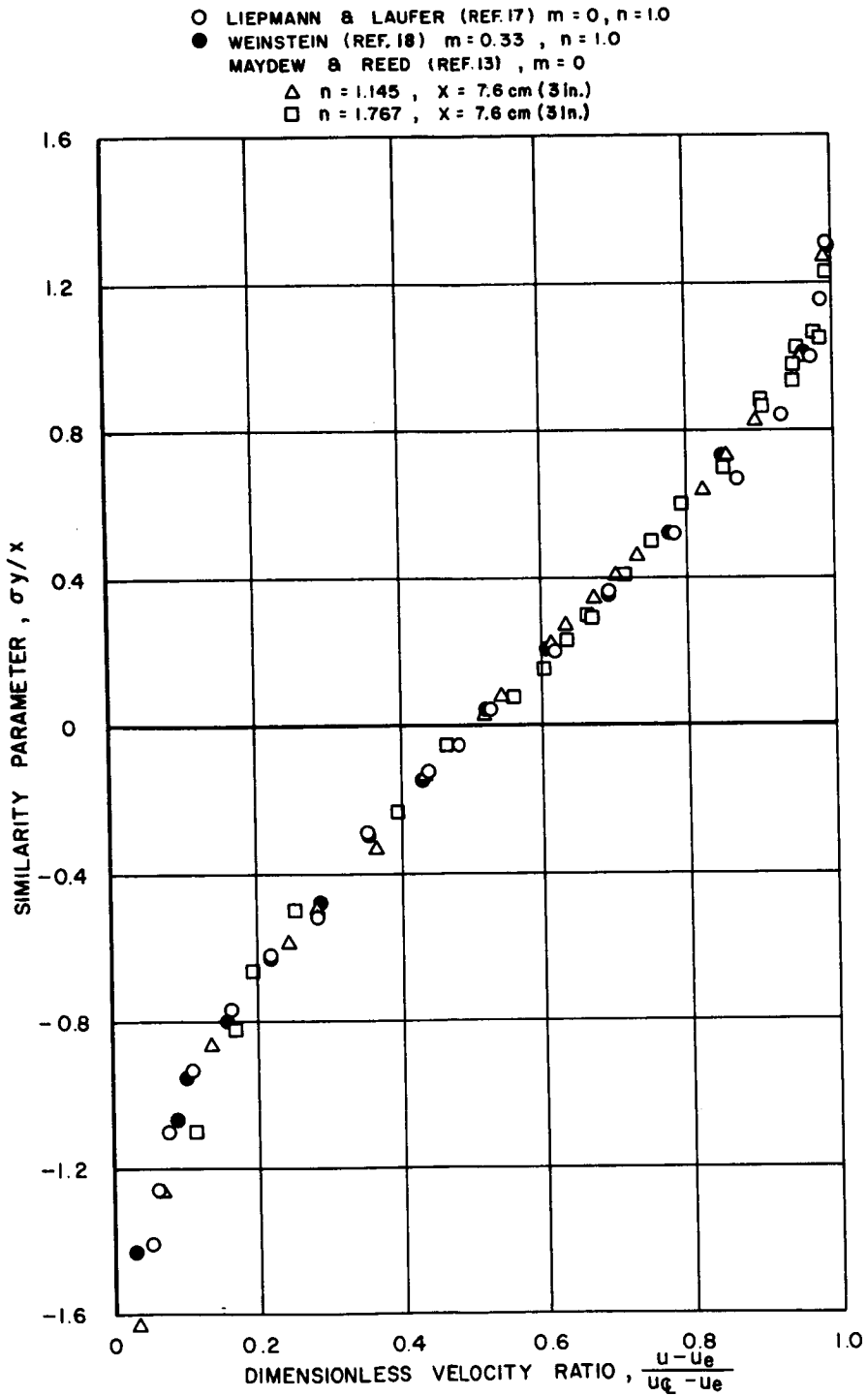


Figure 4.- Correlation of experimental mixing data in core region.

○ LIEPMANN AND LAUFER (REF. 17)

△ SABIN (REF. 19)

◇ YAKOVLEVSKIY (REF. 16)

□ ZHESTOV ET AL. (REF. 20)

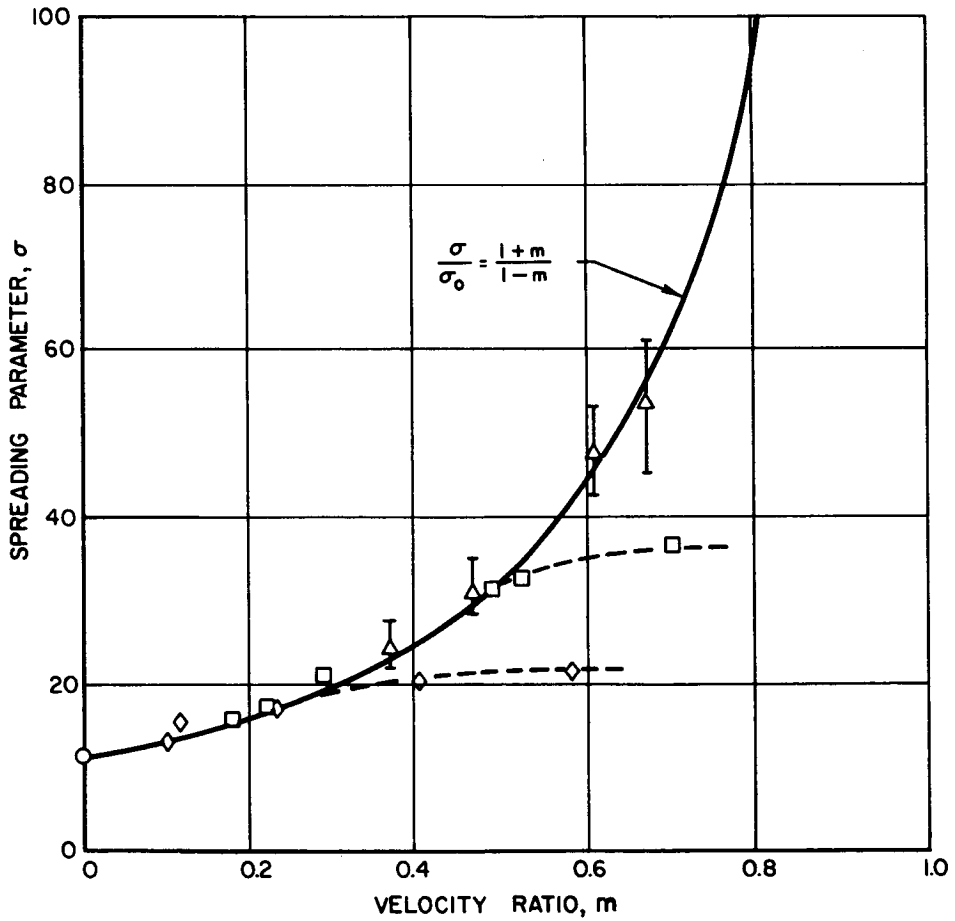


Figure 5.- Effect of velocity ratio on the spreading parameter.

- TOLLMEN (REF. 11)
- ◇ CORDES (REF. 21)
- ▽ OLSON AND MILLER (REF. 14)
- △ LIEPMANN AND LAUFER (REF. 17)
- GOODERUM, WOOD AND BREVOORT (REF. 22)
- ◇ BERSHADER AND PAI (REF. 23)
- ▷ CARY (REF. 24)
- D CRANE (REF. 25)
- ◊ JOHANNESSEN (REF. 26)
- ◊ JOHANNESSEN (REF. 27)
- △ ZUMWALT (REF. 13)
- MAYDEW AND REED (REF. 13)
- ANALYTICAL RESULTS, EQ.(29)

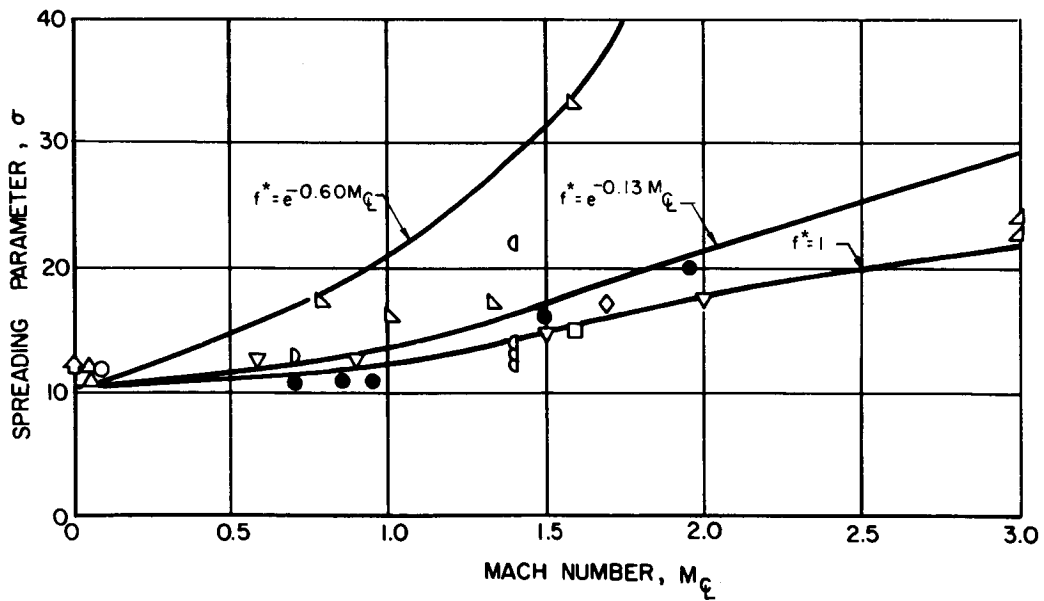


Figure 6.- Effect of Mach number on spreading of turbulent half jets.

LANDIS (REF.28)

SYM	□	▽	◇	△
m	0.25	0.25	0.46	0.50
n	1.300	1.087	1.087	1.300

WILLIS & GLASSMAN (REF.29)

SYM	○	◊	◌	△	⤴	▽	●	◐	▲	⤵
m	0	0.382	0.678	0	0.279	0.496	0.319	0.567	0.234	0.415
n	0.915	0.927	0.952	0.842	0.852	0.876	1.324	1.361	1.218	1.251

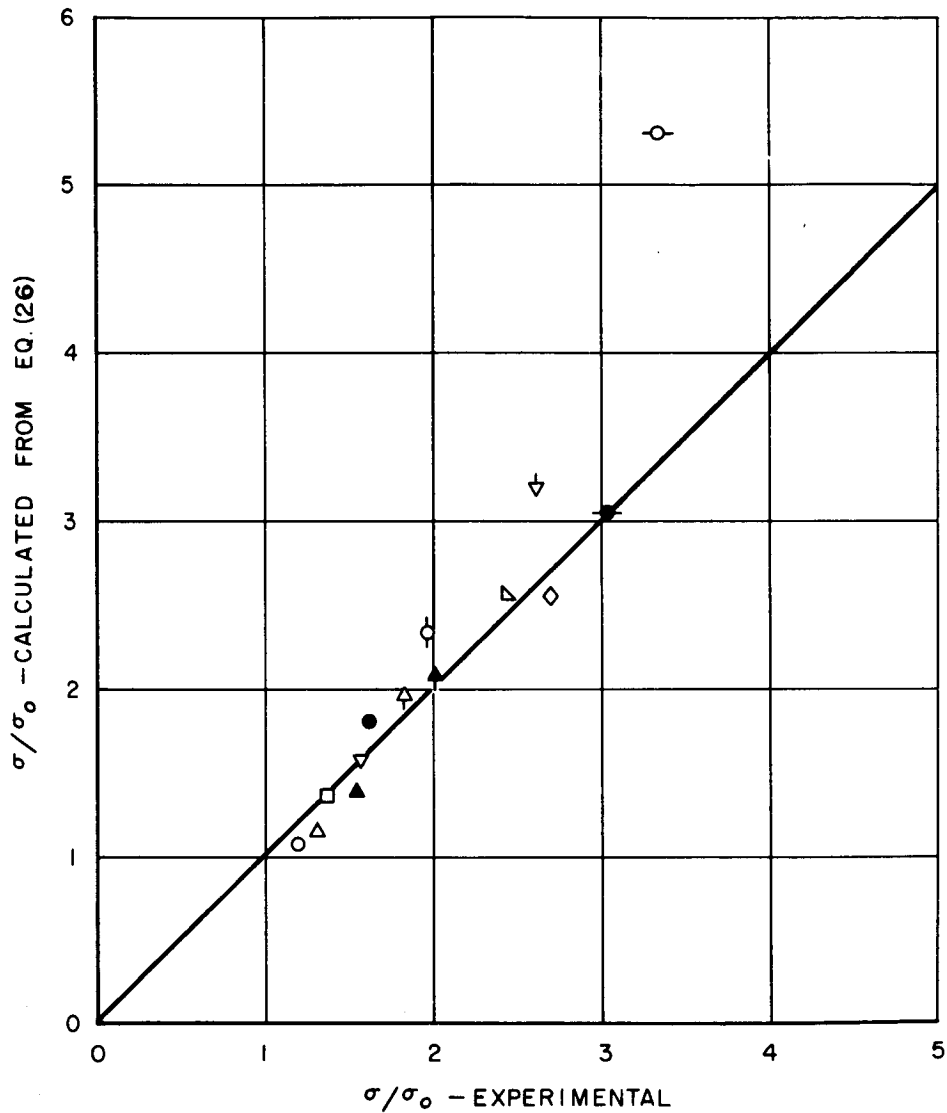


Figure 7.- Comparison of theoretical and experimental spreading parameters in core region.

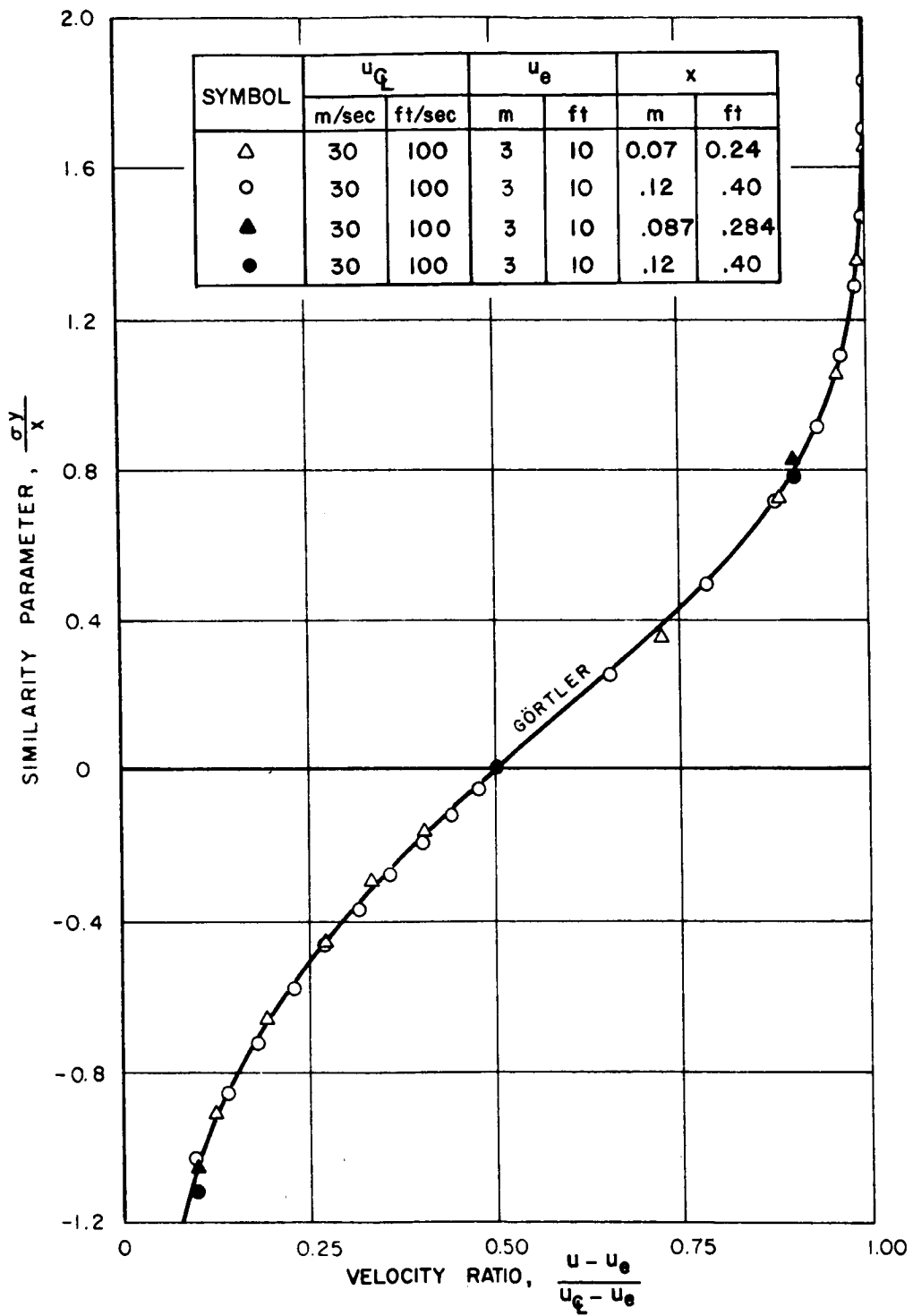


Figure 8.- Comparison of computed results with Görtler profile.

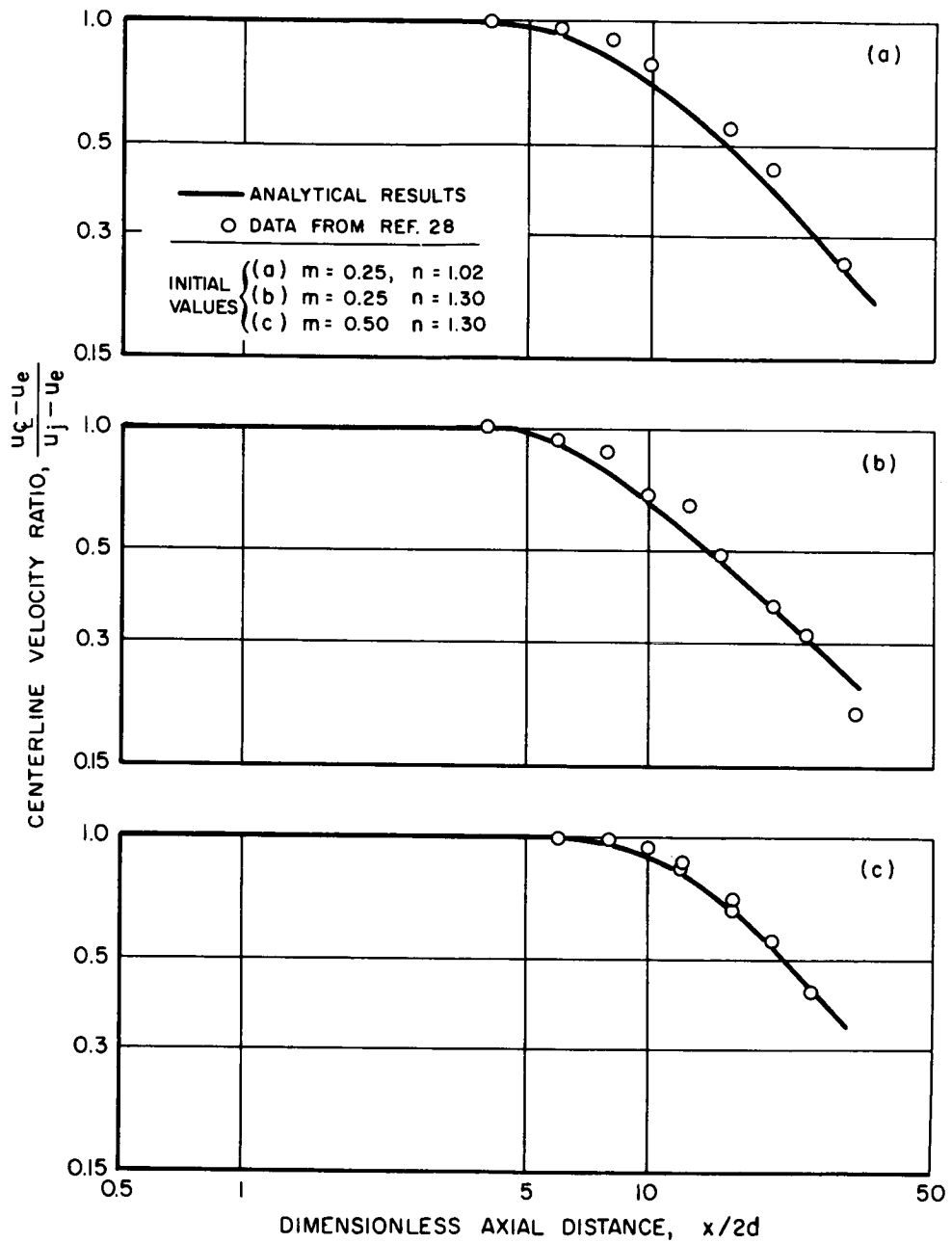


Figure 9.- Centerline velocity distributions for mixing of coaxial incompressible jets.

— ANALYTICAL RESULTS
 ○ DATA FROM REF. 28

INITIAL VALUES { (a) $m = 0.5, n = 1.19$, AIR JETS
 (b) $m = 0.5, n = 1.19$, HELIUM ADDED TO JET

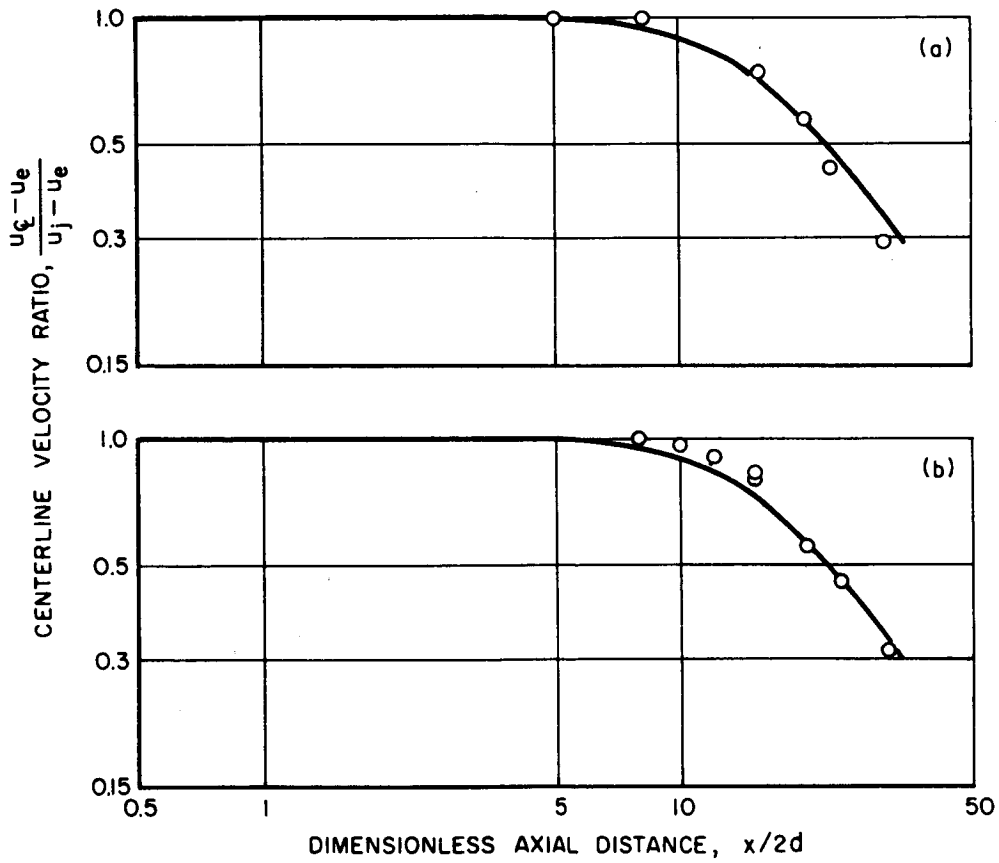


Figure 10.- Centerline velocity distributions for mixing of coaxial incompressible jets.

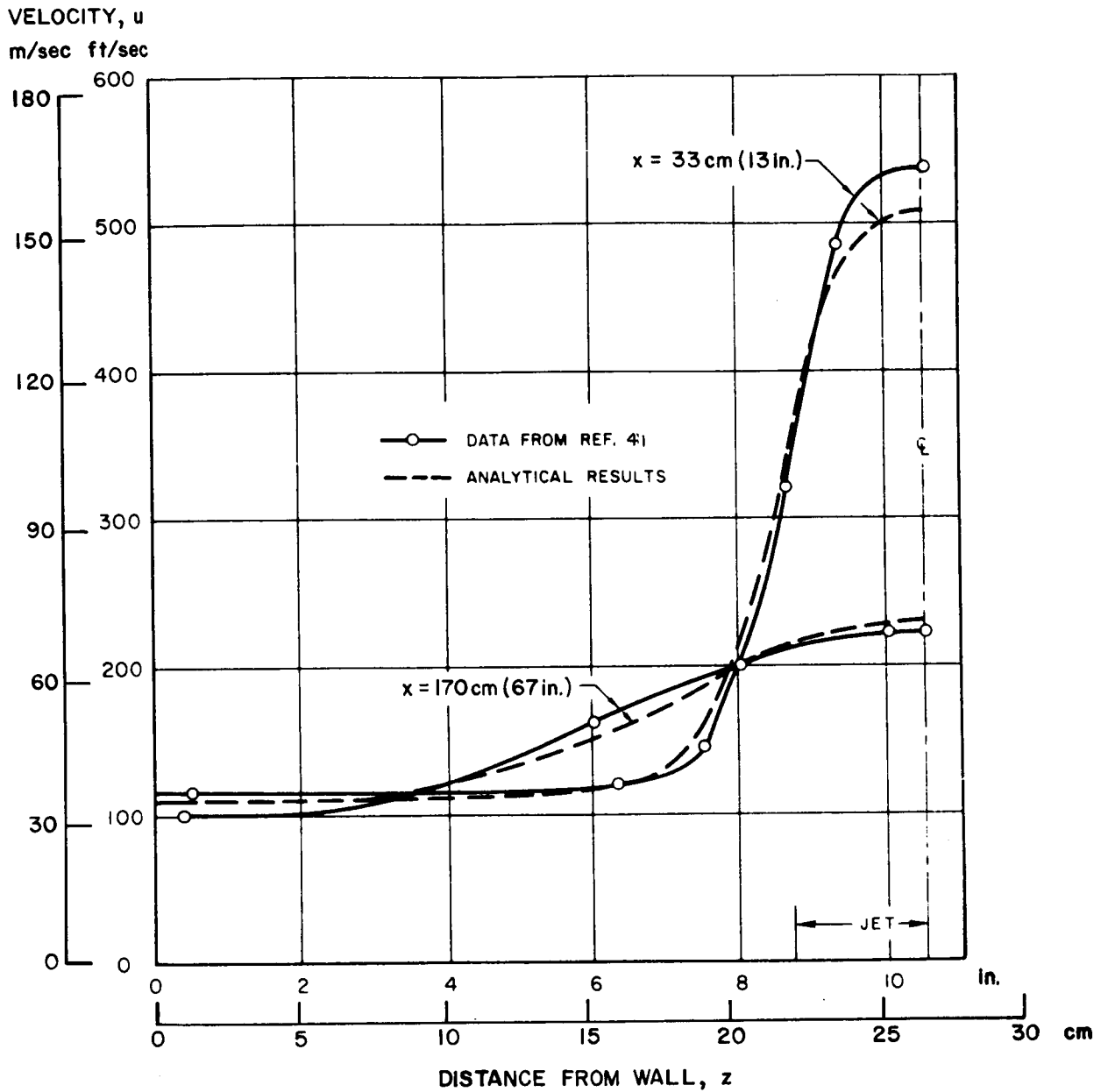


Figure 11.- Radial velocity profiles for subsonic mixing in a 53-cm-diameter (21-in.) tube.

TEMPERATURE, T

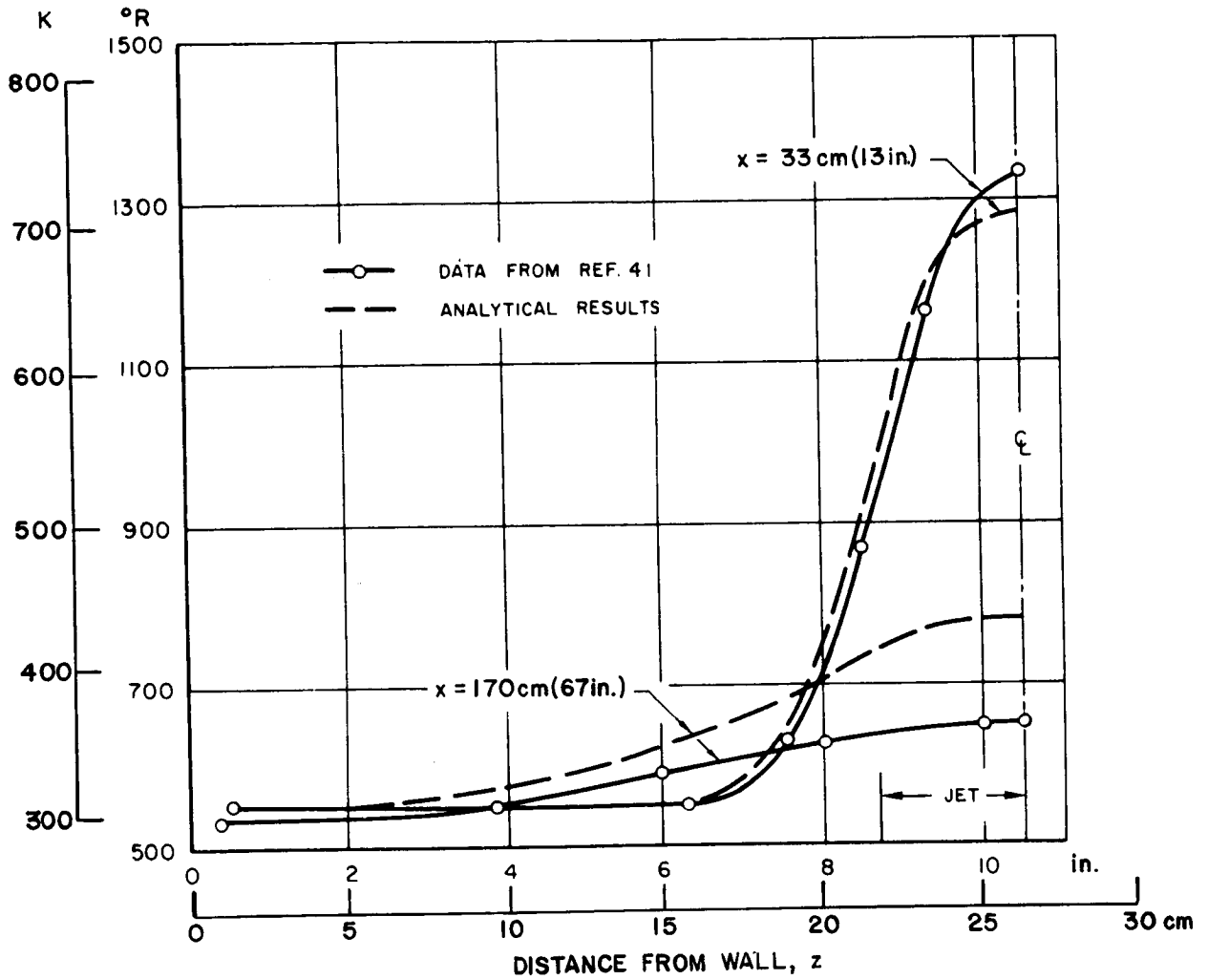


Figure 12.- Radial temperature profiles for subsonic mixing in a 53-cm-diameter (21-in.) tube.

— ANALYTICAL RESULTS

DATA FROM REF. 42

- $x/d = 4.01$
- △ $x/d = 11.03$
- $x/d = 16$

VELOCITY, u
m/sec ft/sec

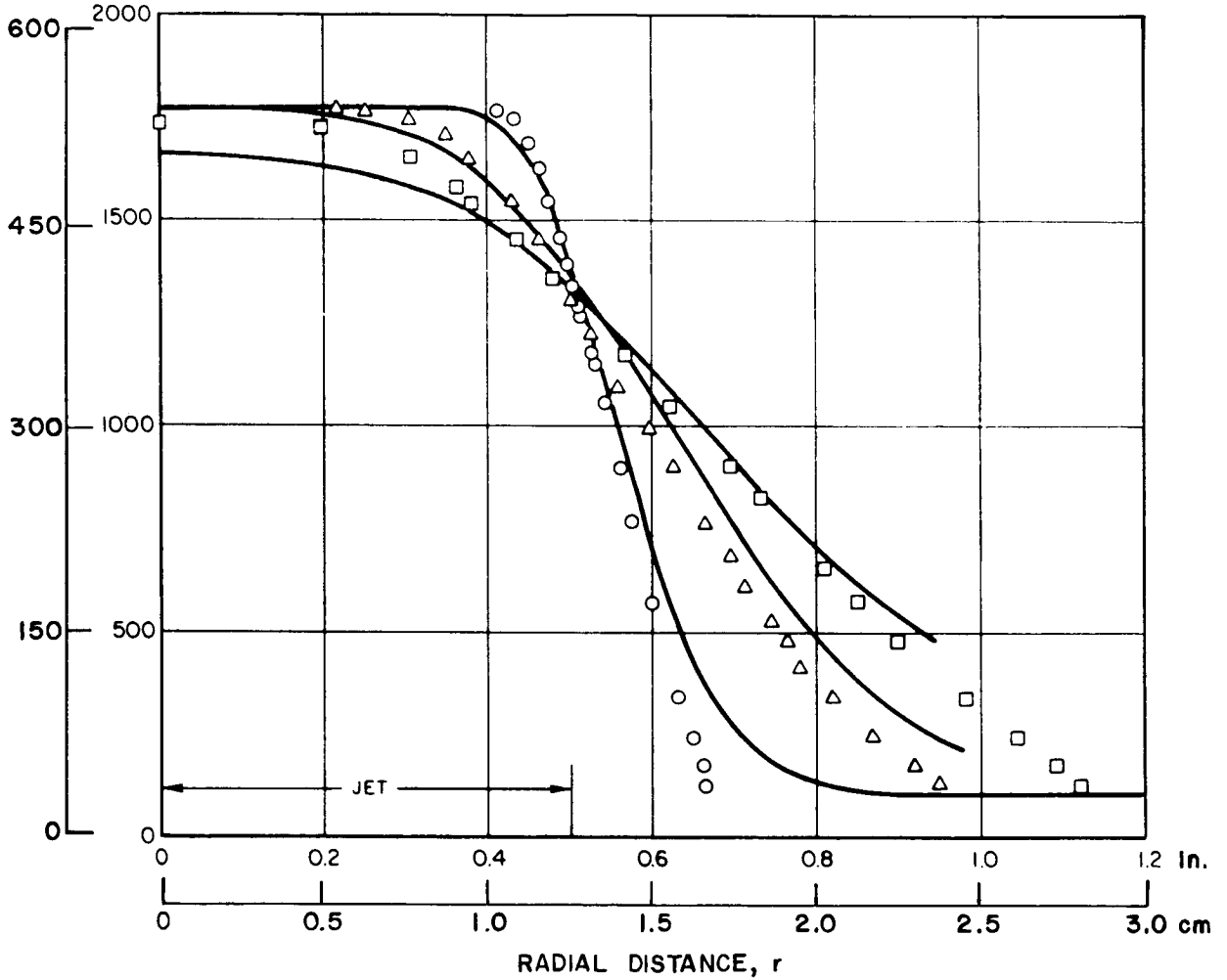


Figure 13.- Mixing of a submerged Mach 2.22 air jet.

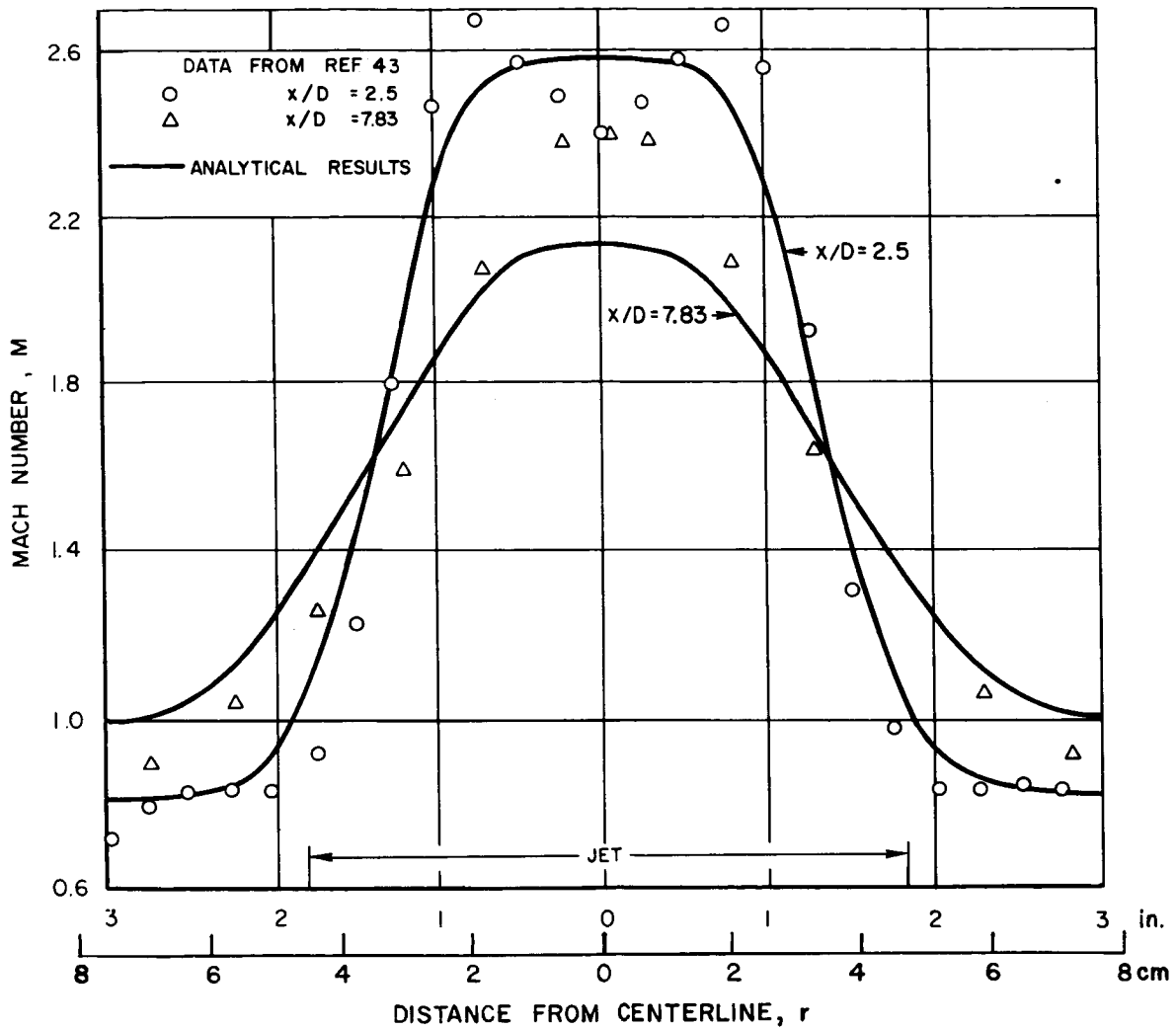


Figure 14.- Mixing of a Mach 2.6 jet with a subsonic external stream in a 15-cm-diameter (6-in.) tube.

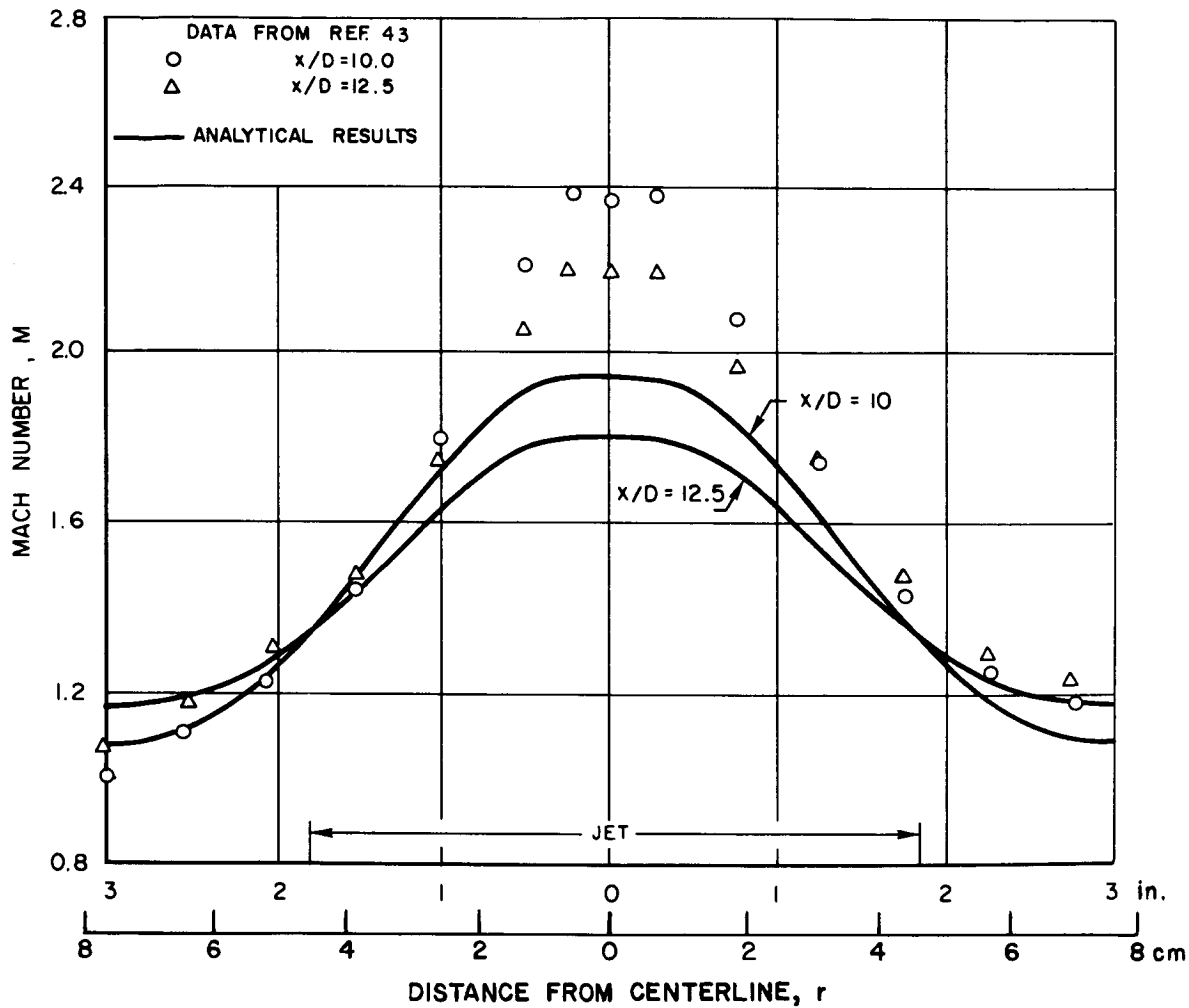


Figure 15.- Mixing of a Mach 2.6 jet with a subsonic external stream in a 15-cm-diameter (6-in.) tube.

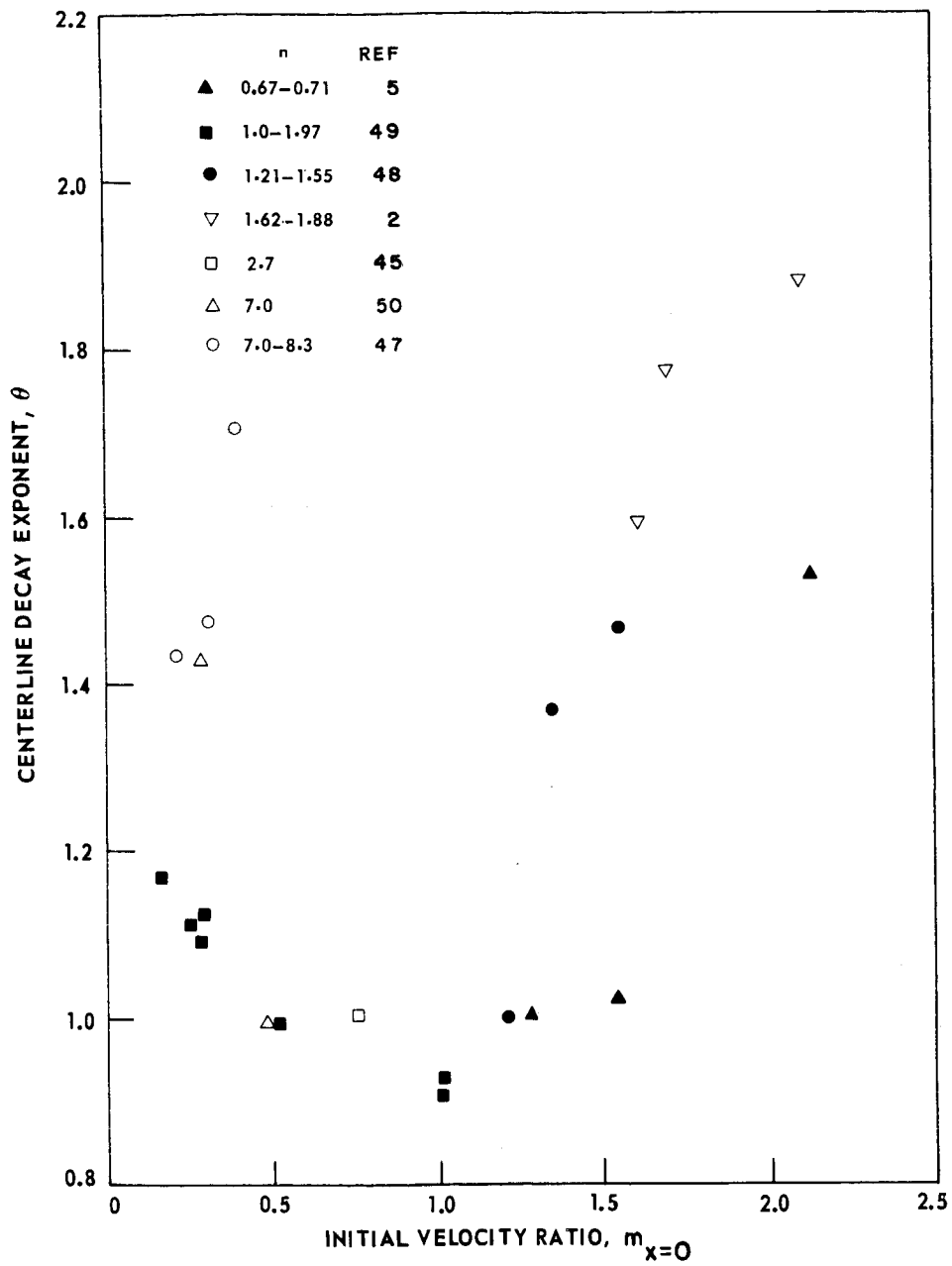


Figure 16.- Correlation of centerline decay exponent.

DISCUSSION

M. V. Morkovin: Will you ask the awkward question or will I?

H. McDonald: I think Dr. Cohen made it quite clear as to how he viewed the present procedure, that is a pragmatic industrial procedure which is designed to effect some calculations, and as he points out, that any developed turbulence model, I don't want to put words in Dr. Cohen's mouth – any developed turbulence model will have to take certain effects into account before they can be of use to people like the propulsion group at United Aircraft Corporation and in addition will have to explain certain things which the present model does in an albeit empirical manner. Is that correct, Dr. Cohen?

L. S. Cohen: Yes.

M. V. Morkovin: You talk about these preturbulent mechanisms. Well, how many parameters do you really have for the preturbulent mechanism – how do you choose for a given situation? What is the upstream boundary-layer effect or what? I mean, is it just because you have seen it before and you know you are a good cook, or do you have some real mechanisms in mind?

L. S. Cohen: Well I think your comment about being a good cook is part of it, certainly. As I mentioned, it turns out accidentally the only thing you really have to select in the model is the m_1 parameter.

M. V. Morkovin: Look, what is the mechanism – you said the mechanism?

L. S. Cohen: I am sorry, I did not mean to imply that what I am calling preturbulence is just a pot into which I am throwing all of my ignorance. What I am saying is, there is some initial turbulence level which, I suppose if we were wise enough, we would go in and measure in all of these difficult cases. In deriving this model, we simply assumed that this initial turbulence level could be sustained somehow in the flow and that this m_1 parameter is directly relatable to this initial turbulence level. The mechanism by which this initial turbulence is produced, I have no idea how it is produced initially. I just think we do not know what the causes are but the effect is a particular initial turbulence level so that no mechanism is put forth for the production of this initial turbulence level.

S. W. Zelazny: I have a comment concerning your second slide and that was the mass fraction decay exponent. You had shown a plot and the plot showed a definite dip in that decay exponent for velocity ratios near unity. I think maybe you might be putting a little bit more into the data interpretation than we have a right to expect, primarily because if you look at the data that are available that enable us to calculate that decay exponent, you will find that most of the data are restricted to about 20 diameters downstream. Some of the data that you used, for example Alpineri's data, did show a decay exponent in the

ballpark of unity. I do not think 20 diameters is sufficiently far downstream to really call it an asymptotic decay and that is what we are looking for.

L. S. Cohen: I guess I agree with what you are saying, but I really do not have any comment on it, I think that your point is well taken.