TURBULENT KINETIC ENERGY EQUATION AND FREE MIXING*

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INTRODUCTION

The present work on calculation of free shear flows was carried out to investigate the usefulness of several concepts which were previously successfully applied to wall flows. The method belongs to the class of differential approaches. The turbulence is taken into account by the introduction of one additional partial differential equation, the transport equation for the turbulent shear stress. The structure of turbulence is modeled after Bradshaw et al. (ref. 1). This model has been used successfully in boundary layers and its applicability to other flows is demonstrated in this contribution. An earlier attempt to use this approach for calculation of free flows was made by Laster (ref. 2). The work reported here differs substantially from that of Laster in several ways. The most important difference is that the region around the center line is treated by invoking the interaction hypothesis (ref. 3) (concerning the structure of turbulence in the regions separated by the velocity extrema). The compressibility effects on shear layer spreading at low and moderate Mach numbers were investigated. In the absence of detailed experiments in free flows, the evidence from boundary layers that at low Mach numbers the structure of turbulence is unaffected by the compressibility was relied on. The present model was tested over a range of self-preserving and developing flows including pressure gradients using identical empirical input. The dependence of the structure of turbulence on the spreading rate of the shear layer $d\delta/dx$ was established.

16

SYMBOLS

a_1,G,L	defined by equations (2)
c_p	specific heat at constant pressure
L_o	width to the half velocity point on the profile
M	Mach number
p	mean pressure

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$\overline{q^2} = \overline{u_i' u_i'}$

r	recovery factor
T	temperature
U, V	mean velocity components
U_1, U_2	external velocities at edges of a mixing layer
ΔU	maximum velocity difference across shear layer
u', v', w'	fluctuating velocity components

$\tilde{V} = V + \frac{\overline{\rho' v'}}{\rho}$

$W = \frac{\Delta U}{U_1}$

X, Y	coordinate axes
x, y	distances along axes
α	angle of characteristic
γ	ratio of specific heats
δ	shear layer thickness defined as distance between points where $\tau = 0.05\tau_m$

$\overline{\frac{d\delta}{dx}} = \frac{d\delta/dx}{(d\delta/dx)_{\text{still-air jet}}}$

ϵ	dissipation rate
θ	momentum thickness
ρ	mean density
σ	spreading parameter for free shear layers

$\tau = -\left(\overline{u'v'} + \frac{\overline{\rho' u' v'}}{\rho}\right)$

τ^+ and τ^- shear stress profiles of "simple" layers

Superscripts:

a exponent

' fluctuating quantities

Subscripts:

$\mathfrak{C}$ center-line value

i index

J initial jet value

m,max maximum value

1/2 half velocity point

ANALYSIS

Equations and the Model of Turbulence

The governing equations considered are the continuity, momentum, and turbulent kinetic energy equations:

$$\left.\begin{aligned} &\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \\ &\left(U\frac{\partial}{\partial x} + V\frac{\partial}{\partial y}\right)U = -\frac{1}{\rho}\frac{dp}{dx} - \frac{\partial\overline{u'v'}}{\partial y} \\ &\left(U\frac{\partial}{\partial x} + V\frac{\partial}{\partial y}\right)\frac{\overline{q^2}}{2} = -\overline{u'v'}\,\frac{\partial U}{\partial y} - \frac{\partial}{\partial y}\left(\frac{\overline{p'v'}}{\rho} + \frac{1}{2}\,\overline{q^2v'}\right) - \epsilon \end{aligned}\right\} \tag{1}$$

The turbulent kinetic energy equation contains three additional correlations not appearing in the other two equations. To close this system, assumptions about the structure of turbulence would have to be made, where by structure a given relation between the local values of two turbulent quantities is understood. As in Bradshaw et al. (ref. 1), three relations are used to define the structure:

$$\tau = a_1\overline{q^2} \tag{2a}$$

$$\epsilon = \frac{\tau|\tau|^{1/2}}{L} \tag{2b}$$

$$\frac{\overline{p'v'}}{\rho} + \frac{1}{2}\,\overline{q^2v'} = G\tau\left|\tau_{max}\right|^{1/2} \tag{2c}$$

The functions a_1, L, and G are specified by algebraic expressions and the local length and velocity scales of turbulence are assumed to be proportional, respectively, to δ and $|\tau_m|^{1/2}$.

By utilizing relations (2), the conservation equation for $\overline{q^2}$ may be converted into an empirical transport equation for τ

$$\left(U\frac{\partial}{\partial x} + V\frac{\partial}{\partial y}\right)\tau = 2a_1\left[\tau\frac{\partial U}{\partial y} - \frac{\partial}{\partial y}\left(G\tau\left|\tau_{max}\right|^{1/2}\right) - \frac{\tau|\tau|^{1/2}}{L}\right] + \tau\left(\frac{U}{a_1}\frac{\partial a_1}{\partial x} + \frac{V}{a_1}\frac{\partial a_1}{\partial y}\right) \tag{3}$$

The choice of convective type diffusion (eq. (2c)) over gradient type in free flows is supported by mixing layer experiments. (See refs. 4 and 5.) The positions of zero diffusion and maximum kinetic energy do not coincide, a fact for which gradient diffusion cannot account for. The two points are separated by a distance of the order of 5 percent of the shear layer thickness. Also, the free shear flows exhibit strong large-scale motions which made the convective diffusion important.

There is much experimental evidence to support the relation (2a). An examination of a wide range of experimental data showed that the value of a_1 varies within a small range depending on the flow considered. The only difficulty in this formulation occurs in the vicinity of a velocity extremum, as discussed in the next paragraph. Relation (2b) is a logical extension, based on equation (2a), of

$$\epsilon = \frac{\left(\overline{q^2}\right)^{3/2}}{L}$$

which is a commonly accepted model.

Flows With Velocity Extrema

The formulation presented was applied to free mixing layers with the simple choice of $a_1(y) = \text{Constant}$ as in boundary layers. In flows with velocity extrema, the shear stress changes sign in the vicinity of the velocity extremum. The turbulent kinetic energy, by definition, does not change sign and this fact precludes the use of constant a_1. Also, the shear stress equation is singular at the point where $a_1 = 0$ (its solution being regular) and for this reason presents numerical difficulties. (See ref. 2.)

To avoid this problem, the suggestion of Bradshaw (ref. 3) is used and the flow with velocity extremum is regarded as two adjoining "simple" shear layers which interact only through the mean velocity profile. Each layer has its own shear profile and the algebraic sum of these profiles in the region of overlap gives the shear profile of the complete flow. One reason for looking at the flow from this viewpoint is that if the structure of turbulence in each layer is unaffected by the presence of the adjoining layer (does not actively participate in the interaction), then a simple tool for calculating more complex flows such as jets, wakes, and wall jets is obtained. There are two such simple or basic shear layer flows – the boundary layer and the mixing layer – and it is proposed to regard all other thin shear layer flows as combinations of these two. The empirical functions in each layer of a complex flow were to be the same, or nearly the same, as in the corresponding simple shear layer and, as a result, the task of determining them would be simplified. (The actual difference between jets and mixing layers can be seen in figures 1 and 2.) Another reason is that this point of view allows a simple explanation of the regions of "negative production" of turbulent kinetic energy which occur in asymmetric flows. The technique can be utilized for calculating these flows which otherwise require the a priori knowledge of the point of vanishing shear when the original formulation is used. The same shortcoming affects all models which involve the eddy-viscosity concept in one form or another.

The idea was applied to the duct flows (ref. 3) and very good results were obtained with the empirical functions which were developed for boundary layers. In this work the same approach is applied to jets and wakes.

Empirical Functions of the Structure

The empirical functions of the structure of turbulence in mixing layers $\left(\text{with } U_2/U_1 = 0\right)$ were derived from the experimental data (refs. 4 and 6) and then refined by comparison of the velocity and shear profiles with the experiments. (See fig. 1.) The simple choice of constant $a_1 = 0.15$ and $L/\delta = 0.09$ was found to be adequate for good results. The shape of the diffusion function G was obtained by integration of the diffusion in the turbulent kinetic energy balance. The proper values of the empirical functions for $U_2/U_1 \neq 0$ were deduced from calculations by comparison with the results of Spencer (ref. 7). The empirical functions were found to be dependent on the velocity ratio of the mixing layer. This dependence may be correlated with $d\delta/dx$, the spreading rate of the shear layer.

The same model was applied to jets and wakes as outlined. The two adjoining layers are calculated as two separate layers. They share joint U and V profiles but they have separate shear profiles. The only difference from the mixing-layer program appears in the momentum equation where the shear profiles are superposed to give the "true" shear profile in the region of overlap:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\partial \tau^+}{\partial y} + \left(\frac{\partial \tau^-}{\partial y}\right)_{\text{other layer}} \tag{4}$$

Beyond the velocity maximum, the layer experiences a negative production in the shear equation which limits the region of overlap.

The program is written, at the present time, for symmetric jets and wakes because of lack of experimental data. However, the concept is not restricted in any way to symmetric flows and its importance lies in its ability to treat and explain asymmetric flows.

The calculations show that in free flows the structure of the "simple" layer is affected by the interaction. The interaction tends to modify more the magnitude than the shape of the empirical functions. Thus, the a_1 and L/δ were retained constant and the shape of the diffusion function G was slightly altered. (See fig. 2.) The comparison with experiments indicates again the dependence of the structure on the spreading rate $d\delta/dx$. The limiting case for $d\delta/dx = 0$ is shown in broken lines in figure 2.

The results show the usefulness of the interaction concept for calculation of free shear flows. The required modifications of the empirical functions are not large and indicate that the flows with velocity extrema may be regarded as weakly interacting adjoining shear layers. There appears to be more interaction of turbulent structure in the free flows than in the ducts and the explanation may be sought in the behavior of the shear stress in the vicinity of the velocity extremum. (See fig. 3.) The shear stress profiles of free flows overlap significantly more and the value of the shear on the center line is typically $0.55\tau_{max}$, against $0.1\tau_{max}$ in the duct, and thus causes a stronger interaction.

All the results presented are calculated with the same input, the structure being a function of y/δ and $d\delta/dx$ alone.

Compressible Flow

The governing equations for the compressible flow are much more complicated than their incompressible counterpart. If the restriction is made to include only low and moderate Mach number flows, many of the new correlations appearing in the compressible equations may be neglected on the basis of order-of-magnitude arguments. (See ref. 8.) This neglect simplifies the equations which may then be written in boundary-layer form as follows:

$$\frac{\partial U}{\partial x} + \frac{\partial \tilde{V}}{\partial y} + \frac{U}{\rho}\frac{\partial \rho}{\partial x} + \frac{\tilde{V}}{\rho}\frac{\partial \rho}{\partial y} = 0$$

$$U\frac{\partial U}{\partial x} + \tilde{V}\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\partial \tau}{\partial y} + \frac{\tau}{\rho}\frac{\partial \rho}{\partial y}$$

$$U\frac{\partial \tau}{\partial x} + \left(\tilde{V} + 2a_1 G\left|\tau_m\right|^{1/2}\right)\frac{\partial \tau}{\partial y} = \tau\left[2a_1\left(\frac{\partial U}{\partial y} - \left|\tau_m\right|^{1/2}\frac{\partial G}{\partial y} - \frac{G\left|\tau_m\right|^{1/2}}{\rho}\frac{\partial \rho}{\partial y} - \frac{\left|\tau\right|^{1/2}}{L}\right) + \frac{U}{a_1}\frac{\partial a_1}{\partial x} + \frac{\tilde{V}}{a_1}\frac{\partial a_1}{\partial y}\right]$$

where

$$\tau = a_1\left(\overline{q^2} + \frac{\overline{\rho' q^2}}{\rho}\right)$$

$$L = \frac{\tau\left|\tau\right|^{1/2}}{\epsilon}$$

$$\frac{1}{2}\overline{p'v'} + \frac{1}{2}\overline{q^2 v'} + \frac{1}{2\rho}\overline{\rho' q^2 v'} = G\tau\left|\tau_m\right|^{1/2}$$

The inclusion of compressibility presents two additional tasks. The structure of turbulence in compressible flow and the density variation across the layer have to be determined.

For the values of the empirical functions in compressible flows, the suggestion (ref. 9) is relied on that the turbulence is convected passively by the mean flow as long as the local Mach number of the root-mean-square fluctuations $\left(\overline{\rho u'^2}/\gamma p\right)^{1/2}$ is much less than unity. The inference is that the structure of turbulence is Mach number independent and the "incompressible" values of the empirical functions may be used. This assumption is based on the analysis of experiments in boundary layers, where this condition is satisfied up to moderate Mach numbers ($M < 5$).

In the Mach number range to which the present approach is limited, a good approximation for the density profile can be obtained from the Crocco formula used in boundary layers:

$$c_pT + 0.5rU^2 = \text{Constant}$$

which together with the equation of state yields $\rho = \rho(U)$. It eliminates the need for an additional equation and may be incorporated into the incompressible version of the program with only small modifications.

Compressibility effects due to temperature or density differences were also incorporated by assuming, respectively, similarity between temperature and velocity profiles and between mass fractions and velocity profiles. That again yields a relation $\rho = \rho(U)$ which is useful for small compressibility effects. Large effects will require the solution of a separate equation.

METHOD OF SOLUTION

With the diffusion term in the kinetic energy equation modeled to be of the convective type, the set of equations becomes hyperbolic. There are two choices of solving this system, either by a procedure suitable for parabolic equations or to use the method of characteristics. The first approach is advantageous if it is intended to introduce additional equations, for example, for compressible flows or for a more involved model of turbulence. In the present work, it was decided to use the mathematically simpler method of characteristics used already in the boundary-layer calculations. (See ref. 1.) The model of turbulence and the empirical functions developed in the course of this work would be the same if an alternative method of solution of these equations is used.

The accuracy of the calculations is governed by the number of points on the profile and by the number of iterations used to improve the interpolation along the characteristics. However, even if several iterations are used, the momentum and mass balance are not preserved because of numerical inaccuracies and lead to a momentum thickness gain of the order of 0.1 percent per station. To avoid accumulation of error, the grid size is being readjusted by that very small amount after every step so that momentum is preserved. The mass balance is then very well preserved – to within 1 percent on a typical run. The largest inaccuracies occur for flows issuing into a small external stream. One of the characteristic angles near the edge tends to $\alpha = \arctan (V/U)$ and precludes calculation of mixing with still air unless some special numerical treatment for this boundary is introduced. The problem at the still-air edge affects other methods of solution as well. Calculation of mixing with small external stream has some practical limitations. The step size in the x-direction is inversely proportional to $\tan \alpha_{max}$ and, therefore, for economical calculation, it is necessary to maintain at least $U_1 = 0.05U_{max}$ which gives $\tan \alpha_{max} \approx 1$. The region of very small external stream also suffers from larger than average errors in mass balance.

COMPARISON WITH EXPERIMENTS

The model developed for the mixing layers can be used for all velocity ratios. Calculations for three ratios are compared with experiments below:

(a) Mixing into still air. (See Liepmann and Laufer (ref. 6) and Bradshaw and Ferriss (ref. 4) and also figure 4.) Note that the computed profile has a small external stream rather than still air on its edge; thus, the spreading rate is reduced somewhat.

(b) Mixing with a parallel moving stream. (See Spencer (ref. 7) and figures 5 and 6.)

The jet program is capable of handling both jets and wakes in the presence of pressure gradients by using the same empirical input. Some predictions compared with experiments are

(a) Jet mixing with still air. (See Bradbury (ref. 10) and figure 7.) Both the experiment and the calculations had actually a small external stream which does not affect the nondimensional profiles.

(b) Small deficit wake. (See Townsend (ref. 11).) The region in which Townsend made his measurements is far from complete self-preservation as documented by the difference between the measured shear profile and the profile required for self-preservation. (See fig. 8.) The calculations compare well in the region investigated by Townsend and show self-preservation far downstream. (See fig. 9.) The small excess jet tends toward the same results as the small deficit wake when $W \rightarrow 0$.

When the external flow varies as $U_1 \propto x^a$, there are ranges of the exponent a for which the momentum equation allows self-preservation of jets or wakes. The following two flows fall in that category:

(c) Wakes in a pressure gradient – investigated by Gartshore (ref. 12) who found them approximately self-preserving. The calculations show that, although the flow has the tendency to conform to the self-preservation, it drifts steadily away from it (fig. 10).

(d) Self-preserving jet in a pressure gradient. Value of the exponent $a = -\frac{1}{2.4}$ was used for the calculations. The results obtained are plausible, although there are no experiments to support them. (See fig. 11.)

APPENDIX

COMMENTS ON TEST CASES

This appendix presents comments on some of the test cases presented in figures 12 to 19.

Test Case 5 (Hill and Page)

The free shear layer was measured in the early stages of its development from a boundary layer separated over a cavity. At the separation the boundary layer was turbulent and had a momentum thickness of 0.0793 cm. The last measured station was at $x = 20.95$ cm or $x/\theta = 264$. The dimensions of the cavity and of the orientations of the axes X,Y are not clear in reference 13. Depending on the details of the geometry, significant backflow and transverse pressure gradient may have been present. In view of these uncertainties, it is difficult to comment on the disagreement of the calculations and of the experiment and also on the lack of spreading of the experimental profile on the low velocity side.

In any case, as the boundary layer was turbulent at the separation the shear layer was probably not in the self-preserving range even at the last station. (See Bradshaw, ref. 14.) In the calculations the maximum shear first increased above the "fully developed" value (overshoot). Its decrease toward the value far downstream was slow and monotonical. At $x = 20.95$ cm, its value was still higher by an amount on the order of 10 percent. Also the spreading rate $1/\sigma$ was still substantially higher at that station than far downstream.

Test Case 13 (Bradbury)

The calculations were not overly sensitive to the initial shear. An increase of the initial shear by 66 percent produced less than 1-percent difference in W at $x/D = 300$.

Test Case 14 (Chevray and Kovasznay)

In a wake very close to the trailing edge, the structure of turbulence may be expected to be that of a boundary layer rather than that of a free flow. The question remains how long does it take for the structure to undergo the change from one regime to another. The examination of the data of reference 15 showed that the mean velocity and turbulent quantities take on their wakelike shapes already by the distance $x = 20$ cm and possibly even earlier. It seems reasonable to expect that also the structure of turbulence came close to the wake type by that distance. This possibility was tested by starting the calculations at three different points $x_0 = 0$, 20, and 50 cm by using the experimental profiles as the input. The results do confirm the expectation since although the latter two calculations differ from the first one they agree with each other. (See figs. 12(a) and 12(b).) It appears that the present model has difficulties near the trailing edge where the structure is still that of the boundary layer. The calculated and experimental profiles at $x = 240$ cm agree well when the calculations start at $x = 20$ cm (fig. 12(c)) or at $x = 50$ cm. From the figures it is also clear that the momentum thickness of the computed and measured profiles are not equal. A check on the momentum thickness at several stations revealed the following variation of θ and gives an idea about the general accuracy of the experiment:

x	0	20	50	240
θ	0.578	0.606	0.585	0.557

Test Case 16 (Demetriades (ref. 16))

The results of the calculations were found to be rather sensitive to the initial shear in contrast to the test case 13. A 20-percent change of the initial shear produced a substantial difference at $x/4\theta = 2000$ and this difference $\left(\text{as represented by the value of } \tau_m/\Delta U^2\right)$ still persisted very far downstream. This behavior was found both in compressible and incompressible flows. This lack of tendency "to forget" of small deficit wakes is very interesting and suggests that convection and diffusion of turbulence in these flows have a large influence on the flow development.

The presented results were obtained by using a recovery factor of 0.9.

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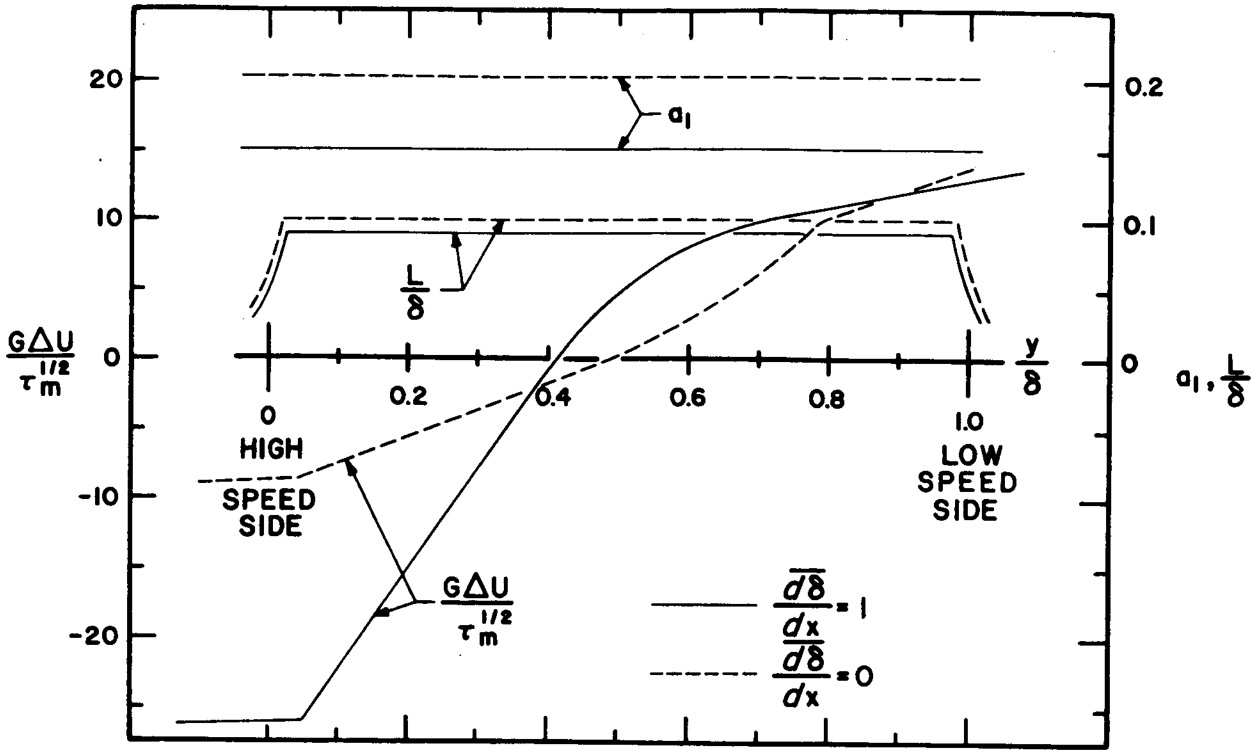


Figure 1.- Free shear layers – empirical functions (structure).

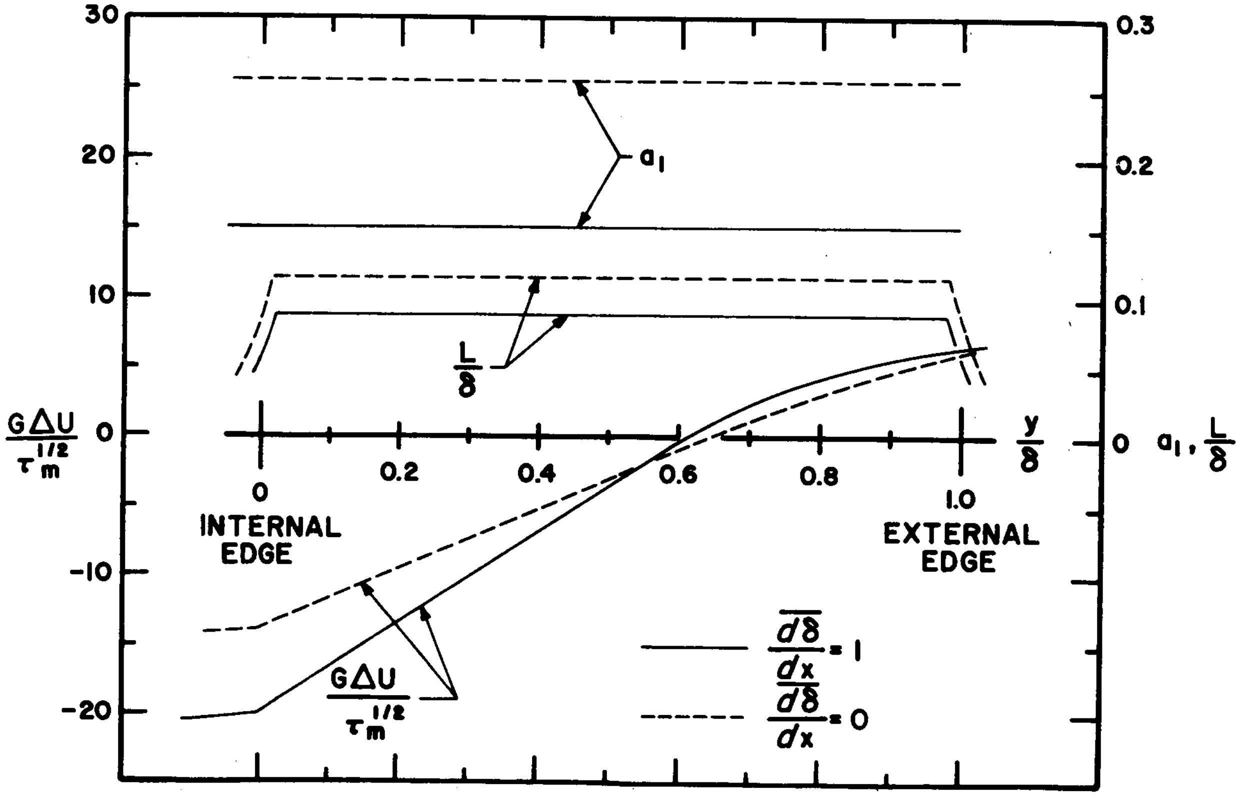


Figure 2.- Jets and wakes – empirical functions (structure).

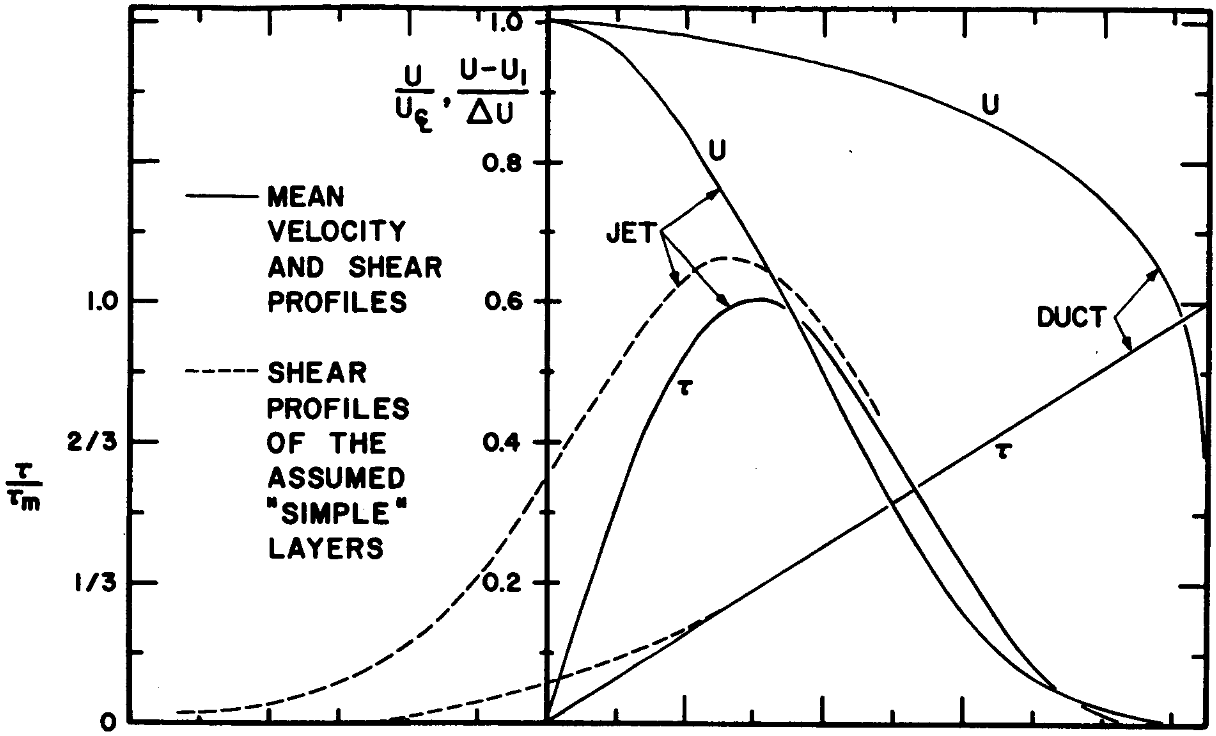


Figure 3.- Interaction approach – jets compared with ducts.

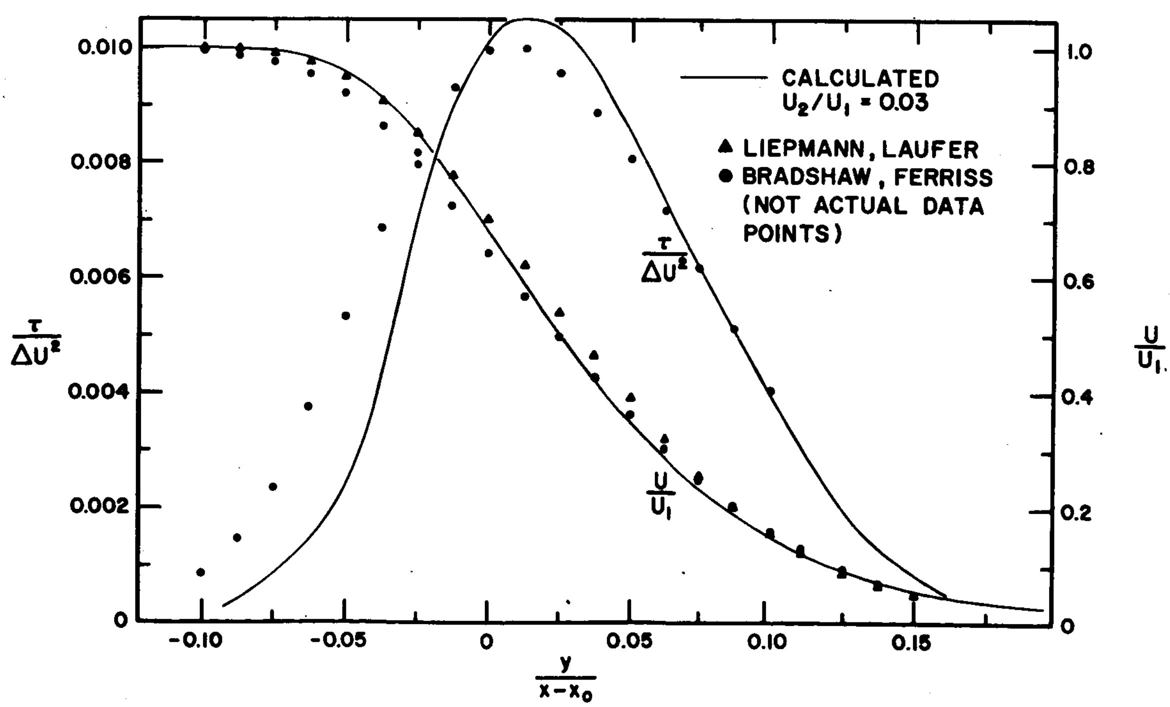


Figure 4.- Free shear layer; $U_2/U_1 = 0$.

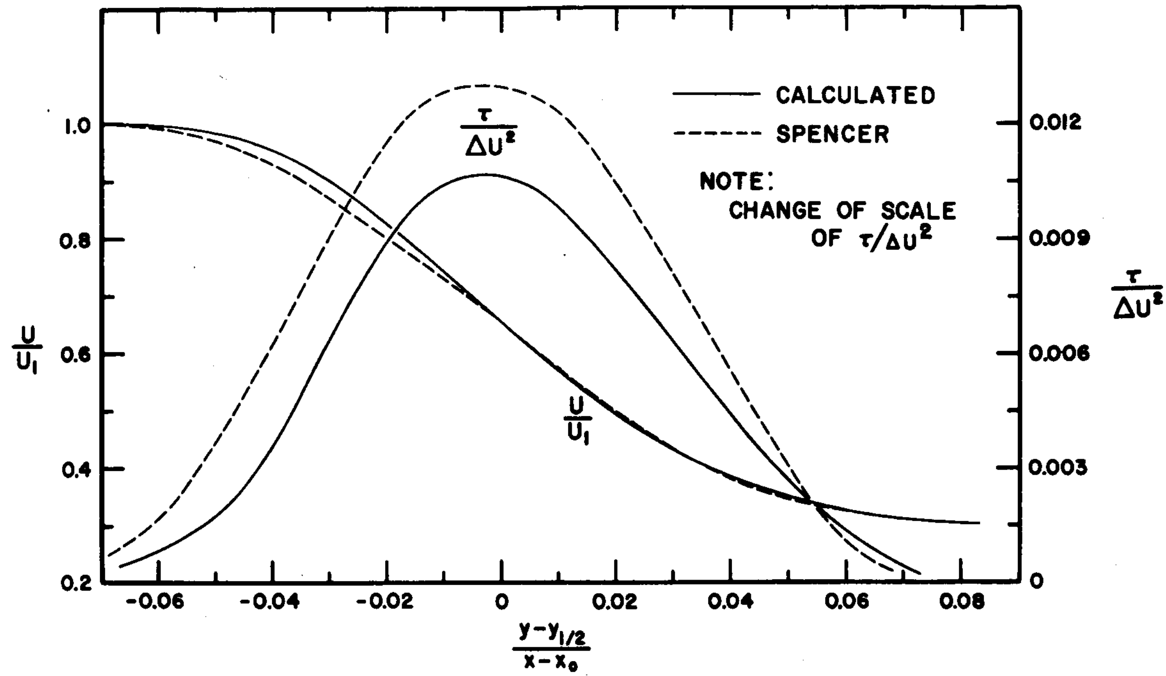


Figure 5.- Free shear layer; $U_2/U_1 = 0.3$.

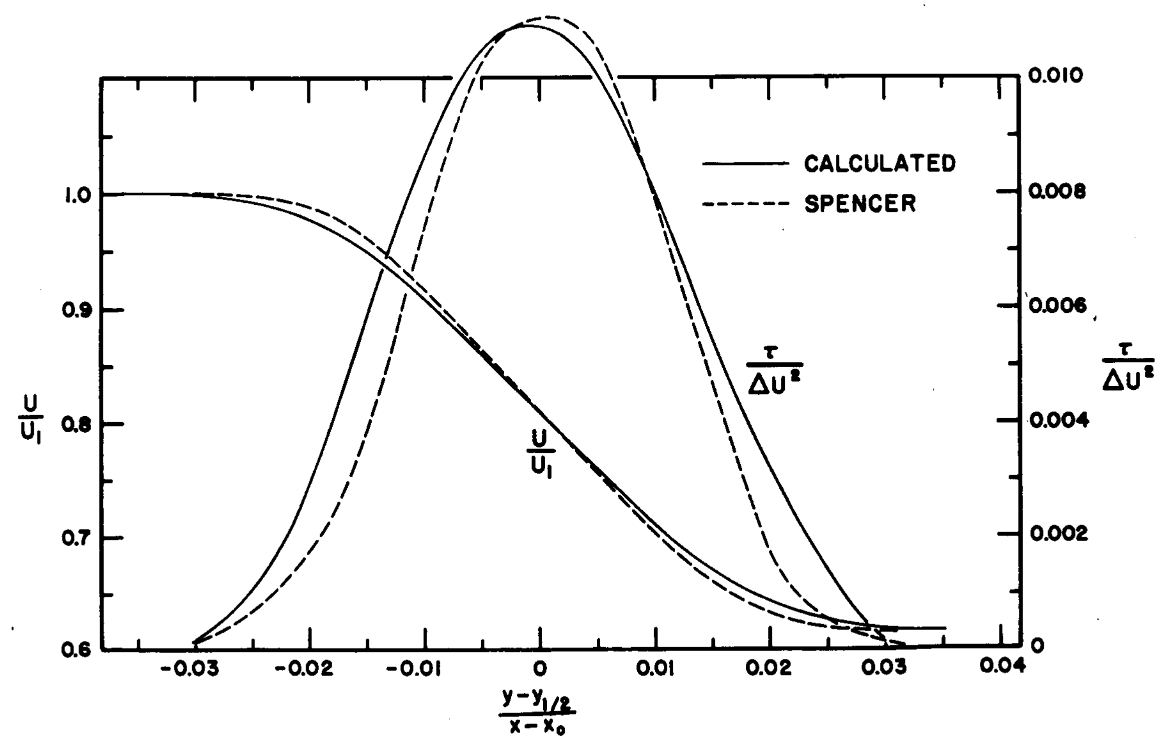


Figure 6.- Free shear layer; $U_2/U_1 = 0.61$.

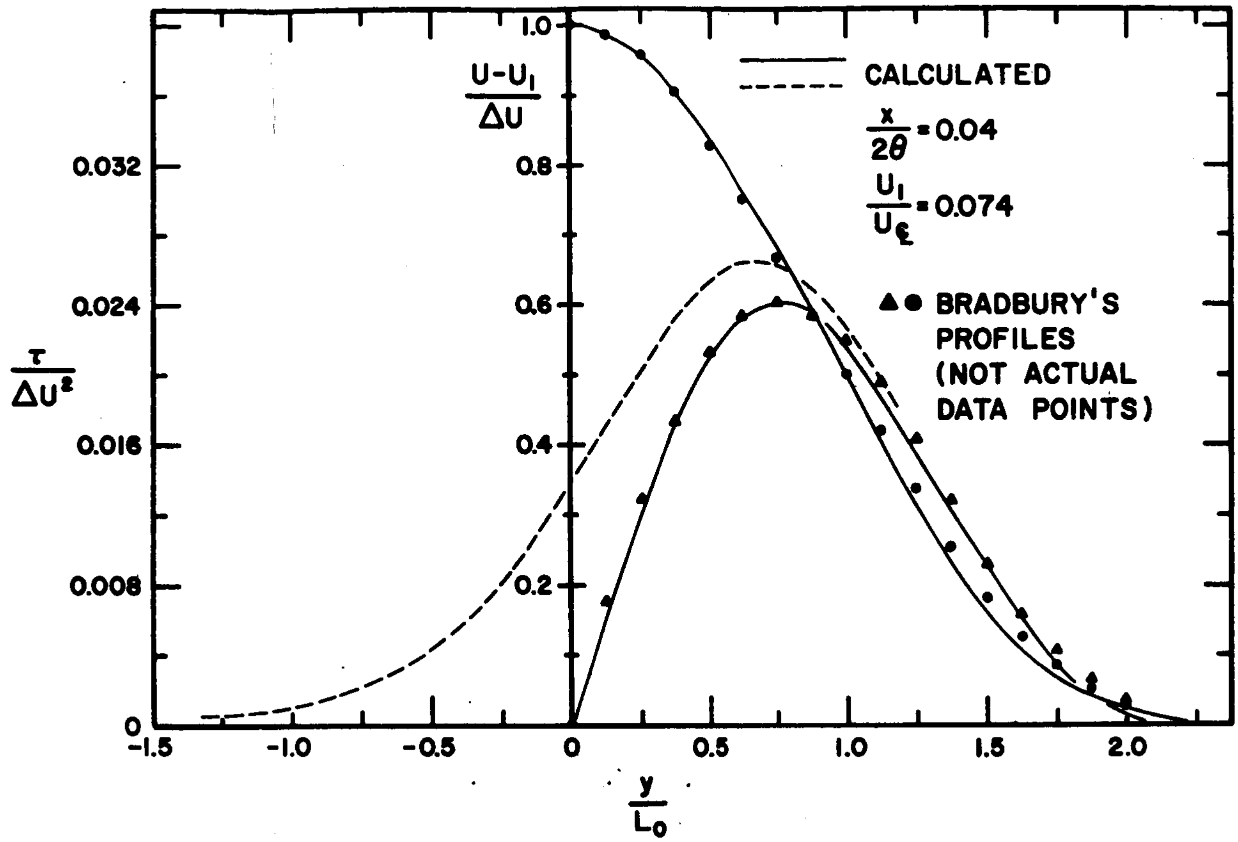


Figure 7.- Two-dimensional jet with small external flow.

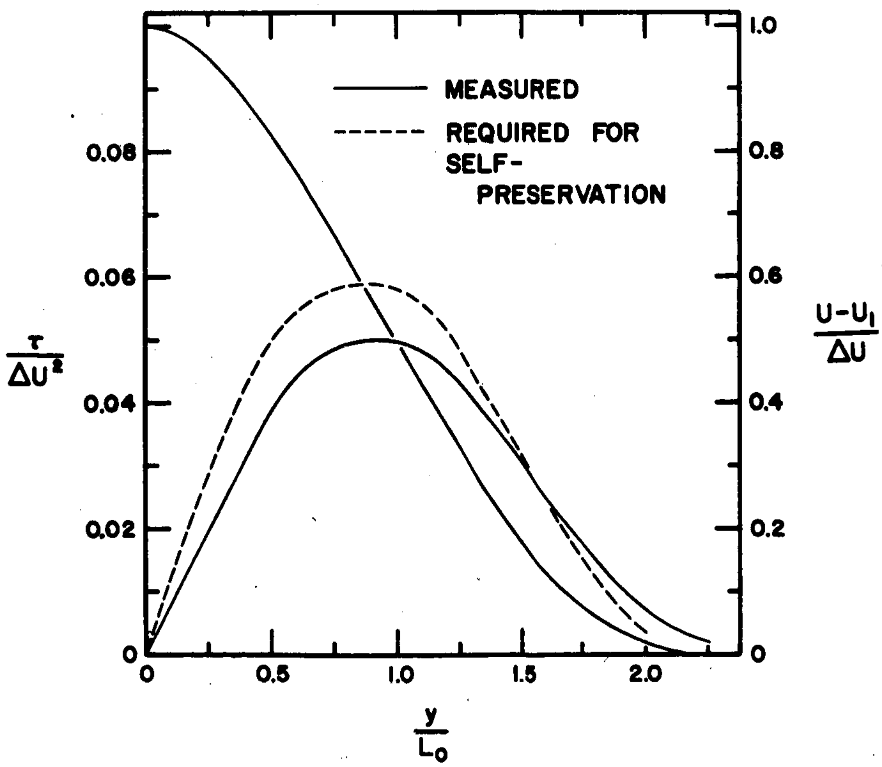


Figure 8.- Small-deficit two-dimensional wake – Townsend.

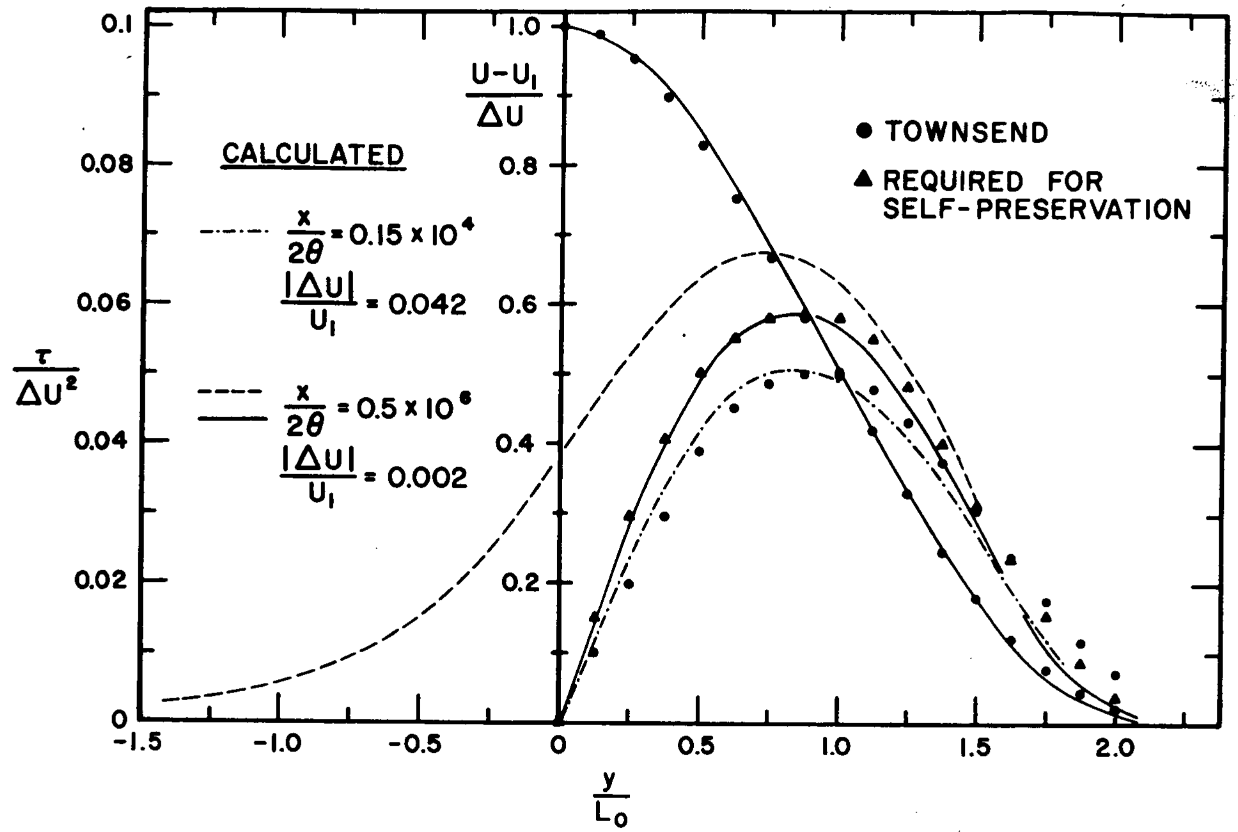


Figure 9.- Small-deficit two-dimensional wake – comparison with calculations.

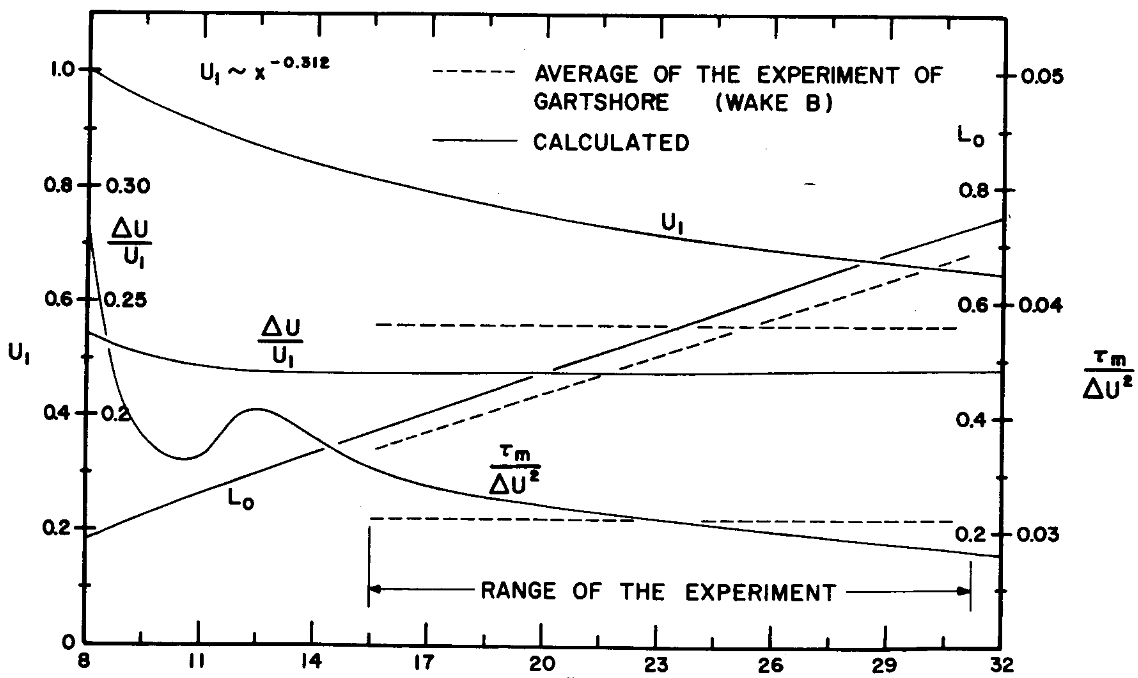


Figure 10.- Two-dimensional wake in "self-preserving" pressure gradient.

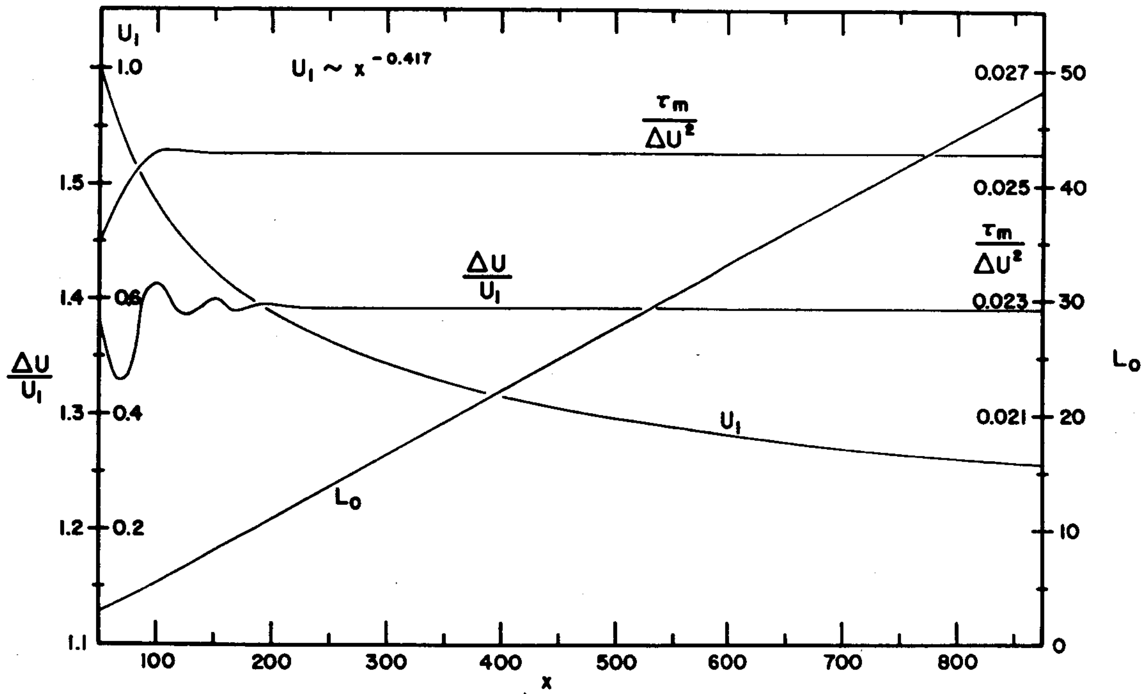
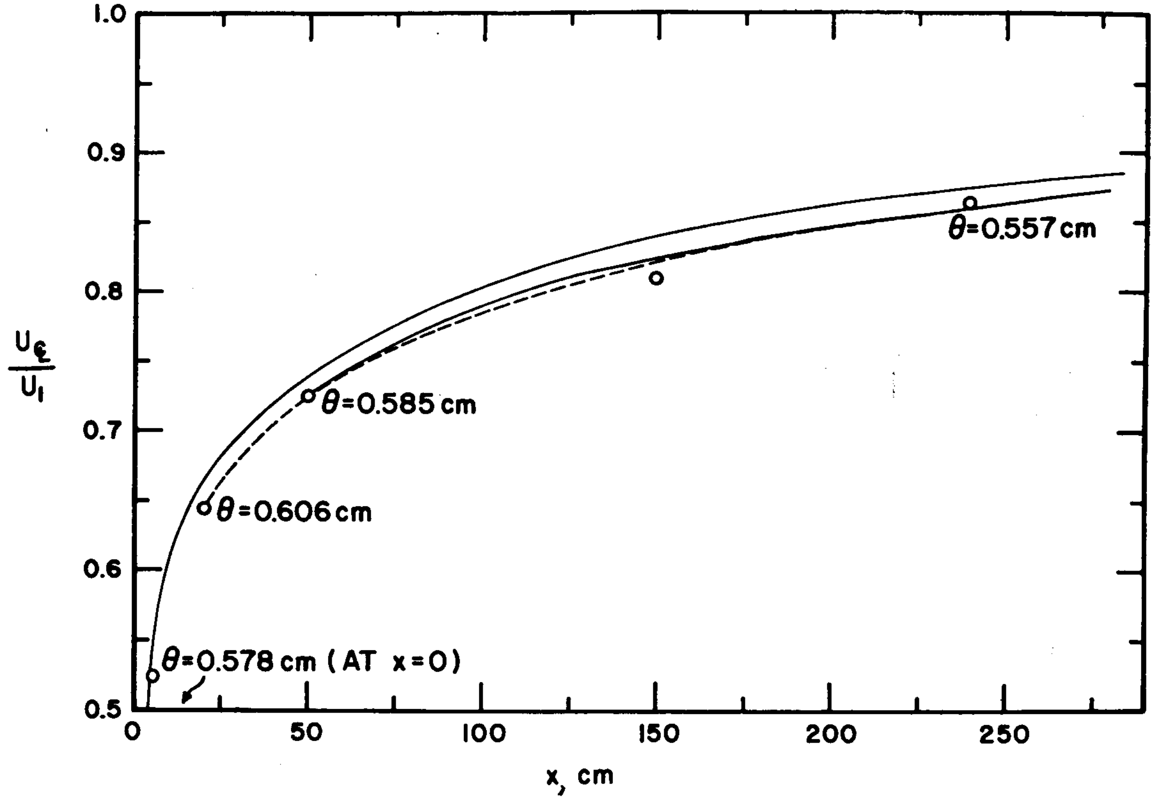
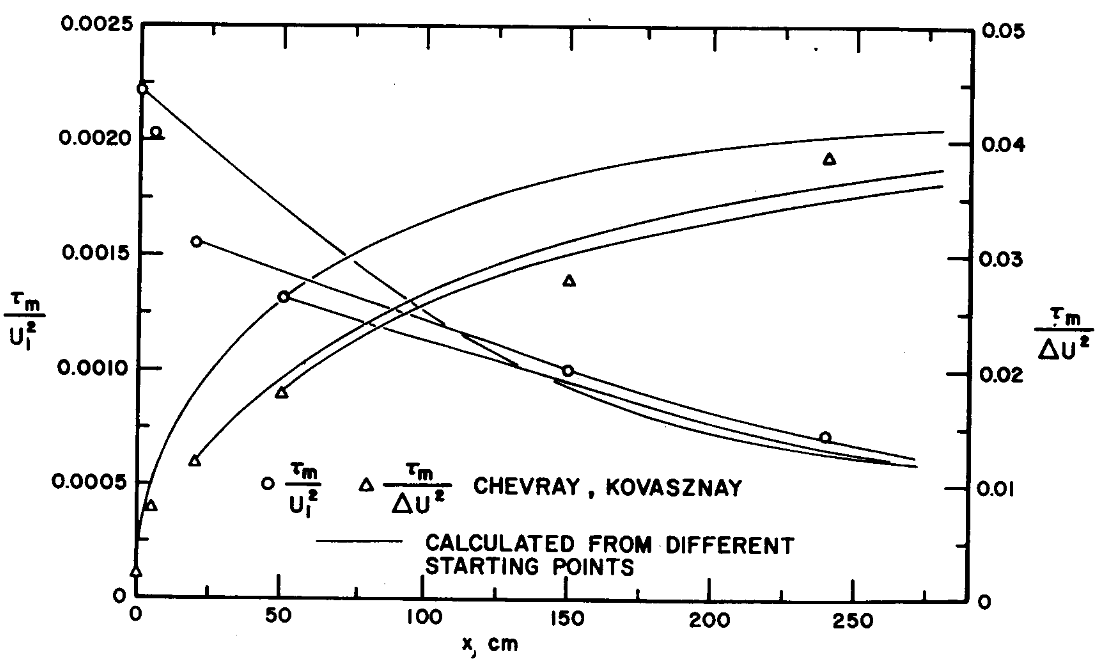


Figure 11.- Two-dimensional jet in "self-preserving" pressure gradient.

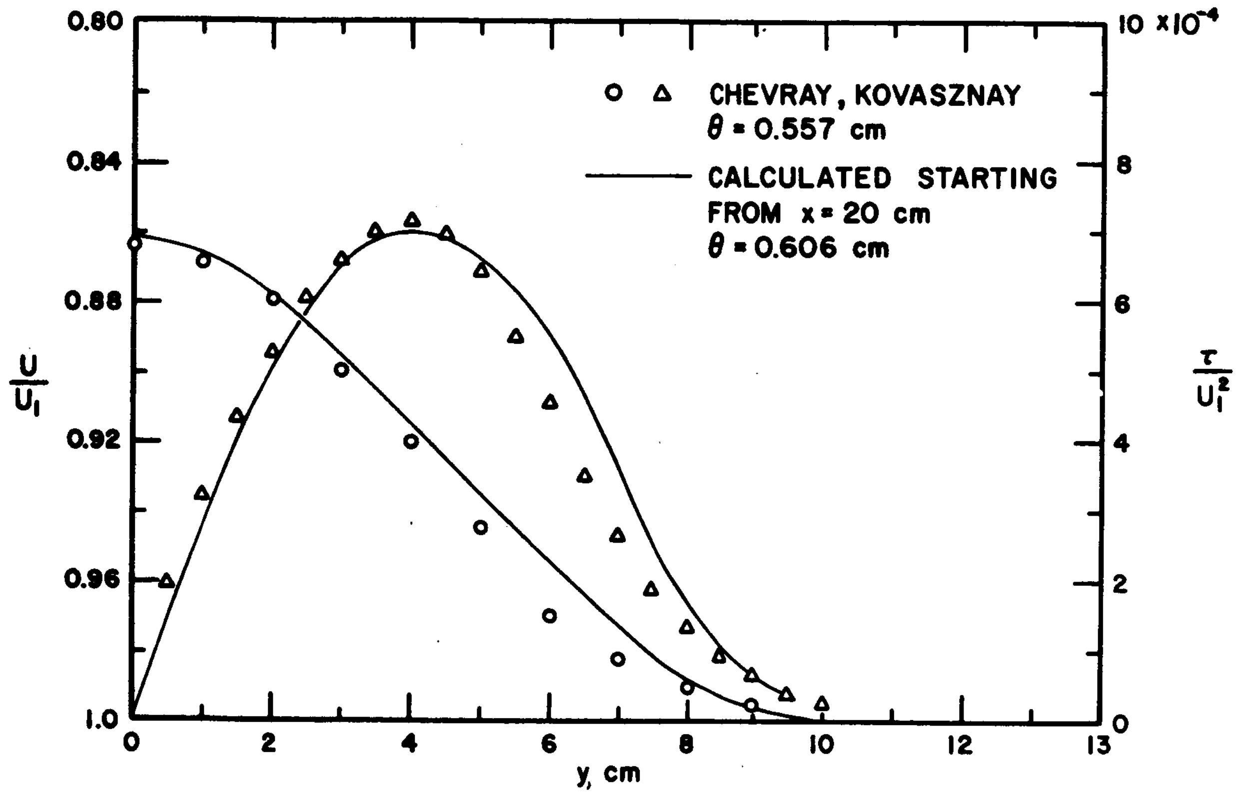


(a) Center-line velocity.



(b) Maximum shear.

Figure 12.- Two-dimensional wake of a thin flat plate.



(c) Profiles at x = 240 cm.

Figure 12.- Concluded.

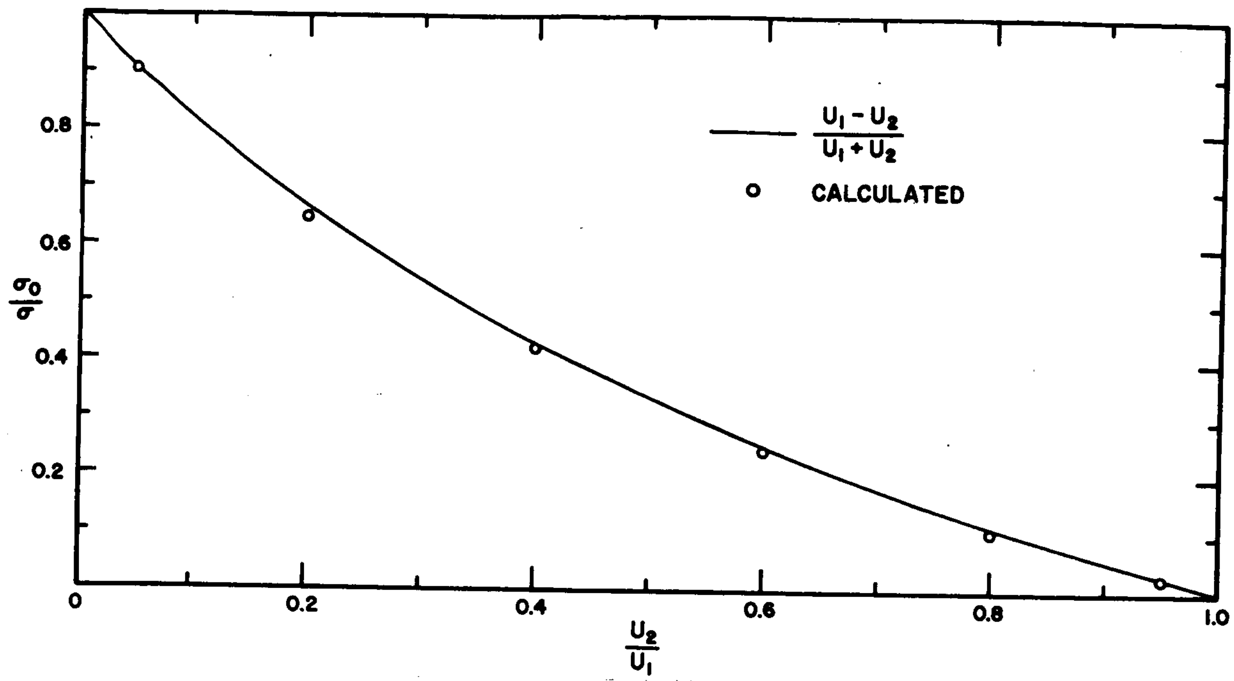


Figure 13.- Test case 1. Free shear layers – effect of the velocity ratio.

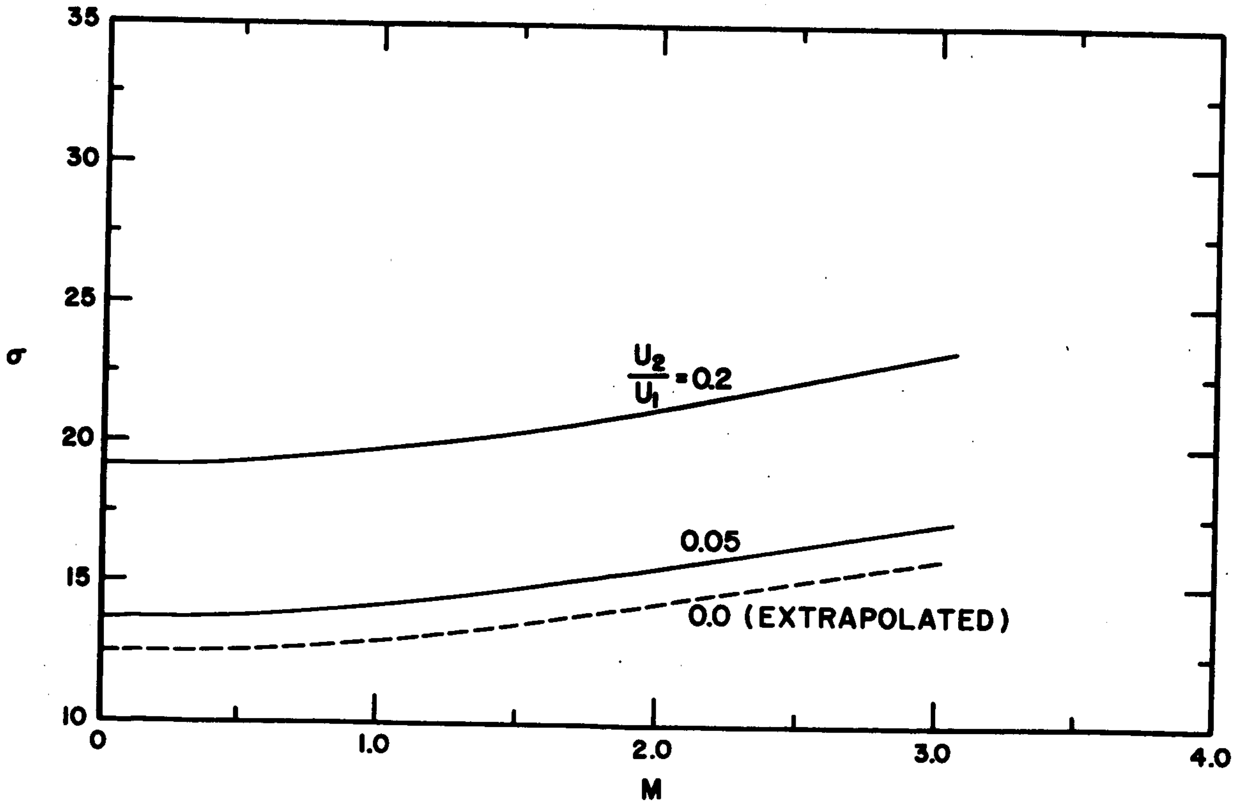


Figure 14.- Test case 2. Free shear layers – Mach number effect.

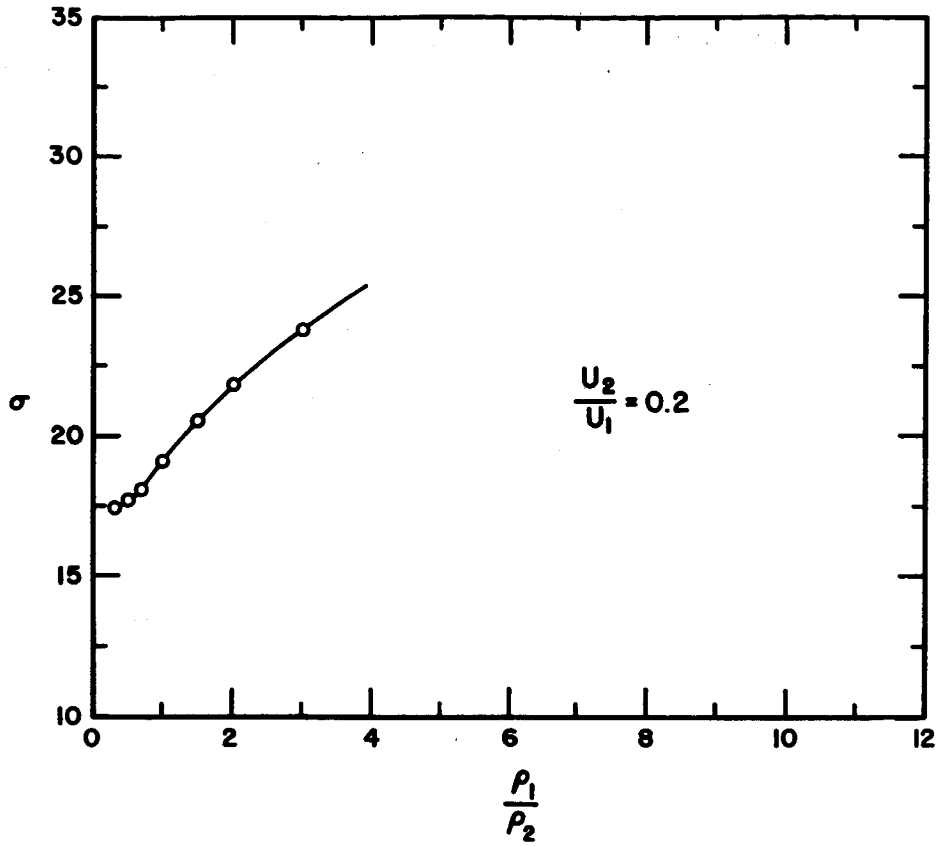


Figure 15.- Test case 3. Free shear layers – effect of the density ratio.

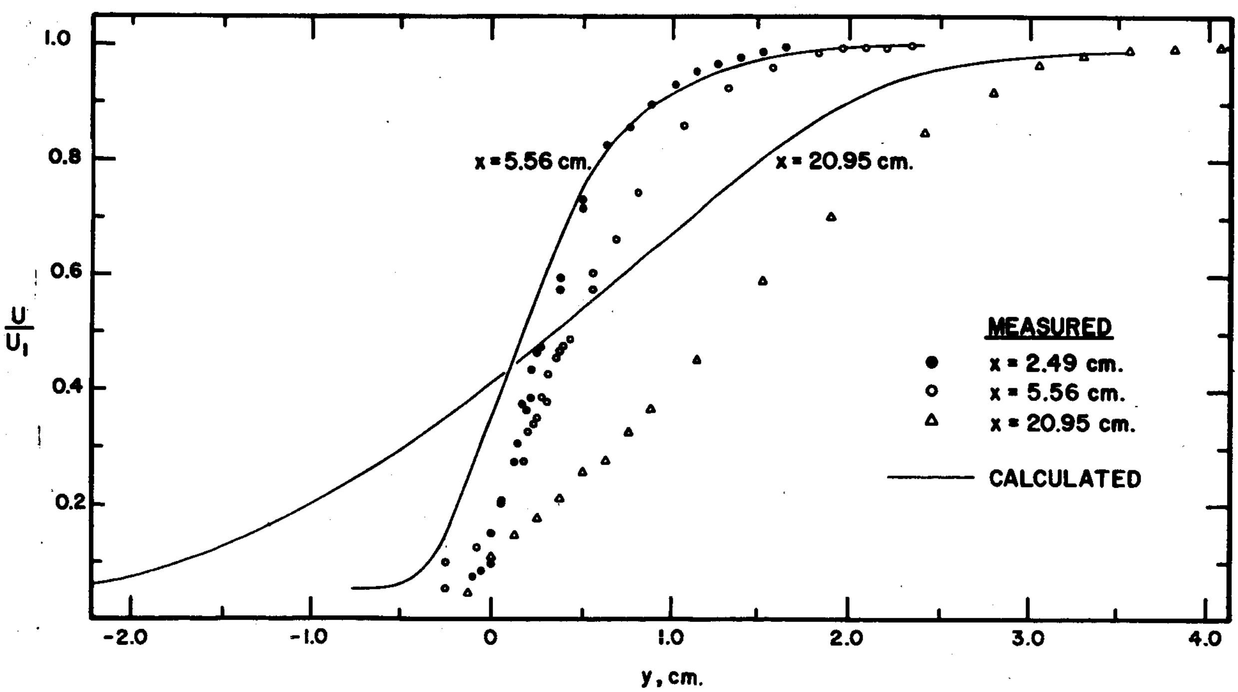


Figure 16.- Test case 5. Initial development of compressible free shear layer.

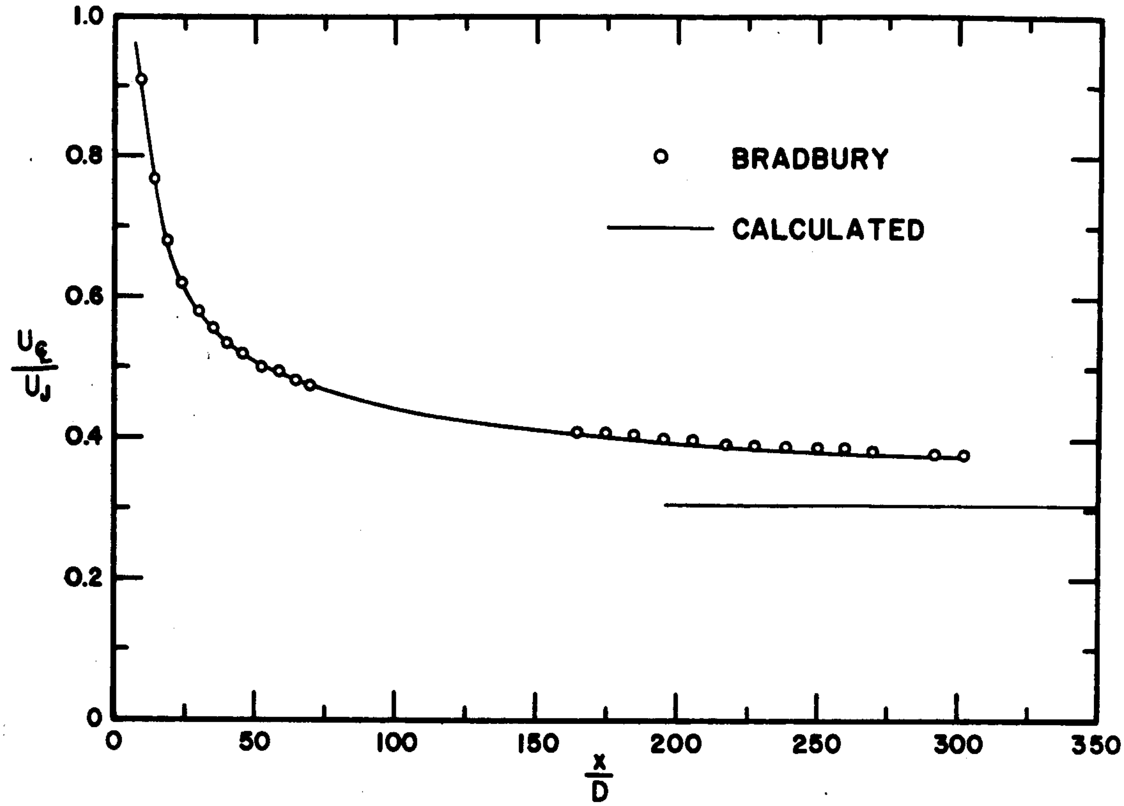


Figure 17.- Test case 13. Two-dimensional jet in a moving stream.

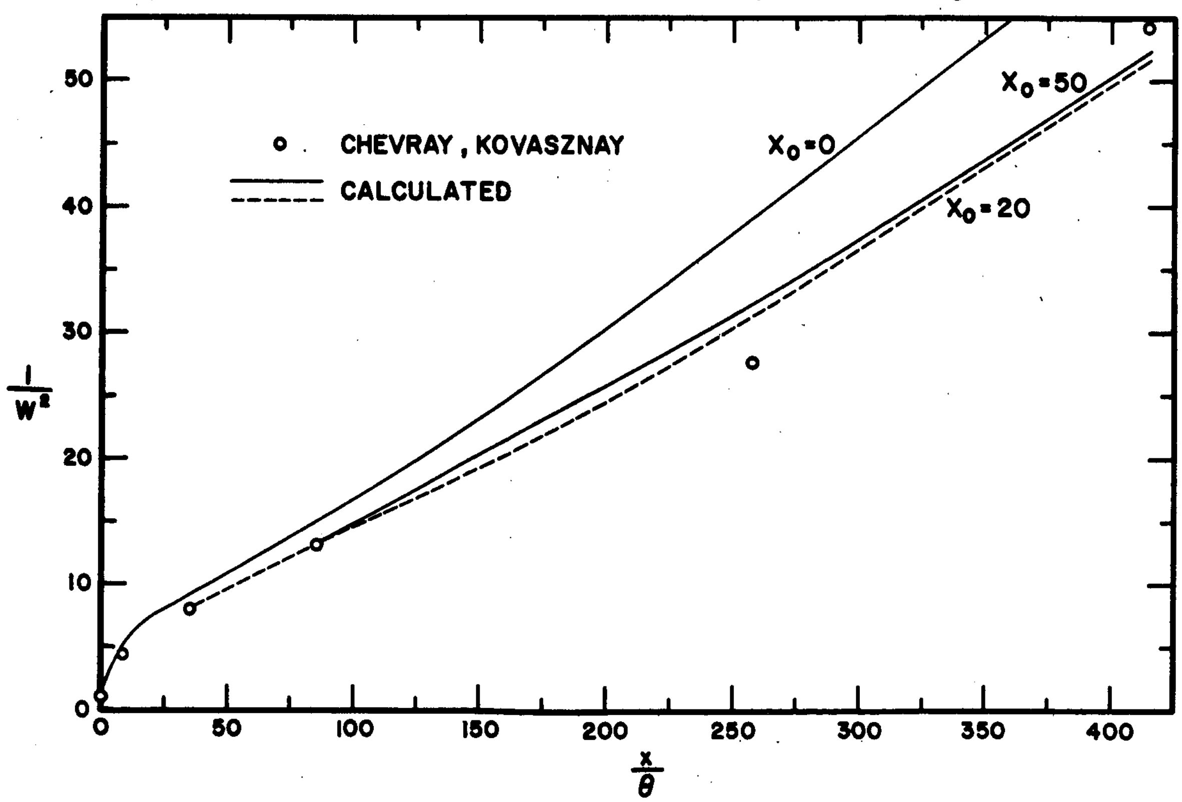


Figure 18.- Test case 14. Two-dimensional wake of a thin flat plate.

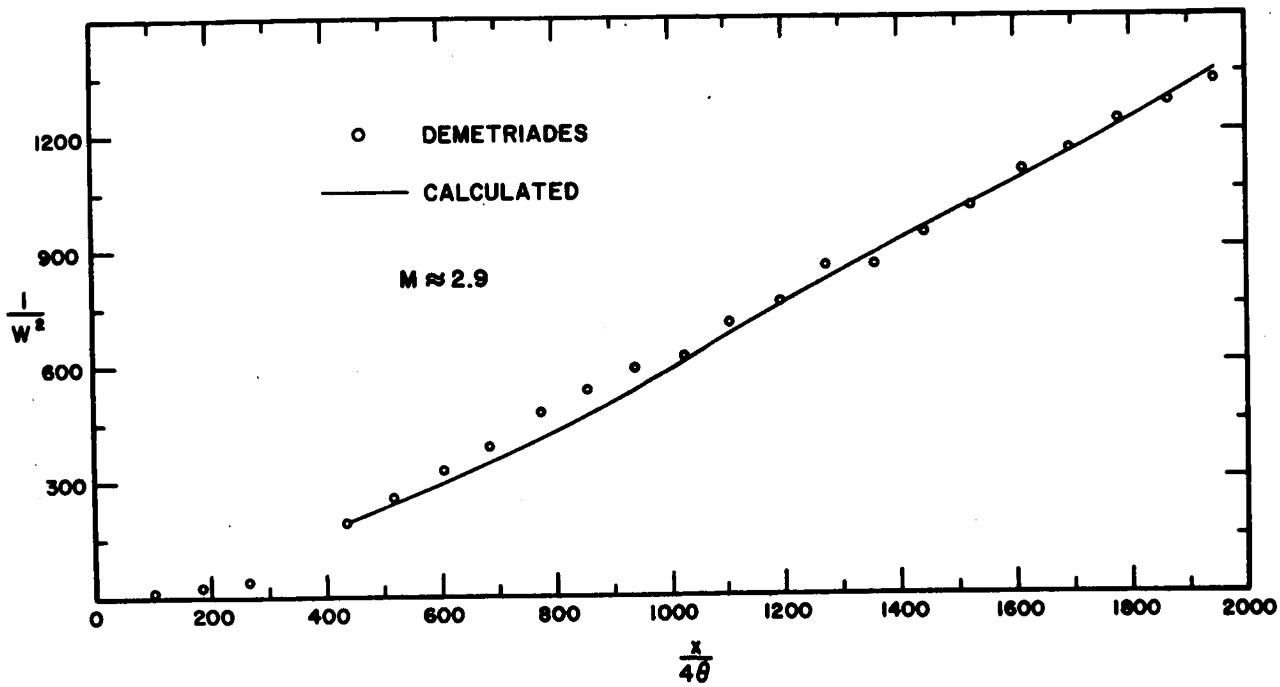


Figure 19.- Test case 16. Two-dimensional supersonic wake.

DISCUSSION

B. E. Launder: I would like to ask whether your method can be extended to cope with axisymmetric flows?

T. Morel: Presumably, one could certainly try to do that, and I would expect it would work. However, there we have to talk, strictly speaking, about an infinite number of interactions. That means we would lose the nice mental picture we were basing this on.

B. E. Launder: Doesn't it look then, that yours is a rather complicated way of doing something very simple? A more conventional simulation of the shear stress equation would permit one to treat both axisymmetric and plane shear flows.

T. Morel: The flows that we want to calculate are certainly not very simple and that is the reason we have all gathered here. We know so much more about the kinetic energy equation that we thought it worthwhile to pursue our work in this direction. Further, as I pointed out in the presentation, the interaction approach has a very important consequence. It allows us to use the kinetic energy equation to close the system without having to rely on the eddy viscosity to obtain the shear stress. This fact alone makes this work certainly worthwhile.

G. L. Mellor: One comment here – it looks like you are taking a perfectly good energy equation and turning it into a shear stress equation. And yet there closely exists a perfectly good shear stress equation. I think you get into trouble when you do that, as evidenced by qualitative argument required to avoid the jump in sign of a, when going from one sign at a channel to the other.

T. Morel: First, the exact shear stress equation is not necessarily a perfectly good equation. There are terms which we do not know enough about. The kinetic energy equation is very well documented; our results certainly seem to support that. Second, the pressure-rate-of-strain correlation in the exact shear stress equation is usually modeled as a sum of production and dissipation. When all the terms are modeled, that equation looks the same as our equation! You can ask Brian Launder about that. To your last question, if you view the flow from the point of view of separate layers, there is no jump of a_1 within either one of them. And that is precisely the point we are making.

A. Roshko: Yes, I would like to comment on Launder's comment. This question of whether what works for two-dimensional flow will work with axisymmetric flow is not so clear. For example, I have a feeling that these shear layers, in particular, have the large structure that has a lot of two-dimensionality in it. In fact our measurements show that. In a fully developed axisymmetric flow, I think that is going to have a very different structure. I think it is an instability structure. I don't think it will be axisymmetric. I think it will be skewed, and random. Therefore I am sure that the physics will be the same for those two flows.

P. T. Harsha: Your figure 11 shows some pretty mystifying wiggles in your parameter $\Delta u / u_1$. Can you explain them?

T. Morel: Well they don't mean much. You start out with some initial conditions which are away from self-preservation and watch what happens. If it is a flow that likes the self-preservation, it will tend toward it. We started a bit off, and it had to adjust. It is just trying to adjust. That's explainable.

P. A. Libby: Tom, I would like to ask a question, not directly to you, but I think it does raise a question about some of these newer methods, and perhaps some of these other people will straighten me out. For example, in the method you described, if I look at the mathematical structure, I see a first-order differential of τ with respect to y. That raises the possibility of satisfying one boundary condition with respect to τ on some line of x. In a free shear problem, of course, you want to say that τ is 0 at two points, plus or minus infinity. I don't see how you do that. I've raised similar questions with other people; they say you've got to put in molecular viscosity which puts you back into a second-order equation. But of course, if you look at similar free-mixing flows, you cannot leave that molecular viscosity in, because it destroys the similarity. That term wants to vary as $\sqrt{x}$ and the purely turbulent problem wants to go like x itself. Now this, in my view, is just a manifestation of one of the problems that enter when you look at the newer methods of solving turbulent shear problems. I would like to hear what you and other people have to say about this matter. Would you like to comment?

T. Morel: No, I really didn't quite follow what you said. I would like to talk to you afterwards.