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FORTRAN IV COMPUTER PROGRAM FOR CALCULATING CRITICAL SPEEDS OF ROTATING SHAFTS

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16. Abstract

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SUMMARY

This report describes a FORTRAN IV computer program, written for the IBM DCS 7094/7044 computer, that calculates the critical speeds of rotating shafts. The shaft may include bearings, couplings, extra masses (nonshaft mass), and disks for the gyroscopic effect. Shear deflection is also taken into account, and provision is made in the program for sections of the shaft that are tapered. The boundary conditions at the ends of the shaft can be fixed (deflection and slope equal to zero) or free (shear and moment equal to zero). The fixed end condition enables the program to calculate the natural frequencies of cantilever beams. Instead of using the lumped-parameter method, the program uses continuous integration of the differential equations of beam flexure across different shaft sections. The advantages of this method over the usual lumped-parameter method are less data preparation and better approximation of the distribution of the mass of the shaft.

A main feature of the program is the nature of the output. The Calcomp plotter is used to produce a drawing of the shaft with superimposed deflection curves at the critical speeds, together with all pertinent information related to the shaft.

INTRODUCTION

Most methods of calculating critical speeds are based on the lumped-parameter method, such as that of Prohl (ref. 1). This lumped-parameter technique consists in transforming a shaft system into a series of mass points so that their spacing approximates the distribution of the mass in the actual shaft. The next step is to integrate from mass point to mass point for the purpose of determining the shear, moment, slope, and deflection at each mass point. Consequently, in order to represent accurately the mass distribution of a shaft system, a considerable amount of data preparation is necessary.

This is especially true for sections of the shaft that are tapered, where it is necessary to approximate the taper with a series of small increments.

The method described in this report improves on the lumped-parameter method by integrating along each shaft section by means of the fourth-order numerical integration technique of Runge-Kutta. The advantages of this method are that both the time for data preparation and the chance for input error are considerably reduced and that the distribution of the mass of the shaft is better approximated.

The three main sections of this report are STATEMENT OF PROBLEM, COMPUTER PROGRAM, and the appendixes. The first section contains the statement of the problem along with the method of solution. The second section contains the program description, the FORTRAN symbols for the NAMELIST card, and the method in which a shaft system is broken down in sections and stations for proper input to the program. Also, this section presents example problems, together with printed and plotted output, demonstrating the flexibility of the program. Finally, the appendixes contain the definitions of symbols and the listing of the FORTRAN program.

STATEMENT OF PROBLEM

A shaft of variable diameter, mass density, and modulus of elasticity having bearings, couplings, disks, and extra masses (nonshaft mass) is defined to be a complex shaft system. When such a system is rotating steadily at a critical speed ω, the following boundary value problem has a solution:

$$\frac{d}{dx^2}\left(EI\,\frac{d^2Y}{dx^2}\right) = \eta\omega^2 Y \tag{1}$$

where

$$\frac{dV}{dx} = \eta\omega^2 Y \qquad \frac{dM}{dx} = V \qquad M = EI\,\frac{d^2Y}{dx^2} \qquad \frac{dY}{dx} = \theta$$

E(x) modulus of elasticity at x, $lb/in.^2$

I(x) diametral moment of inertia at x, $in.^4$

η(x) mass per unit length of a general cross section, $lb\text{-}sec^2/in.^2$

Y(x) deflection at x

M(x) moment at x

$V(x)$ shear to right of x

$\theta(x)$ slope at x

x axial position on shaft

All symbols are defined in appendix A.

The terminal boundary conditions are introduced by (1) free overhang

$$M(0) = V(0) = 0$$

or (2) fixed end

$$Y(0) = \theta(0) = 0$$

and free overhang

$$M(L) = V(L) = 0$$

The interior boundary conditions are introduced by (1) rigid bearing at station n

$$Y(x) = 0$$

$$\theta, M \text{ continuous}$$

or (2) coupling at station n

$$M(x) = 0$$

$$Y, V \text{ continuous}$$

The interior discontinuities are introduced by (1) flexible bearing at station n

$$V(x^+) = V(x^-) - KY(x)$$

(2) extra mass at station n

$$V(x^+) = V(x^-) + S_n \omega^2 Y(x)$$

or (3) gyroscopic disk at station n

$$M(x^+) = M(x^-) + (A - B)\omega^2 \theta(x)$$

where

K bearing stiffness factor (spring rate), lb/in.

A mass moment of inertia of disk about its axis of symmetry, lb-in. -sec^2

B mass moment of inertia of disk about axis through center of gravity and normal to axis of symmetry, lb-in. -sec^2

S_n nonshaft mass at station n, lb-sec^2/in.

Method of Solution

To obtain the critical speeds of a shaft system, the shaft is transformed into a series of sections and stations. Stations are axial positions on the shaft where disks, bearings, couplings, and extra masses are located or where the formulas for E, η, and I change. A section is the portion of the shaft between two stations. By using the following method on a shaft system broken down in this manner, the critical speeds can be found.

The method can best be illustrated by studying a simple uniform shaft, with no interior boundary conditions or discontinuities. The boundary conditions at both ends of the shaft are free.

Equation (1) is composed of the following first-order linear differential equations of beam flexure:

$$\frac{d}{dx} V(x) = \eta(x)\omega^2 Y(x) \tag{2}$$

$$\frac{d}{dx} M(x) = V(x) \tag{3}$$

$$\frac{d}{dx} \theta(x) = \frac{M(x)}{EI(x)} \tag{4}$$

$$\frac{d}{dx} Y(x) = \theta(x) \tag{5}$$

To obtain a simultaneous solution for this system of equations, the fourth-order Runge-Kutta method is used. The boundary conditions at $x = 0$ are used for the initial conditions. This system of equations is linear and homogeneous. The theorem of the superposition principle states that any linear combination of two solutions of a linear homogeneous differential equation is again a solution. Since $Y(0)$ and $\theta(0)$ are unspecified, we break the problem down into the following two cases:

Case 1	Case 2
$V_{01} = 0$	$V_{02} = 0$
$M_{01} = 0$	$M_{02} = 0$
$\theta_{01} = 0$	$\theta_{02} = 1$
$Y_{01} = 1$	$Y_{02} = 0$

where the subscripts denote the station and case number, respectively.

After the integration across the entire shaft is completed, the solution to the initial-value problem with $Y(0) = Y_0$ and $\theta(0) = \theta_0$ is

$$V_n = V_{n1}Y_0 + V_{n2}\theta_0 \tag{6}$$

$$M_n = M_{n1}Y_0 + V_{n2}\theta_0 \tag{7}$$

$$\theta_n = \theta_{n1}Y_0\theta + \theta_{n2}\theta_0 \tag{8}$$

$$Y_n = Y_{n1}Y_0 + Y_{n2}\theta_0 \tag{9}$$

Now by applying the boundary condition at $x = L$, $V(L) = 0$, in equation (6) and solving for θ_0 and substituting in equation (7), we have:

$$M_{L3} = Y_0\left(M_{n1} - \frac{M_{n2}V_{n1}}{V_{n2}}\right) \tag{10}$$

When an ω is used that makes M_{L3} equal zero, we have satisfied the four boundary conditions and the problem is solved. Hence, by plotting M_{L3}, in equation (10), as a function of the assumed speed ω and noting where M_{L3} becomes zero, the critical speeds are obtained.

Shear Deflection

Equation (5), which defines deflection, considers bending stresses only. There is, however, some additional deflection due to shear. Usually this is negligible; however, in cylindrical shafts with small span-to-depth ratios, stresses are likely to be high. Consequently, the deflection due to shear constitutes a considerable part of the total deflection and is taken into account by Roark (ref. 2) as follows:

$$\frac{d}{dx} Y_s(x) = \frac{F(x)V(x)}{A(x)G(x)} \tag{11}$$

where

$$G(x) = \frac{E(x)}{2(1 + \nu)} \tag{12}$$

$$F(x) = \frac{(7 + 6\nu)(1 + c^2)^2 + (20 + 12\nu)c^2}{6(1 + \nu)(1 + c^2)^2} \quad \text{(ref. 3)} \tag{13}$$

A(x) is the cross-sectional area at x, ν is Poisson's ratio (= 1/3), and $c = D_i/D_o$.

Now subtracting equation (11) from equation (5) yields

$$\frac{d}{dx} Y(x) = \theta(x) - \gamma(x)V(x) \tag{14}$$

where

$$\gamma(x) = \frac{F(x)}{A(x)G(x)}$$

Interior Boundary Conditions

In the previous section a simple uniform shaft was used to illustrate the method of calculating critical speeds. When internal boundary conditions are added (i.e., rigid bearings and couplings), creating a multispan shaft system, the method of calculating critical speeds changes only at the boundary conditions. The explanation of these changes follows.

Rigid bearing. - This type of bearing is infinitely stiff against displacement but offers no restraint to tilting of the shaft. At the point on the shaft where the rigid bearing is located, $x = x_R$, the new boundary conditions are zero deflection, continuous θ and M, and an unknown jump in V.

Equations (6) to (9) define V, M, θ, and Y in the span from $x = 0$ to $x = x_R$. Now when we apply the condition $Y(x_R) = 0$ to equation (9), it follows that

$$Y(0) = C_R \theta(0) \tag{15}$$

where

$$C_R = -\frac{Y_{n2}(x_R)}{Y_{n1}(x_R)}$$

and equations (6) to (9) simplify to

$$V_{n3} = (V_{n2} + C_R V_{n1})\theta_0 \tag{16}$$

$$M_{n3} = (M_{n2} + C_R M_{n1})\theta_0 \tag{17}$$

$$\theta_{n3} = (\theta_{n2} + C_R \theta_{n1})\theta_0 \tag{18}$$

$$Y_{n3} = (Y_{n2} + C_R Y_{n1})\theta_0 \tag{19}$$

Since $V(x_R) = V_R$ is unspecified, the problem is broken down for the next span into the following two cases:

Case 1	Case 2
$V_{R1} = 1$	$V_{R2} = 0$
$M_{R1} = 0$	$M_{+R2} = M_{-R2}(x_R) + C_R M_{-R1}(x_R)$
$\theta_{R1} = 0$	$\theta_{+R2} = \theta_{-R2}(x_R) + C_R \theta_{-R1}(x_R)$
$Y_{R1} = 0$	$Y_{R2} = 0$

For points in this span the solution to the boundary value problem with boundary conditions $\theta(0) = \theta_0$ and $V(x_R) = V_R$ is given by

$$V_{n3} = V_{n1}V_R + V_{n2}\theta_0 \tag{20}$$

$$M_{n3} = M_{n1}V_R + M_{n2}\theta_0 \tag{21}$$

$$\theta_{n3} = \theta_{n1}V_R + \theta_{n2}\theta_0 \tag{22}$$

$$Y_{n3} = Y_{n1}V_R + Y_{n2}\theta_0 \tag{23}$$

Couplings. - At the point on the shaft where a coupling is located, $x = x_C$, the new boundary conditions are zero moment, continuous Y and V, and an unknown jump in θ. Now when we apply the condition $M(x_C) = 0$ to equation (7), it follows that

$$Y(0) = C_C\theta(0) \tag{24}$$

where

$$C_C = -\frac{M_{n2}(x_C)}{M_{n1}(x_C)}$$

and equations (6) to (9) simplify to

$$V_{n3} = (V_{n2} + C_CV_{n1})\theta_0 \tag{25}$$

$$M_{n3} = (M_{n2} + C_CM_{n1})\theta_0 \tag{26}$$

$$\theta_{n3} = (\theta_{n2} + C_C\theta_{n1})\theta_0 \tag{27}$$

$$Y_{n3} = (Y_{n2} + C_CY_{n1})\theta_0 \tag{28}$$

Since $\theta(x_C) = \theta_C$ is unspecified, the problem is broken down for the next span into the following two cases:

Case 1

$V_{C1} = 0$

$M_{C1} = 0$

$\theta_{C1} = 1$

$Y_{C1} = 0$

Case 2

$V_{+C2} = V_{-C2}(x_C) + C_C(x_C)$

$M_{C2} = 0$

$\theta_{+C2} = 0$

$Y_{+C2} = Y_{-C2}(x_C) + C_C Y_{-C1}(x_C)$

This completes the treatment on interior boundary conditions.

Interior Discontinuities

This section treats interior discontinuities introduced by flexible bearings, extra masses (nonshaft mass), and disks attached to the shaft. The flexible bearings and extra masses introduce a discontinuity in shear. A disk attached at some point on the shaft introduces to the rotating system a gyroscopic effect which causes a discontinuity in the moment. The treatment of these three cases follows.

Flexible bearing. - This type of bearing is elastic against displacement but offers no restraint to tilting of the shaft. At the point on the shaft where the flexible bearing is located, the shear is changed by subtracting from it the product of the spring rate K of the bearing and the deflection Y(x)

$$V(x^+) = V(x^-) - KY(x) \tag{29}$$

Extra masses (nonshaft mass). - Masses that are attached to the shaft but do not contribute to the stiffness of the shaft bring about a change in shear at the point on the shaft where they are concentrated. By using the following equation, the shear to the right of the station where the mass is located can be computed:

$$V(x^+) = S\omega^2 Y(x) + V(x^-) \tag{30}$$

where S is the extra mass at x.

Gyroscopic effect of disks attached to shaft. - The gyroscopic effect of disks attached to the shaft can be taken into account for calculating the critical speeds by using the following formula from reference 1:

$$M(x^+) = (A - B)\omega^2\theta(x) + M(x^-) \tag{31}$$

where

A mass moment of inertia of disk about its axis of symmetry

B mass moment of inertia of disk about axis through center of gravity and normal to axis of symmetry

For a flat disk, A and B are given by

$$A = \frac{\pi\rho h D_d}{32}$$

$$B = \frac{\pi\rho h D_d^4}{64}\left[1 + \frac{4}{3}\left(\frac{h}{D_d}\right)^2\right]$$

Deflection Curve Calculation

An important phase of analyzing complex shaft systems is that of studying the deflection curve at a critical speed. One of the important aspects of a deflection curve is that it reveals where the greatest bending of the shaft is taking place at a critical speed. Knowledge of this nature enables the engineer to make the necessary changes in the shaft design to meet the requirements of the design running speed. As an aid to the design engineer, the program superimposes all deflection curves at the critical speeds on a drawing of the shaft.

By using the following formula, a value for Y_{n3} can be obtained for every x_n:

$$Y_{n3} = (Y_{n2} + C_{R,C,F}Y_{n1})\theta_0 \tag{34}$$

where C_R is used from a boundary condition to a rigid bearing and is equal to $-[Y_{n2}(x_R)/Y_{n1}(x_R)]$, C_C is used from a boundary condition to a coupling and is equal to $-[M_{n2}(x_C)/M_{n1}(x_C)]$, and C_F is used from a boundary condition to a free overhang and is equal to $-[V_{n2}(x_L)/V_{n1}(x_L)]$.

It should be noted that the deflection curve produced by equation (34) represents the shape of the deflected shaft centerline at the critical speed. It does not represent the true magnitude of the shaft deflection.

COMPUTER PROGRAM

The critical-speed program is written in FORTRAN IV language for the IBM DCS 7094/7044 computer. It is made up of a main program, six subroutines, and one library subroutine used for plotting, which is not included in this report. The program can solve a multispan shaft system for a maximum of 200 stations and any combination of bearings and couplings up to nine. See appendix B for the listing of the FORTRAN program.

The program will compute the first three critical speeds in the rpm interval specified. If more than three critical speeds are desired, another computer run is made with the rpm interval set at a higher level.

The execution time to compute a critical speed depends, of course, on the shaft length and the number of rpm increments that will be used. For example, in the section on sample computer problems, problem 1 involves a shaft 53.39 inches long with 51 shaft sections. For 150 increments of 200 rpm each, the execution time is approximately 1 minute.

A more detailed explanation of the program follows.

Program Description

Main program CSP. - The main program performs the following functions:

(1) Reads all input data

(2) Writes all input data

(3) Sets switches to regulate the flow of the program

(4) Calculates gyroscopic data (if any)

(5) Calls subroutine MAIN

Subroutine MAIN. - This subroutine (1) sets up initial conditions for numerical integration and then calls subroutine RUNGEK; (2) performs calculations for changes brought about by extra masses, flexible and rigid bearings, couplings, and gyroscopic effects; (3) calculates critical speed and writes converged value; (4) calls subroutines GRA and PPLOT for appropriate curves and drawings.

Subroutine SUBR. - This subroutine is called by RUNGEK to calculate values of the derivatives for equations (2) to (5).

Subroutine PPLOT. - This subroutine prepares data for shaft drawings.

Subroutine GRA. - This subroutine prepares data for the following curves:

(1) Deflection

(2) Bearing against critical speed

Subroutine RUNGEK. - This subroutine (1) performs the numerical integration and (2) saves numerical values of Y at every step for cases 1 and 2.

Subroutine CALPLT. - This subroutine plots the shaft and the superimposed deflection curves and calls CALTIT.

Subroutine CALTIT. - This subroutine writes Hollerith information and numbers on the plots.

FORTRAN Symbols for NAMELIST Card

The FORTRAN symbols for the NAMELIST cards are listed and defined in this section. The underlined symbols are those initialized by the program.

A array for mass moment of inertia of disk about its axis of symmetry

AI array for moments of inertia of sections

AREA array for areas of sections

BB array for mass moment of inertia of disk about axis through center of gravity and normal to axis of symmetry

BRG array for bearing spring constants:
BRG=0 - rigid bearing
BRG=-1 - coupling

DEN array for mass density of disks

DIA array for inside diameter of disks

DRPM set equal to 200

DX array for lengths of shaft sections

E array for modulus of elasticity of shaft sections

IC switch for boundary conditions at $x = 0$:
IC=1 - free overhang
IC=2 - fixed end

ICM switch used for plotting:
ICM=0 - plot desired
ICM=1 - no plot

ID array for inside diameters of shaft sections

IH,KA switches used for reading and writing data:

IH=1 and KA=0 - data to be read in with FORMAT 300

IH=1 and KA=1 - data to be read in with FORMAT 400

IH=0 and KA=0 - data already read in with FORMAT 300 or to be entered through NAMELIST card

IH=0 and KA=1 - data already read in with FORMAT 400 or to be entered through NAMELIST card

IS used as a subscript for BRG array; tells which bearing will be used to form the x-axis of the bearing-stiffness-against-critical-speed plot; IS=1

ISK switch for the gyroscopic effect:

ISK=0 - no gyroscopic data

ISK=1 - gyroscopic effect to be considered

ISK=2 - gyroscopic effect to be considered; data in computer

IXZ switch used to write out data for the rpm-against-excess-moment curve:

IXZ=0 - no writeout

IXZ=1 - data desired

KA see IH

KB total number of bearings and couplings

KL array for switches that tells program how gyroscopic data will enter:

KL=0 - THIC, DEN, DIA, and ODIA

KL=1 - A and BB

L number of shaft sections

LOC array for locations of disks for gyroscopic effect

MOD switch used to modify shaft data:

MOD=0 - no modifications

MOD=1 - modifications to be made; program reads in shaft DATA cards and returns to beginning of program

ND number of disks

NNN used to identify run number for critical-speed-against-bearing-stiffness plot; NNN=1

OD array for outside diameters of shaft sections

ODIA array for outside diameters of disks

RHO array for mass densities of shaft sections

RPM starting rpm (revolutions per minute); RPM=1

RPMF upper limit of rpm interval; RPMF=50000

RU percent used to determine maximum step size in integration; maximum step = length of shaft multiplied by RU; RU=.01

SCALE variable used for scaling plots:

SCALE=1 - full scale

SCALE=2 - double size

SCALE=.5 - one-half scale, etc.

(Because of the limitations of the Calcomp plotter, a maximum of a 5-inch radius can be drawn.)

SS array for extra masses

STA array for station numbers of bearings and couplings

TAID array for inside diameters of tapered sections

TAOD array for outside diameters of tapered sections

THIC array for thicknesses of disks

DATA INPUT

All computer runs require a title card, a NAMELIST card, and a set of shaft data cards. The title card is used for problem identification on the printed and plotted output. The NAMELIST card contains all the input parameters and program switches. The shaft data cards contain the physical and geometric properties of the shaft.

Two formats, 300 and 400, are used to read in the shaft data cards. FORMAT 300 is used for the outside and inside diameters of each shaft section, along with the physical properties of the shaft. FORMAT 400 is used for the cross-sectional areas and the moments of inertia of each shaft section, along with the physical properties of the shaft. See the section Preparation of Shaft Data Cards.

The order in which the title card, NAMELIST card, and shaft data cards enter the computer is illustrated by figure 1.

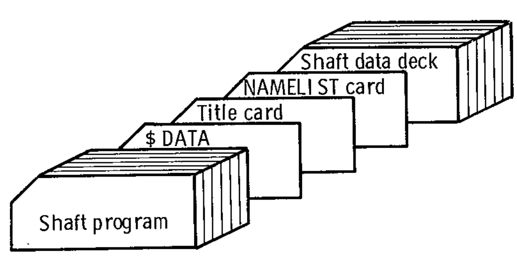

Figure 1. - Deck setup.

Computer Output

The program produces printed and plotted output. The printed output gives all the input data and the critical speeds. The program will also print the data necessary to plot the rpm-against-excess-moment curve. This option is used to confirm the converged critical speeds and is automatically printed whenever a discontinuity occurs in the excess moment.

The plotted output is not only beneficial for analysis but also for simplicity in record-keeping. The plotting routine produces a drawing of the shaft with superimposed deflection curves at the critical speed together with all pertinent information related to the shaft. Another form of plotted output is the critical-speed-against-bearing-stiffness plot. For illustrations of the output, see the section Sample Problems with Output.

In addition to the output already described, the program also prints the following error messages:

(1) Inside diameter is greater than outside diameter - shaft data card *.[1]

(2) Outside diameter of disk at station * is equal to or less than zero.

(3) Location of disk is out of range.

(4) Station location is out of range.

(5) No convergence on critical speed occurred after 35 iterations.

Preparation of Shaft Data Cards

The shaft cards contain the physical and geometric properties of the shaft. The procedure for the preparation of these cards for both tapered and untapered shafts follows:

Shaft without taper. - Figure 2 is a scaled drawing of a multispan shaft system of a two-stage turbine. The system contains four bearings and two couplings. Also, there are eight extra masses[2] (nonshaft mass) and one disk. To prepare this shaft system for the computer program, the following procedure is used:

Step 1: Vertical lines are drawn perpendicular to the shaft where bearings, couplings, extra masses, and disks are located.

---

[1]Asterisk denotes station number or card number to be printed by program.

[2]The program does not allow for an extra mass, bearing, or disk that is located at the initial end of the shaft $x = 0$. To achieve the equivalent, simply make section 0-to-1 small (0.00001) and place the bearing, disk, or extra mass at station 1.

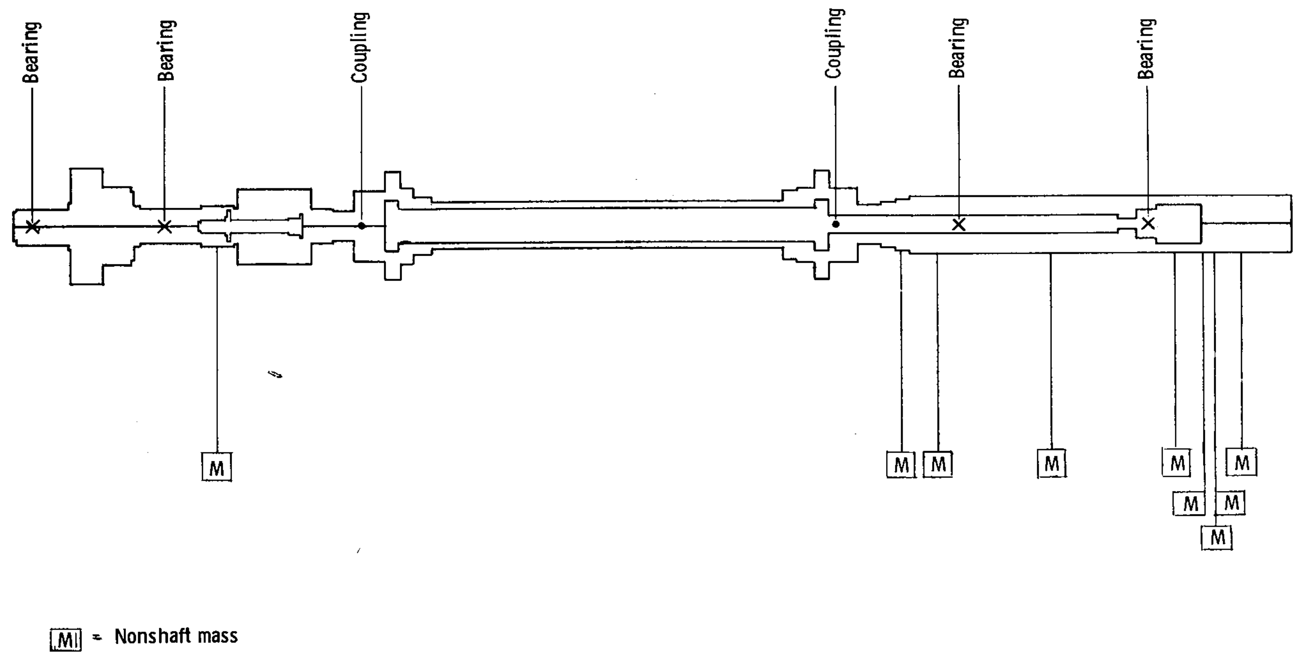

Figure 2. - Multispan shaft system for a two-stage turbine (nontapered shaft).

Step 2: Vertical lines are drawn perpendicular to the shaft where a change in every cross section of the shaft occurs, including the points $x = 0$ and $x = L$.

Step 3: Starting at the left end of the shaft, number the vertical lines 0 to n. The numbered vertical lines represent stations, and the segments between stations are sections. (Figure 3 illustrates this procedure for the first 25 sections of the shaft system of figure 2.)

Step 4: Fill out the shaft data sheet (fig. 4) with the appropriate values for each section and station.

Shaft with taper. - Figure 5 is a scaled drawing of a three-span shaft system of a seven-stage compressor with a tapered portion. The system contains two bearings and 15 extra masses.

For the tapered portion the outside and inside diameters of the left end are placed in columns 6 to 10 and 11 to 15, respectively, of the shaft data sheet (fig. 4). The outside and inside diameters of the right end are placed in columns 26 to 30 and 31 to 35, respectively. Figure 6 illustrates the procedure for the first 10 sections of the shaft system of figure 5, and figure 7 is the data sheet.

Blade preparation. - Because of the many variations in blade configurations, no set procedure is presented for the breakdown of a blade for proper input to the computer program.

In general, a blade is broken down into different sections; and each section contains the cross-sectional area, moment of inertia, modulus of elasticity, and mass density. Figure 8 shows a breakdown of a blade, and figure 9 is the blade data sheet.

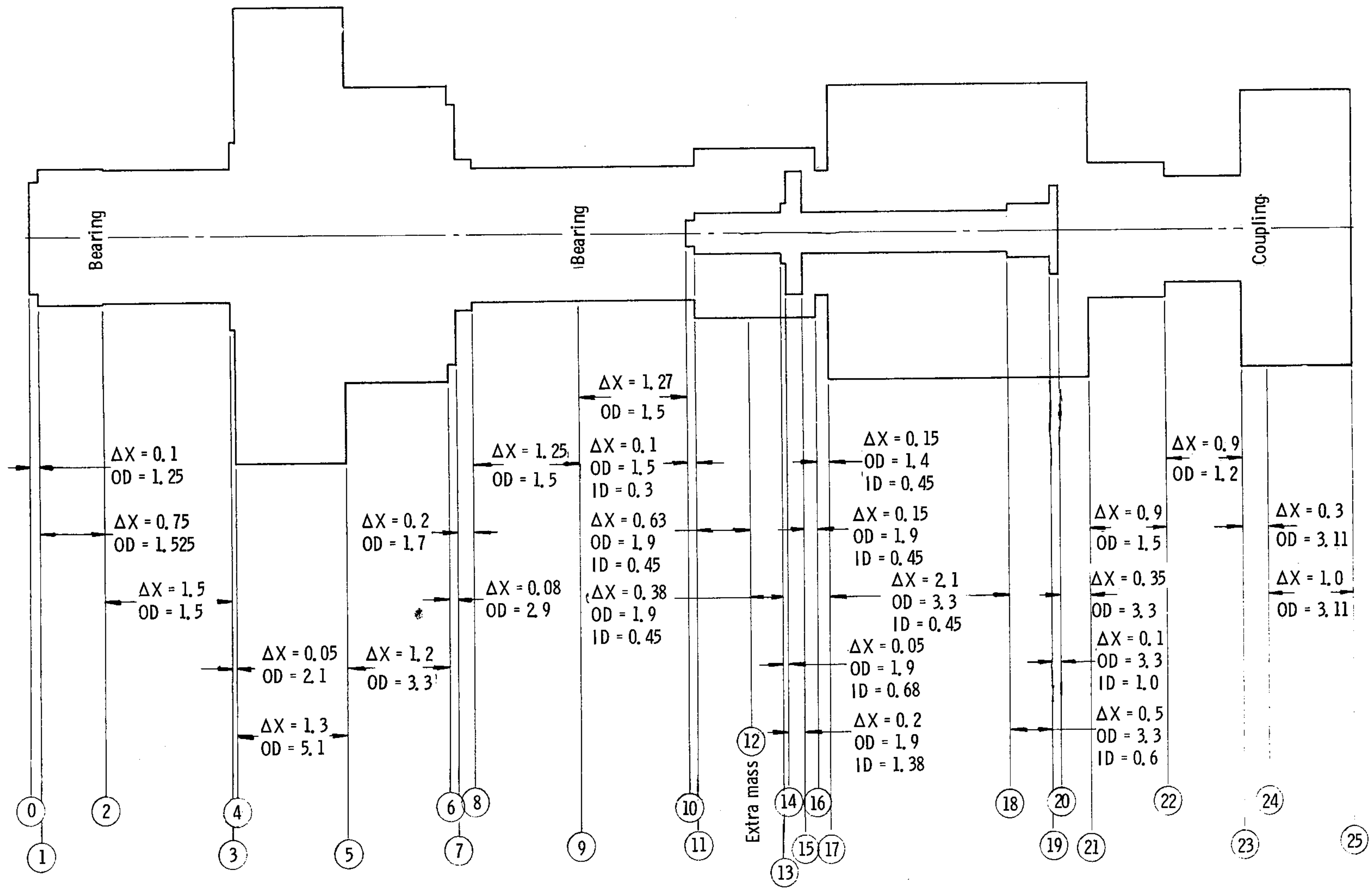

Figure 3. - Detailed breakdown of first 25 sections of multispan shaft system for a two-stage turbine (fig. 2).

Shaft section	Change in axial position, x, in.					Outside diameter, in.					Inside diameter, in.					Modulus of elasticity, lb/in.^{2}					Mass density, lb-sec^2/in.^{4}					Tapered outside diameter, in.					Tapered inside diameter, in.					Extra mass, lb-sec^2/in.^{2}, at station-					Station
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
0-1			1				1	2	5							2	8	0	+	6	7	3	3	-	5																1
1-2			7	5			1	5	2	5									+					-																	2
2-3		1	5				1	5											+					-																	3
3-4			0	5			2	1											+					-																	4
4-5		1	3				5	1											+					-																	5
5-6		1	2				3	3											+					-																	6
6-7			0	8			2	9											+					-																	7
7-8			2				1	7											+					-																	8
8-9		1	2	5			1	5											+					-																	9
9-10		1	2	7			1	5											+					-																	10
10-11			1				1	5					3						+					-																	11
11-12			6	3			1	9					4	5					+					-												0	0	5	0	6	12
12-13			3	8			1	9					4	5					+					-																	13
13-14			0	5			1	9					6	8					+					-																	14
14-15			2				1	9				1	3	8					+					-																	15
15-16			1	5			1	9					4	5					+					-																	16
16-17			1	5			1	4					4	5					+					-																	17
17-18		2	1				3	3					4	5					+					-																	18
18-19			5				3	3					6						+					-																	19
19-20			1				3	3				1							+					-																	20
20-21			3	5			3	3											+					-																	21
21-22			9				1	5											+					-																	22
22-23			9				1	2											+					-																	23
23-24			3				3	1	1										+					-																	24
24-25		1					3	1	1										+					-																	25
25-26																			+					-																	26
26-27																			+					-																	27
27-28																			+					-																	28

Figure 4. - Shaft data sheet for two-stage turbine (nontapered shaft).

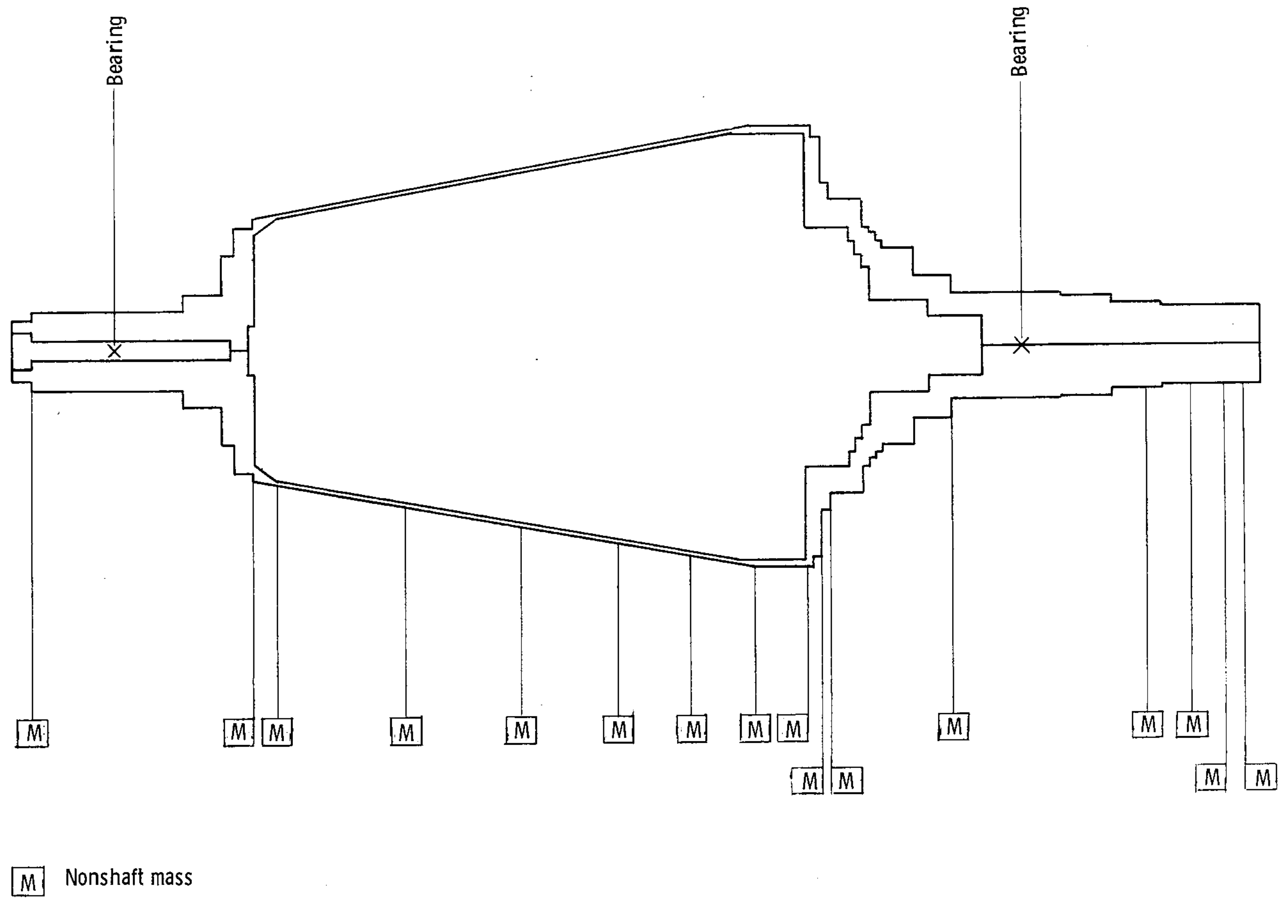

Figure 5. - Seven-stage compressor with tapered portion.

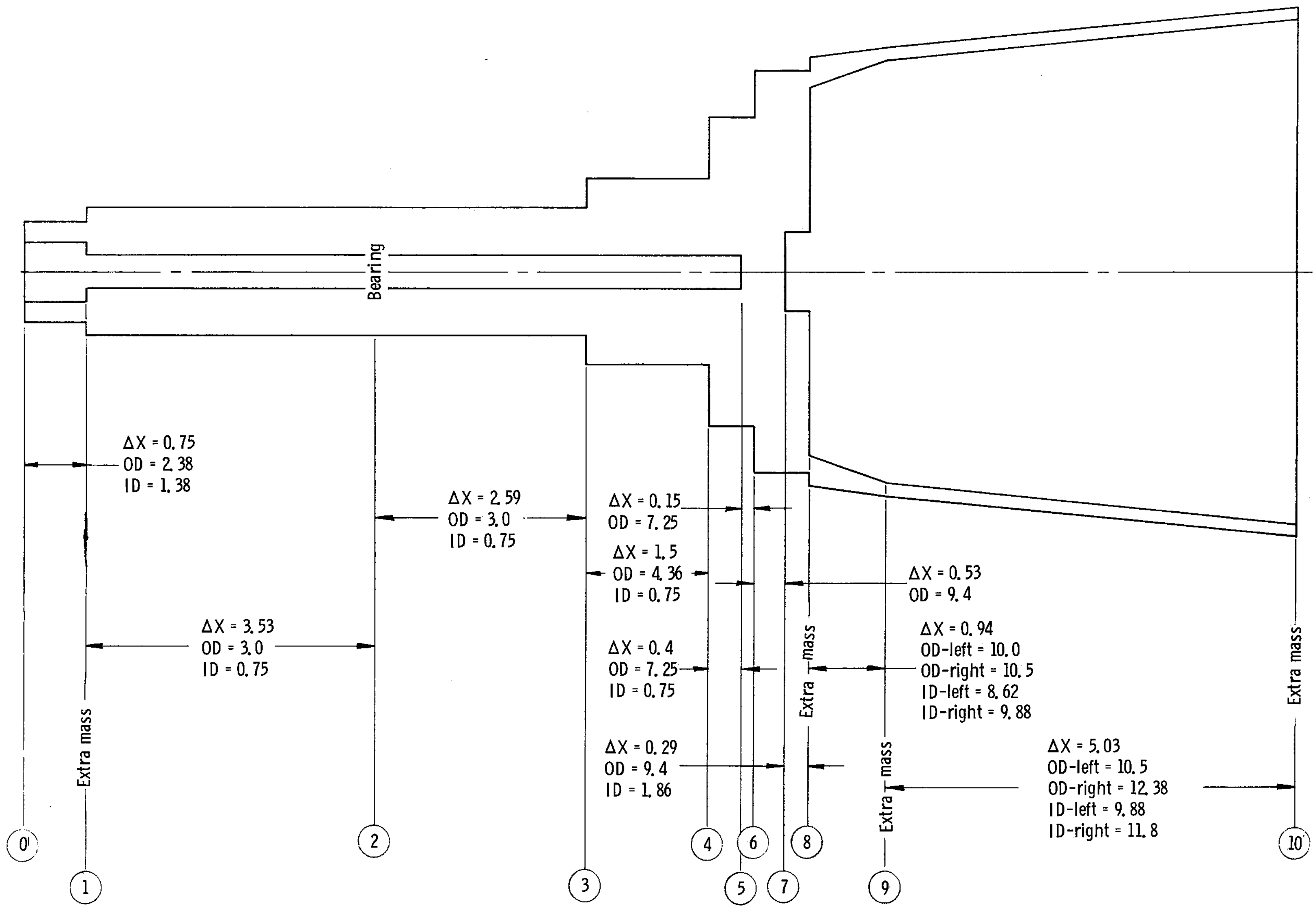

Figure 6. - Detailed breakdown of first 10 sections of seven-stage compressor with tapered portion (fig. 5).

Shaft section	Change in axial position, x, in.					Outside diameter, in.					Inside diameter, in.					Modulus of elasticity, $lb/in.^2$					Mass density, $lb\text{-}sec^2/in.^4$					Tapered outside diameter, in.					Tapered inside diameter, in.					Extra mass $lb\text{-}sec^2/in.^2$, at station-					Station
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
0-1			7	5			2	3	8			1	3	8		2	9	0	+	6	7	4	0	-	5											0	1	4	8		1
1-2		3	5	3			3						7	5			↓		+			↓		-																	2
2-3		2	5	9			3						7	5			↓		+			↓		-																	3
3-4		1	5				4	3	6				7	5			↓		+			↓		-																	4
4-5			4				7	2	5				7	5			↓		+			↓		-																	5
5-6			1	5			7	2	5			0					↓		+			↓		-																	6
6-7			5	3			9	4				0					↓		+			↓		-																	7
7-8			2	9			9	4				1	8	6			↓		+			↓		-												0	1	5	5		8
8-9			9	4		1	0					8	6	2			↓		+			↓		-		1	0	5				9	8	8		0	7	6	8		9
9-10		5	0	3		1	0	5				9	8	8			↓		+			↓		-		1	2	3	8		1	1	8			1	1	3	9		10
10-11																			+					-																	11
11-12																			+					-																	12
12-13																			+					-																	13
13-14																			+					-																	14
14-15																			+					-																	15
15-16																			+					-																	16
16-17																			+					-																	17
17-18																			+					-																	18
18-19																			+					-																	19
19-20																			+					-																	20
20-21																			+					-																	21
21-22																			+					-																	22
22-23																			+					-																	23
23-24																			+					-																	24
24-25																			+					-																	25
25-26																			+					-																	26
26-27																			+					-																	27
27-28																			+					-																	28

Figure 7. - Shaft data sheet for seven-stage compressor with tapered portion.

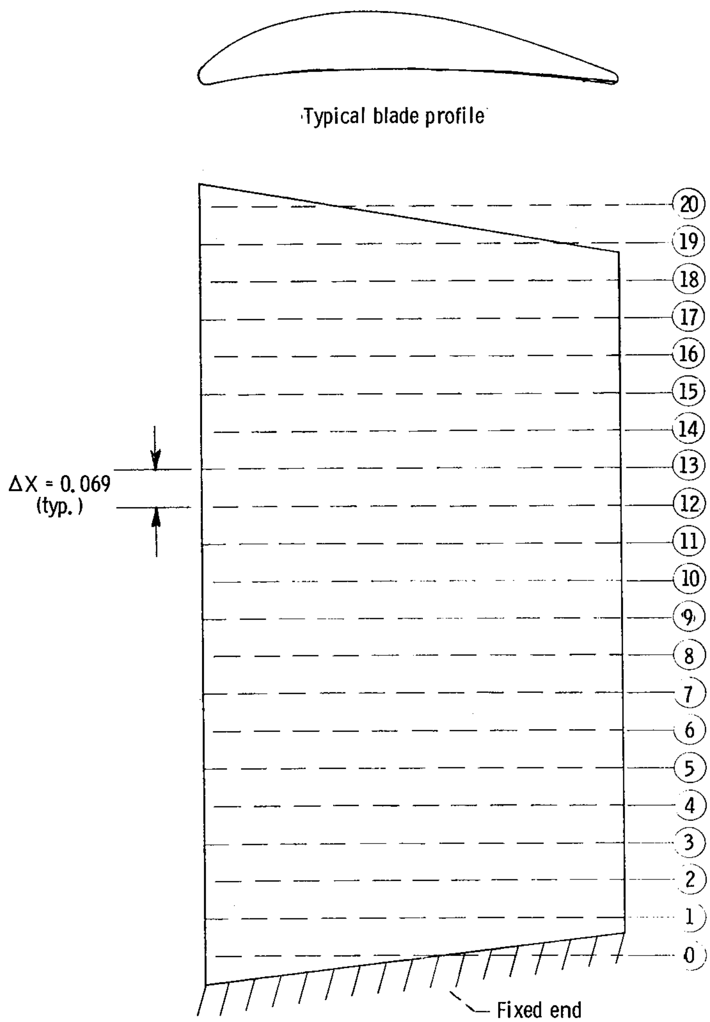

Figure 8. - Breakdown of a compressor blade.

Blade section	Change in axial position, x, in.						Area, in.^{2}									Moment of inertia, in.^{4}									Modulus of elasticity, E, lb/in.^{2}					Mass density, lb-sec^2/in.^{4}					Extra mass, lb-sec^2/in.^{4}, at station-						Station
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
0-1		0	6	9					4	9				-	1			1	7	8			-	4	2	7	5	+	6	7	5	0	-	5							1
1-2									4	7	7			-				1	6	1			-	4				+					-								2
2-3									4	6	5			-				1	4	7			-	4				+					-								3
3-4									4	5	2			-				1	3	4			-	4				+					-								4
4-5									4	4	0			-				1	2	3			-	4				+					-								5
5-6									4	8	8			-				1	1	3			-	4				+					-								6
6-7									4	1	7			-				1	0	4			-	4				+					-								7
7-8									4	0	4			-				9	5	1			-	5				+					-								8
8-9									3	9	1			-				8	6	4			-	5				+					-								9
9-10									3	7	8			-				7	8				-	5				+					-								10
10-11									3	6	5			-				7	0	1			-	5				+					-								11
11-12									3	5	3			-				6	3				-	5				+					-								12
12-13									3	4	0			-				5	6	4			-	5				+					-								13
13-14									3	2	7			-				5	0	3			-	5				+					-								14
14-15									3	1	3			-				4	4	6			-	5				+					-								15
15-16									3					-				3	9	2			-	5				+					-								16
16-17									2	8	6			-				3	4	2			-	5				+					-								17
17-18									2	7	2			-				2	9	6			-	5				+					-								18
18-19									2	5	9			-				2	5	9			-	5				+					-								19
19-20			↓						2	4	8			-				2	3	2			-	5		↓		+			↓		-								20
20-21														-									-					+					-								21
21-22														-									-					+					-								22
22-23														-									-					+					-								23
23-24														-									-					+					-								24
24-25														-									-					+					-								25
25-26														-									-					+					-								26
26-27														-									-					+					-								27
27-28														-									-					+					-								28

Figure 9. - Blade data sheet.

Sample Problems with Output

The purpose of this section is twofold: (1) to show the flexibility of the program, and (2) to illustrate specifically the preparation of the NAMELIST card for particular problems. The following six problems will be considered:

(1) Multispan shaft with bearings and couplings, as well as a disk for the gyroscopic effect, and extra masses

(2) Successive cases using same shaft data but different bearing constants

(3) Problem 1 with modified shaft data

(4) Free-free, single-span shaft

(5) Set of cases to produce bearing-stiffness-against-critical-speed plot

(6) Blade calculation with fixed-end boundary condition (cantilever beam and non-rotating uncoupled bending vibration)

The program has built-in standard options; and consequently, they need not be placed on the NAMELIST card when a specific case is run. These standard options are indicated by an underscore in the section FORTRAN Symbols for NAMELIST Card. For example, ICM=0 is underscored and therefore indicates that a plot will always be part of the output. If, however, no plot is desired, ICM=1 must be put on the NAMELIST card for the specific problem being run.

Problem 1 - multispan shaft. - The NAMELIST card described here contains the proper variables and constants that will enable the program to compute a critical speed for the shaft system in figure 2. The procedure for problem 1 is as follows:

Title card	TWO-STAGE TURBINE
NAMELIST card	$SHAFT L=51, KB=6, SCALE=.25, STA=2,9,24,34,41,45, BRG=2*50.E+03,2*-1,500.E+03,100.E+03, ISK=1, ND=1, KL=1, A=.80994, BB=.79379, LOC=50$
Shaft data cards	obtained from shaft system (fig. 2)
where	
L=51	number of shaft sections
KB=6	total number of bearings and couplings
SCALE=.25	number used for scaling plots
STA=2,9,24,34,41,45	station numbers of bearings and couplings STA(1)=2 STA(2)=9

BRG=2*50. E+03, 2* -1, 500. E+03, 100. E+03	array for bearing spring constants BRG(1)=50. E+03 denotes flexible bearing at station 2 BRG(2)=50. E+03 denotes flexible bearing at station 9 BRG(3)=-1 denotes coupling at station 24 BRG(4)=-1 denotes coupling at station 34 BRG(5)=500. E+03 denotes flexible bearing at station 41 BRG(6)=500. E+03 denotes flexible bearing at station 45
ISK=1	gyroscopic effect to be considered
ND=1	one disk for gyroscopic effect
KL=1	switch that tells that data for gyroscopic effect will enter with values of A and BB known
A=.	array for mass moment of inertia of disk about its axis of symmetry
BB=	array for mass moment of inertia of disk about axis through center of gravity and normal to axis of symmetry
LOC=50	station number where disk is located

The following variables and constants need not be placed on the NAMELIST card because they are set by the computer program: DRPM, ICM, IC, RPM, RPMF, IH, and KA. If the program is to compute the values of A and BB, the NAMELIST card should be

```
$SHAFT L=51, KB=6, SCALE=.25, STA=2,9,24,34,41,45,
BRG=2*5. E+04,2* -1,5. E+05,1. E+05, ISK=1, ND=1, LOC=50,
ODIA=5.5, DEN=.266E03, DIA=0, THIC=1.5$
```

where ODIA, DIA, THIC, and DEN are the outside and inside diameters, the thickness, and the mass density of the disk.

Figures 10 and 11 are the printed and computer-plotted output of problem 1. Figure 12 is a hand-drawn plot of excess moment as a function of rpm, for problem 1.

TWO- STAGE TURBINE

X	OD	ID	EW	E	RHO	TADD	TAID
0.1000	1.2500	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.7500	1.5250	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.5000	1.5000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.0500	2.1000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.3000	5.1000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.2000	3.3000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.0800	2.9000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.2000	1.7000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.2500	1.5000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.2700	1.5000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.1000	1.5000	0.3000	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.6300	1.9000	0.4500	0.005060	0.28000E+08	0.73300E-03	-0.	-0.
0.3800	1.9000	0.4500	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.0500	1.9000	0.6800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.2000	1.9000	1.3800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.1500	1.9000	0.4500	-0.	0.10000E+08	0.25900E-03	-0.	-0.
0.1500	1.4000	0.4500	-0.	0.10000E+08	0.25900E-03	-0.	-0.
2.1000	3.3000	0.4500	-0.	0.10000E+08	0.25900E-03	-0.	-0.
0.5000	3.3000	0.6000	-0.	0.10000E+08	0.25900E-03	-0.	-0.
0.1000	3.3000	1.0000	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.3500	3.3000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.9000	1.5000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.9000	1.2000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.3000	3.1100	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.0000	3.1100	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.6500	4.7500	2.2200	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.5100	3.1800	1.3600	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.7900	2.4000	1.3600	-0.	0.28000E+08	0.73300E-03	-0.	-0.
14.5000	2.0000	1.3600	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.1700	2.0000	1.3600	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.5200	3.0000	1.3600	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.7400	3.1900	1.3600	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.6000	4.7500	2.2500	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.4900	3.1000	0.8800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.7500	3.1000	0.8800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.0200	1.5600	0.8800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.6000	2.0000	0.8800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.2800	2.1500	0.8800	0.001400	0.28000E+08	0.73300E-03	-0.	-0.
0.2800	2.1500	0.8800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
1.1800	2.5000	0.8800	0.000643	0.28000E+08	0.73300E-03	-0.	-0.
0.6600	2.5000	0.8800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
4.1000	2.5000	0.8800	0.000908	0.28000E+08	0.73300E-03	-0.	-0.
2.6800	2.5000	0.8800	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.8600	2.5000	0.5000	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.6300	2.5000	1.3100	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.2000	2.5000	1.3100	-0.	0.28000E+08	0.73300E-03	-0.	-0.
0.6600	2.5000	1.6600	0.000643	0.28000E+08	0.73300E-03	-0.	-0.
1.2500	2.5000	1.6600	0.004300	0.28000E+08	0.73300E-03	-0.	-0.
0.4000	2.5000	-0.	0.015000	0.28000E+08	0.73300E-03	-0.	-0.
1.1400	2.5000	-0.	0.013700	0.28000E+08	0.73300E-03	-0.	-0.
2.2200	2.5000	-0.	-0.	0.28000E+08	0.73300E-03	-0.	-0.

BOUNDARY CONDITIONS---SHEAR AND MOMENT EQUAL ZERO AT X=0 AND L

BRG AT STATION 2K= 0.500000E+05

BRG AT STATION 9K= 0.500000E+05

COUPLING AT STATION 24

COUPLING AT STATION 34

BRG AT STATION 41K= 0.500000E+06

BRG AT STATION 45K= 0.100000E+06

TOTAL MASS = 0.19590678E+00

THIS RUN CONSIDERS GYROSCOPIC EFFECT

DISK AT STATION 50 A-B= 0.161500E-01

A DISCONTINUITY OCCURS AT 15000.IN THE EXCESS MOMENT VS RPM CURVE

DATA FOR THIS CURVE FOLLOWS

1ST CRITICAL SPEED = 4084.

2ND CRITICAL SPEED = 8696.

3RD CRITICAL SPEED = 15769.

0.	0.42578142E+16	200.	0.10601078E+10	400.	0.26270325E+09	600.	0.11496511E+09
800.	0.63256416E+08	1000.	0.39330808E+08	1200.	0.26344510E+08	1400.	0.18525405E+08
1600.	0.13462204E+08	1800.	0.10002868E+08	2000.	0.75405860E+07	2200.	0.57310430E+07
2400.	0.43670570E+07	2600.	0.33179008E+07	2800.	0.24977620E+07	3000.	0.18484066E+07
3200.	0.13291924E+07	3400.	0.91103281E+06	3600.	0.57266344E+06	3800.	0.29823668E+06
4000.	0.75740273E+05	4200.	-0.10407595E+06	4400.	-0.24843330E+06	4600.	-0.36304434E+06
4800.	-0.45248193E+06	5000.	-0.52044806E+06	5200.	-0.56997019E+06	5400.	-0.60354906E+06
5600.	-0.62327231E+06	5800.	-0.63089362E+06	6000.	-0.62790281E+06	6200.	-0.61557500E+06
6400.	-0.59500831E+06	6600.	-0.56715800E+06	6800.	-0.53286000E+06	7000.	-0.49285013E+06
7200.	-0.44778150E+06	7400.	-0.39823638E+06	7600.	-0.34473738E+06	7800.	-0.28775487E+06
8000.	-0.22771575E+06	8200.	-0.16500613E+06	8400.	-0.99979125E+05	8600.	-0.32956375E+05
8800.	0.35767375E+05	9000.	0.10592550E+06	9200.	0.17727375E+06	9400.	0.24959113E+06
9600.	0.32267925E+06	9800.	0.39636200E+06	10000.	0.47048712E+06	10200.	0.54492625E+06
10400.	0.61957662E+06	10600.	0.69436575E+06	10800.	0.76925337E+06	11000.	0.84423800E+06
11200.	0.91936487E+06	11400.	0.99473450E+06	11600.	0.10705186E+07	11800.	0.11469808E+07
12000.	0.12245032E+07	12200.	0.13036270E+07	12400.	0.13851142E+07	12600.	0.14700417E+07
12800.	0.15595423E+07	13000.	0.16570448E+07	13200.	0.17646734E+07	13400.	0.18879704E+07
13600.	0.20352747E+07	13800.	0.22210095E+07	14000.	0.24723412E+07	14200.	0.28471443E+07
14400.	0.34942161E+07	14600.	0.49467520E+07	14800.	0.11623892E+08	15000.	-0.11240952E+08
15200.	-0.25793776E+07	15400.	-0.98175519E+06	15600.	-0.31028778E+06	15800.	0.57058683E+05

Figure 10. - Printed output for problem 1 (multispan shaft with bearings and couplings, as well as a disk for the gyroscopic effect, and extra masses).

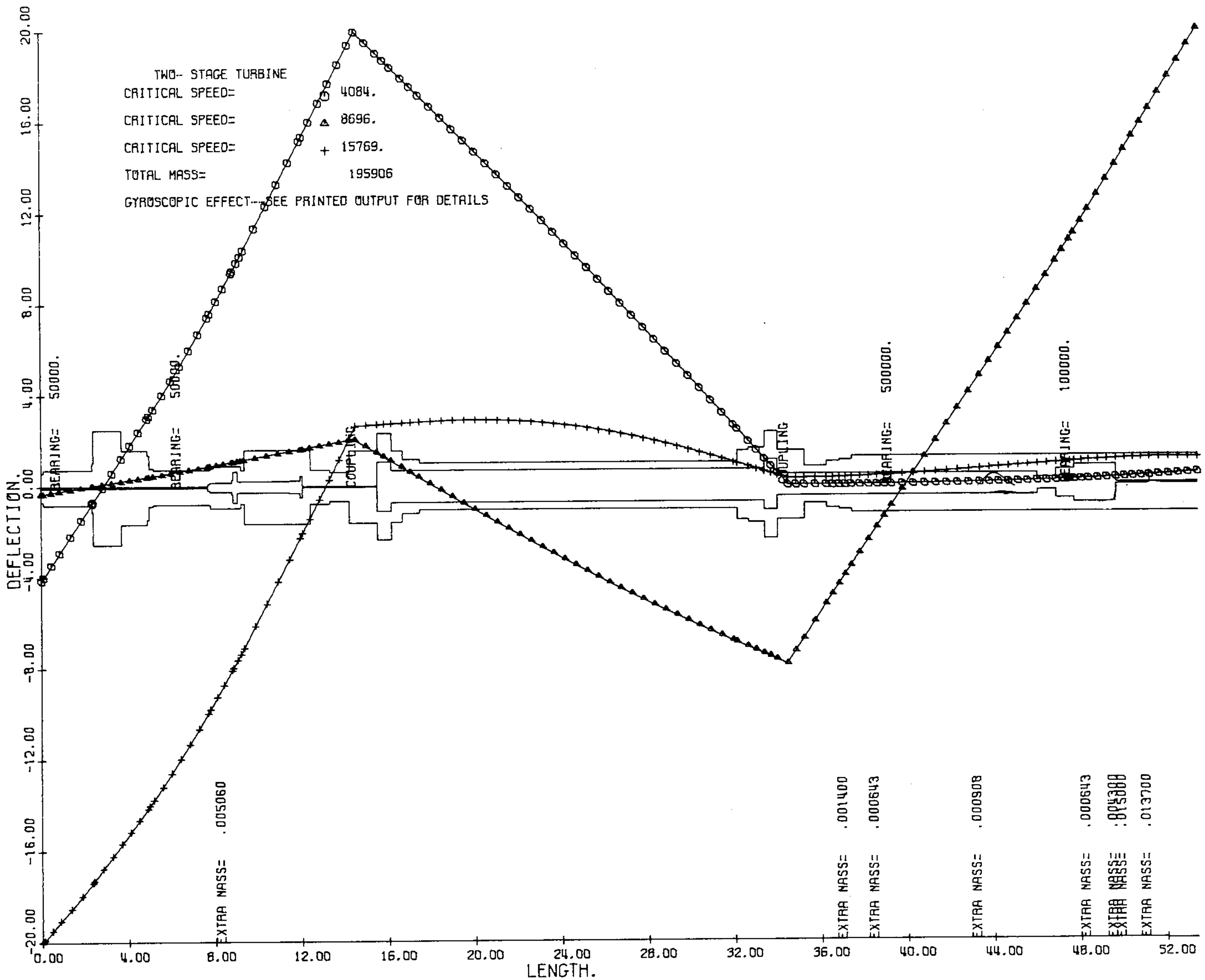

Figure 11. - Plotted output for problem 1. Total mass, 0.195906. For details of gyroscopic effect, see figure 10.

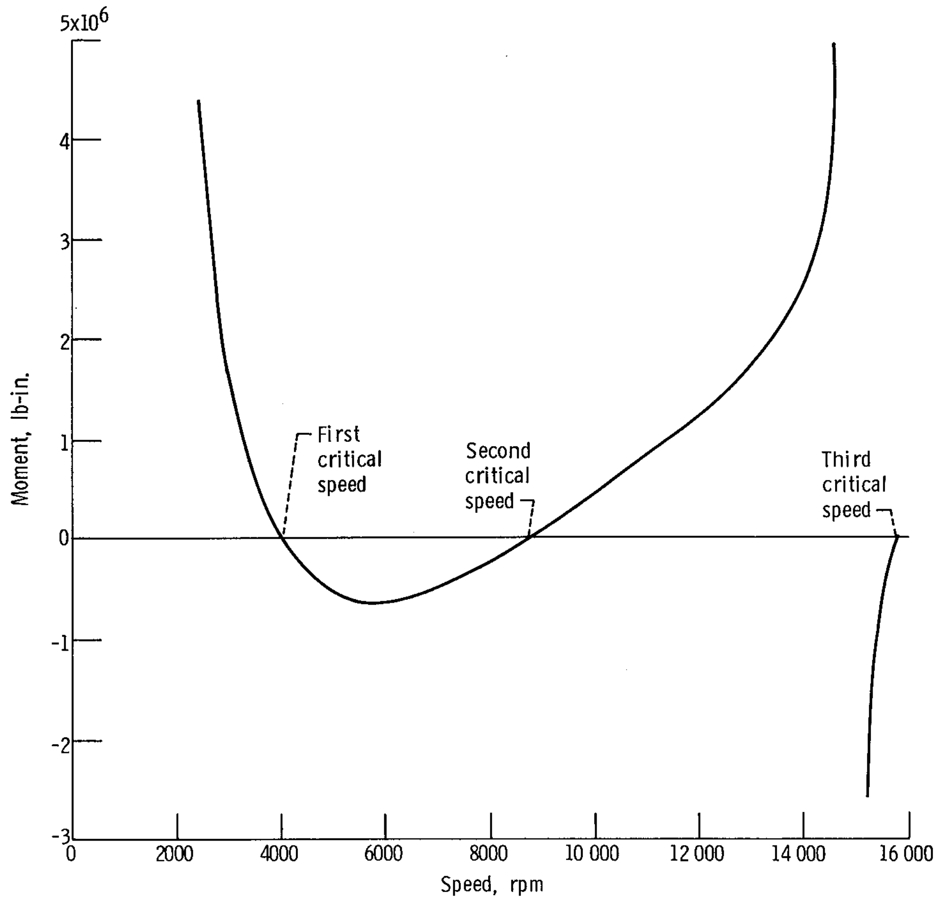

Figure 12. - Hand-drawn plot of excess moment as function of rpm, for problem 1.

Problem 2 - successive cases using same shaft data. - The procedure for problem 2 is as follows:

For case 1:

Title card	same as for problem 1
NAMELIST card	same as for problem 1
Shaft data cards	same as for problem 1

For case 2:

Title card	TWO-STAGE TURBINE BRG AT STA2=6.E+06
NAMELIST card	$SHAFT IH=0, SCALE=.25, ISK=2, BRG(1)=6.E+06$

where

IH=0	switch that indicates shaft data are in computer
ISK=2	switch that indicates gyroscopic data are in computer

BRG(1)=6. E+06 — new spring constant at station 2 (all other BRG values remain unchanged)

Problem 3 - problem 1 with modified shaft data. - A change from 2.5 inches to 3.0 inches will be made on the outside diameter for sections 39-40 and 50-51 inclusive. The procedure for problem 3 is as follows:

Title card	TWO-STAGE TURBINE - ORIGINAL DATA
NAMELIST card	$SHAFT L=51, MOD=1$
Shaft data cards	Same as for problem 1, where MOD=1 is the switch that returns flow of program to beginning, after shaft data cards are read
Title card	TWO-STAGE TURBINE - MODIFIED SHAFT DATA
NAMELIST card	Same as for problem 1, plus IH=0, OD(40)=12*3. $
OD(40)=12*3.	outside diameter of 3 inches for 12 consecutive sections starting at section 39-40

Problem 4 - free-free, single-span shaft. - The procedure for problem 4 is as follows:

Title card	TWO STAGE TURBINE - NO BEARINGS OR COUPLINGS
NAMELIST card	$SHAFT L=51, KB=0, SCALE=.25$
Shaft data cards	Same as for problem 1

Problem 5 - bearing-stiffness-against-critical-speed plot. - The NAMELIST cards for this problem illustrate the ability of the program to produce a bearing-stiffness-against-critical-speed plot. This plot requires six computer runs that use the same shaft data cards. The scale on the x-axis of this plot is determined by changing the spring constant of the same bearing for each run. The first computer run has the smallest bearing constant, the second has the next larger, and so forth. The data used for these runs represent the shaft system of figure 5.

Case 1:

Title card	SEVEN-STAGE COMPRESSOR - CASE 1
NAMELIST card	$SHAFT L=39, STA=2,31, BRG=2*300.E+03, SCALE=.25, KB=2$
Shaft data cards	obtained from shaft system of figure 2

Case 2:

Title card	CASE 2
NAMELIST card	$SHAFT IH=0, NNN=2, BRG=2*500.E+03, SCALE=.25$

Case 3:

Title card	CASE 3
NAMELIST card	$SHAFT IH=0, NNN=3, BRG=2*700.E+03, SCALE=.25$

Case 4:

Title card	CASE 4
NAMELIST card	$SHAFT IH=0, NNN=4, BRG=2*100.E+04, SCALE=.25$

Case 5:

Title card	CASE 5
NAMELIST card	$SHAFT IH=0, NNN=5, BRG=2*150.E+04, SCALE=.25$

Case 6:

Title card	CASE 6
NAMELIST card	$SHAFT IH=0, NNN=6, BRG=2*200.E+04, SCALE=.25$

where NNN is the number used to identify run number and is used only when a bearing-stiffness-against-critical-speed plot is desired. Figures 13 and 14 are printed and computer-plotted output of case 1. Figure 15 is the bearing-stiffness-against-critical-speed plot for cases 1 to 6.

Problem 6 - blade calculation with fixed end boundary condition. - This problem involves a cantilever beam having nonrotating, uncoupled bending vibration. The procedure for problem 6 is as follows:

Title card	BLADE CALCULATION - FIXED END
NAMELIST card	$SHAFT L=20, KB=0, IC=2, KA=1,DRPM=1000., RPMF=75000. $

where

IC=2	switch for fixed end boundary condition at $x = 0$
KA=1	data read in by FORMAT 400
Blade data cards	obtained from blade illustrated in figure 8

Figure 16 is the printed output of problem 6.

TEST PROBLEM TAPERED SHAFT

X	OD	ID	EW	E	RHO	TAOD	TAID
0.7500	2.3800	1.3800	0.014800	0.29000E+08	0.74000E-03	-0.	-0.
3.5300	3.0000	0.7500	-0.	0.29000E+08	0.74000E-03	-0.	-0.
2.5900	3.0000	0.7500	-0.	0.29000E+08	0.74000E-03	-0.	-0.
1.5000	4.3600	0.7500	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.4000	7.2500	0.7500	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.1500	7.2500	0.	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.5300	9.4000	0.	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.2900	9.4000	1.8600	0.015500	0.29000E+08	0.74000E-03	-0.	-0.
0.9400	10.0000	8.6200	0.076800	0.29000E+08	0.74000E-03	10.5000	9.8800
5.0300	10.5000	9.8800	0.113900	0.29000E+08	0.74000E-03	12.3800	11.8000
4.6500	12.3800	11.8000	0.116800	0.29000E+08	0.74000E-03	14.0000	13.5000
3.8600	14.0000	13.5000	0.103400	0.29000E+08	0.74000E-03	15.2000	14.7000
3.0000	15.2000	14.7000	0.086400	0.29000E+08	0.74000E-03	16.2000	15.7000
2.5000	16.2000	15.7000	0.086600	0.29000E+08	0.74000E-03	16.8400	16.2400
2.1700	16.8400	16.2400	0.002500	0.29000E+08	0.74000E-03	-0.	-0.
0.2500	16.8400	9.2400	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.4100	15.9600	9.2400	0.015200	0.29000E+08	0.74000E-03	-0.	-0.
0.3000	12.5000	9.2400	0.105200	0.29000E+08	0.74000E-03	-0.	-0.
0.7800	11.1600	9.2400	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.2700	11.1600	8.1600	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.2400	11.1600	7.2200	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.0700	11.1600	6.1000	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.1500	9.1200	6.1000	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.0600	8.4400	6.1000	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.2800	8.4400	3.6200	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.2400	7.9000	3.6000	-0.	0.29000E+08	0.74000E-03	-0.	-0.
1.2700	7.5000	3.6200	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.5800	5.5600	3.6200	-0.	0.29000E+08	0.74000E-03	-0.	-0.
1.0000	5.5600	2.2500	0.014400	0.29000E+08	0.74000E-03	-0.	-0.
1.1000	4.0000	2.2500	-0.	0.29000E+08	0.74000E-03	-0.	-0.
1.8400	4.0000	-0.	-0.	0.29000E+08	0.74000E-03	-0.	-0.
1.5100	4.0000	-0.	-0.	0.29000E+08	0.74000E-03	-0.	-0.
1.9900	3.9200	-0.	-0.	0.29000E+08	0.74000E-03	-0.	-0.
1.1300	3.2500	-0.	0.055400	0.29000E+08	0.74000E-03	-0.	-0.
0.8400	3.2500	-0.	-0.	0.29000E+08	0.74000E-03	-0.	-0.
0.9600	3.0000	-0.	0.010200	0.29000E+08	0.74000E-03	-0.	-0.
1.2800	3.0000	-0.	0.006700	0.29000E+08	0.74000E-03	-0.	-0.
0.8100	3.0000	-0.	0.079200	0.29000E+08	0.74000E-03	-0.	-0.
0.7600	3.0000	-0.	-0.	0.29000E+08	0.74000E-03	-0.	-0.

BOUNDARY CONDITIONS---SHEAR AND MOMENT EQUAL ZERO AT X=0 AND L

BRG AT STATION 2K= 0.300000E+06

BRG AT STATION 31K= 0.300000E+06

TOTAL MASS = 0.14907389E+01

1ST CRITICAL SPEED = 5694.

2ND CRITICAL SPEED = 9031.

3RD CRITICAL SPEED = 25432.

Figure 13. - Printed output for problem 5 (bearing-stiffness-against-critical-speed plot), case 1.

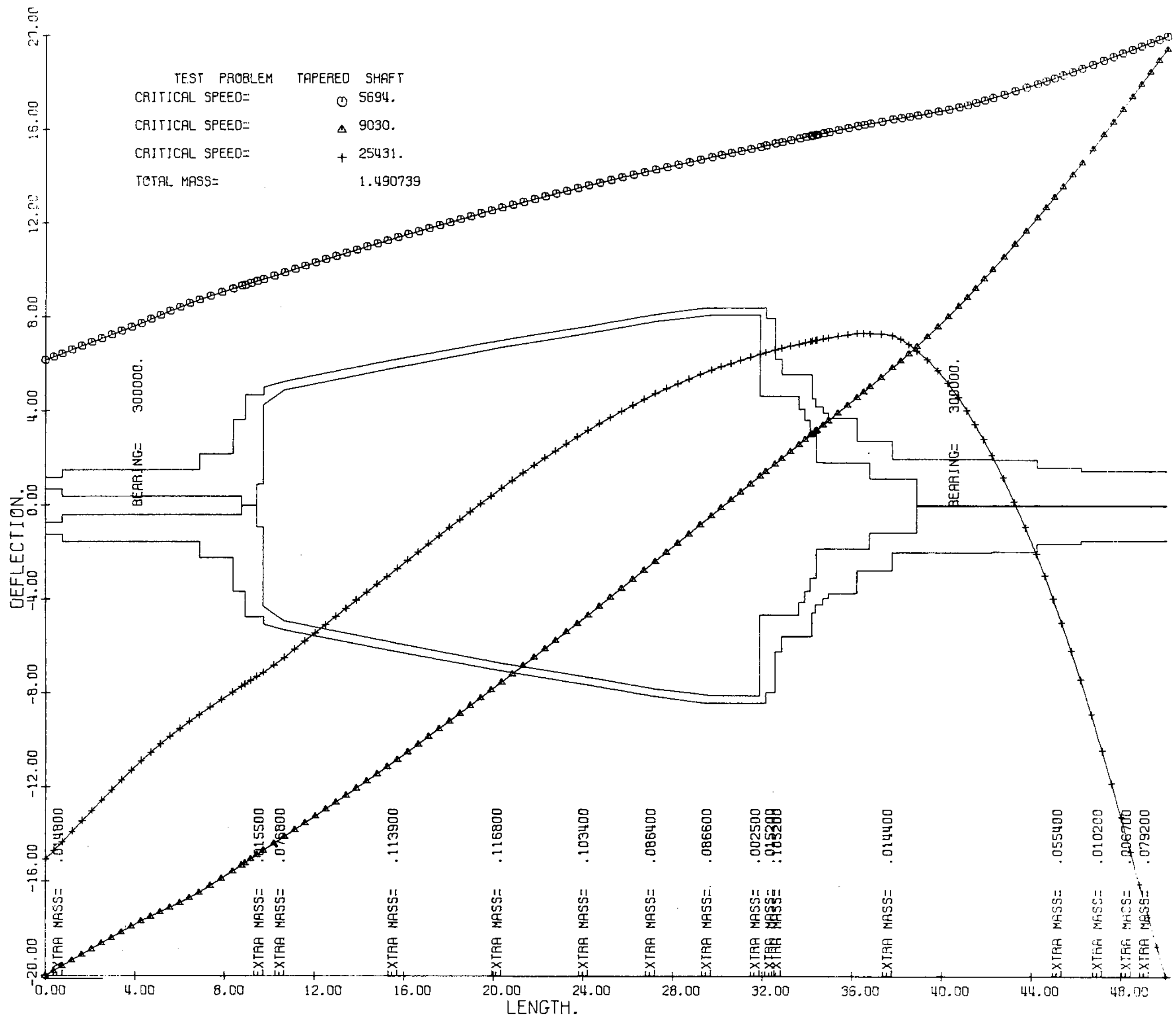

Figure 14. - Plotted output for problem 5, case 1. Total mass, 1.490739.

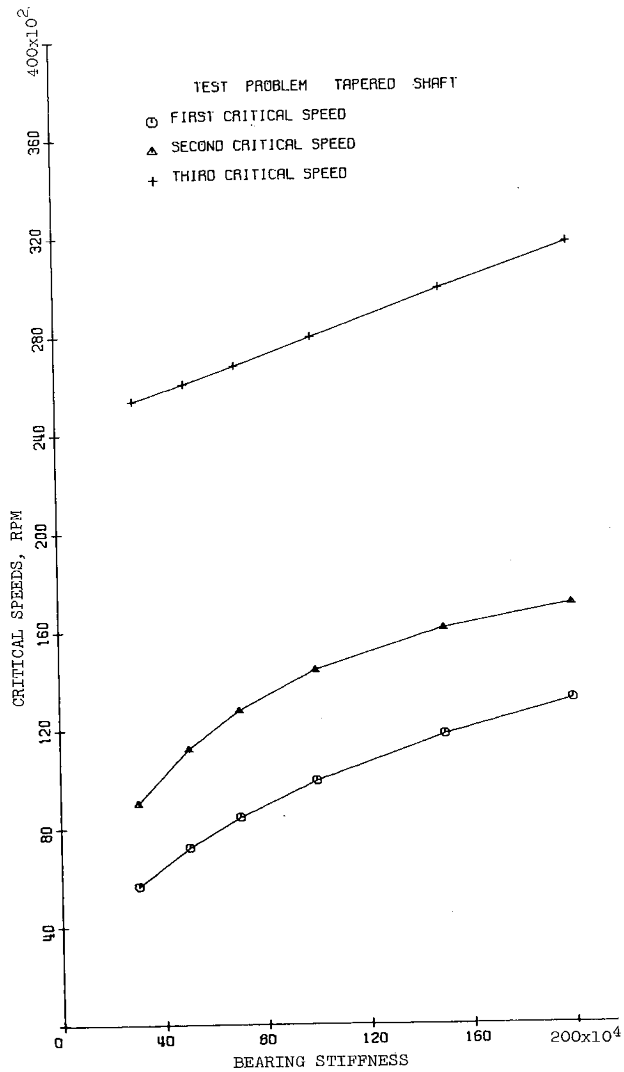

Figure 15. - Bearing stiffness as function of critical speed for all six cases.

BLACE CALCULATION---CANTILEVER BEAM--FIXED END

X	AREA	I	EW	E	RHO
0.06900	0.490E-01	0.178E-04	-0.	0.27500E+08	/.75000E-03
C.06900	0.477E-01	0.161E-04	-0.	0.27500E+08	0.75000E-03
C.06900	0.465E-01	0.147E-04	-0.	0.27500E+08	0.75000E-03
C.C6900	0.452E-01	0.134E-04	-0.	0.27500E+08	0.75000E-03
C.06900	0.440E-01	0.123E-04	-0.	0.27500E+08	0.75000E-03
C.06900	0.488E-01	0.113E-04	-0.	0.27500E+08	0.75000E-03
C.06900	0.417E-01	0.104E-04	-0.	0.27500E+08	0.75000E-03
C.C6900	0.404E-01	0.951E-05	-0.	0.27500E+08	0.75000E-03
C.06900	0.391E-01	0.864E-05	-0.	0.27500E+08	0.75000E-03
C.C6900	0.378E-01	0.780E-05	-0.	0.27500E+08	0.75000E-03
0.06900	0.365E-01	0.701E-05	-0.	0.27500E+08	0.75000E-03
C.06900	0.353E-01	0.630E-05	-0.	0.27500E+08	0.75000E-03
C.06900	0.340E-01	0.564E-05	-0.	0.27500E+08	0.75000E-03
C.06900	0.327E-01	0.503E-05	-0.	0.27500E+08	0.75000E-03
C.06900	0.313E-01	0.446E-05	-0.	0.27500E+08	0.75000E-03
0.06900	0.300E-01	0.392E-05	-0.	0.27500E+08	0.75000E-03
C.06900	0.286E-01	0.342E-05	-0.	0.27500E+08	0.75000E-03
0.06900	0.272E-01	0.296E-05	-0.	0.27500E+08	0.75000E-03
C.06900	0.259E-01	0.259E-05	-0.	0.27500E+08	0.75000E-03
0.06900	0.248E-01	0.232E-05	-0.	0.27500E+08	0.75000E-03

BOUNDARY CONDITIONS---DEFLECTION AND SLOPE EQUAL ZERO AT X=0
AND SHEAR AND MOMENT EQUAL ZERO AT X=L

THIS RUN HAS NO BEARINGS OR COUPLINGS

1ST CRITICAL SPEED = 70135.

Figure 16. - Printed output for problem 6 (blade calculation with fixed end boundary condition).

CONCLUDING REMARKS

A FORTRAN IV computer program using a modified method for calculating critical speeds of rotating shafts has been described. Its main features are flexibility, plotted computer output, and a minimum amount of data preparation. The program is easy to use and is a useful tool for design engineers concerned with rotating machinery.

The program can be adapted to any computer system that uses FORTRAN IV, but a plotting routine must be supplied by the user.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 18, 1973,
501-24.

APPENDIX A

SYMBOLS

A	mass moment of inertia of disk about its axis of symmetry, lb-in.-sec^2
B	mass moment of inertia of disk about axis through center of gravity and normal to axis of symmetry, lb-in.-sec^2
D_d	diameter of disk, in.
D_i	inside diameter of shaft section, in.
D_o	outside diameter of shaft section, in.
$E(x)$	modulus of elasticity at x, lb/in.^{2}
$F(x)$	shear coefficient for shear deflection equation
$G(x)$	modulus of rigidity at x, lb/in.^{2}
h	thickness of disk, in.
$I(x)$	diametral moment of inertia at x, in.^{4}
K	bearing stiffness factor (spring rate), lb/in.
L	length of shaft, in.
$M(x)$	moment for a general cross section, lb-in.
$M(x^+)$	moment to right of a station, lb-in.
$M(x^-)$	moment to left of a station, lb-in.
M_{n1}, M_{n2}, M_{n3}	moment for solutions 1, 2, and 3 at station n
S_n	nonshaft mass at station n, lb-sec^2/in.
$V(x^+)$	shear to right of a station, lb
$V(x^-)$	shear to left of a station, lb
V_{n1}, V_{n2}, V_{n3}	shear for solutions 1, 2, and 3 at station n, lb
x	station or axial position on shaft
$Y(x)$	deflection for a general cross section, in.
$Y_s(x)$	shear deflection for a general cross section, in.
Y_{n1}, Y_{n2}, Y_{n3}	deflection for solutions 1, 2, and 3 at station n, in.
$\eta(x)$	mass per unit length of a general cross section, lb-sec^2/in.^{2}
$\theta(x)$	slope at a general cross section (nondimensional)

$\theta_{n1}, \theta_{n2}, \theta_{n3}$	slope for solutions 1, 2, and 3 at station n (nondimensional)
ν	Poisson's ratio
ρ	mass density, $lb\text{-}sec^2/in.^4$
ω	speed of rotation or frequency of vibration, rad/sec

APPENDIX B

LISTING OF FORTRAN PROGRAM

```
C***********************************************************************
C      CRITICAL SPEED PROGRAM -- CSP.
C      READS AND WRITES ALL INPUT DATA. SETS
C      SWITCHES TO REGULATE THE FLOW OF THE
C      PROGRAM.  CALCULATES GYROSCOPIC DATA (IF ANY)
C***********************************************************************
       REAL ID(200), MASS(200)
       INTEGER STA(10)
       COMMON ID,MASS,STA
       COMMON/CLPLOT/XPEN,YPEN,NX,NY,IPEN,XLABEL(10),YLABEL(10)
       COMMON RAT(10),CS(3), DX(200),OD(200),SS(200),E(200),RHO(200),
      1TAOD(200),TAID(200),AREA(200),AI(200),DEL(200),BRG(10),BC(10),
      2ZZ(200),EI(200),GAM(200),LOC(200),THIC(200),DEN(200),ODIA(200),
      3 DIA(200),CRIT(6,3),KO(10),DY(58),X(500),Y1(500),Y2(500),KL(200),
      4EM(400),YP(10),SL(200),SLL(200),ON(10),TITLE(12)
       COMMON PI,KEY,NNN,IC,SCALE,ICM,ISK,IJJ,KA,INN,IXY,RPM , NO,NUM,
      1TP,RP,KB,   WW,IR,J,KK,DRPM,FF,L ,TOTAL,IXZ,NOO,RPMF,MOD
       DIMENSION A(200),BB(200)
       NAMELIST/SHAFT/DX,OD,ID,SS,E,RHO,TAOD,TAID,AREA,AI,BRG,STA,KB,
      1RPM,DRPM,IH,IC,SCALE,ICM,ISK,KA,KL,L,ND,LOC,THIC,DEN,ODIA,DIA,A,
      2BB,NOO,DEL,IS,NNN,IXZ,RPMF,MOD,RU
 300   FORMAT(3F5.3,2E5.1,2F5.3,F5.5)
 400   FORMAT(F6.5,2E9.5,2E5.1,F6.5)
  7    FORMAT(1H0,30X,60HBOUNDARY CONDITIONS---DEFLECTION AND SLOPE EQUAL
      1 ZERO AT X=0/39H AND SHEAR AND MOMENT EQUAL ZERO AT X=L)
 100   FORMAT(1H0,5X,1HX,9X,2HOD,8X,2HID,9X,2HEW,11X,1HE,12X,3HRHO,10X,
      14HTAOD,10X,4HTAID)
  11   FORMAT(1X,3F10.4,F12.6,2E14.5,2F10.4)
 401   FORMAT(1H0,5X,1HX,9X,4HAREA,8X,1HI,9X,2HEW,11X,1HE,12X,3HRHO)
 402   FORMAT(1X,F10.5,2E12.3,F12.6,2E14.5)
 4000  FORMAT(1H0,5X,12HTOTAL MASS =E15.8)
  6    FORMAT(1H1,30X,66HBOUNDARY CONDITIONS---SHEAR AND MOMENT EQUAL
      1 ZERO AT X=0 AND L)
 202   FORMAT(1H0,10X,14HBRG AT STATIONI3,2HK=E15.6)
 200   FORMAT(1H0,10X,20HRIGID BRG AT STATIONI4)
 203   FORMAT(1H0,10X,20HCOUPLING  AT STATIONI4)
 2500  FORMAT(1H0,10X,62HINSIDE DIAMETER GREATER THAN OUTSIDE DIAMETER-SH
      1AFT DATA CARD I5)
 2501  FORMAT(1H0,20X,37HTHIS RUN HAS NO BEARINGS OR COUPLINGS)
 2503  FORMAT(1H0,10X,35HOUTSIDE DIAMETER OF DISK AT STATION,I4,1H=,F9.5)
 2505  FORMAT(1H0,10X,29HLOCATION OF DISK OUT OF RANGE,2X,8HSTATION=I4)
 2502  FORMAT(1H0,10X,29HSTATION LOCATION OUT OF RANGE,2X,8HSTATION=I4)
 3000  FORMAT(1H0,15X,36HTHIS RUN CONSIDERS GYROSCOPIC EFFECT)
 3002  FORMAT(1H0,15X,15HDISK AT STATIONI4,5X,4HA-B=E15.6)
 1000  FORMAT(12A6)
 2000  FORMAT(1H1,12A6)
       PI=3.1415927
       CC=PI/32.
 5     DO 990 IV=1,10
       KO(IV)=0.
 990   RAT(IV)=0.
```

```
      DC 888 I=1,3
888   CS(I)=0.
      DO 992 I=1,200
992   KL(I)=0
      IS=1
      RL=.C1
      IIZ=C
      RPMF=500C0.
      MOD=C
      RPM=.1
      DRPM=200.
      KEY=0
      NNN=1
      IH=1
      IC=1
      SCALE=1.
      ICM=C
      ISK=C
      IJJ=1
      KA=0
      INN=1
      IXY=C
      IXZ =0
      READ(5,1C00) TITLE
      WRITE(6,2C00) TITLE
      READ(5,SHAFT)
      ON(NNN)=BRG(IS)
      IF(IH.EQ.1.AND.KA.EQ.0) GO TO 67
      IF(IH.EQ.0.AND.KA.EQ.0) GO TC 78
      IF(IH.EQ.1.AND.KA.EQ.1) GO TC 79
      GO TC 87
67    READ(5,3C0)(DX(I),OD(I),ID(I),E(I),RHO(I),TAOD(I),TAID(I),SS(I),
     1I=1,L)
78    WRITE(6,100)
      WRITE(6,11)  (DX(I),CD(I),ID(I),SS(I),E(I),RHO(I),TAOD(I),
     1TAID(I),I=1,L)
      DO 919 I=1,L
      IF(ID(I).GT.OD(I).OR.TAID(I).GT.TAOD(I)) GO TO 991
      GO TO 919
991   WRITE(6,2500) I
      IIZ=1
919   CONTINUE
      IF(IIZ.EC. 1) GO TO 5
      GO TC 90
79    READ(5,4C0)(DX(I),AREA(I),AI(I),E(I),RHO(I),SS(I),I=1,L)
87    WRITE(6,401)
      WRITE(6,402)(DX(I),AREA(I),AI(I),SS(I),E(I),RHO(I),I=1,L)
      ICM=1
90    IF(MOD.EC. 1)GO TO 5
      NOO=(RPMF-RPM)/DRPM
      NO = NOO
      TP=C.
      DO 55 I=1,L
55    TP=DX(I)+ TP
      STEP=TP *RL
      DO 951 I=1.L
      IF(STEP .LE.DX(I))  GO TO 806
      DEL(I)=1.
      GC TC 951
```

```
806   I5=CX(I)/STEP +.999
      DEL(I)=I5
951   CONTINUE
      SUM=C.
      DO 950 I=1,L
950   SUM=DEL(I)+SUM
      NUM=SUM+1.
      IF(NNN.GT.1) GO TO 700
      DO 443 I=1,6
      DO 443 IV=1,3
443   CRIT(I,IV)=0.
700   RP=RPM
      LP=KB+1
      DO 805 JZ=LP,10
      BRG(JZ)=C.
805   STA(JZ)=C
      GO TO(10,20),IC
10    BC(3)=0.
      BC(4)=0.
      BC(5)=0.
      BC(6)=.0C01
      BC(7)=0.
      BC(8)=0.
      BC(9)=.0C01
      BC(10)=0.
      WRITE(6,6)
      GO TO 133
20    BC(3)=.0C01
      BC(4)=0.
      BC(5)=0.
      BC(6)=0.
      BC(7)=0.
      BC(8)=.0C01
      BC(9)=0.
      BC(10)=0.
      WRITE(6,7)
133   IF(KB.EQ.0) GO TO 41
      DO 66 I=1,KB
      IF(BRG(I).EQ. 0.) GO TO 43
      IF(BRG(I).EQ.-1.) GO TO 44
      WRITE(6,202) STA(I),BRG(I)
      IF(STA(I).GT.L .OR.STA(I).LE.0) GO TO 666
      GO TO 66
43    WRITE(6,200) STA(I)
      IF(STA(I).GT.L .OR.STA(I).LE.0) GO TO 666
      GO TO 66
44    WRITE(6,203) STA(I)
      IF(STA(I).GT.L .OR.STA(I).LE.0) GO TO 666
      GO TO 66
 666  WRITE(6,2502) STA(I)
      IIZ=1
66    CONTINUE
      IF(IIZ.EQ. 1) GO TO 5
      GO TO 77
41    WRITE(6,2501)
77    IF(ISK-2) 82,38,82
82    DO 80 KZ=1,200
80    ZZ(KZ)=0.
38    IF(KA.EQ. 0) GO TO 33
      DO 62 I=1,L
```

```
      MASS(I)=AREA(I)*RHO(I)
      EI(I)=E(I)*AI(I)
62    GAM(I)=0.
      GO TO 61
33    DO 25 I=1,L
      IF(TAOD(I).EQ. 0.) GO TO 24
      SL(I)=(TAOD(I)-OD(I))/DX(I)
      GO TO 25
24    SL(I)=0.
25    CONTINUE
      DO 26 I=1,L
      IF(TAOD(I) .EQ. 0.) GO TO 23
      SLL(I)=(TAID(I)-ID(I))/DX(I)
      GO TO 26
23    SLL(I)=0.
26    CONTINUE
      TOTAL=0.
      DO 556 I=1,L
      TIL=PI*RHO(I)*DX(I)/3.
      IF(TAOD(I).EQ.0.) GO TO 555
      VX=TIL*(OD(I)**2/4.+OD(I)*TAOD(I)/4.+TAOD(I)**2/4.)   +SS(I)
      TOTAL=TOTAL+VX-TIL*(ID(I)**2/4.+ID(I)*TAID(I)/4.+TAID(I)**2/4.)
      GO TO 556
555   TOTAL=(OD(I)**2-ID(I)**2)*PI*DX(I)*RHO(I)/4.+TOTAL    +SS(I)
556   CONTINUE
      WRITE(6,4000)  TOTAL
61    IF(ISK.EQ.0) GO TO 88
      IF(ISK.EQ. 2) GO TO 72
      DO 71 I=1,ND
      NA=LOC(I)
      IF(LOC(I).GT.L.OR.LOC(I).LE.0)   GO TO 748
      IF(KL(I) .EQ. 1) GO TO 48
      IF(ODIA(I).GT.0.) GO TO 894
      WRITE(6,2503) LOC(I),ODIA(I)
      GO TO 5
894   D=CC*THIC(I)*DEN(I)*(ODIA(I)**4- DIA(I)**4)
      CD=.5*D*(1.+4.*THIC(I)**2/(3.*(ODIA(I)**2- DIA(I)**2)))
      ZZ(NA)=D-CD
      GO TO 71
48    ZZ(NA)=A(I)-BB(I)
      GO TO 71
748   WRITE(6,2505) LOC(I)
      IIZ=1
71    CONTINUE
      IF(IIZ.EQ.1) GO TO 5
72    WRITE(6,3000)
      DO 73 I=1,ND
      NA=LOC(I)
      WRITE(6,3002) LOC(I),ZZ(NA)
73    CONTINUE
88    CALL MAIN
      GO TO 5
      END
```

```
C***********************************************************************
C                    MAIN SUBROUTINE
C       SETS UP INITIAL CONDITIONS FOR
C       NUMERICAL INTEGRATION AND THEN CALLS SUBROUTINE
C       RUNGEK.  PERFORMS CALCULATIONS FOR CHANGES
C       BROUGHT ABOUT BY EXTRA MASSES, FLEXIBLE AND
C       RIGID BEARINGS, COUPLINGS AND GYROSCOPIC
C       EFFECTS.  CALCULATES CRITICAL SPEEDS AND
C       WRITES CONVERGED VALUE.  CALLS
C       SUBROUTINE GRA AND PPLOT FOR APPROPRIATE
C       CURVES AND DRAWINGS.
C***********************************************************************
      REAL ID(200), MASS(200)
      INTEGER STA(10)
      EXTERNAL SUBR
      COMMON ID,MASS,STA
      DIMENSION XA(500)
      COMMON RAT(10),CS(3), DX(200),OD(200),SS(200),E(200),RHO(200),
     1TAOD(200),TAID(200),AREA(200),AI(200),DEL(200),BRG(10),BC(10),
     2ZZ(200),EI(200),GAM(200),LOC(200),THIC(200),DEN(200),ODIA(200),
     3 DIA(200),CRIT(6,3),KO(10),DY(58),X(500),Y1(500),Y2(500),KL(200),
     4EM(400),YP(10),SL(200),SLL(200),ON(10),TITLE(12)
      COMMON PI,KEY,NNN,IC,SCALE,ICM,ISK,IJJ,KA,INN,IXY,RPM , NO,NUM,
     1TP,RP,KB,   WW,IR,J,KK,DRPM,FF,L ,TOTAL,IXZ,NOO,RPMF,MOD
      COMMON/CLPLOT/XPEN,YPEN,NX,NY,IPEN,XLABEL(10),YLABEL(10)
 400  FORMAT(1H0,26HA DISCONTINUITY OCCURS AT ,F10.0,33HIN THE EXCESS MO
     1MENT VS RPM CURVE/1H0,27HDATA FOR THIS CURVE FOLLOWS)
 501  FORMAT(1H0,75X,20H1ST CRITICAL SPEED =F8.0)
 502  FORMAT(1H0,75X,20H2ND CRITICAL SPEED = F8.0)
 503  FORMAT(1H0,75X,20H3RD CRITICAL SPEED = F8.0)
 504  FORMAT(1H0,15X,22HNO CRITICAL SPEED FROM,F8.0,2HTO,F8.0)
 506  FORMAT(1H0,16HDID NOT CONVERGE)
 4000 FORMAT(1X,4(F8.0,E15.8))
 100  WW=(2.*RP*PI /60.)**2
      DO 99 I=3,10
 99   DY(I)=BC(I)
      IR=1
      J=1
      KK=1
      DY(1)=0.
      X(J)=0.
      Y1(J)=DY(6)
      Y2(J)=DY(10)
      IM=1
      IU=1
      DO 50 K=1,L
      OD(200)=X(J)
      KK=K
      DY(2)=DX(K)/DEL(K)
      IT=DEL(K)
      DO 90 IVP=1,IT
 90   CALL RUNGEK(8,DY,SUBR)
      IF(SS(K).EQ.0.) GO TO 44
      DY(3)=SS(K)*WW*DY(6)+DY(3)
      DY(7)=SS(K)*WW*DY(10)+DY(7)
 44   IF(STA(IU).EQ.KK) GO TO 45
      GO TO 96
 45   IF(BRG(IU))81,82,83
```

```
83    DY(3)=DY(3)-BRG(IU)*DY(6)
      DY(7)=DY(7)-BRG(IU)*DY(10)
      GO TO 95
82    DY(9)=DY(9)-DY(10)*DY(5)/DY(6)
      DY(8)=DY(8)-DY(4)*DY(10)/DY(6)
      RAT(IR)=DY(10)/DY(6)
      KO(IR)=J
      IR=IR+1
      Y2(J)=0.
      Y1(J)=0.
      DY(10)=0.
      DY(7)=0.
      DY(3)=.0001
      DY(4)=0.
      DY(5)=0.
      DY(6)=0.
      GO TO 95
81    DY(7)=DY(7)-DY(3)*DY(8)/DY(4)
      DY(10)=DY(10)-DY(6)*DY(8)/DY(4)
      RAT(IR)=DY(8)/DY(4)
      KO(IR)=J
      IR=IR+1
      Y2(J)=DY(10)
      Y1(J)=0.
      DY(8)=0.
      DY(9)=0.
      DY(3)=0.
      DY(4)=0.
      DY(5)=.0001
      DY(6)=0.
95    IU=IU+1
96    IF(LOC(IM).EQ.KK) GO TO 97
      GO TO 50
97    DY(4)=ZZ(K)*DY(5)*WW+DY(4)
      DY(8)=ZZ(K)*DY(9)*WW+DY(8)
      IM=IM+1
50    CONTINUE
      RAT(IR)=DY(7)/DY(3)
      KO(IR)=J
      IF(IXY.EQ.1) GO TO 295
      EM(INN)=DY(3)*DY(8)-DY(4)*DY(7)
      IF(INN.LT.2) GO TO 299
      IF(EM(INN)*EM(INN-1))298,299,299
298   SO=RP
      RP=(EM(INN)*DRPM)/(EM(INN-1)-EM(INN))+RP
      W3=RP
      JKK=0
      IXY=1
      W1=SO-DRPM
      W2=EM(INN-1)
      W5=SO
      W6=EM(INN)
      GO TO 100
 295  W4=DY(3)*DY(8)-DY(4)*DY(7)
      IF(ABS(W4).GE.ABS(EM(INN)).OR.ABS(W4).GE.ABS(EM(INN-1))) GO TO 300
      IF(JKK.EQ. 35) GO TO 2968
      JKK=JKK+1
      IF(W2*W4) 2200,2200,2201
2200  W5=W3
      W6=W4
```

```
      GO TO 2202
2201  W1=W3
      W2=W4
2202  PRP=W3
      RP=-W6*(W1-W5)/(W2-W6)+W5
      W3=RP
      IF(ABS(PRP-RP).LE.9.) GO TO 2967
      GO TO 100
2967  CRIT(NNN,IJJ)=PRP
      IXY=0
      RP =SO
      GO TO 803
2968  WRITE(6,506)
      IXY=0
      GO TO 39
803   CS(IJJ)=CRIT(NNN,IJJ)
      KEY=0
      CALL GRA
705   IJJ=IJJ+1
      IF(IJJ.EQ.4) GO TO 78
299    INN=INN+1
      RP=RP+DRPM
      NO=NO-1
      IF(NO.NE.0) GO TO 100
78    INN=INN+1
      OON=NOO
      ZN= OON*DRPM +RPM
      IF(CS(1).EQ. 0.)  GO TO 666
      WRITE(6,501)  CS(1)
      IF(CS(2) .EQ. 0.)  GO TO 667
      WRITE(6,502)  CS(2)
      IF(CS(3) .EQ. 0.)  GO TO 667
      WRITE(6,503)  CS(3)
667    GO  TO  39
666   WRITE(6,504) RPM,ZN
39    KEY=1
      CALL PPLOT
      CALL GRA
      NP=INN-1
      ICU=INN-2
      IF(IXZ .EQ. 0)  GO TO 451
      XA(1)=RPM
       DO  25  I=1,ICU
25    XA(I+1)=XA(I)+DRPM
      WRITE(6,4000) (XA(I), EM(I), I=1,NP)
451   IF(NNN.EQ.6) GO TO 900
      RETURN
900   KEY=-1
      CALL GRA
      RETURN
300   WRITE(6,400)  SO
      IXZ=1
      IXY=0
      RP=SO
      GO TO 299
      END
```

```
C*****************************************************************************
C                       SUBROUTINE GRA
C       PREPARES DATA FOR THE DEFLECTION
C       AND BEARING STIFFNESS VS CRITICAL
C       SPEEDS CURVES
C*****************************************************************************
      SUBROUTINE GRA
      REAL ID(200), MASS(200)
      INTEGER STA(10)
      COMMON IC,MASS,STA
      COMMON RAT(10),CS(3), DX(200),OD(200),SS(200),E(200),RHO(200),
     1TAOD(200),TAID(200),AREA(200),AI(200),DEL(200),BRG(10),BC(10),
     2ZZ(200),EI(200),GAM(200),LOC(200),THIC(200),DEN(200),ODIA(200),
     3 DIA(200),CRIT(6,3),KO(10),DY(58),X(500),Y1(500),Y2(500),KL(200),
     4EM(400),YP(10),SL(200),SLL(200),ON(10),TITLE(12)
      COMMON PI,KEY,NNN,IC,SCALE,ICM,ISK,IJJ,KA,INN,IXY,RPM , NO,NUM,
     1TP,RP,KB,    WW,IR,J,KK,DRPM,FF,L ,TOTAL,IXZ,NOO,RPMF,MOD
      COMMON/SPECL/TEST,ORGSET,SPASET
      COMMON/CLPLOT/XPEN,YPEN,NX,NY,IPEN,XLABEL(10),YLABEL(10)
      DIMENSION           BELX(10),BELY(10),CA(18),KKK(40),P(20),DEF(500)
     1,AZ(500),ABELX(10),ABELY(10)
      DATA ( ABELX(K),K=1,10)/6H      ,6H      ,6H      ,6H      ,
     1 6HLENGTH,6H.     ,6H      ,6H      ,6H      ,6H      /
      DATA ( ABELY(L),L=1,10)/6H      ,6H      ,6H      ,6H      ,
     1 6HDEFLEC,6HTION. ,6H      ,6H      ,6H      ,6H      /
      DATA (  BELY(L),L=1,10)/6H      ,6H      ,6H      ,6HCRITIC,
     1 6HAL    ,6HSPEEDS,6H      ,6H      ,6H      ,6H      /
      DATA ( BELX (K),K=1,10)/6H      ,6H      ,6H      ,6HBEARIN,
     1 6HG     ,6HSTIFFN,6HESS   ,6H      ,6H      ,6H      /
      IF(KEY)99,90,777
 99   IF(CRIT(1,1).EQ. 0.)GO TO 777
      DO 100 I=1,6
      AZ(I)=ON(I)
      AZ(I+6)=AZ(I)
 100  AZ(I+12)=AZ(I)
      II=1
      DO 76 I=1,10
      XLABEL(I)=BELX(I)
 76   YLABEL(I)=BELY(I)
      DO 20 KV=1,3
      DO 20 I=1,6
      CA(II)=CRIT(I,KV)
      IF(CA(II).NE.0.) GO TO 20
      CA(II)=CA(II-1)
      AZ(II)=AZ(II-1)
 20   II=II+1
      KKK(1)=1
      KKK(2)=1
      KKK(3)=3
      KKK(4)=1
      KKK(5)=1
      KKK(6)=6
      KKK(7)=6
      KKK(8)=6
      P(1)=3.
      P(2)=10.
      P(5)=10.
      P(8)=10.
      DO 77 I=9,13
```

```
77    P(I)=0.
      P(14)=90.
      XPEN=0.
      YPEN=0.
      NX=-60
      NY=60
      FF=2.
      ORGSET=0.
      IF(ICM.EQ. 1) GO TO 777
      CALL CALPLT(AZ,CA,KKK,P)
      GO TO 777
90    DO 91 I=1,NUM
91    AZ(I)=X(I)*SCALE
      NP=1
      LZ=1
      N=KC(LZ)
79    DO 88 I=NP,N
88    DEF(I)=Y2(I)-Y1(I)*RAT(LZ)
      LZ=LZ+1
      IF(KC(LZ).EQ.0) GO TO 74
      NP=N+1
      N=KC(LZ)
      GO TO 79
74    TEST=50.
      ORGSET=1
      DO 600 I=1,10
      XLABEL(I)=ABELX(I)
600   YLABEL(I)=ABELY(I)
      KKK(1)=4
      KKK(2)=1
      KKK(3)=1
      KKK(4)=1
      KKK(5)=IJJ
      KKK(6)=J
      P(1)=1.
      P(2)=TP*SCALE
      P(3)=0.
      P(4)=TP*SCALE
      P(5)=10.
      P(6)=-5.
      P(7)=5.
      P(8)=10.
      DO 877 I=9,13
877   P(I)=0.
      P(14)=90.
      XPEN=0.
      YPEN=0.
      NX=-60
      NY=60
      HI=0.
      FF=1.
      DO 333 I=1,J
333   HI=AMAX1(ABS(DEF(I)),HI)
      DO 334 I=1,J
334   DEF(I)=5./HI*DEF(I)
      IF(ICM.EQ. 1) GO TO 777
      CALL CALPLT(AZ,DEF,KKK,P)
777   RETURN
      END
```

```
      SLBRCUTIKE PPLOT
C*****************************************************************************
C                       SUBROUTINE PPLOT
C      PREPARES DATA FOR SHAFT DRAWING
C      CALPLT IS A GENERAL PURPOSE ROUTINE FOR PRODUCING PLOTS ON THE
C      CALCCMP MODEL 670 DIGITAL PLOTTER.
C      CALL CALFLOT(XDOWN,YACROS,KKK,P)
C      WHERE--    XDOWN IS AN ARRAY CONTAINING THE DATA FOR THE ABSCISSA
C                 YACROS IS AN ARRAY CONTAINING THE DATA FOR THE ORDINATE
C                 KKK AND P ARE ARRAYS THAT CONTAIN PLOTTING INFORMATION
C      THERE ARE TWO COMMON BLOCKS
C         COMMCN/CLPLOT/
C         CCMMCN/SPECL/
C      PLCT,SYMBOL,NUMBER,AND AXIS   ARE SUBROUTINES USED BY CALPLT
C*****************************************************************************
      INTEGER STA(10)
      REAL ID(200), MASS(200)
      COMMON IC,MASS,STA
      COMMON RAT(10),CS(3), DX(200),OD(200),SS(200),E(200),RHO(200),
     1TAOD(200),TAID(200),AREA(200),AI(200),DEL(200),BRG(10),BC(10),
     2ZZ(200),EI(200),GAM(200),LOC(200),THIC(200),DEN(200),ODIA(200),
     3 DIA(200),CRIT(6,3),KO(10),DY(58),X(500),Y1(500),Y2(500),KL(200),
     4EM(400),YP(10),SL(200),SLL(200),ON(10),TITLE(12)
      COMMON PI,KEY,NNN,IC,SCALE,ICM,ISK,IJJ,KA,INN,IXY,RPM , NO,NUM,
     1TP,RP,KB,   WW,IR,J,KK,DRPM,FF,L ,TOTAL,IXZ,NOO,RPMF,MOD
      COMMON/SPECL/TEST,ORGSET,SPASET
      COMMON/CLPLOT/XPEN,YPEN,NX,NY,IPEN,XLABEL(10),YLABEL(10)
      DIMENSION DD(400),ODL(1600),Z(200),ZO(400),KKK(40),P(20)
      IF(ICM.NE.0) GO TO 38
      FF=C.
      TEST=50.
      ORGSET=1.
      JJJP=1
      DO 120 I=1,KB
      JC=STA(I)
      TOT=C.
      DO 131 KFP=1,JC
  131 TOT=DX(KFP)*SCALE+TOT
      YP(JJJP)=TOT
  12C JJJP=JJJP+1
      JP=1
      DO 501 IJ=1,L
      DD(JP)=IC(IJ)
      ODL(JP)=OD(IJ)*.5*SCALE
      JP=JP+1
      IF(TAID(IJ).EQ.0.) GC TO 80
      DD(JP)=TAID(IJ)
      GO TC 40
   80 DD(JP)=IC(IJ)
   40 IF(TAOD(IJ).EQ.0.) GC TO 81
      ODL(JP)=TAOD(IJ)*.5*SCALE
      GO TC 501
   E1 ODL(JP)=ODL(JP-1)
  501 JP=JP+1
      DO 88 IA=1,L
   E8 Z(IA)=DX(IA)*SCALE
      ZC(1)=0.
      JP=2
      DO 48 IA=1,L
      ZO(JP)=ZC(JP-1)+Z(IA)
```

```
      JP=JP+1
      ZO(JP)=ZO(JP-1)
 48   JP=JP+1
      KH=2*L
      DO 86 IA=1,KH
      KM=KH+IA
 86   ODL(KM)=OD(IA)*.5*SCALE
      KMM=KM+1
      DO 108 IA=1,KM
      ODL(KMM)=-ODL(IA)
 108  KMM=KMM+1
      KKK(1)=4
      KKK(2)=0
      KKK(3)=4
      KKK(4)=1
      KKK(6)=KH
      P(1)=1.
      P(2)=TP*SCALE
      P(3)=0.
      P(4)=TP*SCALE
      P(5)=10.0
      P(6)=-5.
      P(7)=5.
      P(8)=10.0
      DO 77I=9,13
 77   P(I)=0.
      P(14)=90.
      XPEN=0.0
      YPEN=0.0
      NX=-60
      NY=60
      CALL CALPLT(ZO,ODL,KKK,P)
      CALL CALTIT
      RG=1./SCALE
      P(6)=-5./SCALE
      P(7)=5./SCALE
      CALL AXIS(P(9),P(10),XLABEL,NX,P(2),P(13),P(3),RG,P(8))
      RG=(P(7)-P(6))/P(5)
      CALL AXIS(P(11),P(12),YLABEL,NY,P(5),P(14),P(6),RG,P(8))
      CALL PLOT(P(2)+10.,0.,-3)
 38   RETURN
      END
```

```
C***********************************************************************
C                     SUBROUTINE SUBR
C     THIS SUBROUTINE IS CALLED BY RUNGEK
C     TO CALCULATE VALUES OF THE DERIVATIVES
C     FOR THE EQUATIONS OF BEAM FLEXURE.
C***********************************************************************
      REAL ID(200), MASS(200)
      INTEGER STA(10)
      COMMON ID,MASS,STA
      COMMON RAT(10),CS(3), DX(200),OD(200),SS(200),E(200),RHO(200),
     1TAOD(200),TAID(200),AREA(200),AI(200),DEL(200),BRG(10),BC(10),
     2ZZ(200),EI(200),GAM(200),LOC(200),THIC(200),DEN(200),ODIA(200),
```

```
  3 DIA(200),CRIT(6,3),KO(10),DY(58),X(500),Y1(500),Y2(500),KL(200),
  4EM(400),YP(10),SL(200),SLL(200),ON(10),TITLE(12)
   COMMON PI,KEY,NNN,IC,SCALE,ICM,ISK,IJJ,KA,INN,IXY,RPM , NO,NUM,
  1TP,RP,KB,   WW,IR,J,KK,DRPM,FF,L ,TOTAL,IXZ,NOO,RPMF,MOD
   KK=KK
   IF(KA.EQ. 1) GO TO 80
   AA=OD(KK)+SL(KK)*(DY(1)-OD(200))
   EE=ID(KK)+SLL(KK)*(DY(1)-OD(200))
   G=3.*E(KK)/8.
   TZ=(EE/AA)**2
   A2=AA*AA
   D2=EE*EE
   CROSS=(A2-D2)*PI/4.
   F=(9.*(1.+TZ)**2+24.*TZ)/(8.*(1.+TZ)**2)
   GAM(KK)=F/(G*CROSS)
   A4=A2*A2
   D4=D2*D2
   MASS(KK)=(A2-D2)*PI*RHO(KK)/4.
   EI(KK)=(A4-D4)*PI/64.*E(KK)
80 DY(11)=MASS(KK)*DY(6)*WW
   DY(12)=DY(3)
   DY(13)=DY(4)/EI(KK)
   DY(14)=DY(5)-DY(3)*GAM(KK)
   DY(15)=MASS(KK)*DY(10)*WW
   DY(16)=DY(7)
   DY(17)=DY(8)/EI(KK)
   DY(18)=DY(9)-DY(7)*GAM(KK)
   RETURN
   END
```

```
      SUBRCUTIKE RUNGEK(NN,Y,SUBR)
C************************************************************************
C                       SUBROUTINE RUNGEK
C      PERFORMS THE NUMERICAL INTEGRATION
C THE DIMENSION CF Y IN MAIN MUST BE AT LEAST 7*N+2
C   Y(1)=X, Y(2)=DX, Y(3),....,Y(N+2)=Y1,....,YN
C   Y(N+3),....,Y(2N+2)=Y1P,....,YNP
C  CALL SUBR IN MAIN INITIALLY TO SET UP THE INITIAL FIRST DERIVATIVES
C************************************************************************
      REAL ID(200), MASS(200)
      INTEGER STA(10)
      COMMON IC,MASS,STA
      COMMON RAT(10),CS(3), DX(200),OD(200),SS(200),E(200),RHO(200),
     1TAOD(200),TAID(200),AREA(200),AI(200),DEL(200),BRG(10),BC(10),
     2ZZ(200),EI(200),GAM(200),LOC(200),THIC(200),DEN(200),ODIA(200),
     3 DIA(200),CRIT(6,3),KO(10),DY(58),X(500),Y1(500),Y2(500),KL(200),
     4EM(400),YP(10),SL(200),SLL(200),ON(10),TITLE(12)
      COMMON PI,KEY,NNN,IC,SCALE,ICM,ISK,IJJ,KA,INN,IXY,RPM , NO,NUM,
     1TP,RP,KB,   WW,IR,J,KK,DRPM,FF,L ,TOTAL,IXZ,NOO,RPMF,MOD
      DIMENSION Y(1)
      CALL SUBR
      N=NN
      DO 10 I=1,N
      I1=6*N+2+I
```

```
10 Y(I1)=Y(I+2)
   DO 12 I=1,N
   I1=N+2+I
   I2=2*N+2+I
12 Y(I2)=Y(2)*Y(I1)
   Y(1)=Y(1)+Y(2)/2.
   DO 14 I=1,N
   I1=2*N+2+I
   I2=6*N+2+I
14 Y(I+2)=Y(I2)+.5*Y(I1)
   CALL SUBR
   DO 16 I=1,N
   I1=N+2+I
   I2=3*N+2+I
16 Y(I2)=Y(2)*Y(I1)
   DO 18 I=1,N
   I1=3*N+2+I
   I2=6*N+2+I
18 Y(I+2)=Y(I2)+.5*Y(I1)
   CALL SUBR
   DO 20 I=1,N
   I1=N+2+I
   I2=4*N+2+I
20 Y(I2)=Y(2)*Y(I1)
   Y(1)=Y(1)+Y(2)/2.
   DO 22 I=1,N
   I1=4*N+2+I
   I2=6*N+2+I
22 Y(I+2)=Y(I2)+Y(I1)
   CALL SUBR
   DO 24 I=1,N
   I1=N+2+I
   I2=5*N+2+I
24 Y(I2)=Y(2)*Y(I1)
   DO 26 I=1,N
   I1=2*N+2+I
   I2=3*N+2+I
   I3=4*N+2+I
   I4=5*N+2+I
   I5=6*N+2+I
26 Y(I+2)=Y(I5)+(Y(I1)+2.*Y(I2)+2.*Y(I3)+Y(I4))/6.
   J=J+1
   Y1(J)=Y(6)
   Y2(J)=Y(10)
   X(J)=Y(1)
   RETURN
   END
```

```
      SUBROUTINE CALTIT
C***********************************************************************
C                     SUBROUTINE CALTIT
C     WRITES HOLLERITH INFORMATION AND NUMBERS
C     ON PLOTS.
C***********************************************************************
      REAL ID(200), MASS(200)
      INTEGER STA(10)
      COMMON ID,MASS,STA
      COMMON RAT(10),CS(3), DX(200),OD(200),SS(200),E(200),RHO(200),
     1TACC(200),TAID(200),AREA(200),AI(200),DEL(200),BRG(10),BC(10),
     2ZZ(200),EI(200),GAM(200),LOC(200),THIC(200),DEN(200),ODIA(200),
     3 DIA(200),CRIT(6,3),KO(10),DY(58),X(500),Y1(500),Y2(500),KL(200),
     4EM(400),YP(10),SL(200),SLL(200),ON(10),TITLE(12)
      COMMON PI,KEY,NNN,IC,SCALE,ICM,ISK,IJJ,KA,INN,IXY,RPM , NO,NUM,
     1TP,RP,KB,    WW,IR,J,KK,DRPM,FF,L ,TOTAL,IXZ,NOO,RPMF,MOD
      IF(FF.EQ.2.) GO TO 155
      IF(FF.EQ.1.) GO TO 177
 176  CALL SYMBOL(1.,9.5,.1,TITLE,0.,72)
      XXX=0.
      DO 50 I=1,L
      XXX=DX(I)*SCALE+XXX
      IF(SS(I).EQ.0.) GO TO 50
      Q=SS(I)
      CALL SYMBOL(XXX,0.,.1,11HEXTRA MASS=,90.,11)
      CALL NUMBER(XXX,1.2,.1,Q,90.,6)
 50   CONTINUE
      YY=9.3
      IF(CS(1).EQ.0.) GO TO 40
      KVV=IJJ-1
      DO 10 I=1,KVV
      CALL SYMBOL(1.,YY,.1,15HCRITICAL SPEED=,0.,16)
      II=I
      QQ=CS(I)
      CALL SYMBOL(3.3,YY,.1,II,0.,-1)
      CALL NUMBER(3.5,YY,.1,QQ,0.,0)
 10   YY=YY-.3
 40   CALL SYMBOL(1.,YY,.1,11HTOTAL MASS=,0.,11)
      CALL NUMBER(3.5,YY,.1,TOTAL,0.,6)
      IF(ISK .EQ. 0)  GO TO 35
      YY=YY-.3
      CALL SYMBOL(1.,YY,.1,49HGYROSCOPIC EFFECT--SEE PRINTED OUTPUT FOR
     1DETAILS,0.,49)
 35   IF(KB.EQ.0)  GO TO 177
      DO 17 I=1,KB
      IF(BRG(I)) 16,19,18
 18   YPP=YP(I)
      CALL SYMBOL(YPP,5.,.1,8HBEARING=,90.,9)
      QQ=BRG(I)
      CALL NUMBER(YPP,6.0,.1,QQ,90.,0)
      GO TO 17
 19   YPP=YP(I)
      CALL SYMBOL(YPP,5.,.1,13HRIGID BEARING,90.,13)
      GO TO 17
 16   YPP=YP(I)
      CALL SYMBOL(YPP,5.,.1,8HCOUPLING,90.,8)
 17   CONTINUE
      GO TO 7
 177  CALL SYMBOL(1.,9.5,.1,TITLE,0.,72)
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7    RETURN
155  CALL SYMBOL(1.,9.5,.1,TITLE,0.,72)
     CALL SYMBOL(1.,9.2,.1,1,0.,-1)
     CALL SYMBOL(1.2,9.2,.1,20HFIRST CRITICAL SPEED,0.,21)
     CALL SYMBOL(1.,8.9,.1,2,0.,-1)
     CALL SYMBOL(1.2,8.9,.1,21HSECOND CRITICAL SPEED,0.,22)
     CALL SYMBOL(1.,8.6,.1,3,0.,-1)
     CALL SYMBOL(1.2,8.6,.1,20HTHIRD CRITICAL SPEED,0.,21)
     GO TO 7
     END
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