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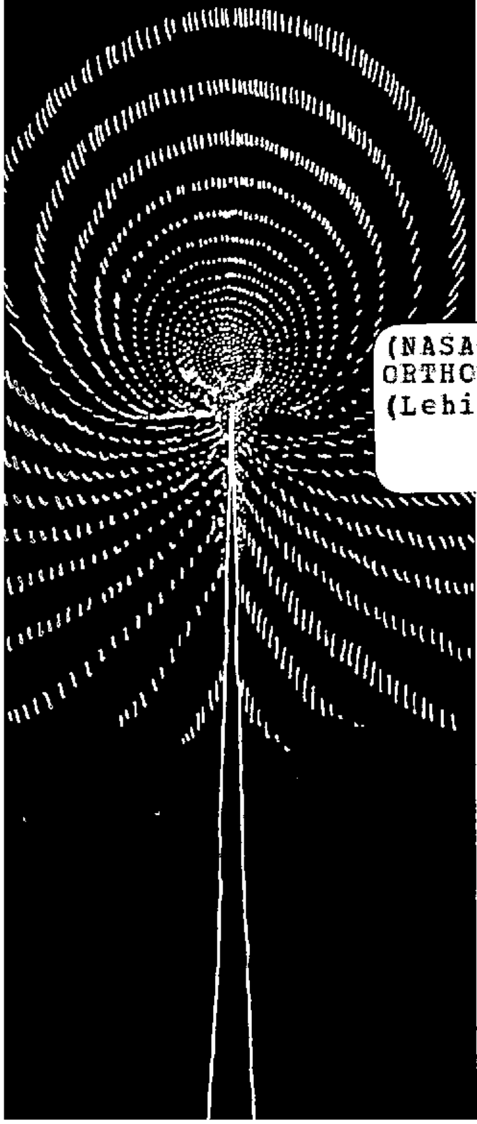

LEHIGH UNIVERSITY

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BY

F. ERDOGAN
M. RATWANI
U. YUCEOGLU

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ON THE EFFECT OF ORTHOTROPY IN A CRACKED CYLINDRICAL SHELL*

F. Erdogan, M. Ratwani, and U. Yüceoglu
Lehigh University, Bethlehem, Pa.

Abstract. A pressurized cylindrical shell containing a longitudinal crack is considered. It is assumed that the material has a special orthotropy, namely that the shear modulus of the sheet may be evaluated from the measured Young's moduli and the Poisson's ratios rather than being an independent material constant. Two examples, one for a mildly orthotropic (titanium) and the other for a strongly orthotropic (graphite) material approximately satisfying the condition of special orthotropy are given. The results show that the stress intensity factors are rather strongly dependent on the degree of orthotropy.

SOME REMARKS ON THE FORMULATION OF THE PROBLEM

A detailed treatment of the linear bending theory of anisotropic shallow shells may be found in [1-3]. Assuming that through a proper superposition the problem has been reduced to a perturbation problem with the crack surface tractions as the only external loads, in an eight order theory the differential equations for an orthotropic shallow cylindrical shell may be expressed as

$$D_1 \nabla_1^4 w(x_1,x_2) - \frac{a^2}{R}\frac{\partial^2}{\partial x_1^2}F(x_1,x_2) = 0 \ ,$$

$$\nabla_2^4 F(x_1,x_2) + \frac{hE_2 a^2}{R}\frac{\partial^2}{\partial x_1^2}w(x_1,x_2) = 0 \ , \qquad (1.a,b)$$

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where a,R,h are the dimensions of the crack and the shell (see the insert in Figure 1),

$$x_1 = X/a \ , \qquad x_2 = Y/a \ , \tag{2}$$

F is a stress function and w is the displacement component normal to the surface. The operators $\nabla_1^{\ 4}$ and $\nabla_2^{\ 4}$ are defined by

$$\nabla_1^{\ 4} = \frac{\partial^4}{\partial x_1^{\ 4}} + 2[\nu_2 + 2(1-\nu_1\nu_2)\frac{G_{12}}{E_1}]\frac{\partial^4}{\partial x_1^{\ 2}\partial x_2^{\ 2}} + \frac{E_2}{E_1}\frac{\partial^4}{\partial x_2^{\ 4}} \ ,$$

$$\nabla_2^{\ 4} = \frac{\partial^4}{\partial x_1^{\ 4}} + 2(\frac{E_2}{2G_{12}} - \nu_2)\frac{\partial^4}{\partial x_1^{\ 2}\partial x_2^{\ 2}} + \frac{E_2}{E_1}\frac{\partial^4}{\partial x_2^{\ 4}} \ . \tag{3.a,b}$$

The notation for the orthotropic elastic constants are defined by the following stress strain relations:

$$\varepsilon_{11} = \frac{1}{E_1}(\sigma_{11} - \nu_1\sigma_{22}) \ , \qquad \varepsilon_{22} = \frac{1}{E_2}(\sigma_{22} - \nu_2\sigma_{11}) \ ,$$

$$\varepsilon_{12} = \frac{1}{2G_{12}}\sigma_{12} \ , \qquad \frac{\nu_1}{E_1} = \frac{\nu_2}{E_2} \ , \qquad D_1 = E_1h^3/[12(1-\nu_1\nu_2)] \ , \tag{4.a-e}$$

where x_1 and x_2 are the principal directions of orthotropy and are taken respectively along the axial and circumferential directions. The stress and moment resultants are related to F and w through the usual expressions (see [3] or [4]).

In solving the problem, for example by expressing F and w in terms of appropriate Fourier integrals, (1) may be reduced to a system of two fourth order ordinary differential equations. The characteristic function of this system will then be an 8th degree polynomial the coefficients of which will be functions of the transform variable. For the problem to be analytically tractable it is essential that the roots of the characteristic equation be obtainable in closed form. Apparently, for the anisotropic shells in general and for the orthotropic shells in particular this is not possible. In order to express the roots in closed form the operators ∇_1^4 and ∇_2^4 must be properly factorized. From (3) it is clear that these operators can indeed be factorized and may be expressed in the following form

$$\nabla_1^4 = \left(\frac{\partial^2}{\partial x_1^2} + \sqrt{E_2/E_1}\,\frac{\partial^2}{\partial x_2^2}\right)^2 = \nabla_2^2 , \tag{5}$$

provided the elastic constants satisfy the following conditions:

$$[\nu_2 + 2(1-\nu_1\nu_2)\frac{G_{12}}{E_1}]\sqrt{E_1/E_2} = 1 ,$$

$$\left(\frac{E_2}{2G_{12}} - \nu_2\right)\sqrt{E_1/E_2} = 1 . \qquad (6.a,b)$$

By direct substitution it may easily be shown that the conditions (6) are satisfied if

$$G_{12} = \frac{(E_1E_2)^{\frac{1}{2}}}{2[1 + (\nu_1\nu_2)^{\frac{1}{2}}]} \tag{7}$$

Considering also the relation (4.d), this means that the sheet material has only three independent constants. Such a material is said to be specially orthotropic. The results given in this note will then be valid only for those materials in which the measured value of G_{12} and that calculated from (7) in terms of measured E_i and ν_i, $(i=1,2)$ are in reasonably good agreement.

Changing the variables once more as

$$x_1 = x \; , \qquad (E_1/E_2)^{\frac{1}{4}}\, x_2 = y \; , \tag{8}$$

(5) becomes

$$\nabla_1{}^4 = \nabla_2{}^4 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 = \nabla^4 \; . \tag{9}$$

With (9), (1) becomes identical to the differential equations for isotropic shells in which $D = Eh^3/[12(1-\nu^2)]$ and E are replaced by D_1 and E_2, respectively. The problem may then be solved by following, for example, the same procedure as that described in [4] or [5] and will not be repeated here.

STRESS STATE AROUND THE CRACK TIP AND STRESS INTENSITY FACTORS

Following very closely the technique described in [4], for a pressurized shell the membrane and bending components of the stresses in the close neighborhood of the crack tip may be obtained as

$$\sigma_{yy}{}^m(r,\theta) = A_m \frac{k_p}{\sqrt{2r}} \frac{1}{4} \left(5\cos\frac{\theta}{2} - \cos\frac{5\theta}{2}\right) + 0(r^{\frac{1}{2}}) \; ,$$

$$\sigma_{xx}{}^m(r,\theta) = A_m \frac{k_p}{\sqrt{2r}} \frac{c^2}{4} \left(3\cos\frac{\theta}{2} + \cos\frac{5\theta}{2}\right) + 0(r^{\frac{1}{2}}) \; ,$$

$$\sigma_{xy}{}^m(r,\theta) = A_m \frac{k_p}{\sqrt{2r}} \frac{c}{4} \left(\sin\frac{\theta}{2} - \sin\frac{5\theta}{2}\right) + 0(r^{\frac{1}{2}}) \; , \tag{10.a-c}$$

$$\sigma_{yy}{}^b(r,\theta,z) = A_b \frac{k_p}{\sqrt{2r}} \frac{z}{2h} \Big[\Big(8\nu_o - 8\nu_c + \frac{5\nu_o\nu_c}{c^2}\Big)\cos\frac{\theta}{2}$$

$$- \frac{\nu_o\nu_c}{c^2} \cos\frac{5\theta}{2}\Big] + 0(r^{\frac{1}{2}}) \; ,$$

$$\sigma_{xx}{}^{b}(r,\theta,z) = A_b \frac{k_p}{\sqrt{2r}} \frac{2z}{h} [(8 + \frac{5\nu_c}{c^2} - 8\nu_c c^2 + 3\nu_2\nu_c)\cos\frac{\theta}{2}$$

$$- \frac{\nu_c}{4c^2}(1-\nu_2 c^2)\cos\frac{5\theta}{2}] + 0(r^{\frac{1}{2}}) ,$$

$$\sigma_{xy}{}^{b}(r,\theta,z) = A_b \frac{k_p}{\sqrt{2r}} \frac{z}{h} \frac{a^2 h^3 G_{12}}{12D_1} [\frac{\nu_c}{c^2}\sin\frac{5\theta}{2}$$

$$- (8 + \frac{\nu_c}{c^2})\sin\frac{\theta}{2}] + 0(r^{\frac{1}{2}}) , \tag{11.a-c}$$

where (r,θ) are the polar coordinates at the crack tip, z is the coordinate perpendicular to and measured from the neutral surface of the shell, and

$$k_p = \frac{p_o R}{h}\sqrt{a} , \qquad c = (E_1/E_2)^{\frac{1}{4}} , \qquad \nu_o = c^2 - \nu_1 ,$$

$$\nu_c = c^2 - (\nu_1 + \frac{h^3 G_{12}}{3D_2}) , \qquad D_2 = \frac{E_2 h^3}{12(1-\nu_1\nu_2)} . \tag{12}$$

Here p_o is the internal pressure. Thus, k_p is the corresponding stress intensity factor in the flat plate, and A_m and A_b represent stress intensity factor ratios defined by

$$A_m = k_m^s/k_p , \qquad A_b = k_b^s/k_p \tag{13.a,b}$$

where k_m^s and k_b^s are the membrane and bending components of the stress intensity factor in the shell.

NUMERICAL EXAMPLES

The elastic constants of the orthotropic shells which are considered as examples are shown in Table 1. The Table also shows the "average shear modulus" calculated from

$$G_{av} = \frac{(E_1E_2)^{\frac{1}{2}}}{2[1 + (\nu_1\nu_2)^{\frac{1}{2}}]} . \tag{14}$$

Table 1. Elastic Constants of the Materials

	Titanium	Graphite
E_1 (psi)	1.507×10^7	1.5×10^6
E_2 (psi)	2.08×10^7	40×10^6
ν_1	0.1966	0.0075
ν_2	0.2714	0.2000
G_{12}	6.78×10^6	4.0×10^6
G_{av}	7.15×10^6	3.73×10^6

Table 1 indicates that for the two materials under consideration G_{12} and G_{av} are sufficiently close to justify the assumption of special orthotropy. The calculated stress intensity factor ratios based on this assumption are shown in Tables 2 and 3 and Figures 1-4. In the examples E_1 is taken in the axial direction. The tables also show the related (orthotropic) shell parameter λ_o calculated from

$$\lambda_o = [12(1-\nu_1\nu_2)E_2/E_1]^{\frac{1}{4}}\ a/(Rh)^{\frac{1}{2}} . \tag{15}$$

Since λ_o is dependent on the elastic constants, in comparing the orthotropic results for different material orientation and the results for the isotropic material, $a/(hR)^{\frac{1}{2}}$ is used as the independent variable.

Table 2. The Stress Intensity Factor Ratios for a Pressurized Titanium Cylinder

$(E_1/E_2) = 0.724$

λ_o	$a/(Rh)^{1/2}$	A_m	A_b
1	0.5025	1.1962	0.1453
2	1.0050	1.6053	0.2645
3	1.5075	2.0795	0.3091
4	2.0100	2.5638	0.2853
5	2.5126	3.0419	0.1983
6	3.0151	3.5094	0.0590
7	3.5176	3.9648	-0.1255
8	4.0201	4.4081	-0.3493
$(E_1/E_2) = 1.380$			
1	0.5903	1.1962	0.2010
2	1.1807	1.6053	0.3643
3	1.7711	2.0794	0.4265
4	2.3615	2.5638	0.3927
5	2.9518	3.0420	0.2734
6	3.5422	3.5094	0.0808
7	4.1326	3.9649	-0.1743
8	4.7229	4.4082	-0.4835

Table 3. The Stress Intensity Factor Ratios for a Pressurized Graphite Cylinder

$(E_1/E_2) = 0.0375$

λ_o	$a/(Rh)^{1/2}$	A_m	A_b
1	0.2365	1.1928	0.0217
2	0.4730	1.5906	0.0464
3	0.7095	2.0493	0.0645
4	0.9460	2.5179	0.0726
5	1.1826	2.9818	0.0697
6	1.4191	3.4373	0.0564
7	1.6556	3.8833	0.0338
8	1.8922	4.3196	0.0027
$(E_1/E_2) = 26.66$			
1	1.2214	1.1928	0.5787
2	2.4428	1.5906	1.2393
3	3.6642	2.0493	1.7220
4	4.8856	2.5179	1.9368
5	6.1070	2.9818	1.8593
6	7.3284	3.4373	1.5049
7	8.5498	3.8833	0.9007
8	9.7712	4.3196	0.0726

For the purpose of comparison, Figures 1-4 also show the results for an isotropic shell (with a Poisson's ratio of 1/3) which are given in [5]. It is seen that in the specially orthotropic shells the stress intensity factors are strongly dependent on the modulus ratio E_1/E_2, and A_m and A_b generally increase with decreasing E_1/E_2. However, this does not necessarily mean a reduction in the resistance of the shell to cleavage as E_1/E_2 decreases (i.e., as the shell becomes stiffer in circumferential direction). For this, one also has to consider the fracture strength of the shell in the plane parallel to E_1 as a function of E_1/E_2. Intuitively, it is expected that this strength too would increase as E_1/E_2 decreases.

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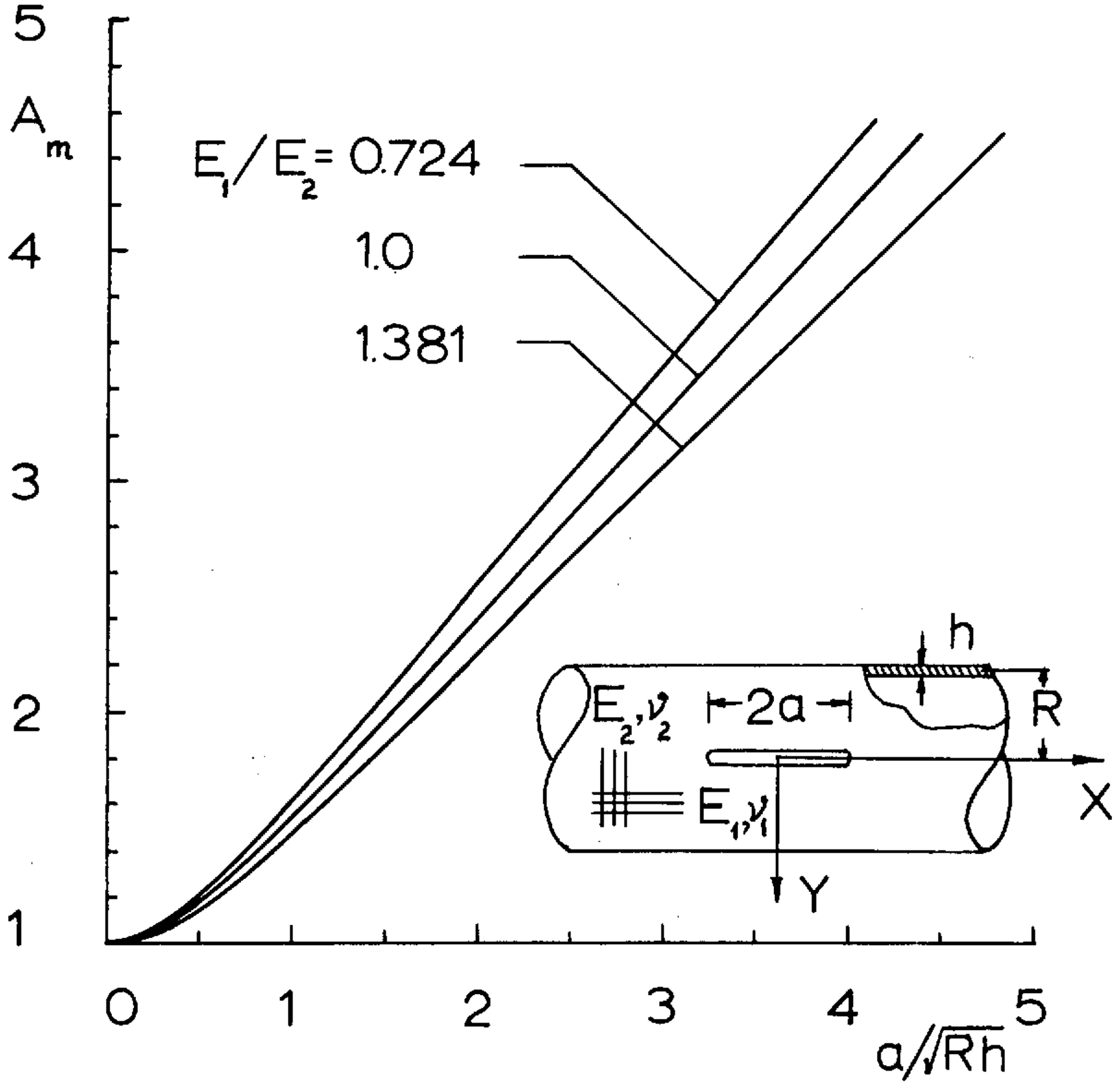

Figure 1. Membrane component of the stress intensity factor ratio for a pressurized Titanium cylinder.

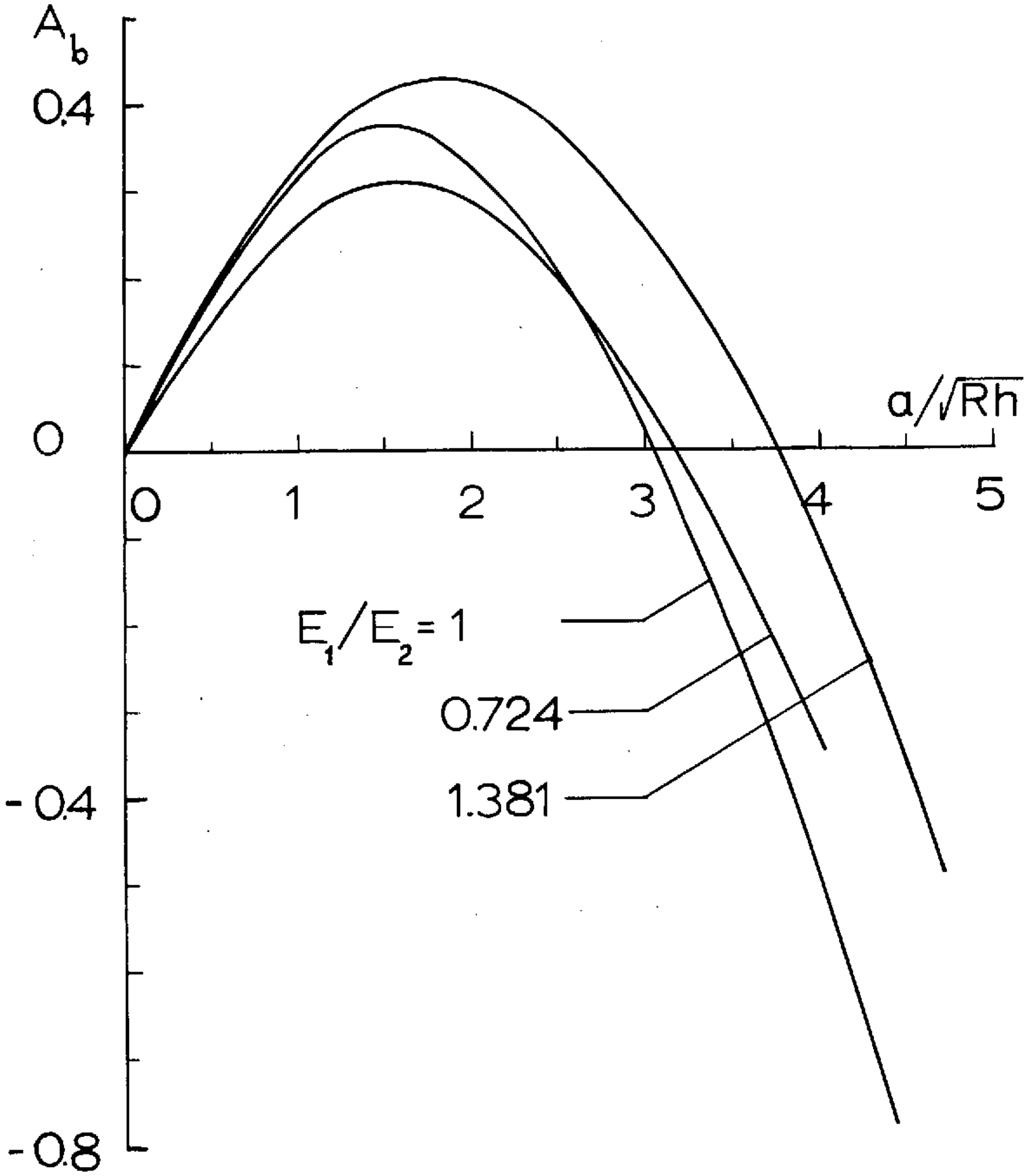

Figure 2. Bending component of the stress intensity factor ratio for a pressurized Titanium cylinder.

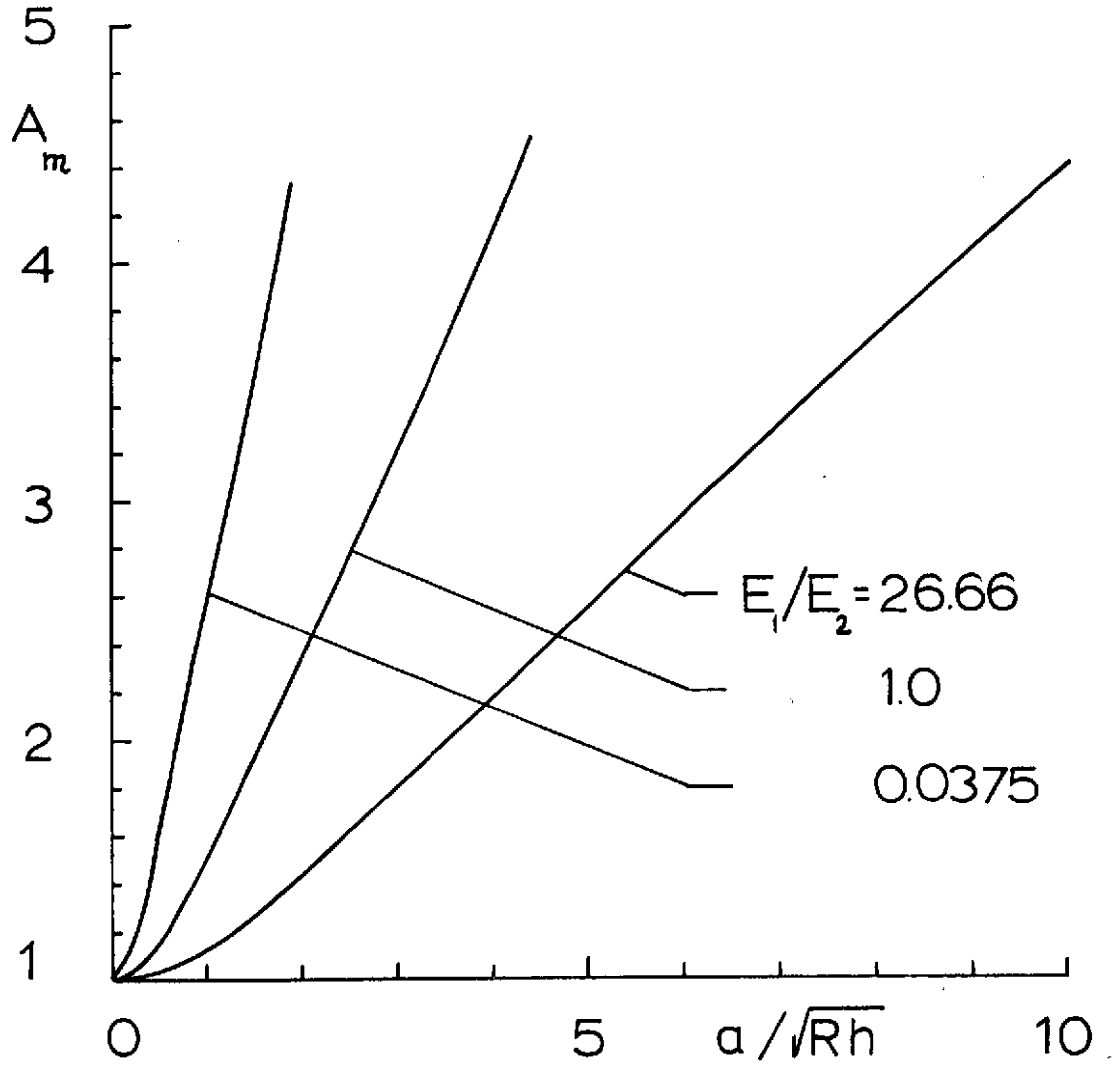

Figure 3. Membrane component of the stress intensity factor ratio for a pressurized Graphite cylinder.

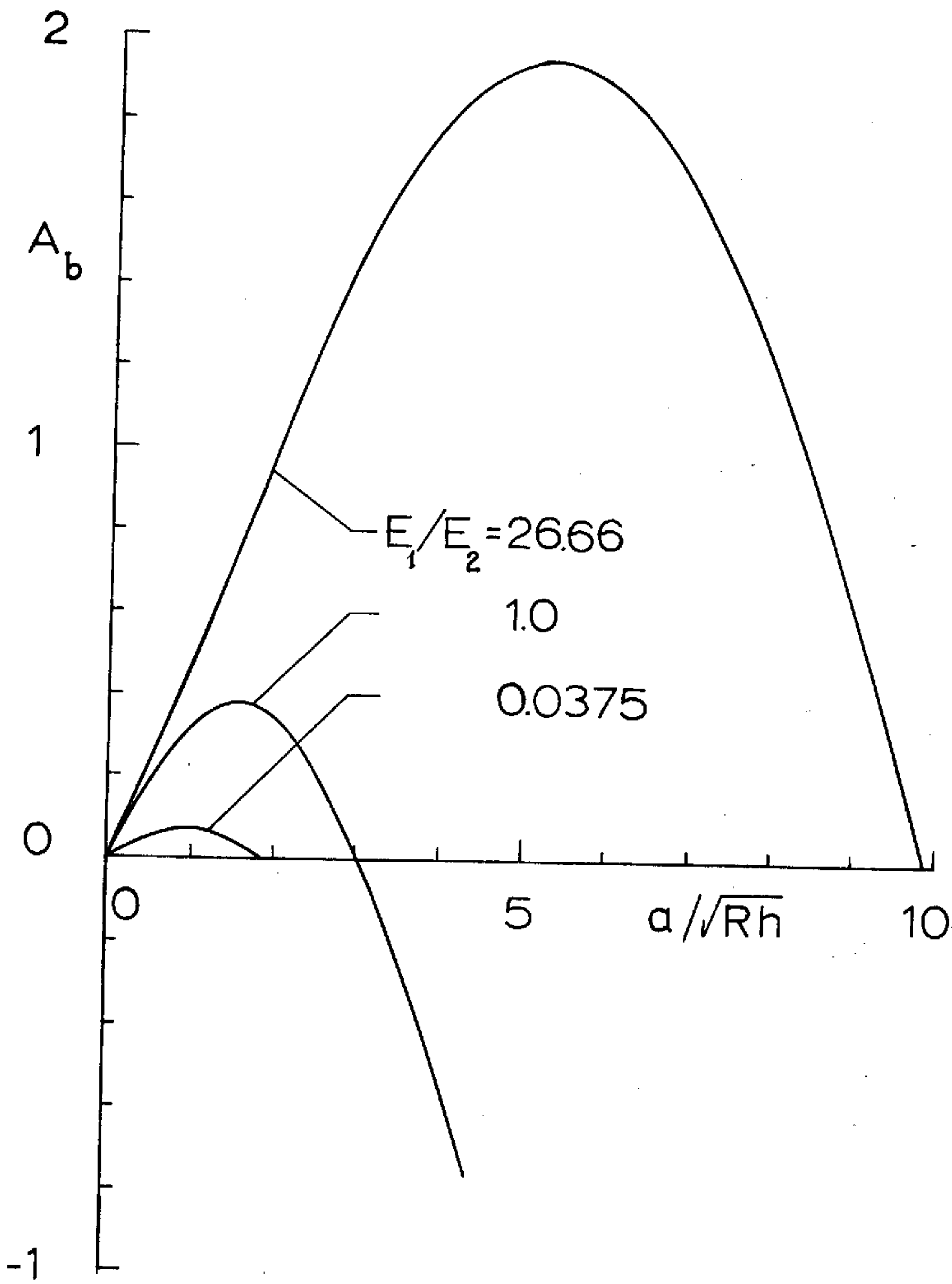

Figure 4. Bending component of the stress intensity factor ratio for a pressurized Graphite cylinder.