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ABSTRACT

An analysis of the orbital inclination of the Beacon Explorer C spacecraft over a period of nearly five months in 1970 has revealed a very clear and distinctive perturbation caused by the earth and ocean tides. The perturbation has a full amplitude of about 1.8 seconds of arc and a period of about 85 days. This amplitude is approximately 15% smaller than would be expected from the solid-earth tide alone and is shifted slightly in phase. The perturbation can be represented almost exactly by a Love number of $k_2 = 0.245$ with a phase lag of 3.2 degrees. The data used in this analysis were laser range measurements obtained by a NASA Goddard Space Flight Center tracking system in Greenbelt, Maryland

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INTRODUCTION

During the summer and fall of 1970 a NASA Goddard Space Flight Center laser tracking system tracked the Beacon Explorer C spacecraft from the Goddard Optical Facility in Greenbelt, Maryland. The tracking system provided distance measurements to the satellite at a rate of one per second with a precision of 30 to 50 cm on as many as four consecutive passes each day, weather permitting. The satellite, Beacon Explorer C, carries laser retro-reflectors and is in a near circular orbit at an altitude of about 1000 km with an inclination to the equator of 41 degrees.

The laser range measurements to the satellite from this single tracking station have been used to determine its orbit for nearly five months in 1970 and because the latitude of the tracking station is similar to the orbital inclination, the inclination is the best determined parameter. It is the inclination that has been studied here. The original purpose of this investigation was to try and observe the change in latitude of the tracking station arising from polar motion (ref. 1) but in the course of the analysis the perturbation of the inclination by the earth and ocean tides had to be accounted for.

From a point in space the basic tidal response of the earth to the gravitational attraction of the moon appears as a small elongation of the earth's shape (bulge) in the general earth-moon line. A similar elongation, or distortion, is caused by and directed roughly towards the sun. To a satellite, the tides appear as a distortion of the gravitational field arising from the tidal bulge which is associated with the motions of the sun and moon rather than the rotation of the earth. Thus, the perturbations of a satellite orbit arising from the tides are primarily determined by the motion of the satellite and its orbit with respect to the moon and sun rather than with the motion of the earth. Hence, to a satellite there are no significant perturbations associated with the large semi-diurnal and diurnal tides that are observed on the earth but only with the tidal (and other) terms of longer period, such as a few weeks to years. For the Beacon Explorer C satellite the largest terms in the perturbation of the inclination have periods of 85 days, one coming from the sun and the other from the moon, and amount to the time required for the orbital plane of BE-C to rotate once (about 4.25 degrees/day) with respect to the First Point of Aries and the node of the moon's orbit. The other major terms have periods of 34 days and 10 days, corresponding to half the rotation period of the orbital plane with respect to the sun and moon, respectively.

For a solid earth the magnitude of the tidal perturbations of the orbit would be directly proportional to the magnitude of the physical distortion of the surface by the tides, representable in magnitude by Love's number, k, where k is the ratio of the tidal potential to the tide rising potential. However, for the real earth, the magnitudes of the perturbing forces are probably modified by the additional influences of the oceans and the true nature of these forces as they affect a satellite are not presently fully understood.

THE ORBITAL INCLINATION

The orbital inclination of Beacon Explorer C has been determined on twenty-eight occasions in a period of nearly five months. Only laser range measurements from the Goddard tracking station were used in the analysis and each determination of the orbital inclination was obtained from data collected on four consecutive passes of the satellite, spanning a period of 6 hours. The values of the inclination over the five month period showed considerable variation (about 20 arcseconds), as was expected, but after subtracting the perturbations caused by the earth's gravity field, the sun and moon's gravity and solar radiation pressure, and allowing for the change in latitude of the station arising from polar motion, a variation of 2 arcseconds still remained. This remaining variation had a very distinctive structure which can be almost entirely explained as earth tidal perturbations.

The effect of the earth tides on the orbit has been calculated numerically from the simplest of tide potential models, that is, a second degree spherical harmonic expansion about an axis off-set slightly from the direction of the tide rising body to allow for a phase lag in the tidal response of the earth. In this representation no differentiation has been made between the continental crust and the oceans. The effect of both the solar and lunar earth-tides have been included in the orbital computations with the restriction that the Love number, k_2, and the phase, φ_2, are the same for both the lunar and solar tides.

The theoretical perturbation has been compared with the observed inclination residuals for several values of k_2 and φ_2. The initial comparisons were all made with $\varphi_2 = 0$ from which it soon became apparent that k_2 was around 0.25. Prior to this analysis we had anticipated that k_2 would be in the range 0.29 to 0.31 because these are the values that are being obtained from surface measurements, such as horizontal pendulums (ref. 2) and seismicity (ref. 3). For all values of k_2 near 0.25 the differences between the observed and computed values of the inclination showed systematic variation and it became necessary to introduce a phase lag (φ_2) into the perturbations. After a number of tests the values of

$$k_2 = 0.245 \pm 0.005 \quad \text{and} \quad \varphi_2 = 3.2 \pm 0.5 \text{ degrees}$$

were obtained as the best values where the fit to the data was 0.045 arcseconds standard deviation. The essential effect of introducing the phase was to reduce the standard deviation of fit to the data by about 35%. Neglecting the phase altogether led to the same optimum value of k_2 but with a larger standard error. Figure 1 shows the residuals to the inclination (after all but the earth-tides have been removed) and the theoretical tidal perturbation for the optimum values of k_2 and φ_2. The agreement between the laser data and theory is clearly evident.

The gravitational field model used in this analysis was GEM 1 (ref. 4) with modified values for the 19th degree and the 13th order terms with which the BE-C satellite is resonant. GEM 1 is a field complete to degree and order 12 with selected higher resonance terms and zonals out to degree 21. When the original GEM 1 (19, 13) resonant terms were used in this analysis the residuals about the tidal curve showed a periodic structure of 0.063 arcseconds and period 5.5 days. This pattern was used to derive new values of the 19th degree and 13th order terms and the data were then re-analyzed. The adjustment to the (19, 13) terms amounted to approximately 15% in the cosine term and 20% in the sine term. It is with the GEM 1 gravity field containing these two modified coefficients that the present results have been obtained.

DISCUSSION

The amplitude of the observed perturbation of the orbit is smaller than expected if the true value of Love's number, k_2, for the earth is around 0.3, as is generally accepted. Hence, the simple model of the earth-tides upon which the theoretical curve was based must be inadequate. However, the detailed agreement between the observed points and the theory necessitates the form of the perturbation for the BE-C satellite to be a second degree spherical harmonic. Thus, the perturbing influence that is modifying the computed tidal variation can only have a cancelling effect and in no way modify the form of the disturbing function for the satellite. Munk (ref. 5) has indicated that this effect can probably be attributed to the ocean tides and recent work by Lambeck and Cazenave (ref. 6) corroborate this view. Lambeck and Cazenave suggest that certain terms in the tidal potential function of the oceans cause perturbations of the orbit almost identical in form to the solid-earth tide. However, the magnitudes of these ocean perturbations have a different variation with orbital parameters than the solid-earth perturbations. Thus, they suggest the observed tidal perturbations will depend on the orbit of the satellite being studied; and the determinations of k_2 (from satellites) so far (ref. 7, 8, 9) strongly support this viewpoint. Thus, orbital perturbation techniques may soon be in a position to provide quantitative information on certain components of the ocean tides.

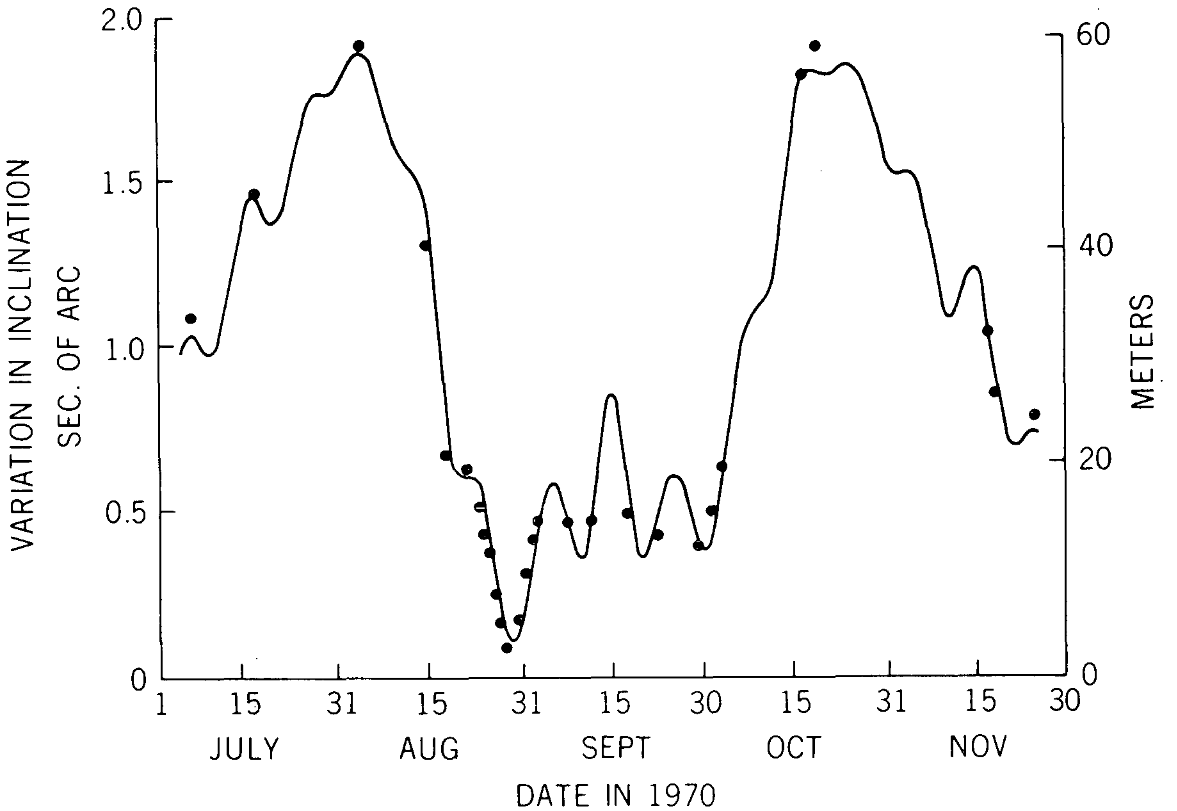

Figure 1. The perturbation of the orbital inclination of the Beacon Explorer C spacecraft arising from the earth and ocean tides. The dots are values of the orbital inclination derived from laser range measurements. The continuous line shows the theoretical perturbation computed from a second degree spherical harmonic representation of the solar and lunar tides with Love's number $k_2 = 0.245$ and a phase lag of 3.2 degrees. The right hand vertical axis shows the variation in inclination projected onto the earth's surface. The rms deviation of the observed points to the theoretical curve is 0.045 arcseconds (1.38 meters).

The analysis reported here is probably the most precise determination to date of the amplitude and phase lag of the earth and ocean tides from the tidal perturbations of a satellite. The use of high precision laser range measurements (30 to 50 cm) at a high repetition rate (1 per second) permitted the computation of very short orbital arcs of 6 hours duration. This short averaging time permitted some of the fine structure of the perturbation to be revealed. The good agreement between observation and theory clearly shows that the tidal disturbing potential (for the Beacon Explorer C satellite) can be almost entirely represented by two second degree spherical harmonics off-set by a few degrees from the directions of the sun and moon.

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