FINAL REPORT

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Theoretical Study of Nonlinear Waves and Shock-Like Phenomena in Hot Plasmas

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This contract supported theoretical studies in basic plasma physics during the years 1966 - 1971. This was a critical period in the development of the UCLA Plasma Physics Group, and the support of graduate and postdoctoral students, together with faculty summer salaries and a modest amount of numerical computation provided by this contract, played a key role in the development of both plasma physics and plasma physicists.

The work was centered on nonlinear waves and shock-like phenomena in hot plasmas, and 16 reports, both published and unpublished, summarize the principal results obtained. We list below the titles and authors of those research reports which were wholly or partially supported by this contract, and in the case of those which have been published, we also give the journal reference. It should be noted that, whereas the emphasis in most of this work was on basic plasma physics, the phenomena studied are almost all pertinent to space physics applications, and in fact, under other NASA support (NGL 05-007-190 S4), members of the UCLA Plasma Physics Group have also studied specific problems of magnetospheric and solar wind plasma physics.

"Linear and Nonlinear Theory of Grid Excitation of Low Frequency Waves in a Plasma," G.L. Johnston

The steady state response of an infinite, uniform expanse of hot, rarefied plasma to low-frequency excitation by a pair of idealized parallel plane grids is investigated. The Vlasov equation for grid excitation of longitudinal waves is considered both in the linearized (infinitesimal amplitude) limit and in the weakly nonlinear case of small but finite amplitude disturbances.

In the linearized theory, the Fourier inversion integral for the spatial behavior of the potential excited in the plasma by a pair of grids with finite spacing is evaluated by first transforming the variable of integration from the Fourier transform variable k to $\zeta = \omega_o/ka_i$, where ω_o is the driving frequency and a_i is the ion thermal speed, and then separating the integrand into two parts, one principally responsible for the ion acoustic wave near the grid and the other for the electron wave at large distances from the grid. For driving frequencies small in comparison with the ion plasma frequency, the resulting integrals are evaluated numerically along appropriate deformed contours of integration. The path of steepest descents is chosen for the ion integral; it includes a residue contribution from the pole which gives the dominant behavior of the ion wave. In both cases, the integrands undergo small phase changes over the chosen contours. Numerical results are presented; in the dipole limit, the results of Gould are recovered.

A perturbation series expansion of the potential and of the species distribution functions in the (nonlinear) Vlasov equation yields a hierarchy of equations associated with a smallness parameter proportional to the amplitude of excitation. In each order the equations are linear in the perturbation quantities of that order and have driving terms composed of quadratic combinations of lower order quantities. For sufficiently small amplitudes of excitation, the principal contributions to the response come from the first and second order equations. In the first order, the linearized Vlasov equation is obtained. In the second order, the steady-state response consists of zero frequency and double frequency components. In the manner of Landau, the second order equations are Laplace-Fourier transformed and resulting velocity integrals are analytically continued and expressed in terms of the plasma dispersion function. Confining our attention to driving frequencies small in comparison with the ion plasma frequency, and approximating the driving terms (linearized response) by their dominant pole component, we obtain the Fourier inversion integrals for the steady-state response. The integrals are evaluated numerically by methods developed in treating the linearized problem. The double frequency component of the response is strongly damped, like an "ion wave" but has a slow phase variation with distance, like an "electron wave". The zero frequency component is a polarization of the plasma unaccompanied by species current densities.

Ph.D. Dissertation

"Temperature Gradient Instabilities in Axisymmetric Systems," C. S. Liu

The temperature gradient effect on the stability of a plasma in the "Maximum -J Configuration" is studied and two possible instabilities due to temperature gradient are found in the axisymmetric system.

Published in Phys. Fluids 12, 1489 (1969)

"Low Frequency Interchange Instabilities of the Ring Current Belt," C. S. Liu

The stability of the ring current belt against low-frequency, electrostatic perturbations is analyzed, with the effect of a finite electric field along the magnetic line of force included. We find four possible instabilities - the hydromagnetic interchange, the trapping instability, the low-frequency flute mode, and the resonant interchange; the conditions for their occurrence in the ring current belt are derived. Due to the parallel electric field associated with the instabilities, their main consequence is to precipitate charged particles into the ionosphere, leading to ring current pressure relaxation. Their possible relevance to observations is discussed.

Published in J. Geophys. Res. 75, 3789 (1970)

"A New Representation for the Conductivity Tensor of a Collisionless Plasma in a Magnetic Field," B. D. Fried and C. L. Hedrick

Conventional expressions for the conductivity tensor of a collisionless plasma in a uniform magnetic field involve infinite sums of Bessel function terms, each of which is associated with an harmonic of the cyclotron frequency. For many problems, it is much more convenient to have closed form expressions for these infinite sums and we show here that a suitable approach to the solution of the linearized Vlasov equation leads directly to such closed forms, thus eliminating the necessity for subsequent resumming. All elements of the conductivity tensor are expressed in terms of a single function of $A(v_z,v_\perp)$ and its first two derivatives with respect to $v_\perp$. The results are valid for any particle distribution function since the integrations over v_z and $v_\perp$ are not carried out explicitly. The conventional expressions, involving sums over harmonics, can be easily recovered at any point by making use of a standard Bessel function identity. In addition to providing an economical derivation of results important for the case of a uniform plasma, this approach also facilitates the treatment of a plasma with a small density gradient, the new functions associated with each additional order in $(\nabla n/n)$ being again expressible in terms of A.

Published in Festschrift for Gregor Wentzel, University of Chicago Press, 1969

"Effect of Fokker-Planck Collisions on Plasma Wave Echoes," G. L. Johnston

The effect on plasma wave echoes of Coulomb collisions described by a complete Fokker-Planck collision term is determined theoretically by application, in first and second orders, of the particular integral of a linearized kinetic equation which is capable of describing a Fokker-Planck collisional kinetic equation in the resonant region.

Published in Physics of Fluids 13, 136 (1970)

"Linear and Nonlinear Theory of Grid Excitation of Low-Frequency Waves in a Plasma," G. L. Johnston

The steady-state excitation of longitudinal waves by a pair of idealized grids immersed in a collisionless plasma and driven at a frequency small compared with the ion plasma frequency is investigated theoretically. In linear theory the Fourier-inversion integral which determine the spatial behavior of the potential in the plasma is expressed as a sum of two integrals which embody the interactions of phase-velocity components of the wave with ions and electrons. An appropriate choice of the deformed contour of integration permits evaluation of the response as the sum of the residue of the dominant "ion-acoustic" pole and of the two branch-cut integrals. A perturbation-series expansion of the potential and the species distribution functions in the (nonlinear) Vlasov equation yields a hierarchy of equations. In each order the equations are linear in the perturbation quantities of that order and have driving terms composed of quadratic combinations of lower-order quantities. For sufficiently small amplitude of excitation the principal contributions to the response come from the first-order (linearized Vlasov) equation and the second-order equation. In second order the steady-state response consists of zero-frequency and double-frequency components. The second-order equations are Laplace-Fourier transformed and resulting velocity integrals are expressed in terms of plasma dispersion functions. By approximating the driving terms by their dominant-pole component, one can express the steady-state double-frequency response as a single Fourier-inversion integral. As in the linear problem, the integral can be evaluated as the sum of a residue and of "ion-like" and "electron-like" branch-cut integrals. Numerical results are presented for the linear and nonlinear cases.

Unpublished

"Theory of Stability of Large Amplitude Periodic (BGK) Waves in Collisionless Plasmas," M. V. Goldman

A method is developed for examining the stability of a large amplitude periodic (BGK) wave, E_o, in a collisionless plasma. Vlasov's equation is integrated by the method of characteristics to yield a polarization charge density response, ρ_1, linear in a small-amplitude field E_1, but nonlinear in E_o. The susceptibility linking ρ_1 and E_1 is expressed in terms of the exact orbits of trapped and untrapped particles in the field E_o, distributed in energy according to the assumed BGK distribution function, f_o. These susceptibilities couple the Fourier components of E_1 in the usual mode-coupling fashion, but trapping effects are now included. For fields E_o which are not too large, the mode-coupling problem reduces to finding the zeroes of a 2×2 or 3×3 determinant. Distribution functions which are localized at the bottom of the potential energy troughs of E_o give identically the growing side-band instability of Dawson, Kruer, and Sudan.

Published in Physics of Fluids 13, 1281 (1970)

"Interaction of Quasi-Transverse and Quasi-Longitudinal Waves in an Inhomogeneous Vlasov Plasma", C.L. Hedrick, Jr.

We consider the interaction of quasi-longitudinal and quasi-transverse modes in an inhomogeneous Vlasov plasma for frequencies between the electron cyclotron frequency and the maximum value of the upper-hybrid frequency. Using a formalism which parallels that used for a homogeneous plasma, we solve the Vlasov equation and Maxwell's equations for propagation perpendicular to the dc magnetic field and obtain a set of coupled differential equations for the components of the electric field. These equations are truncated so that they are correct to order $k^2\rho_c{}^2$ and $k\kappa\rho_c$ where k indicates -i d/dx operating on the electric field, $\kappa = (1/n_0)(dn_0/dx)$, n_0 is the density and ρ_c is the gyroradius. Fourth order wave equations for the longitudinal component of the electric field E_x, as well as for the transverse component E_y, are solved for a linear density profile and connection formulae are derived. Comparison of the solutions for E_x and those for E_y provides considerable insight into the coupling which occurs near points where the frequency equals the local upper-hybrid frequency. Outside these coupling regions we show that the fourth order wave equation decouples into two second order systems: one for the quasi-longitudinal waves and one for the quasi-transverse waves. Using an idealized density profile for a bounded plasma, we obtain analytic expressions for the transmission and reflection coefficients. These expressions depend strongly on the number of "wavelengths" of the quasi-longitudinal modes between the points where the upper hybrid condition is satisfied, as well as the number of "wavelengths" of the quasi-transverse modes between these same points. In contrast to previous treatments, all approximations are made in a consistent and systematic fashion and the results apply to a much wider frequency range.

Ph.D. Thesis

"The Analytic and Asymptotic Properties of the Plasma Dispersion Function", Alfredo Baños, Jr. and George L. Johnston

This paper gives a systematic account of the analytic and asymptotic properties of the plasma dispersion function. The basic properties of the function are derived from its definition as the Hilbert transform of the Gaussian and then, invoking the fact that the function is a solution of the confluent hypergeometric equation with parameters $a = c = \frac{1}{2}$, additional properties are derived, such as a loop integral representation which lends itself readily to the proper interpretation of the Stokes phenomenon associated with its asymptotic expansion. The major portion of this effort is devoted to an analysis of the behavior of the remainder in the asymptotic expansion of the function, which goes much beyond the usual order of magnitude estimates or upper bounds, and actually provides asymptotic formulas for the computation of the ratio of the remainder to the first discarded term in the asymptotic expansion.

"High-frequency Hall Current Instability", K. Lee, C.F. Kennel and J.M. Kindel.

In a partially ionized gas, if electrons and ions have a relative drift V_d perpendicular to a static B field, Farley and Buneman have shown that an electrostatic wave with propagation vector K perpendicular to B and wavelength $\lambda \gtrsim 0.375$ meter will be unstable if $K \cdot V_d/K$ exceeds the ion thermal speed. We have extended Farley's calculation for K perpendicular to B by the following procedure. First, we examine modes of higher frequencies and shorter wavelengths. Second, we retain the term $K^2 \lambda_{DE}{}^2$ (λ_{DE} is the electron Debye length), since this term cannot be neglected for short wavelength modes. This makes the dispersion relation density dependent. Third, we obtain plots of growth rate versus frequency for various values of $K \cdot V_d/K$. With the use of the Culler-Fried on-line computer system, the dispersion relation has been solved numerically by contour integration. There are four results of the numerical calculation. First, there is an electron density threshold for excitation of higher frequency modes. Second, for a fixed electron density of $3.5 \times 10^5/cm^3$, the fastest growing mode shifts to higher frequency as $K \cdot V_d/K$ increases. For $K \cdot V_d/K$ equal to three times the ion thermal speed, $3a_r$, the fastest growing mode oscillates at $\omega_r \simeq (\Omega_E \Omega_I)^{\frac{1}{2}}$ can be excited for $K \cdot V_d/K \gtrsim 2.3a_I$ and current density greater than 1.0×10^{-5} amp/m^2. These are reasonable values for the equatorial electrojet and the auroral electrojet in the ionosphere during disturbed times.

Published in Radio Science, 6, 209, (1971).

"Spatial Cyclotron Damping", C.L. Olson

To examine spatial electron cyclotron damping in a uniform Vlasov plasma, we note that the plasma response to a steady-state transverse excitation consists of several terms (dielectric-pole, free-streaming, and branch-cut), but that the cyclotron-damped pole term is the dominant term for $z > \ell \simeq c/\omega_{ce}$ provided $(\omega_{pe}/\omega_{ce})^2(c/a) >> 1$. If the latter inequality does not hold, then the free-streaming and branch-cut terms persist well past $z = c/\omega_{ce}$ as ω_1 approaches ω_{ce}, making experimental measurement of cyclotron damping essentially impossible. Considering only $(\omega_{pe}/\omega_{ce})^2(c/a) >> 1$, we show how collisional effects should be estimated and how a finite-width excitation usually has little effect on the cyclotron-damped part of the response. We establish criteria concerning collisional damping, measurable damping length sizes, and allowed uncertainty in the magnetic field $\underset{\sim}{B}_o$. Results of numerical calculations, showing the regions in the appropriate parameter spaces that meet these criteria, are presented. From these results, one can determine the feasibility of, or propose parameter values for, an experiment designed to measure spatial cyclotron damping. It is concluded that the electron temperature T_e should be at least 1 ev., and preferably 10 ev. or higher, for a successful experiment.

Published in Physics of Fluids, 15, 160, (1972).

"Electromagnetic Plasma Wave Propagation Along a magnetic Field," Craig L. Olson

Part I: Steady-state excitation of transverse plasma waves along a magnetic field.

The linearized response of a Vlasov plasma to the steady-state excitation of transverse plasma waves along an external magnetic field is examined. Previous research is reviewed. Assuming a delta-function excitation mechanism, and performing a detailed Vlasov-Maxwell equation analysis using Fourier-Laplace transforms, the plasma response is found to consist of three terms: a branch-cut term, a free-streaming term, and a dielectric-pole term(s). These terms are examined analytically, and numerically evaluated for a case in which the driving frequency (ω_1) is slightly below the electron cyclotron frequency (ω_{ce}). In addition to the least-damped pole term, it is shown that the free-streaming term is always significant and the branch-cut term is significant when $\omega_1 \approx \omega_{ce}$. The infinite sequence of pole terms (that result from the infinite number of roots of the appropriate transverse dispersion relation) is shown to be negligible except at positions very close to the place of excitation. Effects of Krook model collisions are investigated, as well as effects of a finite-width excitation mechanism.

Part II: Steady-state transverse plasma wave echoes.

The phenomenon of plasma wave echoes, introduced by Gould, Malmberg, O'Neil, and Wharton for the case of longitudinal electrostatic waves, is extended to the case of transverse plasma waves that propagate along an external magnetic field. It is shown that a transverse echo results in lowest order only when one excitation is transverse and the other is longitudinal. For this case, the second-order (nonlinear) plasma echo response is computed from the Vlasov-Maxwell equations up to an integral over the velocity variable v_z. Transverse echo characteristics are discussed and several experiments are suggested. The integral over v_z is evaluated by the method of steepest descent; the results are explained physically and also evaluated numerically for some specific cases. Several extensions are considered: (1) effects of Fokker-Planck collisions and finite-width excitation mechanisms, (2) effects of temperature anisotropy ($T_\perp \neq T_z$), and (3) effects of propagation just off-axis ($k_\perp \neq 0$). Lastly, transverse echoes for the case of no external magnetic field are examined.

In addition, several appendices concerning Part I and Part II are presented, the most important of these being (1) a thorough investigation of the roots of the transverse dispersion relation ($\underline{k} \parallel \underline{B}_0$) for real ω and complex k, and (2) the method of steepest descent as used in evaluating the branch-cut and phase-mixing integrals that occur in Part I and Part II.

Ph.D. Thesis

"Ion Acoustic Waves in a Multi-ion Plasma," B.D. Fried, R.B. White, and T.K. Samec

An exact treatment of the multi-species ion acoustic dispersion relation is given for an Argon/Helium plasma. Phase velocity and damping are obtained as a function of ion-electron temperature ratio and relative densities of the two species. There are two important modes in the plasma, with quite different phase velocities in general, which we refer to as principal heavy ion mode and principal light ion mode. Which of these is dominant depends on the relative densities of the two components, but in general the light ion mode becomes important for surprisingly small light ion contamination. Approximate analytic expressions are derived for damping rates and phase velocities and their domains of valicity are investigated. Relevance of the results for the investigation of collisionless shocks is discussed.

Published in Phys. Fluids 14, 2388 (1971).

"Nonlinear Evolustion and Saturation of an Unstable Electrostatic Wave," B.D. Fried, C.S. Liu, R.W. Means, and R.Z. Sagdeev

The nonlinear development and saturation of a single Langmuir wave driven unstable by a gentle bump in the tail of the distribution function in a collisionless plasma is studied by treating the resonant particles numerically. Over a wide range of parameter values, the amplitude of the potential ϕ is found to saturate at such a level that the ratio $g \equiv \omega_b/\gamma_0 \simeq 3.2$, where $\omega_b = (ek^2\phi/m)^{1/2}$ is the bounce frequency of the trapped particles in the wave trough and γ_0 is the linear growth rate, approximately given by the classical Landau value. In view of the importance of inverse Landau damping for many instabilities, this work should have wide applicability and the results should be suitable for direct experimental tests.

Submitted for publication, Doklady.

"Cross-Field Current-Driven Ion Acoustic Instability," P.J. Barrett, B.D. Fried, C.F. Kennel, J.M. Sellen, and R.J. Taylor

This instability occurs when electrons and ions have a relative streaming velocity, along x, $v_D > c_s = (T_e/M)^{1/2}$, across a weak magnetic field, b_z. If $k_x r_{ci} >> 1$ and $\omega << \omega_{ce}$, the spatial growth rate is enhanced by a factor k_x/k_z over the value at $B_z = 0$. Measurements of growth rate (Im k/Re k) have been carried out in two different plasma configurations as a function of both B_z and ω . The results are consistent with the linear dispersion relation for this instability.

Published in Phys. Rev. Lett. 28, 337 (1972).

"Structure of Ion Acoustic Solitons and Shock Waves in a Two-Component Plasma," R.B. White, B.D. Fried, F.V. Coroniti

Time independent solitary waves and shocks are investigated in a two-component plasma using a fluid model and kinetic theory. It is found that very small concentrations of a light ion can drastically alter the structure, changing the potential maximum ϕ_m by an order of magnitude. For a fixed Mach number, a critical density ratio of light to heavy ions is found at which ϕ_m changes discontinuously from a value large enough to reflect the light ions to one which allows them to traverse the shock front and enter the downstream flow. The downstream oscillatory structure normally seen in a shock is completely quenched by dissipation due to light ion reflection at concentrations of 3 - 8% He in an A plasma for typical T_e/T_i and Mach number values.

Published in Physics of Fluids 15, 1484 (1972).