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A MATHEMATICAL MODEL CONCERNING REFLECTANCE FROM A ROW CROP

by

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ABSTRACT. This report extends the recent work of Allen, Gayle, and Richardson (1970) and Suits (1972) to compute directional reflectance from a crop row. A model is constructed which takes into account edge effects and aids in discriminating crops with leaf orientation in preferred directions. This report only contains the development of the mathematical equations. Numerical results will be published in a forthcoming report.

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Contents.

1. Abstract

We have extended the recent work of Allen, Gayle, and Richardson (1970) and Suits (1972) to compute directional reflectance from a row crop. We assume a 'row canopy' to be bounded in the horizontal direction perpendicular to the rows, and in the vertical direction while along the rows the 'row canopy' is assumed to extend to infinity. The problem can be easily adapted to a crop in which the row structure is destroyed by the components of plants in one row running into components of another row. In this case one assumes that the plant canopy extends to infinity in the horizontal direction perpendicular to the rows. The advantage of bounding a plant canopy in horizontal directions is that the edge effects are properly taken into account. The calculations can easily be extended to incorporate a boundary between two fields. The incident solar radiation is decomposed into three cartesian components, one along the rows and two perpendicular to row direction. For horizontal incidence the scattering and absorption coefficients of specular radiation are assumed to be different from those for vertical incidence. Hopefully, the introduction of these new scattering coefficients will help in discriminating crops with leaf orientation in preferred directions. Changes in leaf orientation caused by certain diseases will also show up as changes in reflectance and transmittance.

This report only contains development of mathematical equations. Numerical results based upon these equations will be published in a forthcoming report.

2. Introduction.

Attempts at identifying vegetation types have been made in more than one way. Laboratory experiments performed to study the absorption spectra of a single leaf taken from each vegetation type(see Mc Leod, 1971) have not yielded algorithms suitable for discriminating one type of vegetation from another. The main reason for this is that leaves from most plants contain almost the same amount and type of chlorophyls and water, which are responsible for the absorption spectra. Another approach that has been applied to this problem is to study the scattering and absorption properties of a collection of fresh leaves placed flat on a background of known reflectivity. This approach represents an improvement from the one leaf case as this arrangement is closer to a plant than an individual leaf. However, it is still not close enough to a vegetation field or even to a plant.

The aim in a mathematical model of a plant canopy is the same as in the laboratory experiments, that is to arrive at algorithms which will help in discriminating vegetation types. The mathematical models developed in the past assume a plant to be occupied between two horizontal planes, extending to infinity in all horizontal directions. These models can only give reliable results if the sensors view area is small enough, that is, either the plant canopy is large or a vegetative field being observed has closely spaced plants and a small portion of the field is being viewed at a time. Furthermore, if the field of view contains more than one vegetation type the canopy model needs modification in that boundaries in horizontal directions need to be added.

When the sun is at an angle from the local vertical the light penetrates through a horizontal direction in addition to the vertical direction. Now, consider that the object under observation is a row crop and the sun is at a non zero angle from the local vertical and the azimuthal angle of the sun with respect to the row direction is also non zero. It is obvious that for a complete description of the radiation field we need to solve for the radiation equations in three mutually orthogonal directions, one vertical and two horizontal directions parallel and perpendicular to the rows. Section 3 of this paper is devoted to this problem. In sections 4, 5, and 6 we shall elaborate on additional shadowing of soil between plants, limitations of our model and some experiments that should be performed to gain insight into plant canopy effects. This report only contains development of mathematical equations. Numerical results based upon these equations will be published in a forthcoming report.

3. Review of Previous Plant Canopy Models

Allen and Richardson (1968) were the first to employ the equations of Kubelka and Munk (referred to below as KM) (1931) to a plant canopy. The KM and a large number of related papers were concerned with the study of interaction of light with paints, glass, paper, and plastic materials where the object consists of tightly packed particles. In general light penetrates only a short distance into the material and the lateral extent of the material does not play an important role. For this reason it sufficed to assume that the object extended to infinity in lateral directions.

The only scattering coefficients occuring in the KM work are an absorption coefficient and a back scattering coefficient, both of these coefficients are related to internal diffuse light. Duntley (1942) reported theoretical work of Ryde (1931,1932) who, based on comparison of his work to observed data, concluded that absorption and scattering coefficients of incoming specular light are different from those of internal diffuse light(Since these equations are now being used in plant canopy models it seems appropriate to verify this same result experimentally for plant canopies). The mathematical equations of Duntley (1942) which incorporate this modification contain five unknown constants, two for diffuse light and three for specular. Allen, Gayle and Richardson (1970) used these improved equations to study plant canopy effects. Again, as with the KM work, the plant canopy is assumed to have infinite lateral extent.

Two important assumptions in the KM work are: (1) Lambert Cosine law, i.e. there is no Fresnel reflection. It is not known to the present author if this assumption has been experimentally verified in the case of plants and leaves. (ii) the particles in the layer are regarded as randomly distributed and smaller in size than the thickness of the layer. Referring to the first assumption it is quite possible that some percentage of the plant area acts as a specular reflector. However, because of lack of time this subject will not be dealt with in this report. Concerning the second assumption, one observation can be made by visual examination, that is, in a healthy plant the upper surface of a leaf is, in general, a better reflector than its lower surface. This fact will be incorporated in the present work. Suits (1972) made a useful extension of the work of Allen, Gayle and Richardson in that he defined a vertical leaf area index of a plant. The leaves were assumed to be Lambertian reflectors but the introduction of the new vertical leaf area index leads to expressions of transmitted and reflected radiation which depend upon the sun and sensor angles, thereby yielding non-Lambertian expressions for the reflectance and transmittance. However, Suits has assumed that the reflection and transmission coefficients for specular and diffuse radiation are identical, as can be seen from his expressions (6) and (7).

4. A Model for Interaction of Light with a Row Crop

In this section we shall obtain a self consistent radiation field in a row crop whose plant height, row width and row direction are known. Also known are the angular coordinates (θ_s, ψ_s) of the sun where θ_s represents the zenith angle of the sun and ψ_s its azimuth with respect to the row direction. If I_0 is the intensity of sunlight then the components along the downward vertical (z-axis), along the rows (x-axis) and perpindicular to the rows (y-axis) are $I_0 \mathrm{Cos}\theta_s$, $I_0 \mathrm{Sin}\theta_s \mathrm{Cos}\psi_s$ and $I_0 \mathrm{Sin}\theta_s \mathrm{Sin}\psi_s$ respectively. The coordinates and components are shown in figure 1. Let us denote the attenuated components of specular light as it passes through the canopy by I_{sx}, I_{sy}, I_{sz} and the components of diffuse light in the positive and negative directions along the coordinate axes as I_{+x}, I_{-x}, I_{+y}, I_{-y}, I_{+z}, and I_{-z} respectively. In the vertical direction we introduce μ_z, B_z, F_z as absorption, back scattering and forward scattering coefficients for specular light. The quantities μ_x, B_x, F_x and μ_y, B_y, F_y are similar coefficients for the x and y components of specular light. Most plants have axial symmetry and for this reason we shall assume $\mu_x=\mu_y$, $B_x=B_y$, $F_x=F_y$. For diffuse light in the z direction we assume back scattering coefficient 'B' of I_{+z} to be different from 'B′' the back scattering coefficient for I_{-z}; this we introduce to account for greater reflectance from the upper surfaces of leaves. For the x and y components of diffuse radiation the back scattering coefficient is assumed to be B" different from B or B' as the scattering in horizontal directions depends upon the vertical leaf area index which can be different in magnitude from the horizontal leaf area index. Similarly the absorption coefficient for x,y component equations is assumed to be a new parameter μ'. With these approximations the equations for specular and diffuse light in the three coordinate axes directions can be written as:

Z-Axis:

$$\frac{dI_{sz}}{dz} = -(\mu_z+B_z+F_z)I_{sz} \tag{1}$$

$$\frac{dI_{+z}}{dz} = F_z I_{sz} - (\mu+B)I_{+z} + B'I_{-z} \tag{2}$$

$$-\frac{dI_{-z}}{dz} = B_z I_{sz} - (\mu+B')I_{-z} + BI_{+z} \tag{3}$$

X-Axis:

$$\frac{dI_{sx}}{dx} = -(\mu_x+B_x+F_x)I_{sx} \tag{4}$$

$$\frac{dI_{+x}}{dx} = F_x I_{sx} - (\mu'+B'')I_{+x} + B''I_{-x} \tag{5}$$

$$-\frac{dI_{-x}}{dx} = B_x I_{sx} - (\mu'+B'')I_{-x} + B''I_{+x} \tag{6}$$

Along the y-axis equations can be written simply by replacing I_{sx} by I_{sy}, μ_x by μ_y etc. The boundary conditions along the x-,y-,z-axes are all different. For example, at the top of a plant, which is at $z=0$ and at the sunlit side of the row i.e. $y=0$, and at the sunlit end of a row there is no diffuse light in the positive directions, i.e.

$$I_{+z}(z=0) = 0 \tag{7}$$

$$I_{+y}(y=0) = 0 \tag{8}$$

$$I_{+x}(x=0) = 0 \tag{9}$$

At the bottom of a row we have

$$I_{-z} = Rg(I_{+z}+I_{sz}) \text{ at } z=z_1 \tag{10}$$

At the other side of the row

$$I_{-y} = 0 \quad \text{at } y=y_1 \tag{11}$$

where y_1 is the width of plant cover in a row. At the other end of a row (which is assumed to be of infinite length)

$$I_{-x} = 0 \quad \text{at } x=\infty \tag{12}$$

The solution of equation (1)-(6) and the y-equations subject to boundary conditions (7)-(12) need not be written down as we are only interested in quantities that can be observed with the help of a remote sensor. These quantities are the reflectance and transmittance at five surfaces: $x=0$, $x=\infty$, $y=0$, $y=y_1$, and $z=0$. Reflectance values at $z=0$, $x=0$ and $y=0$ are given by $I_{-z}(z=0)$, $I_{-x}(x=0)$ and $I_{-y}(y=o)$ respectively. Transmittance at $x=\infty$ is 0 and that at $y=y_1$ is

$I_{+y}(y=y_1)$. These quantities are given below:

$$I_{-z}(z=0) = A_3 + A_4 + N\, I_{z0} \qquad (13)$$

where $$I_{z0} = I_0 \cos\theta_s \qquad (14)$$

$$Q_3 = \mu_z + B_z + F_z \qquad (15)$$

$$A_3 = \frac{\mu+B+\alpha_1}{B'} A_1 \equiv LA_1 \qquad (16)$$

$$A_4 = \frac{\mu+B+\alpha_2}{B'} A_2 \equiv MA_2 \qquad (17)$$

$$A_1 = \frac{PI_{z0}(M-Rg)e^{\alpha_2 z_1} + (1-PRg-N)I_{z1}}{e^{\alpha_1 z_1}(L-Rg) - (M-Rg)\,e^{\alpha_2 z_1}} \qquad (18)$$

$$I_{z1} = I_{z0}\exp(-Q_3 z_1) \qquad (19)$$

$$A_2 = \frac{-PI_{z0}(L-Rg)e^{\alpha_1 z_1} - (1-PRg-N)I_{z1}}{e^{\alpha_1 z_1}(L-Rg) - (M-Rg)\,e^{\alpha_2 z_1}} \qquad (20)$$

$$N = \frac{(\mu+B-Q_3)B_z + BF_z}{(\mu-\alpha_1)\ (\mu-\alpha_2)} \qquad (21)$$

$$P = \frac{B'B_z + (\mu+B'+Q_3)F_z}{(\mu-\alpha_1)\ (\mu-\alpha_2)} \qquad (22)$$

$$\alpha_1 = -\tfrac{1}{2}(B-B') + \sqrt{\mu^2+\mu(B+B')+\tfrac{1}{4}(B-B')^2} \qquad (23)$$

$$\alpha_2 = -\tfrac{1}{2}(B-B') - \sqrt{\mu^2+\mu(B+B')+\tfrac{1}{4}(B-B')^2} \qquad (24)$$

and Rg is the reflectivity of the ground.

$$I_{+y}(y=y_1) = A_{12}e^{\beta y_1} + A_{22}e^{-\beta y_1} + \frac{\mathrm{ANUM}\ 4}{\mathrm{Den}\ 3} I_{y1} \qquad (25)$$

where

$$A_{12} = \frac{P_2 I_{y0} M_2 e^{-\beta y_1} + (1-N_2) I_{y1}}{L_2 e^{\beta y_1} - M_2 e^{-\beta y_1}} \equiv \frac{\text{ANUM 2}}{\text{DEN 4}} \tag{26}$$

$$A_{22} = \frac{-P_2 I_{y0} L_2 e^{\beta y_1} - (1-N_2) I_{y1}}{\text{DEN 4}} \tag{27}$$

$$P_2 = \frac{B''B_2 + (\mu'+B''+Q_2)F_y}{\text{DEN 3}} \tag{28}$$

$$\text{DEN 3} = \mu'(\mu'+B'') - Q_2^2 \tag{29}$$

$$I_{y0} = I_0 \operatorname{Sin} \theta_s \operatorname{Sin} \psi_s \tag{30}$$

$$I_{y1} = I_{y0} e^{-Q_2 y_1} \tag{31}$$

$$Q_2 = \mu_y + B_y + F_y \tag{32}$$

$$\beta = \sqrt{\mu'(\mu'+2B'')} \tag{33}$$

$$L_2 = \frac{\mu'+B''+\beta}{B''} \tag{34}$$

$$M_2 = \frac{\mu'+B''-\beta}{B''} \tag{35}$$

$$N_2 = \frac{(\mu'+B''-Q_2)B_y + B''F_y}{\text{DEN 3}} \tag{36}$$

$$I_{-y}(y=0) = L_2 A_{12} + M_2 A_{22} + N_2 \tag{37}$$

If the row structure does not exist and plants merge into each other $y_1 \to \infty$, $A_{12}=0$ and $A_{+y}(y=y_1 \equiv \infty)=0$. However, $I_{-y}(y=0)$ is not necessarily equal to zero. There is no transmittance at the end of the row. The reflectance at $x=0$ is given by

$$I_{-x}(x=0) = M_1 A_{21} + N_1 \tag{38}$$

where

$$M_1 = M_2 \text{ from axial symmetry} \tag{39}$$

$$N_1 = N_2 \tag{40}$$

$$A_{21} = -P_2 I_{x0} \tag{41}$$

$$I_{x0} = I_0 \text{ Sin } \theta_s \text{ Cos } \psi_s \tag{42}$$

The input into the sensor depends upon its location relative to the position and orientation of the rows. If θ_D, ψ_D are the angular coordinates of the detector then 'R_D' the input into the detector is given by

$$R_D = I_{-x(x=0)} \text{Sin}\theta_D \text{Cos}\psi_D + I_{+y(y=y_1)} \text{Sin}\theta_D \text{Cos}\psi_D + I_{-z(z=0)} \text{ Cos } \theta_D \tag{43}$$

if $2\pi > \psi_D > \frac{3\pi}{2}$,

$$R_D = A_{-x(x=0)} \text{Sin}\theta_D \text{Cos}\psi_D + I_{-y(y=0)} \text{Sin}\theta_D \text{Sin } \psi_D + I_{-z(z=0)} \text{ Cos } \theta_D \tag{44}$$

if $\frac{\pi}{2} > \psi_D > 0$,

$$R_D = A_{-y(y=0)} \text{Sin}\theta_D \text{Sin}\psi_D + I_{-z(z=0)} \text{Cos}\theta_D \tag{45}$$

if $\pi > \psi_D > \frac{\pi}{2}$,

$$R_D = A_{+y(y=y_1)} \mathrm{Sin}\theta_D \mathrm{Sin}\psi_D + A_{-z(z=0)} \mathrm{Cos}\theta_D \tag{46}$$

if $\frac{3\pi}{2} > \psi > \pi$

5. Shadowing of Soil Between Plants

In the notation of the last section the plant height is z_1 and the solar zenith angle is θ_s. The length of the shadow is $z_1 \tan\theta_s$. If $\psi_s = 0$ this shadow does not cover the open soil between the rows. If $\psi_s \neq 0$ the length of the shadow between the rows is $|z_1 \tan\theta_s \operatorname{Sin}\psi_s|$. If the total area of a sensors view is A and $\frac{t}{100}$ A, i.e. t % of A, is plant cover then the solar azimuthal angle $\psi_s \neq 0$ gives rise to an additional ground cover, reducing the magnitude of the open soil area to:

$$A_{soil} = A\left(1 - \frac{t}{100} - \frac{|z_1 \tan\theta_s \operatorname{Sin}\psi_s| \cdot t}{100 y_1}\right) \tag{47}$$

Therefore, as long as, $A_{soil} > 0$ the soil reflectance has to be taken into account separately provided also

$$|z_1 \tan\theta_D \operatorname{Sin}\psi_D| < \left(\frac{100}{t} - 1\right) y_1$$

This later inequality comes from the condition that the detector's viewing angles are such that some of the open soil is directly 'visible' to the detector. The contribution of open soil reflectance can be taken into account by a simple modification of the expression for $I_{-z(z=0)}$. The new expression for $I_{-z(z=0)}$ is denoted by I_{-z} and is given by:

$$I_{-z} = \left(1 - \frac{A'(soil)}{A}\right) \times (\text{R.H.S. of 13}) + \frac{A'(soil)}{A} \times Rg \tag{48}$$

where

$$A'(soil) = A\left(1 - \frac{t}{100} - \frac{|z_1 \tan\theta_s \operatorname{Sin}\psi_s| + |z_1 \tan\theta_D \operatorname{Sin}\psi_D|}{100 y_{1/t}}\right) \tag{49}$$

if the detector and the sun are on opposite sides of each other i.e. if ψ_D is in the range of $0 < \psi_D < \pi$ then ψ_s is in the range $\pi < \psi_s < 2\pi$. However, if the detector and the sun are such that ψ_D and ψ_s are in the same range of angles $0-\pi$ and $\pi-2\pi$ then I_{-z} is modified as follows:

$$I_{-z} = \left(1 - \frac{A'(\text{soil})}{A}\right) \times (\text{R.H.S. of 13}) + \frac{A'(\text{soil})}{A} \times Rg$$

where

$$A'(\text{soil}) = A\left(1 - \frac{t}{100} - \frac{z_2 t}{100\, y_1}\right) \qquad (50)$$

and z_2 is larger of the quantities $z_1 \tan\theta_s \operatorname{Sin}\psi_s$ and and $z_1 \tan\theta_D \operatorname{Sin}\psi_D$. Implicit in the above expressions is the assumption that the soil is a Lambertian reflector.

For an accurate treatment of this problem one should follow Suits and compute contribution from each element of the canopy and integrate over the height and width of the tree. However, due to lack of time this task will not be performed in this report.

6. Critique

The present model does not contain at least two features that should be included to make better predictions from reflectance. These are:

(1) Surface reflections from leaves which are smooth and may not be Lambertian.

(2) In the area of a field covered by a plant, there are, in general, holes through which sunlight falls unattenuated on the ground. At these places there is increased ground reflectance and the boundary condition (10) needs to be modified.

7. Experiments that Should Be Performed

Some of the assumptions made in this theoretical work are based on everyday common sense and not on experience gained from experiments with plant canopies. In order to evaluate the effect of these assumptions or to make assumptions that are based on true experience we propose that experiments should be performed to test the following properties and effects in plant canopies:

(1) Does a plant canopy or a portion of it act as a Fresnel reflector?

(2) Are the scattering and absorption coefficients of diffuse light different from those of specular light? Do these coefficients depend linearly or non-linearly on the light intensity?

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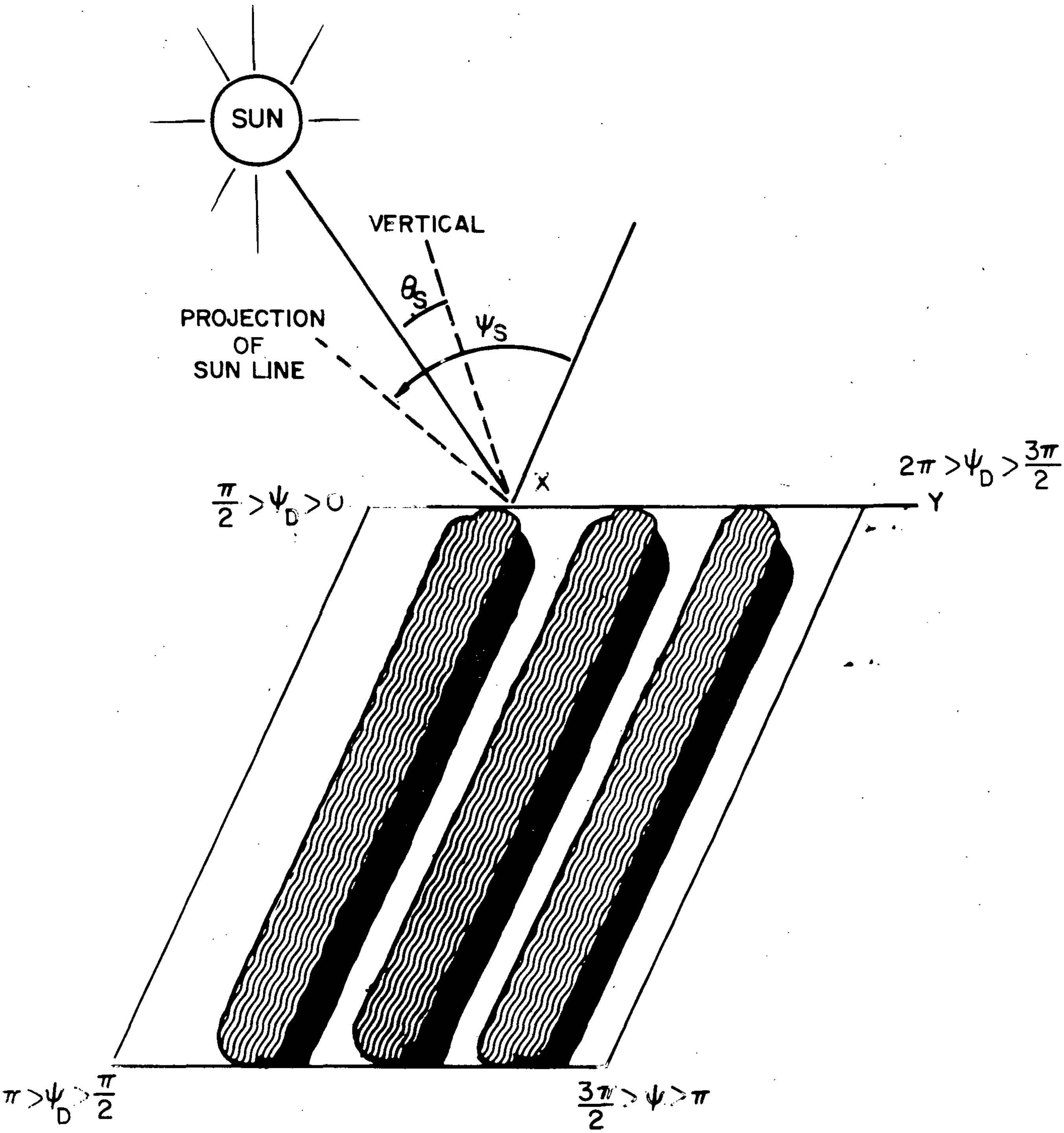

Figure 1. ROW CROP AND ANGULAR COORDINATES

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