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AN IMPROVED METHOD FOR DESIGN OF EXPANSION-CHAMBER MUFFLERS WITH APPLICATION TO AN OPERATIONAL HELICOPTER

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16. Abstract

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A field test of a muffler designed with the aid of this method was conducted on a helicopter with a known exhaust-noise problem. When the exhaust noises of the helicopter with a standard exhaust system and a similar helicopter with a muffler system installed were compared for hover flight conditions, the muffler system was found to reduce the exhaust noise by approximately 11 dB(A). No significant degradation in the engine performance was observed.

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MUFFLERS WITH APPLICATION TO

AN OPERATIONAL HELICOPTER

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SUMMARY

An improved method for the design of expansion-chamber mufflers is described and applied to the task of reducing exhaust noise generated by a helicopter. The method is an improvement of standard transmission-line theory in that it accounts for the effect of the mean exhaust-gas flow on the acoustic-transmission properties of a muffler system, including the termination boundary condition. The method has been computerized, and the computer program includes an optimization procedure that adjusts muffler component lengths to achieve a minimum specified desired transmission loss over a specified frequency range. A printout of the program is included together with a user-oriented description.

A field test of a muffler designed with the aid of this method was conducted on a helicopter with a known exhaust-noise problem. When the exhaust noises of the helicopter with a standard exhaust system and a similar helicopter with a muffler system installed were compared for hover flight conditions, the muffler system was found to reduce the exhaust noise by approximately 11 dB(A). No significant degradation in the engine performance was observed.

INTRODUCTION

There is an increasing awareness of noise pollution on the part of the general public, especially noise caused by aircraft. In particular, users of certain types of general aviation aircraft may be compelled to restrict operations and/or modify their aircraft to comply with existing or forthcoming noise legislation that will specify upper limits on external noise levels (ref. 1). In addition, measurements have indicated that internal noise levels of 11 light, twin-engine, fixed-wing aircraft exceed the currently accepted damage-risk criterion for hearing loss. Analysis of this noise indicated that engine-exhaust noise was the primary cause of both unacceptably high cabin-noise levels and radiated far-field noise (ref. 2).

The commonly accepted solution for excessive reciprocating-engine exhaust noise is the installation of an appropriate reactive-muffler system. Reactive-muffler design efforts in the past have relied upon acoustic-transmission-line theory as a design guide. However, these efforts have been characterized, for the most part, by trial-and-error techniques. Since trial-and-error methods tend to be expensive and inflexible, there is a need to provide a more rational basis for muffler design.

The aircraft-muffler designer must achieve a number of goals. The primary goal is to achieve a specified exhaust-noise reduction over a selected frequency range. Other goals include the minimization of power loss due to back pressure and the muffler volume. Factors of importance are the total weight, weight distribution, geometrical layout of the muffler and exhaust system, and the service life. Thus, to produce a viable muffler system requires not only a knowledge of applicable acoustic theory but also a knowledge of various operational constraints and their interrelationships.

This paper describes an improved analytical design method for expansion-chamber mufflers and indicates how the method is applied. The method, which was developed by Alfredson (ref. 3), includes the effect of mean exhaust-gas flow. In the present paper the theory has been formulated as an extension of the standard transmission-line theory whose primary shortcoming has been thought to be the failure to account for mean-flow effects (refs. 3, 4, 5, and 6).

The second objective of this paper is to describe a computer program that was originally developed by Alfredson. A discussion is also given of the application of the computer program to the problem of designing an expansion-chamber muffler for an aircraft engine. The paper concludes by presenting the results of this application to a helicopter.

SYMBOLS

Values are given both in SI Units and in U.S. Customary Units. Measurements and calculations were made in U.S. Customary Units.

$A_{k,j}, B_{k,j}, C_{k,j}, D_{k,j}$ elements of impedance-parameter matrix

D_1, D_2 denominators of elements of impedance-parameter matrix

c average sound speed in exhaust gas

f_i ith harmonic of engine firing frequency

h	enthalpy
I	acoustic intensity
j	imaginary unit, $\sqrt{-1}$
k	wave number, $2\pi/\lambda$
L	length of expansion-chamber component
M	average Mach number
N	final component of expansion-chamber muffler system
N_1, N_2, N_3, N_4	numerators of elements of impedance-parameter matrix
p	average total or fluctuating pressure over cross section
p_i	fluctuating pressure at station i
R	reflection factor
S	cost function (eq. (29))
S_i	cross-sectional area at station i
s	entropy
T	temperature
t	time
V	average total velocity over duct cross section
V_i	fluctuating velocity at station i
X_i	transformed variable corresponding to component lengths

Z	specific acoustic impedance
γ	ratio of specific heats
δ	irreversible pressure loss
λ	wavelength
Π	series multiplication
ρ	mass density of exhaust medium
Σ	series summation
ϕ	phase angle between incident and reflected wave
ω	circular frequency

Subscripts:

b	branch pipe
f	presence of mean flow
i	station designation, matrix-element index
k,j	identification of impedance-parameter element
o	stagnation value
t	termination value

Superscripts:

(+)	wave propagating in forward direction
(-)	wave propagating in negative direction

4

Notation:

^   amplitude of fluctuating quantity

—   normalization by ρc

Abbreviations:

dB   sound pressure level in decibels (ref. 0.0002 dyne/cm^2)

dB(A)   A-weighted sound pressure level (ref. 0.0002 dyne/cm^2)

MAP   manifold pressure

PWL   acoustic power level

TL   acoustic transmission loss

MUFFLER DESIGN METHOD

Review of Transmission-Line Theory

An improved muffler design method has been developed which is an extension of transmission-line theory to include the effects of flow, termination impedance, extended inlet and outlet pipes, and yielding walls. The details underlying the development of this method are given in references 3, 4, and 5. In this section the method is described through the use of the concepts of transmission-line theory. Then, in the following section a description of the computer program and its use will be given. The reader concerned only with the application of the computer program may proceed to the section "Computer Program" without loss of continuity.

The application of the electrical transmission-line analogy to acoustic filter design was successfully accomplished in this country by Stewart in 1923 (as discussed in ref. 7). The applicability of this design tool to mufflers for reciprocating engines used in aircraft was investigated experimentally in the early fifties by Davis, Stokes, Moore, and Stevens (ref. 6). Original credit for the development of the transfer-matrix approach to four-terminal (or two-port) physical systems is due to Strecker and Feldtkeller (ref. 8). The widespread use of the transfer-matrix approach in mechanical, fluid, and thermal systems is largely due to the contributions of Pipes (ref. 9). In Japan this approach was extended to engine-exhaust muffler systems by Igarashi, Toyama, Miwa, and Arai (refs. 10, 11, and 12). Although transmission-line theory is well developed, it is of

interest to review briefly the fundamental ideas and assumptions of this theory to place in better perspective the improved design method discussed in this paper.

The key assumptions underlying transmission-line theory in its simplest application to muffler design are

(1) Fluctuating pressures are small relative to the average pressure in the system.

(2) The duct and muffler walls are rigid.

(3) Only plane waves are propagated $\left(\text{i.e., } p^{(\pm)} = \pm \rho c V^{(\pm)}\right)$.

(4) The temperature of the medium is constant with time and spatially uniform throughout the muffler.

(5) The average velocity of the medium is zero.

(6) Viscosity effects at the duct and muffler walls are negligible.

(7) Fluctuating quantities are decomposable into sinusoidal components of the form $\hat{p}e^{j\omega t}$.

Operating principle.- Reactive mufflers, of which expansion chambers are a special case, function by causing reflected waves to be propagated back toward the acoustic source. In the case of expansion chambers, these reflected waves are generated by abrupt changes of cross-sectional area. The objective in design of expansion-chamber mufflers is to arrange combinations of duct area changes in the most propitious way for reflecting acoustic energy back into the source over the desired frequency range. The simplest practical expansion-chamber muffler is shown in figure 1. It consists of two cross-sectional-area changes separated by a length L. The amount of acoustic energy associated with the reflected pressure wave $p_1^{(-)}$ is a function of the cross-sectional area ratio S_2/S_1, whereas the length L determines the acoustic frequency at which the maximum reflection occurs.

Muffler performance.- The ratio of fluctuating pressure to fluctuating volume velocity of a forward-going plane wave averaged over a duct cross section through which the wave is propagating is a useful quantity for describing the noise-reduction performance of a reactive-muffler system and is called the analogous characteristic acoustic impedance of the duct, or simply the characteristic impedance when the meaning is clear from the context. At an isolated discontinuity in duct area, the change in characteristic impedance of the duct is related to the amount of acoustic energy reflected at the discontinuity. The total impedance change associated with a series of discontinuities may be calculated from an appropriate set of boundary conditions relating fluctuating quantities across the discontinuities, or it may be determined from laboratory tests. When all the impedance changes in a muffler system are known, the input impedance at the muffler inlet may be calculated by starting at the tailpipe and working systematically backward to the inlet. If

the source impedance is known, the total radiated power of the system may be determined and compared with the radiated power before the muffler system is attached. This comparison is accomplished by taking the ratio of the radiated power before the muffler is attached to that after the muffler is attached. This ratio is called the insertion loss of the muffler system, in analogy with the electrical filter theory. The insertion loss, as a function of frequency, completely characterizes a muffler system insofar as its noise-reducing ability is concerned.

Source-impedance effects.- If the amplitude of the incident wave arriving at the muffler inlet from the source is equal to the amplitude of the incident wave when the muffler system is absent, then the source is said to be nonreflecting and the effective source impedance is equal to the duct impedance. For reciprocating-engine exhaust systems in general, this condition seems unlikely to hold for all frequencies of interest; however, experimental and/or analytical studies to date have not provided reliable information on source impedance. In its absence, the muffler-system designer is forced to rely on transmission loss as a measure of muffler-system performance.

Transmission loss.- Transmission loss is the ratio of the incident acoustic power to the transmitted power through the muffler. It is a single number (for each frequency of interest) and does not permit the prediction of acoustic performance of a complete engine and exhaust muffler system. Furthermore, transmission loss is not directly measurable and is influenced by the termination boundary condition. Its advantages are that it is conceptually simple and does provide a figure of merit when comparing muffler systems with identical terminations, hence the traditional propensity for its use. Also, to the extent that the source can be considered nonreflecting, the transmission loss is an indication of the insertion loss to be expected.

Matrix analysis.- The algebraic complexities involved in the analysis of muffler performance make the use of matrix methods attractive. These methods have been taken over from transmission-line theory and can be easily adapted to high-speed computers. Also, the concept of the "impedance-parameter matrix" provides a better understanding of system component interaction. The impedance-parameter matrix relates the total fluctuating pressure and velocity at one station in a muffler system to the total fluctuating pressure and velocity at a different station in the system. For example, the inlet and outlet total fluctuating pressures and velocities for the expansion chamber of figure 1 (i.e., the sum of the respective forward- and backward-going wave pressures) can be related by a matrix equation of the form

$$
\begin{bmatrix} p_k \\ v_k \end{bmatrix} = \begin{bmatrix} A_{k,j} & B_{k,j} \\ C_{k,j} & D_{k,j} \end{bmatrix} \begin{bmatrix} p_j \\ v_j \end{bmatrix}
\tag{1}
$$

The elements of the square matrix can be related to the impedances of the system at the stations where the fluctuating pressures and velocities are observed. Such a matrix is built up from similar matrices characterizing the behavior of the muffler elements. For instance, continuity of pressure and volume velocity for the simple area discontinuity shown in figure 2(a) gives

$$
\begin{bmatrix} p_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{S_1}{S_2} \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}
\tag{2}
$$

and for a section of constant-diameter pipe of length L, the appropriate relationship is

$$
\begin{bmatrix} p_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos kL & j\rho c \, \sin kL \\ \dfrac{j}{\rho c} \sin kL & \cos kL \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}
\tag{3}
$$

For a branch component (see fig. 2(b)) that can be treated as a lumped branch impedance Z_b, the fluctuating pressures and velocities are related by

$$
\begin{bmatrix} p_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{\rho c}\dfrac{S_2}{S_3} \\ \overline{Z_b} & \dfrac{S_1}{S_3} \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}
\tag{4}
$$

If there are N sections in series, then the fluctuating pressure and velocity at the inlet section N can be related to the pressure and velocity at the outlet section 1 by an impedance-parameter matrix which is equal to the product of all the component matrices as follows:

$$
\begin{bmatrix} A_{N,1} & B_{N,1} \\ C_{N,1} & D_{N,1} \end{bmatrix} = \prod_{i=1}^{N-1} \begin{bmatrix} A_{i+1,i} & B_{i+1,i} \\ C_{i+1,i} & D_{i+1,i} \end{bmatrix}
\tag{5}
$$

Transmission-loss calculation.- For transmission-loss calculations it is convenient to express the fluctuating pressure and velocities throughout the system in terms of the fluctuating pressure amplitude associated with the incident wave at the tailpipe termina-

tion $p_1^{(+)}$. By using the boundary condition for the radiation impedance at the tailpipe exit

$$p_1 = \rho c \overline{Z}_t V_1 \tag{6}$$

the fluctuating pressure and velocity at the exit may be expressed as

$$\begin{bmatrix} p_1 \\ \\ V_1 \end{bmatrix} = \frac{2}{\overline{Z}_t + 1} \begin{bmatrix} \overline{Z}_t \\ \\ 1/\rho c \end{bmatrix} p_1^{(+)} \tag{7}$$

The radiation impedance $\overline{Z}_t$ may be calculated (ref. 13) or measured; however, assuming $\overline{Z}_t$ is known, equation (7), together with the impedance-parameter matrix given by equation (5), implies a knowledge of the fluctuating pressure and velocity at the muffler system inlet from which the pressure associated with the incident wave at the muffler inlet can be shown to be simply

$$p_N^{(+)} = \frac{1}{2}\left(p_N + \rho c V_N\right) \tag{8}$$

The transmission loss is given in terms of the areas and incident-wave pressures at the muffler system inlet and termination as

$$TL = 10 \log_{10} \frac{S_N\left[\hat{p}_N^{(+)}\right]^2}{S_1\left[\hat{p}_1^{(+)}\right]^2} = 20 \log_{10} \frac{\hat{p}_N^{(+)}}{\hat{p}_1^{(+)}} + 10 \log_{10} \frac{S_N}{S_1} \tag{9}$$

Since $p_N^{(+)}$ is proportional to $\hat{p}_1^{(+)}$, the transmission loss is independent of $p_1^{(+)}$ and any convenient numerical value, say unity, may be assumed for $\hat{p}_1^{(+)}$ for computational purposes.

Inadequacies of transmission-line theory.- The laboratory data of reference 6 indicated good agreement between measured and calculated transmission losses for expansion chambers applicable to aircraft engines. These data were taken at room temperature, with no mean flow, and with a loudspeaker capable of producing a pure-tone sound pressure level of approximately 140 dB in the muffler inlets. Also, nonreflecting terminations

were used for the muffler outlets. When similar mufflers were tested on an aircraft engine, the exhaust-noise reduction fell short of that predicted from the transmission-loss calculations (ref. 6). The authors of reference 6 believed that the discrepancies were related to mean-flow effects and the violation of the small-amplitude assumption of linear acoustics. The recent work reported in references 3, 4, 5, and 14 suggested that for expansion-chamber mufflers, the nonlinear effect due to large acoustic pressures was negligible for design purposes. Further investigations reported in references 3, 4, and 5 suggested that the effect of mean gas flow on the termination boundary condition and other impedance discontinuities was the most significant factor neglected in the standard transmission-line theory. Consequently, mean-flow effects were included in the analysis of expansion-chamber performance. The results of this analysis will be summarized in the following section.

Effect of Mean Gas Flow on Muffler Performance

The improved muffler design method described herein involves the effect of the mean flow on the impedance changes at duct-area and branch discontinuities. Also, the effect on the tailpipe termination impedance is included. The effect of flow on the internal impedance changes is a straightforward extension of transmission-line theory using the linearized equations for energy, mass, and in some cases momentum conservation across area discontinuities. The single most significant effect of the flow is associated with the radiation characteristics of the tailpipe.

Tailpipe radiation.- In a duct with a termination characterized by the unflanged-pipe reflection factor R, the relation between the incident-wave pressure amplitude $\hat{p}^{(+)}$ and the reflected-wave pressure amplitude $\hat{p}^{(-)}$ is given by

$$\hat{p}^{(-)} = Re^{j\phi}\hat{p}^{(+)} \tag{10}$$

where ϕ is the phase angle between the incident and reflected wave. Thus the net intensity in the pipe is given by

$$I = \frac{\left[\hat{p}^{(+)}\right]^2}{2\rho c}\left(1 - R^2\right) \tag{11}$$

where $\hat{p}^{(+)}$ is the pressure amplitude of the forward-going wave. For a mean flow with Mach number M, the acoustic intensity becomes

$$I_f = \frac{\left[\hat{p}^{(+)}\right]^2}{2\rho c}\left[(1 + M)^2 - (1 - M)^2 R^2(M)\right] \tag{12}$$

10

In this equation (see ref. 3) the reflection factor becomes a function of Mach number. An increase in acoustic radiation occurs as a result of the mean flow. To appreciate this effect, consider the ratio of I_f to I expressed in decibels:

$$\text{Increase in PWL} = 10 \log \frac{I_f}{I} = 10 \log \frac{(1 + M)^2 - (1 - M)^2 R^2(M)}{1 - R^2(0)} \tag{13}$$

where $R(M)$ is now a reflection factor which depends upon the Mach number of the mean flow. The measurements of reference 12 suggest a 3- to 5-percent increase in the reflection factor and no change in phase angle for Mach number and wave-number ranges typical of exhaust systems. Thus $R(M)$ in the numerator of equation (13) can be replaced by the reflection factor for zero mean flow $R(0)$ with a maximum error of approximately 2 dB. Thus, to a good approximation, the increase in the radiated acoustic power level relative to that for zero mean flow is given by

$$\text{Increase in PWL} \approx 10 \log \frac{(1 + M)^2 - (1 - M)^2 R^2(0)}{1 - R^2(0)} \tag{14}$$

This equation is plotted in figure 3 with the Mach number as a parameter. From this plot it is seen that the radiated power increases significantly relative to that for zero mean flow when the reflection factor approaches 1 and as the mean-flow Mach number increases. Typical exhaust-pipe diameters, exhaust-gas sound speeds, and exhaust-noise frequencies are such that the reflection factor is greater than about 0.975. For a typical exhaust-flow Mach number of 0.1, the plot of figure 3 implies an increase of 8 dB or more in the radiated acoustic power over that for zero mean flow.

Area contraction.- References 3 and 5 indicated that a one-dimensional description of wave propagation on either side of a duct discontinuity would be adequate for muffler design provided that the mean-flow effect was included. For flow area contractions the isentropic energy-balance equation was linearized to obtain one equation relating fluctuating pressure and velocity on either side of the discontinuity. This resulting equation is given as

$$p_2 + \rho c M_2 V_2 = p_1 + \rho c M_1 V_1 \tag{15}$$

where the subscripts 1 and 2 denote the downstream and upstream stations, respectively, as indicated in the schematic of figure 4(a). From mass conservation across the area discontinuity a second linearized equation is found to be

$$S_2 M_2 p_2 + \rho c S_2 V_2 = S_1 M_1 p_1 + \rho c S_1 V_1 \tag{16}$$

If these equations are formulated in a matrix format and solved for the upstream fluctuating pressure and velocity, there results

$$
\begin{bmatrix} p_2 \\ \\ V_2 \end{bmatrix} =
\begin{bmatrix} 1 & \rho c M_1 \dfrac{1 - \left(\dfrac{S_1}{S_2}\right)^2}{1 - M_1^2 \left(\dfrac{S_1}{S_2}\right)^2} \\ \\ 0 & \dfrac{S_1}{S_2} \dfrac{1 - M_1^2}{1 - M_1^2 \left(\dfrac{S_1}{S_2}\right)^2} \end{bmatrix}
\begin{bmatrix} p_1 \\ \\ V_1 \end{bmatrix} \tag{17}
$$

Thus, the effect of the mean flow is to introduce a new element into the impedance-parameter matrix and a correction for the element which relates the velocities. Clearly, when the Mach number becomes zero, the matrix reduces to the form of that in equation (2).

Area expansion.- Experimental evidence discussed in reference 3 suggests that the energy relation connecting fluid state properties across a sudden flow area expansion is nonisentropic. However, an adiabatic condition can be assumed to hold. Hence the stagnation enthalpy would be conserved across such a discontinuity; that is,

$$dh_0 = T \, ds + \frac{dp}{\rho} + V \, dV = 0 \tag{18}$$

The entropy term $T \, ds$ has been shown in reference 15 to be given by

$$T \, ds \approx \frac{d\delta}{\rho(\gamma - 1)} \tag{19}$$

where δ can be regarded as an irreversible fluctuating pressure loss due to the non-isentropic nature of the process. Also, the isentropic relation between the fluctuating density and fluctuating pressure no longer holds, but instead is replaced by

12

$$\rho = \frac{p + \delta}{c^2} \tag{20}$$

By using the relation (19), equation (18) can be integrated across the discontinuity from the downstream to the upstream side, as indicated in the schematic of figure 4(b). There results after linearization

$$p_2 + \rho c M_2 V_2 = p_1 + \rho c M_1 V_1 - \frac{\delta}{\gamma - 1} \tag{21}$$

From mass conservation and the use of equation (20) to eliminate the fluctuating density,

$$S_2 M_2 p_2 + \rho c S_2 V_2 = S_1 M_1 p_1 + \rho c S_1 V_1 + S_1 M_1 \delta \tag{22}$$

Because of the introduction of the unknown quantity δ, a third equation must be derived. This can be obtained from momentum conservation across the discontinuity, which gives after linearizing and the use of equation (20)

$$\left(S_1 + S_2 M_2{}^2\right) p_2 + 2\rho c S_2 M_2 V_2 = \left(S_1 + S_1 M_1{}^2\right) p_1 + 2\rho c S_1 M_1 V_1 + S_1 M_1{}^2 \delta \tag{23}$$

where the momentum source term due to the discontinuity has been taken to be $p_2 \left(S_1 - S_2\right)$, which is based on experimental evidence indicated in reference 3. When this equation is solved for δ and substituted into equations (21) and (22), the upstream fluctuating pressure and velocity can again be related to the corresponding downstream quantities by an impedance-parameter matrix whose elements are given by

$$A_{k,j} = \frac{1 + \left[\left(\frac{S_1}{S_2}\right)^2 (\gamma - 1) - 2\gamma \frac{S_1}{S_2} + \gamma\right] M_1{}^2}{D_1} \tag{24a}$$

$$B_{k,j} = \frac{\rho c \left\{ 2\left(1 - \frac{S_1}{S_2}\right) M_1 + (\gamma - 1)\left[\left(\frac{S_1}{S_2}\right)^2 - 2\frac{S_1}{S_2} + 1\right] M_1{}^3 \right\}}{D_1} \tag{24b}$$

$$C_{k,j} = \frac{\frac{1}{\rho c}\left[\gamma \frac{S_1}{S_2}\left(\frac{S_1}{S_2} - 1\right)\right]M_1^{\ 3}}{D_1}$$ (24c)

$$D_{k,j} = \frac{\frac{S_1}{S_2} + \frac{S_1}{S_2}\left(\frac{S_1}{S_2} - 2\right)M_1^{\ 2} + \left[\frac{S_1}{S_2}\left(\frac{S_1}{S_2} - 1\right)(\gamma - 1)\right]M_1^{\ 4}}{D_1}$$ (24d)

where

$$D_1 = 1 + \left\{(\gamma - 1)\left[1 + \left(\frac{S_1}{S_2}\right)^2\right] + \frac{S_1}{S_2}\left(\frac{S_1}{S_2} - 2\gamma\right)\right\}M_1^{\ 2} + \left[(\gamma - 1)\left(\frac{S_1}{S_2}\right)^2\left(\frac{S_1}{S_2} - 1\right)\right]M_1^{\ 4}$$ (24e)

Again, these equations are seen to reduce to a form equivalent to equation (2) when the Mach number approaches zero.

Area contraction with a branch.- Practical expansion-chamber mufflers usually have extended inlet and outlet pipes, as shown in the schematic of figures 4(c) and (d). Such geometrical arrangements perform acoustically as a side branch with impedance Z_b. The side branch is indicated as region ② in figures 4(c) and (d). The value of Z_b, which is specified at the junction of regions ①, ②, and ③, is controlled by the rigidity of the end wall and by the branch length. The branch impedance $\overline{Z}_b$ is related to the reflection factor R_b and phase angle ϕ_b at the branch entrance by the equation

$$\overline{Z}_b = \frac{1 + R_b e^{j\phi_b}}{1 - R_b e^{j\phi_b}}$$ (25)

If the reflection factor and phase angle at the end cap are denoted by R and ϕ, respectively, then $R_b = R$ for no energy dissipation between the entrance and end cap. Also, $\phi_b = \phi - 2kL$, where L is the branch length and the coordinate direction is that shown in figures 4(c) and (d). By the appropriate selection of the various branch lengths in a muffler system, the transmission-loss characteristic can be tailored to specifications within certain limits. For a branch impedance associated with a flow area contraction shown schematically in figure 4(c), the impedance-parameter matrix for the simple area contraction can be generalized to include the branch impedance. The result is

14

$$\begin{bmatrix} p_3 \\ \\ \\ v_3 \end{bmatrix} = \begin{bmatrix} \dfrac{\left[1 - \dfrac{1}{\overline{Z}_b}\dfrac{S_1}{S_3}\dfrac{S_2}{S_3}M_1 - \left(\dfrac{S_1}{S_3}\right)^2 M_1{}^2\right]}{1 - \left(\dfrac{S_1}{S_3}\right)^2 M_1{}^2} & \dfrac{\rho c\left\{\left[1 - \left(\dfrac{S_1}{S_3}\right)^2\right]M_1 + \dfrac{1}{\overline{Z}_b}\dfrac{S_1}{S_3}\dfrac{S_2}{S_3}M_1{}^2\right\}}{1 - \left(\dfrac{S_1}{S_3}\right)^2 M_1{}^2} \\ \\ \dfrac{\dfrac{1}{\rho c}\dfrac{S_2}{S_3}\dfrac{1}{\overline{Z}_b}}{1 - \left(\dfrac{S_1}{S_3}\right)^2 M_1{}^2} & \dfrac{\dfrac{S_1}{S_3} + \left(\dfrac{1}{\overline{Z}_b}\dfrac{S_2}{S_3}\right)M_1 + \dfrac{S_1}{S_3}M_1{}^2}{1 - \left(\dfrac{S_1}{S_3}\right)^2 M_1{}^2} \end{bmatrix} \begin{bmatrix} p_1 \\ \\ \\ v_1 \end{bmatrix} \tag{26}$$

Note that if Z_b approaches infinity (i.e., the end wall becomes rigid and the branch length becomes zero), then the impedance-parameter matrix reduces to that of equation (17). Furthermore, if the mean-flow Mach number approaches zero, then the result becomes equivalent to that of equation (4).

Area expansion with a branch.- If the muffler inlet pipe is extended, then a branch impedance becomes associated with an area expansion. When this impedance is included in the boundary condition for an area expansion, the elements for the impedance-parameter matrix relating upstream and downstream fluctuating pressures and velocities become

$$A_{k,j} = \frac{N_1 + \dfrac{1}{\overline{Z}_b}\dfrac{S_1}{S_3}\dfrac{S_2}{S_3}\left[M_1{}^2 + (\gamma - 1)M_1{}^3 + \dfrac{S_1}{S_3}M_1{}^4\right]}{D_1 + D_2} \tag{27a}$$

$$B_{k,j} = \frac{N_2 - \dfrac{\rho c}{\overline{Z}_b}\dfrac{S_1}{S_3}\dfrac{S_2}{S_3}\left[2M_1{}^2 - (\gamma - 1)M_1{}^4\right]}{D_1 + D_2} \tag{27b}$$

$$C_{k,j} = \frac{N_3 - \dfrac{1}{\rho c\overline{Z}_b}\dfrac{S_2}{S_3}\left(1 + \gamma M_1{}^2\right)}{D_1 + D_2} \tag{27c}$$

15

$$D_{k,j} = \frac{N_4 - \frac{1}{\overline{Z}_b} \frac{S_2}{S_3} \left[2M_1 + (\gamma - 1)M_1^3 \right]}{D_1 + D_2} \tag{27d}$$

where N_1, N_2, N_3, and N_4 are the respective numerators of equations (24a, b, c, and d), and D_1 is given by equation (24e). The quantity D_2 is associated with the branch impedance $\overline{Z}_b$ and is given by

$$D_2 = \frac{1}{\overline{Z}_b} \frac{S_2}{S_3} \left[\left(\frac{S_1}{S_3} - 2 \right) M_1 + \left(\frac{S_1}{S_3} \right)^2 M_1^3 \right] \tag{27e}$$

Again, when Z_b becomes large, these equations are seen to reduce to equations (24), as expected. It was found by the authors of references 4, 5, and 6 that yielding walls could significantly affect muffler performance; consequently, in references 4 and 5 the impedance of the end caps was altered in a trial-and-error procedure to force the mathematical models for the branched elements to conform with measurements. For a flow area contraction the reflection factor and phase angle of the branch impedance at the end cap were taken as 0.8 and 0.01 radian, respectively, whereas for flow area expansion the reflection factor was taken to be 1.0 and the phase angle as 0.01 radian. The lower reflection factor for area contraction was used because the isentropic flow condition across that type of discontinuity does not allow the occurrence of acoustic energy dissipation, which in fact is known to exist but to a much lesser extent than for an expansion type of discontinuity. This completes the formulations of the impedance-parameter matrix for the various discontinuities encountered in a muffler system consisting of a series of cascaded expansion chambers.

Finite-length section.- To relate the fluctuating pressure and velocity at the upstream side of a discontinuity to the fluctuating pressure and velocity at the downstream side of the next discontinuity joined by a constant-diameter duct (see fig. 5) carrying a mean flow with Mach number M_3, equation (3) is modified as follows:

$$\begin{bmatrix} p_4 \\ \\ v_4 \end{bmatrix} = \begin{bmatrix} \cos\left(\dfrac{kL}{1 - M_3^2}\right) & j\rho c \, \sin\left(\dfrac{kL}{1 - M_3^2}\right) \\ \\ \dfrac{j}{\rho c} \sin\left(\dfrac{kL}{1 - M_3^2}\right) & \cos\left(\dfrac{kL}{1 - M_3^2}\right) \end{bmatrix} \begin{bmatrix} p_3 \\ \\ v_3 \end{bmatrix} \tag{28}$$

16

Clearly, for the typical mean-flow Mach numbers of 0.1 encountered in exhaust systems, this correction is relatively insignificant.

Model comparisons.- Figure 6 shows a comparison of transmission loss calculated by the theoretical model described in this paper with the standard transmission-line theory discussed in reference 6. The muffler configuration chosen for illustrative purposes is shown in the sketch in the figure along with relevant geometrical data. Note that there is a significant reduction of transmission loss in the frequency range 200 to 350 Hz due to both the mean-flow effects and the yielding end cap (reflection factor 0.8 for flow area contraction). On the other hand there is an increase in transmission loss above 350 Hz over that for no mean flow.

COMPUTER PROGRAM

A computer program called EXRSIL has been written that incorporates the effects of the mean flow as well as estimated reflection factors for the muffler end walls associated with flow area contractions. This program will enable a user to design expansion-chamber mufflers of up to four stages. To increase the usefulness of the program as a design tool, an optimization subroutine has been included that will adjust all muffler component lengths to approach a specified minimum transmission-loss characteristic within imposed component-length constraints. The physical meanings of the inputs are described in this section and the detailed formats are described in appendix A. Appendix B contains a listing of the program.

Program Inputs

Input quantities for the computer program consist of the following:

(1) Desired minimum transmission loss at three wavelengths

(2) Acoustic sound speed and wavelength in the exhaust system

(3) Gas-flow Mach number in exhaust tailpipe

(4) Optimization attempts

(5) Number of expansion chambers

(6) Length constraints and initial lengths for all expansion-chamber components

(7) Cross-sectional areas of expansion chamber, annular region, and tailpipe

Desired transmission loss.- The desired minimum transmission-loss characteristic in decibels is deduced with the help of narrow-band analysis of far-field exhaust noise over the relevant operating range of engine load and rotational speed. The engine should preferably be equipped with an exhaust pipe of the same length as the final com-

plete muffler system. A representative narrow-band spectrum of exhaust-noise-dominated helicopter noise is shown in figure 7. Prominent engine-bank firing frequencies are labeled in the figure as f_1, f_2, and so forth. The test conditions for this spectrum will be discussed in the section "Application to Helicopter." The frequency range over which the muffler is to be effective and the amount of transmission loss desired at each frequency are largely a matter of engineering judgment. Since the low-frequency noise components are less objectionable, it is important to transform the narrow-band noise levels to A-weighted levels to reduce the required minimum transmission losses at the lower frequencies. To see how the minimum desired transmission loss is specified, the bar graph of figure 8 should be studied. This bar graph represents the important exhaust noise components of the spectrum shown in figure 7.

In figure 8, the shaded bars represent the abstracted narrow-band levels associated with the engine firing frequencies shown in the measurement represented in figure 7. The overall level associated with the exhaust noise was found to be 96 dB. The solid bars represent the A-weighted levels for which the overall level was found to be 82 dB(A). The arrows represent the desired levels to which the lowest, intermediate, and highest frequency A-weighted components are to be reduced. These desired exhaust noise levels are decided upon from a determination of the levels of other dominating noise sources and, possibly, other considerations such as volume and weight. The computer program constructs by linear interpolation the desired transmission-loss characteristic as a function of $\log_{10} \lambda$, as shown by the solid line in figure 9. This interpolation is based upon three pairs of numbers consisting of transmission losses and wavelengths corresponding to the firing frequencies indicated by the arrows in figure 8. The dashed line in figure 9 is the computed transmission loss for an optimized muffler which will be discussed in the section "Application to Helicopter."

Wavelength and Mach number.- The wavelengths at which the designer specifies a desired transmission loss must be calculated from the speed of sound in the exhaust gas estimated from the mean temperature distribution throughout the proposed muffler system. Gas-flow Mach number must likewise be measured or estimated for the tailpipe of the proposed muffler system.

Optimization attempts.- This input specifies the number of different directions of the gradient vector taken in cost-function space (see section "Optimization Subroutine") to achieve a minimum cost function. If a zero value of the cost function is achieved, then the number of optimization attempts should be increased and/or adjustments to geometric constraints considered.

Geometrical inputs.- The remainder of the inputs concern the geometry of the muffler system. The volume and shape constraints are specified by the number of expansion chambers, the muffler stage component lengths, and the cross-sectional areas of the tail-

pipe, chamber, and annulus. It should be noted that the cross-sectional areas cannot be varied within a stage or between stages. There are four component lengths per muffler stage as indicated in figure 10. Constraints on these lengths are specified independently for each stage together with an initial length for each stage component.

Program Outputs

Program outputs are the following:

(1) Input data listing

(2) Computer-generated desired minimum transmission-loss characteristic

(3) Record of optimization attempts

(4) Optimized component lengths

(5) Computed transmission-loss characteristic

Input data listing.- This section of output simply tabulates the minimum, initial, and maximum values of all component lengths working from the tailpipe to the inlet. Also given are the cross-sectional areas and tailpipe Mach number.

Desired transmission loss.- The minimum desired transmission-loss characteristic is listed along with the frequency and wavelength as determined from the mean temperature of the exhaust gas.

Optimization attempts.- The next listing shows a record of the optimization attempts which consists of the cost functions and the corresponding component lengths at a particular point in the optimization process. The message NEW POINT indicates the change of direction of the gradient vector in the cost-function space.

Optimized component lengths.- This listing shows the component lengths for the final configuration together with the final cost function.

Computed transmission loss.- This section of output lists the computed transmission loss of the final configuration at the same wavelengths and frequencies as was done for the desired transmission loss.

Engineering Judgments

Reduction of low-frequency exhaust noise generally deteriorates with decreasing expansion-chamber length. Thus it is mandatory to establish the maximum muffler length available if low-frequency noise is a problem. Most practical muffler systems will incorporate flow-reversing bends to achieve necessary compactness, as shown in the schematic of figure 10. The transmission loss of such bends is estimated to be less than 2 dB when transmitting into a nonreflecting termination over the frequency range of interest in

reactive-muffler design. Thus, the effect of smooth bends should be relatively unimportant for design purposes.

The muffler geometry also affects the back pressure at the engine exhaust ports. Since an increase in mean back pressure has been found to correlate roughly with degradation of multicylinder-engine performance, mean back pressure is conventionally used as an indicator of possible loss in engine performance. Mean back pressure is minimized in purely expansion-chamber mufflers by making bends as smooth as possible and by alining inlet and outlet pipes, as shown in figure 10. No attempt is made in this program to compute magnitude of mean back pressure.

All component-length constraints should be mutually consistent and compatible with the space requirements. If some component lengths must be held constant, constraints must still be specified to bracket the initial length so that a significant variation cannot occur. Maximum lengths of internal pipes should not be such that their separation is less than 1 pipe diameter and preferably 1 chamber diameter.

Optimization Subroutine

The transmission loss of a muffler system is treated by the optimization subroutine as a function of 4N variables, where N is the number of muffler stages. By systematically changing the various component lengths and comparing the resulting transmission losses with the desired minimum transmission losses at 151 equally spaced values of the logarithm of the wavelength, the subroutine attempts to minimize a cost function. This cost function is defined by

$$S\left(L_1, L_2, \ldots, L_{4N}\right) = \sum_{i=1}^{151} \left[(TL)_d\left(\lambda_i\right) - (TL)_c\left(\lambda_i\right)\right]^2 \tag{29}$$

where $(TL)_d\left(\lambda_i\right)$ is the desired transmission loss at the wavelength λ_i and $(TL)_c\left(\lambda_i\right)$ is the computed transmission loss at the same wavelength for some given configuration. If $(TL)_d\left(\lambda_i\right) \leqq (TL)_c\left(\lambda_i\right)$ for a particular wavelength, then that term is omitted.

The cost-function independent variables L_i must all have boxlike constraints imposed on them, that is,

$$L_{min} < L_i < L_{max} \tag{30}$$

On the other hand, the optimization procedure must operate with independent variables that are unconstrained. This condition is fulfilled by defining a new set of variables X_i by means of the transformation

20

$$\sin^2 X_i = \frac{L_i - L_{min}}{L_{max} - L_{min}} \tag{31}$$

The details of the process by which the procedure systematically alters the component lengths are discussed in reference 3 and will not be given here; however, the process is basically a modification and extension of the steepest-descent method that results in an improved rate of convergence. From the user's standpoint, he can regard the subroutine as a minimization process that operates by following a prescribed direction vector in the cost-function space until a minimum is achieved; then the direction vector is changed or updated whereupon a search for a new minimum will begin. This event is signified in the output listing by the heading NEW POINT. The number of new points (or optimization attempts) to be taken is set by the user by reading in a value of the integer variable NO, which is usually taken to be between 5 and 10. It is the nature of the optimization procedure to find the minimum cost function which is nearest the initial configuration of component lengths. Since other more significant or preferable minimums may exist, several different sets of initial configurations should be tried. At the end of the last minimum search, a normal return of control to the main program will be indicated by the message SEARCH COMPLETED, NORMAL RETURN.

This completes the description of the computer program and its use. In the remainder of this paper an evaluation of the results obtained upon applying the program to the design of a muffler for a helicopter will be given.

APPLICATION TO HELICOPTER

The purpose of this section is to describe the design and to present the measured noise-reduction performance of a muffler for a helicopter engine exhaust. The muffler design was accomplished with the aid of the previously described computer program.

Apparatus and Methods

Test helicopter.- A photograph of the test helicopter for which a muffler system was designed is shown in figure 11. The physical characteristics are given in table I. The aircraft was powered by a six-cylinder, horizontally opposed engine. The exhaust gases from each bank of three cylinders are routed through a manifold and then out a straight stack to the side and rear of the helicopter, as shown in the photograph of figure 12. The engine fundamental firing frequency is defined to be $\frac{N_c \times \text{rot. speed}}{120}$ for a four-stroke operating cycle, where N_c is the number of cylinders. For the manifold

geometry of this particular engine, it was convenient to correlate the exhaust noise components with harmonics of the engine-bank fundamental firing frequency, in which case N_c was taken as 3.

Acoustic measurements.- Initial base-line acoustic measurements were conducted for the helicopter with the engine equipped with the standard exhaust manifold system shown by the photograph of figure 12 and hereinafter called the standard helicopter. The acoustic signals emitted by the helicopter were measured with commercially available, piezoelectric, ceramic-type microphones and were recorded by a multichannel, frequency-modulated tape recorder at a tape speed of 0.76 m/sec (30 in/sec) and a center frequency of 54 kHz. The frequency response of the complete measurement system was within 3 dB over the frequency range 12 Hz to 12 kHz.

The microphones were positioned at a radius of 30.5 m (100 ft) from the aircraft every 18° over one quadrant. A complete 360° noise survey of the helicopter noise was obtained by hovering the helicopter first at 0°, then 90°, 180°, and finally at 270°. The measurements were conducted over flat terrain at the approach end of runway 17 at Langley Air Force Base.

Muffler Design and Installation

Design procedure.- The computerized analytical muffler design method and required engineering judgments have already been described elsewhere in this report. In this section specific design considerations for the test helicopter will be discussed.

Weight and volume limitations indicated that a single muffler that would handle the exhausts from both engine banks would be preferable to a muffler for each bank. Also, it was anticipated that the Y-connector used for combining the two streams of engine exhaust gas would provide some noise reduction of the engine odd-harmonic firing frequencies and some amplification of the engine even-harmonic firing frequencies. This possibility permitted a muffler design with a less demanding transmission-loss performance at the engine-bank fundamental firing frequency. This relaxed performance requirement is evidenced by the 9 dB desired minimum transmission loss at the engine-bank fundamental firing frequency indicated by the arrow in figure 8.

Back-pressure considerations.- Engine back pressure produced by the muffler system was not computed or measured; however, an effort was made to minimize back pressure by making the total exhaust-pipe area as large as was practical. Also, the inlet and outlet pipes in all the chambers were arranged so that the center lines coincided insofar as was practical. Finally, sudden flow-direction changes were avoided as much as possible.

22

Muffler configuration.- The computer-generated component lengths and a sketch of the corresponding geometrical configuration are shown in figure 13. It is important to understand that the general configuration is determined by the designer. The designer must specify constraints on the overall lengths, cross-sectional areas, relative arrangement and number of chambers, and the handling of the internal flow. The computer program adjusts the various component lengths within imposed constraints to achieve as nearly as possible a desired transmission-loss characteristic. The configuration shown in figure 13 produces the transmission-loss characteristic shown by the dashed line in figure 9, which compares favorably with the minimum desired transmission-loss characteristic shown by the solid line in figure 9. The muffler system was designed to produce an exhaust-noise reduction of 15 to 20 dB(A).

Installation.- The installation of the Y-connector and muffler is shown in figures 14 and 15, respectively. The exhaust gases were combined with the Y-connector then led into the muffler inlet, as shown in figure 15. This particular mounting arrangement allowed minimum disturbance of the aircraft weight balance and prevented operating problems relative to the high-temperature exposed surfaces. The muffler volume was 0.057 m^3 (2.0 ft^3) and the weight of the entire system was 21.3 kg (47 lb).

Results and Discussion

Exhaust-noise reduction.- Total radiated noise from the helicopter was measured with the muffler system installed (hereinafter called the modified helicopter) as was done for the standard helicopter configuration. The narrow-band (4-Hz) spectrum of the noise from the modified helicopter is shown in figure 16 by the solid curve, and the narrow-band spectrum for the standard helicopter from figure 7 is shown superimposed in figure 16 by the dashed line to facilitate evaluation of the muffler-system effectiveness. Clearly, the dominant components of the exhaust noise have been eliminated. At the low frequencies, rotor noise now appears to dominate the spectrum. In the higher frequency range, 320 to 520 Hz, there is a 3 to 5 dB increase in the spectrum floor which may be attributable to self-noise (see ref. 16) generated by the muffler system. The overall exhaust-noise reduction obtained was approximately 11 dB(A), whereas the desired reduction was 15 to 20 dB(A).

Estimated muffler insertion losses.- The spectrum information shown in figure 7 and similar data with the Y-connector only installed were used to estimate the insertion losses of the muffler at the engine-bank firing frequencies. These insertion losses are listed in the last column of table II. Also listed are the measured sound pressure levels under comparable test conditions corresponding to the standard exhaust system, Y-connector only, and Y-connector with muffler. From these tabulations the estimated muffler insertion losses were obtained. It is important to note that the listed insertion

losses are lower bounds on the actual muffler insertion losses because of possible contributions from other discrete-frequency noise sources radiating at approximately the same frequencies as the tailpipe exit.

Estimated transmission losses.- If at a particular frequency the amplitude of the incident wave in the exhaust pipe outlet is not changed by the addition of the muffler, then the insertion loss is equal to the transmission loss at that frequency. Assuming this condition to hold, the estimated insertion losses of table II were equated to muffler transmission losses and plotted on the computed transmission-loss curve for the muffler. The results for the first four engine-bank firing frequencies are shown in figure 17. For frequencies 2, 3, and 4 the computed transmission losses are high by 5 to 15 dB. For the higher engine-bank firing frequencies, the estimated insertion losses are negative, indicating that the muffler is amplifying at these frequencies by 9 to 14 dB. Plausible explanations for this behavior include both source-impedance effects and self-generated noise by the muffler at discrete frequencies (ref. 16). The mathematical model described in this paper fails to account for these effects.

Engine performance loss.- Accurate back-pressure measurements for the modified exhaust system were not obtained; however, the aircraft operator did not observe any significant degradation of engine performance for hover flight conditions.

CONCLUDING REMARKS

An improved design method for expansion-chamber mufflers has been described and applied to a helicopter exhaust-noise problem. The resulting three-stage expansion-chamber muffler together with the exhaust-pipe Y-connector for combining the exhaust gases from all cylinders reduced the exhaust noise by 11 dB(A). Experimental comparisons between the noise levels for the standard and modified exhaust systems indicated that the muffler was generating self-noise and/or interacting with the source impedance to result in an amplification of 9 to 14 dB at the fifth and sixth engine-bank firing frequencies.

The muffler volume was 0.057 m^3 (2.0 ft^3) and the weight was 21.3 kg (47 lb), which were within the limits for a flightworthy system. There was no significant loss of engine performance as judged by the aircraft operator.

Langley Research Center,
    National Aeronautics and Space Administration,
        Hampton, Va., June 12, 1973.

APPENDIX A

COMPUTER PROGRAM EXRSIL

Language: FORTRAN

Purpose: To compute transmission loss at 151 values of wavelength for multiple-stage expansion-chamber mufflers. The program makes a specified number of attempts to optimize the transmission loss in accordance with a desired minimum transmission-loss characteristic. The effect of mean flow is included in the program.

Use: Data cards are prepared to be read by statements shown below:

    READ 8, NØ, C
    READ 10, S1, S2, S3
    READ 10, AMACH
    READ 10, XYZ(1,1),XYZ(1,2)
    READ 22, NUMBER
    READ 10, G, (1),X(1),H(1)
    8 FORMAT (I10,F10.3)
    10 FORMAT (3F10.5)
    22 FORMAT (15)

Input variables are

NØ   Number of optimization attempts

C   Sound speed in exhaust gas of proposed muffler, ft/sec

S1   Tailpipe cross-sectional area, ft^2

S2   Annular cross-sectional area, ft^2

S3   Chamber cross-sectional area, ft^2

AMACH   Mach number of mean flow in tailpipe of muffler

$XYZ(I,1)$ = I=1, 2, 3   Wavelengths, ft (minimum, intermediate, maximum)

$XYZ(I,2)$ I=1, 2, 3   Desired minimum transmission losses at above wavelengths, dB

NUMBER   Number of stages in muffler system (should not exceed 4)

$G(I),X(I),H(I)$ I=1,2,3,4   Minimum, initial, and maximum lengths of muffler-stage components starting at tailpipe and working toward inlet, ft (note that there will be four lengths per chamber)

Restrictions: Tailpipe Mach number should be in the approximate range 0.05 to 0.15. The ratio of expansion-chamber cross-sectional area to tailpipe cross-sectional area should not exceed approximately 6. The number of expansion-chamber stages should not exceed 4.

<u>Method</u>: The theoretical basis used in this program is given in references 3, 4, 5 and described briefly in this paper.

<u>Accuracy</u>: Within the limitations of the theoretical model, the accuracy of the computed transmission losses is for practical purposes a function only of the accuracy of the input data.

<u>References</u>: See references 3, 4, and 5 of this report.

<u>Run time</u>: 200 to 600 sec

<u>Storage</u>: 42,000$_8$ (CDC 6000 series)

PROGRAM LISTING

```
      PROGRAM EXRSIL (INPUT,OUTPUT)
C     THIS PROGRAM WAS ORIGINATED BY DR.R.J. ALFREDSON AND IS BASED ON
C     EMPIRICAL DATA FOR A MACH NO RANGE .05 TO .15 AND EXPANSION RATIOS
C     UP TO 6.
      COMMON/ONE/NS,FS,T,JP,TST,STP
      COMMON/TWO/W,W1,W2,W3
      COMMON/THREE/G(20),X(20),H(20),AL(20,20),AU(20,20),DU(20,20),D(20),U(20)
      COMMON/FOUR/NO,IROOT,TOL
      COMMON/FOB/S1,S2,S3,SVEL,AM(20),VAL(200,3),NUMBER
      COMMON/TEN/XYZ(3,2),AIM(151),AAA
C     READ NUMBER OF OPTIMIZATION ATTEMPTS AND EXHAUST GAS SOUND SPEED
      READ 8,NO,C
    8 FORMAT(I10,F10.3)
C     READ AREAS OF TAILPIPE,ANNULUS,AND EXPANSION CHAMBER
      READ 10,S1,S2,S3
C     READ MACH NUMBER OF GAS FLOW IN TAILPIPE
      READ 10,AMACH
C     READ THREE SETS OF WAVELENGTHS AND TRANSMISSION LOSSES
      DO 23 I=1,3
   23 READ 10, XYZ(I,1),XYZ(I,2)
C     FORM IN ARRAY AIM DESIRED MINIMUM TRANSMISSION LOSSES AT 151 VALUES OF
C     WAVELENGTH
      ABC=ALOG10(XYZ(1,1))
      AAB=ALOG10(XYZ(2,1))
      BCA=ALOG10(XYZ(2,1)/XYZ(1,1))
      CAB=ALOG10(XYZ(3,1)/XYZ(2,1))
C     VARIABLE *AAA* IS AN INCREMENTAL LOGARITHMIC WAVELENGTH GOT BY
C     DIVIDING WAVELENGTH RANGE INTO AN EQUAL NUMBER OF PARTS
      AAA=0.0066667*ALOG10(XYZ(3,1)/XYZ(1,1))
      RAB=ABC
      DO 25 I=1,151
      AIM(I)=XYZ(1,2)+(RAB-ABC)/BCA*(XYZ(2,2)-XYZ(1,2))
C     NEXT STATEMENT CAUSES SHIFT FROM POSITIVE SLOPE SIDE OF DESIRED
C     TRANSMISSION LOSS CURVE TO NEGATIVE SLOPE OF CURVE
      IF(RAB.GT.AAB) GO TO 26
   25 RAB=RAB+AAA
```

```
C     THIS DO LOOP STORES VALUES IN AIM FROM NEGATIVE SLOPE SIDE OF
C     TRANSMISSION LOSS CURVE
26    DO 27 N=1,151
      RAB=RAB+AAA
27    AIM(N)=XYZ(2,2)+(XYZ(3,2)-XYZ(2,2))*(RAB-AAB)/CAB
      RAB=ABC
C     READ IN NUMBER OF CHAMBERS NOT TO EXCEED FOUR
      READ 8,NUMBER
      IROOT=-1
C     NN=NUMBER OF AXIAL DIMENSIONS FOR SILENCER(4 PER CHAMBER)
      NN=4*NUMBER
C     NS=NUMBER OF VARIABLES ENTERING THE COST FUNCTION
      NS=NN
C     READ DIMENSION CONSTRAINTS(MINIMUM,INITIAL VALUE,MAXIMUM)
      DO 20 I=1,NN
20    READ 10, G(I),X(I),H(I)
10    FORMAT(3F10.5)
      AM(1)=AMACH
      AM(3)=AMACH*S1/S3
C     PRINT DETAILS OF INITIAL CONFIGURATION
      PRINT 12
12    FORMAT(1H0,25X*RANGES AND INITIAL LENGTHS OF SILENCER COMPONENTS*)
      PRINT 11
11    FORMAT(1H0,12X*TAILPIPE                       EXTENDED OUTLET
     1 EXPANSION CHAMBER                 EXTENDED INLET*/1H0,4X*MIN   INITIAL
     2INITIAL      MAX      MIN      INITIAL     MAX      MIN      INITIAL
     3   MAX      MIN      INITIAL     MAX*)
      PRINT 5,(G(I),X(I),H(I),I=1,NN)
5     FORMAT(12F10.4)
      PRINT 14,S1,S2,S3
14    FORMAT(1H0,14HTAILPIPE AREA=F10.5,18H  ANNULAR AREA=F10.5,28H
     1  EXPANSION CHAMBER AREA=F10.5)
      PRINT 3,AMACH
3     FORMAT(1H0,21HTAILPIPE MACH NUMBER=F10.5)
      PRINT 50
50    FORMAT(/38H    DESIRED MINIMUM TRANSMISSION LOSS/1H0,33HWAVELENGT
     1H  DECIBELS    FREQUENCY)
C     THIS DO LOOP PRINTS THE DESIRED MINIMUM TRANSMISSION LOSS
      DO 28 I=1,151
      FRED=10.0**RAB
C     SPEED OF SOUND ESTIMATED FOR PROPOSED MUFFLER SYSTEM IS USED HERE
      FREQ=C/FRED
      PRINT 17,FRED,AIM(I),FREQ
28    RAB=RAB+AAA
      CALL OPTIM
```

28

```
C
      PRINT DETAILS OF OPTIMUM SILENCER
      PRINT 15
15 FORMAT(30X*OPTIMUM LENGTHS OF SILENCER COMPONENTS IN FEET*)
      PRINT 13, (X(I),I=1,NN)

13 FORMAT(1H0,9HTAILPIPE=F10.4,24H     EXTENDED OUTLET(1)=F10.4,26H
  1 EXPANSION CHAMBER(1)=F10.4,23H     EXTENDED INLET(1)=F10.4/1H0,
  29HCON PIPE=F10.4,24H     EXTENDED OUTLET(2)=F10.4,26H     EXPANSIO
  3N CHAMBER(2)=F10.4,23H     EXTENDED INLET(2)=F10.4/1H0,9HCON PIPE=
  4F10.4,24H     EXTENDED OUTLET(3)=F10.4,26H     EXPANSION CHAMBER(3
  5)=F10.4,23H     EXTENDED INLET(3)=F10.4)
      PRINT 21 ,FS
21 FORMAT(5X20HFINAL COST FUNCTION=F10.3)
      PRINT 52
52 FORMAT(1H0*FREQUENCY IS COMPUTED FOR SONIC VELOCITY ESTIMATED FOR
  1EXHAUST GAS TEMPERATURE*)
      PRINT 51
51 FORMAT(/25H           TRANSMISSION LOSS/1H0,33HWAVELENGTH     DECIBELS
  1 FREQUENCY)
      DO 16 I=1,151
      FRED=10.0**(VAL(I,1))
      FREQ=C/FRED
16 PRINT 17,FRED,VAL(I,2),FREQ
17 FORMAT(3F10.3,2X)
      STOP
      END

C
      SUBROUTINE AUX(V)
C     CALCULATES TL FOR 151 VALUES OF WAVELENGTH AND THE COST FUNCTION
      COMPLEX PI,PR,E
      DIMENSION V(20),W(20)
      COMMON/ONE/NS,FS,T,JP,TS1,STP
      COMMON/THREE/G(20),X(20),H(20),AL(20,20),DG(20,20),D(20),U(20)
      COMMON/FOB/S1,S2,S3,SVEL,AM(20),VAL(200,3),NUMBER
      COMMON/TEN/XYZ(3,2),AIM(151),AAA
C     COORDINATE TRANSFORMATION
      DO 10 I=1,NS
10 W(I)=G(I)+(H(I)-G(I))*SIN(V(I))*SIN(V(I))
      RAD=SQRT(S1/2.1412)
      RAB=ALOG10(XYZ(1,1))
C     CALCULATE TL AT 151 VALUES OF WAVELENGTH
      DO 11 I=1,151
      WLTH=10.0**RAB
      AMACH=AM(1)
```

```
C     RADIATION CONDITION
      CALL  RADFLAN(WLTH,RAD,AMACH,PI,PR,R,TH)
      II=0
C     TAIL PIPE OR PIPE SEPARATING  SILENCERS
      DO 12 IJ=1,NUMBER
      AMACH=AM(1)
      II=II+1
      AL1=W(II)
      CALL PIPE(WLTH,AL1,AMACH,PI,PR)
C     EXTENDED OUTLET
      II=II+1
      AL1=W(II)
      CALL   EXTOLET(WLTH,S1,S2,S3,AMACH,AL1,PI,PR)
C     PIPE BETWEEN INLET AND OUTLET
      AMACH=AM(3)
      II=II+1
      AL1=W(II)-W(II-1)-W(II+1)
      CALL PIPE(WLTH,AL1,AMACH,PI,PR)
C     EXTENDED INLET
      II=II+1
      AL1=W(II)
      CALL  EXTNLET(WLTH,S3,S2,S1,AMACH,AL1,PI,PR)
C     CALC  TL FOR EACH WAVELENGTH
   12 CONTINUE
      CALL FINDCUT(PI,WA,WW,WWW,WWWW)
      VAL(I,1)=RAB
      VAL(I,2)=WWW*WWW
      VAL(I,2)=10.0*ALOG10(VAL(I,2))
C     GAS FLOW NOISE LIMITS MAX  TL TO ABOUT 45 DB
      IF(VAL(I,2).LT.-45.0)GO TO 11
      VAL(I,2)=45.0
   11 RAB=RAB+AAA
C     FORM COST FUNCTION----SUMS OF SQUARES LESS THAN  AIM
      FS=0.0
      DO 15 I=1,151
      IF(VAL(I,2).GT.AIM(I)) GO TO 15
      FS=FS+(AIM(I)-VAL(I,2))*(AIM(I)-VAL(I,2))
   15 CONTINUE
      RETURN
      END
```

```
C
C
      SUBROUTINE RADFLAN(WLTH,RAD,AMACH,PI,PR,R,TH)
      THIS S/R CALCULATES REF.AND PHASE ANGLE,R,TH.
      INCIDENT WAVE,PI=1.0000,0.0000,REFLECTED WAVE= R.PI.EIC
      COMPLEX PI,PR
      PI=(1.0000,0.0000)
      P=6.2832*RAD/WLTH
      P1=P
      P2=P1*P
      P3=P2*P
      P4=P3*P
      P5=P4*P
      P6=P5*P
      TH=3.0885347-0.72673573*P1-0.168190S*P2+0.0631405174566*P3
      PR=CMPLX(COS(TH),SIN(TH))
      RZERO=0.9925+0.05142*P1-0.9360*P2+1.0654*P3-0.6833*P4+0.2289*P5-0
     1.030427*P6
      R=RZERO
      PR=R*PR
      RETURN
      END


      SUBROUTINE PIPE(WLTH,ALENGTH,AMACH,PI,PR)
      COMPLEX E,PI,PR
      THIS CALCULATES PI AND PR AT ONE OF A PIPE GIVEN PI AND PR AT
      THE OTHER END. PI AND PR REPLACE ORIGINAL VALUES
      AK=6.2832/(WLTH*(1.0+AMACH))
      AKA=-6.2832/(WLTH*(1.0-AMACH))
      E=CMPLX(COS(AK*ALENGTH),SIN(AK*ALENGTH))
      PI=PI*E
      E=CMPLX(COS(AKA*ALENGTH),SIN(AKA*ALENGTH))
      PR=PR*E
      RETURN
      END


      SUBROUTINE EXTCLET(WLTH,S1,S2,S3,AMACH,ALENGTH,PI,PR)
      COMPLEX PI,PR,PPI,PPR,TRANS,REFL,E
      THIS S/R CALCULATES INCIDENT AND REFLECTED WAVES IN CHAMBER AT
      PIPE END GIVEN PIAND PR IN PIPE AT CHAMBER END
      AAMACH=AMACH*S1/S3
      CALL PIPE(WLTH,ALENGTH,AMACH,PI,PR)
      AK=6.2832/WLTH
      TH=Q.
```

```
      E=CMPLX(COS(TH-2.0*AK*ALENGTH),SIN(TH-2.0*AK*ALENGTH))
      PPI=(1.0-E)/(1.0+E)
      TRANS=S2*(1.0+AMACH+PR/PI*(1.0-AMACH))*PPI
     1+(S1+S3)*(1.0+AMACH)
     1+(S3-S1)*PR/PI*(1.0-AMACH)
      TRANS=TRANS/(2.0*S3*(1.0+AMACH))
      REFL=(1.0+AMACH+PR/PI*(1.0-AMACH))/TRANS-1.0-AAMACH
      REFL=REFL/(1.0-AAMACH)
      PI=PI*TRANS
      PK=PI*REFL
      RETURN
      END

C
C     SUBROUTINE EXTNLET(WLTH,S1,S2,S3,M1,AL1,PI,PR)
C     THIS SUBROUTINE CALCULATES PI AND PR IN PIPE AT END OF CHAMBER
C     GIVEN PI AND PR IN CHAMBER AT PIPE END. PI AND PR REPLACE
      ORIGINAL VALUES ,    **    ACIABATIC
      COMPLEX A(4,4),B(4,1),DETERM
      COMPLEX PI,PR,TRANS,REFL,E,EE,GG,GH
      DIMENSION IPIVGT(4),INDEX(4,2)
      REAL M1,M3
      M3=M1*S1/S3
      E=PR/PI
      TH=0.
      AK=6.2832/WLTH*2.0*AL1
      EE=CMPLX(COS(TH+AK),SIN(TH+AK))
      N=4
      M=1
      MAX=4
      DO 100 I=1,4
      B(I,1)=(0.0,0.0)
      DO 100 J=1,4
  100 A(I,J)=(0.0,0.0)
      A(1,1)=CMPLX(1.0+M3,0.0)
      A(1,2)=CMPLX(1.0-M3,0.0)
      A(1,4)=(2.5,0.0)
      A(2,1)=CMPLX(S3*(1.0+M3),0.0)
      A(2,2)=CMPLX(-S3*(1.0-M3),0.0)
      A(2,3)=S2*(1.0-EE)
      A(2,4)=CMPLX(-S1*M1,0.0)
      A(3,1)=(1.0,0.0)
      A(3,2)=(1.0,0.0)
      A(3,3)=-(1.0+EE)
```

```
      A(4,1)=CMPLX((S1+S3*(2.0*M3+M3*M3)),0.0)
      A(4,2)=CMPLX((S1+S3*(M3*M3-2.0*M3)),0.0)
      A(4,4)=CMPLX(-M1*M1,0.0)
      B(1,1)=1.0+M1+PR/PI*(1.0-M1)
      B(2,1)=S1*(1.0+M1)-PR/PI*S1*(1.0-M1)
      B(4,1)=S1*(1.0+M1)*(1.0+M1)+PR/PI*(1.0-M1)*(1.0-M1)*S1
      CALL CXINV(A,N,B,M,DETERM,IPIVOT,INDEX,MAX,ISCALE)
      PR=PI*B(2,1)
      PI=PI*B(1,1)
      CALL PIPE(WLTH,ALL,M3,PI,PR)
      RETURN
      END

      SUBROUTINE FINDOUT(E,W,WW,WWW,WWWW)
C     THIS S/R CALCULATES THE MODULUS(WWW) AND PHASE(WWWW) OF A COMPLEX
C     NUMBER, E, WHOSE REAL AND IMAGINARY PARTS ARE W AND WW
C     RESPECTIVELY
      COMPLEX E
      W=REAL(E)
      WW=AIMAG(E)
      WWW=SQRT(W*W+WW*WW)
      WWWW=ATAN2(WW,W)
      RETURN
      END

      SUBROUTINE OPTIM
      DIMENSION D1(20),D2(20),D3(20),X1(20),X2(20),X3(20),C2(20),C(20)
      COMMON/ONE/NS,FS,T,JP,TST,STP
      COMMON/TWO/W,W1,W2,W3
      COMMON/THREE/G(20),X(20),H(20),AL(20,20),DG(20,20),D(20),U(20)
      COMMON/FOUR/NUMBER,IROOT,TOL
      IRA=0
      PK= 0.01
      EPS=0.1E-6
      JP=0
      JK=0
      T=0.0
      JS=-1
      STP=0.01
      W=0.1E-9
      ITER=0
      W1=0.1E-11
```

```
      W2=0.1E-7
      W3=0.1E-8
      DO 5 I=1,NS
    5 X(I)=ASIN(SQRT((X(I)-G(I))/(H(I)-G(I))))
      DO 20 I=1,NS
      DO 20 J=1,NS
      AL(I,J)=0.0
   20 D(I)=0.0
      DO 25 I=1,NS
      AL(I,I)=1.0
   25 DG(I,I)=1.0
      PRINT30
   30 FORMAT(28H1         OPTIMISATION OF SYSTEM)
      PRINT 35
   35 FORMAT(1H0,9X*COST FUNCTION   TP    EO    EC    EI
     1CP   EC    EI                             EC    EC    EI    EI*)
      CALL AUX(X)
      CALL WRITE(X)
      IF(FS .NE. 0.0) GO TO 38
      PRINT 36
      DO 18 I=1,NS
   18 X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
   36 FORMAT(54H    INITIAL MUFFLER MEETS DESIRED TRANSMISSION LOSS)
      RETURN
   38 CONTINUE
      U1=FS
      DO 45 I=1,NS
      STC=X(I)
      X(I)=X(I)+100.0*W
      CALL AUX(X)
      U2=FS
      U(I)=(U2-U1)/(100.0*W)
   45 X(I)=STO
      ST=0.0
      DO 50 I=1,NS
   50 ST=ST+U(I)*U(I)
      ST=SQRT(ST)
      DO 55 I=1,NS
   55 D(I)=-U(I)/ST
      DO 60 I=1,NS
   60 U(I)=-D(I)
      DO 65 I=1,NS
```

```
65   X(I)=X(I)+STP*C(I)
     CALL AUX(X)
     CALL WRITE(X)
     CALL UPDATE
     W=W1
70   DO 75 I=1,NS
     D1(I)=D(I)
75   Q(I)=X(I)
80   DO 85 I=1,NS
85   D(I)=0.0
     DO 90 I=1,NS
     DO 90 J=1,NS
90   D(I)=D(I)+AL(I,J)*U(J)
     DO 95 I=1,NS
95   D(I)=D(I)*JS
     S1=0.0
     DO 100 I=1,NS
100  S1=S1+D(I)*U(I)
     IF(S1.LE.0.1E-7) GO TO 110
     DO 105 I=1,NS
     DO 105 J=1,NS
     AL(I,J)=0.0
     AL(I,1)=1.0
105  DG(I,1)=1.0
     GO TO 80
110  CONTINUE
     S2=0.0
     DO 120 I=1,NS
120  S2=S2+U(I)*D(I)
     S2=SQRT(S2)
     IF(IROOT.GT.3) GO TO 145
     IF(S2.GT.0.01*EPS) GO TO 130
     PRINT 125
125  FORMAT(27H   SMALL DIRECTION VECTOR)
     IRA=IRA+1
     IF(IRA.NE.3) GO TO 130
     DO 17 I=1,NS
17   X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
     RETURN
130  CONTINUE
     I2=0
     DO 135 I=1,NS
135  IF(TST*ABS(U(I)).LT.EPS)I2=I2+1
     IF(I2.NE.NS) GO TO 145
     PRINT 140
```

```
140 FORMAT(19H      SMALL GRADIENT)
    IRA=IRA+1
    IF(IRA.NE.3) GO TO 145
    DO 8 I=1,NS
  8 X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
    RETURN
145 CONTINUE
    STO=STP
    U1=FS
    TS1=0.0
    TS2=0.0
    DO 150 I=1,NS
150 TS1=TS1+D1(I)*C1(I)
    TS1=SQRT(TS1)
    TS2=S2
    YS=0.6667*STP*TS1/TS2
    ICT=0
    ICOUNT=0
155 DO 160 I=1,NS
    X1(I)=X(I)
160 X2(I)=X1(I)+YS*C(I)
    CALL AUX(X2)
    CALL WRITE(X2)
    U2=FS
    IF(U2.LT.U1) GO TO 235
165 DO 170 I=1,NS
    X3(I)=X2(I)
170 X2(I)=0.6667*X1(I)+0.3333*X3(I)
    U3=U2
    ICOUNT=ICOUNT+1
    IF(ICOUNT-8) 230,175,175
175 ICT=ICT+1
    ICOUNT=0
    IF((ICT.EQ.2).AND.(NS.EQ.1)) GO TO 210
    GO TO (220,176,220,210),ICT
176 DO 180 I=1,NS
    D2(I)=-D(I)
180 C2(I)=0.0
    IF(ABS(X1(I)).GT.0.1E-8) GO TO 185
    C2(I)=X1(I)+0.100
    GO TO 190
185 C2(I)=X1(I)
190 DO 195 I=1,NS
```

```
195 D3(I)=C2(I)-D2(I)*C2(I)*C2(I)
    TP=0.0
    DO 200 I=1,NS
200 TP=TP+D3(I)*C3(I)
    TP=SQRT(TP)
    DO 205 I=1,NS
205 D(I)=D3(I)*S2/TP
    GO TO 155
210 PRINT 215
215 FORMAT(43H    EXCESSIVE NUMERICAL ERRORS IN GRADIENT)
    FS=U1
    DO 16 I=1,NS
16  X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
    RETURN
220 DO 225 I=1,NS
225 D(I)=-D(I)
    GO TO 155
230 CALL AUX(X2)
    CALL WRITE(X2)
    U2=FS
    IF(U2-U1)260,260,165
235 DO 240 I=1,NS
240 X3(I)=3.0*X2(I)-2.0*X1(I)
    CALL CHECK(X3,C,JUMP)
    IF(JUMP.EQ.0) GO TO 250
    DO 245 I=1,NS
245 X(I)=X2(I)
    FS=U2
    GO TO 280
250 CALL ALX(X3)
    U3=FS
    CALL WRITE(X3)
    IF(U3.GE.U2) GO TO 260
    DO 255 I=1,NS
    X1(I)=X2(I)
255 X2(I)=X3(I)
    U1=U2
    U2=U3
    GO TO 235
```

```
260   CONTINUE
      S1=0.0
      S2=0.0
      DO 265 I=1,NS
      FRED=X2(I)-X1(I)
      SAM=X3(I)-X1(I)
      S1=S1+FRED*FRED
      S2=S2+SAM*SAM
265   A=SQRT(S1)
      B=SQRT(S2)
      ZNUM=B*B*U2-A*A*U3-(B*B-A*A)*U1
      DENOM=B*U2-A*U3-(B-A)*U1
      STP=0.5*ZNUM/DENOM
      DO 270 I=1,NS
270   X(I)=X1(I)+(STP/TS2)*D(I)
      CALL AUX(X)
      IF(FS.LT.U2) GO TO 280
      DO 275 I=1,NS
275   X(I)=X2(I)
      FS=U2
280   CONTINUE
      PRINT 285
285   FORMAT(13H        NEW POINT/)
      IF(FS.NE.0.0) GO TO 286
      DO 287 I=1,NS
287   X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
      RETURN
286   CALL WRITE(X)
      ITER=ITER+1
      IF(ITER.NE.NUMBER) GO TO 295
      PRINT 290
290   FORMAT(36H        SEARCH COMPLETED,NORMAL RETURN)
      DO 14 I=1,NS
14    X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
      RETURN
295   IF(IROOT.LT.0) GO TO 305
      IF(FS.GT.TOL) GO TO 305
      PRINT 300
300   FORMAT(35H        FUNCTION SMALLER THAN TOLERANCE)
      DO 12 I=1,NS
12    X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
      RETURN
```

```
305 CONTINUE
    I1=0
    DO 310 I=1,NS
    PT=ABS(SIN(Q(I))*SIN(Q(I))-SIN(X(I))*SIN(X(I)))
    IF(PT.LT.0.000025) I1=I1+1
310 CONTINUE
    IF(I1.NE.NS) GO TO 320
    PRINT 315
315 FORMAT(54H    STEP SIZE LESS THAN 0.01PERCENT OF VARIABLE RANGE)
    IRA=IRA+1
    IF(IRA.LT.3) GO TO 320
    DO 10 I=1,NS
10  X(I)=G(I)+(H(I)-G(I))*SIN(X(I))*SIN(X(I))
    RETURN
320 CONTINUE
    STP=0.0
    DO 325 I=1,NS
    FRED=X(I)-G(I)
    STP=STP+FRED*FRED
325 STP=SQRT(STP)/TS2
    JK=JK+1
    IF(JK.LT.3) GO TO 330
    JP=JP+1
    JK=0
    CALL ALTERW
330 T=T+STP*TS2
    CALL UPDATE
    GO TO 70
    END

    SUBROUTINE GRAD
    DIMENSION DEL(20)
    COMMON/ONE/NS,FS,T,JP,TST,STP
    COMMON/TWO/W,W1,W2,W3
    COMMON/THREE/G(20),X(20),H(20),AL(20,20),DG(20,20),D(20),U(20)
    W=ABS(W)
    TK=0.01
    IF(ABS(FS).GT.0.1E-7) GO TO 2
    DO1 I=1,NS
1   DEL(I)=0.000001
    GO TO 4
2   DO 3 I=1,NS
    DEL(I)=2.0*SQRT(W*ABS(FS)/DG(I,I))
3   IF(DEL(I).GT.0.00001) DEL(I)=0.000001
```

39

```
3 CONTINUE
  U4=FS
4 U1=FS
  DO 7 I=1,NS
  STO=X(I)
  X(I)=X(I)+DEL(I)
  CALL AUX(X)
  U2=FS
  U(I)=(U2-U1)/DEL(I)
  IF(ABS(U(I)).LT.0.1E-10) GO TO 5
  FRED=2.0*DEL(I)*DG(I,I)/ABS(U(I))
  IF(FRED.LT.TK) GO TO 6
5 X(I)=STO-DEL(I)
  CALL AUX(X)
  U3=FS
  U(I)=(U2-U3)/(2.0*DEL(I))
6 X(I)=STO
7 CONTINUE
  FS=U1
  TST=0.0
  DO 8 I=1,NS
8 TST=TST+U(I)*U(I)
  TST=SQRT(TST)
  DO 9 I=1,NS
9 U(I)=U(I)/TST
  RETURN
  END

  SUBROUTINE UPDATE
  DIMENSION P(20),Y(20),T(20,20),C(20,20),S(20,20),F(20)
  COMMON/ONE/NS,FS,U,JP,TST,STP
  COMMON/THREE/G(20),X(20),H(20),AL(20,20),DG(20,20),D(20),U(20)
  DO 1 I=1,NS
1 P(I)=U(I)
  CALL GRAD
  IF(NS.EQ.1) GO TO 14
  DO 2 I=1,NS
2 Y(I)=U(I)-P(I)
  R=0.0
  B=0.0
  DO 3 I=1,NS
  B=B+Y(I)*U(I)
```

```
  3 R=R+P(I)*D(I)
    S1=1.0/(B*STP)
    S2=R/(B*B)
    S3=STP/B
    DO 5 I=1,NS
    DO 5 J=1,NS
  5 T(I,J)=S3*D(I)*D(J)
    DO 6 I=1,NS
    DO 6 J=1,NS
    C(I,J)=0.0
    S(I,J)=0.0
  6 F(I)=0.0
    DO 7 I=1,NS
    DO 7 J=1,NS
  7 F(I)=F(I)+AL(I,J)*Y(J)
    S4=0.0
    DO 8 I=1,NS
  8 S4=S4+Y(I)*F(I)
    DO 9 I=1,NS
    DO 9 J=1,NS
  9 S(I,J)=F(I)*Y(J)
    DO 10 I=1,NS
    DO 10 K=1,NS
    DO 10 J=1,NS
 10 C(I,K)=C(I,K)+S(I,J)*AL(K,J)
    DO 11 I=1,NS
    DO 11 J=1,NS
 11 AL(I,J)=AL(I,J)+T(I,J)-C(I,J)/S4
    DO 12 I=1,NS
 12 DG(I,I)=DG(I,I)+Y(I)*Y(I)*(S1-S2)+2.0*P(I)*Y(I)/B
    DO 13 I=1,NS
 13 DG(I,I)=ABS(DG(I,I))
    RETURN
 14 AL(1,1)=1.0
    DG(1,1)=1.0
    RETURN
    END

    SUBROUTINE ALTERm
    COMMON/UNE/NS,FS,T,JP,TST,STP
    COMMCN/TWO/W,W1,W2,W3
    GO TO (1,2,3),JP
  1 T1=T
    W=W2
```

```
      RETURN
 2    T2=T
      W=W3
      RETURN
 3    T3=T
      IF(T1.LT.T2) GO TO 4
      T2=T1
      W=W1
 4    W=W2
 5    IF(T2.LT.T3) GC TO 6
      W2=W
      W1=0.01*W2
      W3=100.0*W2
      W=W1
      JP=0
      T=0.0
      RETURN
 6    W2=W3
      W1=0.01*W2
      W3=100.0*W2
      W=W1
      JP=0
      T=0.0
      RETURN
      END

      SUBROUTINE CHECK(X3,Q,JUMP)
      DIMENSION X3(20),Q(20),V(20),W(20),Z(20)
      COMMON/ONE/NS,FS,T,JP,TST,STP
      COMMON/THREE/G(20),X(20),H(20),AL(20,20),DG(20,20),D(20),U(20)
      DO 1 I=1,NS
      V(I)=G(I)+(H(I)-G(I))*SIN(Q(I))*SIN(G(I))
 1    W(I)=G(I)+(H(I)-G(I))*SIN(X3(I))*SIN(X3(I))
      JUMP=0
      DC 2 I=1,NS
      Z(I)=0.20*(H(I)-G(I))
 2    IF(ABS(W(I)-V(I)).GT.Z(I)) JUMP=JUMP+1
      RETURN
      END
```

```
SUBROUTINE WRITE(V)
DIMENSION V(20),W(20)
COMMON/ONE/NS,FS,T,JP,TST,STP
COMMON/THREE/G(20),X(20),H(20),AL(20,20),DG(20,20),D(20),U(20)
DO 1 I=1,NS
1 W(I)=G(I)+(H(I)-G(I))*SIN(V(I))*SIN(V(I))
PRINT 2,FS,(W(I),I=1,NS)
2 FORMAT(E20.4,20F9.4)
RETURN
END
```

RANGES AND INITIAL LENGTHS OF SILENCER COMPONENTS

TAILPIPE			EXTENDED OUTLET			EXPANSION CHAMBER			EXTENDED INLET		
MIN	INITIAL	MAX	MIN	INITIAL	MAX	MIN	INITIAL	MAX	MIN	INITIAL	MAX
1.0000	2.0000	2.5000	.1500	.2000	.3500	.7500	.8000	.8500	.0500	.1000	.2500
.0630	.0800	.1000	.1000	.2500	.3000	1.6000	1.6500	1.7000	.8000	1.0000	1.1000
1.9500	2.0000	2.0500	.3000	.4000	.5000	2.4500	2.5000	2.5500	1.3000	1.4500	1.6500

TAILPIPE AREA= .02190   ANNULAR AREA= .10810   EXPANSION CHAMBER AREA= .13140

TAILPIPE MACH NUMBER= .1000

DESIRED MINIMUM TRANSMISSION LOSS

WAVELENGTH	DECIBELS	FREQUENCY
1.500	15.000	1333.333
1.528	15.224	1308.558
1.557	15.448	1284.243
1.587	15.672	1260.380
1.617	15.896	1236.961
1.647	16.120	1213.976
1.679	16.345	1191.419
1.710	16.569	1169.281
1.743	16.793	1147.554
1.776	17.017	1126.231
1.809	17.241	1105.304
1.844	17.465	1084.756
1.879	17.689	1064.610
1.914	17.913	1044.628

1.950	18.137	1025.413
1.987	18.361	1006.360
2.025	18.585	987.660
2.063	18.810	969.308
2.102	19.034	951.297
2.142	19.258	933.621
2.183	19.482	916.273
2.224	19.706	899.247
2.266	19.930	882.538
2.309	20.154	866.139
2.353	20.378	850.045
2.397	20.602	834.250
2.443	20.826	818.749
2.489	21.050	803.535
2.536	21.275	788.604
2.584	21.499	773.951
2.633	21.723	755.570
2.683	21.947	745.456
2.734	22.171	731.604
2.785	22.395	718.010
2.838	22.619	704.669
2.892	22.843	691.575
2.947	23.067	678.725
3.002	23.291	666.113
3.059	23.515	653.736
3.117	23.740	641.588
3.176	23.964	629.667
3.236	24.188	617.967
3.298	24.412	606.484
3.360	24.636	595.215
3.424	24.860	584.155
3.489	25.084	573.300
3.555	25.308	562.648
3.622	25.532	552.193
3.690	25.756	541.932
3.760	25.980	531.863
3.832	26.205	521.980
3.904	26.429	512.281
3.978	26.653	502.762
4.053	26.877	493.420
4.130	27.101	484.252
4.208	27.325	475.253
4.288	27.549	466.423
4.369	27.773	457.756

449.250	27.997	4.452
440.902	28.221	4.536
432.710	28.445	4.622
424.670	28.670	4.710
416.779	28.894	4.799
409.034	29.118	4.890
401.434	29.342	4.982
393.975	29.566	5.076
386.654	29.790	5.173
379.470	30.014	5.271
372.418	30.238	5.370
365.498	30.462	5.472
358.707	30.686	5.576
352.042	30.910	5.681
345.500	31.135	5.789
339.080	31.359	5.898
332.780	31.583	6.010
326.596	31.807	6.124
320.528	32.031	6.240
314.572	32.255	6.358
308.727	32.479	6.478
302.990	32.703	6.601
297.360	32.927	6.726
291.835	33.151	6.853
286.412	33.375	6.983
281.090	33.600	7.115
275.867	33.824	7.250
270.741	34.048	7.387
265.710	34.272	7.527
260.773	34.496	7.670
255.928	34.720	7.815
251.172	34.944	7.963
246.505	34.366	8.113
241.925	34.004	8.267
237.429	33.642	8.424
233.018	33.280	8.583
228.688	32.917	8.746
224.438	32.555	8.911
220.268	32.193	9.080
216.175	31.831	9.252
214.158	31.469	9.427
208.216	31.107	9.605
204.347	30.745	9.787
200.550	30.382	9.973

196.824	30.020	10.161
193.166	29.658	10.354
189.577	29.296	10.550
186.055	28.934	10.750
182.557	28.572	10.953
179.205	28.210	11.160
175.875	27.848	11.372
172.607	27.485	11.587
169.399	27.123	11.806
166.252	26.761	12.030
163.163	26.399	12.258
160.131	26.037	12.490
157.155	25.675	12.726
154.235	25.313	12.967
151.369	24.950	13.213
148.557	24.588	13.463
145.796	24.226	13.718
143.087	23.864	13.977
140.428	23.502	14.242
137.819	23.140	14.512
135.258	22.778	14.787
132.745	22.415	15.066
130.278	22.053	15.352
127.858	21.691	15.642
125.482	21.329	15.939
123.150	20.967	16.240
120.862	20.605	16.548
118.616	20.243	16.861
116.412	19.880	17.180
114.249	19.518	17.506
112.125	19.156	17.837
110.043	18.794	18.175
107.998	18.432	18.519
105.991	18.070	18.870
104.022	17.708	19.227
102.089	17.345	19.591
100.192	16.983	19.962
98.330	16.621	20.340
96.503	16.259	20.725
94.710	15.897	21.117
92.950	15.535	21.517
91.223	15.173	21.924
89.528	14.810	22.339
87.864	14.448	22.762

23.193	14.086	86.232
23.632	13.724	64.629
24.080	13.362	63.057
24.536	13.000	81.514
25.000	12.638	79.999

OPTIMISATION OF SYSTEM

COST FUNCTION	TP	CP	EC	EO	EI	CP	EC	EO	EI	CP	EC	EO	EI
2.7831E+03	2.0000	.0800	.8000	.2000	.1000	2.0000	1.6500	.2500	1.0000	2.0000	2.5000	.4000	1.4500
2.7627E+03	2.0015	.0800	.8002	.2000	.1006	1.9998	1.6500	.2506	.9994	1.9998	2.5000	.3996	1.4523
2.7489E+03	2.0026	.0801	.8004	.2003	.1009	1.9997	1.6501	.2509	.9990	1.9997	2.4999	.3993	1.4538
2.7209E+03	2.0043	.0801	.8007	.1999	.1017	1.9995	1.6501	.2517	.9982	1.9995	2.4999	.3987	1.4568
2.6640E+03	2.0092	.0802	.8014	.1999	.1032	1.9990	1.6502	.2532	.9965	1.9990	2.4998	.3975	1.4629
2.5498E+03	2.0180	.0804	.8027	.1998	.1062	1.9981	1.6504	.2561	.9932	1.9981	2.4996	.3951	1.4750
2.3410E+03	2.0353	.0807	.8053	.1996	.1125	1.9962	1.6508	.2617	.9865	1.9962	2.4991	.3904	1.4985
NEW POINT													
2.3410E+03	2.0353	.0807	.8053	.1996	.1125	1.9962	1.6508	.2617	.9865	1.9962	2.4991	.3904	1.4985
2.1082E+03	2.0556	.0813	.8089	.1993	.1223	1.9942	1.6528	.2692	.9812	1.9942	2.4986	.3863	1.5266
NEW POINT													
2.1082E+03	2.0556	.0813	.8089	.1993	.1223	1.9942	1.6528	.2692	.9812	1.9942	2.4986	.3863	1.5266
1.9933E+03	2.0567	.0817	.8109	.1986	.1319	1.9924	1.6560	.2735	.9782	1.9924	2.4991	.3788	1.5364
1.8966E+03	2.0589	.0825	.8150	.1974	.1517	1.9888	1.6622	.2814	.9721	1.9888	2.4999	.3642	1.5544
NEW POINT													
1.8966E+03	2.0589	.0825	.8150	.1974	.1517	1.9888	1.6622	.2814	.9721	1.9888	2.4999	.3642	1.5544
1.7858E+03	2.0519	.0827	.8033	.2062	.1693	1.9845	1.6637	.2845	.9615	1.9845	2.5028	.3724	1.5634
NEW POINT													
1.7858E+03	2.0519	.0827	.8033	.2062	.1693	1.9845	1.6637	.2845	.9615	1.9845	2.5028	.3724	1.5634
1.7618E+03	2.0533	.0829	.7963	.2120	.1827	1.9811	1.6630	.2865	.9602	1.9811	2.5054	.3779	1.5696
NEW POINT													
1.7618E+03	2.0533	.0829	.7963	.2120	.1827	1.9811	1.6630	.2865	.9602	1.9811	2.5054	.3779	1.5696

SEARCH COMPLETED, NORMAL RETURN

OPTIMUM LENGTHS OF SILENCER COMPONENTS IN FEET

TAILPIPE= 2.0533	EXTENDED OUTLET(1)= .2120	EXPANSION CHAMBER(1)= .2120	EXTENDED INLET(1)= .1827
CON PIPE= .0829	EXTENDED OUTLET(2)= .2865	EXPANSION CHAMBER(2)= 1.6630	EXTENDED INLET(2)= .9602
CON PIPE= 1.9811	EXTENDED OUTLET(3)= .3779	EXPANSION CHAMBER(3)= 2.5054	EXTENDED INLET(3)= 1.5696

FINAL COST FUNCTION= 1761.771

FREQUENCY IS COMPUTED FOR SONIC VELOCITY ESTIMATED FOR EXHAUST GAS TEMPERATURE

TRANSMISSION LOSS

WAVELENGTH	DECIBELS	FREQUENCY
1.500	45.000	1333.333
1.528	45.000	1308.558
1.557	40.716	1284.243
1.587	18.791	1260.380
1.617	15.452	1236.961
1.647	17.821	1213.976

1.679	18.530	1191.419
1.710	18.794	1169.231
1.743	26.465	1147.554
1.776	40.340	1126.231
1.809	45.000	1105.304
1.844	45.000	1084.766
1.879	45.000	1064.610
1.914	45.000	1044.828
1.950	45.000	1025.413
1.987	45.000	1006.360
2.025	45.000	987.660
2.063	45.000	969.308
2.102	45.000	951.297
2.142	45.000	933.621
2.183	40.749	916.273
2.224	28.786	899.247
2.266	30.861	882.538
2.309	27.575	866.139
2.353	27.131	850.045
2.397	27.780	834.250
2.443	25.219	818.749
2.489	29.219	803.535
2.536	35.953	788.604
2.584	40.150	773.951
2.633	42.841	759.570
2.683	44.423	745.456
2.734	45.000	731.604
2.785	45.000	718.010
2.838	44.652	704.669
2.892	43.377	691.575
2.947	41.318	678.725
3.002	38.226	666.113
3.059	33.479	653.736
3.117	24.685	641.588
3.176	16.788	629.667
3.236	25.289	617.567
3.298	26.356	606.434
3.360	25.924	595.215
3.424	30.745	584.155
3.489	36.424	573.300
3.555	40.578	562.648
3.622	43.519	552.193
3.690	45.000	541.932
3.760	45.000	531.863

521.980	45.000	3.832
512.281	45.000	3.904
502.762	45.000	3.978
493.420	43.684	4.053
484.252	41.254	4.130
475.253	38.161	4.208
466.423	34.393	4.288
457.756	30.260	4.369
449.250	27.305	4.452
440.902	26.690	4.536
432.710	26.475	4.622
424.670	25.187	4.710
416.779	22.219	4.799
409.034	19.518	4.890
401.434	24.054	4.982
393.975	28.947	5.076
386.654	32.405	5.173
379.470	34.916	5.271
372.418	36.789	5.370
365.498	38.188	5.472
358.707	39.199	5.576
352.042	39.859	5.681
345.500	40.164	5.789
339.080	40.075	5.898
332.780	39.510	6.010
326.556	38.350	6.124
320.528	36.449	6.240
314.572	35.668	6.358
308.727	30.074	6.478
302.990	27.113	6.601
297.360	27.533	6.726
291.835	29.399	6.853
286.412	30.809	6.983
281.090	31.618	7.115
275.867	31.960	7.250
270.741	31.950	7.387
265.710	31.662	7.527
260.773	31.146	7.670
255.928	30.430	7.815
251.172	29.537	7.963
246.505	28.486	8.113
241.925	27.304	8.267
237.429	26.045	8.424
233.018	24.813	8.583
228.688	23.784	8.746

224.438	23.175	8.911
220.268	23.111	9.080
216.175	23.511	9.252
212.158	24.172	9.427
208.216	24.913	9.605
204.347	25.626	9.787
200.550	26.260	9.973
196.824	26.800	10.161
193.166	27.245	10.354
189.577	27.599	10.550
186.055	27.870	10.750
182.597	28.065	10.953
179.205	28.193	11.160
175.875	28.258	11.372
172.607	28.267	11.587
169.399	28.224	11.806
166.252	28.135	12.030
163.163	28.002	12.258
160.131	27.828	12.490
157.155	27.618	12.726
154.235	27.372	12.967
151.369	27.093	13.213
148.557	26.784	13.463
145.796	26.444	13.718
143.087	26.076	13.977
140.428	25.682	14.242
137.819	25.261	14.512
135.258	24.814	14.787
132.745	24.343	15.066
130.278	23.848	15.352
127.858	23.329	15.642
125.482	22.787	15.939
123.150	22.221	16.240
120.862	21.632	16.548
118.616	21.019	16.861
116.412	20.383	17.180
114.249	19.723	17.506
112.126	19.057	17.837
110.043	18.327	18.175
107.998	17.590	18.519
105.991	16.825	18.870
104.022	16.033	19.227
102.089	15.210	19.591

19.962	14.355	100.192
20.340	13.467	98.330
20.725	12.544	96.503
21.117	11.583	94.710
21.517	10.582	92.950
21.924	9.540	91.223
22.339	8.454	89.528
22.762	7.326	87.864
23.193	6.158	86.232
23.632	4.958	84.629
24.080	3.741	83.057
24.536	2.537	81.514
25.000	1.393	79.999

REFERENCES

1. Anon.: Noise Control Act of 1972. Public Law 92-574, Oct. 1972.

2. Tobias, Jery V.: Noise in Light Twin-Engine Aircraft. Sound & Vib., vol. 3, Sept. 1969, pp. 16-19.

3. Alfredson, R. J.: The Design and Optimization of Exhaust Silencers. Ph. D. Thesis, Inst. Sound & Vib. Res., Univ. of Southampton, July 1970.

4. Alfredson, R. J.; and Davies, P. O. A. L.: The Radiation of Sound From an Engine Exhaust. J. Sound & Vib., vol. 13, no. 4, Dec. 1970, pp. 389-408.

5. Alfredson, R. J.; and Davies, P. O. A. L.: Performance of Exhaust Silencer Components. J. Sound & Vib., vol. 15, no. 2, Mar. 22, 1971, pp. 175-196.

6. Davis, Don D.; Stokes, George M.; Moore, Dewey; and Stevens, George L., Jr.: Theoretical and Experimental Investigation of Mufflers With Comments on Engine-Exhaust Muffler Design. NACA Rep. 1192, 1954. (Supersedes NACA TN's 2893 and 2943.)

7. Stewart, George W.; and Lindsay, Robert Bruce: Acoustics. D. Van Nostrand Co., Inc., 1930.

8. Strecker, F.; and Feldtkeller, R.: Grundlagen der Theorie des allgemeinen Vierpols. Elek. Nach. Tech., vol. 6, no. 3, Mar. 1929, pp. 93-112.

9. Pipes, Louis A.; and Harvill, Laurence R.: Applied Mathematics for Engineers and Physicists. Third ed., McGraw-Hill Book Co., Inc., c.1970.

10. Igarashi, Juichi; and Toyama, Masasuke: Fundamentals of Acoustical Silencers. I — Theory and Experiment of Acoustic Low-Pass Filters. Rep. No. 339, Aeronaut. Res. Inst., Univ. of Tokyo, Dec. 1958.

11. Miwa, Toshisuke; and Igarashi, Juichi: Fundamentals of Acoustical Silencers. II — Determination of Four Terminal Constants of Acoustical Elements. Rep. No. 344, Aeronaut. Res. Inst., Univ. of Tokyo, May 1959.

12. Igarashi, Juichi; and Arai, Masaaki: Fundamentals of Acoustical Silencers. III — Attenuation Characteristics Studied by an Electric Simulator. Rep. No. 351, Aeronaut. Res. Inst., Univ. of Tokyo, Feb. 1960.

13. Levine, Harold; and Schwinger, Julian: On the Radiation of Sound From an Unflanged Circular Pipe. Phys. Rev., vol. 73, no. 4, Feb. 15, 1948, pp. 383-406.

14. Sacks, M. P.; and Allen, D. L.: Effects of High-Intensity Sound on Muffler Element Performance. J. Acoust. Soc. Amer., vol. 52, no. 3, pt. 1, Sept. 1972, pp. 725-731.

15. Mungur, P.; and Gladwell, G. M. L.:   Acoustic Wave Propagation in a Sheared Fluid
     Contained in a Duct.   J. Sound & Vib., vol. 9, no. 1, Jan. 1969, pp. 28-48.

16. Fujimoto, Shigeo: Experiment on Exhaust Muffler by Pulsating Flow.   NASA
     TT F-14,295, 1972.

TABLE I

PHYSICAL CHARACTERISTICS OF THE STANDARD HELICOPTER

Main rotor:

Diameter, m (ft) 11.3 (37.13)

Number of blades 2

Blade chord, m (in.) 0.28 (11.0)

Airfoil . . . . NACA 0015

Blade area, m^2 (ft^2) 3.18 (34.27)

Disk area, m^2 (ft^2) 100.8 (1085)

Solidity 0.0314

Tip speed, m/sec (ft/sec) 210.3 690

Design operating speed, rpm 356

Tail rotor:

Diameter, m (ft) 1.7 (5.67)

Number of blades 2

Blade chord, m (in.) 0.10 (4.13)

Blade area, m^2 (ft^2) 0.209 (2.25)

Disk area, m^2 (ft^2) 2.35 (25.3)

Design operating speed, rpm 1920

General:

Normal gross weight, kg (lb) 1111 (2450)

Empty weight, kg (lb) 716.6 (1580)

Overall length, m (ft) 13.29 (43.62)

Power, kW (hp) 194 (260)

Maximum level airspeed, knots 85

Gear ratios:

Main rotor mast 0.111

Tail rotor 0.6

Cooling fan 1.5

Engine characteristics:

Number of cylinders 6

Operating cycle 4

Exhaust-gas temperature at MAP of 48 cm Hg, K (°F) 1090 (1500)

Gas-flow Mach number at MAP of 48 cm Hg and area of 0.0045 m^2 . . . . . . 0.1

TABLE II.- ESTIMATED INSERTION LOSS OF MUFFLER OBTAINED FROM

MEASURED SOUND PRESSURE LEVELS OF STANDARD AND

MODIFIED EXHAUST SYSTEM

Engine-bank firing-frequency harmonic	Sound pressure level, dB, for –			Estimated insertion loss of muffler, dB
	Standard exhaust system	Y-connector only	Y-connector and muffler	
1	94.5	81	69.5	11.5
2	83.0	94	69.5	24.5
3	88.5	65	56.5	8.5
4	77.0	74	64.2	9.8
5	66.6	54	68.0	-14
6	72.5	60	69.5	-9.5
7	76.2	---	69.5	-----
8	70.0	---	64.2	-----
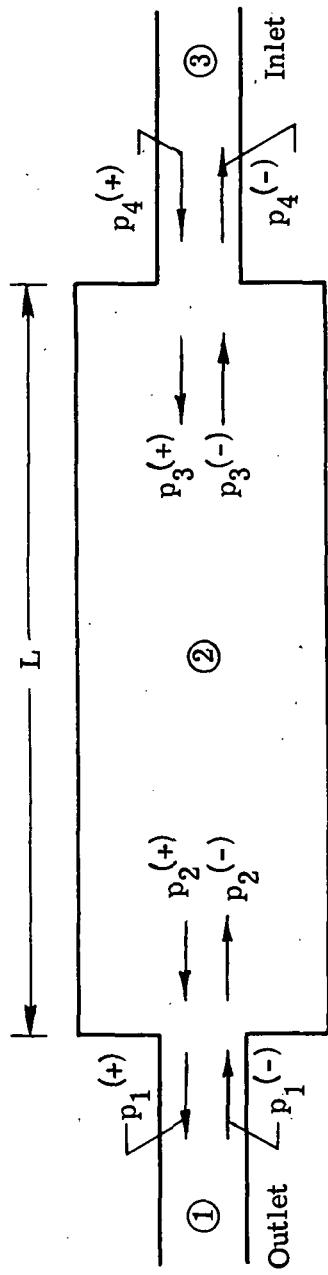
Area expansion ratio S_2/S_1

Forward-going waves $p^{(+)}$

Backward-going waves $p^{(-)}$

Total fluctuating pressure $\quad p = p^{(+)} + p^{(-)}$

Figure 1. - Simple expansion-chamber muffler with descriptive notation.
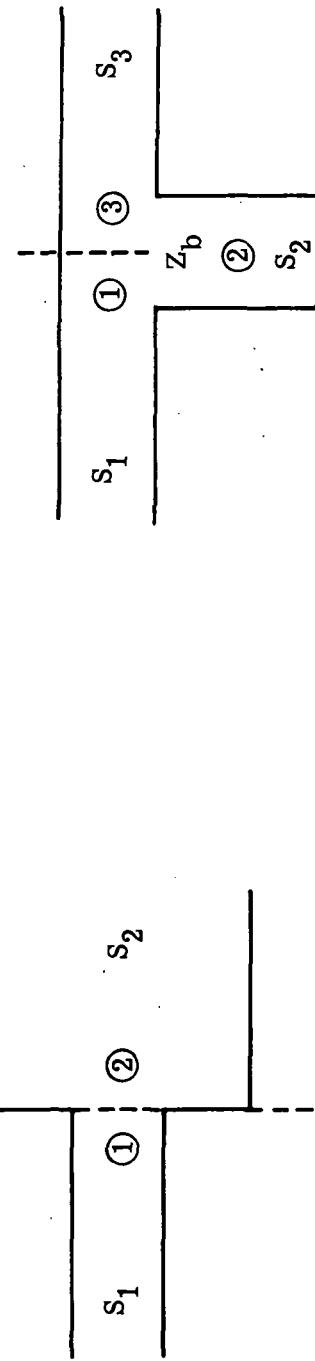
(a) Area expansion or contraction discontinuity.

(b) Side branch discontinuity.

Figure 2. - Simple duct discontinuities.
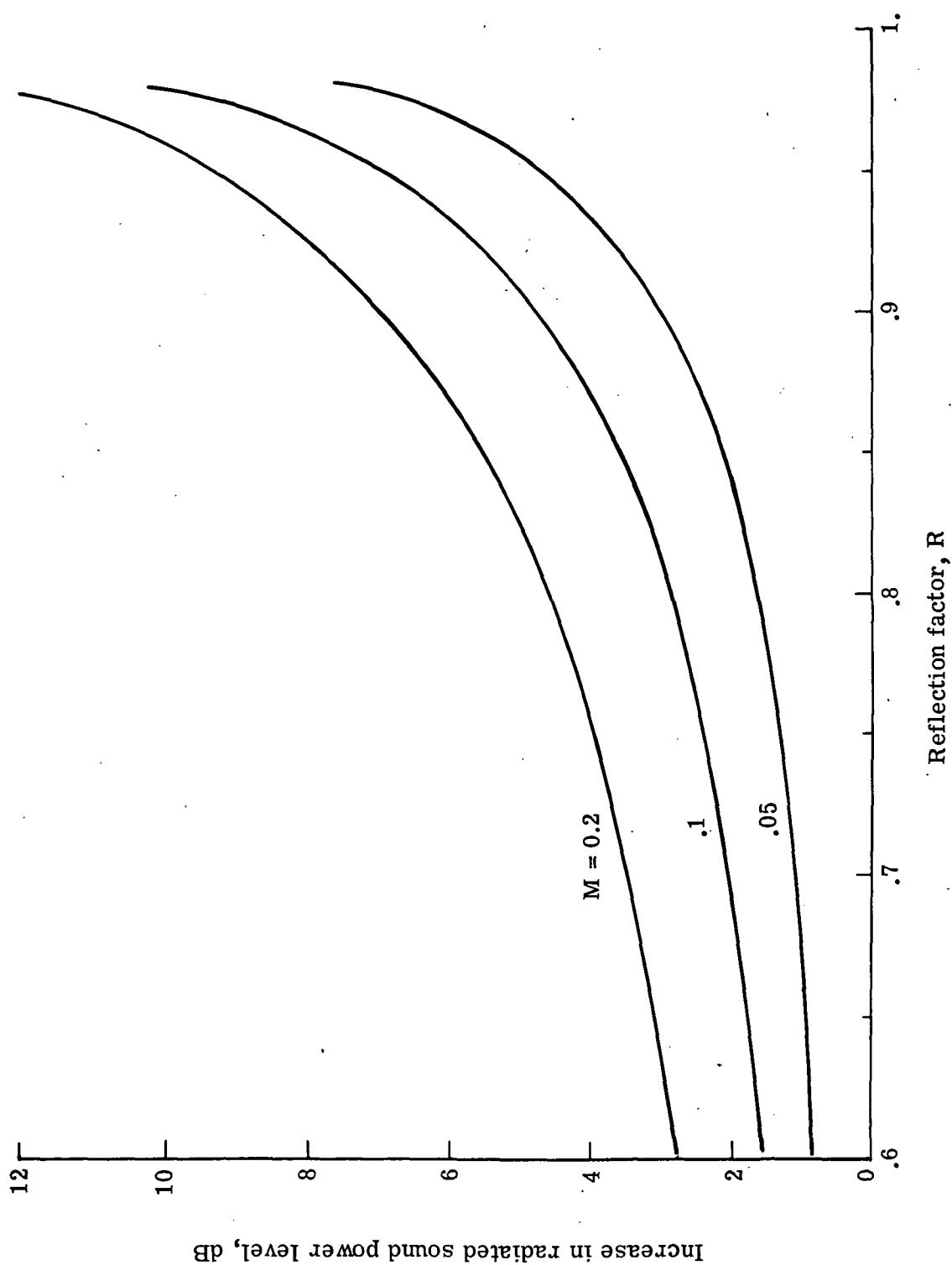
Figure 3.- Increase in radiated power level from tailpipe due to transport effect of mean gas flow, with Mach number as a parameter.

57
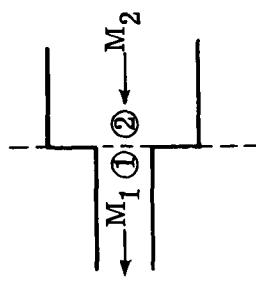
(a) Flow area contraction.
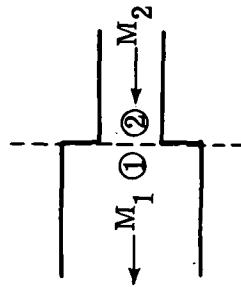
(b) Flow area expansion.
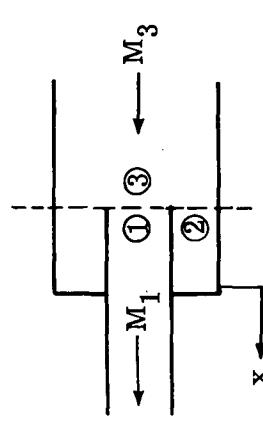
(c) Flow area contraction with branch impedance.
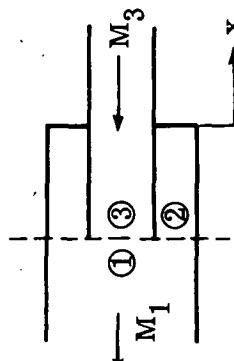
(d) Flow area expansion with branch impedance.

Figure 4.- Schematic diagrams of area discontinuities.
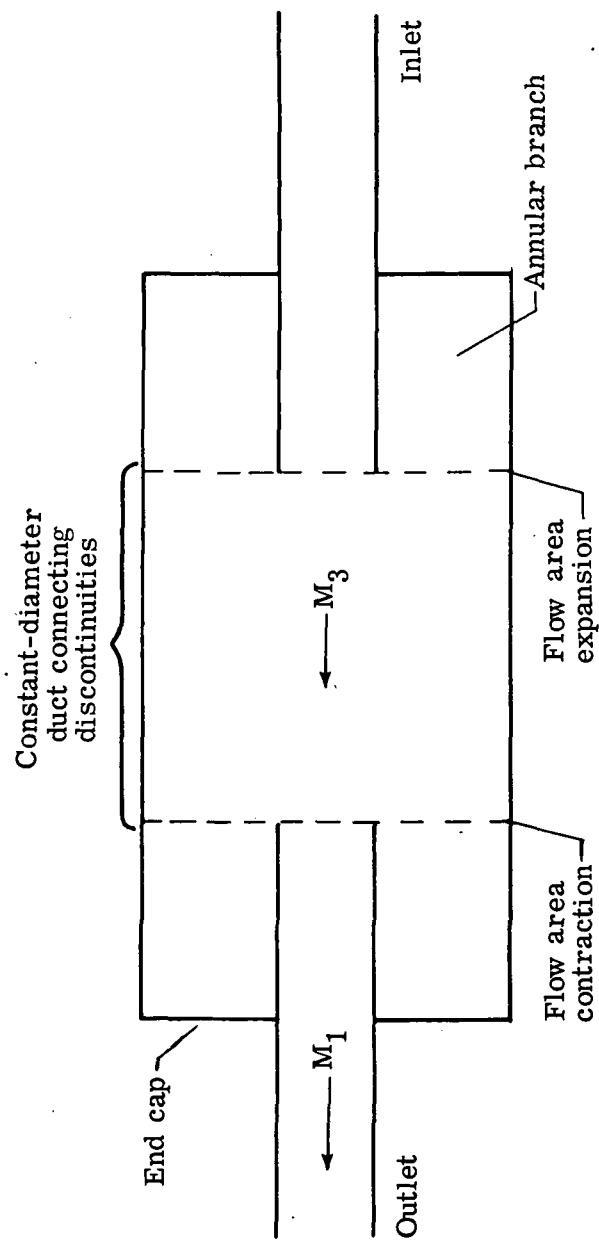
Figure 5.- Expansion-chamber muffler with extended inlet and outlet pipes.

Inlet

Annular branch

Constant-diameter
duct connecting
discontinuities

M_3

Flow area
expansion

Flow area
contraction

End cap

M_1

Outlet
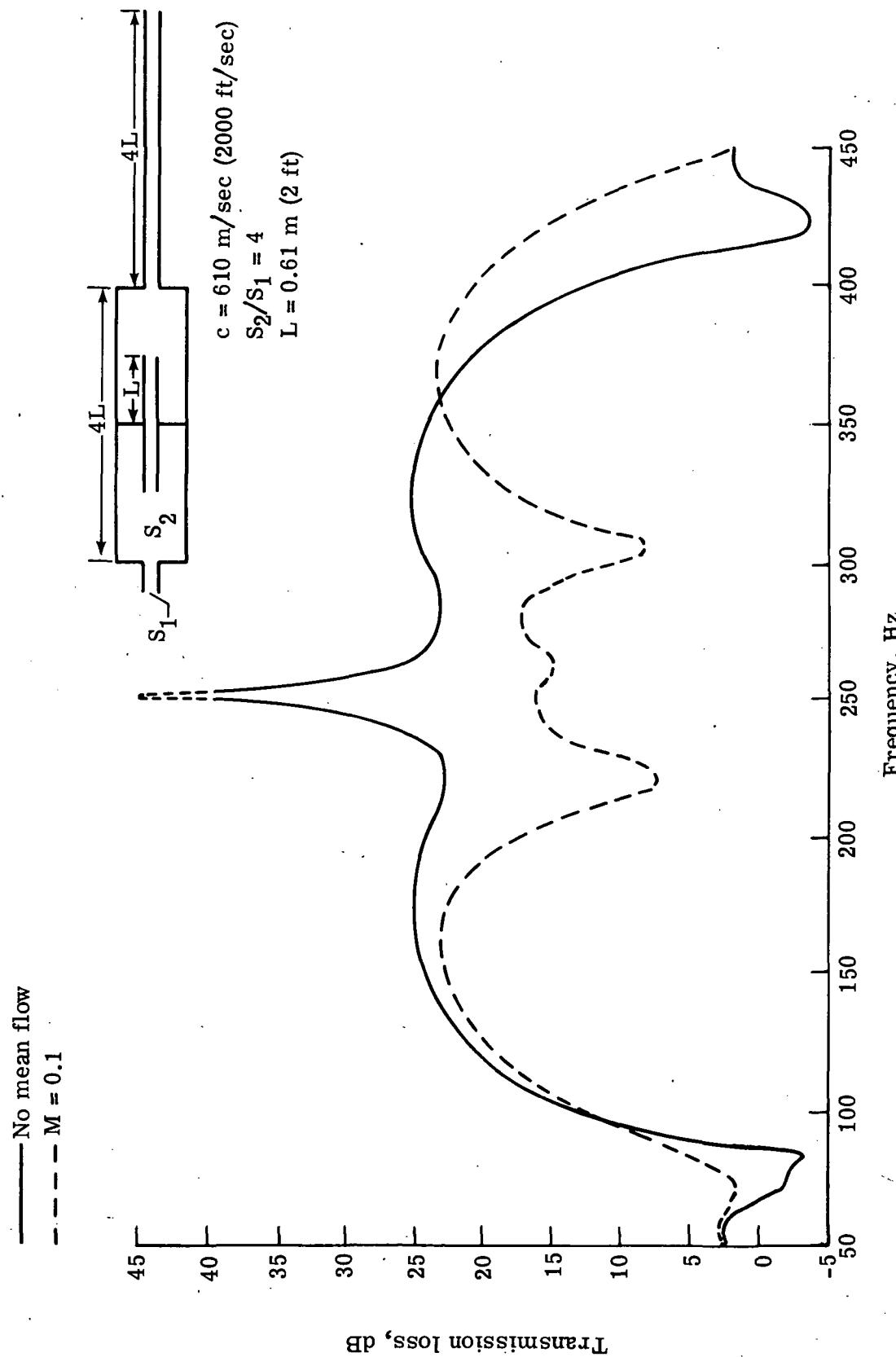
Figure 6.- Comparison of transmission loss with and without flow.
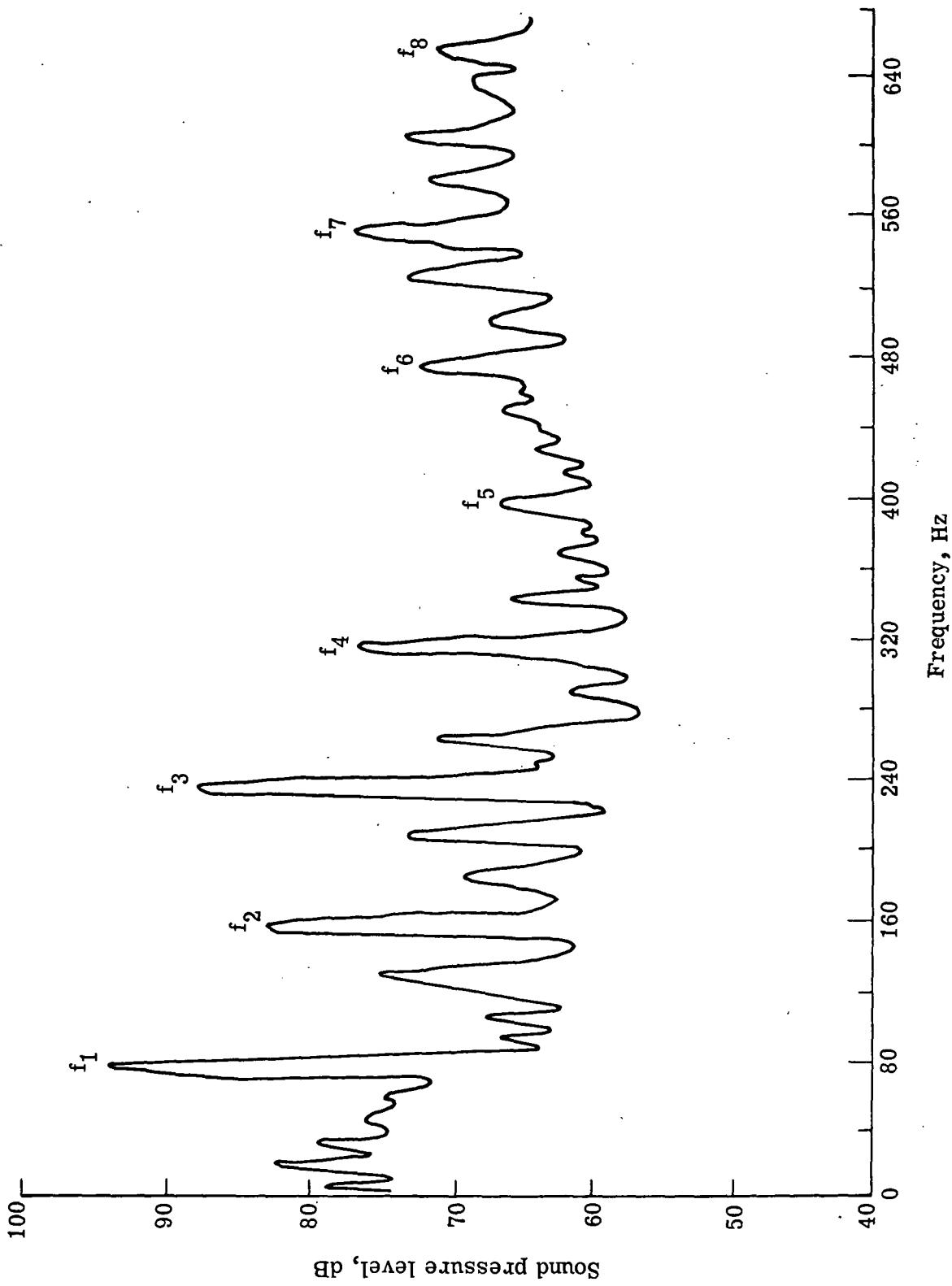
Figure 7.- Narrow-band (4 Hz) spectrum of helicopter noise emission at 30.5 m (100 ft), an angle of 90° to nose of helicopter, and a hover height of 1.83 m (6 ft).
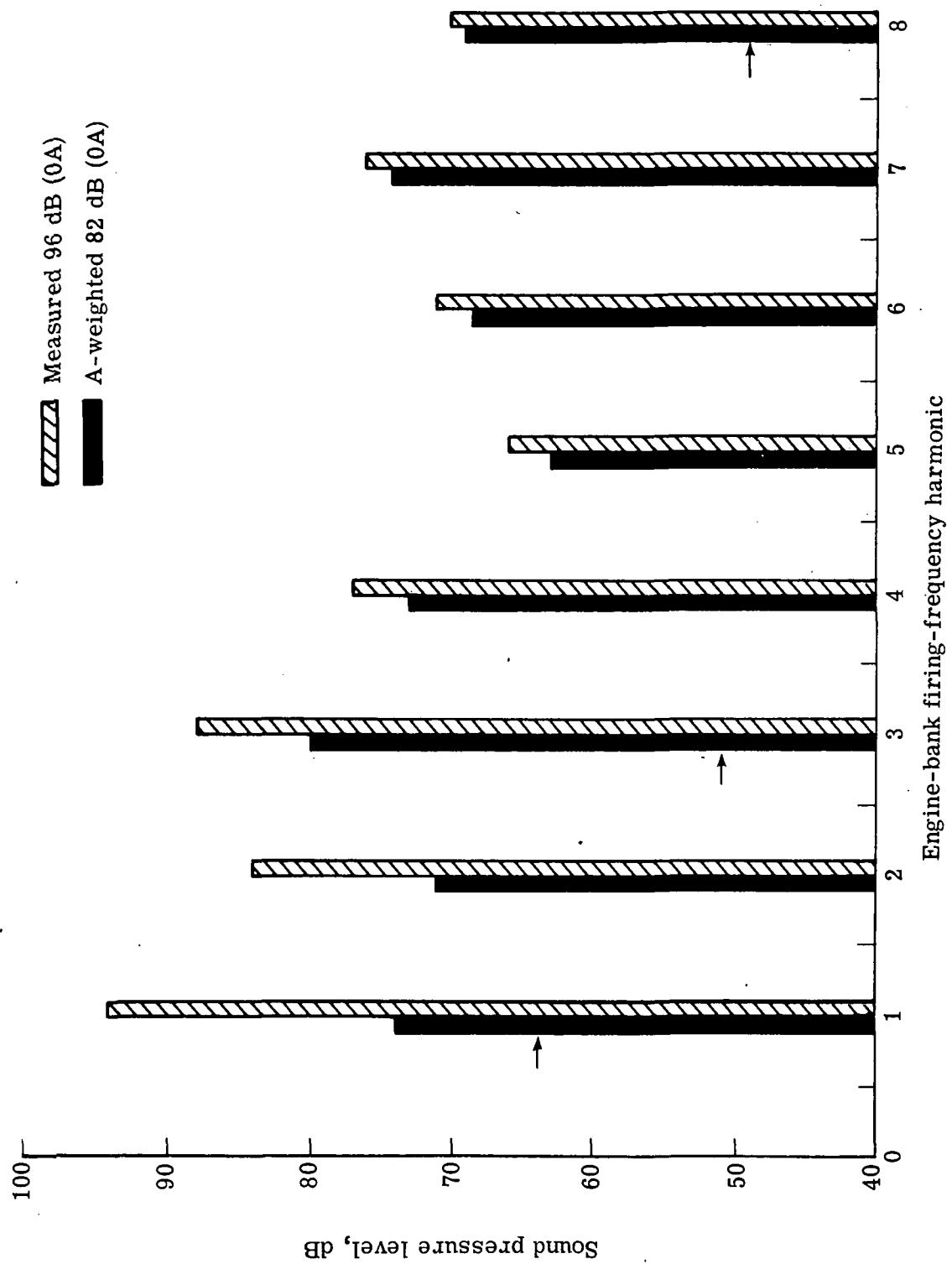
Figure 8.- Bar graph of significant exhaust noise components.
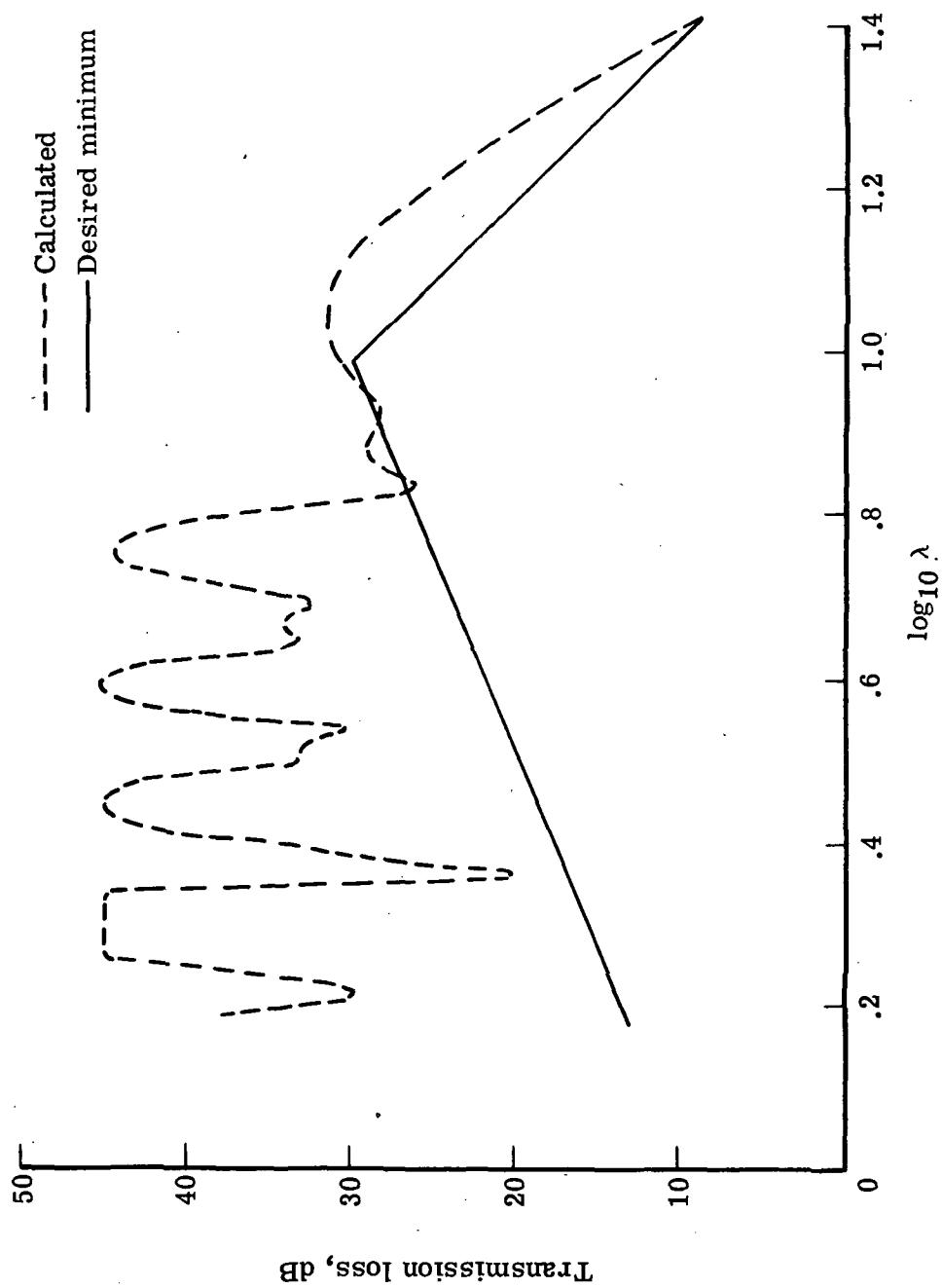
Figure 9. - Computed performance of optimum muffler configuration II.
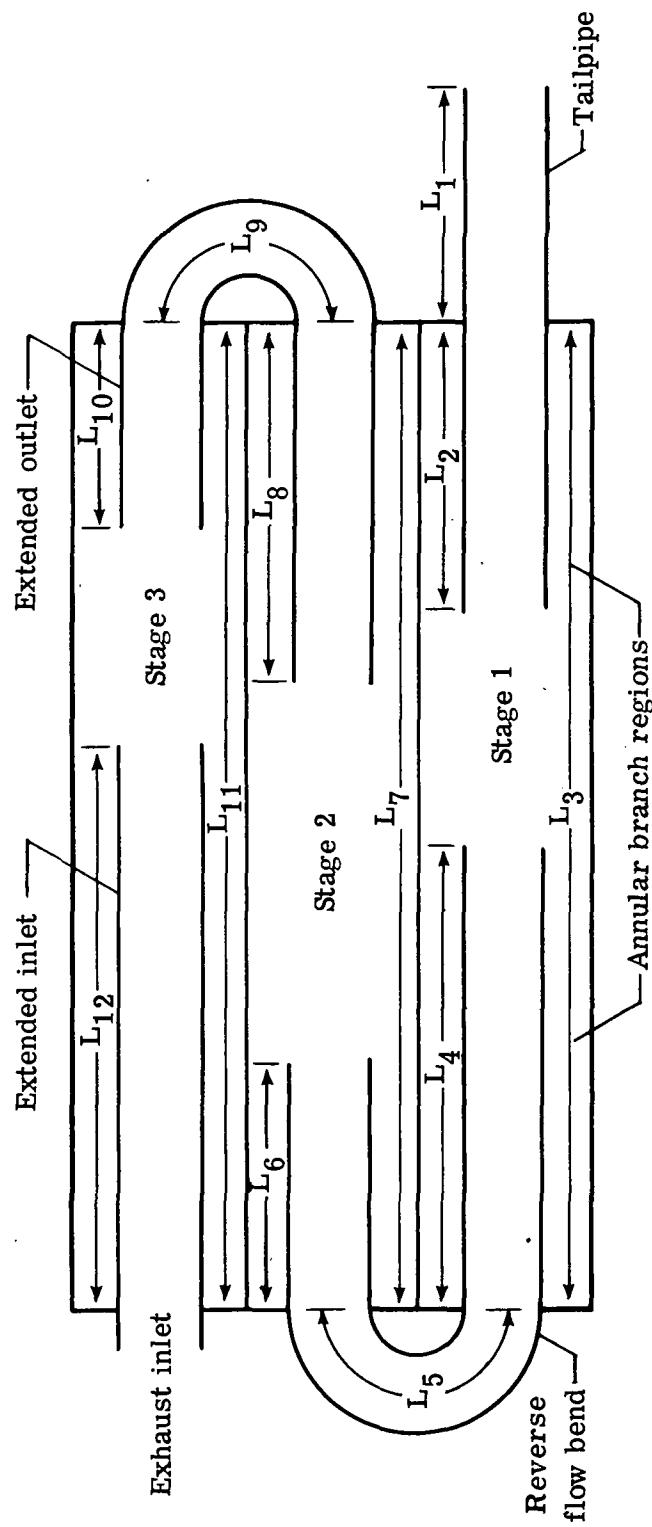
Figure 10.- Schematic of three-stage muffler with bends to conserve space.
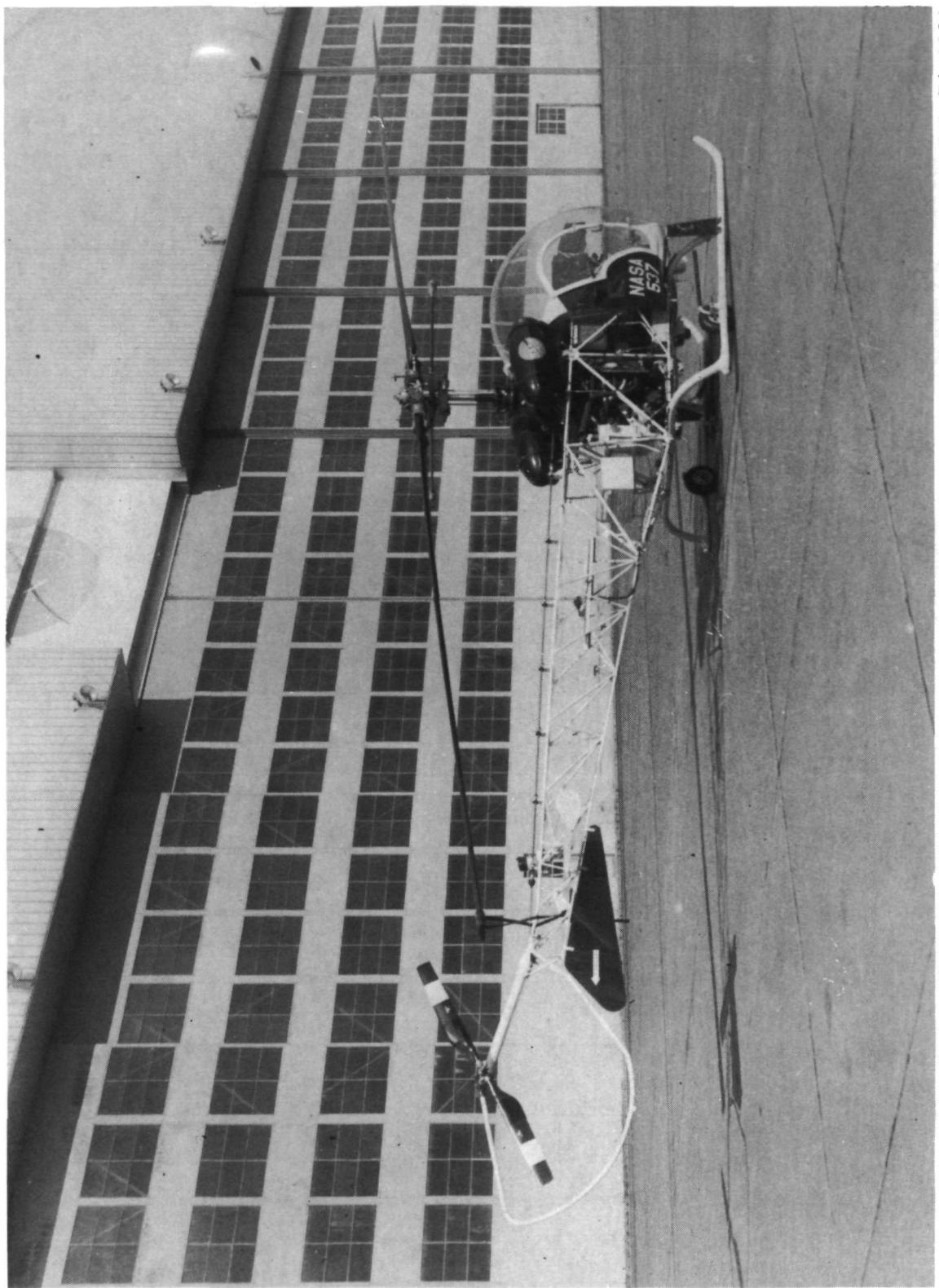
Figure 11.- Test helicopter for which muffler system was designed.
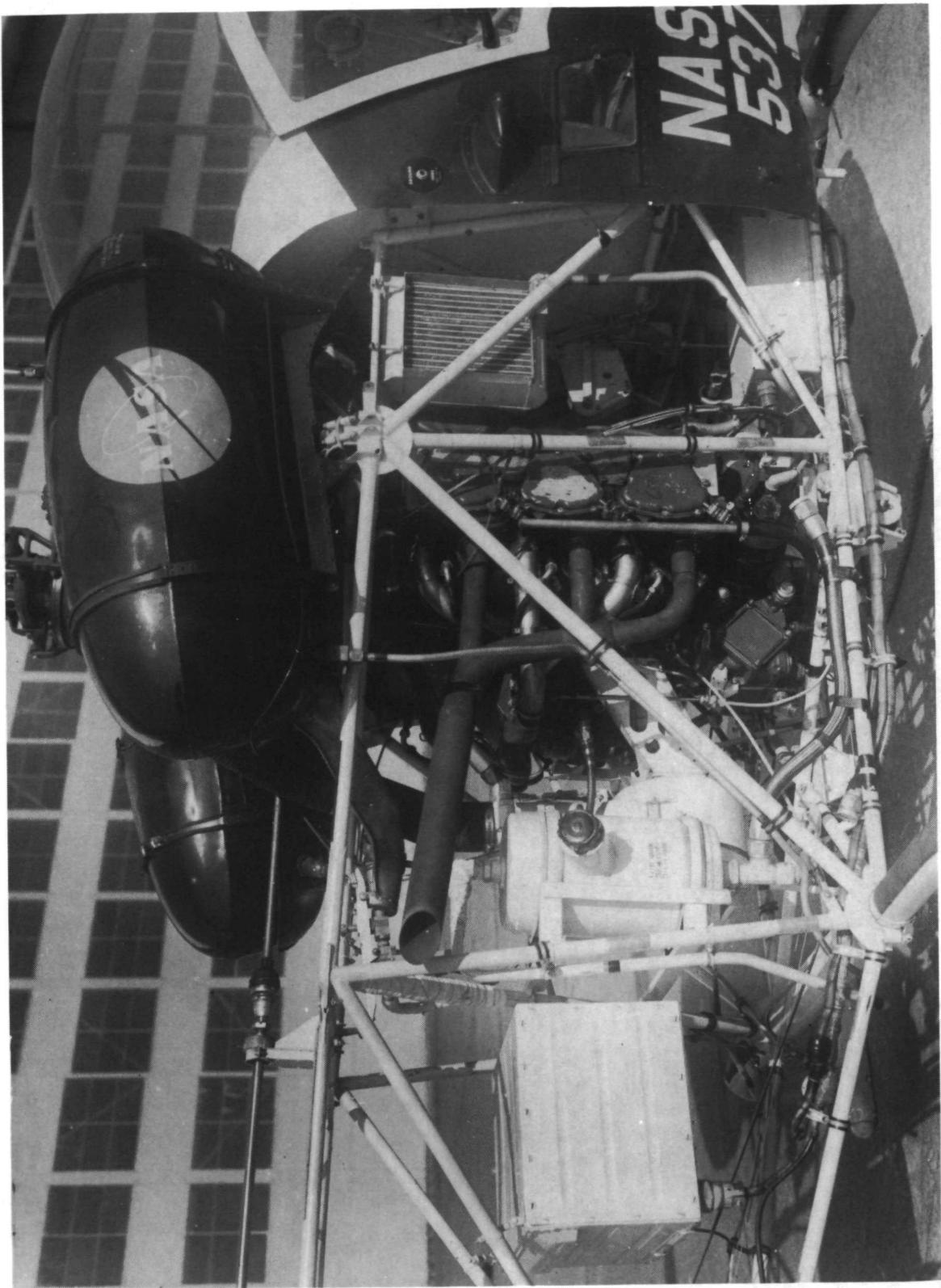
Figure 12.- Test helicopter with standard exhaust system for one three-cylinder bank shown.

L-71-9390

Component	Length, m	Area, m^2
1. Tailpipe	0.60	0.002
2. Extended outlet	.06	.002
3. First chamber	.25	.019
4. Extended inlet	.04	.002
5. Connector	.01	.002
6. Extended outlet	.08	.002
7. Second chamber	.51	.019
8. Extended inlet	.30	.002
9. Connector	.61	.002
10. Extended outlet	.13	.002
11. Third chamber	.76	.019
12. Extended inlet	.44	.002
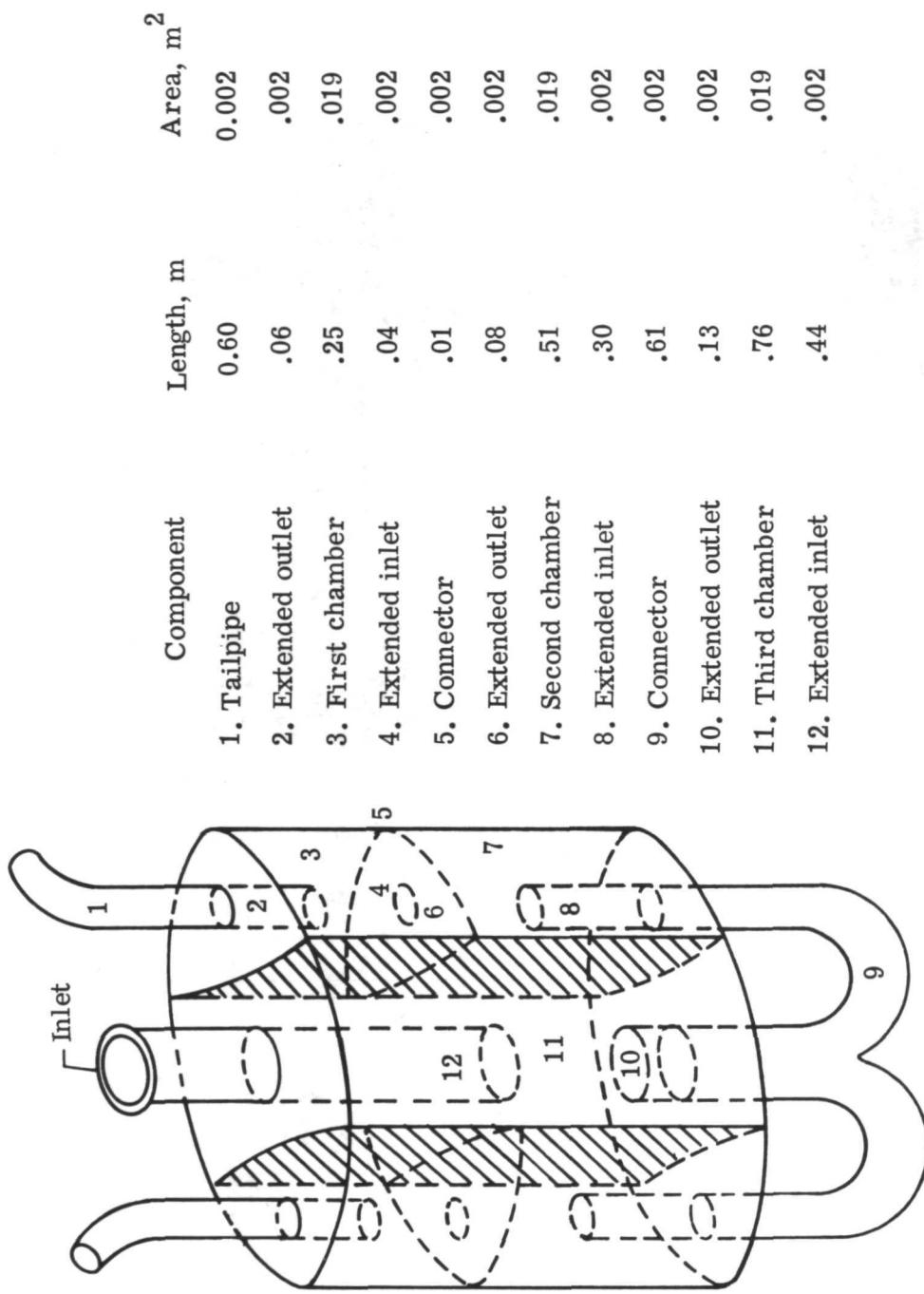
Figure 13. - Schematic of muffler configuration.

Figure 14.- Y-connector installation.
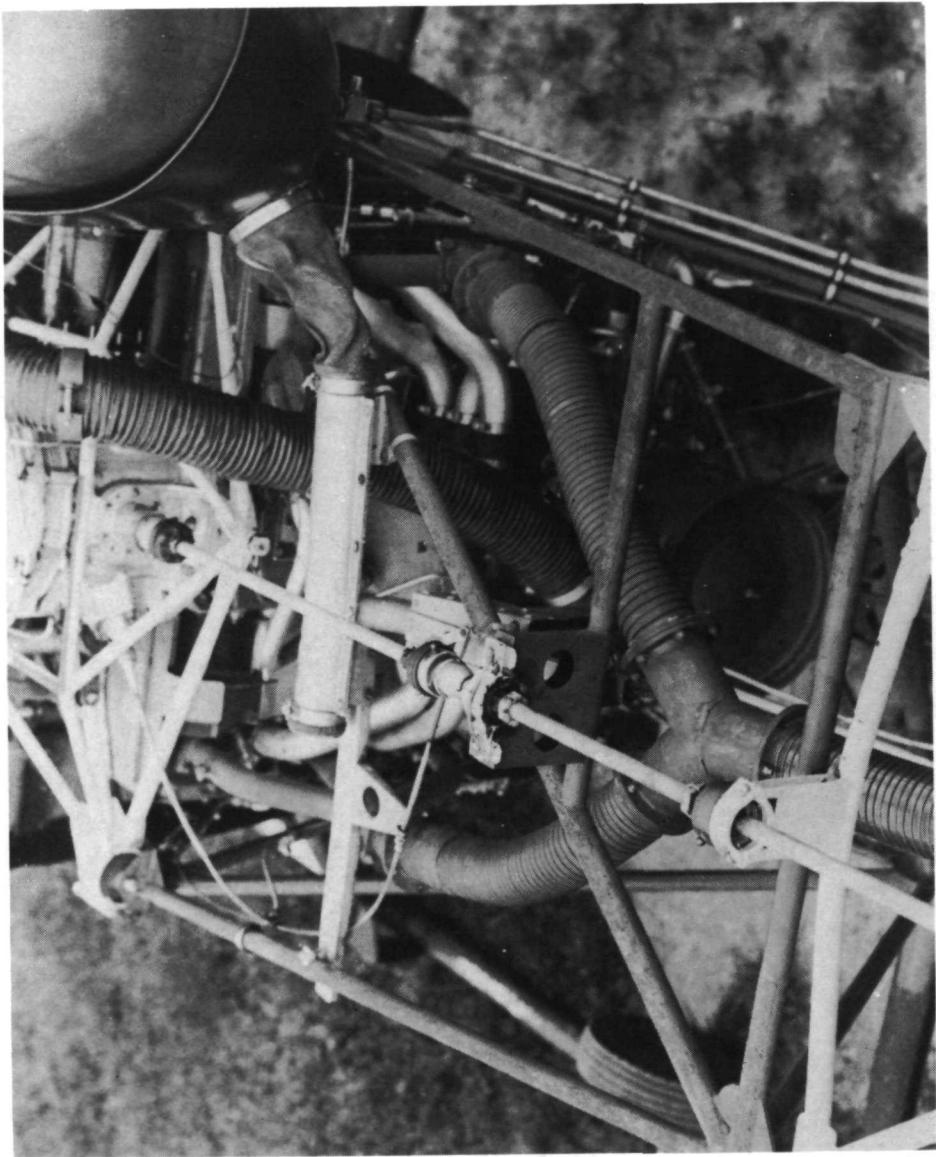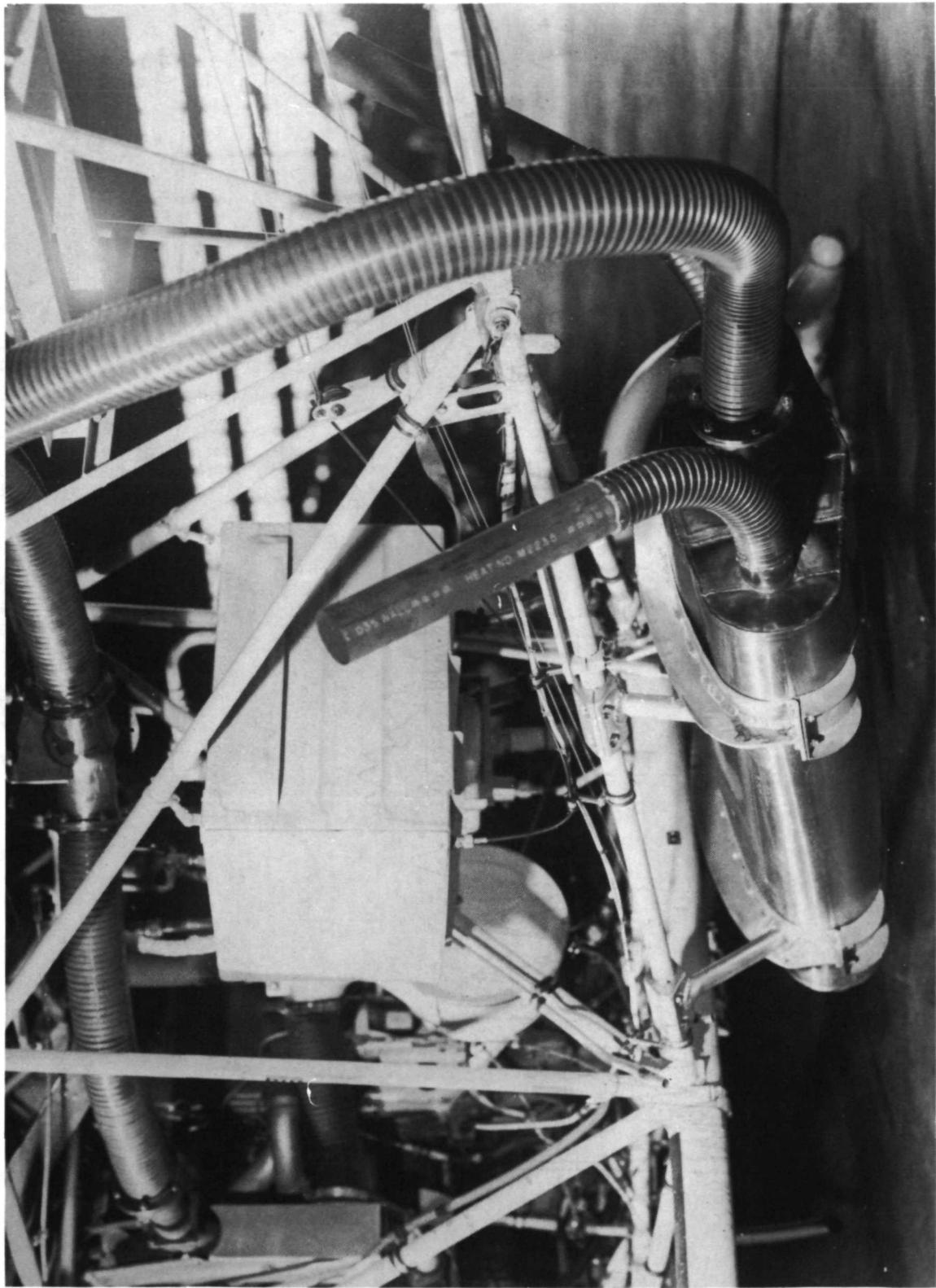
Figure 15.- Muffler installation.
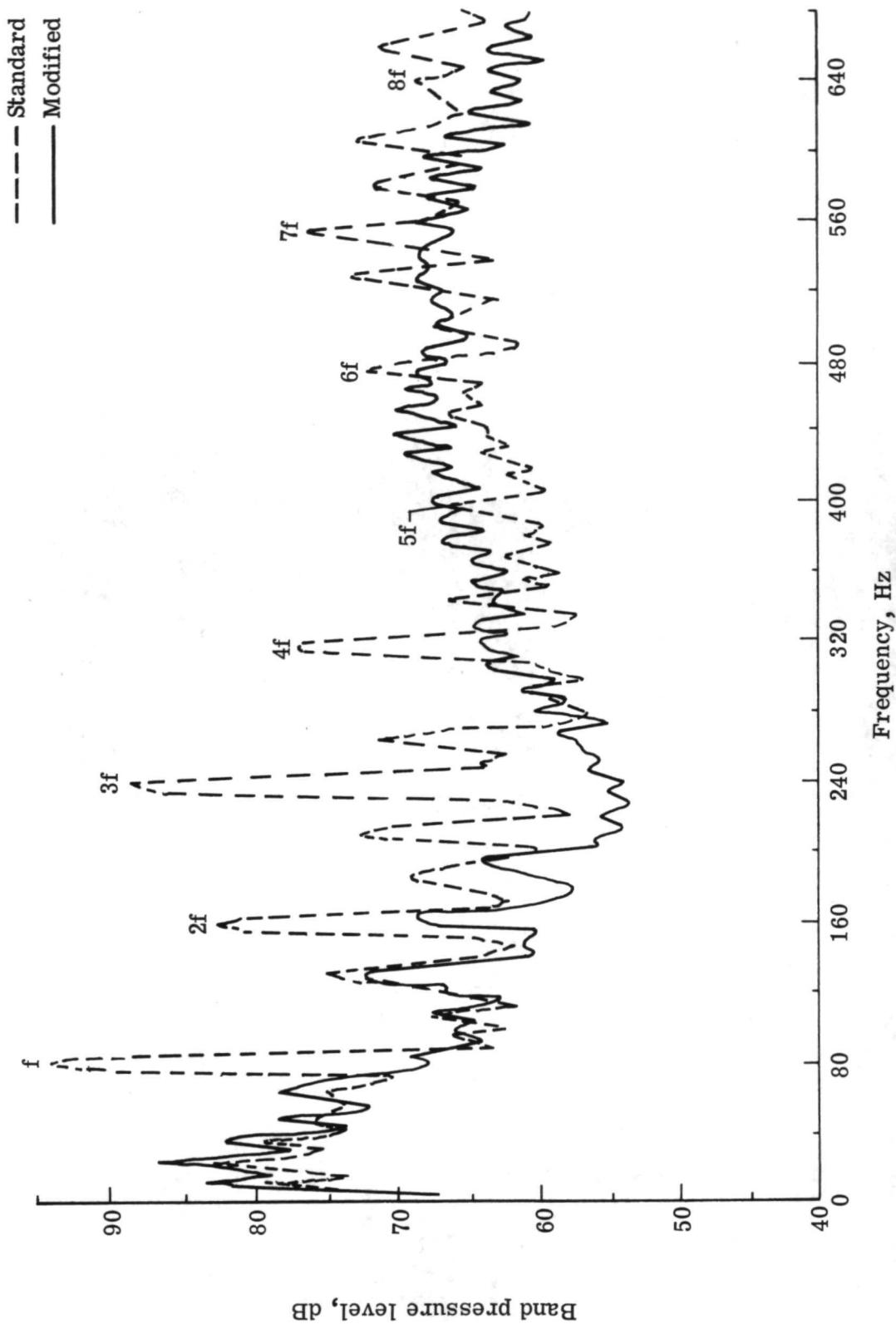
Figure 16.- Narrow-band (4 Hz) spectra of noise emitted by standard and modified helicopter.
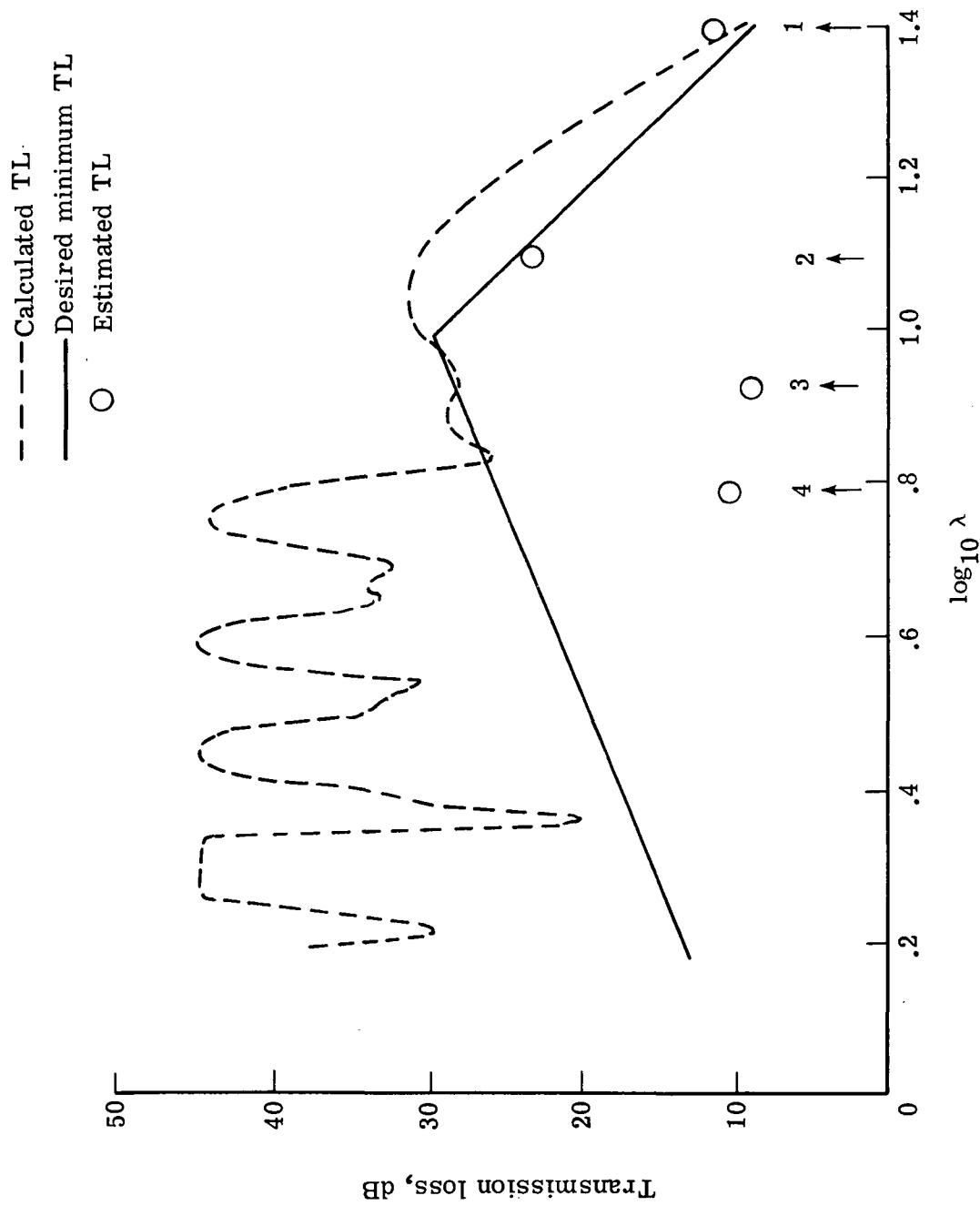
Figure 17.- Comparison of calculated transmission losses with estimated transmission losses. (Arrows denote firing-frequency harmonic.)

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