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TEST STAND FOR PRECISE MEASUREMENT OF IMPULSE AND THRUST VECTOR OF SMALL ATTITUDE CONTROL JETS

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16. Abstract

A test stand which accurately measures the impulse bit and thrust vector of reaction jet thrusters used in the attitude control system of space vehicles has been developed by Ames Research Center. It can be used to measure, in a vacuum or ambient environment, both impulse and thrust vector of reaction jet thrusters using hydrazine or inert gas propellants.

The ballistic pendulum configuration was selected because of its accuracy, simplicity, and versatility. The pendulum is mounted on flexure pivots rotating about a vertical axis at the center of its mass. The test stand has the following measurement capabilities: impulse of 4×10^{-5} to 4.4 N-sec (10^{-5} to 1.0 lb-sec) with a pulse duration of 0.5 msec to 1 sec; static thrust of 0.22 to 22 N (0.05 to 5 lb) with a 5-percent resolution; and thrust angle alinement of 0.22 to 22 N (0.05 to 5 lb) thrusters with 0.01° accuracy.

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SYMBOLS

C	instrument display counts per degree of pendulum rotation, counts
I_O	moment of inertia of the pendulum without added inertia, kg-m^2
I_1, I_2	moment of inertia of the pendulum with different added inertia, kg-m^2
K	rotary spring constant, m-N/rad
p	impulse, N-sec
r	distance from pendulum axis to thruster, m
T	period of the pendulum, sec
T_1, T_2	period of the pendulum with different added moments of inertia, sec
θ	pendulum excursion, deg
θ_m	maximum pendulum excursion, deg
τ	torque, N-m

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SUMMARY

A test stand for measuring the impulse of reaction jets for attitude control systems has been designed and developed at Ames Research Center. It is capable of measuring jet impulses using various types of propellants in either a vacuum or sea-level environment.

During development of the test stand, several designs were evaluated. A design based on the ballistic pendulum concept was selected because it is simple and versatile. The ballistic pendulum directly measures the impulse of a single short pulse of an attitude control thruster. The pendulum arm is supported by flexure pivots and rotates about a vertical axis at the center of the pendulum mass. The entire thruster system including propellant tank, line, regulator, and thruster is mounted on the pendulum arm and is part of the pendulum.

The test stand has been used to measure impulses from 1.2×10^{-2} to 4.4 N-sec (2.7×10^{-3} to 1 lb-sec). Four calibration techniques were used: (1) the calculation of impulse based on the pendulum periods for two different inertias, (2) the calculation of impulse based on the measured spring constant and pendulum period, (3) the measurement of electromagnetic impulses imparted by the calibration impulse generator, and (4) calculation of impulse based on momentum imparted from a falling steel ball. A comparison of results obtained from independent impulse calibration techniques assures the accuracy of the impulse measurements.

Two other parameters of the jet thruster can be measured on this test stand: thrust angle and static thrust. The thrust angle can be measured under vacuum or sea-level conditions to an accuracy of 0.01°. The static thrust measurements on the test stand are not influenced by the usual pressure or temperature-induced stresses in the thruster system because the complete thruster system, including the fuel tank and fuel line, is mounted on the pendulum arm.

INTRODUCTION

A novel test stand, which accurately measures the impulse bit and thrust vector of jet thrusters used for the attitude control of space vehicles, has been developed at Ames Research Center. The test stand has been used in the development of attitude control systems.

The impulse bit is the time integral of the thrust of the reaction jet each time the jet is pulsed. It is an important performance parameter for both design and performance evaluation of attitude control systems using reaction jets for control torquing. The reaction jets are usually operated in a pulse mode (on-off). They are used to provide torque for attitude control of both three-axis-stabilized and spin-stabilized spacecraft. The jet is pulsed to maintain the spacecraft attitude within a desired tolerance band. The pointing accuracy, frequency of oscillation, and propellant consumption all depend on the impulse bit. In spin-stabilized spacecraft (such as Pioneer), the accuracy of orientation is determined by impulse bit size, impulse bit centroid, and alinement of the thrust vector.

Because attitude control design and operational performance depend on the characteristics of the impulse bit (to be referred to as impulse), a study program was undertaken at Ames in April 1966 to determine the best way to measure the impulse of reaction jets.

To increase the flexibility of the test system in evaluating thruster performance, it was considered desirable to include the capability of measuring the thrust vector and static thrust of the jet in a vacuum. The resulting test-stand design objectives, in order of priority, were:

(1) Impulse measuring capability of 4×10^{-5} to 4 N-sec (10^{-5} to 1.0 lb-sec) for reaction jets using either hydrazine or inert gas propellants with pulse duration of 0.5 msec to 1 sec. Measurements to be performed in a vacuum without mechanical or electrical modification to the jet.

(2) Measurement of static thrust from 0.22 to 22 N (0.05 to 5 lb) with a 5 percent resolution.

(3) Thrust angle alinement capability of 0.22 to 22 N (0.05 to 5 lb) thrusters within an accuracy of 0.01°.

To select a system that would meet these test-stand design objectives, analytical studies were conducted on three different measurement concepts: a system that sensed and integrated thrust; a system using an air-bearing table with a rate sensor; and ballistic pendulum systems.

The concept based on the integration of measured thrust was eliminated because thrust measurement would be distorted by the dynamics of the thruster test-stand system and the response characteristics of the thrust-sensing instrument. The air-bearing system was eliminated because rate could not be sensed with sufficient resolution and because of operational problems involved in using an air-bearing system in a vacuum. The ballistic pendulum concepts showed the greatest promise for accurately measuring impulse, static thrust, and the thrust angle. Further, a ballistic pendulum permits the entire thruster system, including lines, regulator, and storage tanks, to be mounted on a common platform so that interactions between or relative movement of the components do not affect the accuracy of the measurement.

Basically, the ballistic pendulum measures impulse because the swing of the pendulum after the jet is pulsed is proportional to impulse. Ballistic pendulums have long been used to measure the velocity of bullets by measuring momentum and then dividing the momentum by the mass of the bullet. Another long used application of a ballistic pendulum is the ballistic galvanometer, which is used to measure the charge of a capacitor by measuring the impulse generated when the capacitor is charged or discharged through a coil in a magnetic field. Impulse, the time integral of force, is equivalent to and has the same physical dimensions as momentum. A simplified mathematical analysis of the basic theory used in designing the test stand to measure the impulse bit is given in appendix A. Appendix B is an analysis of impulse measurement errors.

The first pendulum configuration tested used a torsion wire suspension. The axis was not placed at the center of gravity of the pendulum because, if it were, an impulse would make the pendulum translate as well as rotate. The translational and rotational energy modes would interact so that maximum excursion of the pendulum could not be used to measure impulse. To eliminate this interaction, the axis was located at the center of percussion of the pendulum relative to the thruster nozzle (ref. 1). It was theorized that when the thruster was fired the axis would remain fixed because the linear velocity of the axis resulting from rotation around the center of gravity would be equal and opposite to the translational velocity of the center of gravity.

Rotational displacement was sensed by strain gages consisting of constantan wire wrapped around and bonded to the upper support wire. A series of tests were conducted on this configuration. It was concluded that a torsion wire pendulum rotating about its center of percussion is too sensitive to seismic noise. Translatory seismic noise of the support wire formed a couple with the center of gravity, producing a rotation.

To eliminate this problem, a new ballistic pendulum configuration was designed and evaluated. In the new design, the pendulum is mounted at its center of gravity on flexure pivots. The pivots limit the movement of the pendulum to rotational motion about the vertical axis. A capacitive position transducer senses the position of the pendulum. A peak holding circuit detects the maximum displacement angle of the pendulum. An expedient method of checking the impulse calibration in a vacuum is provided by a calibration circuit and electromagnetic driver that generate known impulses. This system has been presently operational in the NASA Ames Pneumatics Laboratory since August 1968.

After this work was completed, a technical note by A. H. Pfeffer of the European Space Research and Technology Centre, which describes similar measurements made with a ballistic pendulum, was brought to the attention of the authors (ref. 2).

DESCRIPTION OF THE SYSTEM

The ballistic pendulum system (fig. 1) was developed to measure the impulse and thrust vector of attitude control jets. It consists of the pendulum, the instrumentation located on the test stand, and the instrumentation system used with the test stand. Each of these is described in the following sections.

Pendulum

The ballistic pendulum rotates about a vertical axis (see figs. 2 and 3) and is supported by two flexure pivots 0.17 m (6-1/2 in.) apart. The pivots hold a vertical moving part (part A, fig. 2) to a support post that is attached to the baseplate of the test stand. A long, narrow horizontal plate is attached to the top of part A. The plate extends 0.36 m (14 in.) from the axis in one direction and 0.18 m (7 in.) in the other. The thruster to be tested is mounted on the long end of the horizontal plate. The fuel tank and other thruster system components are mounted on the horizontal plate so that it is roughly balanced about the axis of rotation. Small weights are added and moved as necessary for final balancing of the pendulum.

Several advantages result from having the pendulum rotate about a vertical axis. The flexures can support the greatest load in an axial direction. The spring rate does not change with supported load in the axial direction. A slight imbalance of the pendulum does not change the restoring force about the vertical axis. Thruster parts, balance weights, and accessories used with the pendulum are easily mounted on the horizontal plate. The horizontal plate is easy to remove and replace on the pendulum, which permits mounting the thruster components at any convenient work place and without risking damage to the flexures.

The flexure pivots are commercially available. They have a nominal spring rate of 2.3×10^{-3} N-m (0.2 in.-lb) per radian and rotate ±30° (fig. 4). Interchangeable, stiffer flexures are used when the load exceeds 7 kg (15 lb).

Three or four leveling screws, as convenient for the particular test setup, pass through threaded holes in the baseplate of the test stand and rest on the bottom of the vacuum chamber. The vacuum chamber is a steel bell jar, 0.6 m (2 ft) high, with a 0.6 m (2 ft) inside diameter.

Instrumentation on the Test Stand

Position sensor– The position sensor measures the displacement of the pendulum. It is a parallel-plate, variable capacitor somewhat like the tuning capacitor of a radio (fig. 5). The moving part is a horizontal, pie-shaped aluminum plate attached at the narrow end to the bottom of the vertical moving part (part A, fig. 2) of the pendulum. Two pairs of stationary plates, electrically insulated from each other, are arranged so that when the pendulum moves the capacitance between the moving plate and one pair of stationary plates increases while the capacitance between the moving plate and the other pair of stationary plates decreases by an equal amount. The capacitance levels range from about 5 to about 32 pF. The position-sensing capacitor is connected to an oscilloscope plug-in unit so that it forms a part of a bridge (figs. 6 and 7). The capacitances vary linearly with pendulum rotation so that the output signal of the bridge varies linearly with the rotation of the pendulum.

Triaxial cables are used between the position sensor and the bridge unit so that the cable capacitance is not across the active arms of the bridge (fig. 7). The capacitance between the inner shield and the center wires is across the nonactive, low-impedance bridge arms and the capacitance between the inner shield and the outer grounded shield is across the bridge output. The latter capacitance can be tuned out by means of a parallel variable inductance; however, this was not necessary with the 4 ft cables used on this system. The inner cable shielding is attached to shielding plates between the stationary plates of the position sensor and the grounded metal case. Acrylic plates are used as spacers between the various stationary plates and the case. Screws, passing through spring washers, provide adjustment of the stationary plates of the position sensor relative to the baseplate.

Load cell for measuring static thrust– The measuring element of the load cell is a cantilever beam with semiconductor strain gages. The load cell capacity is 0.23 kg (0.5 lb); however, it will withstand forces up to 4.5 kg (10 lb) without damage. Static force is measured by placing the load cell stand on the baseplate so that the pendulum presses against the load cell.

Leads– Eight flexible leads are provided between the base and the moving pendulum for monitoring and controlling the thruster components. These are ribbons of a silver-platinum alloy 0.13 mm (0.005 in.) thick and 0.39 mm (0.015 in.) wide. They cross the axis of rotation of the pendulum between the upper and lower pivots. They add less than 0.25 percent to the spring constant of the pendulum. The ribbons permit enough motion between the pendulum parts so that the flexure pivots can be interchanged without disconnecting the ribbons. The ribbons have been used for command signals to the thruster, monitoring temperature and pressure of the propellant, heating the propellant, and driving the force generator when calibrating for impulse measurement.

Calibration impulse generator– The calibration impulse generator imparts an electromagnetic impulse to the pendulum arm for calibration. A rectangular coil is mounted on the underside of the moving horizontal plate, below the thruster, so that the lower segment of the coil passes between the poles of a stationary permanent magnet. The magnet is supported by the ends of three leveling screws that pass through threaded holes in the baseplate. The screw ends are approximately spherical; one seats in a conical depression in the bottom of the magnet, one in a slot having a triangular cross section, and a third on a flat surface. This kinematic design permits easy leveling of the magnet and permits the magnet to be removed and replaced without changing the leveling adjustment. The magnet is made of soft steel except for the cylindrical part which is Alnico.

Eddy current damper– The pendulum has an adjustable eddy current damper, consisting of an electro magnet mounted on the baseplate and a horizontal copper plate attached to an arm extending from the vertical moving part. When the pendulum moves, the plate moves between the poles of the magnet. Damping is adjusted by varying the current in the electromagnet.

Accessory pendulum arm for measuring the thrust vector– For thrust angle measurement, the impulse pendulum arm is replaced by an accessory pendulum arm. This arm has a 0.05m by 0.05m (2-in. by 2-in.) plate attached above the end of the longer part of the arm by a flexure. The flexure and an adjusting screw permit the small plate to be rotated about an axis along the pendulum arm. The thruster is mounted on the small plate so that it exhausts straight up. A precision first surface plane mirror, mounted on a stand so that the bottom of the stand is at precisely right angles to the mirror, is placed on the small plate beside the thruster, so that the angle of the small plate from horizontal can be measured with a theodolite.

Instrumentation System Used With the Test Stand

Several instruments and a central control panel are used with the pendulum to make measurements. An oscilloscope plug-in unit is used with the pendulum position sensor. A control unit contains a peak holding circuit for the position sensor signal and an amplifier and related circuitry to drive the calibration impulse generator (fig. 6). A digital time delay generator times the impulse calibration pulses. An adjustable constant current power supply energizes the eddy current damping magnet. An Ames indicating millivolt potentiometer is used with a load cell for measuring the thrust. A theodolite is used in thrust angle measurement. A digital voltmeter displays maximum pendulum excursion and is used in calibration procedures.

Oscilloscope plug-in unit– The oscilloscope plug-in unit completes the capacitive position sensor bridge. This unit also contains a 25-kHz bridge excitation oscillator, a phase-sensitive demodulator, a signal amplifier, and bridge balance and sensitivity controls.

Central control unit– Operational amplifiers are used to remove the high common mode voltage of the pendulum position signal from the oscilloscope plug-in unit, to amplify the signal, and to filter out high-frequency noise before the signal is fed to a digital peak holding circuit and then to a digital voltmeter (DVM). The DVM displays the maximum excursion of the slowly swinging pendulum after the thruster is pulsed. A pushbutton resets the peak holding circuit for the next impulse measurement. A function switch on the control panel (fig. 8) can be set so that the DVM displays the position transducer signal before it enters the peak holding circuit. This function is used when calibrating the system.

A precisely timed, controlled-current pulse to drive the calibration impulse generator is obtained from the digital time delay generator, the pulse voltage regulator circuit, and the current amplifier (fig. 6). The pulse voltage regulator circuit is shown in figure 9 and the current amplifier in figures 10(a) and 10(b). When the output of the time delay generator is negative, the output of operational amplifier 1 (at point A in figs. 9 and 10) is equal and opposite to the voltage across diode Z_1; otherwise, it is zero.

The output of operational amplifer 2 (figs. 10) is zero when the output of operational amplifier 1 (fig. 9) is zero. When a negative pulse from the time delay generator causes a voltage at point A, operational amplifier 2 provides a controlled current through the drive coil. Operational amplifier 2 has an output of ±36 V up to 5 amp.

For large currents, operational amplifier 2 serves as a current source, with the drive coil in the feedback loop. For small currents, it acts as a voltage source, and precision resistors that are quite large compared to the coil resistance regulate the current. Tests show that the dynamic response is adequate in this configuration, which is used for small currents so that the voltage output of operational amplifier 2 will be large compared to drift and noise. These configurations are shown in figures 10(a) and 10(b).

IMPULSE CALIBRATION METHODS

Four different calibration techniques for impulse measurement were used in the development of the ballistic pendulum: dropping a ball on the pendulum, timing the pendulum with different moments of inertia, timing the pendulum and measuring the spring constant, and pulsing with calibration impulse generator. Although the ball drop test was not refined, it is discussed because it appears to be a valid method of calibration. The other three methods were refined and used to calibrate the pendulum. In the second and third methods, measurements are made which can be used to determine the moment of inertia and spring constant of the pendulum; or the results of the measurements can be substituted directly into formulas that give impulse as a function of pendulum travel. The fourth method uses the electromagnetic calibration impulse generator. A description of the calibration methods is given in appendix C.

TESTS TO VERIFY THE ACCURACY OF THE PENDULUM

Two tests were made to demonstrate the accuracy of the pendulum. First, the calibration impulse generator was excited by current pulses having different magnitudes and durations and the results compared. Second, the pendulum was calibrated by three different methods and the results compared. The three calibration methods compared were timing the pendulum with different moments of inertia, timing the pendulum and measuring the spring constant, and using the calibration impulse generator.

Calibration Impulse Generator Tests Using Different Combinations of Pulse Length and Drive Current

Since impulse is the time integral of thrust, the same impulse may be obtained with various combinations of thrust and thrust duration. For this test, the electromagnetic impulse generator was used to impart three different impulse levels to the pendulum. At each level, calibration measurements were made with two combinations of pulse duration and current-generated thrust, each of which was calculated to give the same impulse. Each pulse length/current combination at each impulse level was repeated three times. There was a spread of about ±1 percent between the readings at each level. As a result, it was felt that the impulse generator developed impulses sufficiently accurate that it could be used for calibrating the system.

Comparing Results of Independent Calibration Methods

A second test was conducted that compares three calibration methods: timing the pendulum using different moments of inertia, timing the pendulum and measuring the spring constant, and using the calibration impulse generator. In this test, the impulse was generated by the impulse generator and was measured by the other two calibration methods. The resulting values were compared.

To conduct this test, the static current through the impulse coil of the impulse generator was adjusted to give a 0.0294 N force at a point 0.264 m (10-3/8 in.) from the pendulum axis. This force was measured by placing the load cell so that it pressed against the pendulum arm at the 0.264 m (10-3/8 in.) point. The time delay generator was set for 1 sec, so that the impulse was calculated to be 0.0294 N-sec (6.614×10^{-3} lb-sec). In comparing this with a calibration made by timing the period of the pendulum with different moments of inertia, which gave an impulse of 0.0289 N-sec (6.49×10^{-3} lb-sec), it was determined that there was a difference of 1.7 percent. An error of 0.1 sec in timing 10 swings of the pendulum would cause an error of 0.6 percent. According to a calibration by timing the period and measuring the spring constant with the load cell, the measured impulse was 0.0289 N-sec (6.50×10^{-3} lb-sec). The closeness of these measurements led to the conclusion that the ballistic pendulum would measure impulse with sufficient repetitive accuracy to be used in the testing of attitude control thrusters. It was further concluded that the impulse generator was sufficiently accurate to be used as the primary calibration method.

OPERATION OF THE SYSTEM

Three measurements may be made with the ballistic pendulum system: an impulse measurement, a static thrust measurement, and a thrust angle measurement. Before measuring the impulse, the pendulum must be calibrated by one of the methods previously discussed.

Impulse Measurement

To measure impulse, the pendulum is first brought to rest by the eddy current damper. The damper power supply is slowly turned all the way down. If it is turned down too rapidly, the changing field of the damping magnet induces currents in the damping plate which interact with the damping magnet, causing the pendulum to swing. If the impulse generator is used for calibration, the damper can be left on at a setting that results in something less than critical damping. After the pendulum is brought to rest, the peak holding circuit reset button on the control panel is pressed and the thruster is pulsed. The resulting pendulum excursion, which is proportional to the impulse, is indicated by the output of the peak holding circuit displayed on the DVM. Figures 3 and 11 show the test setup for impulse measurement.

Static Thrust Measurement

Static thrust is measured by placing the load cell so that the pendulum contacts the sensing element in line with the thrust vector of the jet. The damping current is increased to prevent the pendulum from oscillating as the jet is fired to measure static thrust. Figure 12 shows the setup for static thrust measurements.

Thrust Angle Measurement

The thruster is mounted on an attachment at the end of the pendulum arm so that the thrust is downward (fig. 13). The leveling screws of the pendulum baseplate are adjusted until adding a weight at the thruster location does not displace the pendulum. This ensures that when the thruster fires in the vertical direction the pendulum will not move. Then the plate on which the thruster is mounted is adjusted until firing the thruster does not cause the pendulum to rotate. At this time, the thruster must be thrusting straight down, or at least in a vertical plane in which both the line of thrust and the axis of the pendulum are located. There is a precision first surface mirror mounted on a stand that rests on the plate to which the thruster is attached. The stand holds the surface of the mirror at exactly 90° to the mounting plate. After the adjustments described above have been made, the angle of the mirror from the vertical is measured with a theodolite. The thruster is then rotated 90° about its axis and the procedure repeated to obtain the orthogonal component of the angle.

PERFORMANCE OF THE SYSTEM

Impulse Test

Impulse data obtained with the thrusters on the test stand have been used to define the thruster model used in computer simulation and analysis programs supporting the design, development, and operational flight of spacecraft attitude control systems. The test-stand data have been used to correlate the performance of the analytical thruster model with thruster performance under different operating conditions.

The test stand has been used to evaluate the impulse performance of the hydrazine thruster design for the Pioneer spacecraft which is presently on its way to Jupiter. The purpose of the evaluation was to accurately determine the impulse performance of the thrusters under various environmental conditions, at different catalyst bed temperatures, and for various on-time commands. Knowledge of the thruster impulse performance is essential to Pioneer mission operations because it is used in calculating attitude pointing of the spacecraft during the open-loop orientation mode.

The impulse test stand has been used to support the design, development, and flight operations of the Solar Pointing Aerobee Rocket Control System (SPARCS). The analog model, used in the preflight tests of SPARCS electronics, incorporates the thruster model derived from the impulse measurements made on the test stand. In another application, test information was provided to aid in the analysis of a SPARCS flight failure. Test information (fig. 14) was provided to compare the analog thruster model used in the preflight test simulator with the performance of the thruster when tested under various environments and with different propellants. As shown in figure 14, even though the same thruster unit was used in the tests, the performance was significantly different when different propellants were used or when operating at different ambient pressures including a vacuum.

Thrust Angle Tests

An indication of how accurately the alignment of the thrust vector can be determined using the modified impulse test stand is illustrated in figure 15. This figure is a sensitivity plot in which the angle the thrust vector makes with the normal to the reference mounting plate is plotted as a function of the pendulum displacement angle. The curve is a plot of performance data taken from the tests conducted on a development thruster to determine the misalinement of the thrust vector to its mounting base. The tests were begun with the thruster base parallel to a reference plane that is normal to the rotational axis of the pendulum. The rotational axis of the pendulum was alined with the Earth's center of gravity. The base was first rotated along the X axis (fig. 16) to determine the γ component.

Next the thruster was rotated 90° about an axis parallel to the Z axis and the test procedure repeated to determine the β component. The tests were conducted in the vacuum chamber. A theodolite was used to autocollimate through the window of the chamber with a mirror normal to the thruster base. In this manner, the angular change of the thruster base with a reference plane normal to the rotational axis of the pendulum was determined in a vacuum. A 4.5 N (1 lb) thruster was pulsed for 150 msec. In figure 15, the intersection of the curve with the abscissa (zero pendulum displacement) indicates the error angle in one of the planes orthogonal to the reference plane.

Each cluster of points represents three thruster test firings taken at the same thruster base angle. The width of the curve is defined by drawing two parallel straight lines at the extreme points of the cluster. The maximum scatter in the data indicates that the repeatability of the thrust vector is within 1 arcmin.

RECOMMENDATIONS

To improve the accuracy and facilitate the use of the test system, it would be useful to complete a force-measuring servo-loop containing the angular displacement transducer, amplifiers, and the electromagnetic force generator for measuring steady thrust. The system could be switched to this mode of operation, whenever desired, while the pendulum is in a vacuum environment.

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APPENDIX A

THEORY OF IMPULSE MEASUREMENT USING A HORIZONTAL BALLISTIC PENDULUM

Two mathematical approaches were used to show the relationship of the maximum pendulum displacement to the impulse imparted by the thruster. In one approach, the pendulum was represented by a differential equation of motion. In the other, a constant energy system was assumed. However, both derivations resulted in the same basic impulse equation for the pendulum.

DIFFERENTIAL EQUATIONS OF MOTION

The equation of motion is derived by summing torques around the center of rotation of the pendulum. It is assumed that the pendulum has no friction or damping and therefore can be characterized by an inertia (I), a spring constant (K), and a jet thrust ($F(t)$):

$$F(t)r = I\ddot{\theta} + K\theta \tag{A1}$$

If the Laplace of equation (A1) is taken, the system transfer function can be written:

$$\frac{\theta(S)}{F(S)} = \frac{r/I}{S^2 + K/I} \tag{A2}$$

If it is assumed that the jet firing period is short relative to the response characteristics of the test stand, the forcing function $F(S)$ can be approximated by the jet impulse times a unit impulse function.

$$F(S) = P = \left[\int_{o}^{t} F(t)dt\right] \tag{A3}$$

From the inverse transform of Laplace equation (A2), the time response of the pendulum can be written in terms of the jet impulse and the displacement of the pendulum:

$$\theta = \frac{Pr}{\sqrt{KI}} \sin\sqrt{K/I}\, t \tag{A4}$$

The period of the pendulum can be determined by equating the argument of the sine term in equation (A4) to 2π radians or one cycle:

$$T = 2\pi\sqrt{I/K} \tag{A5}$$

The maximum displacement angle of the pendulum occurs when the sine term in equation (A4) is a maximum of 1. The impulse for the maximum angle is:

$$P = \frac{\theta_m\sqrt{KI}}{r} \tag{A6}$$

Substituting the inertia of the pendulum, I, from equation (A5) into the impulse equation (A6), a mathematical expression for impulse can be written as a function of maximum displacement (θ_m), spring constant (K), moment arm (r), and pendulum period (T):

$$P = \frac{KT\theta_m}{2\pi r} \tag{A7}$$

CONSTANT ENERGY SYSTEM

In the constant energy approach, the system equation relating impulse to maximum displacement angle can be derived by assuming a constant energy system. The impulse applied to the system during jet firing can be expressed in terms of angular momentum and in terms of angular velocity of the pendulum:

$$P = \frac{I\dot{\theta}}{r} \tag{A8}$$

If the impulse time is short compared to the period of the pendulum, the energy of the system at termination of the thruster firing is approximately all kinetic:

$$E_K = \frac{1}{2}\, I(\dot{\theta})^2 \tag{A9}$$

At the maximum displacement angle, the kinetic energy of the pendulum is converted entirely into potential energy. The potential energy can be expressed in terms of displacement angle and spring constant:

$$E_P = \frac{K\theta_m{}^2}{2} \tag{A10}$$

The kinetic energy equation (A9) for the pendulum at thrust termination is equated to the potential energy equation of (A10). By substituting for the angular rate term, from the momentum equation of (A8), the resulting mathematical representation of the system is the same as that derived using the differential equation of motion:

$$P = \theta_m \frac{\sqrt{KI}}{r} \tag{A11}$$

APPENDIX B

SOME SOURCES OF MEASUREMENT ERROR AND THEIR EFFECTS ON THE PENDULUM

Possible sources of impulse measurement error include air damping of the pendulum, nonlinearity of the spring rate of the flexure pivots, and the thrust duration.

DAMPING

If the motion of the pendulum is represented by a second order linear differential equation, and if the ratio of the magnitudes of successive swings in the same direction is A, then it can be shown that a sufficiently accurate compensation for damping can be made by multiplying the magnitude of the first swing after an impulse by $A^{1/4}$. The accuracy of this correction is greater when A is closer to 1, and is 0.023 percent when A is 1.1.

A, as measured, was 1.02. $A^{1/4}$ was then 1.005, and the effect of damping was 0.5 percent. This damping effect was considered small enough to be ignored in the tests conducted with this system.

There is no error from damping when the calibration impulse generator is used, since the effect of damping when calibrating from a known impulse is the same as when measuring an impulse.

DURATION OF THE PULSE

The equations for measuring impulse with a ballistic pendulum are derived with the assumption that impulse from the thruster imparts momentum to the pendulum when the pendulum is at rest. Actually, the thrust from the thruster continues for some time after the pendulum starts to move.

A realistic worse case would be with a rectangular pulse where the equation of motion is

$$I\ddot{\theta} + K\theta = \tau \quad \text{for} \quad 0 < t < t_O \tag{B1}$$

where I is the moment of inertia, K is the rotary spring constant, θ is the rotary displacement, τ is the torque applied during the pulse, and t_O is the pulse duration.

After time t_O, the equation of motion is

$$I\ddot{\theta} + K\theta = 0 \tag{B2}$$

The solution of equation (B1) with the initial condition $\theta(0) = \dot{\theta}(0) = 0$ is

$$\theta = \frac{\tau}{K}(1 - \cos \omega t)$$

where

$$\omega = \sqrt{\frac{K}{I}}$$

so that

$$\dot{\theta} = \frac{\omega\tau}{K} \sin \omega t$$

If t_o is substituted for t in these expressions for θ and $\dot{\theta}$ and the results are used as initial conditions at time t_o, the solution of equation (B2) is

$$\theta = \frac{\tau}{K} [\sin \omega t_o \sin \omega t + (\cos \omega t_o - 1)\cos \omega t]$$

$$= \frac{\tau}{K} [\sin \omega t_o \sin \omega t + \cos \omega t_o \cos \omega t - \cos \omega t]$$

$$= \frac{\tau}{K} [\cos(\omega t_o - \omega t) - \cos \omega t]$$

By differentiating,

$$\dot{\theta} = \frac{\tau\omega}{K} [\sin(\omega t_o - \omega t) + \sin \omega t]$$

To find the maximum excursion the derivative is set equal to zero so that:

$$\sin(\omega t - \omega t_o) = \sin \omega t$$

This is satisfied when

$$\omega t = \frac{\pi}{2} + \frac{\omega t_o}{2}$$

Then

$$\theta_m = \frac{\tau}{K} \left[\cos\left(\frac{\pi}{2} - \frac{\omega t_o}{2}\right) - \cos\left(\frac{\pi}{2} + \frac{\omega t_o}{2}\right)\right]$$

$$= \frac{2\tau}{K} \sin \frac{\omega t_o}{2}$$

For small values of t_o,

$$\theta_m \approx \frac{\omega\tau}{K} \quad t_o = \frac{\tau t_o}{\sqrt{IK}} = \frac{rP}{\sqrt{IK}} = \theta_1$$

The equation $\theta_m = rP/\sqrt{MK}$ is used in measuring impulse with the ballistic pendulum. The difference between θ_m and θ_1 causes an error in the measurement for impulse. The ratio of the error in the impulse measurement to the impulse measurement is

$$\frac{\theta_m - \theta_1}{\theta_1} = \frac{\delta - 2 \sin \delta/2}{\delta}$$

where $\delta = \omega t_o$.

$$2 \sin \frac{\delta}{2} = \delta - \frac{2\delta^3}{2^2 3!} + \frac{2\delta^5}{2^5 5!} - \frac{2\delta^7}{2^7 7!} + \ldots$$

$$= \delta \frac{\delta^3}{4 \cdot 6} + \frac{\delta^5}{16 \cdot 120} - \frac{\delta^7}{64 \cdot 5040} + \ldots$$

Then

$$\frac{\theta_1 - \theta max}{\theta_1} = \frac{\delta^2}{4 \cdot 6} + \frac{\delta^4}{16 \cdot 120} - \frac{\delta^7}{64 \cdot 5040} + \ldots$$

$$\delta = \omega t_o = 2\pi F t_o = 2\pi \frac{t_o}{T}$$

where t_o/T is the ratio of the impulse period to the period of the pendulum, and

$$\frac{\theta_1 - \theta max}{\theta_1} = \frac{\pi^2 (t_o/T)^2}{6} + \frac{\pi^4 (t_o/T)^4}{120} - \frac{\pi^6 (t/T)^6}{5040} + \ldots$$

Using the above, some typical values of $\theta_1 - \theta max/\theta_1$ for different ratios of pulse duration to period of oscillation of the pendulum are:

$\frac{t_o}{T}$	0.05	0.07	0.1	0.15	0.2	0.25
$\frac{\theta_1 - \theta max}{\theta_1}$	0.004	0.008	0.016	0.037	0.066	0.10

The longest impulse considered in designing the pendulum was 1 sec. The period was always greater than 10 so that $t_o/T < 0.1$. Thus, the error from a finite pulse length of 1 sec is less than 1.6 percent.

APPENDIX C

CALIBRATION METHODS

BALL DROP

A ball was dropped onto a curved ramp on the pendulum near the thruster position. The ramp was constructed so that the ball left the ramp in a horizontal direction normal to the pendulum arm. The ball then followed a trajectory in which it fell a measured vertical distance to the baseplate of the test stand. The ball fell on the back side of a piece of carbon paper, marking a point on a paper below the carbon, indicating the horizontal distance the ball traveled after leaving the pendulum. The time of the drop was computed from the vertical distance, and the horizontal distance was measured. From these figures, the horizontal velocity was computed and multiplied by the mass of the ball to obtain momentum. Calculations were made that show that the ball does not roll down over the edge of the ramp but leaves horizontally. Thirteen drops were made, eight using a 20 mm (3/4 in.) diameter steel ball and five using a 10 mm (3/8 in.) diameter steel ball. Impulses varied from 2.2×10^{-3} N-sec (5×10^{-4} lb-sec) to 1.9×10^{-2} N-sec (4.2×10^{-3} lb-sec). The manner of releasing the ball (the 20 mm ball by hand and the 10 mm from tweezers) may account for most of the difference between the impulse measurements obtained. This method was discarded in favor of the other methods studied because considerable refinement would be required to make it usable.

TIMING THE PENDULUM WITH DIFFERENT MOMENTS OF INERTIA

To calibrate by timing the pendulum with different moments of inertia:

(1) The period of the pendulum is timed with a 0.9 kg (2 lb) weight 0.3 m (12 in.) from the axis and a 1.8 kg (4 lb) weight 0.15 m (6 in.) from the axis on the other side. (The weights have threaded rods that screw into threaded holes in the horizontal moving plate.) A stop watch is used to time 10 cycles. The oscilloscope trace is used to determine when the pendulum passes through its rest position.

(2) The weights are then moved 0.2 m (8 in.) and 0.1 m (4 in.) from the axis, respectively, and the period of the pendulum timed again.

Impulse is measured with the calibration weights on the pendulum to avoid any possible change of flexure characteristics with change in the supported load. The weights are in the inboard position during impulse measurement because the pendulum will be more sensitive. In a test with a thruster and related components that weighed about 6.4 kg (14 lb), the period of oscillation with the weights outboard was about 15 sec and with the weights inboard about 13 sec.

In practice, the spring constant K and moment of inertia are not usually computed; instead, pendulum displacement versus impulse is computed directly. In the following, T_1 is the period with the weights in the inboard position and the T_2 the period with the weights in the outboard position.

The corresponding moments of inertia added by the weights are I_1 and I_2; I_0 is the moment of inertia without the added weights. From the equation for the period of a pendulum, we obtain

$$T_1 = 2\pi \sqrt{\frac{I_0 + I_1}{K}} \quad \text{and} \quad T_2 = 2\pi \sqrt{\frac{I_0 + I_2}{K}} \tag{C1}$$

Solving for K gives

$$K = \frac{4\pi^2 (I_1 - I_2)}{T_1{}^2 - T_2{}^2}$$

From equation (C1),

$$\sqrt{I_0 + I_1} = \frac{T_1 \sqrt{K}}{2\pi}$$

It is shown in appendix A that

$$P = \theta_m \frac{\sqrt{KI}}{r}$$

so that when $I = I_0 + I_1$, $P = \theta_m [\sqrt{K(I_0 + I_1)}]/r$ where P is impulse, θ_m the maximum pendulum excursion, and r the distance from the axis of rotation to the thruster. Substituting from the above expression for K and $I_0 + I_1$ gives:

$$P = \frac{2\pi T_1 (I_1 - I_2)}{r(T_1{}^2 - T_2{}^2)} \theta_m$$

where θ_m is in radians. The moment of inertia added by each weight can be considered to be its mass times the square of the distance to the pendulum axis since the moment of inertia of each weight about its own center of gravity is subtracted in obtaining this formula for P.

The term C is the counts change in the DVM display for each degree of pendulum rotation. To determine C, the reset button is pressed and the pendulum moved through a given angle (usually about 15°). For this purpose, there are marks every 5° along the outer edge of the damping plate, and a stationary plate with index marks is attached to the baseplate. The formula for impulse is

$$P = \frac{\pi^2 T_1 (I_1 - I_2) \text{counts}}{r(T_1{}^2 - T_2{}^2) 90C} \tag{C2}$$

TIMING THE PENDULUM AND MEASURING THE SPRING CONSTANT

For this method, the calibration weights are not used. The period of the pendulum is timed. Then the load cell used to measure static force is used with the degree marks on the damping plate to determine the spring constant of the pendulum. The period of the pendulum is

$$T = 2\pi \sqrt{\frac{I}{K}}$$

Impulse as a function of a ballistic pendulum rotation is

$$P = \frac{\theta m \sqrt{KI}}{r}$$

Substituting and solving for P, we obtain

$$P = \frac{\theta m KT}{2\pi r}$$

CALIBRATION IMPULSE GENERATOR

For this method, the load cell is used to measure the torque exerted by the electromagnetic driver for a given drive coil current setting. The digital time delay generator is then set for the time to obtain the desired impulse. When a calibration impulse is desired, the reset button is pressed and the button on the time delay generator is pressed. The maximum excursion is displayed on the DVM. This is the only calibration method that permits use of the eddy current damper while impulse is being measured. Use of the damper reduces test time because the pendulum oscillations are damped rapidly. The damping magnet current is set the same for both calibration pulses and impulse measurement. A setting slightly less than critical damping is recommended.

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1. Den Hartog, J. P.: Mechanics. Dover Pub., 1961, pp. 246-247.

2. Pfeffer, A. H.: Performance of a Cold-Gas Jet at Short Pulse Lengths. European Space Research Technology Centre Technical Note ESRO TN-95, Aug. 1969.

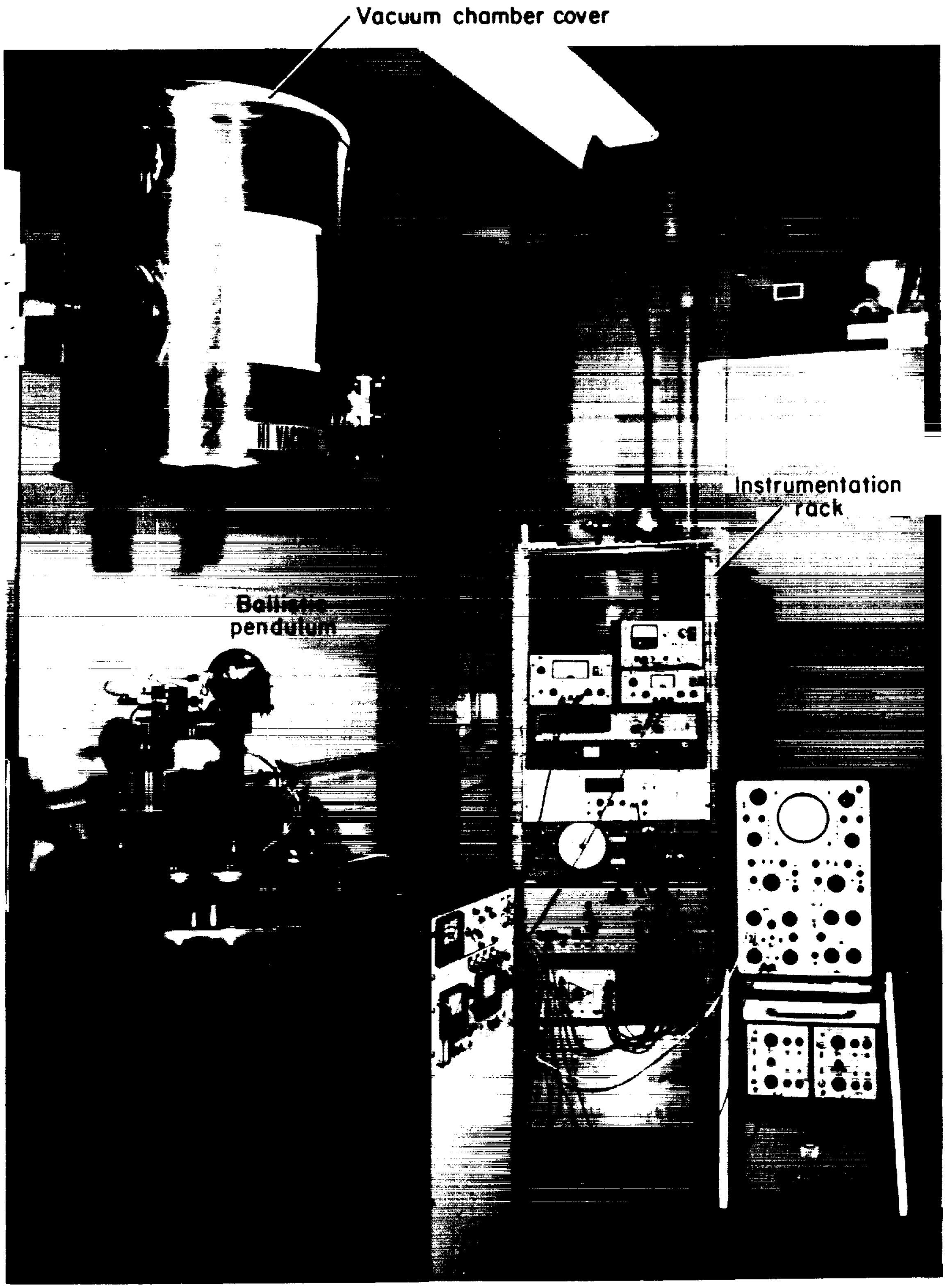


Figure 1.– Overall view of test stand and instrumentation rack.

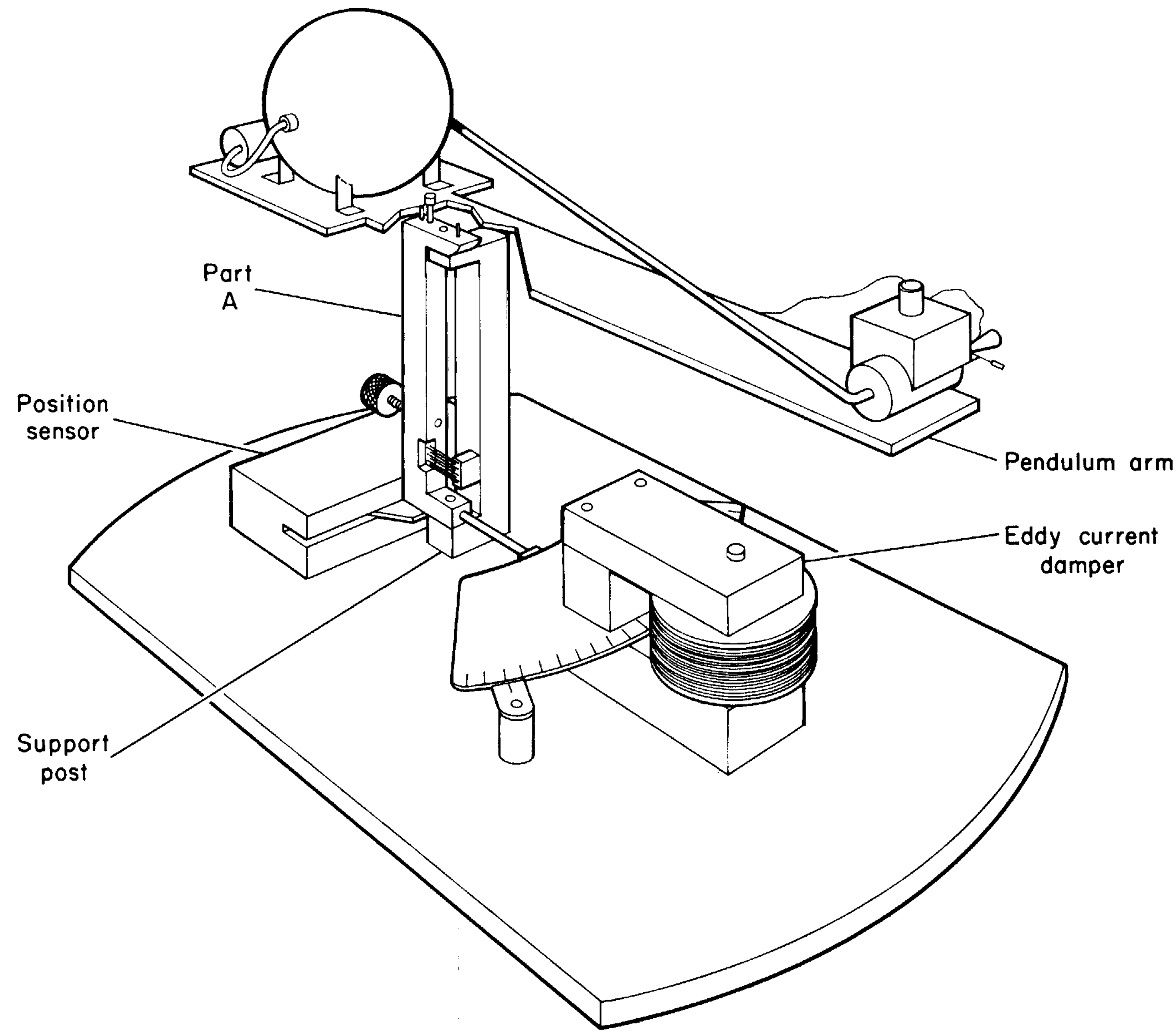


Figure 2.– Sketch of test stand.

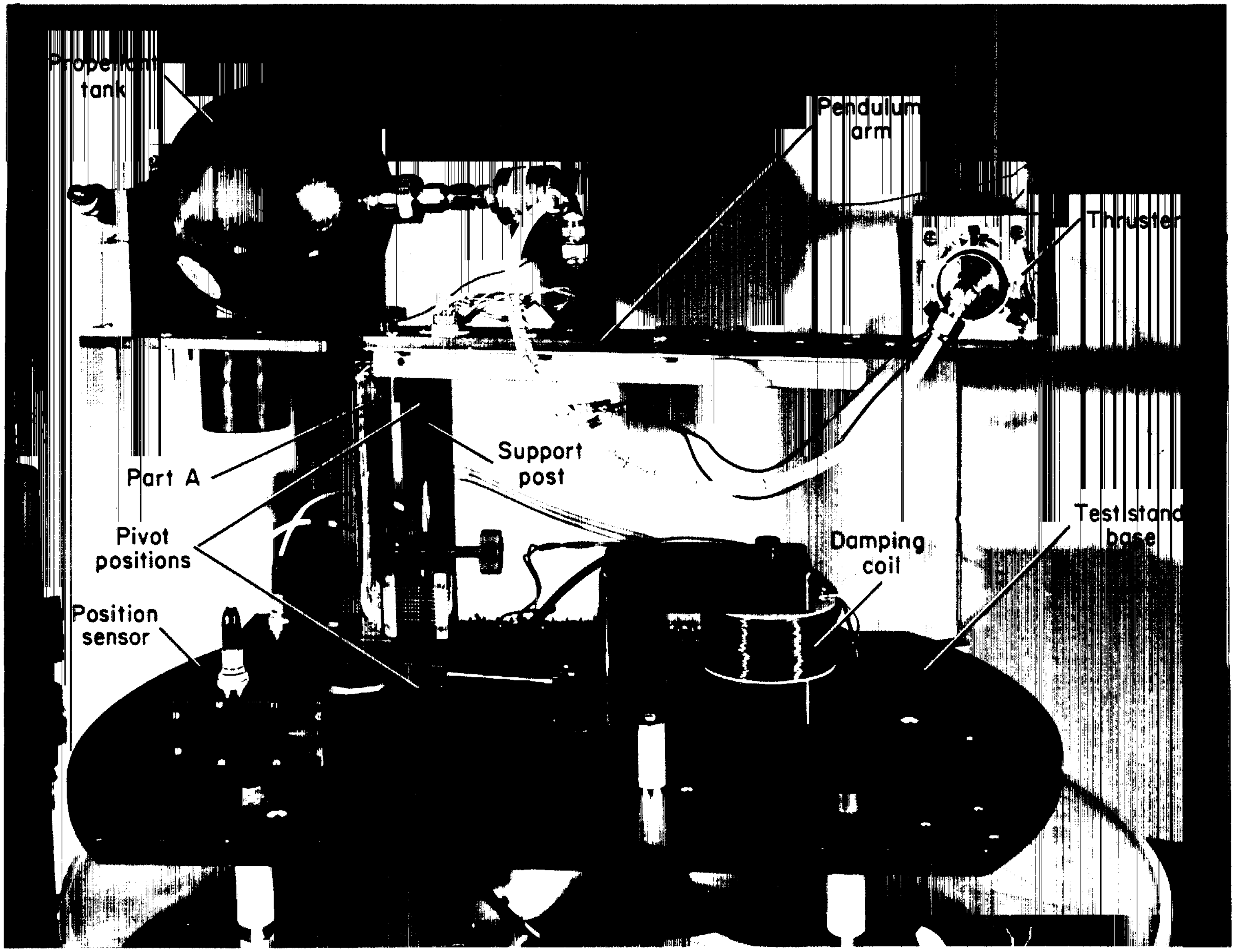


Figure 3.– Test stand set-up for impulse measurement.

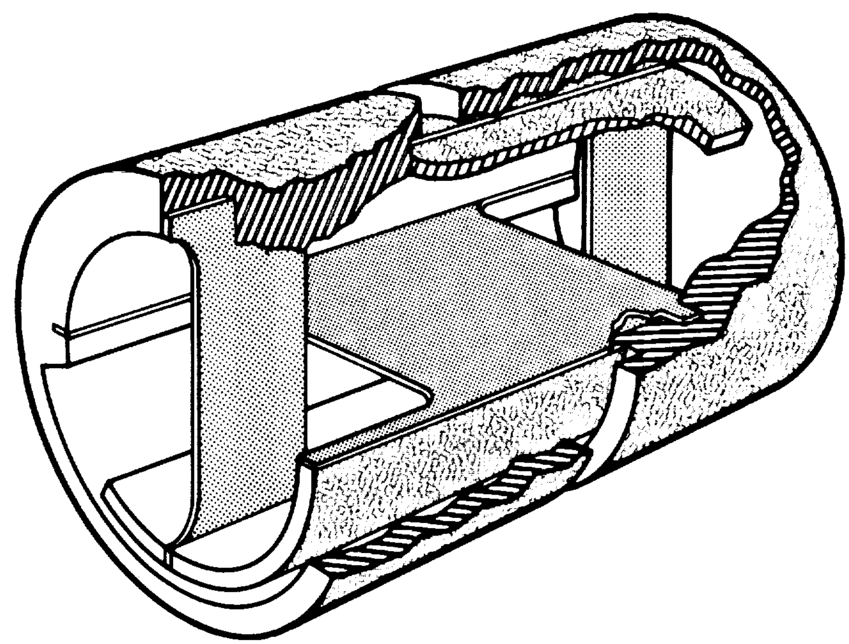


Figure 4.– Cut-away view of cross-flexure pivot.

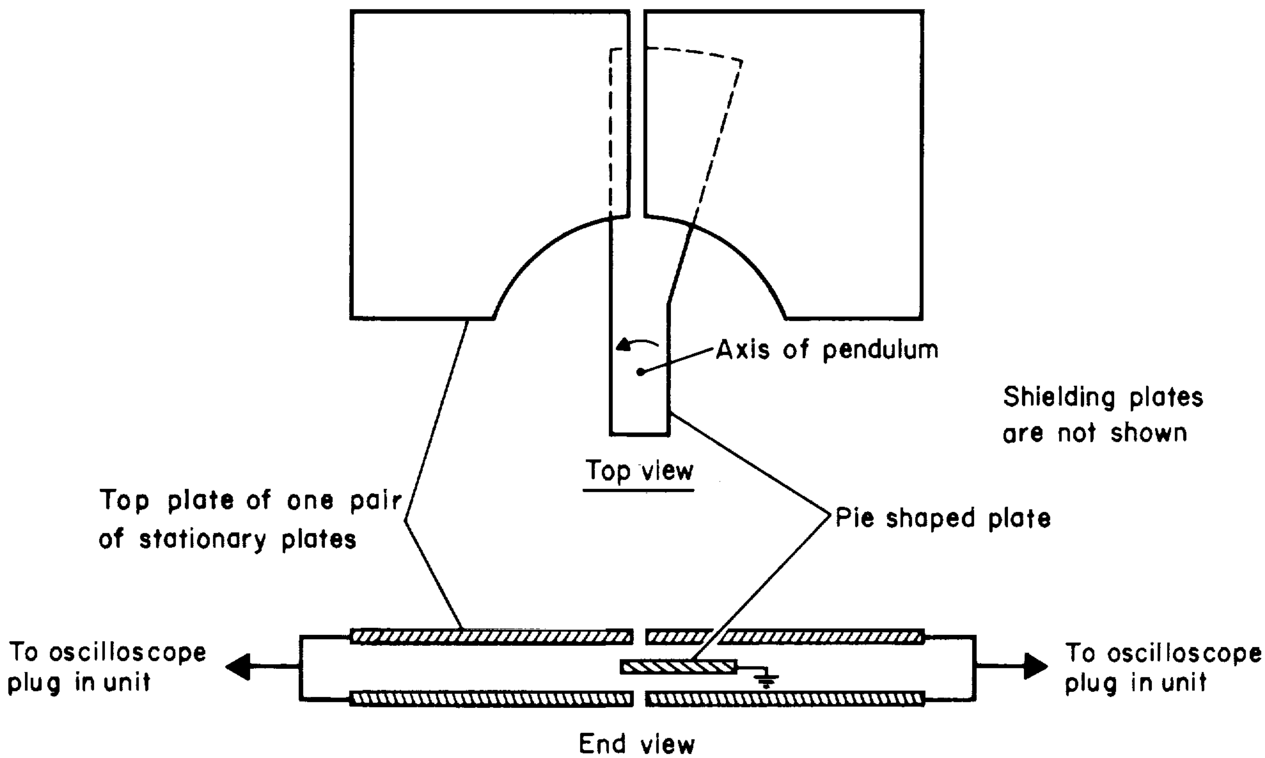
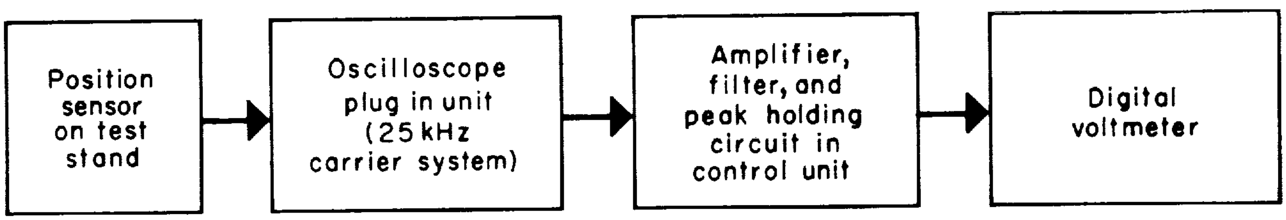
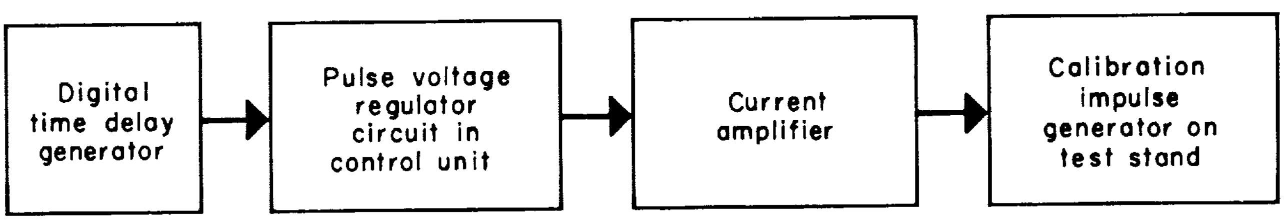


Figure 5.– Diagram of capacitance position transducer.



Impulse measuring system



Impulse generating system

Figure 6.– Block diagram of impluse measuring instrumentation.

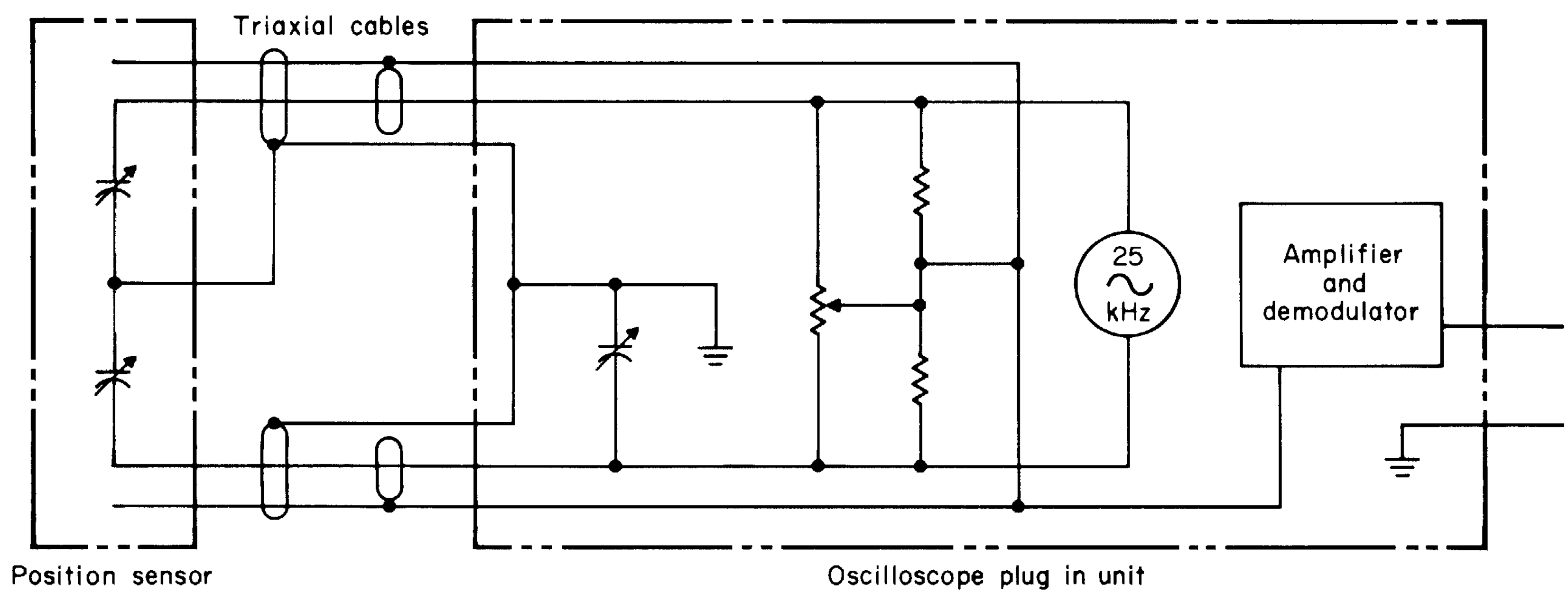


Figure 7.– Cable shielding for position transducer.

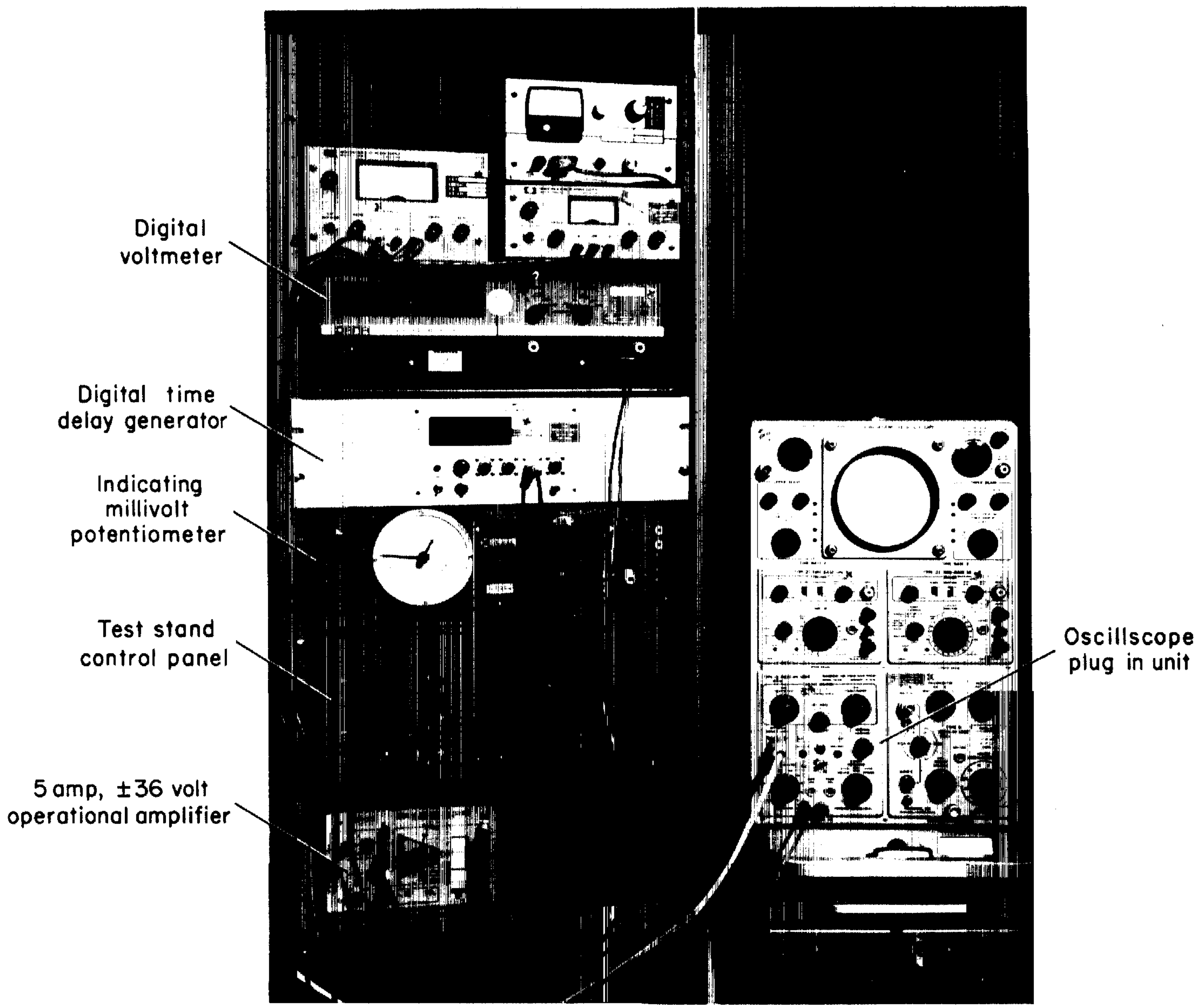


Figure 8.– Control panel.

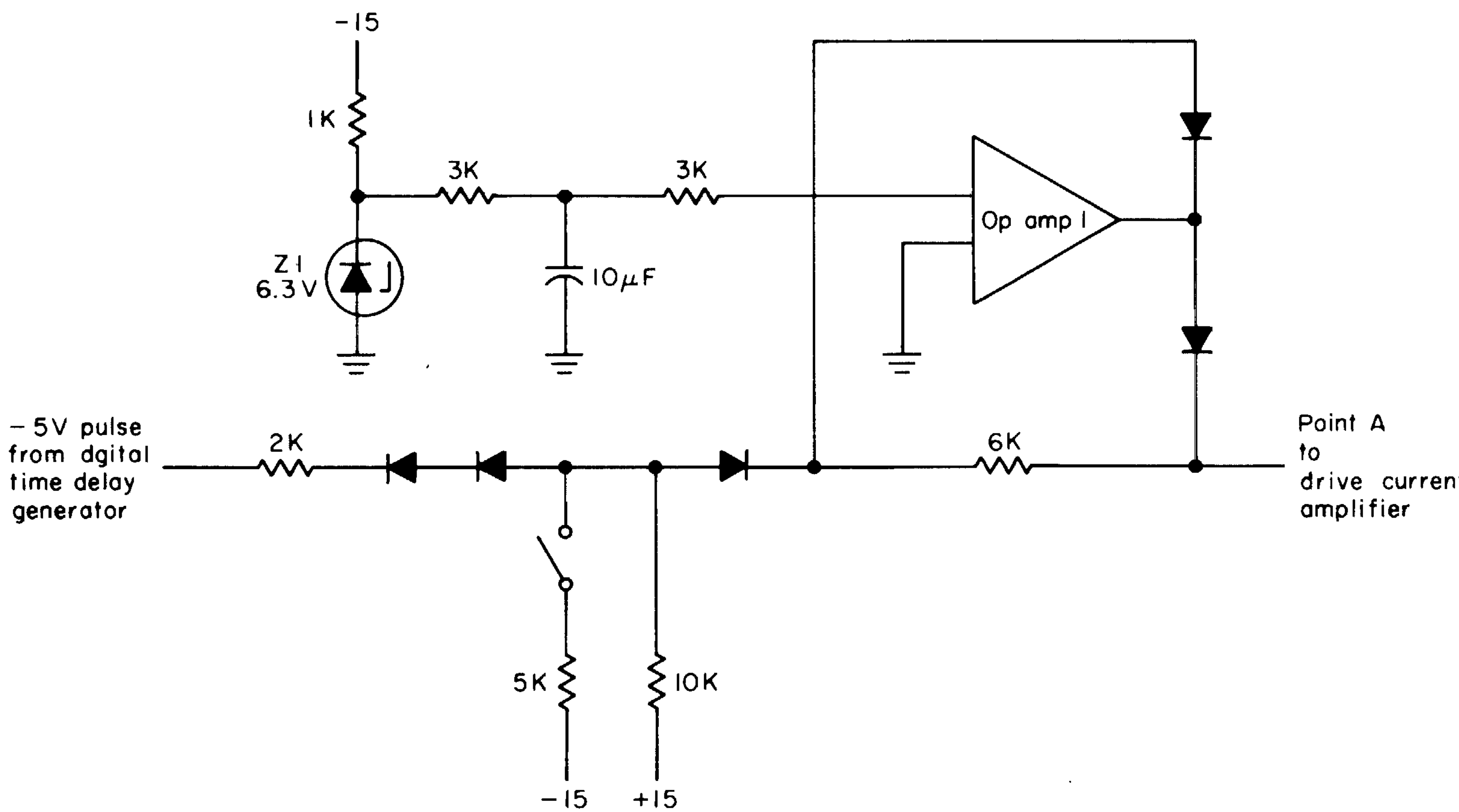


Figure 9.– Pulse voltage regulator circuit.

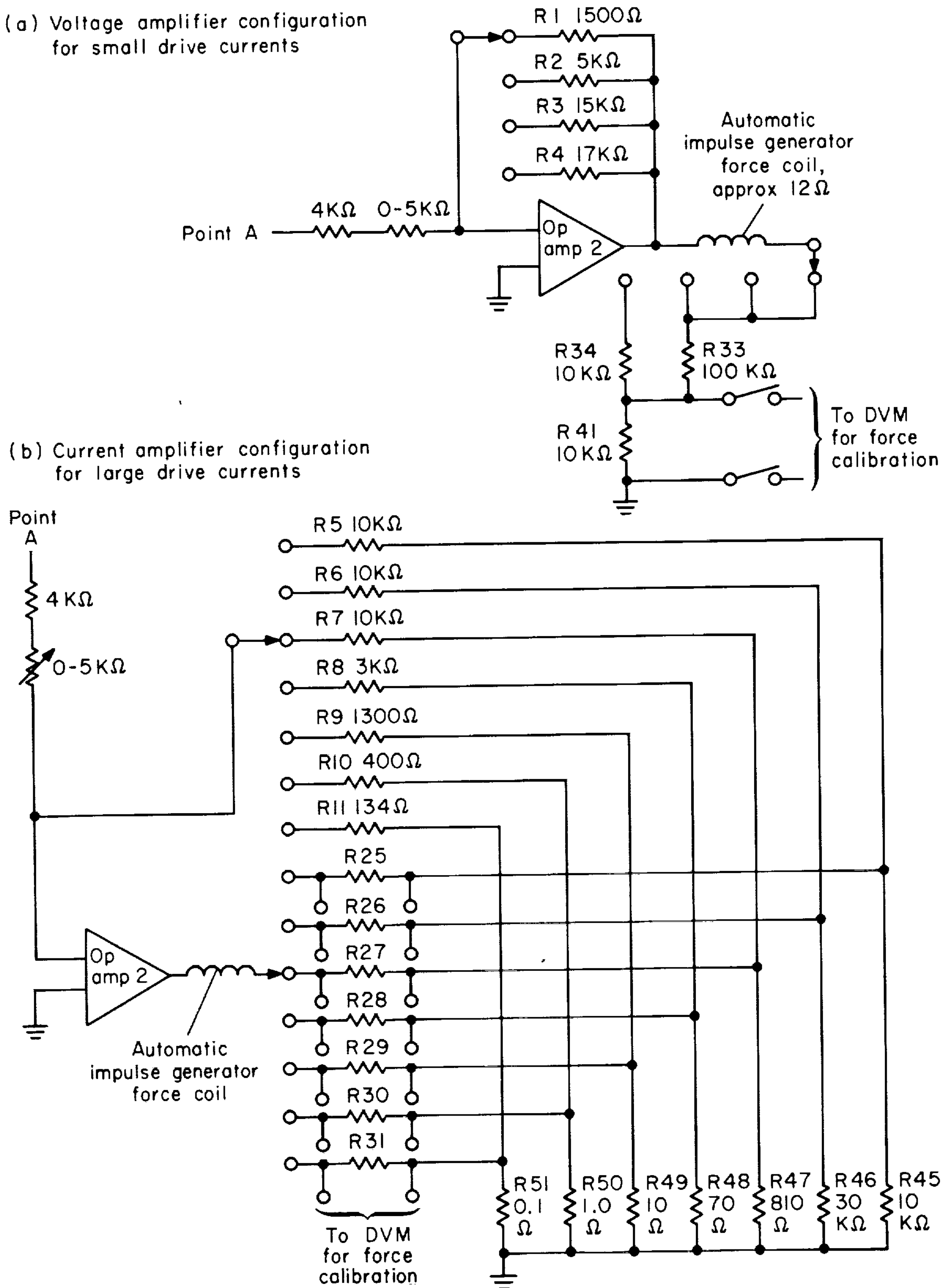


Figure 10.– Amplifier configurations for driving calibration impulse generator.

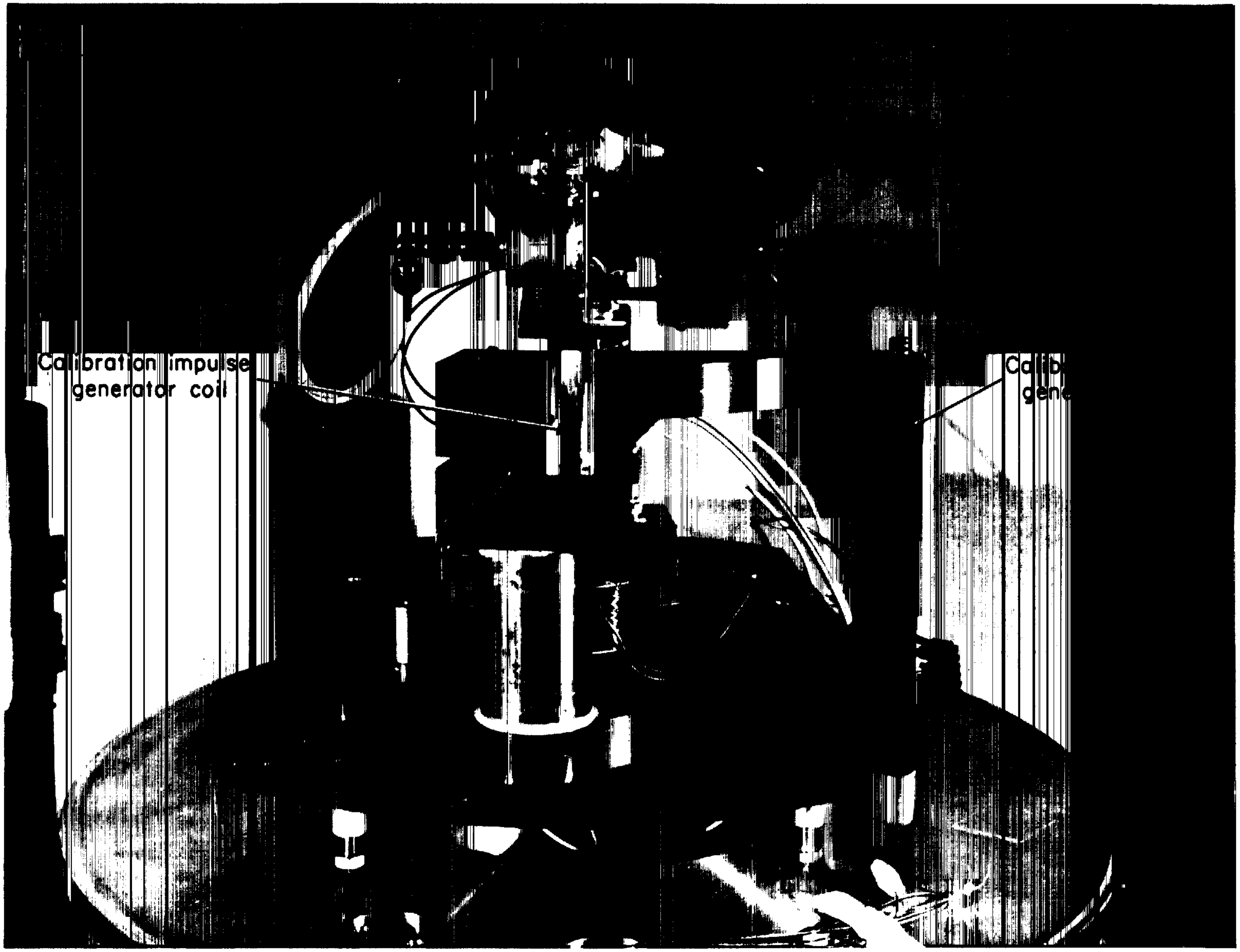


Figure 11.– Test stand with calibration impulse generator.

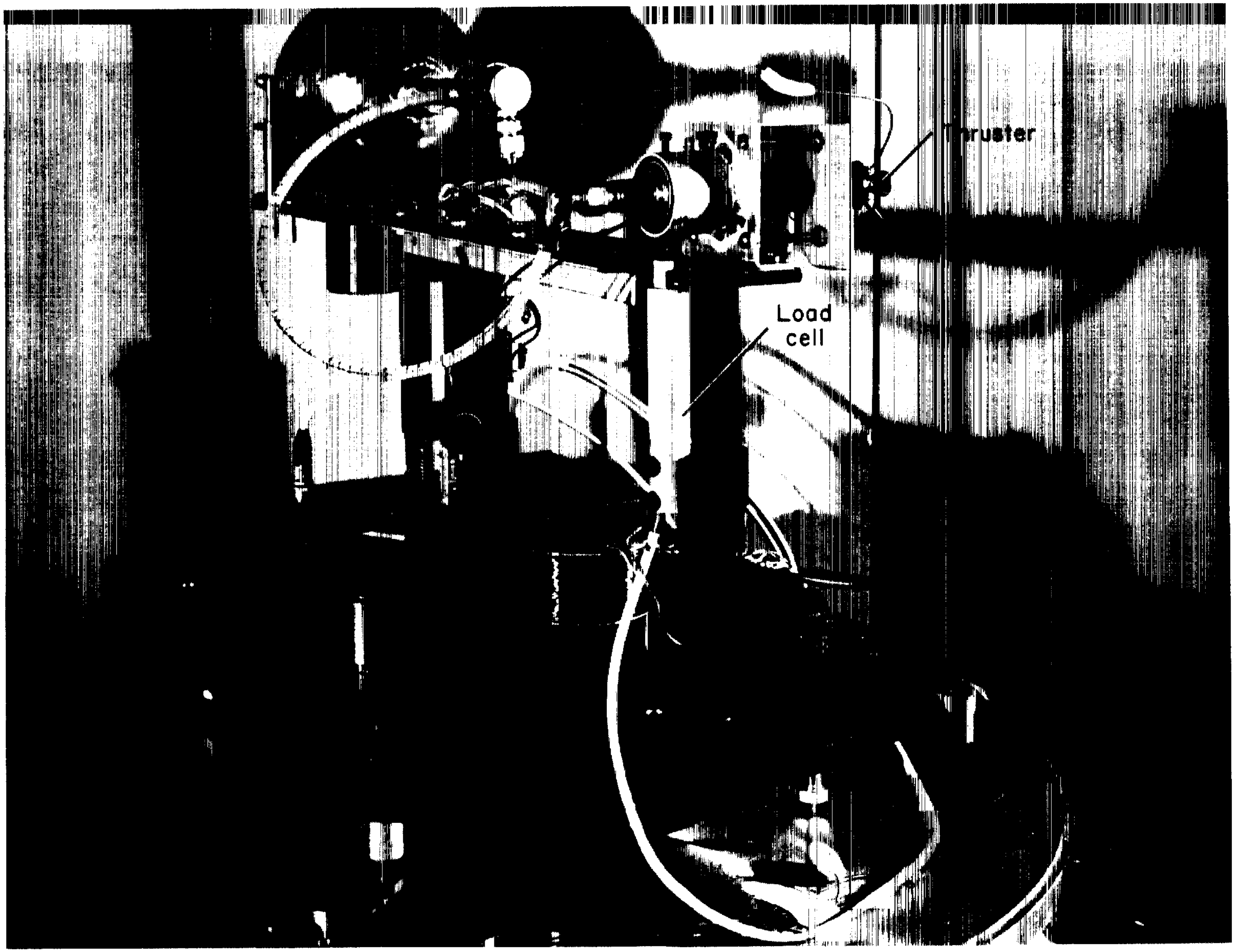


Figure 12.– Test stand setup for static thrust measurement.

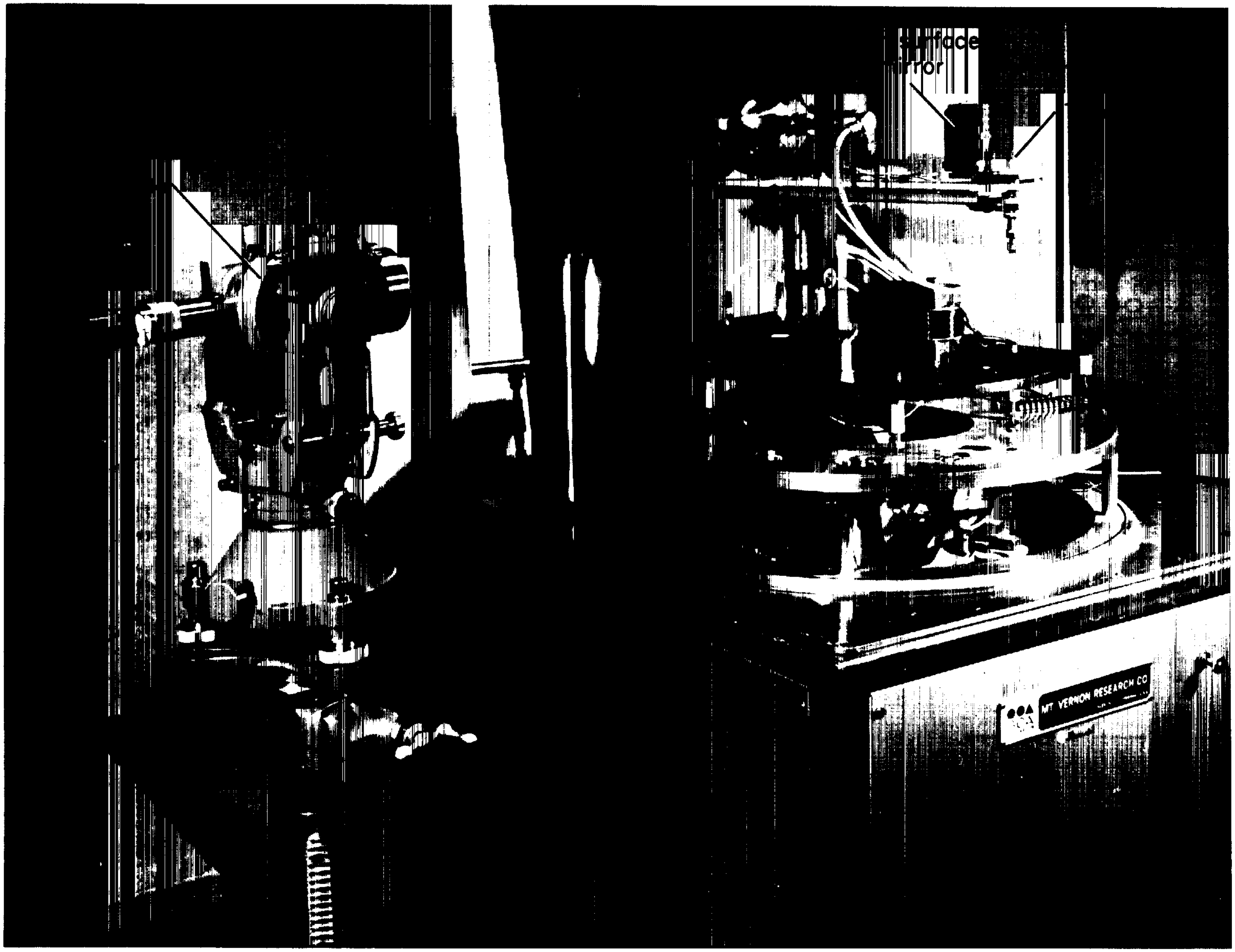


Figure 13.– Test stand setup for thrust angle measurement.

Symbol	Propellant gas	Test environment	Supply pressure-absolute
□	Freon	Vacuum	3.24×10^5 N/m^2 (47 psi)
△	Analog simulation of thruster		3.24×10^5 N/m^2 (47 psi)
○	Air	Vacuum	3.24×10^5 N/m^2 (47 psi)
◇	Air	Ambient	4.27×10^5 N/m^2 (62 psi)

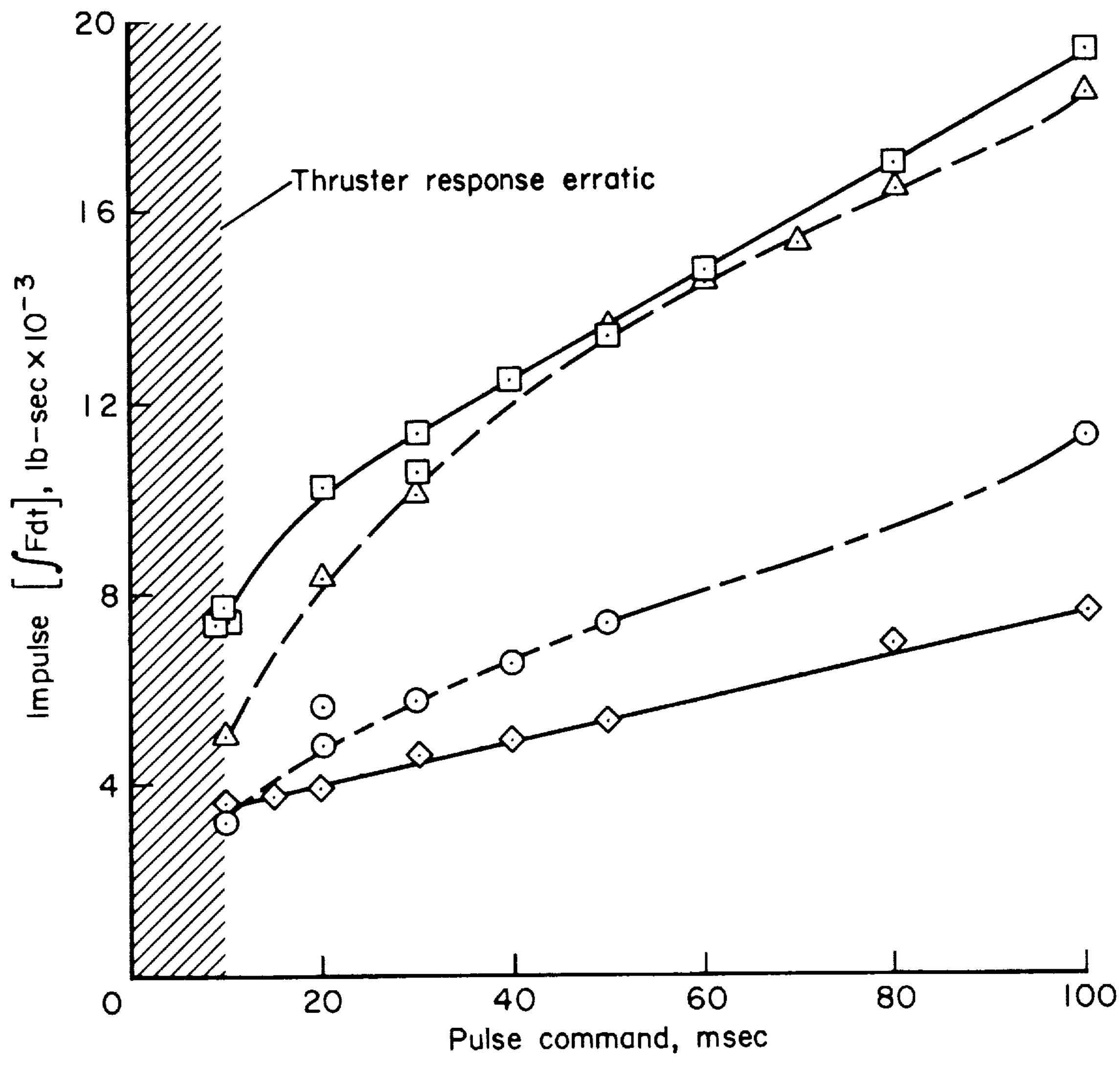


Figure 14.– Typical performance plot for SPARCS thruster.

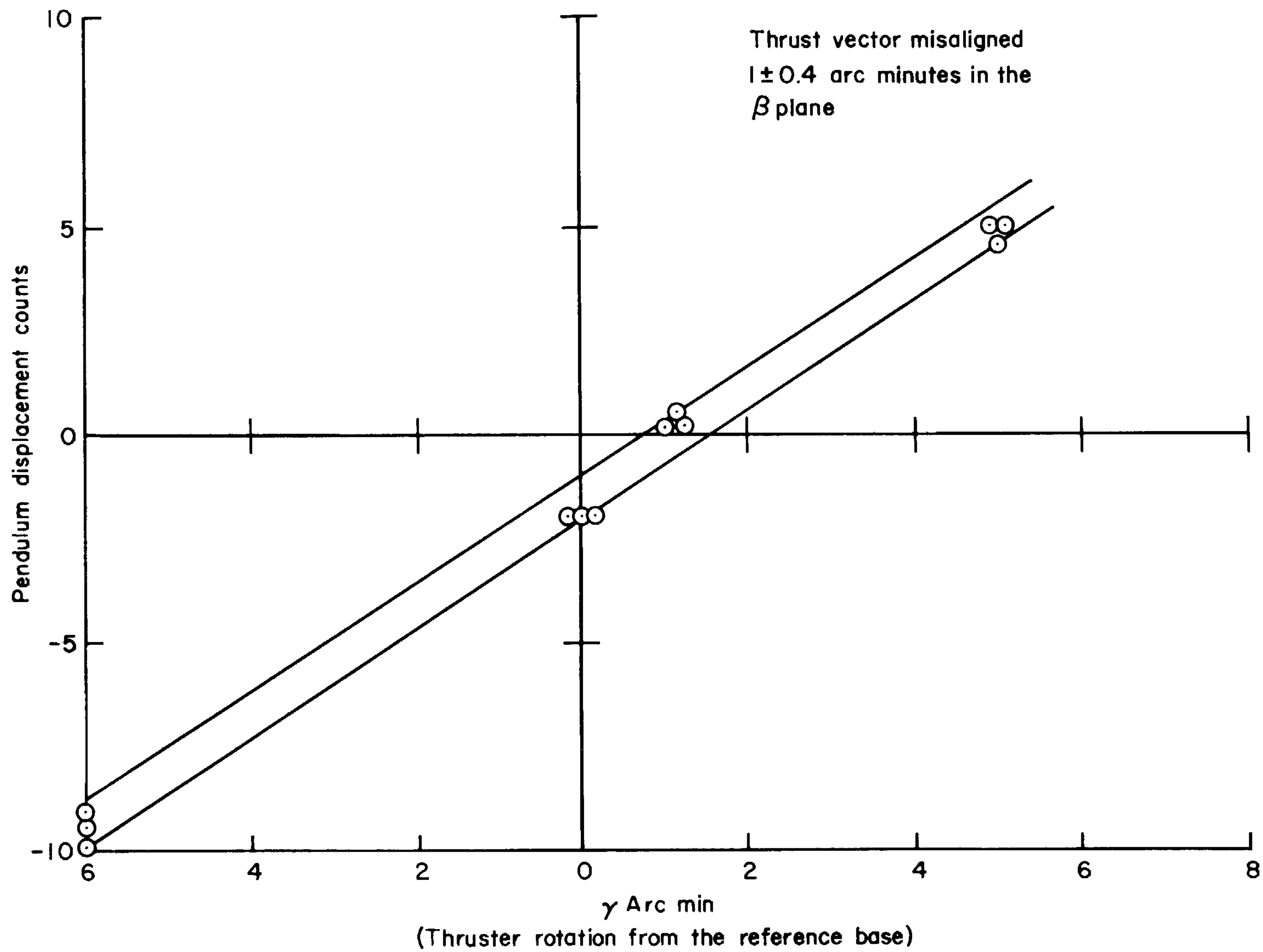


Figure 15.– Plot of thrust vector measurement.

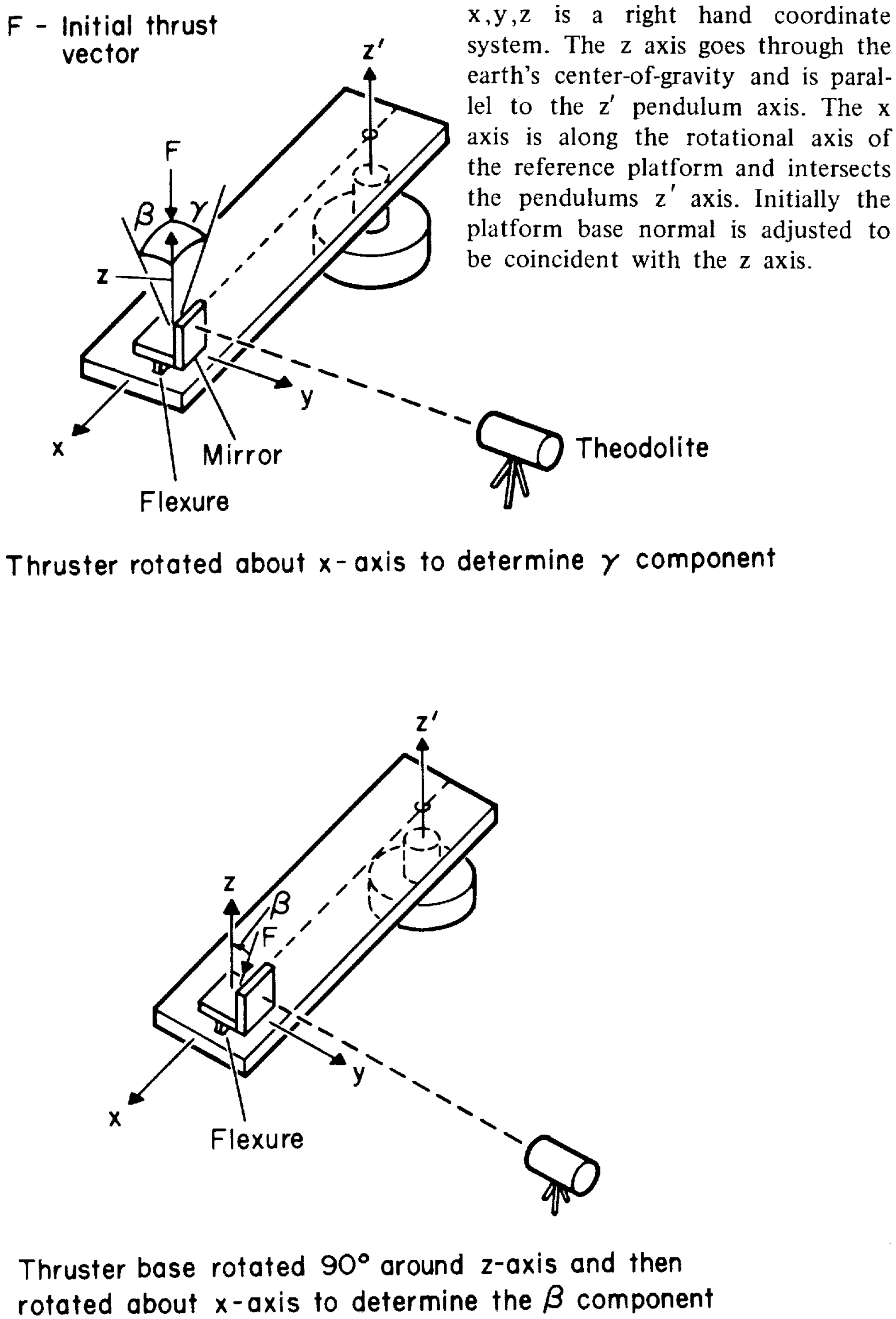


Figure 16.– Geometry applicable to thrust vector measurements.

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