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SEVENTH PROGRESS REPORT

on

CALIBRATION AND EVALUATION OF SKYLAB ALTIMETRY FOR GEODETIC DETERMINATION OF THE GEOID (Contract NAS9-13276), EPN 400), September 1 to September 30, 1973

to

NASA Johnson Space Center
Principal Investigation Management Office
Houston, Texas 77058

from

BATTELLE
Columbus Laboratories

October 12, 1973

A. G. Mourad - Principal Investigator, D. M. Fubara - Co-Investigator
A. H. Byrns, Code TF6 - NASA/JSC Technical Monitor

BATTELLE
Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

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PROGRESS

The status of work initiated in the previous report period is as follows:

(1) Analysis of geoidal ground truth has progressed to the last stage which requires the final values for the orbit ephemeris for completion.

(2) The development of the computer program for sequential least squares update solution of data from repeated tracks and subsequent Skylab missions has been completed. The next stage is to validate it with simulated data.

(3) Extensive investigation into error analysis and performance criteria, which occupied the major effort in this period, was completed. This is discussed in details under "Data Processing and Results".

(4) Print outs of some Skylab altimeter data for EREP passes were received on the last working day of the month, and briefly reviewed. These data are those captioned S072-1, S072-6, S072-7, S072-8, S072-9 as set forth in Reference 1. One set of S072-2 data for EREP pass #9 only was received. The orbit ephemeris, specifically, GMT correlated X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$ coordinates associated with the altimeter ranges have not been received. Further details of data and documents received are in Appendix B.

DATA PROCESSING AND RESULTS

Performance Criteria

Appendix A describes the mathematical developments for the analytical data processing procedure to be used in the real data processing.

In the geodetic calibration and evaluation of Skylab altimetry for geoid determination, the basic inputs are (1) the altimeter ranges, (2) associated orbit ephemeris, and (3) geoidal information used as geodetic control or "bench mark". The outputs are (1) the altimeter residual bias or calibration constant required to give a correct geoidal scale and (2) the geoidal profile deduced from the altimetry. In general, whether it is from a spacecraft or an aircraft, altimetry is a geometric "leveling" operation. Therefore, the limiting factors to the absolute accuracy of the end result are the accuracies of (a) the "bench marks", and the altimeter ranges, and/or (b) the orbit determination and the altimeter ranges. However, in the latter case, a few geodetic (geodial) controls or geodetic bench marks must be used to permit the recovery of any residual altimeter bias(es) or calibration constant(s), check for instrumental drifts and systematic errors in orbit determination, in order to prevent these factors from giving a false scale to the deduced geoid.

Methodology

The investigation was conducted with simulated data. The altimeter ranges were simulated from actual SKYBET ephemeris. Random errors, representing measurement errors, were generated and added to the simulated ranges. A constant bias was then added to each range. In different phases of the investigation, the magnitude of the bias was arbitrarily varied betwwen 15 m. and 234 m. The data were then processed according to the analytical developments described in Appendix A to see how well the added bias can be recovered from the generalized least squares solution used. The recovered bias represents the calibration constant in real data, if any exists. The weighting function estimate for the bias recovery was also varied for each added bias to determine how errors in estimating the weights would affect the accuracy of recovery of calibration constant. The weight for the calibration constant was made inversely proportional to the square of an apriori estimate, PB, of the bias. The magnitude of this apriori estimate was varied between the true value and one tenth of the true value of the actual bias introduced.

Cases C-1, 2, ...7, of Tables 1, 2 and 3 show some of the results from which conclusions were drawn with regards to the recovery of the calibration constants and geoidal heights. These results are, of course, subject to the conditions imposed by the apriori precision estimates, PA, assigned to the geodetic (geoidal) control and PC assigned to the simulated ranges, as indicated for each case. The concept for assigning weights to parameters as well as observations is developed in Section 1.2 of Appendix A. The orbital arc used in the simulation is about 10,000 Km long but only 25 uniformly distributed data points were used.

TABLE 1. PERFORMANCE CRITERIA FOR COMPUTING GEODETIC CALIBRATION CONSTANT
(All values are in meters)

Case Number	PA	PC	Aporiori Estimate of Bias	Additive Calibration Constant	
				True Value	Computed Value
C-1	±2 - ±12	±1.7	15.0	-15.0	-15.2
C-2	±2 - ±12	±1.7	201.0	-15.0	-15.3
C-3	±2 - ±12	±1.7	201.0	-234.0	-234.0
C-4	±15	±1.7	201.0	-234.0	-234.0
C-5	±15	±1.7	27.0	-234.0	-231.1
C-6	±15	±1.7	108.0	-234.0	-233.9
C-7	±15	±1.7	54.0	-234.0	-233.3

TABLE 2. CORRELATION BETWEEN RECOVERY OF CALIBRATION CONSTANT AND DEDUCED GEOIDAL HEIGHTS
(All values are in meters)

Point	Geoidal Heights							
	True Value	Case C-1	Case 2-2	Case C-3	Case C-4	Case C-5	Case C-6	Case C-7
		Analytically Deduced from Simulated Altimetry						
1	-25.0	-24.6	-24.6		-24.1	-21.2	-24.0	-23.4
2	-24.0	-23.3	-23.3		-22.6	-19.7	-22.5	-21.9
3	-29.0	-27.7	-27-8		-27.4	-24.5	-27.3	-26.7
4	-30.0	-30.3	-30.4		-31.5	-28.6	-31-4	-30.8
5	-30.0	-30.3	-30.4		-30.1	-27.2	-30.0	-29.4
6	-34.5	-33.4	-33.5		-33.2	-30.3	-33.1	-32.5
7	-44.0	-45.7	-45.8	As in	-45.6	-42.7	-45.4	-44.9
8	-46.2	-45.3	-45.4	Cases	-45.1	-42.3	-45.0	-44.4
9	-52.5	-51.4	-51.5	C-1 and	-51.2	-48.4	-51.1	-50.5
10	-52.0	-53.4	-53.5	C-2 to	-53.3	-50.4	-53.2	-52.6
11	-47.0	-46.1	-46.2	Within	-45.9	-43.1	-45.8	-45.2
12	-46.0	-44.8	-44.9	±0.1	-44.6	-41.8	-44.5	-43.9
13	-45.0	-46.3	-46.4		-46.2	-43.3	-46.1	-45.5
14	-36.0	-35.2	-35.3		-35.0	-32.1	-34.9	-34.3
15	-32.0	-33.5	-33.6		-33.4	-30.5	-33.3	-32.7
16	-28.0	-29.3	-29.6		-29.2	-26.3	-29.1	-28.5
17	-25.0	-23.5	-23.6		-23.3	-20.5	-23.2	-22.6
18	-24.8	-26.1	-26.2		-26.0	-23.1	-25.9	-25.3
19	-22.5	-23.9	-24.0		-23.8	-20.9	-23.6	-23.1
20	-20.5	-19.5	-19.6		-19.3	-16.4	-19.2	-18.6
21	-18.0	-16.9	-17.0		-16.7	-13.8	-16.6	-16.0
22	-15.5	-17.2	-17.3		-17.1	-14.2	-16.9	-16.4
23	-13.5	-15.0	-15.1		-14.9	-12.0	-14.8	-14.2
24	-11.0	-9.9	-10.0		-9.7	-6.8	-9.6	-9.0
25	-8.5	-9.4	-9.5		-9.9	-7.0	-9.8	-9.2
Average Standard Error		±0.58	+0.41	+0.63	±0.87	±6.07	±1.54	±3.04

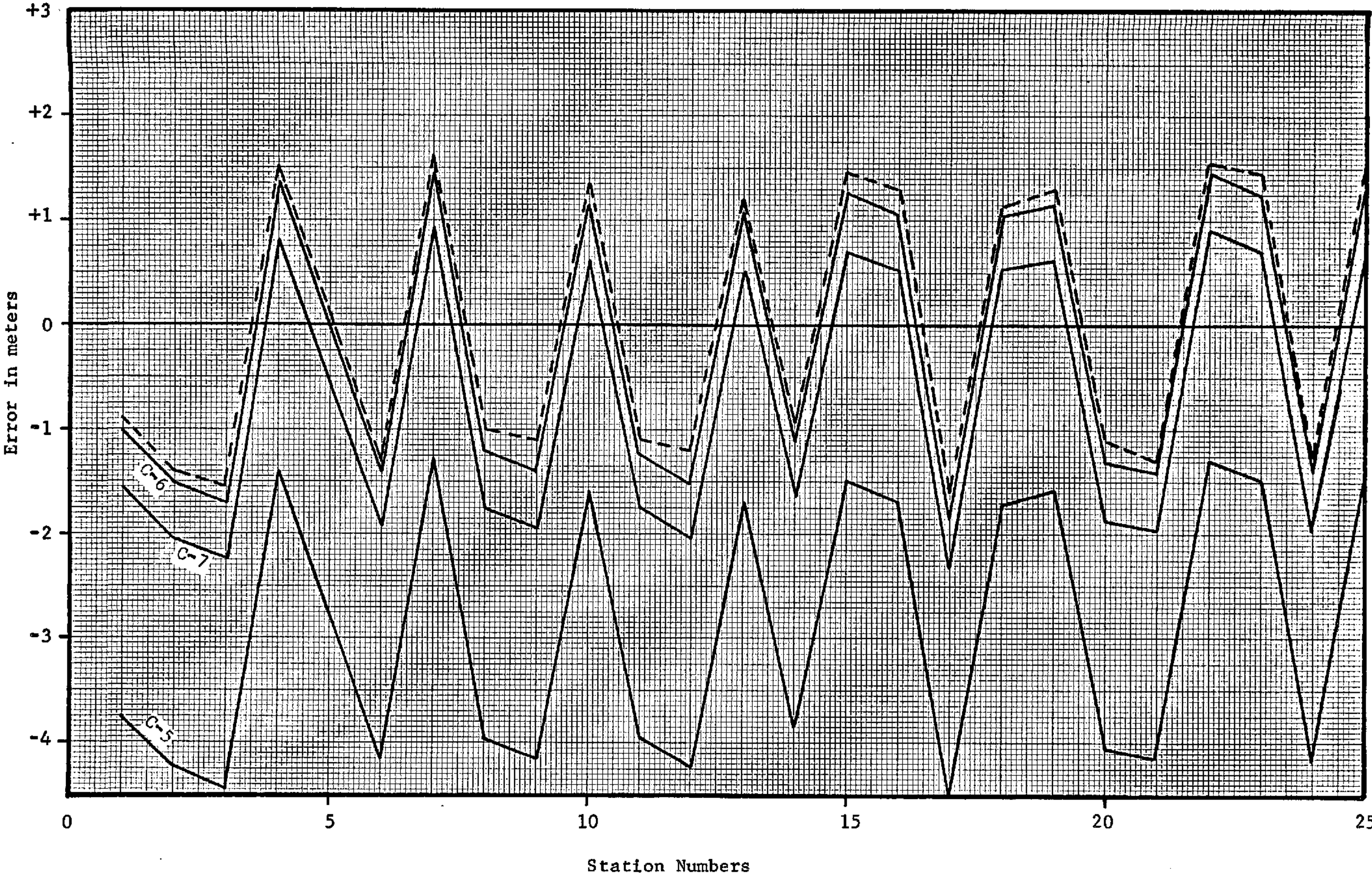

FIGURE 1. CORRELATION BETWEEN RECOVERY OF ALTIMETER CALIBRATION FACTOR AND ERRORS IN DEDUCED GEOID. (CF. TABLES 1 AND 2; CASES C-1 THROUGH C-4 NEARLY IDENTICAL TO C-6 TO WITHIN ±0.5m). THE DASHED LINES REPRESENT THE TRUE RANDOM ERRORS INTRODUCED INTO THE SIMULATED RANGES.

CONCLUSIONS

Based on this simulated data investigation, the main physical factors that influence the performance criteria or the ability to deduce accurate geoid heights and the geodetic calibration constant are (a) the precision of the radar ranges; (b) the use of some reliable geoidal controls whose errors are smaller than the total altimetry system bias; and (c) the apriori estimate (PB) of the bias, should satisfy the condition that 60% Actual Bias $\leq$ PB $\leq \infty$ to permit reliable recovery of the geodetic calibration constant. In other words the weight which equals $1/PB^2$ has to satisfy the inequality $\left(\frac{5}{3}\text{ of Actual Bias}\right)^2 >$ Weight > 0. This is in keeping with the developments in Section 1.2 of Appendix A, and gives the limits of the inequality $\infty > P > 0$ as is applicable to satellite altimetry data processing for geoid computation. Specifically,

(1) Even if all other errors were eliminated, the errors in the computed geoidal heights approximately equal the residual or random measurement errors in the altimeter ranges. (See Figure 1 and Table 2.) That is, the better the precision of the altimeter, the better the precision of the computed geoid and its subsequent applications in oceanography, geophysics, earth gravity model improvement, etc.

(2) Any unmodelled or improperly modelled or inaccurate recovery of system bias or calibration constant results into a computed geoid with scale error. The mean value of the errors in the computed geoid heights is equal to the error in the computed calibration constant as shown in Table 1 and Figure 1. In satellite altimetry, the total system's bias arise from the altimeter, orbit determination, correction for sea state where the magnitude is significant, data processing procedures, and geoidal controls used. These error sources are not easily separable unless extremely precise orbit determination is used at the altimeter calibration test areas.

(3) As should be expected, the computed parameters from the least squares adjustment and their standard errors are stochastically independent. The standard errors are more sensitive to the errors in weight estimates than to errors in the derived parameters. Consequently, statistical tests on variance factor ratios and their confidence intervals as developed in Reference 2 will have to be used on the real S-193 altimeter data.

This performance criteria investigation sets the guide lines for the conditions necessary for achieving the objectives of the project during data processing, and the types of results to be expected under various data conditions.

PROBLEMS

The long awaited first look data from SL-2 have begun to arrive in paper print out forms. This is the least desirable form for economical and speedy data processing on electronic computers. One of our most inportant data requirements, the Skylab ephemeris in geocentric X, Y, Z coordinates, has not yet been received. The geodetic latitude, longitude and altitude received could have been a useful substitute except that, as was pointed out in the last progress report, the current NASA/JSC computational accuracy for the altimeter geodetic altitude is inadequate for our particular project. Furthermore, even in the paper print out form, data S072-2 (See Reference 1) which have information on altimeter geodetic altitude have been received for one EREP pass only. On the data sheets received, there appears to be a mix up in describing the groundtrack by EREP pass or orbit numbers which vary on the different maps received.

RECOMMENDATIONS

(1) As stipulated in the proposal, our main data requirements which should be met as soon as possible, include:

(a) computer compatible tapes (7 tracks,556 - 800 BPI) that, among other things, contain date, mission number, groundtrack or EREP pass number, altimeter ranges in linear metric units, mode and sub-mode of altimeter data, housekeeping data in engineering units, the GMT (year, month, day, hour, minutes, seconds to 10^{-4}) and geocentric true of data X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$ of each altimeter range, the angular difference between center of sensor FOV and subsatellite point, the corresponding geodetic latitudes, longitudes and altitudes.

Mean values of altimeter ranges are more desirable but the averaging time interval should not exceed one second of time. If mean values of altimeter ranges are furnished, the associated standard deviations should be given. The corresponding GMT, X, Y, Z, and angle of FOV off the nadir must be given. Parameters of the reference ellipsoid used in computing the geodetic latitude, longitude and altitude should be given.

(b) If all the above data are furnished on tape, then we do not require the S-193 CCT captioned S071-1 (See Reference 1). However, we still need the following: S072-1, 2, 3, 7 (S072-4, 5, 6 are merely desirable) and S073-6, 7, 8, 10.

(c) The tracking stations and their coordinates used in orbit computations. The types of data used, their precision estimates, limits of satellite elevation angles at each station, orbit computation residuals are highly desirable data and are requested. Such information is required for additional error analysis and confidence estimates in the final answers, investigation of possible altimeter drift, inter-comparison of final results from different passes and/or missions over the same ground track.

(d) A GE report, "S-193 Calibration Data Report", Document No. 72SD 4207, Rev. D, Vol. 1B, 22 March 1973, is requested.

NEXT PERIOD AND SUMMARY OUTLINE

First look analysis of data from EREP pass 9 will be initiated. (That is the only pass for which our minimum data requirements have been received). Data from other passes will be anlayzed as they become available. We look forward to expedited action on our data requirements so that we can catch up with the "Milestone Plan".

TRAVEL

On account of the recommendation in the last status report and discussions with the NASA/JSC Technical Monitor, the PI and 2 co-investigators will travel to NASA/Wallops Station for consultations with Mr. Joe McGoogan and his Skylab altimeter group, during the next reporting period. For the same reason a similar travel to NASA/JSC, Houston is contemplated, if necessary, to resolve various data irregularities and requirements already reported.

REFERENCES

1. NASA/JSC, "Skylab Program EREP Investigators Information Book", Document No. MSC-07874, April, 1973.
2. Fubara, D. M. J., "Geoditic Numerical and Statistical Analysis of Data", Bulletin Geodesique, No. 108, June, 1973.

APPENDIX A

APPENDIX A

GEODETIC CALIBRATION AND EVALUATION OF SKYLAB ALTIMETRY FOR DETERMINATION OF THE GEOID

1.0 ANALYTICAL DATA HANDLING FORMULATIONS

1.1 Condition Equation of Intrinsic Parameters

Each measured altimeter range R_i^o with an associated measurement residual v_i is intrinsically related to (1) X_s, Y_s and Z_s (the satellite coordinates at the instant of measurement) and (2) the geoidal undulation N_i^a (of the subsatellite point) based on a reference ellipsoid of parameters a, and e, and (3) the biases in all measurement systems involved. The condition equation for this intrinsic relationship can be stated as:

$$v_i + R_i^o(1 + \Delta f) - D_i \pm N_i^o + \Delta N_i = 0 \qquad (1)$$

where

$\Delta f = f_1$(biases in X_s, Y_s, Z_s, the altimeter and sea state measurement) is the total system calibration constant,

$N_i^a = N_i^o + \Delta N_i$ (N_i^o is an approximate value for N_i^a)

and

$D_i = f_2\ (X_s, Y_s, Z_s, a, e).$

The exact functional mathematical expression for Δf is unknown. Because R_i^o is a function of X_s, Y_s and Z_s, and sea state, the expression of Δf as a function R_i^o stated in Equation (1) is valid and simple. Furthermore, R_i^o thus becomes the coefficient of Δf in the observation equation and its variability in magnitude lends numerical stability to the resultant system of normal equations for the least squares data processing.

D_i is essentially the geodetic height of the satellite above the chosen reference ellipsoid and is given by

$$D_i = (X_s^2 + Y_s^2)^{1/2} \text{ Sec } \varphi - a(1-e^2\text{Sin}^2\varphi)^{-1/2} \tag{2a}$$

or

$$D_i = Z_s \text{Cosec } \varphi - a(1-e^2\text{Sin}^2\varphi)^{-1/2}(1+e^2) \tag{2b}$$

However, usually φ in Equation (2) is not known and has to be derived from

$$\varphi_i = \tan^{-1}\left[\frac{Z_s + e^2 a(1-e^2\text{Sin}^2\varphi)^{-1/2}}{(X_s^2 + Y_s^2)^{1/2}}\right] \tag{3}$$

Equation (3) is not solved directly. Solving for D_i and φ_i from the given X_s, Y_s and Z_s is done iteratively. By putting $D_i = 0$, the first approximation for φ_i is

$$\varphi \approxeq \tan^{-1}\left[Z_s(X_s^2 + Y_s^2)^{-1/2}(1-e^2)^{-1/2}\right] \tag{4}$$

This φ is then used in Equation (3) which is iteratively solved from $i = 1, \ldots n$ until

$\varphi_n - \varphi_{n-1} \leq \Delta\varphi$ which is usually set at $\Delta\varphi = 0.001$ arc second.

Thereafter, D_i is computed from Equation (2a) or (2b).

1.2 Generalized Least Squares and Sequential Adjustment Model

Equation (1) can be rewritten in matrix form as

$$F_1\ (X_1^a, X_2^a, L_1^a) = 0, \tag{5}$$

subject to the normalized weighting functions P_1, P_2 and P_3 associated with X_1, X_2 and L_1, respectively. Relating Equations (1) and (5) explicitly,

$$X_1^a = N_i^o + \Delta N_i \tag{6}$$

$$X_2^a = \Delta f \tag{7}$$

$$L_1^a = R_i^o + V_i \tag{8}$$

In this model, all parameters and measurements of the mathematical model are treated as "measurements" and weighted accordingly. Thus, constants (fixed variables) have infinitely large weights ($P = \infty$) because they need no corrections (residuals) and as residuals tend towards zero, the corresponding weight approaches infinity. Unknown parameters (free variables) in the classical sense have weights $P = 0$. All other "measurements" have finite weights $0 < P < \infty$. This mathematical model for the generalized least squares processing of experimental data is based on works of Helmert [1892], Schmid and Schmid [1964], Fubara [1969 and 1973]. The superscript "a" denotes the exact true values of the "measurements". Usually, these true values are not known. Instead, the corresponding measured or approximate values X_1^o, X_2^o, and L_1^o with associated variance-covariances P_1^{-1}, P_2^{-1}, and P_3^{-1}, are estimated or measur measured. Therefore, Equation (5) can be rewritten in the form

$$F_2 \left[(X_1^o + \Delta_1), \ (X_2^o + \Delta_2) \ , \ (L_1^o + V_1) \right] = 0 \tag{9}$$

where

$$X_1^a = X_1^o + \Delta_1$$

$$X_2^a = X_2^o + \Delta_2$$

$$L_1^a = L_1^o + V_1$$

The linearized form of Equation (9) is

$$A_1\Delta_1 + B_1\Delta_2 + C_1V_1 + F_2 \ (X_1^o, X_2^o, L_1^o) = 0 \tag{10}$$

A_1, B_1, and C_1 are the first partial derivatives in a Taylor series expansion of Equation (9), associated with X_1^o, X_2^o, and L_1^o, respectively, while Δ_1, Δ_2, and V_1 are the correction parameters to be determined. Eliminating the lengthy matrix algebra steps in between, it can be shown that the least squares solution of Equation (10) to derive the corrections Δ_1, Δ_2, and V_1 to "measured" X_1^o and X_2^o and L_1^o, is

$$\Delta_1 = -N^{-1}A_1^*M_1^{-1}W_1 \tag{11}$$

where * indicates a matrix transpose,

$$M_1 = \left(B_1P_2^{-1}B_1^* + C_1P_3^{-1}C_1^*\right) \tag{12}$$

$$N = \left(P_1 + A_1^*M_1^{-1}A_1\right) \tag{13}$$

$$W_1 = F_1 \left(X_1^o, X_2^o, L_1^o\right) \tag{14}$$

and

$$\Delta_2 = P_2^{-1}B_1^*M_1^{-1} \left[A_1 \left(A_1^*M_1^{-1}A \right)^{-1}A_1^*M_1^{-1} - I \right] W_1 \tag{15}$$

$$V_1 = P_3^{-1} C_1^* K_1 \tag{16}$$

where

$$K_1 = -M_1^{-1} \left(A_1 \Delta_1 + W_1 \right) . \tag{17}$$

The variance factor σ_o is given by either

$$\sigma_o = \left(-K_1^* W_1 / df \right)^{1/2} \tag{18}$$

or

$$\sigma_o = \left[\left(\Delta_1^* P_1 \Delta_1 + \Delta_2^* P_2 \Delta_2 + V_1^* P_3 V_1 \right) / df \right]^{1/2} , \tag{19}$$

where

df = number of degrees of freedom[the number of observations minus the rank of the matrix Equation (10)].

The variance-covariance matrices can be shown to be for Δ_1,

$$V\Delta_1 = \sigma_o^2 \left[P_1 + A_1^* \left(B_1 P_2^{-1} B_1^* + C_1 P_3^{-1} C_1^* \right)^{-1} A_1 \right]^{-1} , \tag{20}$$

for Δ_2,

$$V\Delta_2 = \sigma_o^2 P_2^{-1} B_1^* M_1^{-1} \left[I - A_1 \left(A_1^* M_1^{-1} A_1 \right)^{-1} A_1^* M_1^{-1} \right] B_1 P_2^{-1} ; \tag{21}$$

and for V_1,

$$VV_1 = \sigma_o^2 P_3^{-1} C_1^* M_1^{-1} \left[I - A_1 \left(A_1^* M_1^{-1} A_1 \right)^{-1} A_1^* M_1^{-1} \right] C_1 P_3^{-1} . \tag{22}$$

Sequential least squares adjustment with parameter weighting permits the addition of new observations, L_2^o, (or subtraction of old observations), to update previous solutions and parameter estimates without recomputing previous steps. It may also include estimation of new additional parameters, X_3^a, which are functionally related to the old parameters, X_1^a. These features are effected by the addition of equations of type based on Equation (10) in the form.

$$A_2\Delta_1 + B_2\Delta_3 + C_2V_2 + F_3\left(X_1^o, X_3^o, L_2^o\right) = 0 \tag{23}$$

Denoting the previous solution for Δ_1 from Equations (10) and (11) by Δ_1^o, the inclusion and solution of Equation (23) will lead to updating Δ_1^o, by $\delta\Delta$. Thus, the new Δ_1 is given by

$$\Delta_1 = \Delta_1^o + \delta\Delta \ . \tag{24}$$

It can be shown that

$$\delta\Delta = N^{-1}A_2^* \left[A_2N^{-1}A_2^* + M_2 \right]^{-1} \left[A_2N^{-1}A_1^*M_1^{-1}W_1 - W_2 \right] , \tag{25}$$

where M_1 from Equation (12), N from Equation (13), W_1 from Equation (14), Δ_1^o from Equation (5), have been previously computed and

$$M_2 = B_2P_4^{-1}B_2^* + C_2P_5^{-1}C_2^* \ , \tag{26}$$

$$W_2 = f_3\left(X_1^o, X_3^o, L^o\right) \ . \tag{27}$$

In general, the sequential solution results in updated values at the n^{th} sequence of

$$\Delta_n = \Delta_1^o + \sum_{i=1}^{n-1} \delta_i \ , \tag{28}$$

where

$$\delta\Delta_i = -N_{i-1}^{-1} A_i^* \left[A_iN_{i-1}^{-1} A_i^* \pm M_i \right]^{-1} \left[A_i\Delta_{i-1}^o + W_i \right] , \tag{29}$$

$$N_i^{-1} = N_{i-1}^{-1} - N_{i-1}^{-1} A_i^* \left[A_iN_{i-1}^{-1} A_i^* \pm M_i \right]^{-1} A_iN_{i-1}^{-1} , \tag{30}$$

and the updated variance-covariance matrix is, for Δ_i,

$$\sigma_i^2N_i^{-1} \tag{31}$$

in which

$$\sigma_i = \left[\left(- K_i^*W_i \right) /df \right]^{1/2} \tag{32}$$

Similar expression can be written for Δ_3 and V_2 as in Equations (15) and (16), and (21) and (22).

These computational procedures can be used in all geodetic adjustments, orbit computations, and all experimental data analyses that require rigorous least squares adjustment techniques. Very often, the ordinary least squares adjustment (weighted or unweighted) could lead to either unstable normal equations or inability to solve for all the unknown parameters. Often, utilization of the above approach, together with the inclusion of effective variance-covariances, eliminates such problems. This type of generalized least squares approach to numerical analysis of experimental data is termed the method of "Intrinsic Parameters" [Fubara 1969, 1973].

The main advantages in this approach to numerical analysis of data include:

(1) parameter-weighting which permits more efficient and theoretically rigorous combination and utilization of hybrid data, correct application of error modelling techniques, accurate incorporation of the statistics of the parameters and observations employed;

(2) flexibility in investigating the influences of geometric configruations, spatial data distribution, desirable and necessary quantities and quality of data;

(3) efficient data editing and updating of previous solutions without repeating previous computations thereby saving computer time and storage.

1.3 Establishment of Confidence in Numerical Processing and Final Results

The above algorithm is the type necessary for a generalized application of least squares techniques to numerical and statistical analyses of any experimental data that can be expressed in terms of other parameters that can or cannot be directly measured, including unavoidable systematic errors that are modeled as unknown parameters. Geodetic calibration and evaluation of Skylab altimetry for determination of the geoid falls into this category.

The emphasis is not on merely acquiring experimental data and computing results but also on establishing (1) how good the data are; (2) the adequacy of the numerical processing including the mathematical formulation, stability of equations, estimation of weighting functions; (3) statistical confidence in the derived parameters. These are accomplished, as in Fubara [1973] via the statistical analyses of the residuals, the variance factor, the weight coefficient matrix (the inverse of the normal equation matrix) and the confidence intervals and associated probability for all the numerical quantities as discussed in the text. Investigation of the stability of normal equations is effected through the use of (1) "condition numbers" on matrix norms, [Turing, 1948], [Todd, 1949], [Faddeev & Faddeeva, 1963], (2) random perturbation of normal equation [Fox, 1965], [Fubara 1969, 1973] and/or (3) analysis of correlation coefficients of normal equation inverse. In most cases, any detected instability or poor convergency of solution to unique values can be accomodated by the generalized least squares approach with parameter weighting or equilibrating of the normal equation [Fox, 1965] and [Fubara, 1969, 1972, 1973].

1.3.1 Assessment of Efficiency of Numerical Processing

For the type of weighted least-squares data processing that have been developed, the weighting function is estimated before the adjustment from

$$P = \sigma_o^2 F^{-1} \tag{33}$$

where F is the variance-covariance matrix of the "measurements" and σ_o^2 is the variance factor. Usually the true value of F and hence σ_o^2 is not known. Both can be and are (1) estimated before the adjustment and also (2) computed after the adjustment. A statistical comparison of the pre- and post-adjustment values [Fubara, 1973] is used for assessment of the quality of field data, and the adequacy of mathematical models and weighting criteria. The variance estimate, σ_1^2, with d_1 degrees of freedom, and another independent estimate, σ_2^2 with d_2 degrees of freedom, of the same variance, are each distributed as chi-square. The non-dimensional ratio of the variance factor,

$$\frac{\left(\sigma_1^2 \quad d_1\right)}{\left(\sigma_2^2 \quad d_2\right)} = F_{d_1,d_2} \tag{34}$$

is tabulated in the Fisher's distribution table as function of d_1 and d_2, and a coefficient of $(1 - \frac{1}{2}\alpha)$ as the confidence coefficient. Normally, the variance factor ratio test, at a selected confidence coefficient, consists of rejecting the hypothesis that σ_1^2 and σ_2^2 for the respective d_1 and d_2 degrees of freedom are estimates of the same variance if either

$$F_{d_1,d_2} < F_{d_1,d_2}/(1-\tfrac{1}{2}\alpha) \tag{35}$$

or $$F_{d_1,\,d_2} > F_{d_1,d_2}/(\tfrac{\alpha}{2}) \tag{36}$$

or $$F_{d_1,\,d_2} > F_{d_1,d_2}/(1-\alpha) \tag{37}$$

For a small number of degrees of freedom, the F-test is a weak indicator unless the ratio of the true variances is indeed large. In cases where the true weight for a least squares solution is unknown, it is usual to experiment with many estimated weights each leading to an estimated variance, σ_i^2. The F-test can in this case be used to establish if one assigned weight is significantly different from the other [Mourad and Fubara, et al 1972]. If the variance factor, σ_o^2, computed from the data processing fails the Chi-square or variance factor ratio test, it indicates presence of systematic errors in the mathematical model or the weights used or the observations or all three combined.

Many of the results emanating from a least squares adjustment, such as variance factors, variances, rms, are point estimates. When used alone, these estimates are no longer sufficient for evaluating results of data processing from hybrid measurements or complex experiments. Evaluation by the use of confidence intervals in two- and three-dimensional space, with preselected probability, gives the more detailed and additional information required. The necessary equations are available in Linnik [1961] and Fubara [1969, 1973]. These developments satisfy the analytical data handling procedure set forth in Figure 1 which is Figure T-3 of the proposal.

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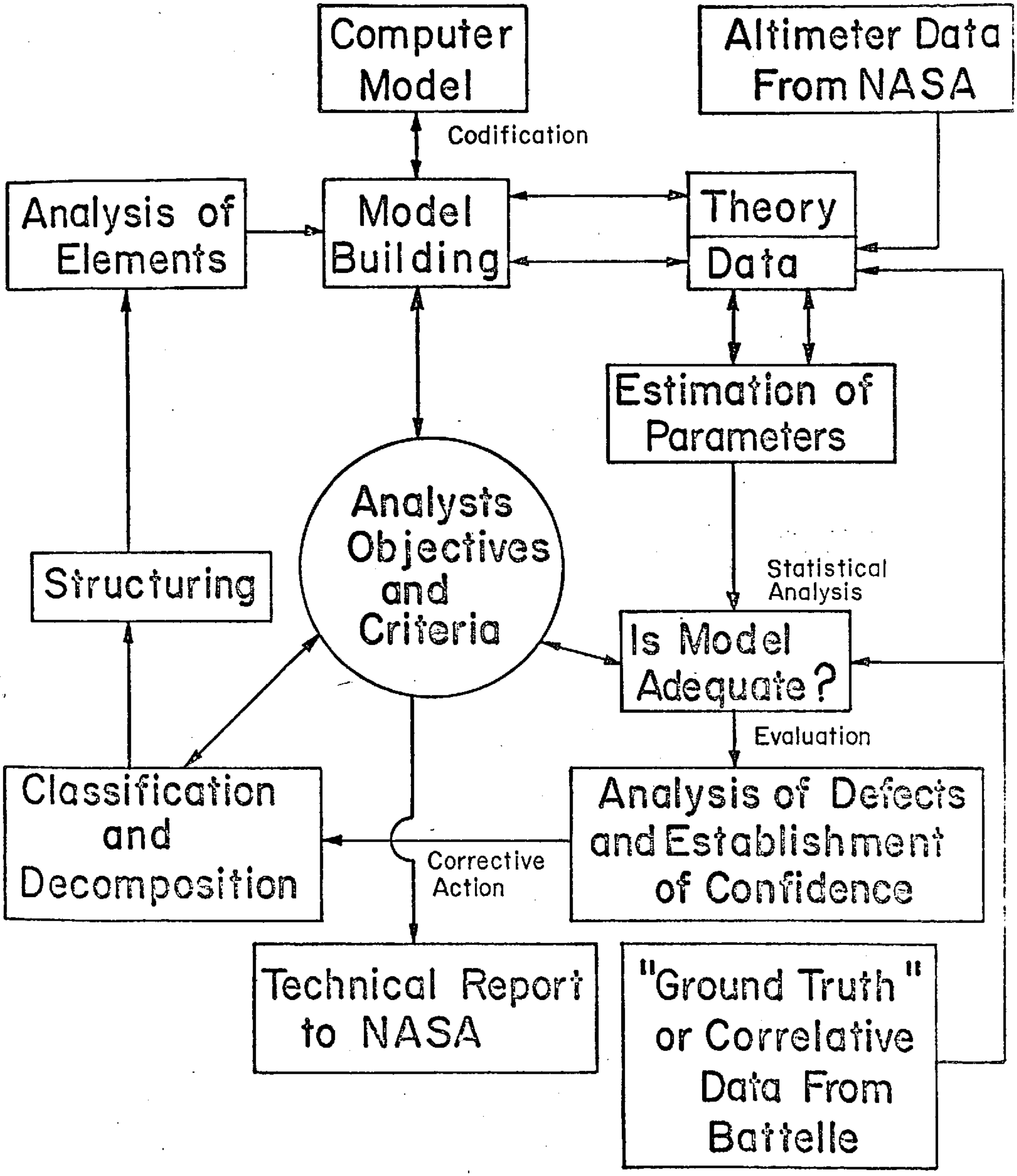

FIGURE 1. ANALYTICAL DATA HANDLING PROCEDURE

APPENDIX B

APPENDIX B

REPORTS AND DATA RECEIVED

(1) "Skylab Instrumentation Calibration Data", Volume IV, Skylab Mission SL-1, EREP Experiment Calibration Data, Prepared by Program Operations Office Test Division Instrumentation Integration Branch, NASA/JSC, MSC-07744, Revision B, August, 1973.

(2) W/O #6749, Skylab 2, S190A, 461682-4-PI; 9 "Prints-1 each, Mag: 10.16, 270/273; 171/185 (2 sets), September 20, 1973.

(3) W/O #6762; Skylab 2, S190A, 461636-4-PI, 9 "Prints-1 each; Mag: 10, 176/190.

(4) 2 Cans B & W Print Skylab 2, S191, #'s 461682 and 461636; 1 each Master Pos. Mags: BH01 and BH02.

(5) W/O #6878, Skylab 2, S190B, 461682-4-PI, 9 "Prints-1 each 8 X 10, Mag: 81 x (color), 357, 366/370 only, September 28, 1973.

(6) Skylab SL/2 Data Books

D.D.C. Accession No.	DPAR	Date/Time START	Date/Time STOP
32-05792	S193B-069-3-7-73-7		
32-05791	S193B-070-3-7-73-7		
32-05722	S193-70-2-4	155:17:11:11	155:17:16:37
32-05721	S193-70-4-9	163:12:56:20	163:13:18:59
32-05718	S193-70-3-7	161:14:28:12	161:14:38:46
32-05719	S193-69-3-7	161:14:28:12	161:14:38:46
32-05730	S193-69-3-6	160:15:03:39	160:15:18:42
32-05731	S193-69-2-6	160:15:03:39	160:15:18:42
32-05720	3-70-4-9-72-1,6,7,9	163:12:56:20	163:13:18:59
32-05791	S193B-070-3-7-73-7	160:14:28:00	160:14:38:46
32-05792	S193B-069-3-7-73-2	160:14:28:00	160:14:38:46