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BACKSCATTERING FROM A TWO-SCALE ROUGH SURFACE WITH APPLICATION TO RADAR SEA RETURN

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16. Abstract

A two-scale composite surface scattering theory was developed without using the non-coherent assumption. The surface is assumed electrically homogeneous and finitely conducting; the surface roughness may be non-uniform geometrically. The special forms of the terms for excluding the non-coherent assumption and the meanings of these terms are discussed. To gain insight into the mechanisms of backscattering, the results are compared with those obtained by previous theories. The comparison with NRL data shows satisfactory agreement for both horizontal and vertical polarization, especially for incident angles larger than 30°. For smaller incident angles, NASA/JSC data have been chosen for comparison and close agreement is again observed.

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FOREWORD

This document reports a surface scattering theory developed by the University of Kansas during the first year of a joint program (RADSCAT) with New York University, General Electric Space Division, and NASA Langley Research Center. This study was performed under Contract NAS 1-10048, issued by the National Aeronautics and Space Administration, Advanced Applications Flight Experiments Office, Langley Research Center, Hampton, Virginia.

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I. SUMMARY

The two-scale composite rough surface model usually considered is one composed of large undulations over which small irregularities are superimposed. This general model may be further subdivided into two other models: (1) the large undulations are larger in dimension than that of the illuminated area so that within the beam of illumination the picture is a tilted perturbed plane; and (2) the large undulations are of such a size that at least several undulations can be found within the beam. The first model is essentially the small perturbation model, since the effect of the tilt can be accounted for either by a change in the angle of incidence or by resolving the incident plane wave into horizontally and vertically polarized components, or by both. The second model is much more complicated and has been approached in most cases with a non-coherent assumption, i.e. the contribution from the small irregularities may be computed by summing powers from the large facets constituting the large undulations. The total contribution from the composite surface is then taken to be that from the large undulations plus that from the small irregularities averaged over the large undulations.

If the non-coherent assumption is not made, the total scattered field from the illuminated area must be computed before evaluating the power which is the approach adopted in this study to calculate both the vertically and horizontally polarized scattering coefficients. The surface is assumed finitely conducting and homogeneous; the surface roughness may be non-uniform. To gain insight into the mechanisms of scatter, results are compared with those obtained by previous theories. The special form of the terms due to excluding the non-coherent assumption and the meaning of such terms are discussed.

Based on Cox and Munk's [9] relation between the mean square surface slope of the sea and wind speed, curves are drawn showing the backscattering coefficient, σ_{pp}, as a function of wind velocity and of the angle of incidence. The comparison with NRL [10,11] data shows satisfactory angular agreement for both horizontal and vertical polarization, especially for incident angles larger than 30°. For smaller incident angles, NASA/MSC [12] data have been chosen for comparison and close agreement is again observed.

II. INTRODUCTION

Many scattering theories [1-4] using two-scale rough surface models have been developed in recent years. The general model is a large undulating surface with small irregularities superimposed. More specifically, two types of models have been considered: (1) the large undulations are larger in dimension than the illuminated area and thus within the beam the picture is a tilted perturbed plane, and (2) the illuminated area contains at least several large undulations. For model (1) the problem remains essentially the same as a small perturbation problem since the effect of any tilt can be accounted for by a change in the angle of incidence and by resolving the incident plane wave into horizontally and vertically polarized components [3]. Such a simple treatment is not possible for model (2) for which two different approaches are in existence: (i) the non-coherent approach where a non-coherent assumption is used to simplify the problem [1], and (ii) the coherent approach where the said assumption is not made [4]. The non-coherent assumption referred to here is the one defined by Semenov [1], i.e. the contribution from the small irregularities may be computed by summing powers from the large facets constituting the large undulations. This assumption implies that the calculation of the contribution from the small irregularities in model (2) is identical to solving the entire problem using model (1). Of course, for model (2), the contribution from the large undulations must also be computed and this is the major difference between the two models when the non-coherent assumption is made. Differences between models (1) and (2) are further magnified if the non-coherent assumption is not made. Thus, the total scattered field must be found by integrating the total field on surface before calculating the power. Hence, it follows that terms due to integrating the first order perturbed field over the large undulations within the illuminated area will show up and give an explicit indication of the interaction between the large and the small scatterers. Such an interaction is restricted to be an average operation in the non-coherent approach and this defines the major difference between approaches (i) and (ii).

This paper discusses another coherent approach for model (2). The composite surface, $\zeta(x,y) = Z(x,y) + s(x,y)$, is assumed to be finitely conducting and homogeneous with $Z(x,y)$ representing the large undulations and $s(x,y)$ the small irregularities. $Z(x,y)$ and $s(x,y)$ are to be generated by independent, stationary, Gaussian random processes.

The approach is a modified Kirchhoff's method employing the equivalent surface field, i.e. the surface field on $Z(x,y)$ estimated by the tangent plane method is modified to include the effect of $s(x,y)$. Once this equivalent field is obtained the problem reduces to a single surface scattering problem, i.e. scattering from the surface $Z(x,y)$. The concept of equivalent field was advanced earlier by Bass and Bocharov [5] for scattering from a single surface. Results obtained by this approach are simpler and reduce more readily to special cases of single surface scattering than Fung and Chan's approach [4], where fields on the composite surface $\xi(x,y)$ were considered.

III. THE SCATTERED FIELD

The far zone scattered field due to an incident plane wave on a rough surface Z(x,y) (Figure 1) is given by a special form of the Stratton-Chu integral [6],

$$\underset{\sim}{E}_s = K_o \underset{\sim}{n}_2 \times \int [\underset{\sim}{n} \times \underset{\sim}{E} - \eta \underset{\sim}{n}_2 \times (\underset{\sim}{n} \times \underset{\sim}{H})] \exp(jk\underset{\sim}{r} \cdot \underset{\sim}{n}_2)\, ds \tag{1}$$

where a time factor of the form, $\exp(j\omega t)$, is understood; $\underset{\sim}{r}$ is the position vector pointing from the origin of the coordinate system to a surface element dS; $\underset{\sim}{n}_2$ is is a unit vector in the direction of observation; R is the distance from the origin to the field point; $\underset{\sim}{E}$, $\underset{\sim}{H}$ are the total electric and magnetic fields on the surface; k is the wave number in air; η is the intrinsic impedance in air, $K_o = -jk \exp(-jkR)/(4\pi R)$, and $\underset{\sim}{n}$ is the local normal to the surface.

The basic problem for finding $\underset{\sim}{E}_s$ is to determine $\underset{\sim}{n} \times \underset{\sim}{E}$ and $\underset{\sim}{n} \times \underset{\sim}{H}$ at any point on Z(x,y). To do so it is necessary to set up a local coordinate system at the point in question. A possible set of local coordinates is

$$\bar{\underset{\sim}{z}} = (-\underset{\sim}{i} Z_x - \underset{\sim}{j} Z_y + \underset{\sim}{k})(Z_x^2 + Z_y^2 + 1)^{-\frac{1}{2}} \tag{2a}$$

$$\bar{\underset{\sim}{y}} = (\bar{\underset{\sim}{z}} \times \underset{\sim}{n}_1) / |\bar{\underset{\sim}{z}} \times \underset{\sim}{n}_1| \tag{2b}$$

$$\bar{\underset{\sim}{x}} = \bar{\underset{\sim}{y}} \times \bar{\underset{\sim}{z}} \tag{2c}$$

where Z_x, Z_y are the partial derivatives of the surface Z(x,y) with respect to x and y respectively, and $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ are the unit vectors of the (x,y,z) coordinates. From the definition of the local coordinates we see that the tangent plane at the point (x,y) on Z(x,y) coincides with the plane $\bar{x}$-$\bar{y}$. If the small irregularities were absent this would be an infinite flat plane. However, with the small irregularities present, this becomes a perturbed plane and the local scattered fields may be found by Rice's theory [7]. This has been done by both Rice [7] and Valenzuela [8]. Hence, let us assume that in the local frame

$$\underset{\sim}{E} = \bar{\underset{\sim}{x}} E_{\bar{x}} + \bar{\underset{\sim}{y}} E_{\bar{y}} + \bar{\underset{\sim}{z}} E_{\bar{z}} \tag{3a}$$

$$\underset{\sim}{H} = \bar{\underset{\sim}{x}} H_{\bar{x}} + \bar{\underset{\sim}{y}} H_{\bar{y}} + \bar{\underset{\sim}{z}} H_{\bar{z}} \;. \tag{3b}$$

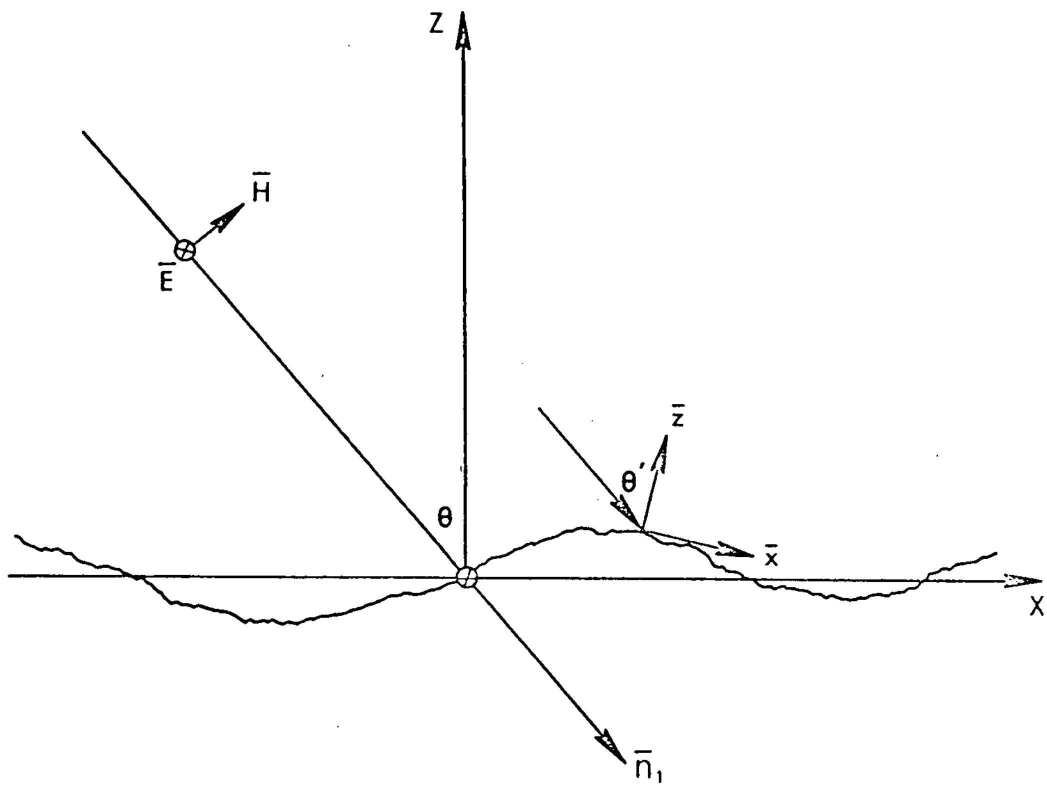

Fig. 1. The geometry of the scattering problem.

Assuming $Z_x^2, Z_y^2 \ll 1$, we may rewrite the backscattered field in terms of the local field components as (see Appendix I)

$$\underset{\sim}{E}_s \simeq -K_0 \int_{-L}^{L}\int_{-L}^{L} \Big\{ \underset{\sim}{j} \big[(E_{\bar{y}} \cos\theta' - \eta H_{\bar{x}}) - (E_{\bar{x}} + \eta H_{\bar{y}} \cos\theta) Z_y / \sin\theta' \big] + (\underset{\sim}{i} \cos\theta + \underset{\sim}{k} \sin\theta) \big[(E_{\bar{x}} + \eta H_{\bar{y}} \cos\theta') + (E_{\bar{y}} \cos\theta - \eta H_{\bar{x}}) Z_{\bar{y}} / \sin\theta' \big] \Big\} \exp(-j \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) \, dx \, dy \qquad (4)$$

where $\underset{\sim}{k}_1 = k\underset{\sim}{n}_1$; $\underset{\sim}{n}_1 = \underset{\sim}{i} \sin\Theta - \underset{\sim}{k} \cos\Theta$; 2L, width of the illuminated area; Θ' is the local angle of incidence.

For computing polarized scattering the Z_y - terms in (4) are unimportant and, therefore, reasonable accuracy may still be achieved by leaving out the Z_y-terms. However, for depolarized scattering the field expressions are complicated by the presence of the Z_y- terms and the local angle appearing in the denominator. In addition, depolarization due to the split of the incident polarization into locally horizontal and vertical components as a result of Z_y also complicates matters. In short, there is as yet no satisfactory method for estimating the depolarized scattering coefficients from (4).

For a horizontally polarized incident plane wave of the form

$$\underset{\sim}{E}_\perp = \underset{\sim}{j} \exp\big[-jk(x \sin\theta - z\cos\theta) \big] ,$$

there correspond two locally incident waves (Appendix I). However, for polarized scattering there is no significant error if we take one of the local incident waves to be

$$\underset{\sim}{E}_\ell = \underset{\sim}{\bar{y}} \exp\big[-jk(\bar{x} \sin\theta' - \bar{z} \cos\theta') \big]$$

and ignore the other one.

By applying Bass and Bocharov's [5] concept to Valenzuela's results [8], the equivalent fields on Z(x,y) up to the first order may be shown to be (see Appendix II)

$$E_{\bar{x}} = \iint T_{\perp}\, uv\, Q' EX \; du\, dv \tag{5a}$$

$$E_{\bar{y}} = (1+R_{\perp}) \exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) - \iint T_{\perp} (u^2+bc)\, Q' EX \; du\, dv \tag{5b}$$

$$\eta H_{\bar{x}} = (1-R_{\perp}) \cos\theta' \exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) + \iint T_{\perp} \left| u^2(b-c)/k + kc \right| Q' EX \; du\, dv \tag{5c}$$

$$\eta H_{\bar{y}} = \iint [\, uv\,(b-c)\, Q'/k\,]\, EX \; du\, dv \quad , \tag{5d}$$

where $EX = \exp(-jux - jvy + jkZ \cos\Theta)$,

$$Q' = j(k'^2 - k^2)\, S(u - k\sin\theta', v)\, [2\pi(k^2 c + k'^2 b)]^{-1}$$

$$T_{\perp} = 1 + R_{\perp}$$

$$b = \begin{cases} (k^2 - u^2 - v^2)^{1/2}, & k^2 \geq u^2 + v^2 \\ -j(u^2 + v^2 - k^2)^{1/2}, & k^2 < u^2 + v^2 \end{cases} ,$$

S(u,v) = two dimensional Fourier Transform of s(x,y),

$$c = \begin{cases} c_o, & \mathcal{I}m(c_o) \leq 0, \; c_o = (k'^2 - u^2 - v^2)^{1/2} \\ -c_o, & \mathcal{I}m(c_o) > 0 \end{cases} ,$$

and $\mathcal{I}m$ means "the imaginary part of."

The limits of integration in (5) are from $-\infty$ to ∞.

Similarly, for a vertically polarized incident plane wave of the form,

$$\underset{\sim}{E}_{\parallel} = (\underset{\sim}{i} \cos\theta + \underset{\sim}{k} \sin\theta) \exp[-jk(x\sin\theta - z\cos\theta)] ,$$

the local fields on Z(x,y) up to the first order are

$$E_{\bar{x}} = (1-R_{\parallel}) \cos\theta' \exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) + \iint T_{\parallel} Q' [\, bu \sin\theta' - (v^2 + bc)\, k \cos\phi'/k'\,]\, EX \; du\, dv \tag{6a}$$

$$E_{\bar{y}} = \iint T_{\parallel} Q' [\, bv \sin\theta' + uvk \cos\phi'/k'\,]\, EX \; du\, dv \tag{6b}$$

$$\eta H_{\bar{x}} = \iint T_{\parallel} Q' [\, uv(c-b) \cos\phi'/k' - vk \sin\theta'\,]\, EX \; du\, dv \tag{6c}$$

$$\eta H_{\bar{y}} = -(1+R_{\parallel}) \exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) + \iint T_{\parallel} Q' [\, ku \sin\theta' + (v^2 c - v^2 b - ck^2) \cos\phi'/k'\,]\, EX \; du\, dv \tag{6d}$$

where $T_{\parallel} = 1 + R_{\parallel}$.

IV. THE SCATTERING CROSS SECTIONS

If we use a linear approximation for the Fresnel reflection coefficients (see Figures 2 and 3) and the local cos Θ' and sin Θ', then

$$R_\perp \simeq R_\perp(\theta) + R_\perp' Z_x \quad ;$$

$$R_{//} \simeq R_{//}(\theta) + R_{//}' Z_x \quad ;$$

$$R_\perp' \simeq -2k\, R_\perp(\theta)\, \sin\theta / (k' \cos\phi) \;;$$

$$R_{//}' \simeq [2k(k'^2 - k^2)\sin\theta] \big/ [k'\cos\phi\,(k'\cos\theta + k\cos\phi)^2] \;;$$

$$\cos\theta' \simeq \cos\theta + Z_x \sin\theta \quad ;$$

$$\sin\theta' \simeq \sin\theta - Z_x \cos\theta \quad ;$$

$$\cos\phi \simeq [1 - (k/k')^2 \sin^2\theta]^{1/2} \;;$$

and (5) and (6) may be substituted into (4) to obtain the backscattered field (Appendix III). The scattering coefficient defined in terms of the scattered field by

$$\sigma_{pp} = 4\pi R^2 \langle \underset{\sim}{E}_{sp} \cdot \underset{\sim}{E}_{sp}^* \rangle / (2L)^2 \qquad (7)$$

can now be computed. ⟨...⟩ is the symbol for ensemble average; * is the symbol for complex conjugate. Some identities for ensemble average useful for evaluating σ_{pp} are

$$\langle Z_x \exp[j v_z (Z - Z')] \rangle = \langle Z_{x'}' \exp[j v_z (Z - Z')] \rangle$$
$$= -j\sigma^2 v_z \frac{\partial\rho}{\partial\alpha} \exp[-\sigma^2 v_z^2 (1-\rho)]$$

$$\langle Z_x Z_{x'}' \exp[j v_z (Z - Z')] \rangle = -\sigma^2 \left[\frac{\partial^2\rho}{\partial\alpha^2} + v_z^2 \sigma^2 \left(\frac{\partial\rho}{\partial\alpha}\right)^2\right] e^{-v_z^2\sigma^2(1-\rho)}$$

$$\langle S(u', v') \frac{\partial S(u,v)^*}{\partial u} \rangle = 2\pi\sigma_1^2 \frac{\partial W(u,v)}{\partial u} \delta(u-u')\delta(v-v')$$

$$\langle S(u', v') S(u,v)^* \rangle = 2\pi\sigma_1^2 W(u,v)\, \delta(u-u')\, \delta(v-v')$$

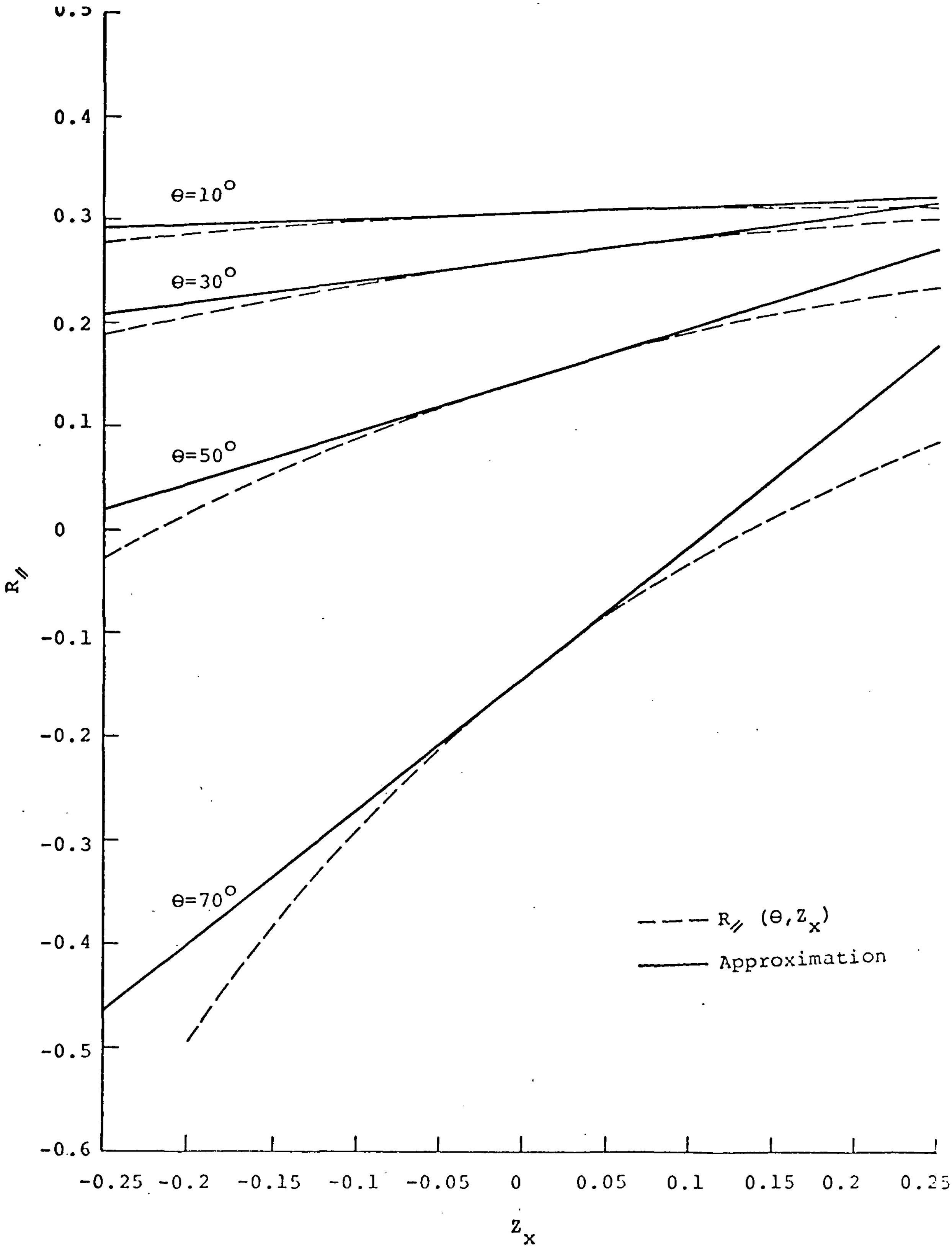

Fig. 2. Comparison between the reflection coefficient $R(\Theta, Z_x)$ and its linear approximation for $\epsilon_r = \sqrt{k'/k} = 3.61$ (a) $R_{\parallel}(0, Z_x)$

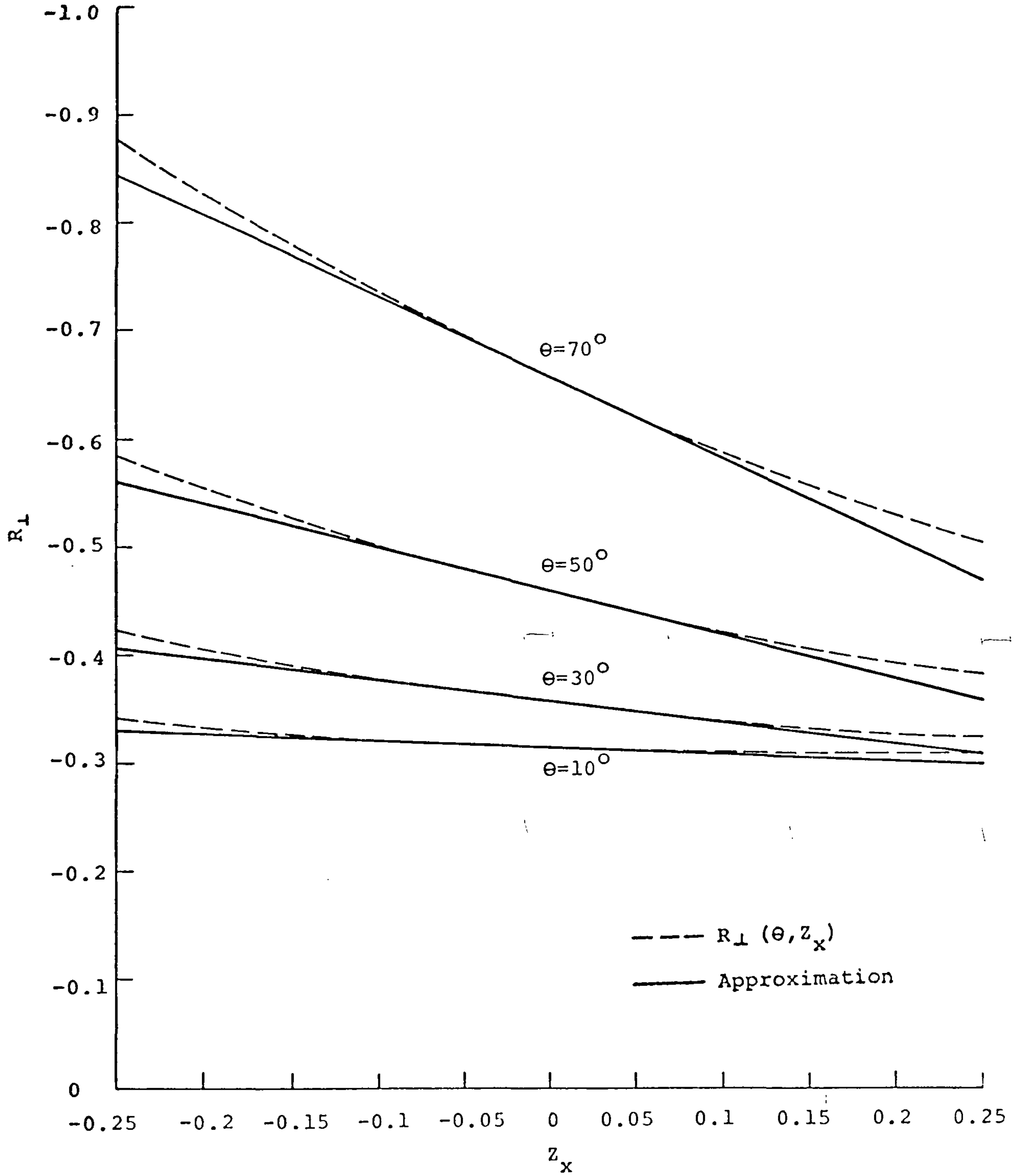

Fig. 2. Comparison between the reflection coefficient $R(\Theta, Z_x)$ and its linear approximation for $\epsilon_r = \sqrt{k'/k} = 3.61$ (b) $R_\perp(\theta, Z_x)$

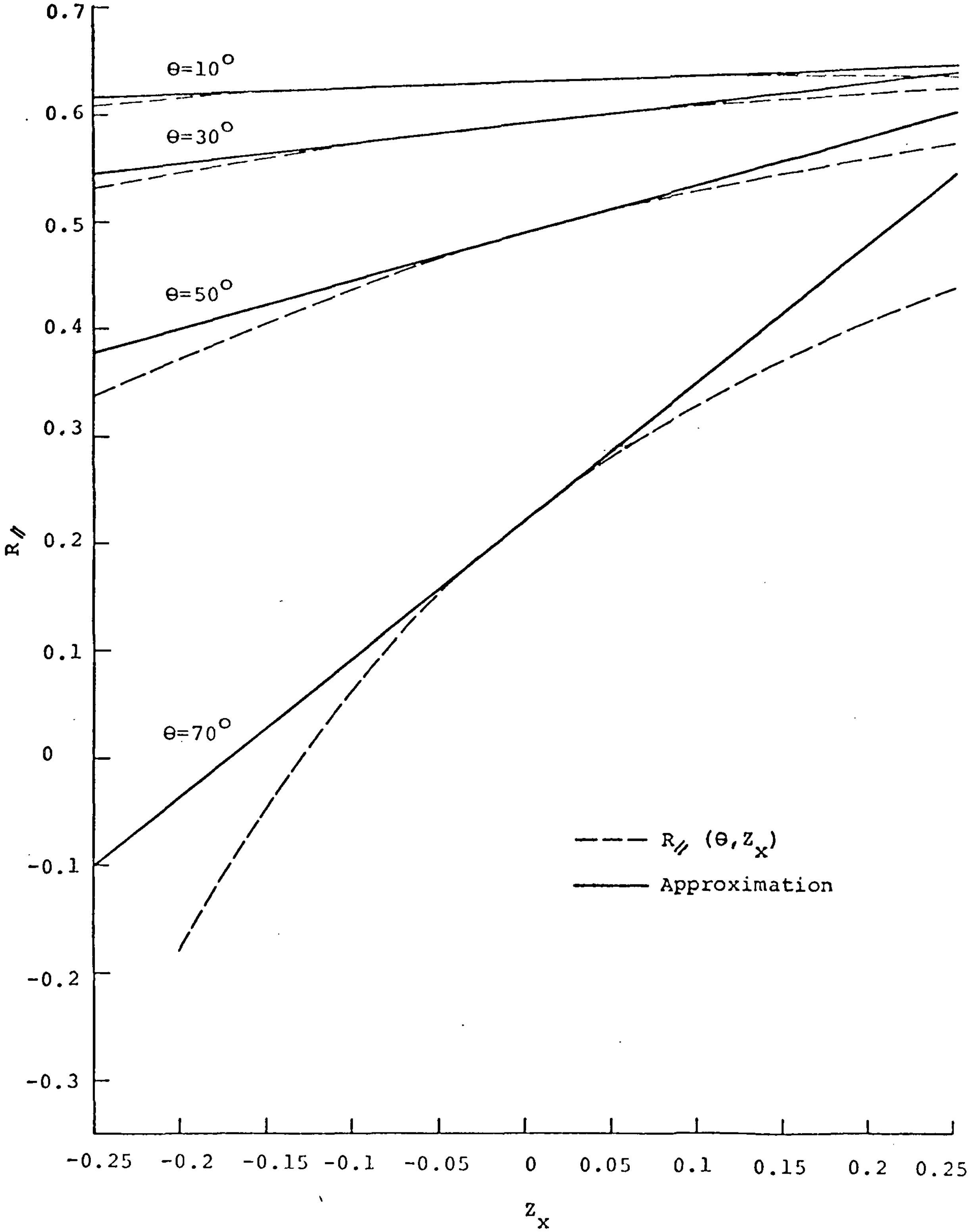

Fig. 3. Comparison between the reflection coefficient $R(\Theta, Z_x)$ and its linear approximation for $\epsilon_r = 20$ (a) $R_{/\!/}(\theta, Z_x)$

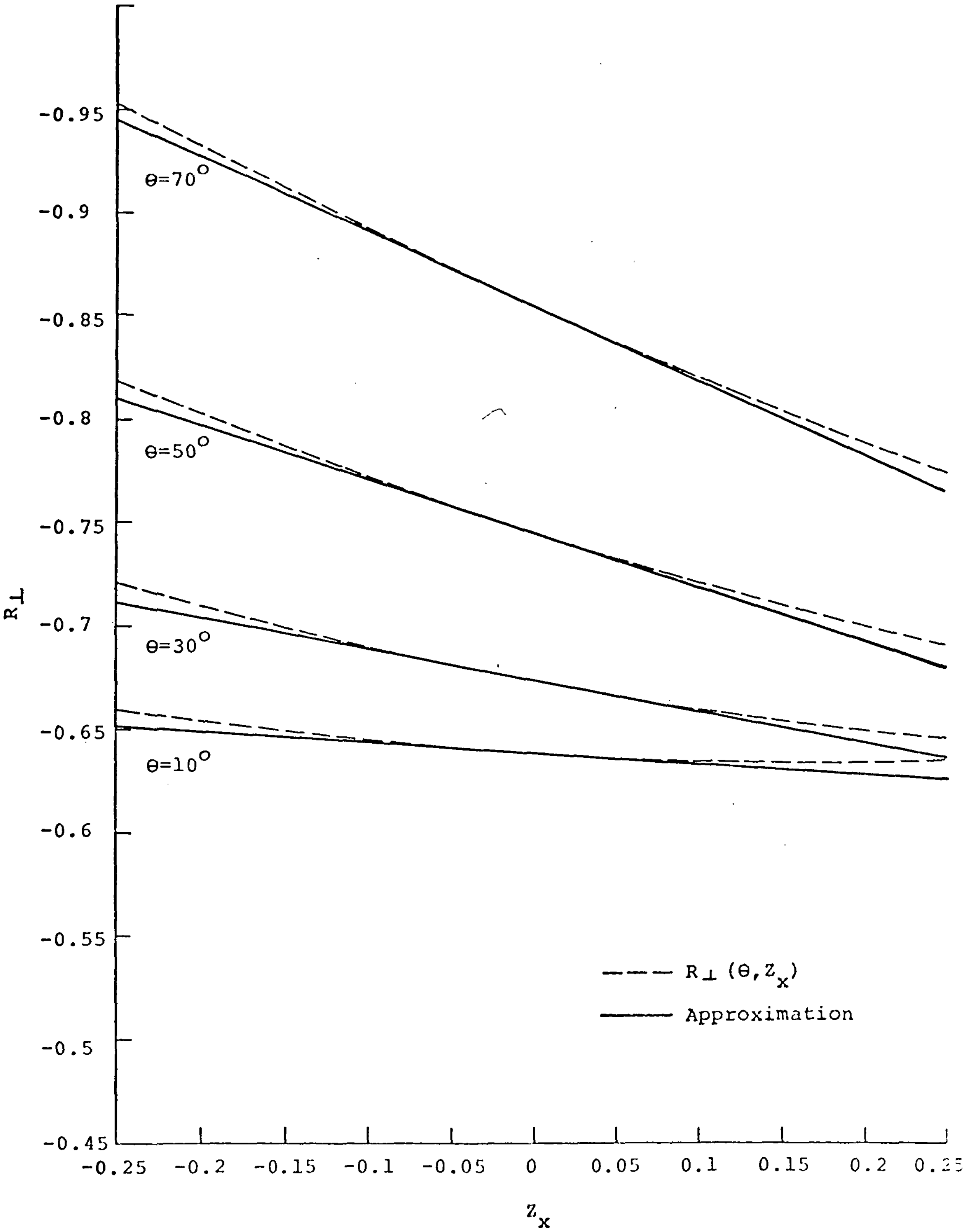

Fig. 3. Comparison between the reflection coefficient $R(\Theta, Z_x)$ and its linear approximation for $\epsilon_r = 20$ (b) $R_\perp(\theta, Z_x)$

where $Z = Z(x,y)$; $Z' = Z(x',y')$; (x,y) and (x',y') represent in general two different points within the illuminated area; (u,v) and (u',v') are similarly defined as (x,y) and (x',y') in the wave number space; $\alpha = x - x'$, $\beta = y - y'$; $\rho = \rho(\alpha,\beta)$ is the correlation coefficient of $Z(x,y)$; σ_1^2, σ^2 are the variances of the surfaces $s(x,y)$ and $Z(x,y)$ respectively; $\delta(\ )$ is the Dirac delta function and $W(u,v)$ is the roughness spsctrum of $s(x,y)$ related to its correlation coefficient $\rho_1(\alpha,\beta)$ by the Fourier transform.

The general form of σ_{pp} may be written in terms of the sum of σ_{1pp} and σ_{2pp} (Appendix IV);

$$\sigma_{1pp} = \frac{k^2}{4\pi}\int_{-2L}^{2L}\int_{-2L}^{2L}\frac{(2L-|\alpha|)(2L-|\beta|)}{(2L)^2}\Big\{|A_{pp}|^2 - j\sigma^2 v_z\,(A_{pp}B_{pp}^* + A_{pp}^*B_{pp})\frac{\partial\rho}{\partial\alpha} - \sigma^2|B_{pp}|^2\Big[\frac{\partial^2\rho}{\partial\alpha^2} + \sigma^2 v_z^2\Big(\frac{\partial\rho}{\partial\alpha}\Big)^2\Big]\Big\} \exp[-j v_x\alpha - v_z^2\sigma^2(1-\rho)]\, d\alpha\, d\beta \tag{8}$$

where $v_x = 2k\sin\Theta$ and $v_z = 2k\cos\Theta$;

$$\sigma_{2pp} = \frac{k^2\sigma_1^2}{2}\int_{-2L}^{2L}\int_{-2L}^{2L}\frac{(2L-|\alpha|)(2L-\beta)}{(2L)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big\{|TC_{pp}|^2 W - j2\sigma^2 v_z\frac{\partial\rho}{\partial\alpha}\,\mathrm{Re}\Big(TC_{pp}\big[(TD_{pp}+R'C_{pp})^* W + (TC_{pp})^* v_z\frac{\partial W}{\partial q}/2\big]\Big) - \sigma^2\Big[\frac{\partial^2\rho}{\partial\alpha^2} + v_z^2\sigma^2\Big(\frac{\partial\rho}{\partial\alpha}\Big)^2\Big]\Big(|TD_{pp}+R'C_{pp}|^2 W + |TC_{pp}|^2 v_z^2\frac{\partial^2 W}{\partial q^2}/4 + v_z\frac{\partial W}{\partial q}\mathrm{Re}\big[TC_{pp}(TD_{pp}+R'C_{pp})^*\big]\Big)\exp\big[-j(u+k\sin\theta)\alpha - j v\beta - K(1-\rho)\big\}\, du\, dv\, d\alpha\, d\beta \tag{9}$$

where Re means "the real part of"

$q = u - k \sin \Theta$

$W = W(q, v)$

$T_p = 1 + R_p$

$R_p = \begin{cases} R_\perp & \text{for horizontal polarization} \\ R_{/\!/} & \text{for vertical polarization} \end{cases}$

$R_p' =$ derivative of T_p with respect to Z_x

$K = 4k^2\sigma^2\cos^2\Theta$.

For horizontal polarization,

$$\sigma_{hh} = \sigma_{1hh} + \sigma_{2hh} \tag{10}$$

in which

$$A_{hh} = 2 R_\perp \cos\theta$$

$$B_{hh} = 2 (R_\perp \sin\theta + R_\perp' \cos\theta)$$

$$C_{hh} = Q [u^2(b-c)/k + kc + (u^2 + bc) \cos\theta]$$

$$D_{hh} = Q (u^2 + bc) \sin\theta$$

$$Q = (k'^2 - k^2) [2\pi (k^2 c + k'^2 b)]^{-1}$$

For vertical polarization,

$$\sigma_{vv} = \sigma_{1vv} + \sigma_{2vv} \tag{11}$$

in which

$$A_{vv} = -2 R_{/\!/} \cos\theta$$

$$B_{vv} = -2 (R_{/\!/}' \cos\theta + R_{/\!/} \sin\theta)$$

$$C_{vv} = Q \{ [u(b + k\cos\theta)] \sin\theta + [\cos\theta(v^2c - v^2b - ck^2) - k(v^2 + bc)] \cos\phi/k' \}$$

$$D_{vv} = Q \{ \sin\theta [ku \sin\theta + (v^2c - v^2b - ck^2)\cos\phi/k'] + k^2 \sin\theta\cos\theta [\cos\theta(v^2c - v^2b - ck^2) - k(v^2 + bc)]/k'^3 \cos\phi - u\cos\theta(b + k\cos\theta) \}$$

The expression for σ_{1pp} is identical to single surface scattering results obtained by the Kirchhoff's method. The form of σ_{1pp} may not appear familiar because most of the cases discussed in the literature are special cases, as noted in the next section. The expression for σ_{2pp} is more complicated; hence, it is best to examine its meaning by considering special cases.

V. SPECIAL CASES AND DISCUSSIONS OF σ_{1pp}, σ_{2pp}

Let us assume that 2L can be chosen so large that within the region of convergence, 2L>> α,β for the integrals in σ_{1pp}. If so, neglecting the edge effect terms we can rewrite σ_{1pp} as

$$\sigma_{1pp} = \frac{k^2}{4\pi} \int_{-2L}^{2L} \int_{-2L}^{2L} \left| A_{pp} + B_{pp} \tan\theta \right|^2 e^{-j v_x \alpha - K(1-\rho)} \, d\alpha \, d\beta . \tag{12}$$

For isotropically rough surface, (12) reduces to

$$\sigma_{1pp} = \frac{k^2}{2} \int_0^{2L} \left| A_{pp} + B_{pp} \tan\theta \right|^2 J_0 (v_x \xi) \, e^{-K(1-\rho)} \, \xi \, d\xi \tag{13}$$

where Jo ( ) is the zero order Bessel function of the first kind. Eq. (13) is the backscatter integral most often discussed in the literature. It is important to note that some of the conditions under which (8) reduces to (13) are loosely defined in terms of inequalities. Hence, the precise region of validity for (13) remains obscure.

It is interesting to note that as L goes to infinity and for sufficiently small K (i.e. small σ/λ), (8) may be approximated, except for a specular-type term, by

$$\sigma_{1pp} = \frac{k^2 K}{2} \left| A_{pp} \right|^2 \int \rho(\xi) \, J_0 (2 k \xi \sin\theta) \, \xi \, d\xi .$$

Thus,

$$\sigma_{1hh} = 8 \, k^4 \sigma^2 \cos^4\theta \left| R_{\perp} \right|^2 W (2 k \sin\theta)$$

$$\sigma_{1vv} = 8 \, k^4 \sigma^2 \cos^4\theta \left| R_{\parallel} \right|^2 W (2 k \sin\theta)$$

where σ_{1hh} is seen to be identical with the first order predictions of the small perturbation theory, [2,8] when the surface under consideration satisfies both assumptions of the Kirchhoff's theory and the small perturbation theory. However, this is not the case with σ_{1vv} indicating that different approaches need not lead to the same results, because the degrees of approximation for different theories are different.

If the small irregularities are absent, $\sigma_{2pp} = 0$ and σ_{pp} reduces to σ_{1pp} as expected.

If the large undulations are absent, i.e. $Z(x,y) = 0$, then σ_{1pp} becomes

$$\sigma_{1pp} = \frac{k^2}{4\pi}\int_{-2L}^{2L}\int_{-2L}^{2L}\frac{(2L-|\alpha|)(2L-|\beta|)}{(2L)^2}\left|A_{pp}\right|^2 e^{-jv_x\alpha}\, d\alpha\, d\beta$$

$$= \frac{k^2|A_{pp}|^2}{4\pi}\int_{-2L}^{2L}(2L-|\alpha|)\, e^{-jv_x\alpha}\, d\alpha$$

which is a specular-type term that behaves like sin x/x.

With $Z(x,y) = 0$, σ_{2pp} becomes

$$\sigma_{2pp} = \frac{k^2\sigma_1^2}{2}\int_{-2L}^{2L}\int_{-2L}^{2L}\frac{(2L-|\alpha|)(2L-|\beta|)}{(2L)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big\{|TC_{pp}|^2$$

$$W(u-k\sin\theta, v)\exp\left[-j(u+k\sin\theta)\alpha - jv\beta\right]\Big\}\, du\, dv\, d\alpha\, d\beta .$$

If 2L can be taken to be infinity, σ_{2pp} reduces to

$$\sigma_{2pp} = 2(\pi k\sigma_1)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|TC_{pp}|^2 W(u-k\sin\theta, v)\,\delta(u+k\sin\theta)\,\delta(v)\, du\, dv. \tag{14}$$

Thus,

$$\sigma_{2vv} = 8k^4\sigma_1^2\left|R_{\parallel}\cos^2\theta + T_{\parallel}^2(k'^2-k^2)\sin^2\theta/2k'^2\right|^2 \cdot W(2k\sin\theta, 0) \tag{15a}$$

$$\sigma_{2hh} = 8k^4\sigma_1^2\cos^4\theta\,|R_{\perp}|^2\, W(2k\sin\theta, 0). \tag{15b}$$

The above results are identical with the first order results obtained by the small perturbation method [2,8] (Appendix V). Note that 2L must be taken to be infinity for σ_{2pp} to reduce to the perturbation results because the mean plane for the perturbation model is an infinite flat plane.

Let us now examine the first term in σ_{2pp} when Z(x,y) is not zero and when 2L can be taken to be infinity. It has the form

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |TC_{pp}|^2 W(u-k\sin\theta, v)$$

$$\exp[-j(u+k\sin\theta)\alpha - jv\beta - K(1-\rho)]\, du\, dv\, d\alpha\, d\beta \qquad (16)$$

The variables u, v may be interpreted as the wave numbers or frequency components for the surface s(x,y). Comparison between (14) and (16) shows that in (14) only a specific pair of u,v values is allowed whereas in (16) all values of u,v are required. This means that the large undulations are responsible for making all frequency components of s(x,y) effective in the scattering process. They also define the appropriate weighting (through K and $\rho(\alpha,\beta)$) on the contributions of the different frequency components of s(x,y). Similar statements can be made about other terms in σ_{2pp} except that they vanish with Z(x,y). Thus, σ_{2pp} is seen to define explicitly the interaction between the large and the small scatterers. This interaction vanishes when either s(x,y) or Z(x,y) is zero. Scattering theories with the non-coherent assumption have this interaction replaced by averaging (15) using the slope distribution of Z(x,y). (The dependence of (15) upon the slopes of Z(x,y) arises when the incident angle is taken to be the local incident angle.) Hence, in such theories the nature of the interaction is assumed rather than calculated.

If the α,β -integrals in σ_{2pp} converge fast enough so that within the region of convergence $2L >> \alpha,\beta$ and if, in addition, edge effects are negligible, then

$$\sigma_{2pp} = \frac{k^2\sigma_1^2}{2}\int_{-2L}^{2L}\int_{-2L}^{2L}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Big\{\Big\{ |TC_{pp} + G(TD_{pp} + R'C_{pp})|^2 W$$

$$+ V_z \frac{\partial W}{\partial q} \operatorname{Re}\Big(GTC_{pp}[TC_{pp} + G(TD_{pp} + R'C_{pp})]^*\Big)$$

$$+ \Big|TGC_{pp}\Big|^2 V_z^2 \frac{\partial^2 W}{\partial q^2}\Big/4 \Big\}$$

$$\exp[-j(u+k\sin\theta)\alpha - jv\beta - K(1-\rho)\Big\}\, du\, dv\, d\alpha\, d\beta \qquad (17)$$

where $G = (u+k \sin \Theta) / v_z$.

For isotropically rough surfaces, (17) may be further reduced to

$$\sigma_{2pp} = \pi k^2 \sigma_1^2 \int_0^{2L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Big\{ \left| T C_{pp} + G\,(T D_{pp} + R' C_{pp}) \right|^2 W$$

$$+ v_z \frac{\partial W}{\partial q} \operatorname{Re}\Big(T G C_{pp} \left[T C_{pp} + G\,(T D_{pp} + R' C_{pp}) \right]^* \Big)$$

$$+ \left| T G C_{pp} \right|^2 v_z^2 \frac{\partial^2 W}{\partial q^2} \Big/ 4 \Big\} J_0 \left(\xi \sqrt{(u + k \sin\theta)^2 + v^2} \right) e^{-K(1-\rho)}$$

$$\cdot\ du\ dv\ \xi\ d\xi\,. \tag{18}$$

To compute the backscattering characteristics with (13) and (18), $\rho(\xi)$ and $W(\underline{K})$ need be specified. As an illustration, we assume that $\rho(\xi)$ is Gaussian and can be approximated by the first two terms of its series expansion about $\xi = 0$ and

$$W(\underline{K}) = (\ell^2/2)\ \exp\left[-\left(\underline{K}\,\ell/2\right)^2\right] \tag{19}$$

where ℓ is the correlation length of the surface, s(x,y). Under these assumptions (13) and (18) become (see Appendix IV)

$$\sigma_{1pp} = (8 m^2 \cos^2\theta)^{-1} \left| A_{pp} + B_{pp} \tan\theta \right|^2 \exp\left[-\tan^2\theta/(2m^2)\right] \tag{20}$$

$$\sigma_{2pp} = \pi k^2 \sigma_1^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Big\{ \left| T C_{pp} + G\,(T D_{pp} + R' C_{pp}) \right|^2 W$$

$$+ \operatorname{Re}\Big(\left[T C_{pp} + G\,(T D_{pp} + R' C_{pp}) \right] T^* C_{pp}^*\, G \Big) V_z \frac{\partial W}{\partial q} \Big\}$$

$$V_z^{-2} m^{-2} \exp\Big\{ -\left[(u + k \sin\theta)^2 + V^2\right] / (2 V_z^2 m^2) \Big\}\, du\, dv\,. \tag{21}$$

Figures 4 and 5 show the general angular behavior of σ_{hh} and σ_{vv} for different values of the rms slopes of Z(x,y) and $k\rho_1$ of s(x,y). Since the major difference between this theory and other scattering theories lies in σ_{2pp}; Figures 6 and 7, σ_{2vv} and σ_{2hh}, are plotted using (21). First order results from the small perturbation theory for a single surface given by (15) are also shown to provide a basis for comparison. In Appendix VI, all the identities used are rewritten for ease of reference.

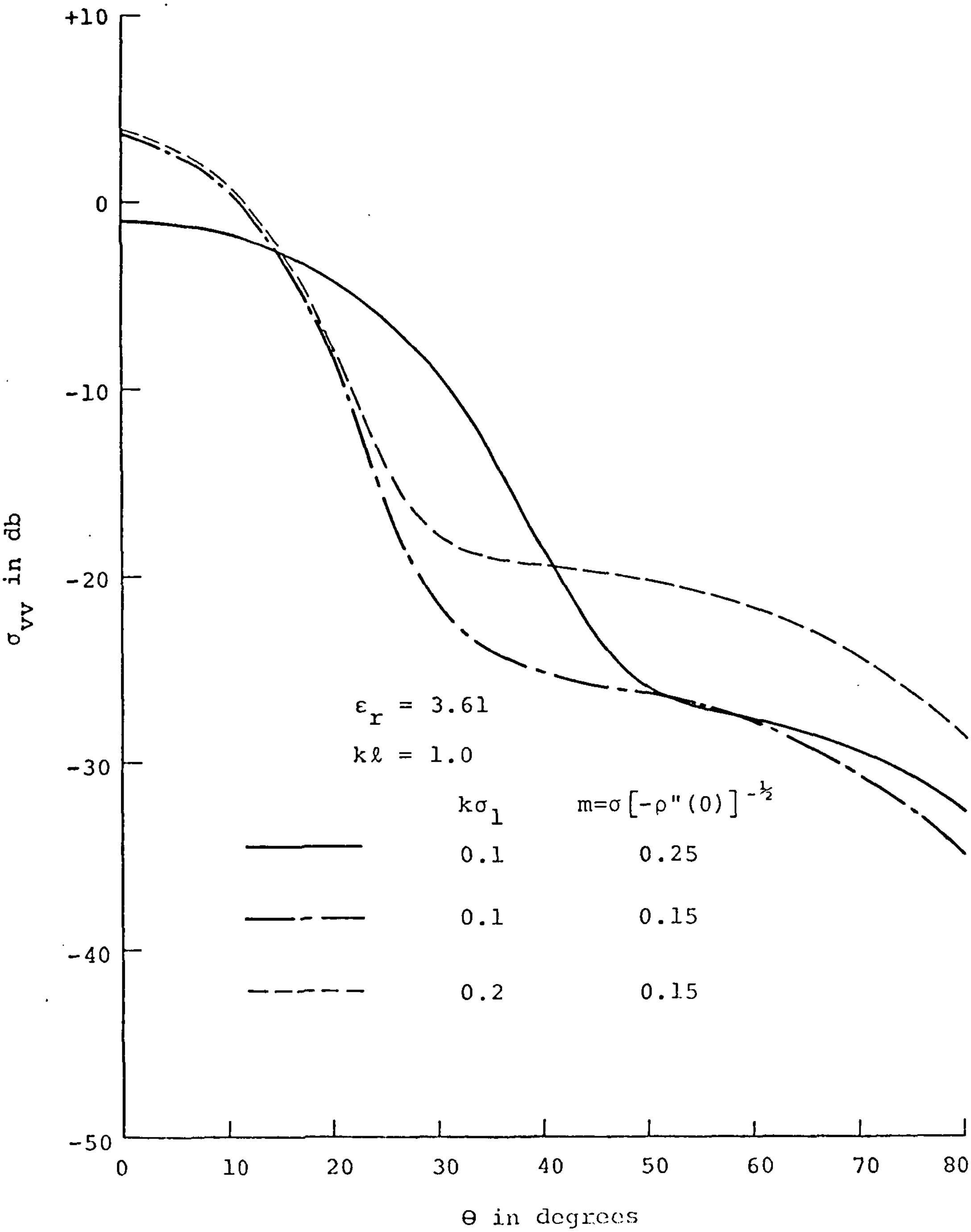

Fig. 4. Angular behavior of σ_{vv} as a function of the rms slope, m, and $k\sigma_1$.

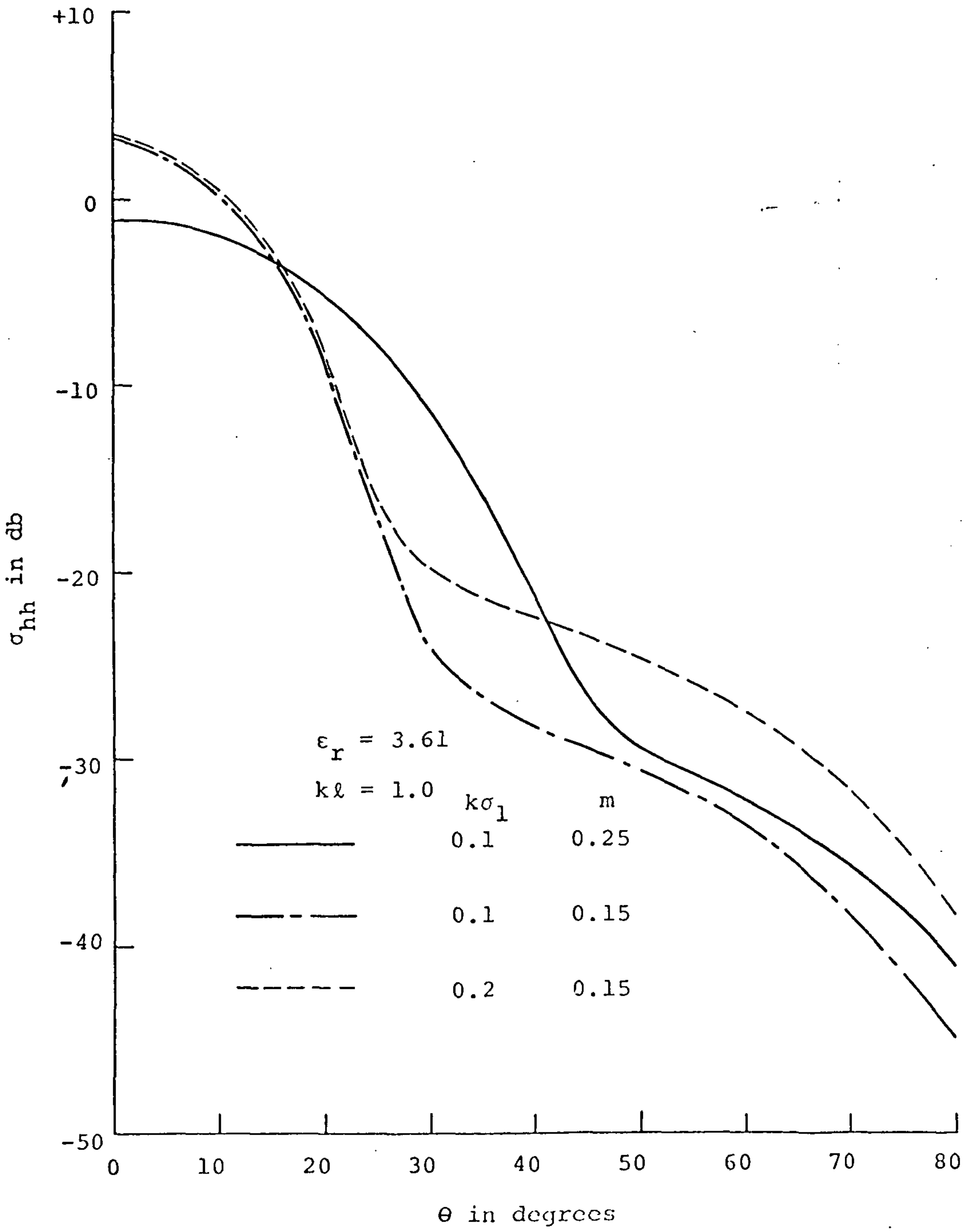

Fig. 5. Angular behavior of σ_{hh} as a function of the rms slope, m, and $k\sigma_1$.

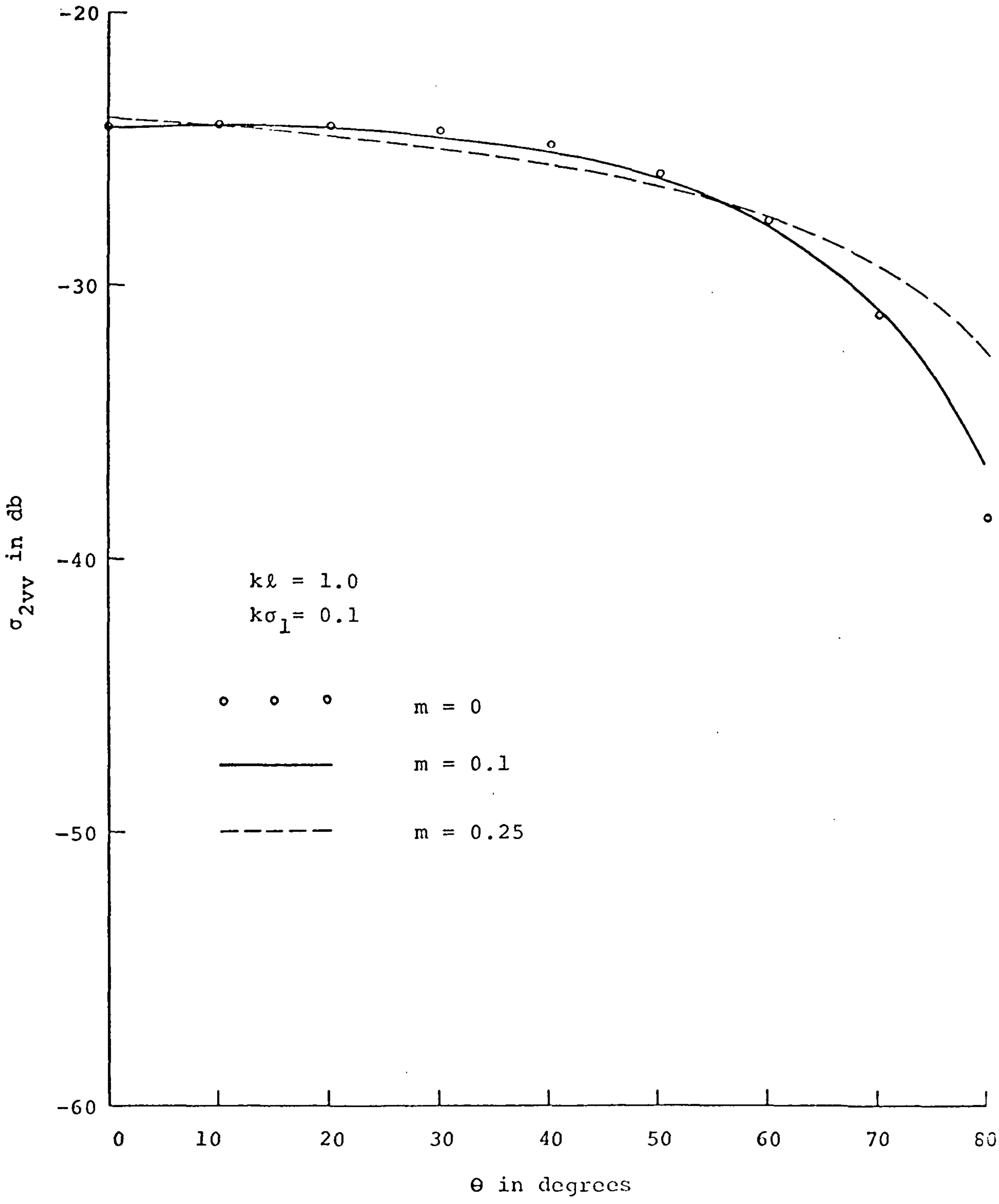

Fig. 6. Angular behavior of σ_{2vv} as a function of the rms slope (a) $\epsilon_r = 3.61$.

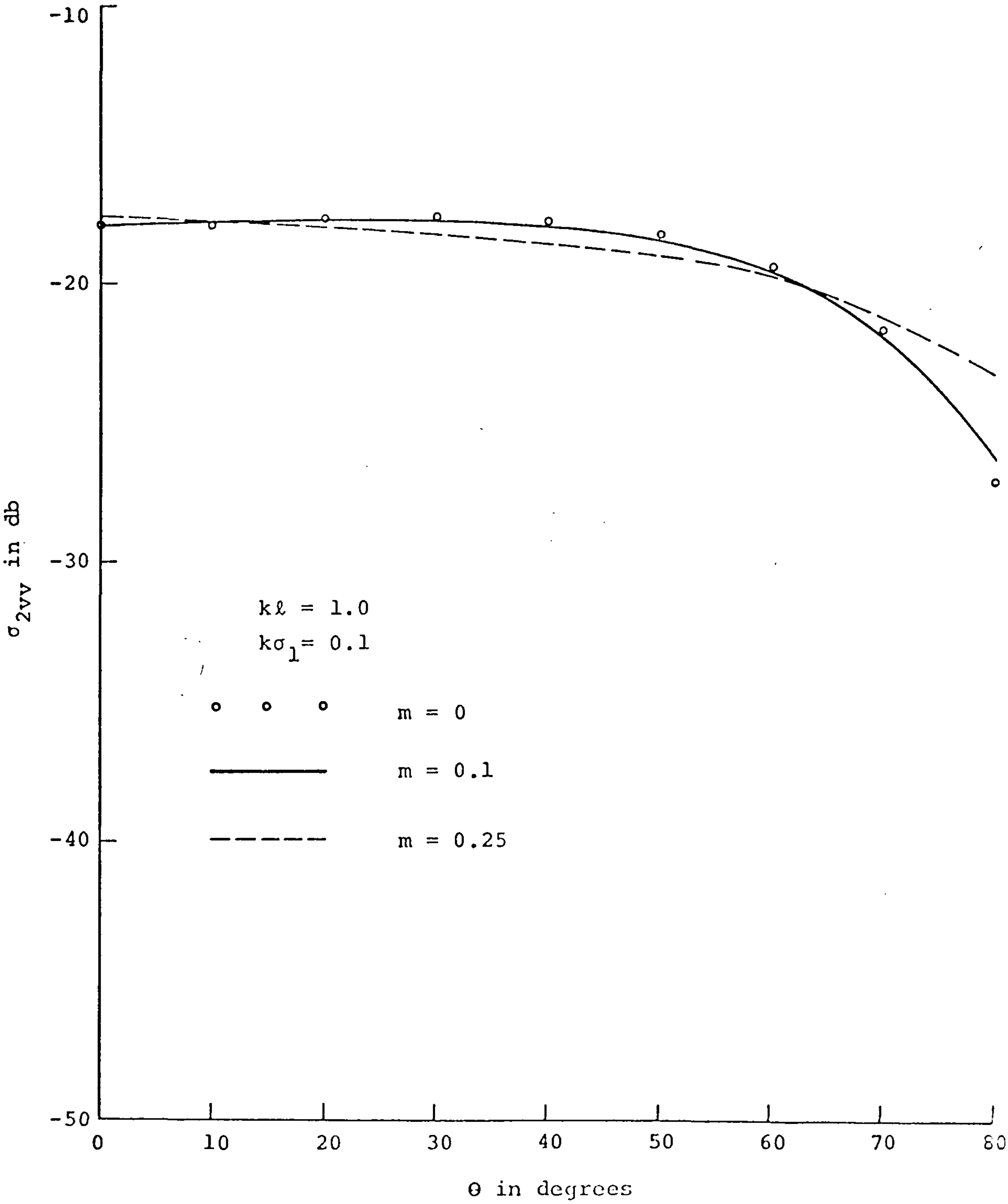

Fig. 6. Angular behavior of σ_{2vv} as a function of the rms slope (b) $\epsilon_r = 20$.

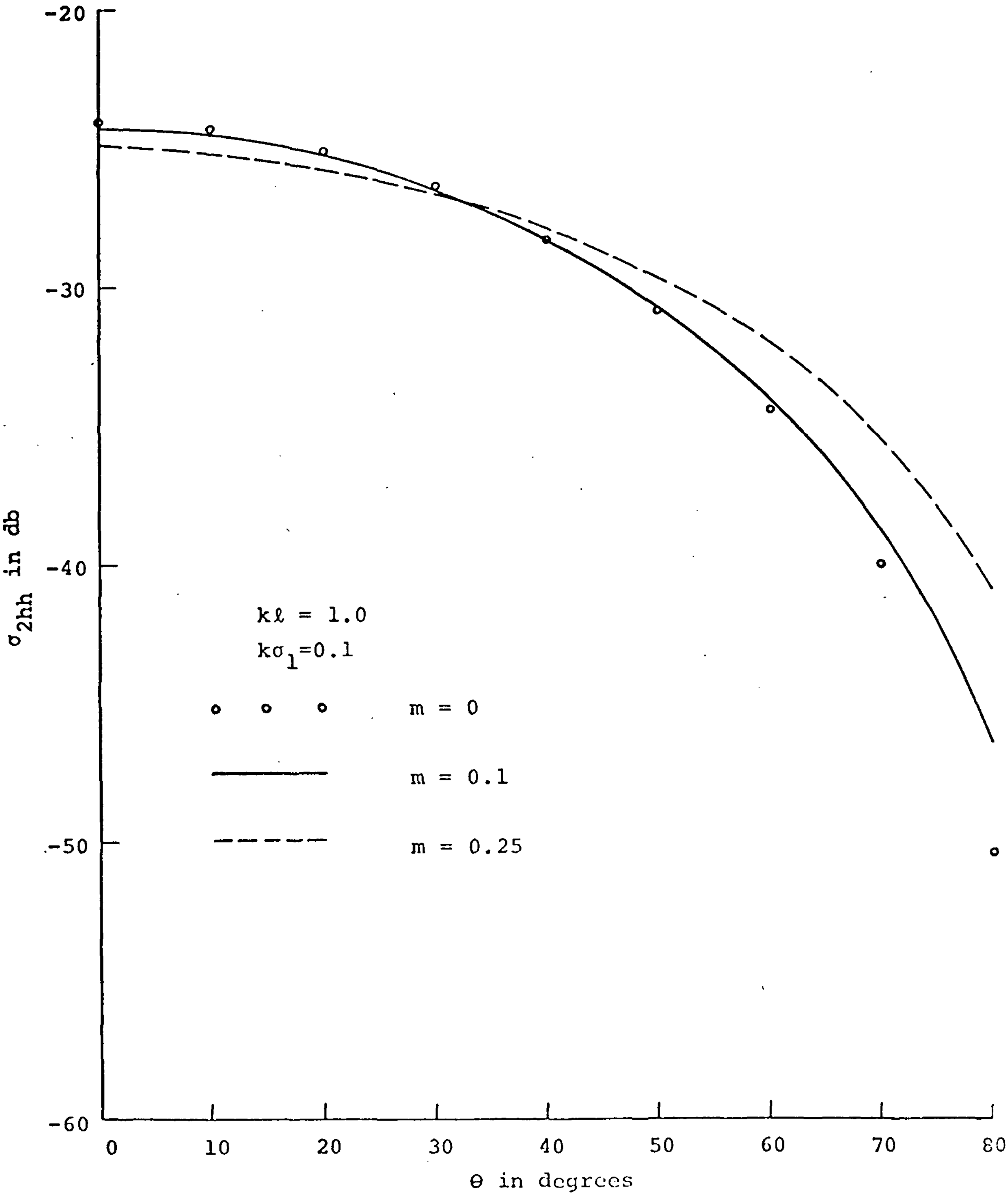

Fig. 7. Angular behavior of σ_{2hh} as a function of the rms slope (a) $\epsilon_r = 3.61$.

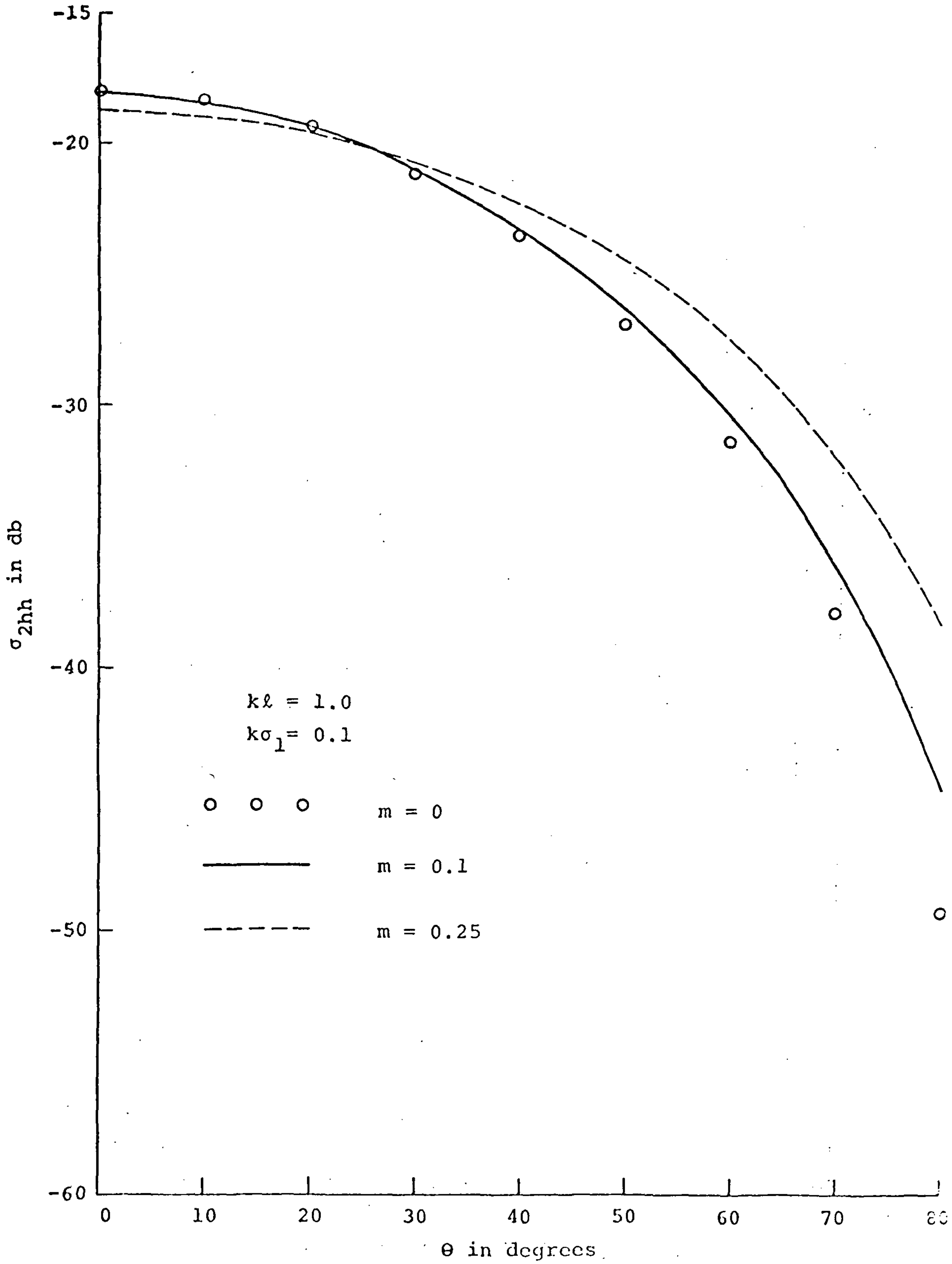

Fig. 7. Angular behavior of σ_{2hh} as a function of the rms slope (b) $\epsilon_r = 20$.

VI. COMPARISON WITH SEA CLUTTER MEASUREMENTS

As an illustration, only the results corresponding to an isotropic rough surface with Gaussian surface height distribution will be compared with the experimental sea data. The assumed surface model although not realistic for the ocean surface, is able to predict the correct angular trend of the backscattering coefficients for both horizontal and vertical polarizations with surface parameters of very reasonable sizes.

An examination of the backscattering coefficients, given by (20) and (21), indicates that the rms surface slope m of the large undulations not only affects the σ_{1pp} term, but also the σ_{2pp} term. So, it is clear that the large undulations influence the returns from the small irregularities as mentioned in the previous section. To suppress as much of the effect from shorter waves as possible, Cox and Munk's [9] mean-squared slope measurement in the presence of oil slicks is adopted to estimate m for different wind conditions (see Appendix VII).

It is generally agreed that the surface spectrum of the small irregularities should vary like $B\underline{K}^{-4}$ ($\underline{K}$ = surface wave number and B = constant). In comparison with experimental data, the value of kl of (19) is assigned to be 2 (Figure 8) so that the correct angular behavior of the Gaussian spectrum approximates $B\underline{K}^{-4}$ well over the angular range, 30° Θ 70°, i.e. $B\underline{K}^{-4}$ is approximated by (19) with $\underline{K} = 2k \sin \Theta$, the Bragg scatter condition. To bring the level into agreement at $\Theta = 60^\circ$, we multiply the Gaussian approximation by the factor of 35.3 . Since a complete information of the increase in intensity of the high frequency part of the sea spectrum is not yet available, the wind dependence of σ_1 cannot be uniquely determined. Oceanographic investigations indicate that the values of B lie in the interval $4.6 \times 10^{-3} \lesssim B \lesssim 3.26 \times 10^{-2}$ [9, 13, 14]. This implies that $k\sigma_1$ should lie in the range from 0.067 to 0.2 when $B\underline{K}^{-4}$ is equated to (19) at 60°. These values of $k\sigma_1$ are consistent with the assumptions of the small perturbation theory.

According to the above arguments, comparisons of computed [(20) and (21)] and measured (NRL) backscatter characteristics of X and L bands for wind speeds of 11-27 knots are shown in Figures 9 through 16. It shows fairly good agreement, especially for incident angles larger than 30°. Since questions have been raised about the accuracy of the absolute levels of the measured scattering coefficient curves from

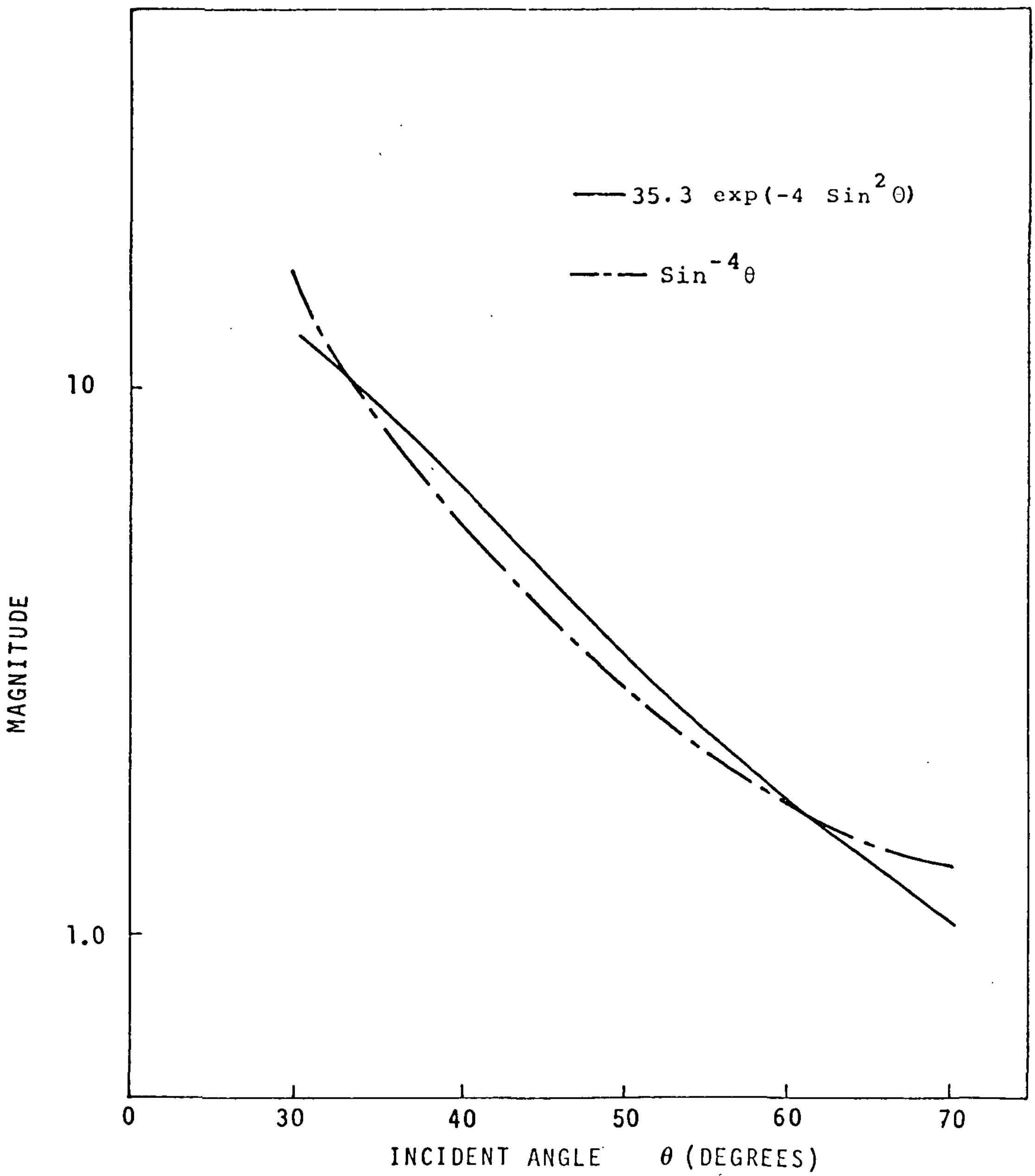

Fig. 8. Comparison of the angular variations of $\sin^{-4}\Theta$ and $35.3 \exp(-4 \sin^2\Theta)$.

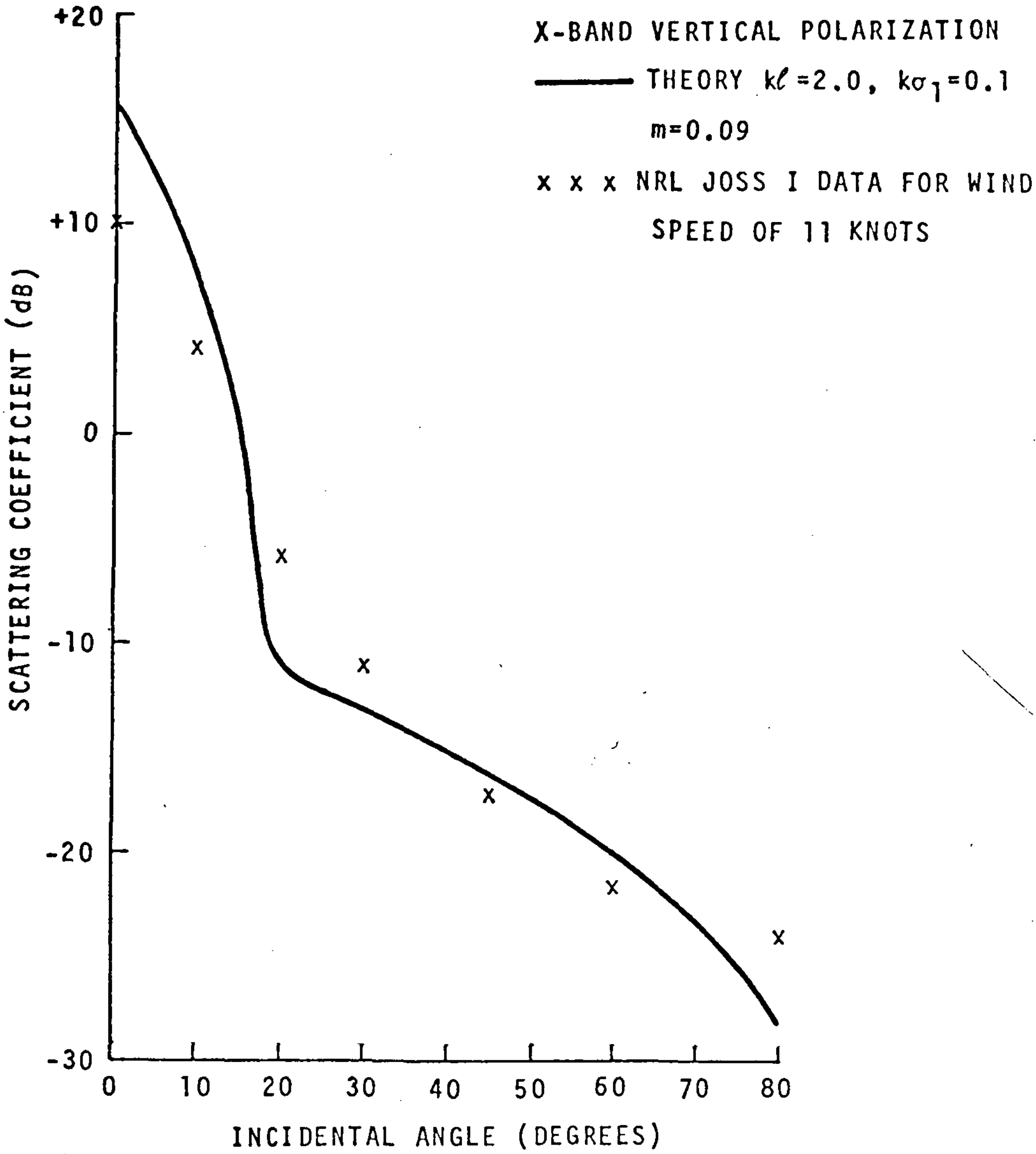

Fig. 9. Comparison of computed and measured backscatter characteristics.

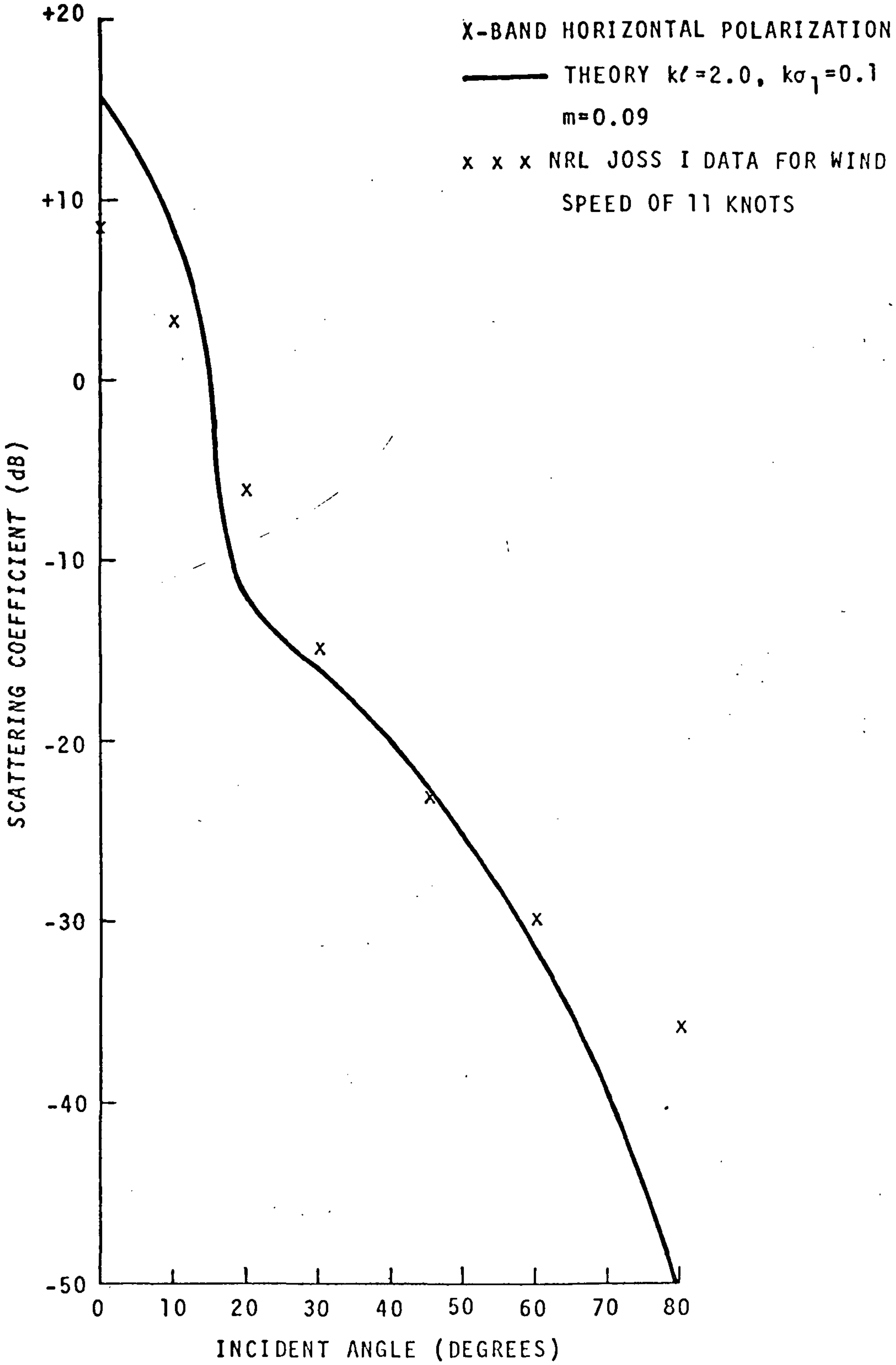

Fig. 10. Comparison of computed and measured backscatter characteristics.

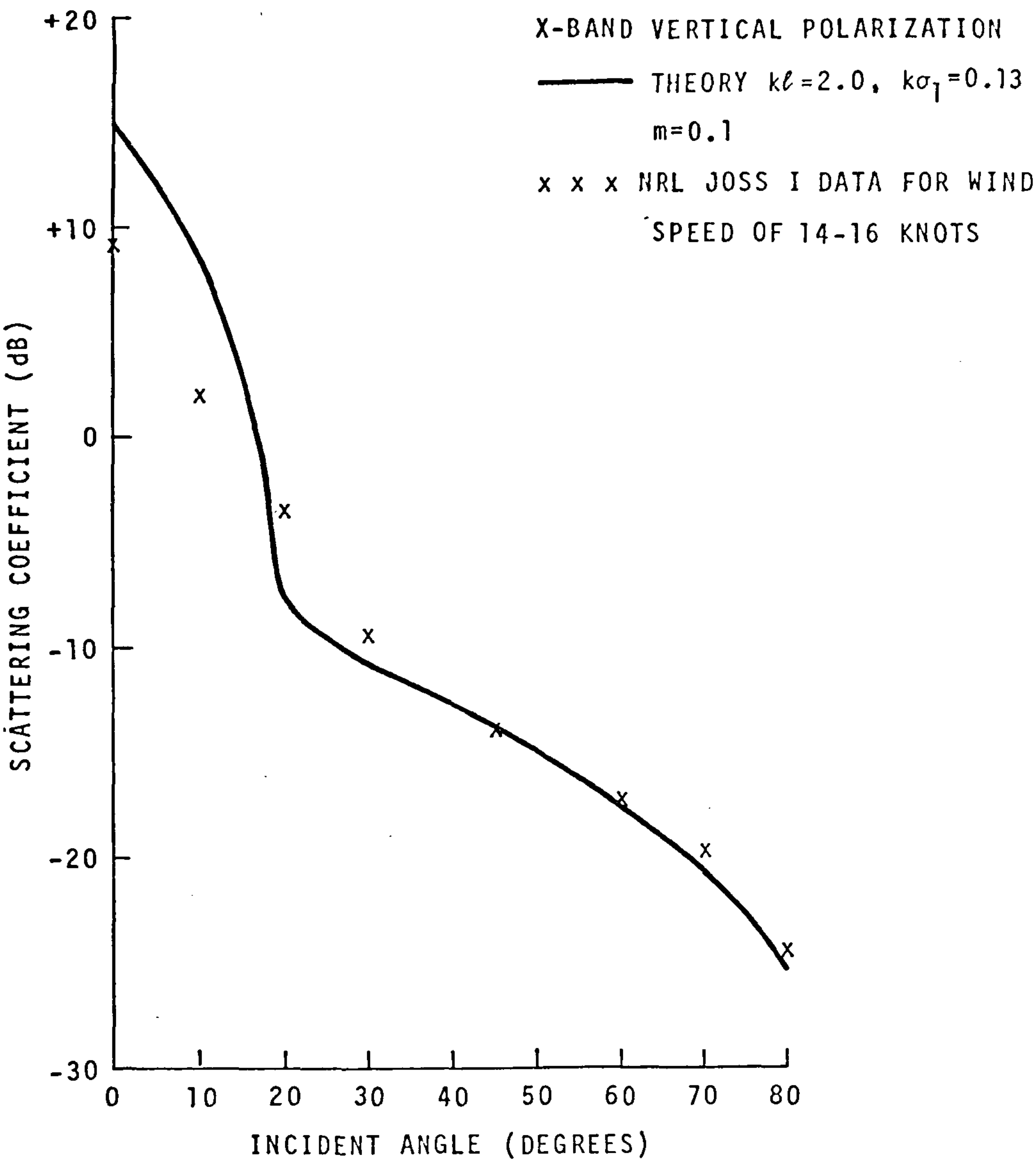

Fig. 11. Comparison of computed and measured backscatter characteristics.

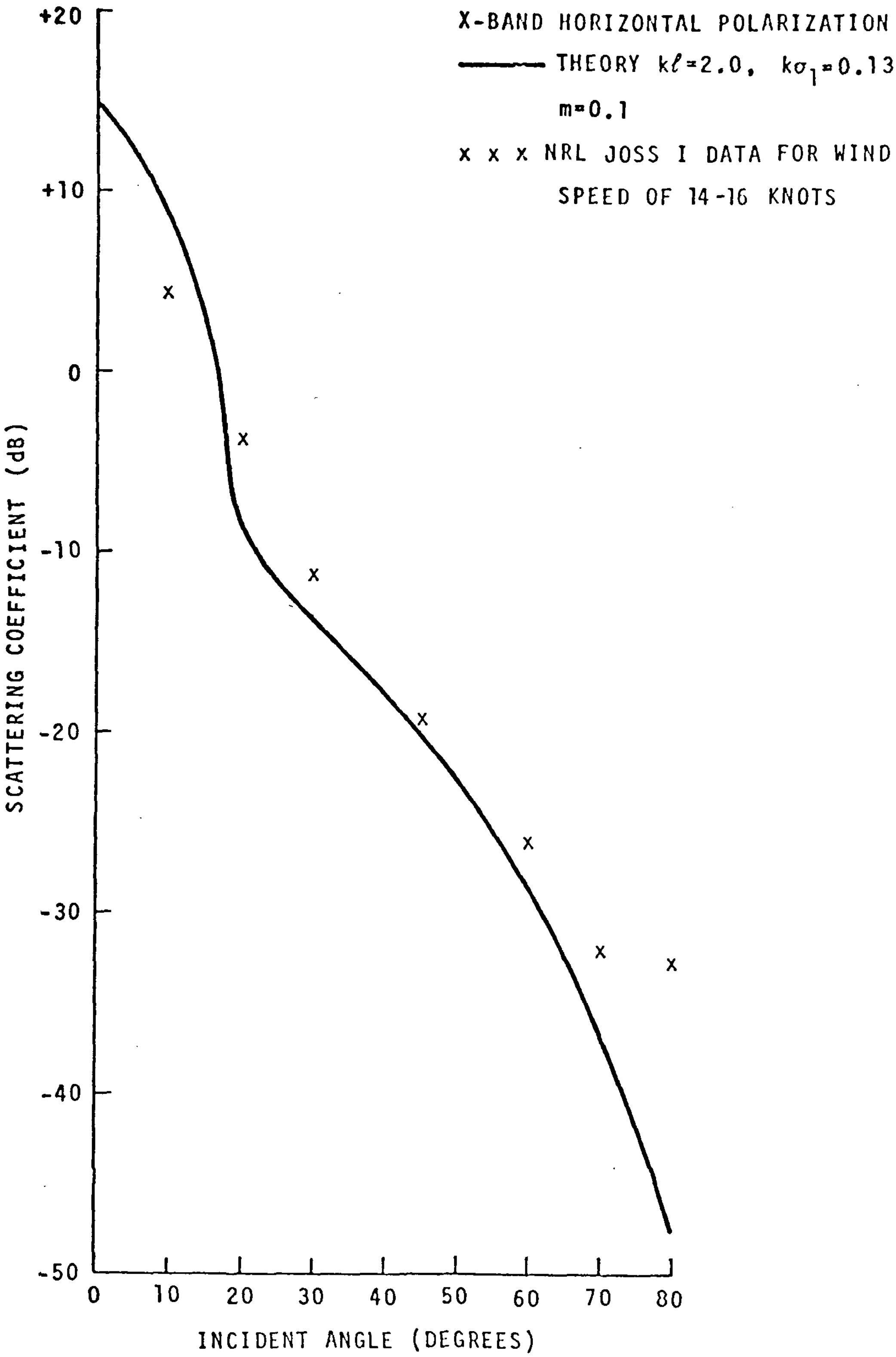

Fig. 12. Comparison of computed and measured backscatter characteristics.

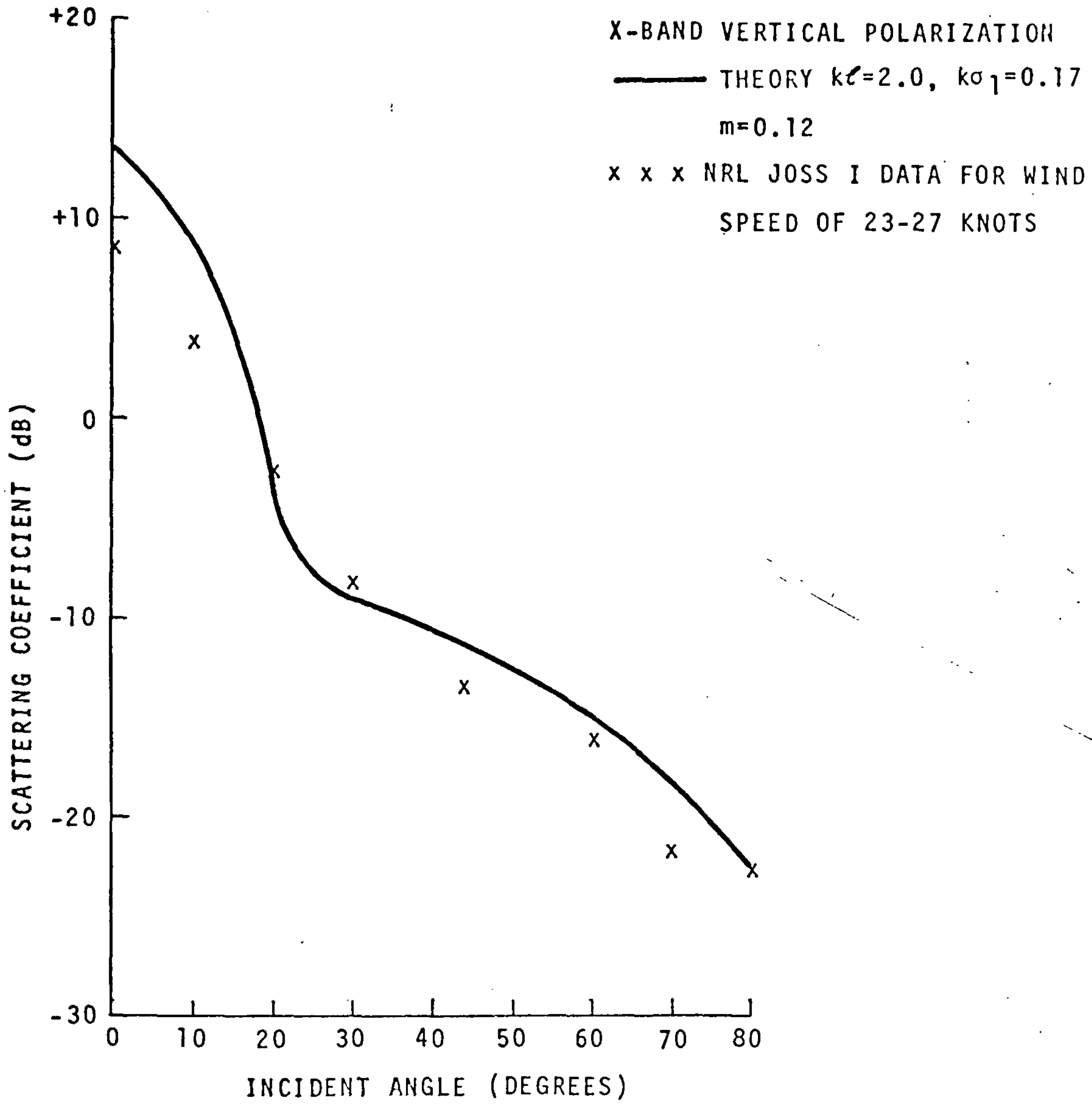

Fig. 13. Comparison of computed and measured backscatter characteristics.

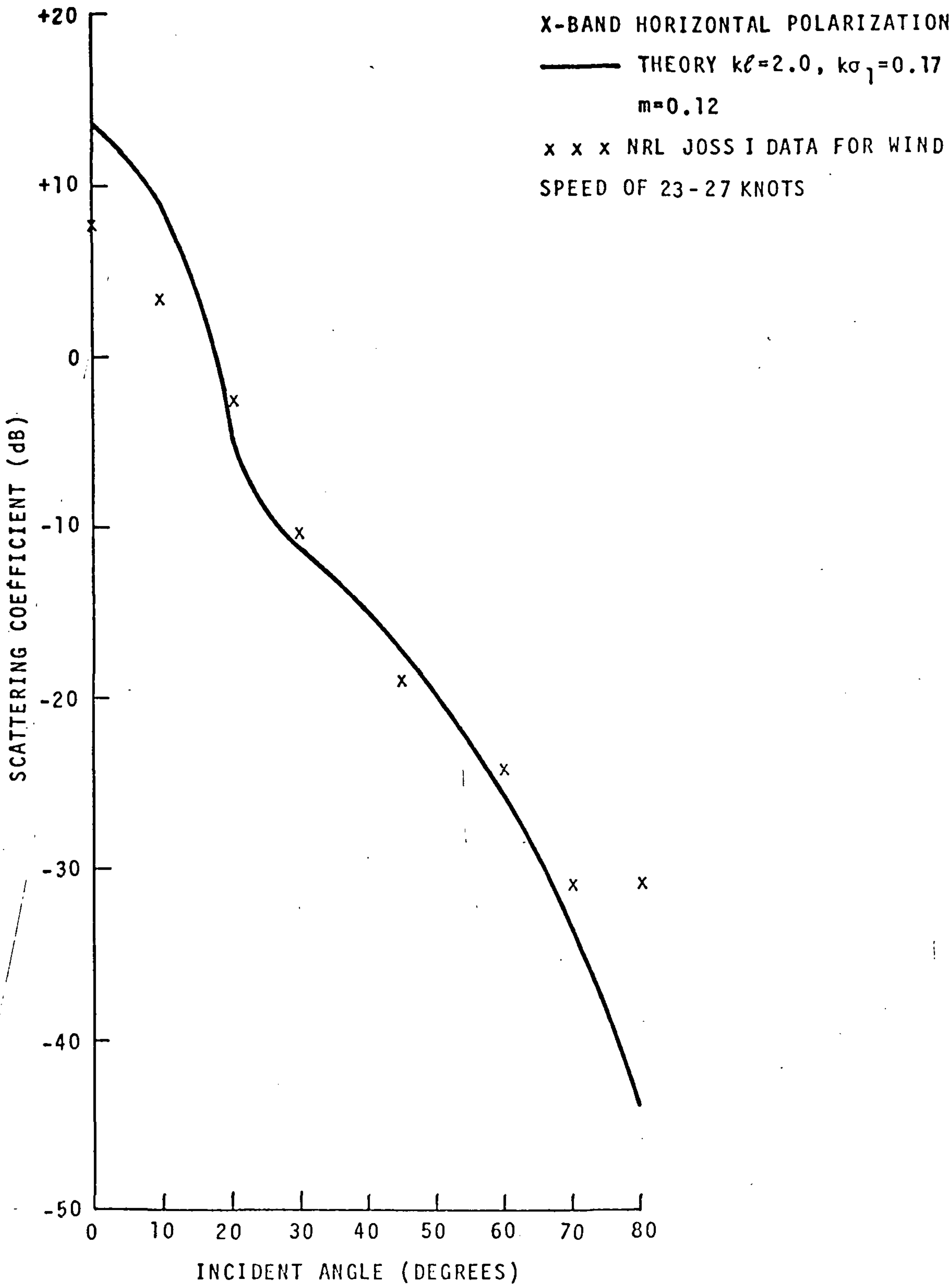

Fig. 14. Comparison of computed and measured backscatter characteristics.

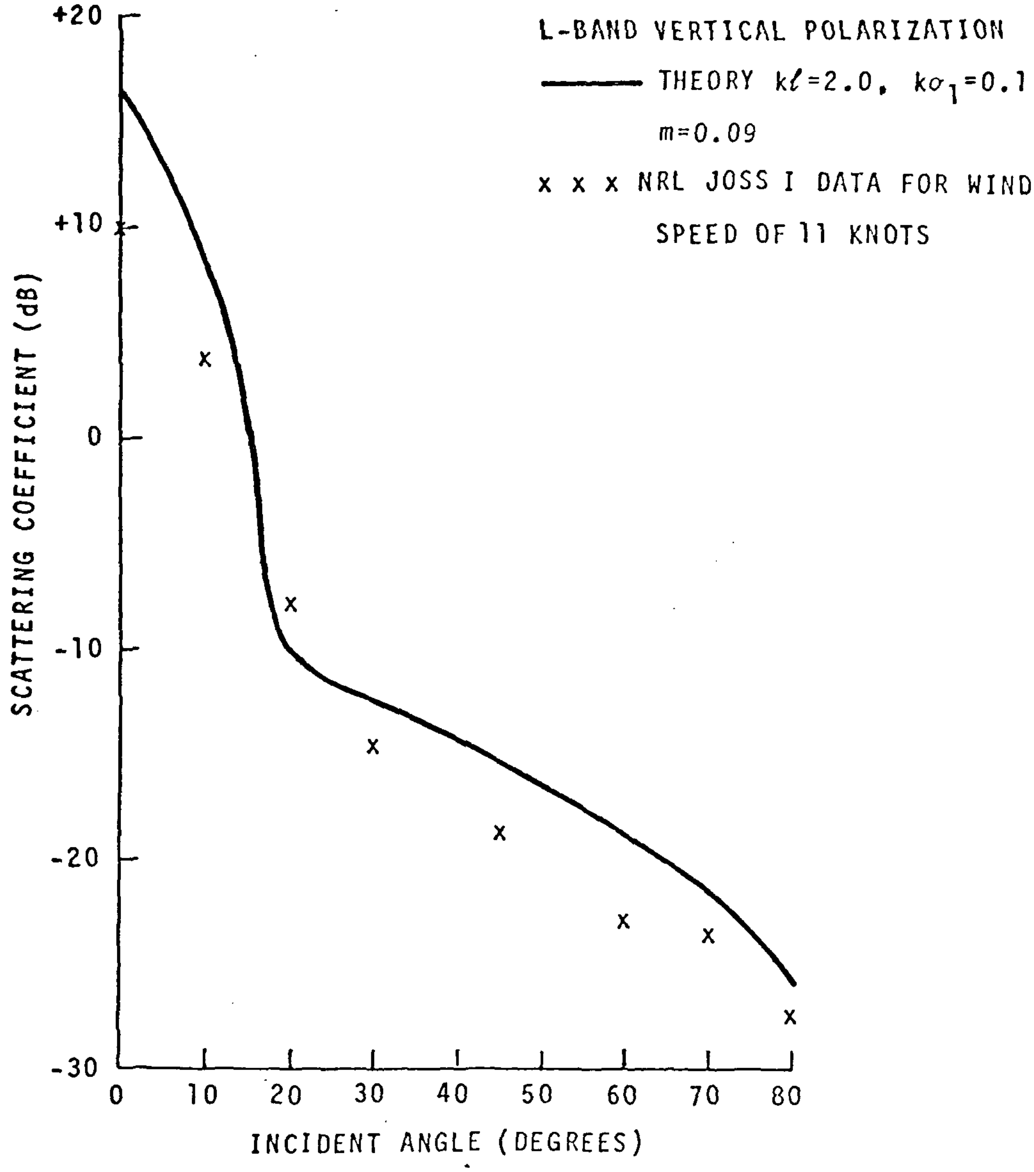

Fig. 15. Comparison of computed and measured backscatter characteristics.

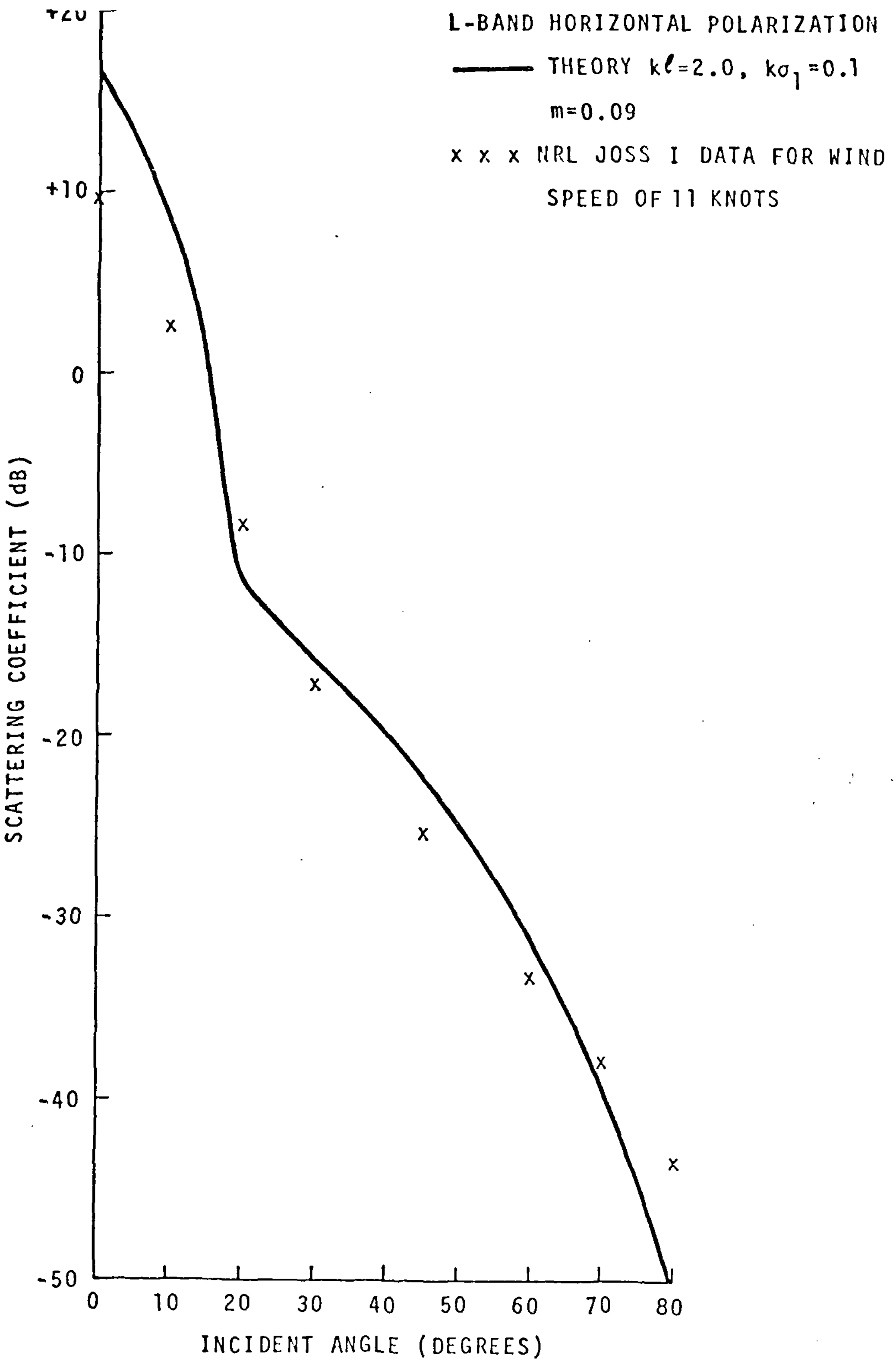

Fig. 16. Comparison of computed and measured backscatter characteristics.

different missions [15], the main intention of these comparisons is to show the angular agreement rather than the agreement in absolute values. However, to allow reference back to the measured data, all levels of the data have been raised by 6 db rather than arbitrarily adjusted for each wind speed to obtain better fit to the theoretical curves. Values of $k\sigma_1$ and m used in the calculation are as follows:

$k\sigma_1$	m	corresponding wind speed (knots)
0.1	0.09	11
0.13	0.1	$14 \sim 16$
0.17	0.12	$23 \sim 27$

It is noted that the value of $k\sigma_1$ increases with increasing wind speed. This observation is in agreement with recent studies of the sea spectrum [16,17]. For smaller incident angles more data points are needed to define the angular shape of the σ_{pp} curves. For this reason NASA/MSC data are chosen in Figure 17. Agreement is observed between measurement and theory with m determined by Cox and Munk's clean sea measurement. More experimental data are needed to explain the discrepancy in using slick and clean sea measurements.

On the basis of the above results, it appears that it is possible to determine the wind dependence of the scatterometric parameters m and σ_1. With sufficient experimental data these parameters may be determined more precisely for different wind speeds.

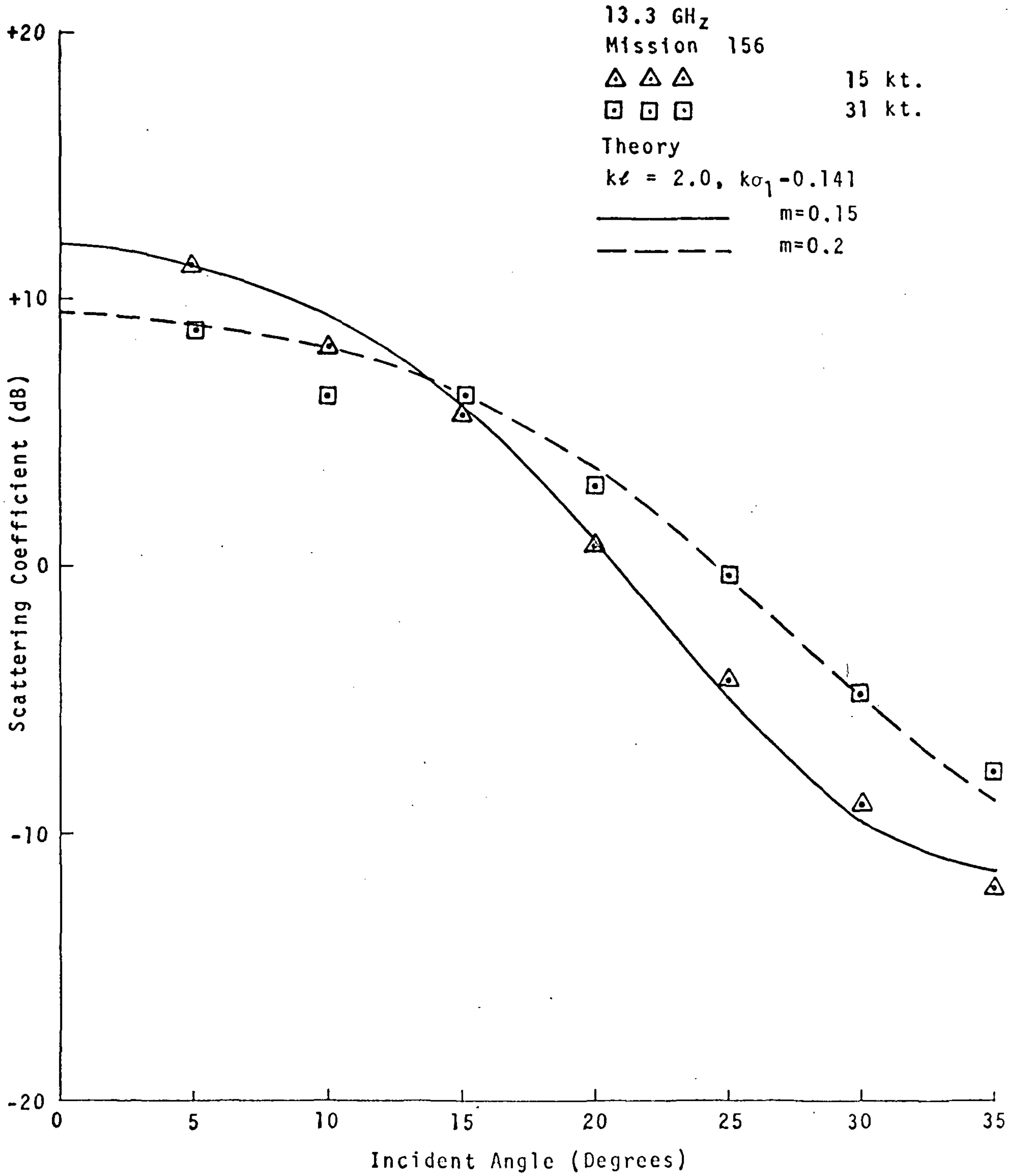

Fig. 17. Comparison of computed and measured backscatter characteristics.

VII. CONCLUSIONS

The results of the present theory indicate an explicit interaction between the large undulations and the frequency components of the small irregularities. As compared with σ_{1pp}, σ_{2pp} which represents this interaction decreases more slowly with the increase of the incident angle. This offers another possible explanation for what has been called the diffuse scattering portion of the angular curve.

APPENDIX I

THE BASIC SCATTERED FIELD EXPRESSION

In this appendix, an expression of the backscattered field in terms of the local field components on surface is derived. The starting point is the modified Stratton-Chu integral:

$$\underset{\sim}{E}_s = K_o \underset{\sim}{n}_2 \times \int [\underset{\sim}{n} \times \underset{\sim}{E} - \eta\, \underset{\sim}{n}_2 \times (\underset{\sim}{n} \times \underset{\sim}{H})] \exp(j\, k\, \underset{\sim}{r} \cdot \underset{\sim}{n}_2)\, ds \qquad \text{(I-1)}$$

Assume

$$\underset{\sim}{E} = \underset{\sim}{\bar{x}} E_{\bar{x}} + \underset{\sim}{\bar{y}} E_{\bar{y}} + \underset{\sim}{\bar{z}} E_{\bar{z}}$$

$$\underset{\sim}{H} = \underset{\sim}{\bar{x}} H_{\bar{x}} + \underset{\sim}{\bar{y}} H_{\bar{y}} + \underset{\sim}{\bar{z}} H_{\bar{z}}$$

where

$$\underset{\sim}{\bar{z}} = \underset{\sim}{n} / |\underset{\sim}{n}|$$
$$= (-\underset{\sim}{i} Z_x - \underset{\sim}{j} Z_y + \underset{\sim}{k})(1 + Z_x^2 + Z_y^2)^{-\frac{1}{2}}$$

$$\underset{\sim}{\bar{y}} = (\underset{\sim}{\bar{z}} \times \underset{\sim}{n}_1)/D_o$$
$$\approx [\underset{\sim}{i} Z_y \cos\theta + \underset{\sim}{j}(\sin\theta - Z_x \cos\theta) + \underset{\sim}{k} Z_y \sin\theta]\, D_o^{-1} \cdot (1 + Z_x^2 + Z_y^2)^{-\frac{1}{2}}$$

$$\underset{\sim}{\bar{x}} = \underset{\sim}{\bar{y}} \times \underset{\sim}{\bar{z}}$$
$$\approx (\underset{\sim}{i} - \underset{\sim}{j} Z_y \cos\theta / D_o + \underset{\sim}{k} Z_x)(1 + Z_x^2 + Z_y^2)^{-1}$$

$$D_o = |\underset{\sim}{\bar{z}} \times \underset{\sim}{n}_1| = [Z_y^2 + (\sin\theta - Z_x \cos\theta)^2]^{\frac{1}{2}}.$$

If Z_x^2, $Z_y^2 << 1$, the local unit coordinate vectors may be approximated as follows:

$$\underset{\sim}{\bar{x}} \approx \underset{\sim}{i} - \underset{\sim}{j} Z_y \cos\theta / \sin\theta' + \underset{\sim}{k} Z_x$$

$$\underset{\sim}{\bar{y}} \approx \underset{\sim}{i} Z_y \cos\theta/\sin\theta' + \underset{\sim}{j} + \underset{\sim}{k} Z_y \sin\theta/\sin\theta'$$

$$\underset{\sim}{\bar{z}} \approx -\underset{\sim}{i} Z_x - \underset{\sim}{j} Z_y + \underset{\sim}{k} \qquad \text{(I-2)}$$

with

$$\sin\theta' \approx \sin\theta - Z_x \cos\theta \quad .$$

For backscattering (i.e. $\underset{\sim}{n}_2 = -\underset{\sim}{n}_1 = -\underset{\sim}{i}\sin\Theta + \underset{\sim}{k}\cos\Theta$) the following relations may be obtained

$$\underset{\sim}{n} \times \underset{\sim}{E} = \underset{\sim}{\tilde{y}}\, E_{\tilde{x}} - \underset{\sim}{\tilde{x}}\, E_{\tilde{y}}$$

$$\underset{\sim}{n}_1 \times \underset{\sim}{\tilde{x}} = -(\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta)\, Z_y \cos\theta/\sin\theta' - \underset{\sim}{j}\cos\theta'$$

$$\underset{\sim}{n}_1 \times \underset{\sim}{\tilde{y}} = (\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta) - \underset{\sim}{j}\, Z_y/\sin\theta'$$

with

$$\cos\theta' \approx \cos\theta + Z_x \sin\theta$$

$$\begin{aligned}\underset{\sim}{n}_2 \times (\underset{\sim}{n} \times \underset{\sim}{E}) &= -\underset{\sim}{n}_1 \times (\underset{\sim}{\tilde{y}}\, E_{\tilde{x}} - \underset{\sim}{\tilde{x}}\, E_{\tilde{y}}) \\ &= -[(\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta)(E_{\tilde{x}} + Z_y\cos\theta\, E_{\tilde{y}}/\sin\theta') \\ &\quad + \underset{\sim}{j}(\cos\theta' E_{\tilde{y}} - Z_y E_{\tilde{x}}/\sin\theta')] \quad .\end{aligned} \qquad (I-3)$$

Similarly,

$$\begin{aligned}\underset{\sim}{n}_2 \times [\underset{\sim}{n}_2 \times (\underset{\sim}{n} \times \underset{\sim}{H})] &= (\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta)(\cos\theta' H_{\tilde{y}} - Z_{\tilde{y}} H_{\tilde{x}}/\sin\theta') \\ &\quad - \underset{\sim}{j}(H_{\tilde{x}} + Z_y\cos\theta\, H_{\tilde{y}}/\sin\theta').\end{aligned} \qquad (I-4)$$

Note that

$$(\underset{\sim}{i}\sin\theta - \underset{\sim}{k}\cos) \times (\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta) = -\underset{\sim}{j}$$

$$\underset{\sim}{n}_1 \times \underset{\sim}{j} = (\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta) \; .$$

Substituting (I-3) and (I-4) into (I-1) we get the backscattered field as

$$\begin{aligned}\underset{\sim}{E}_s = -K_o \int_{-L}^{L}\int_{-L}^{L} \{ &\underset{\sim}{j}[(E_{\tilde{y}}\cos\theta' - \eta H_{\tilde{x}}) - (E_{\tilde{x}} + \eta H_{\tilde{y}}\cos\theta) Z_y/\sin\theta'] \\ &+ (\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta)[(E_{\tilde{x}} + \eta H_{\tilde{y}}\cos\theta') + (E_{\tilde{y}}\cos\theta - \eta H_{\tilde{x}}) Z_y/\sin\theta'] \\ &\cdot \exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r})\, dx\, dy\end{aligned} \qquad (I-5)$$

where

$$\underset{\sim}{k}_1 = k\, \underset{\sim}{n}_1 \quad .$$

By using (I-2) it is possible to find the local incident fields. Thus, for a horizontally polarized incident field of the form $\underset{\sim}{j}\exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r})$, there correspond two local fields, i.e.

$$\underset{\sim}{j}\exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r}) \approx \underset{\sim}{\bar{y}}\exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r}) - (\underset{\sim}{\bar{x}}\cos\theta' + \underset{\sim}{\bar{z}}\sin\theta')\frac{Z_y}{\sin\theta'} \cdot \exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r}) . \tag{I-6}$$

Similarly, a vertically polarized incident field of the form $(\underset{\sim}{i}\cos\Theta + \underset{\sim}{k}\sin\Theta)\exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r})$ may be decomposed into two local fields, i.e.

$$\begin{aligned}(\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta)\exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r}) \\ = (\underset{\sim}{\bar{x}}\cos\theta' + \underset{\sim}{\bar{z}}\sin\theta')\exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r}) \\ + \underset{\sim}{\bar{y}}(Z_y/\sin\theta')\exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r}) .\end{aligned} \tag{I-7}$$

APPENDIX II

FIELDS ON SURFACE Z(x,y)

A. Horizontally Polarized Case

From the small perturbation theory, the total scattered E-field up to the first order in space may be written

$$E_{\bar{x}}(\bar{x},\bar{y},\bar{z}) = \iint T_\perp u v Q' E_x \, du \, dv$$
$$\equiv E_{\bar{x}1} \tag{II-1a}$$

$$E_{\bar{y}}(\bar{x},\bar{y},\bar{z}) = [\exp(jk\bar{z}\cos\theta') + R_\perp \exp(-jk\bar{z}\cos\theta')]$$
$$\cdot \exp(-jk\bar{x}\sin\theta') - \iint T_\perp (u^2 + cb) Q' E_x \, du \, dv$$
$$\equiv E_{\bar{y}o} + E_{\bar{y}1} \tag{II-1b}$$

where

$$E_x = \exp(-ju\bar{x} - jv\bar{y} - jk\cos\theta\,\bar{z})$$
$$Q' = j(k'^2 - k^2) S(u - k\sin\theta', v)(2\pi D)^{-1}$$
$$D = k^2 c + k'^2 b$$
$$T_\perp = 1 + R_\perp .$$

To obtain the zero order E-field on surface Z(x,y) we may apply the tangent plane approximation. Thus, in local coordinates its value at any point on surface is

$$E_{\bar{y}o}(0,0,0) = 1 + R_\perp .$$

To obtain the correct phase relationship between points in (x,y,z) coordinates, the total zero order E-field at a point (x,y) on surface is expressed as

$$E_{\bar{y}o}(x,y,z) = (1 + R_\perp) \exp(-j \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) . \tag{II-2}$$

The form of the first order field is not similar to the zero order in that it does not contain the incident field. Thus, taking E_{x1} as an example, we find the value of the field on surface in local coordinates to be

$$E_{\bar{x}1}(0,0,0) = \iint T_{\perp}\, u v\, Q'\, du\, dv \quad .$$

Following Bass and Bacharov, the total first order field on surface in (x,y,z) coordinates is

$$E_{\bar{x}1}(x,y,z) = \iint T_{\perp}\, u v\, Q' e^{-jux - jvy - jkz\cos\theta}\, du\, dv \quad . \tag{II-3}$$

Thus, the complete set of fields on surface in (x,y,z) coordinates is

$$E_{\bar{x}}(x,y,z) = \iint T_{\perp}\, u v\, Q' EX\, du\, dv \tag{II-4a}$$

$$E_{\bar{y}}(x,y,z) = (1+R_{\perp})\exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) - \iint T_{\perp}(u^2+bc)\, Q' EX\, du\, dv \tag{II-4b}$$

$$\eta H_{\bar{x}}(x,y,z) = \cos\theta'(1-R_{\perp})\exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) + \iint T_{\perp}\left[u^2(b-c)/k + kc\right] Q' EX\, du\, dv \tag{II-4c}$$

$$\eta H_{\bar{y}}(x,y,z) = \iint T_{\perp}\left[uv(b-c)\, Q'/k\right] Q' EX\, du\, dv \tag{II-4d}$$

where

$$EX = \exp(-jux - jvy - jkz\cos\theta) . \tag{II-5}$$

B. Vertically Polarized Case

For a vertically polarized incident plane wave of the form

$$\underset{\sim}{E} = (\underset{\sim}{i}\cos\theta + \underset{\sim}{k}\sin\theta)\exp[-jk(x\sin\theta - z\cos\theta)] \tag{II-6}$$

the local fields on Z(x,y) up to the first order are

$$E_{\bar{x}} = \cos\theta'(1-R_{\parallel})\exp(-j\, \underset{\sim}{k}_1 \cdot \underset{\sim}{r}) + \iint T_{\parallel} Q'\left[bu\sin\theta' - (v^2+bc)\, k\cos\phi'/k'\right] EX\, du\, dv \tag{II-7a}$$

$$E_{\bar{y}} = \iint T_{\parallel} Q' v \left[b \sin\theta' + uk\cos\phi'/k' \right] EX\, du\, dv \tag{II-7b}$$

$$\eta H_{\bar{x}} = \iint T_{\parallel} Q' v \left[u(c-b)\cos\phi'/k' - k\sin\theta' \right] EX\, du\, dv \tag{II-7c}$$

$$\eta H_{\bar{y}} = -(1+R_{\parallel})\exp(-j\,\underline{k}_1\cdot\underline{r}) + \iint T_{\parallel} Q' \left[ku\sin\theta' + (v^2c - v^2b - ck'^2)\cos\phi'/k' \right] EX\, du\, dv \tag{II-7d}$$

where

$$E_x = \exp(-jux - jvy - jkz\cos\theta)$$

$$Q' = j(k'^2 - k^2)\left[2\pi(k'^2b + k^2c)\right]^{-1} S(u - k\sin\theta, v)$$

$$T_{\parallel} = 1 + R_{\parallel}\,.$$

APPENDIX III

INTEGRANDS FOR THE POLARIZED FIELDS

In order to make use of the formulas for computing the scattering coefficients shown in the next appendix, it is necessary to get the integrands of the field expressions in the right format.

The general field expression is of the form

$$E = K \iint (\mathrm{INT}) \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r})\, dx\, dy \qquad \text{(III-1)}$$

where INT for the two polarizations are given below.

A. Integrand for E_{hh}, $(\mathrm{INT})_{hh}$

$$
\begin{aligned}
(\mathrm{INT})_{hh} &= \cos\theta' E_{\tilde{y}} - \eta H_{\tilde{x}} \\
&= \cos\theta' \Big\{ [1 + R_\perp(\theta')] \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) - \iint T_\perp(\theta')(u^2 + bc)\, Q' E X\, du\, dv \Big\} \\
&\quad - \cos\theta' [1 - R_\perp(\theta')] \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) - \iint T_\perp(\theta') [u^2(b-c)/k + kc]\, Q' E X\, du\, dv \\
&\simeq 2 [R_\perp(\theta) + R_\perp' Z_x] (\cos\theta + Z_x \sin\theta) \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) \\
&\quad - \iint T_\perp(\theta') [u^2(b-c)/k + kc + (u^2 + bc)\cos\theta + Z_x \sin\theta (u^2 + bc)] \\
&\quad \cdot Q\, S(\theta')\, E X\, du\, dv \\
&\simeq 2 \{ R_\perp(\theta)\cos\theta + [R_\perp(\theta)\sin\theta + R_\perp' \cos\theta] Z_x \} \\
&\quad \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) - \iint [T_\perp(\theta) + R_\perp' Z_x] \\
&\quad [D_{1hh} + D_{2hh} Z_x] [S(\theta) + S' Z_x] E X\, du\, dv
\end{aligned}
\qquad \text{(III-2)}
$$

where

$$S(\theta') \simeq S(\theta) + S' Z_x \quad , \quad T_\perp(\theta') \simeq T_\perp(\theta) + R_\perp' Z_x$$

$$EX = \exp(-jux - jvy + jkz\cos\theta)$$

$$S' = \frac{\partial S}{\partial Z_x} = [\, \partial S / \partial(u - k\sin\theta) \,] \; k\cos\theta$$

$$D_{1hh} = [\, u^2(b-c)/k + kc + (u^2+bc)\cos\theta \,]\, Q$$

$$D_{2hh} = (u^2+bc)\, Q \sin\theta \quad , \quad Q' = QS(\theta') = QS(u - k\sin\theta', v)$$

$$Q = j(k'^2 - k^2)\,[\, 2\pi(k^2c + k'^2 b)\,]^{-1}.$$

Linear approximations have been used to rewrite results in terms of the incident angle rather than the local angle.

B. Integrand for Evv

$$\begin{aligned}
(INT)_{vv} &= E_{\tilde{x}} + \eta H_{\tilde{y}} \cos\theta' \\
&= \cos\theta' [1 - R_{\parallel}(\theta')] \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) + \iint T_{\parallel}(\theta')\, Q' \\
&\quad [\, bu\sin\theta' - (v^2+bc)\, k\cos\phi'/k' \,]\, EX\, du\, dv \\
&\quad + \cos\theta' \{ -[1 + R_{\parallel}(\theta')] \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) \\
&\quad + \iint T_{\parallel}(\theta')\, Q' [\, ku\sin\theta' + (v^2c - v^2b - ck^2)\cos\phi'/k' \,] \\
&\quad \cdot EX\, du\, dv \} \\
&\simeq -2 \{ R_{\parallel}(\theta)\cos\theta + [\, R_{\parallel}'\cos\theta + R_{\parallel}(\theta)\sin\theta \,] Z_x \} \\
&\quad \cdot \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) + \iint T_{\parallel}(\theta')\, QS(\theta') \\
&\quad \{ u(b + k\cos\theta)\sin\theta' + [\cos\theta(v^2c - v^2b - ck^2) \\
&\quad - (v^2+bc)k]\cos\phi'/k' \}\; EX\, du\, dv \\
&\quad + \iint T_{\parallel}(\theta')\, QS(\theta')\sin\theta\, [\, ku\sin\theta + (v^2c - v^2b - ck^2) \\
&\quad \cdot \cos\phi/k' \,]\, Z_x\, EX\, du\, dv \\
&= -2 \{ R_{\parallel}(\theta)\cos\theta + [\, R_{\parallel}'\cos\theta + R_{\parallel}(\theta)\sin\theta \,] Z_x \} \\
&\quad \cdot \exp(-j\, \underset{\sim}{k_1} \cdot \underset{\sim}{r}) + \iint [\, T_{\parallel}(\theta) + R_{\parallel}' Z_x \,][\, D_{1vv} + D_{2vv} Z_x \,] \\
&\quad [\, S(\theta) + S' Z_x \,]\; EX\, du\, dv
\end{aligned}$$

where

$$T_{/\!/}(\theta') = T_{/\!/}(\theta) + R'_{/\!/} Z_x$$

$$D_{1vv} = A_1 \sin\theta + B_1 \cos\phi$$

$$D_{2vv} = -A_1 \cos\theta + B_1 k^2 \sin\theta \cos\theta / (k'^2 \cos\phi) + C_1$$

$$A_1 = Qu(b + k\cos\theta)$$

$$B_1 = Q\cos\theta(v^2c - v^2b - ck^2)$$

$$C_1 = Q\sin\theta[ku\sin\theta + (v^2c - v^2b - ck^2)\cos\phi / k'].$$

APPENDIX IV

EXPRESSIONS FOR THE SCATTERING COEFFICIENTS

A. Consider a Field Expression of the form

$$E_1 = K_0 \iint_{-L}^{L} (A_{pp} + B_{pp} Z_x) e^{-j v_x x + j v_z z} dx\, dy \quad , \qquad \text{(IV-1)}$$

the product of E_1 and its conjugate is

$$E_1 \cdot E_1^* = |K_0|^2 \iiiint_{-L}^{L} (|A_{pp}|^2 + A_{pp} B_{pp}^* Z'_{x'} + |B_{pp}|^2 Z_x Z'_{x'} + A_{pp}^* B_{pp} Z_x)$$
$$\exp[-j v_x (x - x') + j v_z (z - z')] dx\, dy\, dx'\, dy' . \qquad \text{(IV-2)}$$

Taking ensemble average of the product yields

$$\langle E_1 \cdot E_1^* \rangle = |K_0|^2 \int_{-L}^{L} \int_{-L}^{L} \int_{-L-y'}^{L-y'} \int_{-L-x'}^{L-x'} \Big\{ |A_{pp}|^2 - j (A_{pp} B_{pp}^* + A_{pp}^* B_{pp})$$
$$\cdot v_z \sigma^2 \frac{\partial \rho}{\partial \alpha} - |B_{pp}|^2 \sigma^2 \Big[\frac{\partial^2 \rho}{\partial \alpha^2} + v_z^2 \sigma^2 \Big(\frac{\partial \rho}{\partial \alpha}\Big)^2 \Big] \Big\} \exp[-j v_x \alpha - v_z^2 \sigma^2 (1-\rho)]$$
$$d\alpha\, d\beta\, dx'\, dy'$$
$$= |K_0|^2 \int_{-2L}^{2L} \int_{-2L}^{2L} (2L - |\alpha|)(2L - |\beta|) \Big\{ |A_{pp}|^2 - j (A_{pp} B_{pp}^* + A_{pp}^* B_{pp})$$
$$\cdot v_z \sigma^2 \frac{\partial \rho}{\partial \alpha} - |B_{pp}|^2 \sigma^2 \Big[\frac{\partial^2 \rho}{\partial \alpha^2} + K \Big(\frac{\partial \rho}{\partial \alpha}\Big)^2 \Big] \Big\} \exp[-j v_x \alpha - K (1-\rho)] d\alpha\, d\beta \qquad \text{(IV-3)}$$

where $\alpha = x - x'$; $\beta = y - y'$; A_{pp} and B_{pp} are not functions of α, β ; $K = v_z^2 \sigma^2$; $\sigma^2 \rho = \langle Z Z' \rangle$. The scattering coefficient σ_{1pp} corresponding to the field given in (IV-1) is

$$\sigma_{1pp} \equiv 4\pi R^2 \langle E_1 \cdot E_1^* \rangle / (2L)^2$$
$$= \frac{k^2}{4\pi} \int_{-2L}^{2L} \int_{-2L}^{2L} \frac{(2L - |\alpha|)(2L - |\beta|)}{(2L)^2} \Big\{ |A_{pp}|^2 - j (A_{pp} B_{pp}^* + A_{pp}^* B_{pp})$$
$$\cdot v_z \sigma^2 \frac{\partial \rho}{\partial \alpha} - |B_{pp}|^2 \sigma^2 \Big[\frac{\partial^2 \rho}{\partial \alpha^2} + K \Big(\frac{\partial \rho}{\partial \alpha}\Big)^2 \Big] \Big\} \qquad \text{(IV-4)}$$
$$\cdot \exp[-j v_x \alpha - K (1-\rho)] d\alpha\, d\beta .$$

If the α, β -integrals in σ_{1pp} converge fast enough so that within the region of convergence $2L \gg \alpha, \beta$, and if edge effects are negligible, then

$$\sigma_{1pp} = \frac{k^2}{4\pi}\int_{-2L}^{2L}\int_{-2L}^{2L}\left[|A_{pp}|^2 + \frac{v_x}{v_z}(A_{pp}B_{pp}^* + A_{pp}^*B_{pp}) + \left|\frac{v_x}{v_z}B_{pp}\right|^2\right]$$

$$\cdot \exp[-j v_x\alpha - K(1-\rho)]\, d\alpha\, d\beta$$

$$= \frac{k^2}{4\pi}\int_{-2L}^{2L}\int_{-2L}^{2L}\left|A_{pp} + \frac{v_x}{v_z}B_{pp}\right|^2 \exp[-j v_x\alpha - K(1-\rho)]\, d\alpha\, d\beta \,. \tag{IV-5}$$

For isotropically rough surface, (IV-5) reduces to

$$\sigma_{1pp} = \frac{k^2}{2}\int_0^{2L}\left|A_{pp} + \frac{v_x}{v_z}B_{pp}\right|^2 J_o(v_x\xi)\, e^{-K(1-\rho)}\,\xi\, d\xi \tag{IV-6}$$

where Jo( ) is the zero order Bessel function.

The identities useful for getting (IV-5) from (IV-4) are

$$(1)\ \int_{-2L}^{2L}\int_{-2L}^{2L}\frac{\partial^2\rho}{\partial\alpha^2}\exp[-j v_x\alpha - K(1-\rho)]\,d\alpha\, d\beta = \int_{-2L}^{2L}\frac{\partial\rho}{\partial\alpha}\exp[-j v_x\alpha - K(1-\rho)]\Big|_{-2L}^{2L} d\beta$$

$$+ \int_{-2L}^{2L}\int_{-2L}^{2L}\left[j v_x\frac{\partial\rho}{\partial\alpha} - K\left(\frac{\partial\rho}{\partial\alpha}\right)^2\right]\exp[-j v_x\alpha - K(1-\rho)]\, d\alpha\, d\beta$$

$$(2)\ \int_{-2L}^{2L}\int_{-2L}^{2L}\frac{\partial\rho}{\partial\alpha}\exp[-j v_x\alpha - K(1-\rho)]\,d\alpha\, d\beta = \int_{-2L}^{2L}\frac{1}{K}\exp[-j v_x\alpha - K(1-\rho)]\Big|_{-2L}^{2L} d\beta$$

$$+ \frac{j v_x}{K}\int_{-2L}^{2L}\int_{-2L}^{2L}\exp[-j v_x\alpha - K(1-\rho)]\, d\alpha\, d\beta \,.$$

B. <u>Consider a field expression of the form</u>

$$E_2 = K\iint_{-L}^{L}\left\{\iint_{-\infty}^{\infty}(T + R'Z_x)(D_1 + D_2 Z_x)(S + S'Z_x)\,EXP\,du\,dv\right\}dx\,dy \tag{IV-7}$$

where D_1 and D_2 are both functions of u and v; S' is the derivative of S with respect to Zx;

$$EXP = \exp[-j(u + k\sin\theta)x - jvy + j2k\cos\theta\, z] \,.$$

It is possible to write E_2 in the same form as (IV-1) and identify A_{pp} and B_{pp} with Zx^2-term being ignored. EXP may be taken to play exactly the same role as exp $[-jv_x x + jv_z z]$ without affecting the final result. Note that appropriate subscripts should be attached to D_1, D_2, T, and R, depending upon the polarization states. However, these subscripts have been left out here for simplicity of writing. The basic form of the scattering coefficient for this field is again given by (IV-4). The corresponding coefficient terms can be shown to be (see section C)

$$\langle |A_{pp}|^2 \rangle = 2\pi \sigma_1^2 \iint_{-\infty}^{\infty} |TD_1|^2 W \, du\, dv \qquad \text{(IV-8a)}$$

$$\langle A_{pp} B_{pp}^* \rangle = 2\pi \sigma_1^2 \iint_{-\infty}^{\infty} TD_1 [(TD_2 + R'D_1)^* W + (TD_1)^* \frac{\partial W}{\partial q} \frac{V_z}{2}] \, du\, dv \qquad \text{(IV-8b)}$$

$$\langle |B_{pp}|^2 \rangle = 2\pi \sigma_1^2 \iint_{-\infty}^{\infty} \Big\{ |TD_2 + R'D_1|^2 W + |TD_1|^2 \frac{V_z^2}{4} \frac{\partial^2 W}{\partial q^2} + V_z \frac{\partial W}{\partial q} Re[TD_1 (TD_2 + R'D_1)^*] \Big\} du\, dv \qquad \text{(IV-8c)}$$

where $\langle \ldots \rangle$ is the symbol for ensemble average performed on s(x,y); Re means "the real part of"; * is the complex conjugate sign;

$W = W(q,v)$

$q = u - k \sin \Theta$, $V_z = 2k \cos \Theta$.

Although (IV-1) may be used for (IV-7), it is more convenient to write the scattering coefficient for the field given by (IV-7) directly in terms of D_1 and D_2 instead of A_{pp} and B_{pp}. Thus,

$$\begin{aligned}
\sigma_{2pp} &\equiv 4\pi R^2 \langle E_2 \cdot E_2^* \rangle / (2L)^2 \\
&= \frac{k^2 \sigma_1^2}{2} \iint_{-2L}^{2L} \frac{(2L-|\alpha|)(2L-|\beta|)}{(2L)^2} \Big\{ \iint_{-\infty}^{\infty} |TD_{1pp}|^2 W \\
&\quad - j 2\sigma^2 V_z \frac{\partial \rho}{\partial \alpha} Re \Big(TD_{1pp} [(TD_{2pp} + R'D_{1pp})^* W + (TD_{1pp})^* \frac{\partial W}{\partial q} \frac{V_z}{2}] \Big) \\
&\quad - \sigma^2 \Big[\frac{\partial^2 \rho}{\partial \alpha^2} + K \Big(\frac{\partial \rho}{\partial \alpha}\Big)^2 \Big] \Big(|TD_{2pp} + R'D_{1pp}|^2 W + |TD_{1pp}|^2 \frac{V_z^2}{4} \frac{\partial^2 W}{\partial q^2} \\
&\quad + V_z \frac{\partial W}{\partial q} Re[TD_{1pp} (TD_{2pp} + R'D_{1pp})^*] \Big) \qquad \text{(IV-9)} \\
&\quad \cdot \exp[-j(u + k \sin\theta)\alpha - jv\beta - K(1-\rho)] \, du\, dv \Big\} \, d\alpha\, d\beta .
\end{aligned}$$

If the α, β -integrals converge fast enough so that within the region of convergence $2L >> \alpha, \beta$ and if edge effects are negligible, then for isotropically rough surface, similar to $\sigma_{1pp'}$ (IV-9) reduces to

$$\sigma_{2pp} = \pi k^2 \sigma_1^2 \int_0^{2L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Big\{ \left| T D_{1pp} + G (T D_{2pp} + R' D_{1pp}) \right|^2 W + V_z \frac{\partial W}{\partial q} \operatorname{Re}\Big(G T D_{1pp} [T D_{1pp} + G (T D_{2pp} + R' D_{1pp})]^* \Big) + \frac{V_z^2}{4} \frac{\partial^2 W}{\partial q^2} \left| T G D_{1pp} \right|^2 \Big\} J_0 (\xi \sqrt{(u + k \sin\theta)^2 + v^2})\, du\, dv \cdot \exp[- K (1 - \rho)]\, \xi\, d\xi \qquad \text{(IV-10)}$$

where $G = (u + k \sin \Theta)/ v_z$.

C. Consider a field expression of the form

$$E = K_0 \iint_{-L}^{L} \iint_{-\infty}^{\infty} (A_0 S + B_0 S') Z_x \, EXP\, du\, dv\, dx\, dy , \qquad \text{(IV-11)}$$

then

$$E \cdot E^* = |K_0|^2 \iiiint_{-L}^{L} \Big\{ \iiiint_{-\infty}^{\infty} [|A_0|^2 S S_1^* + A_0 B_0^* S S_1'^* + A_0^* B_0 S^* S_1' + |B_0|^2 S' S_1'^*] Z_x Z_x' \, |EXP|^2 du\, dv\, du'\, dv' \Big\} dx\, dy\, dx'\, dy' \qquad \text{(IV-12)}$$

where Ao, Bo are both functions of u, v; $S = S(u,v)$ and $S_1 = S_1(u',v')$; $EXP = \exp[-j(u + k \sin \Theta) x - jvy + jk \cos \Theta Z)$. The following identities of ensemble average over s(x,y) are needed for getting (IV-13) below:

$$\langle S S_1^* \rangle = 2\pi \sigma_1^2 W (u - k \sin\theta, v)\, \delta(u - u')\, \delta(v - v')$$

$$\langle S S_1'^* \rangle = 2\pi \sigma_1^2 \frac{\partial W}{\partial q} k \cos\theta\, \delta(u - u')\, \delta(v - v')$$

$$\langle S' S_1'^* \rangle = 2\pi \sigma_1^2 k^2 \cos^2\theta \frac{\partial^2 W}{\partial q^2}\, \delta(u - u')\, \delta(v - v')$$

Average (IV-12) first with respect to $s(x,y)$

$$\langle E\cdot E^*\rangle_s = |K_o|^2 \iiiint_{-L}^{L} \Big\{ \iiiint_{-\infty}^{\infty} [\,|A_o|^2 W + (A_o^* B_o + A_o B_o^*) \cdot \frac{\partial W}{\partial q} k\cos\theta + |B_o|^2 k^2 \cos^2\theta \frac{\partial^2 W}{\partial q^2}\,] \, 2\pi\sigma_1^2 \, du\,dv \Big\} Z_x Z'_{x'} \cdot \exp[-j(u+k\sin\theta)(x-x') - jv(y-y') + j2k\cos\theta(z-z')] \, dx\,dy\,dx'\,dy'$$

Then with respect to $Z(x,y)$

(IV-13)

$$\langle E\cdot E^*\rangle_{sz} = 2\pi\sigma_1^2 |K_o|^2 \iint_{-2L}^{2L} (2L-|\alpha|)(2L-|\beta|) \Big\{ \iint_{-\infty}^{\infty} [\,|A_o|^2 W + (A_o^* B_o + A_o B_o^*) \frac{\partial W}{\partial q} k\cos\theta + |B_o|^2 k^2\cos^2\theta \frac{\partial^2 W}{\partial q^2}\,] \, du\,dv \Big\} (-\sigma^2) \Big[\frac{\partial^2 \rho}{\partial \alpha^2} + K \Big(\frac{\partial \rho}{\partial \alpha}\Big)^2 \Big] \exp[-j(u+k\sin\theta)\alpha - jv\beta - K(1-\rho)] \, d\alpha\, d\beta \,.$$

(IV-14)

Comparing (IV-14) and (IV-11) with (IV-3) and (IV-7), we have

$$\langle |B_{pp}|^2 \rangle = 2\pi\sigma_1^2 \iint_{-\infty}^{\infty} [\,|A_o|^2 W + (A_o^* B_o + A_o B_o^*) \frac{\partial W}{\partial q} k\cos\theta + |B_o|^2 k^2 \cos^2\theta \frac{\partial^2 W}{\partial q^2}\,] \, du\,dv$$

$$= 2\pi\sigma_1^2 \iint_{-\infty}^{\infty} \Big\{ |TD_2 + R'D_1|^2 W + 2k\cos\theta \frac{\partial W}{\partial q} \mathrm{Re}[TD_1 \cdot (TD_2 + R'D_1)^*] + |TD_1|^2 k^2 \cos^2\theta \frac{\partial^2 W}{\partial q^2} \Big\} du\,dv$$

as given in (IV-8c) with

$A_o = D_1 R' + TD_2$

$B_o = D_1 T.$

If, in addition, we assume that autocorrelation of the large undulations to be Gaussian and integrate out the Bessel function of (IV-6) and (IV-10), the following expressions result:

$$\sigma_{1pp} = \frac{1}{2}(4\,m^2\cos^2\theta)^{-1}\,|A_{pp} + B_{pp}\tan\theta\,|^2 \exp[-\tan^2\theta/(2m)^2] \quad \text{(IV-15)}$$

$$\sigma_{2pp} = \pi k^2 \sigma_1^2 \iint_{-\infty}^{\infty} \Big\{ |C'_{pp} + G\,D'_{pp}|^2 W + \mathrm{Re}[(C'_{pp} + G\,D'_{pp})\,C'^{*}_{pp}\,G]$$
$$\cdot V_z \frac{\partial W}{\partial q} \Big\}\, V_z^{-2}\, m^{-2} \exp\{-[(u+k\sin\theta)^2 + v^2]/(2V_z^2 m^2)\}\, du\, dv \quad \text{(IV-16)}$$

where $W(\underline{K}) = (\ell^2/2)\exp[-(\underline{K}\ell/2)^2]$ is the roughness spectrum of $s(x,y)$ related to its correlation coefficient by the Bessel transform; ℓ is the correlation length of the surface, $s(x,y)$; $\rho''(0)$ is the second derivative of ρ evaluated at zero;

$$m^2 = \sigma^2\,|\,\rho''(0)\,|$$
$$W = W(\sqrt{q^2 + v^2})$$
$$C'_{pp} = T\,D_{1pp}$$
$$D'_{pp} = T\,D_{2pp} + R'\,D_{1pp}\,.$$

To obtain (IV-15) and (IV-16), we have used the approximation and the identity as shown below

(1) $$\exp[-K(1-\rho)] \simeq \exp[-K\,|\,\rho''(0)\,|\,\xi^2/2]$$

(2) $$\int_0^\infty J_0(V_x\,\xi)\exp(-a\,\xi^2)\,\xi\,d\xi$$
$$= \frac{1}{2a}\exp(-V_x^2/4a)$$

Comparing (IV-1), (IV-7) with (III-2) and (III-5) and using (IV-4) and (IV-9) we get

(1) $$\sigma_{hh} = \sigma_{1hh} + \sigma_{2hh}$$

with

$$A_{hh} = 2 R_{\perp}(\theta) \cos\theta$$

$$B_{hh} = 2 \left[R_{\perp}(\theta) \sin\theta + R_{\perp}' \cos\theta \right]$$

$$D_{1hh} = Q \left[u^2 (b - c)/k + kc + (u^2 + bc) \cos\theta \right]$$

$$D_{2hh} = Q \, (u^2 + bc) \sin\theta \, .$$

(2) $$\sigma_{vv} = \sigma_{1vv} + \sigma_{2vv}$$

with

$$A_{vv} = -2 R_{\parallel}(\theta) \cos\theta$$

$$B_{vv} = -2 \left[R_{\parallel}(\theta) \sin\theta + R_{\parallel}' \cos\theta \right]$$

$$D_{1vv} = Q \Big\{ \left[u (b + k\cos\theta) \right] \sin\theta + \left[\cos\theta \, (v^2 c - v^2 b - c k^2) - (v^2 + bc) k \right] \cos\phi / k' \Big\}$$

$$D_{2vv} = Q \Big\{ \sin\theta \left[k u \sin\theta + (v^2 c - v^2 b - c k^2) \cos\phi / k' \right] + k^2 \sin\theta \cos\theta \left[\cos\theta \, (v^2 c - v^2 b - c k^2) - (v^2 + bc) k \right] / (k'^3 \cos\phi) - u \cos\theta \, (b + k\cos\theta) \Big\} \, .$$

APPENDIX V

IDENTIFICATION OF THE DIFFERENT FORMS OF THE SCATTERING COEFFICIENT FORMULA

To compare σ_{2vv} in (14) with the corresponding Valenzuela's and Wright's scattering coefficients, we rewrite (14) in the following form

$$\sigma_{2vv} = 2(\pi k \sigma_1)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left| T_{//} C_{vv}(u,v) \right|^2 W(u - k\sin\theta, v)$$

$$\delta(u + k\sin\theta)\, \delta(v)\; du\, dv \qquad \text{(V-1)}$$

where

$$T_{//} = \frac{2k'\cos\theta}{k'\cos\theta + k\cos\phi}$$

$$C_{vv} = Q\left\{u(b + k\cos\theta)\sin\theta + [\cos\theta(v^2c - v^2b - ck^2) - k(v^2 + bc)]\cos\phi/k'\right\}$$

$$Q = (k'^2 - k^2)[2\pi(k^2c + k'^2b)]^{-1}$$

$$\cos^2\phi = 1 - k^2\sin^2\theta/k'^2 .$$

By the property of the Dirac delta function it follows that

$u = -k\sin\Theta$

$v = o$.

Thus

$$\sigma_{2vv} = 2(\pi k \sigma_1)^2 \left| T_{//} C_{vv}(-k\sin\theta, 0) \right|^2 W(2k\sin\theta, 0) \qquad \text{(V-2)}$$

where

$$T_{//}C_{vv}(-k\sin\theta, 0) = T_{//}Q\left\{-k\sin\theta(k\cos\theta + k\cos\theta)\sin\theta + [\cos\theta(-k'k^2\cos\phi) - k^2k'\cos\theta\cos\phi]\cos\phi/k'\right\}$$

$$= T_{//}Q\left\{-2k^2\cos\theta(\sin^2\theta + \cos^2\phi)\right\}$$

$$= \left[\frac{2k'\cos\theta}{k'\cos\theta + k\cos\phi}\right]\left[\frac{k'^2 - k^2}{2\pi k k'(k\cos\phi + k'\cos\theta)}\right]\left[-2k^2\cos\theta\left(\sin^2\theta + 1 - \frac{k^2\sin^2\theta}{k'^2}\right)\right]$$

$$= -\frac{2}{\pi}\,\frac{(k'^2 - k^2)k\cos^2\theta[k'^2 + (k'^2 - k^2)\sin^2\theta]}{k'^2(k'\cos\theta + k\cos\phi)^2} . \qquad \text{(V-3)}$$

Substituting (V-3) into (V-2) yields

$$\sigma_{2vv} = 8k^4\sigma_1^2\cos^4\theta \left| \frac{(k'^2 - k^2)[k'^2 + (k'^2 - k^2)\sin^2\theta]}{k'^2(k'\cos\theta + k\cos\phi)^2} \right|^2 W(2k\sin\theta, 0) .$$

(V-4)

σ_{VV} given by Valenzuela and in his notation is $(\sigma_{VV})_{val.} = 8\pi\eta_\circ\cos\theta \langle P_{svv}\rangle$

$$(\sigma_{vv})_{val.} = 8\pi\eta_\circ\cos\theta\langle P_{svv}\rangle$$

$$= 4\pi\beta^4\cos^4\theta\left|\frac{(\epsilon-1)[\epsilon(\sin^2\theta+1)-\sin^2\theta]}{(\epsilon\cos\theta+\sqrt{\epsilon-\sin^2\theta})^2}\right|^2 \qquad \text{(V-5)}$$

$$\cdot\bar{W}(2k\sin\theta, 0)$$

The connections between his notations and the one used in this paper are

$$\beta = k \qquad \text{(V-6a)}$$

$$\epsilon = k'^2/k^2 \qquad \text{(V-6b)}$$

$$\bar{W} = \frac{2\sigma_1^2}{\pi} W \qquad \text{(V-6c)}$$

Substituting (V-6) into (V-5) yields

$$(\sigma_{vv})_{val.} = 4\pi k^4\cos^4\theta\left|\frac{(\frac{k'^2}{k^2}-1)\{\frac{k'^2}{k^2}(\sin^2\theta+1)-\sin^2\theta\}}{(\frac{k'^2}{k^2}\cos\theta+\sqrt{\frac{k'^2}{k^2}-\sin^2\theta})^2}\right|^2$$

$$\frac{2\sigma_1^2}{\pi} W(2k\sin\theta, 0)$$

$$= 8k^4\sigma_1^2\cos^4\theta\left|\frac{(k'^2-k^2)[k'^2+(k'^2-k^2)\sin^2\theta]}{k'^2(k'\cos\theta+k\cos\phi)^2}\right|^2 \qquad \text{(V-7)}$$

$$W(2k\sin\theta, 0)$$

which is identical to (V-4).

Using the notations in this paper, we can rewrite Wright's scattering coefficient as follows

$$(\sigma_{vv})_w = 8k^4\sigma_1^2 |g_{vv}|^2 W(2k\sin\theta, 0)$$
$$= 8k^4\sigma_1^2 \left| R_{//}\cos^2\theta + T_{//}^2(k'^2 - k^2)\sin^2\theta/(2k'^2)\right|^2 W(2k\sin\theta, 0) \qquad \text{(V-8)}$$

where
$$R_{//} = \frac{k'\cos\theta - k\cos\phi}{k'\cos\theta + k\cos\phi} .$$

Substituting $R_{//}$ and $T_{//}$ into g_{vv} yields

$$g_{vv} = R_{//}\cos^2\theta + T_{//}^2(k'^2 - k^2)\frac{\sin^2\theta}{2k'^2}$$
$$= \frac{(k'\cos\theta - k\cos\phi)\cos^2\theta}{k'\cos\theta + k\cos\phi} + \frac{4k'^2\cos^2\theta(k'^2 - k^2)\sin^2\theta}{2k'^2(k'\cos\theta + k\cos\phi)^2}$$
$$= \frac{(k'^2\cos^2\theta - k^2\cos^2\phi)\cos^2\theta + 2\cos^2\theta(k'^2 - k^2)\sin^2\theta}{(k'\cos\theta + k\cos\phi)^2}$$
$$= \frac{\cos^2\theta}{(k'\cos\theta + k\cos\phi)^2}\left[k'^2\cos^2\theta - k^2\cos^2\phi + 2\sin^2\theta(k'^2 - k^2)\right]$$
$$= \frac{\cos^2\theta}{(k'\cos\theta + k\cos\phi)^2}\left[k'^2(1 + \sin^2\theta) - k^2\left(1 - \frac{k^2\sin^2\theta}{k'^2}\right) - 2k^2\sin^2\theta\right]$$
$$= \frac{\cos^2\theta}{(k'\cos\theta + k\cos\phi)^2}\left[(k'^2 + k'^2\sin^2\theta - k^2\sin^2\theta) - \frac{k^2}{k'^2}(k'^2 - k^2\sin^2\theta + k'^2\sin^2\theta)\right]$$
$$= \frac{\cos^2\theta(k'^2 - k^2)(k'^2 + k'^2\sin^2\theta - k^2\sin^2\theta)}{k'^2(k'\cos\theta + k\cos\phi)^2} . \qquad \text{(V-9)}$$

Thus,
$$(\sigma_{vv})_w = 8k^4\sigma_1^2\cos^4\theta \left|\frac{(k'^2 - k^2)[k'^2 + (k'^2 - k^2)\sin^2\theta]}{k'^2(k'\cos\theta + k\cos\phi)^2}\right|^2 W(2k\sin\theta, 0) \qquad \text{(V-10)}$$

which is again identical to (V-4).

In conclusion, when $Z(x,y) = 0$, we get

$$\sigma_{2vv} = (\sigma_{vv})_{val} = (\sigma_{vv})_w$$
$$= 8k^4\sigma_1^2 \left| R_{//}\cos^2\theta + T_{//}^2(k'^2 - k^2)\frac{\sin^2\theta}{2k'^2}\right|^2 W(2k\sin\theta, 0) . \qquad \text{(V-11)}$$

APPENDIX VI

COLLECTION OF IDENTITIES

All the identities used in this report are rewritten in this appendix for ease of reference. The order of appearance of these identities does not correspond to that in the report. All the integral identities are given first and then the identities for ensemble averages.

1. $$S(u,v) = \frac{1}{2\pi}\int_{-L}^{L}\int_{-L}^{L} s(x,y)\ \exp(-jxu - jyv)\,dx\,dy$$

2. $$W(u,v) = \frac{1}{2\pi}\iint \rho_1(\xi,\eta)\ \exp(-ju\xi - jv\eta)\,d\xi\,d\eta$$

3. If $$W(\beta) = \int_0^\infty J_0(\beta\xi)\,\rho_1\,\xi\,d\xi$$

then $$W'(\beta) = \frac{d\,W(\beta)}{d\beta} = -\int_0^\infty J_1(\beta\xi)\,\rho_1\,\xi^2\,d\xi$$

and $$\int_0^\infty \frac{\partial\rho_1}{\partial\xi} J_1(\beta\xi)\exp(b\rho_1)\,\xi\,d\xi \simeq -\beta W(\beta) + \text{edge effect term}$$

4. $$\int_0^\infty x^{\alpha+1}\exp(-ax)\,J_\alpha(bx)\,dx$$
$$= 2a(2b)^\alpha\,\Gamma(\alpha+\tfrac{3}{2}) \Big/ \left[\sqrt{\pi}\,(a^2+b^2)^{\alpha+\frac{3}{2}}\right]$$
$$[\,\text{Re}\ \alpha > -1,\ \text{Re}\ a > |\text{Im}\ b|\,]$$

where $\Gamma(n+1) = n\Gamma(n)$ if $n > 0$

$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$

5. $$\int_0^\infty x^{\alpha+1}\exp(-ax^2)\,J_\alpha(bx)\,dx$$
$$= [\,b^\alpha/(2a)^{\alpha+1}\,]\exp(-b^2/4a)$$

$$[\,\text{Re},\ a > 0,\ \text{Re}\ \alpha > -1\,]$$

6. $$\int_0^{\pi} \exp(jb\cos x)\cos nx\,dx = j^n \pi J_n(b)$$

7. $$\int_0^{2\pi} \sin\theta \exp(\pm jx\sin\theta)\,d\theta = \pm j 2\pi J_1(x)$$

8. $$\int_{\phi}^{\phi+2\pi} \exp(jx\sin\theta + jm\theta)\,d\theta = (-1)^m 2\pi J_m(x)$$

9. $$\int_{\phi}^{\phi+2\pi} \exp(\pm jx\cos\theta \pm jy\sin\theta)\,d\theta = 2\pi J_0(\sqrt{x^2+y^2})$$

10. $$\int_{-\infty}^{\infty} \exp(j\omega t)\,d\omega = 2\pi\delta(t)$$

11. $$\langle Z Z' \rangle = \sigma^2 \rho$$

12. $$\langle s\, s' \rangle = \sigma_1^2 \rho_1$$

13. $$\langle S(u', v') S(u,v)^* \rangle = 2\pi\sigma_1^2 W(u,v)\,\delta(u-u')\,\delta(v-v')$$

14. $$\langle Z_x \exp[jV_z(Z-Z')] \rangle = \langle Z'_{x'} \exp[jV_z(Z-Z')] \rangle$$
$$= -j\sigma^2 V_z \frac{\partial\rho}{\partial\alpha} \exp[-\sigma^2 V_z^2(1-\rho)]$$

where $\alpha = x - x'$.

15. $$\langle Z_x Z'_{x'} \exp[jV_z(Z-Z')] \rangle = -\sigma^2\left[\frac{\partial^2\rho}{\partial\alpha^2} + \sigma^2 V_z^2\left(\frac{\partial\rho}{\partial\alpha}\right)^2\right]\exp[-\sigma^2 V_z^2(1-\rho)]$$

16. $$\left\langle S(u',v') \frac{\partial S(u,v)^*}{\partial u} \right\rangle = 2\pi\sigma_1^2 \frac{\partial W(u,v)}{\partial u}\,\delta(u-u')\,\delta(v-v')$$

17. $$\left\langle \frac{\partial S(u',v')}{\partial u'} \frac{\partial S(u,v)^*}{\partial u} \right\rangle = 2\pi\sigma_1^2 \frac{\partial^2 W(u,v)}{\partial u^2}\,\delta(u-u')\,\delta(v-v')$$

APPENDIX VII

COX AND MUNK (1954) SURFACE MEAN SQUARE SLOPES

1. For clean sea

$$\sigma^2_{cross} + \sigma^2_{up} = 5.12 * 10^{-3} * V + 0.003 \pm 0.004 \quad .$$

2. For slick sea

$$\sigma^2_{cross} + \sigma^2_{up} = 1.56 * 10^{-3} * V + 0.008 \pm 0.004 \quad .$$

In both (1) and (2), V is in meter/second.

For isotropically rough sea surface, we have

$$m^2_{c,R} \equiv \frac{1}{2} \left(\sigma_c^2 + \sigma_u^2 \right)$$

where m_c and m_s are the rms slopes of the clean and the slick sea, respectively. Values of m_c and m_s are given in Table I for different wind speeds.

TABLE I COX N MUNK RMS SURFACE SLOPES

CLEAN SEA
N SLICK SEA

V(M/SEC)	M1	M2	M3
0.500	0.2793E-01	0.5273E-01	0.6914E-01
	0.4689E-01	0.6626E-01	0.7994E-01
2.000	0.6797E-01	0.8136E-01	0.9284E-01
	0.5967E-01	0.7457E-01	0.8695E-01
4.000	0.9869E-01	0.1084E 00	0.1172E 00
	0.7155E-01	0.8438E-01	0.9550E-01
5.000	0.1109E 00	0.1196E 00	0.1277E 00
	0.7681E-01	0.8888E-01	0.9950E-01
6.000	0.1219E 00	0.1298E 00	0.1373E 00
	0.8173E-01	0.9317E-01	0.1033E 00
8.000	0.1414E 00	0.1483E 00	0.1549E 00
	0.9077E-01	0.1012E 00	0.1106E 00
10.000	0.1584E 00	0.1646E 00	0.1706E 00
	0.9899E-01	0.1086E 00	0.1175E 00
12.000	0.1738E 00	0.1795E 00	0.1850E 00
	0.1066E 00	0.1156E 00	0.1239E 00
14.000	0.1880E 00	0.1932E 00	0.1983E 00
	0.1137E 00	0.1221E 00	0.1301E 00
15.000	0.1947E 00	0.1997E 00	0.2047E 00
	0.1170E 00	0.1253E 00	0.1330E 00
16.000	0.2011E 00	0.2061E 00	0.2109E 00
	0.1203E 00	0.1284E 00	0.1359E 00
18.000	0.2135E 00	0.2181E 00	0.2227E 00
	0.1266E 00	0.1343E 00	0.1416E 00
20.000	0.2252E 00	0.2296E 00	0.2339E 00
	0.1327E 00	0.1400E 00	0.1470E 00
23.000	0.2416E 00	0.2457E 00	0.2498E 00
	0.1412E 00	0.1481E 00	0.1547E 00
25.000	0.2520E 00	0.2559E 00	0.2598E 00
	0.1466E 00	0.1533E 00	0.1597E 00

1. FOR EACH WIND SP, THE UPPER LINE IS THE RMS SLOPES FOR CLEAN SEA, N THE LOWER LINE IS FOR SLICK SEA.
2. M1,M2,N M3 ARE THE LOWER,MEDIUM,N UPPER VALUES OF RMS SURFACE SLOPES,RESPECTIVELY.

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