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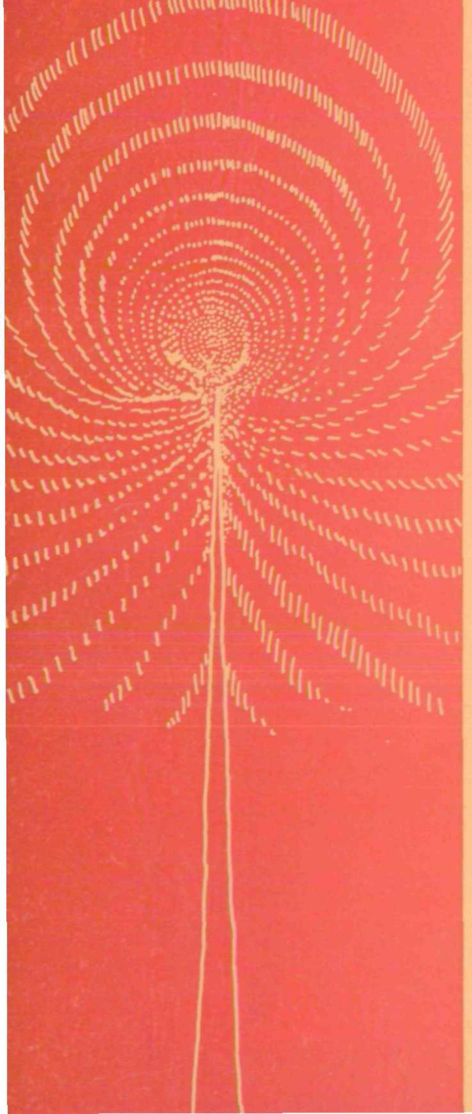

STRAIN ENERGY DENSITY AND
SURFACE LAYER ENERGY FOR
A CRACK-LIKE ELLIPSE

BY

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ABSTRACT

The central purpose of this report is to apply some of the fundamental concepts of sharp crack fracture criteria to cracks and narrow ellipses. The argument rests on the hypothesis that fracture occurs when a small element of material near the tip of the crack has absorbed a critical amount of energy, and then releases it to allow crack extension. This strain energy density theory is also extended to notch boundaries where in addition the energy in a surface layer is calculated and the location of failure initiation is determined.

The concept of a core region near the notch tip, and its consequences, are examined in detail. This stems from a concern that at the crack or notch tip, variations in material behavior, such as localized anisotropy or inhomogeneity, preclude an accurate (physically) local solution, but an analysis presuming a valid elastic solution external to this region, provides an accurate measure of the failure behavior of the material. The size of the core region is left unspecified at present, but subsequent research should provide means to establish its size, perhaps as a parameter depending on the microstructure of the material.

The example treated is that of an elliptical cavity loaded uniformly at a large distance from the hole, and at an angle to the hole; the results are shown to approach that of the crack solution for narrow ellipses, and to display

quite satisfactory agreement with recently published experimental data under both tensile and compressive loading conditions. Results also indicate that in globally unstable configurations in brittle materials, the original loading and notch geometry are sufficient to predict the subsequent crack trajectory with considerable accuracy.

A. INTRODUCTION

The primary area of interest has been that of an attempted correlation of the problems of failure (by fracture) of sharp cracks into a coherent theory where a crack is simply the limiting case of the elliptical cavity, and needs, for the most part, no special consideration. The means by which this is to be accomplished involves the use of a recently proposed theory by Sih (1972a) which requires local knowledge of the strain energy density function as part of the failure criterion.

The concept of a surface layer failure criterion for notches is dealt with as a preliminary requirement to locate the position (approximate) where fracture may be expected to initiate from the notch surface. Once established, the form is set to incorporate knowledge of the local strain energy density field as a means for establishing the failure criterion for the notch at some point in the bulk of the solid near the surface. This requires that a local core region be defined, which may include within its boundaries material anisotropy and inhomogeneity, and possibly geometrically induced singular features of the analytic solution.

The specific example chosen to exemplify the details of the theory is that of an elliptical cavity in a large isotropic, homogeneous medium, subjected to uniform loads applied asymmetrically to the hole. Both the cases of tension and compression are considered, and plots are included detailing the specific behavior expected. The actual elasticity solutions and equations are well known and are not reproduced here.

B. PHYSICAL BOUNDARIES AND SURFACES

It is well known that a physical boundary is not a sharp mathematically defined curve, but a rough surface composed of microscopic pits, holes, ridges, cracks, etc. Materially, the surface is also unlike the bulk of the solid: once a surface is exposed to an environment, gases are adsorbed, chemical alteration occurs (perhaps by oxidation) and various impurities may adhere. In general, thin layers of material foreign to the interior form on the surface and will behave differently from that of the interior. Additionally, forming new surfaces or boundaries, whether by rolling, drawing, or machining, results in layers of material that may be harder, less porous, etc., properties that may significantly differ mechanically from those further inside these regions; it is also expected that a formal boundary likely does not separate surface and interior regions.

At the continuum level, modeling the characteristics mentioned above is largely inhibited by lack of physical understanding, and by the very nature of the continuum approximations. The smooth curve of a continuum cannot describe the microscopic features of a surface as it physically occurs, so the approximate averaged curve is employed to predict behavior in regions away from local disturbances. However, as noted, it is these very surface features that ultimately determine the failure strength of a solid, and to this end a possible means of incorporating the effects of the solid surface into a continuum model is proposed.

In order to include any surface phenomenon into a continuum model, it will be necessary to do so in an averaged way. As an initial analysis, it will be assumed that a thin layer, of approximately continuum dimensions in thickness, extends into the solid from the boundary surface. In this boundary layer will be included any surface inhomogeneities, faults, etc., that characterize the physical surface (Figure 1). To a first approximation, it will be further assumed that the gross elastic properties of the layer are similar enough to those of the interior to permit treatment as an isotropic homogeneous medium. The criterion for failure will be that at some location or region along the boundary layer, the lo-

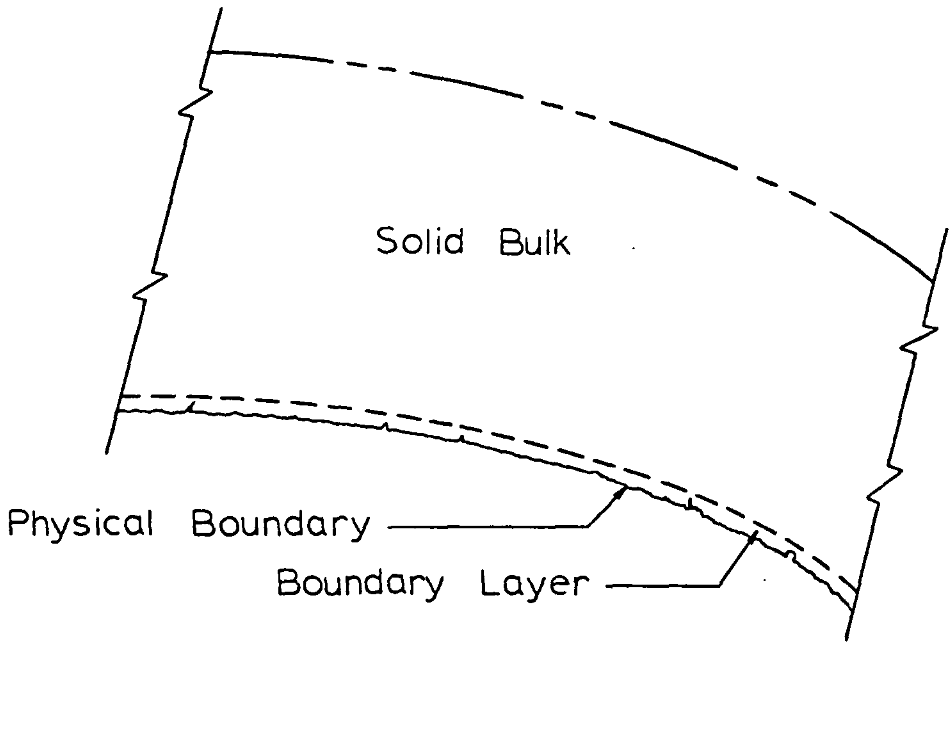


Figure 1 - Free Solid Surface Boundary Layer.

cal energy caused by loading will exceed some materially determined constant, and failure will initiate. Necessarily, this criterion provides but one location point of failure at the surface only, and any subsequent behavior is not expected to emerge from consideration of the surface layer. It may also be noted that failure is expected to occur in a region where the surface layer is in tension, subject to local separation, although local inhomogeneities may alter slightly the actual location of failure as predicted by the model.

Sih (1972b) has suggested that the form of this surface layer energy, represented by γ_e, be given as the product of the local radius of curvature, ρ, the tangential strain at the [continuum] surface, ε_s, and the normal stress, σ_n, on the interior surface of the boundary layer, designated to be of thickness δ; i.e.,

$$\gamma_e = \rho \varepsilon_s \sigma_n \tag{1}$$

where γ_e is the energy contained by a small element at the surface of the notch, for unit thickness of material. The quantities involved in γ_e are shown in Figure 2.

The limitations on geometry are left unspecified. It is expected that the usefulness of the surface layer tension will extend to some limiting sharpness size beyond which another fracture theory will be more precise in accounting for the material behavior.

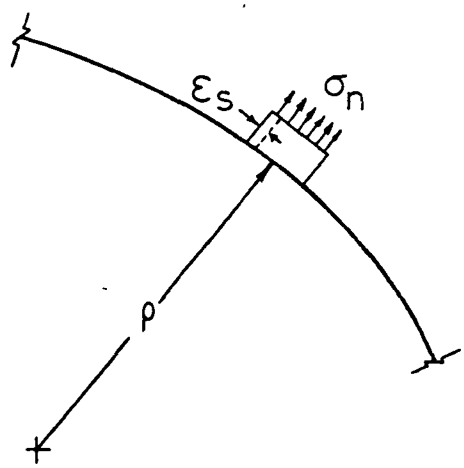


Figure 2 - Local Parameters of the Surface Layer Energy.

For an elastic isotropic material, the tangential strain along the boundary is given by the ratio of the tangential stress, σ_t, to Young's modulus, E;

$$\varepsilon_s = \sigma_t/E \tag{2}$$

The normal stress acting on the interior of the boundary layer, of thickness δ, can be shown to be related to the tangential stress, local radius of boundary curvature, and boundary thickness (to first order in δ);

$$\sigma_n = \sigma_t\delta/\rho \tag{3}$$

Combining expressions (2) and (3) into (1), the surface layer energy now takes the form

$$\gamma_e = \delta\sigma_t^2/E \tag{4}$$

Since neither γ_e nor δ has explicitly been given, it is convenient to form the quantity $\gamma_e E/\delta$ and treat it as a material parameter. A notched specimen may be loaded to failure and from equation (4), $\gamma_e E/\delta$ computed; then for any notch geometry of this material, the failure load may be obtained.

C. THE ELLIPTICAL NOTCH UNDER UNIFORM LOADING

For the purposes of illustrating numerically the behavior of the surface layer tension, consider an elliptical cavity cut into a large sheet uniformly loaded far from the hole at an angle β to the major axis of the ellipse (Figure 3). The boundary surface of the cavity is assumed rough. Around the

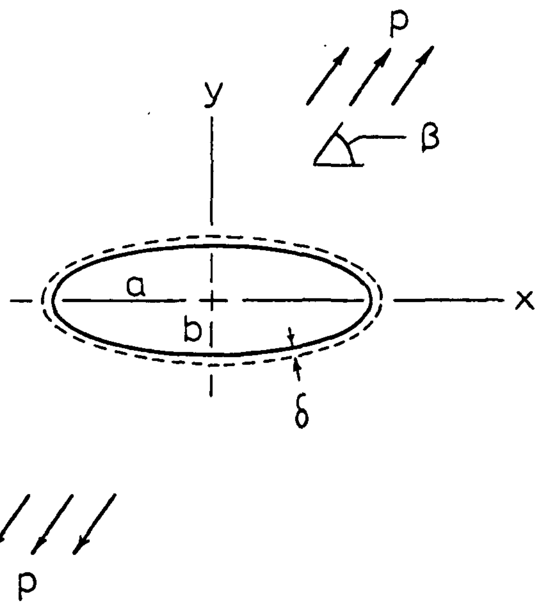
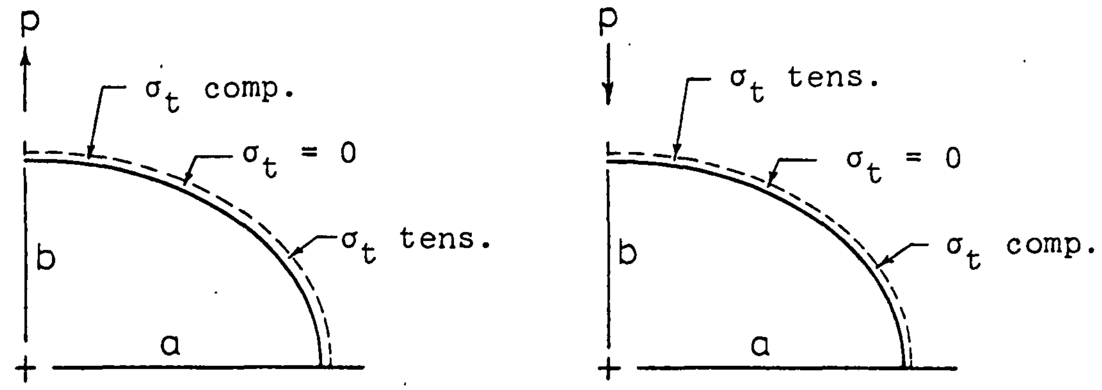


Figure 3 - Loading and Geometry of an Elliptical Cavity.

cavity is depicted a boundary layer of uniform thickness, δ. At some point along the layer, represented by (x,y), or (acosη, bsinη), where η is the eccentric angle for the ellipse, the magnitude of the surface layer energy, γ_e, may be calculated from equation (4) when the local tangential stress is known. In the case of the elliptical hole, the tangential stress is known analytically.

In its present form, equation (4) obscures the difference between tensile and compressive loading. Along the boundary of an elliptical cavity, σ_t may be tensile or compressive, as indicated in Figure 4. However, γ_e remains pos-



(a) Tensile Loading (b) Compressive Loading

Figure 4 - Tangential Stress under Tensile and Compressive (Normal) Loading.

itive at all points along the boundary, and hence, requires that the nature of the loading be dictated in order to select the region in which the surface layer is in tension. As the angle of loading varies, the regions of tensile and compressive surface stresses also shift positions.

If equation (4) is normalized to the form $\gamma_e E/\delta p^2$, where p is the applied uniform stress, then this quantity may be represented as a function of the loading angle β, semi-minor to semi-major axes ratio b/a, and the eccentric angle η.

It has been assumed that failure will occur at the location of maximum surface layer energy (in tension), and both the magnitude and location along the surface vary with angle of loading. Figures 5 and 6 show the normalized failure load, $p\sqrt{\delta/\gamma_e E}$, for various angles of loading and several degrees of sharpness of the tip of the ellipse. In Figure 5, under tension in the minimum load to failure is at $\beta = 90°$, while under compression (Figure 6), the minimum load to failure occurs in the region $\beta = 45°$. The elasticity solution has assumed no surface contact under compression, and the cavity sizes entered are free of this problem. One of the values of the skew-symmetric loading geometry is manifested by the behavior of the elliptical cavity under compression.

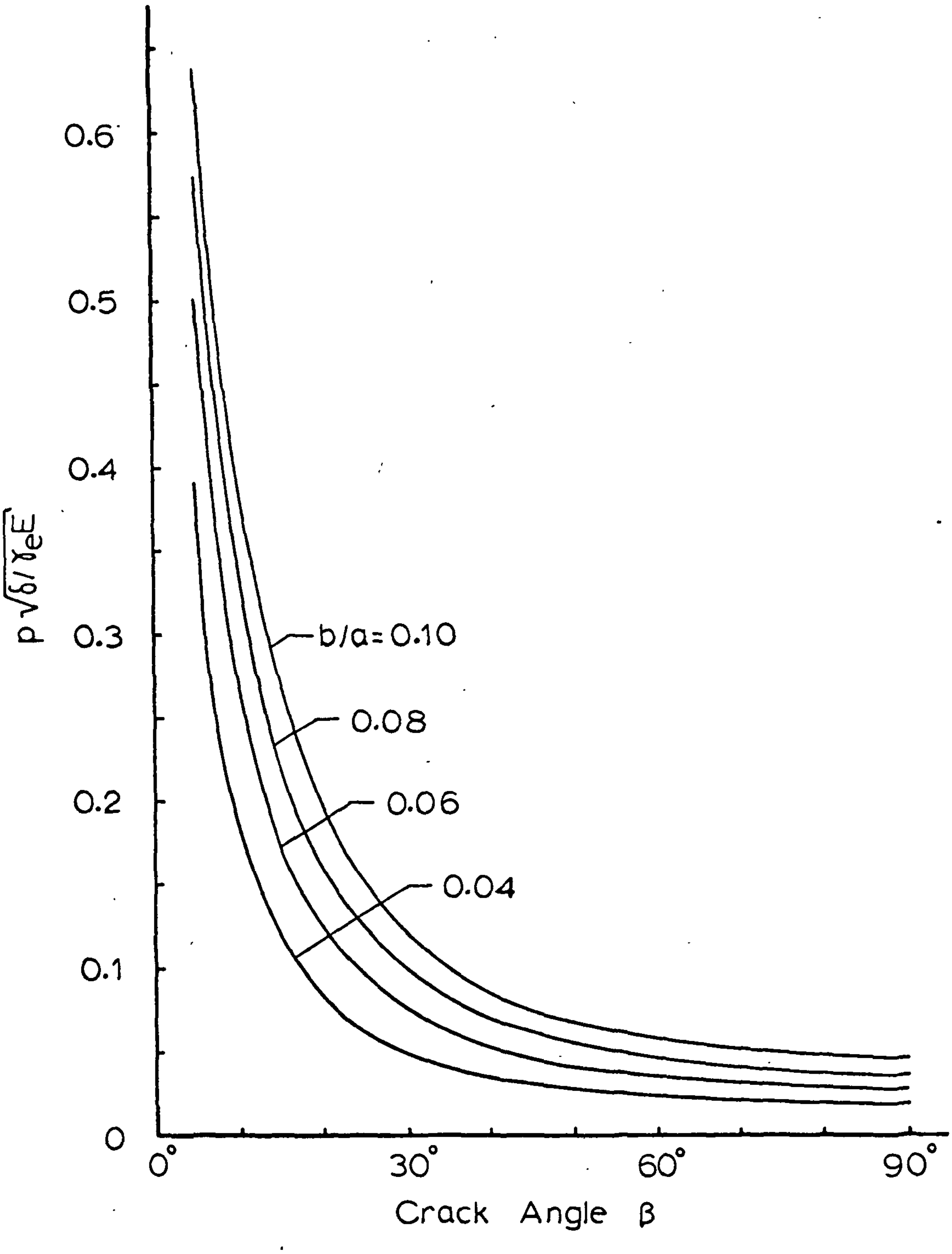


Figure 5 - Variation of Maximum Loading Stress with Angle of Loading; Tension Case.

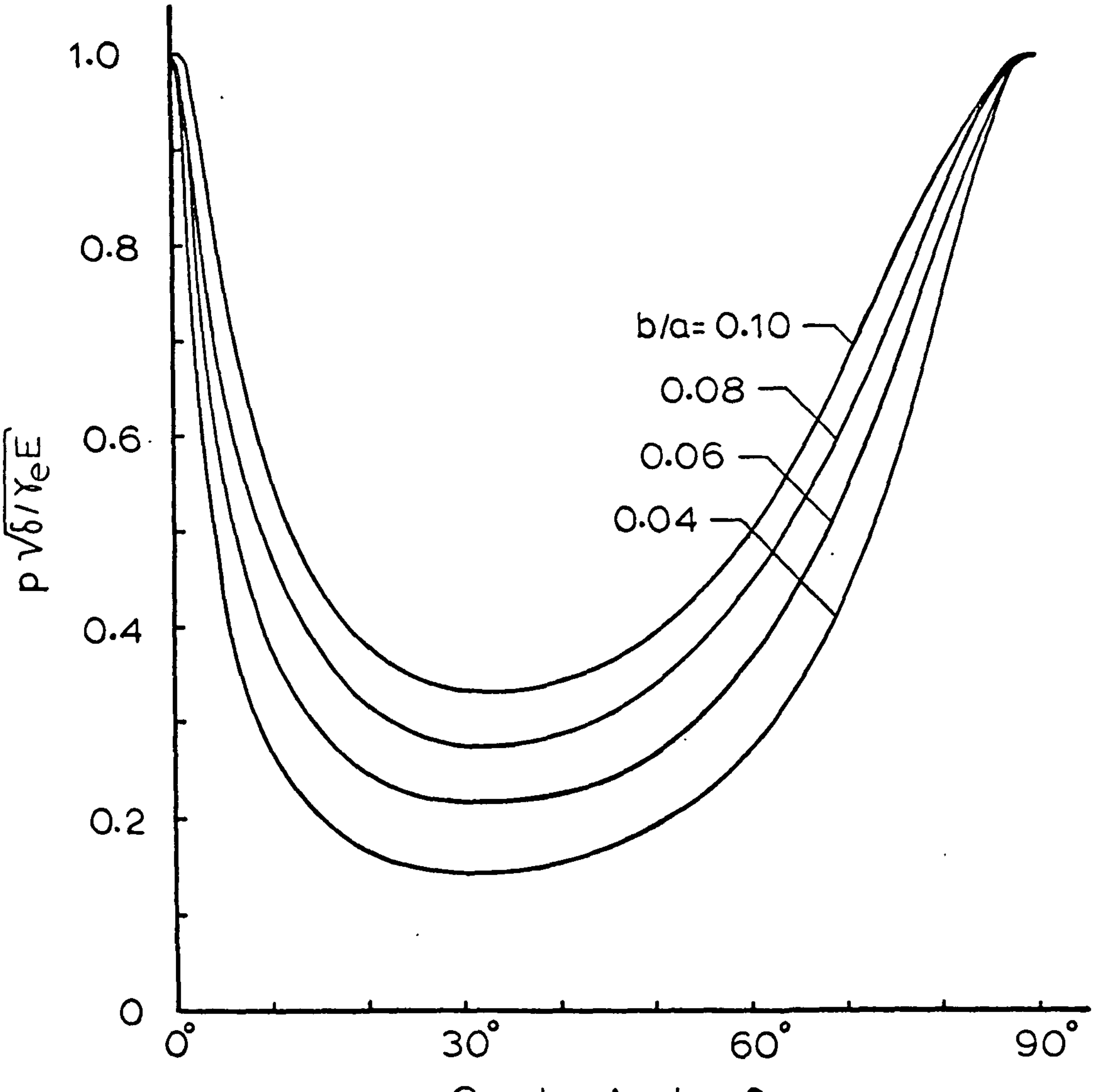


Figure 6 - Variation of Maximum Loading Stress with Angle of Loading; Compression Case.

D. THE S_c-THEORY, THE CORE REGION, AND LOCALIZED FAILURE

As proposed by Sih (1972a), the concept of some critical value of the strain energy density being a material constant may be used to predict fracture of a material, whether at a crack tip, notch tip, re-entrant corner, or in an unflawed structure. In its original form, the theory takes the singular terms of the stresses near a line crack tip, and substitutes them into the strain energy density expression. This results in a quadratic form for the strain energy density function,

$$\frac{dW}{dV} = \frac{1}{r}\left(a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2\right) + \ldots \qquad (5)$$

Here the quadratic

$$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2 \qquad (6)$$

represents the amplitude of the energy density field, where the coefficients a_{ij} $(i,j = 1,2,3)$ vary with a polar angle, θ, measured from the crack tip, and the stress intensity factors k_1, k_2, k_3 are dependent only upon loading and geometric conditions. The quadratic S is postulated to be invariant with respect to k_1, k_2, k_3, and possess the inherent property of a constant. It has been shown, based on potential energy considerations, that crack initiation will start on a radial direction along which the strain energy density is a minimum. In general, for a notch, the form of the strain energy density will not be as concise as that of equation (5), but the evaluation of the energy field will be the same as that for the crack, and the resulting failure loads and angles also determined on the basis of the strain energy density assuming a critical constant value in a given material. It should be noted that in the considerations here, the exact strain energy density function is evaluated, at some finite distance from the notch tip. This has been done since, as alluded, there is no simple expansion of the energy available for a non-sharp notch tip. It is for this reason that as the notch degenerates to a line crack, the evaluation is unaffected, since the locations of the points of interest do not lie on the surface of the notch itself.

A fundamental difficulty in fracture mechanics arises in the use of linear elasticity theory - singular stresses result at sharp corners in the analytical models, stresses which are physically unattainable. In part, this difficulty arises because analytically, one cannot expect the solution to be accurate much closer to the tip of a crack or notch than the minimum continuum element size. It must also be recognized that in a very local region at the crack tip, the physical behavior is unknown, and cannot be incorporated into a mathematical model. In this region, the material is highly strained, may become inhomogeneous, and in general, is not conducive to modelling. In polycrystalline materials, for example, the orientation of a grain may be significant, but unknown from a continuum viewpoint, and statistically, perhaps random. These same limitations hold for sharp notches as well, and likely for blunted notches. But it must be emphasized that both the analytical and physical aberrations mentioned are confined to a very small region near the notch tip, and for this reason are not expected to significantly perturb the analytic solution external to this region.

The initial approach taken here is that of postulating the previously stated aberrant material behavior to be limited to a region local to the notch tip (Figure 7) characterized by a length dimension, r_o. This core region will

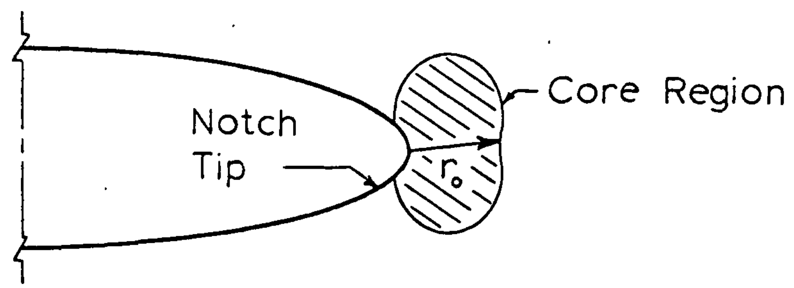


Figure 7 - Notch Tip Core Region.

remain temporarily unspecified in shape. The region is not unlike that proposed by Neuber (1946), and discussed by Orowan (1955). The main purpose is to avoid the requirements of an explicit continuum solution everywhere around the notch. It is immediately obvious that the region in question must be small, since it is expected that apart from this region, the linear elasticity solution of the notch geometry retains its validity. Under these circumstances, the materials to be evaluated are necessarily restricted to be brittle, for otherwise, large local disturbances and energy dissipation occur which must be taken into account by a more refined continuum model. The motivation is that of evaluating the fracture toughness on the basis of material behavior external to the immediate vicinity of the notch tip.

E. AN EXAMPLE - THE ELLIPTICAL CAVITY

1. Introduction and Intentions

For the purposes of illustration, a specific notch geometry has been chosen to demonstrate the methods of analysis and the particular uses for which the resulting information about the strain energy density near the notch tip can be employed. The choice of the uniformly loaded [infinite] solid containing an elliptical [through] cavity is based primarily on the availability of an analytic solution for this geometry, and also on the availability of recent experimental work involving this geometry under both tensile and compressive loading conditions. Additionally, the ease with which the solution is reduced to that of the finite crack geometry permits comparison with this limiting case of blunted notches.

The initial examination indicates favorable agreement with current experimental data. A brief comparison to the maximum stress theory for sharp cracks is also made in light of recently published experimental work.

The details of the stress field around the elliptical cavity under uniform loading are contained in textbooks. Isotropy and homogeneity of material are assumed. The elliptical cavity (in two dimensions - plane strain) is defined by the ratio of minor to major axis half-lengths, b/a, with all other length parameters normalized with respect to the major axis half-length, a. The uniform loading (at infinity) forms an angle of β with the major axis of the cavity, and the angle θ used in the radius vector $\underset{\sim}{r}$ is measured positive counterclockwise, referenced to an axis parallel to the major axis of the cavity. At any given point on the surface of the cavity, the normal angle to the surface forms an angle ϕ with the major axis, and also is measured positive counterclockwise. Figure 8 indicates the geometry described. Only the right hand notch tip will be considered, as antisymmetry of the geometry implicitly allows description of the other notch

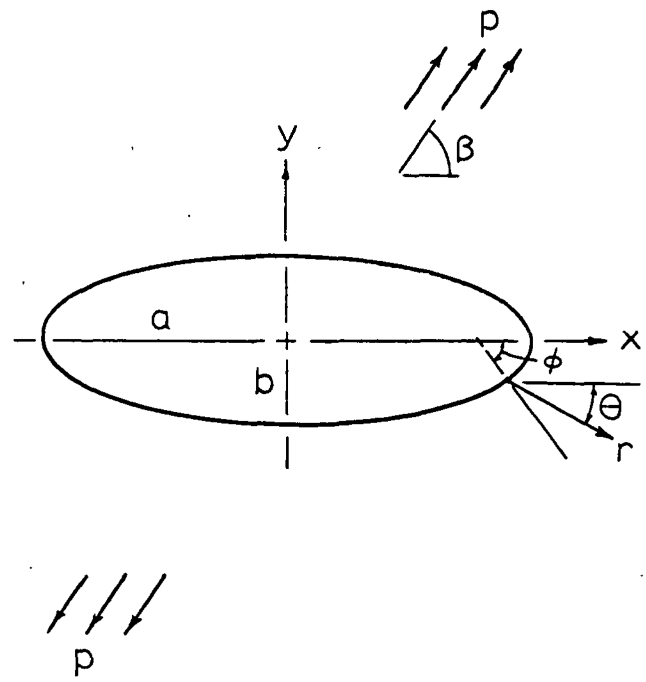


Figure 8 - Geometry and Parameters of Elliptical Cavity and Radius Vector.

tip as well.

The stress field is used in the expression for the (exact) strain energy field surrounding the notch tip. All stationary values of the strain energy density are obtained numerically, as the contours followed do not lend themselves to analytic evaluation.

Since the behavior of the strain energy field is load dependent, the cases of compression and tension are treated separately. There are some fundamental differences in the two cases, and it is intended that separation will clarify these departures without redundancy of explanation.

The strain energy density function is also dependent upon the value of Poisson's ratio, and it will be assigned a value of 0.250, unless otherwise stated. It is clear from the work of Sih (1972a) that there are significant deviations attributable to Poisson's ratio change. These deviations are restricted to magnitudes only, however, and trends remain unchanged. Hence, subsequent discussion will center not on this effect, but on basic observable behavior, restricting changes in the Poisson's ratio to matching experimental specimen material where necessary.

The choice of specific values for the ratio b/a has been governed to large extent by the experimental data available. In other cases, the choice is simply for illustrative purposes, and any further comparison with new data requires new calculations to be performed. It is intended that the choices reflect fairly completely the range of possible behavior for this geometry of notch and loading.

2. Origin of the Radius Vector

The evaluation of the local strain energy density field requires a knowledge of the position of maximum surface layer energy, which coincides with the position of maximum tangential (tensile) stress along the boundary. For the elliptical cavity, this position may be analytically determined: setting the first derivative of σ_t with respect to eccentric angle η to zero, the resulting equation, quadratic in $\tan\eta$, may be solved to give

$$\tan\eta = b\{a\sin^2\beta - b\cos^2\beta \pm \sqrt{a^2\sin^2\beta + b^2\cos^2\beta}\} \;/\; \times$$

$$\times\; a(a+b)\sin\beta\cos\beta \tag{7}$$

The positive root provides the location for failure initiation under compressive loading, and the negative root for failure initiation under tensile loading (Figure 9).

The special case of $\frac{b}{a} = 0$ corresponds to that of a line crack, and in such a case, the radius vector is attached to the crack tip. This has been treated to considerable extent by Sih (1972a), where the asymptotic expansion is considered. Rather than repeat the results of that work, it will be instructive here to discuss possible behavior further from the crack tip.

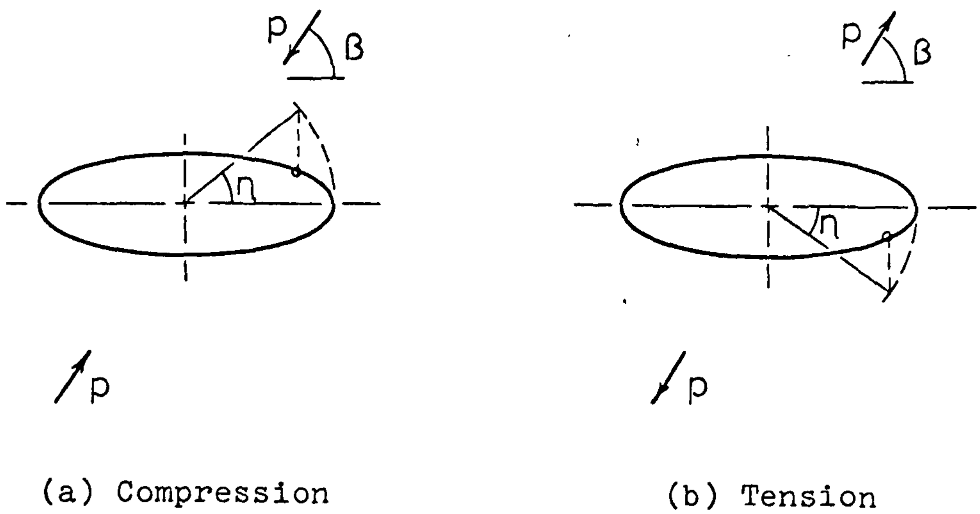


Figure 9 - Location of Points of Initial Failure in Tension and Compression.

3. The Elliptical Cavity in Tension

If an elliptical cavity in an isotropic, homogeneous, linearly elastic solid is uniformly loaded as shown in Figure 8, at an angle β to the major axis, the maximum surface layer tension occurs in the surface region in the fourth quadrant, where $-90° \leq \eta \leq 0°$, (considering right hand tip only). Loading behavior as a function of crack angle, notch size, and position in the medium is the subject of the initial discussion. Then considerations will focus on the initial and subsequent crack trajectories. This order is necessary in light of the earlier criteria established for the expected nature of failure and its relationships to the core region size.

a. Failure Loads in the Medium

For purposes of initial analysis, the applied uniform stress, p, will be assumed to be small, with no core region present, to establish the behavior of the strain energy field.

Although the normalization is slightly different, the strain energy behavior at the surface coincides with that shown earlier in Figure 5, since at the surface, the strain energy is a function only of the one non-zero stress component, the tangential stress, the same one involved in the surface layer tension. As the radius (normalized to the major axis half-length) is made to increase, the located stationary values of the strain energy begin to decrease, indicating that analytically, (exclusive of surface layer energy considerations) the strain energy density assumes its absolute maximum value on the surface of the notch, (Figure 10). In the figure, a family of several curves appears, each curve representing the possible loading behavior of a particular notch at a different distance from the notch surface. It may be observed that in all cases, the minimum

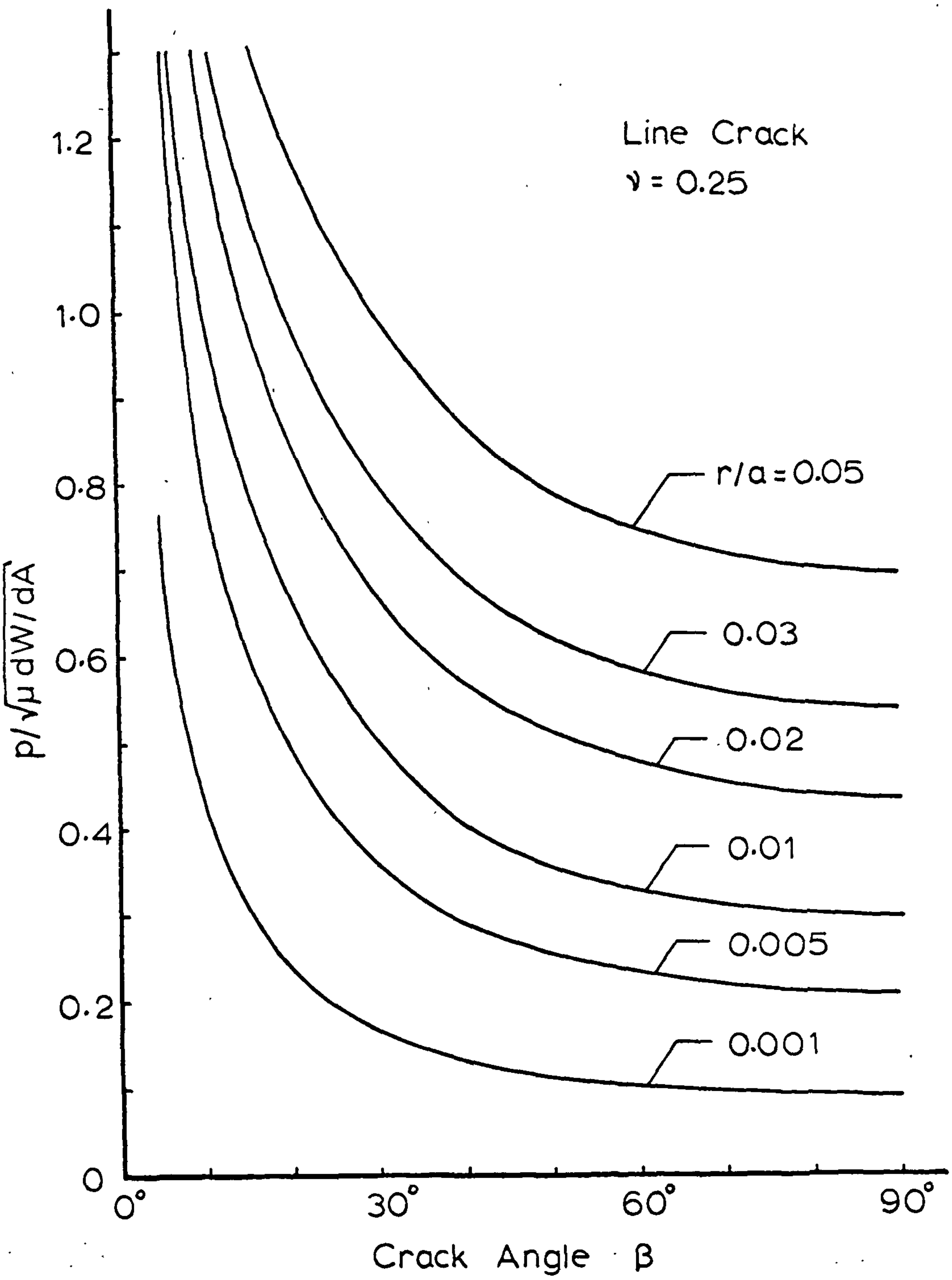


Figure 10 - Variation of Load with Crack Angle:
Line Crack; (Tension).

load required for fracture occurs in the normal loading case (β=90°). Intuitively, one would expect such a result, although some other fracture criteria have been proposed in which the minimum load appears in the vicinity of β = 70°. Palaniswamy (1972) attempted an energy release rate procedure for complex loading, in which a small extension to the main crack was varied in direction until the maximum strain energy release rate for small crack extension was determined; in this case, a minimum was located near 72°. It must be mentioned that in order to solve the stated problem, approximations in the numerical procedures altered the strain energy density function by unknown amounts, assumed small, casting some doubt on the validity of the results, and the extent to which any implications can be made concerning fracture load behavior. A second case in which the minimum load appears near β = 70° occurs in the criterion of maximum tangential stress near the crack tip. Unmentioned in the original paper on the inclined loading of a crack by Erdogan and Sih (1963), Williams and Ewing (1972) pointed out this effect in the loading response. This latter work included an attempt to avoid the local material behavior by applying the criterion a short distance from the crack tip using an additional term in the asymptotic solution. Unfortunately, the truncation errors in the results over-emphasize the minimum load, manifest when compared to the exact solution (Sih and Kipp, 1973). The primary results (normalized to the β=90° load) are shown in Figure 11 for the maximum stress criterion, and

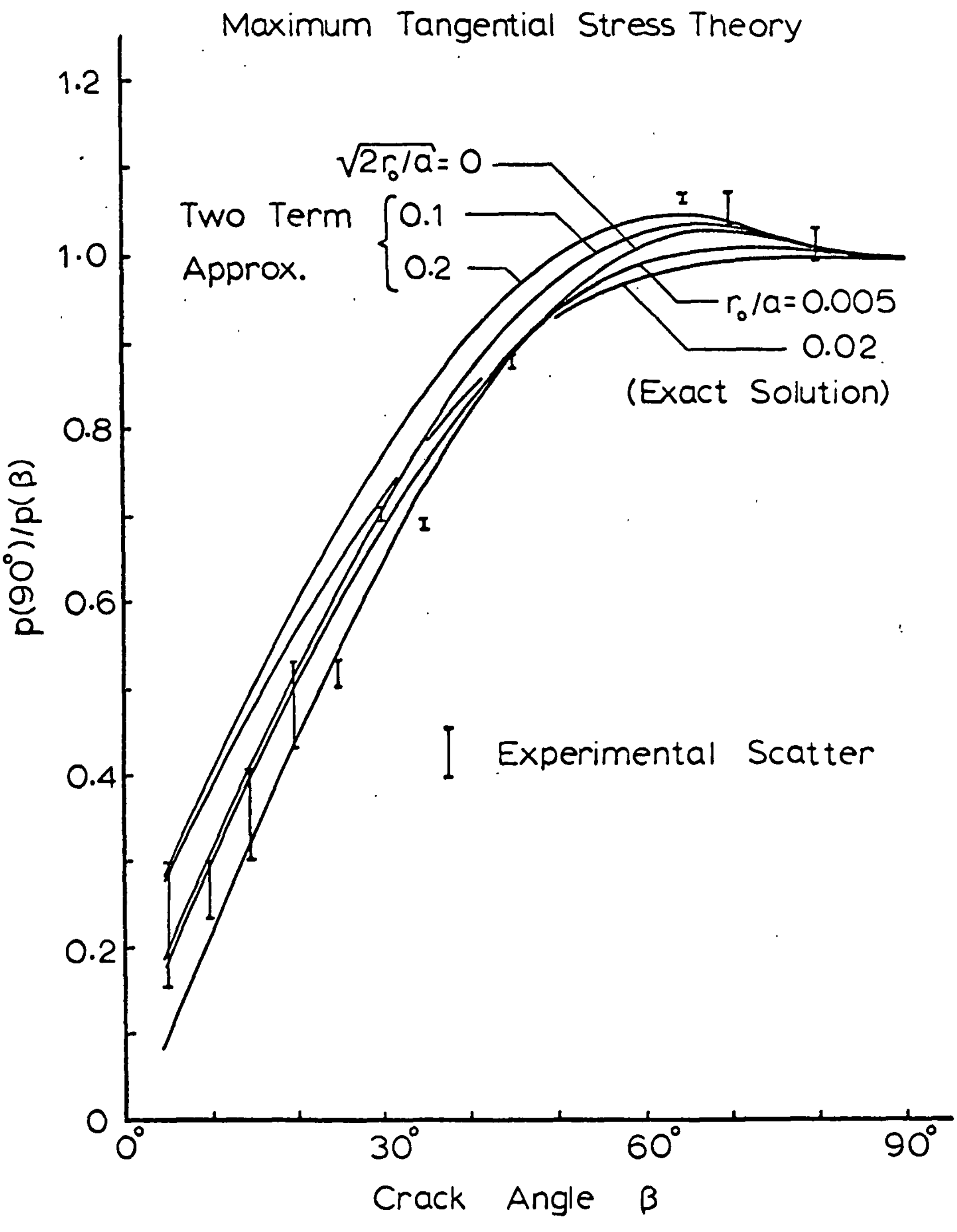


Figure 11 - Variation of Load with Crack Angle: Two Term Approximation and Exact Solution of Maximum Tangential Stress for Line Crack; (Tension).

in Figure 12, where the data obtained by Williams and Ewing is compared to the strain energy density theory for core regions of the same size as those used for the maximum stress criterion. (The data presented appear to display a trend towards minimum load near $\beta=70°$; however, an authors' note indicates that much of the data had to be discarded because of inability to distinguish initial crack growth, so the data is not truly representative of the failure loads as a function of the angle of loading).

b. Fracture Trajectories

Although interest from a safety standpoint centers on loading capacity, and more specifically, on the worst case if possible, the actual post-failure behavior of the fracture provides another insight into the validity of the theory under study. As briefly mentioned earlier, Griffith had suggested that a crack would extend in a direction normal to the maximum tangential stress, and Erdogan and Sih (1963), in applying this criterion obtained striking agreement with their experimental data. In a discussion of this paper, McClintock (1963) suggested the use of the normal angle from the ellipse surface as the directional property, but the result was not in the slightest agreement with the observed behavior. The published data of Williams and Ewing (1972) for initial crack angle (with respect to the plane of the crack) corroborates that of Erdogan and Sih. As discussed with respect to the loading variations with crack angle, a core re-

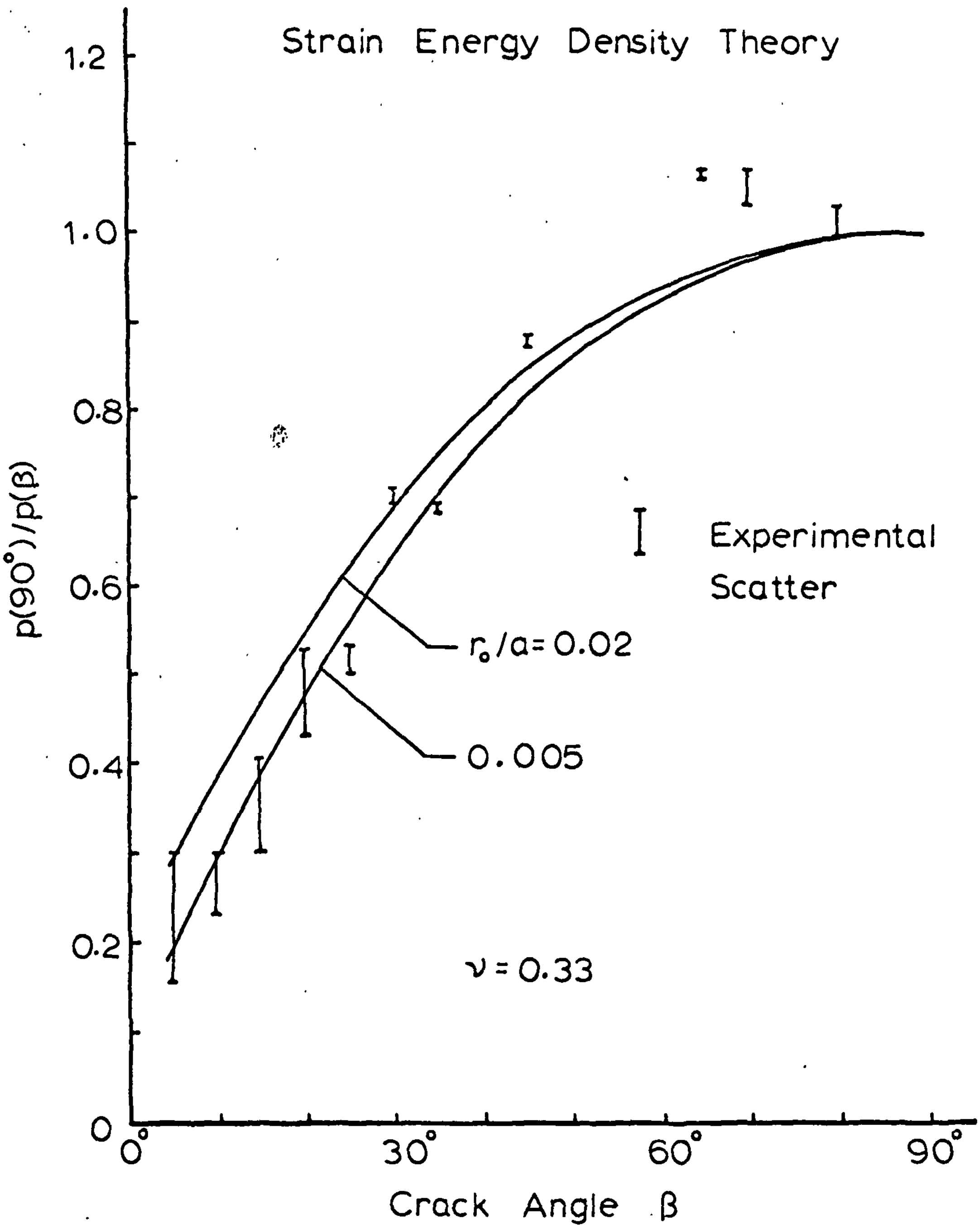


Figure 12 - Variation of Load with Crack Angle: Strain Energy Density Solution for Line Crack; (Tension).

gion concept was applied, and Figure 13 shows the results for the maximum tangential stress, including the corrected (exact) maximum stress solution. Figure 14 illustrates the predictions of the strain energy density function for the same core region dimensions. At $\beta = 90°$, the crack is expected to propagate in its own plane, but as β becomes small, the direction becomes less well defined. At $\beta = 0$, the material reacts as if (in theory) no crack at all were present, and while the material would be expected to break at a normal to the load, the crack solution cannot predict this. Before proceeding further, it is necessary to be more precise in stating how the crack will extend from the tip of the crack or notch. When the stationary values of the strain energy density were determined for various radius vectors, in addition to the load, an angle θ was found corresponding to the load. So for each radius r/a, a curve is generated in the angle θ; Figure 15 reflects this angle for the same ratios of b/a and r/a as were used for the load variations of Figure 10. Once the parameter r_o/a has been established from loading considerations, the angle of fracture, θ_o, is determined from the assumption that there is separation of the material from the surface to the edge of the core region in the direction θ_o, with subsequent fracture proceedings from there.

One must remain cognizant of the non-stationary position of fracture initiation along the notch surface. As

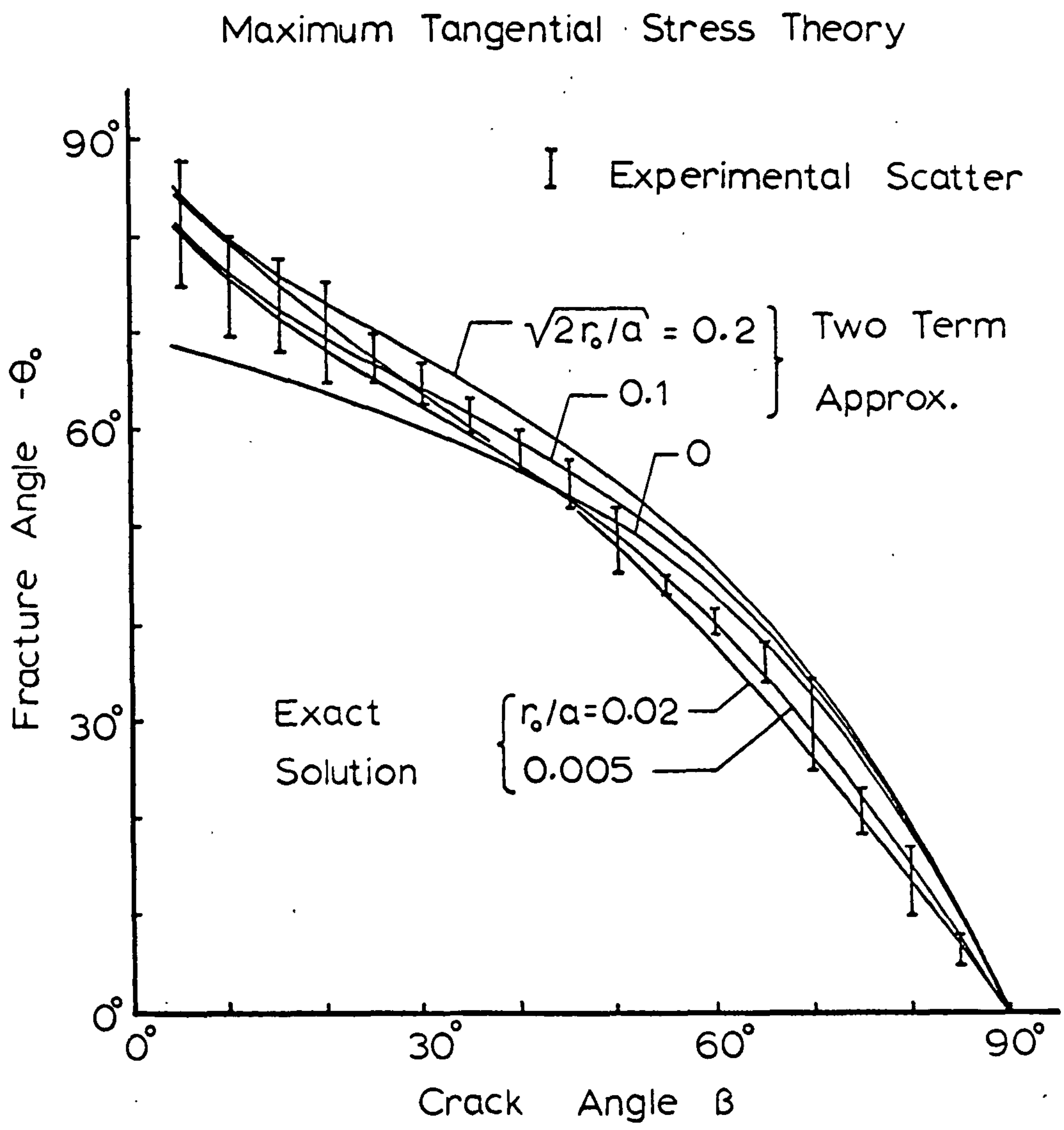


Figure 13 - Variation of Fracture Angle with Crack Angle: Two Term Approximation and Exact Solution of Maximum Tangential Stress for Line Crack; (Tension).

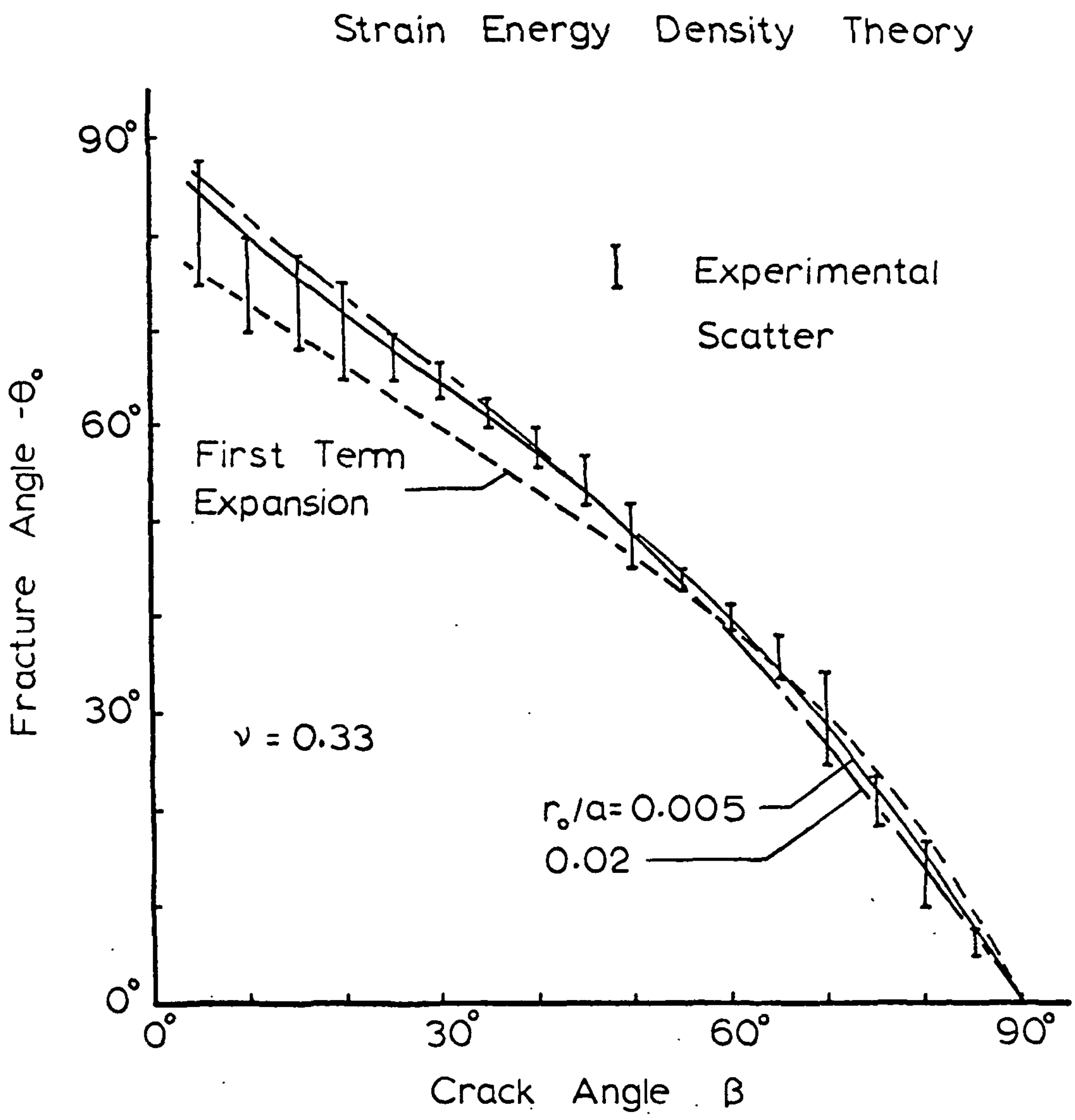


Figure 14 - Variation of Fracture Angle with Crack Angle: Strain Energy Density Solution for Line Crack; (Tension).

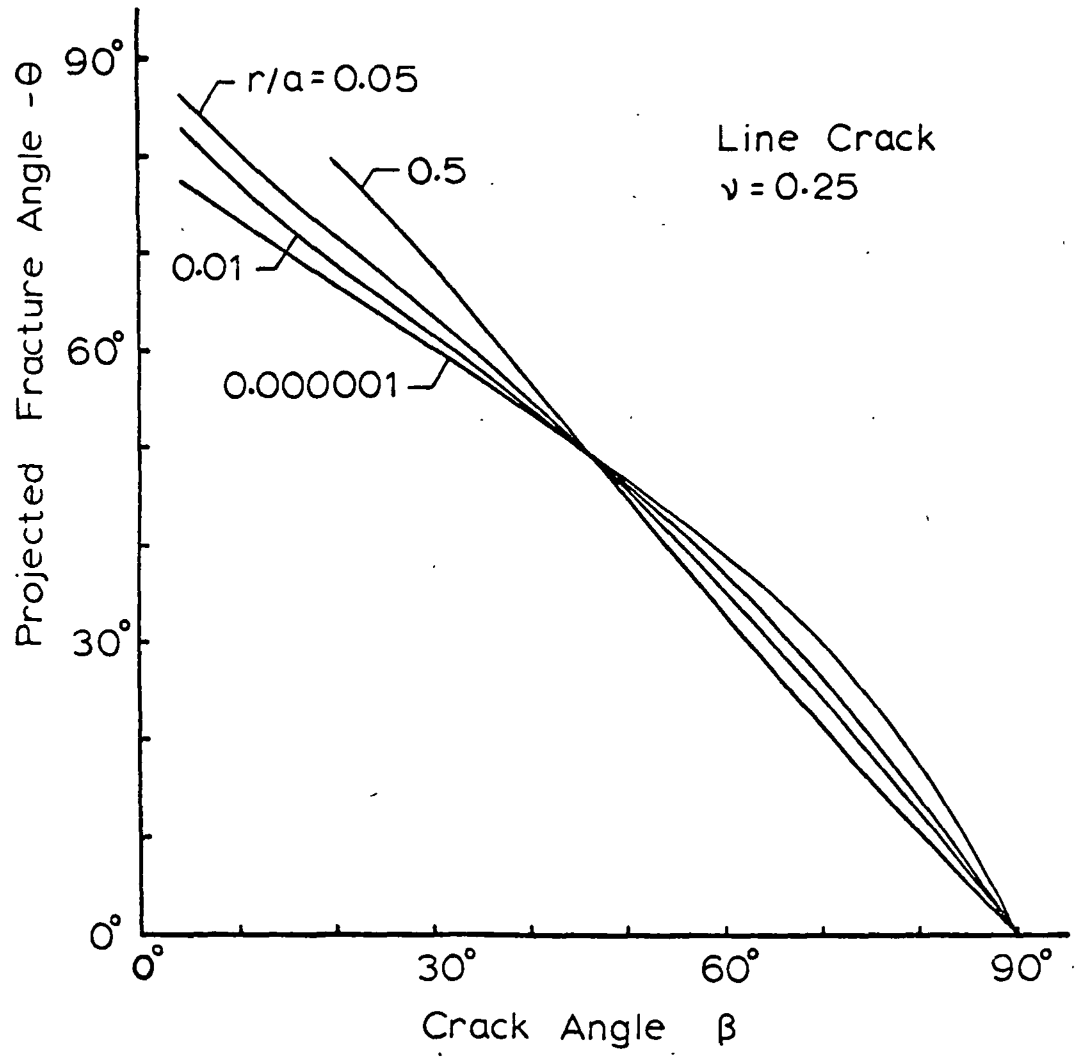


Figure 15 - Variation of Fracture Angle with Crack Angle: Line Crack; (Tension).

the loading angle changes, so does the position of initial failure. One result is that now, when $\beta = 0$, there is a well defined point of failure (because the geometry is now non-trivial), and it is to be expected that the fracture will extend from the minor edge of the cavity along the minor axis direction. The path is, of course, perpendicular to the load, and the behavior, apart from load magnitude, corresponds exactly to that from the major axis notch tip at $\beta = 90°$.

Cotterell (1969) has observed that for notches, rather than immediately turning into a path of fracture coinciding with that of the line crack, the fracture trajectory extends forward from the notch tip, roughly in the same plane, for a distance of approximately one notch tip radius, before turning into the path direction matching that for the line crack. (The ideal line crack has, of course, no tip radius, and would immediately assume its characteristic direction of failure). If it can be assumed that the crack path is predetermined for initially brittle materials in unstable configurations, then the radius vector $\underset{\sim}{r} = \underset{\sim}{r}(\theta)$ should trace out the trajectory along which fracture is expected to occur. In effect, the trajectory will follow a path that restores symmetry to the geometry of fracture.

For a line crack, Figure 16 illustrates the projected paths for several angles of loading. A photograph*

*Appreciation for this photograph is expressed to Dr. T. T. Wang, Bell Telephone Laboratories, Murray Hill, New Jersey.

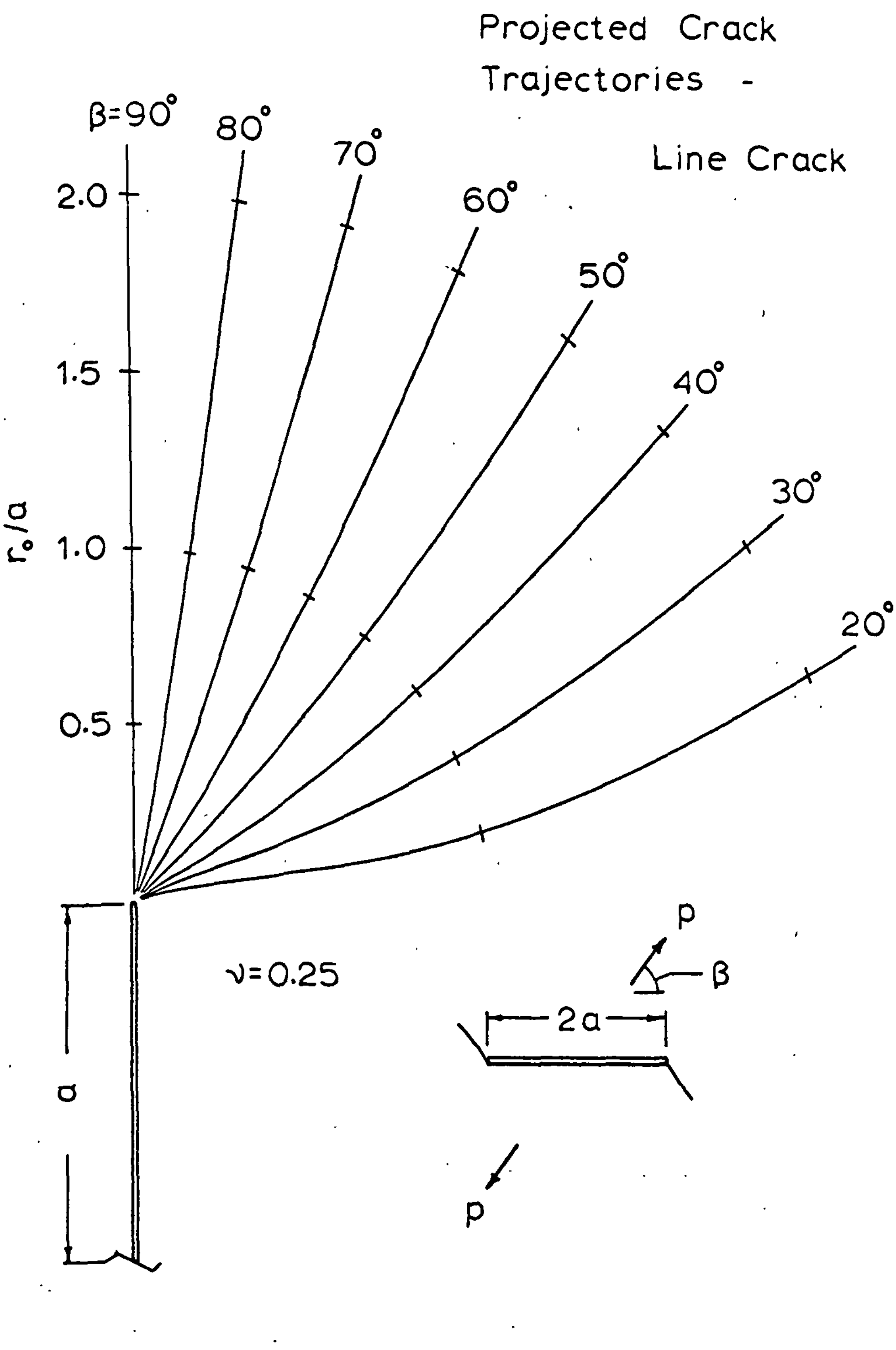


Figure 16 - Projected Fracture Trajectories:
Line Crack; (Tension).

of an actual crack trajectory is shown in Figure 17. The agreement between theory and experiment is very good out to a distance of at least one half crack length.

4. The Elliptical Cavity in Compression

Although much of the behavior to be described in this section corresponds to that of the tensile case, there are a few additional difficulties that must be dealt with in compression. Rather than reiterating the arguments already presented in detail, emphasis will be placed upon the features that tend to complicate the compressive analysis. Were the analysis confined to simply reversing the applied loads in the last section and proceeding as before, there would be no trouble. But immediately, caution must be exercised in requiring that no interpenetration of notch faces occurs. The line crack, for instance, must be reformulated to ensure that sufficient normal stresses occur on the faces to prevent penetration, and this has been attempted by McClintock and Walsh (1962). In addition, it is clear that functional stresses will be developed on the faces if any slip is to occur to allow fracture to progress. In the development here, the notch geometry will be restricted to such dimensions that contact between opposing faces cannot occur.

The geometry of the ellipse is such that in compression, the maximum surface tangential stress that may develop is equal in magnitude to little more than the applied compres-

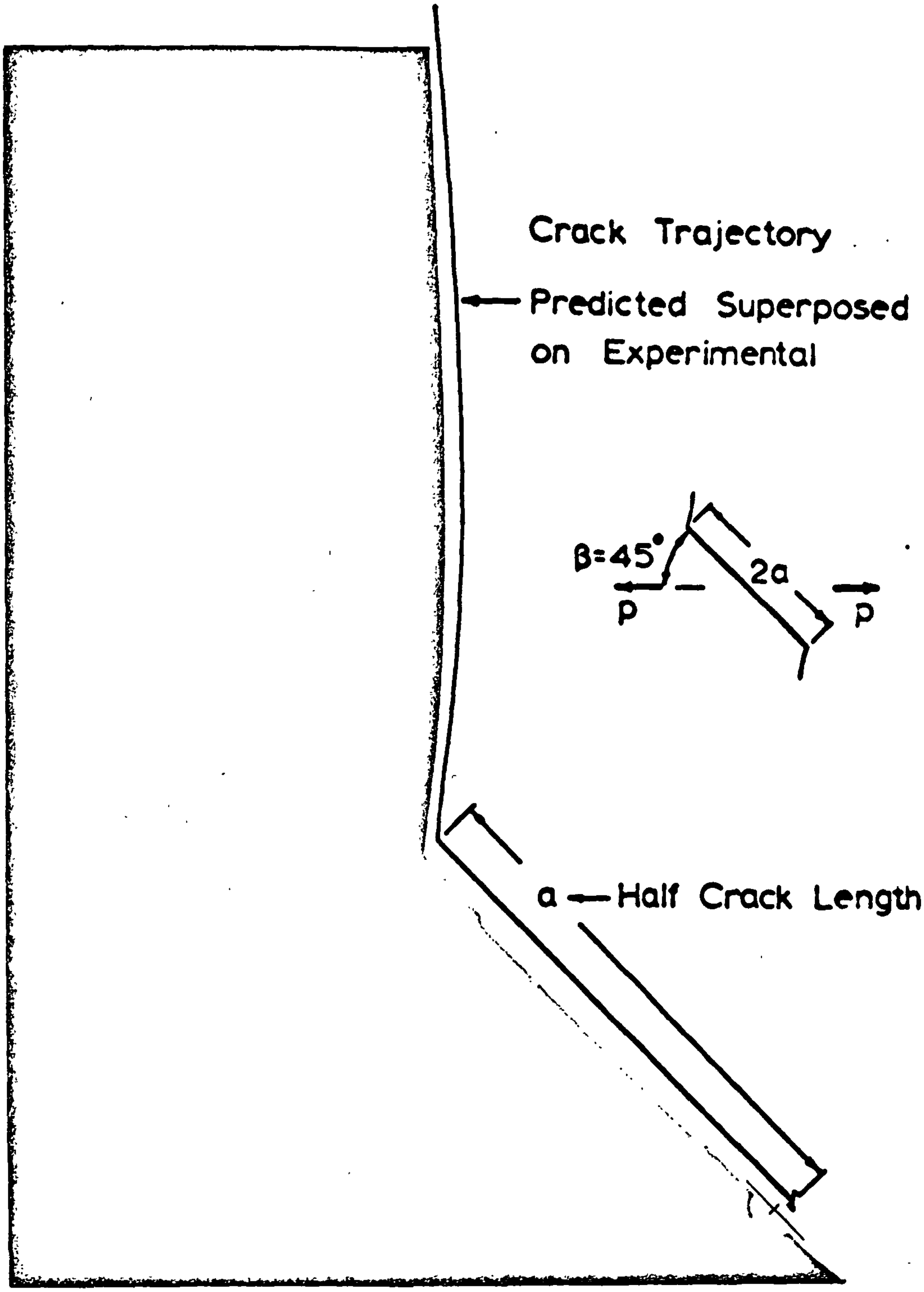


Figure 17 - Predicted and Experimental Fracture Trajectories; β = 45°.

sive load, and elsewhere, tremendous crushing pressures may be developed. The consequence is that some kind of material damage is inevitable, although it does occur at locations away from where a fracture is expected to initiate and grow. A combination of these factors results in a weaker model for fracture prediction than that of the tension case, although limited agreement with experimental data does occur.

As before, assuming a small applied uniform stress, the surface position of maximum energy (maximum surface tangential stress) may be located, and an origin established to evaluate the load behavior as a function of radial distance from the surface. Figure 18 describes the load behavior as a function of notch angle, for various constant radii, r/a.

A characteristic of compressive loading is the minimum load to fracture occurrence in the non-symmetric case. The families of curves plotted indicate a wide range of failure loads over the various values of β from zero to normal, but experimentally, there exists much less variation. Cotterell (1972) has published the experimental data that appear in Figure 19 for an elliptical cavity in glass with proportions b/a = 0.1. The curve r/a = 0 is that of the maximum tangential stress criterion. As the position of failure criterion is moved into the material, there appears quite satisfactory agreement between r/a = 0.005 and 0.008. Hence the energy criterion may be expected to provide quite reasonable failure predictions. On the basis of the data, one

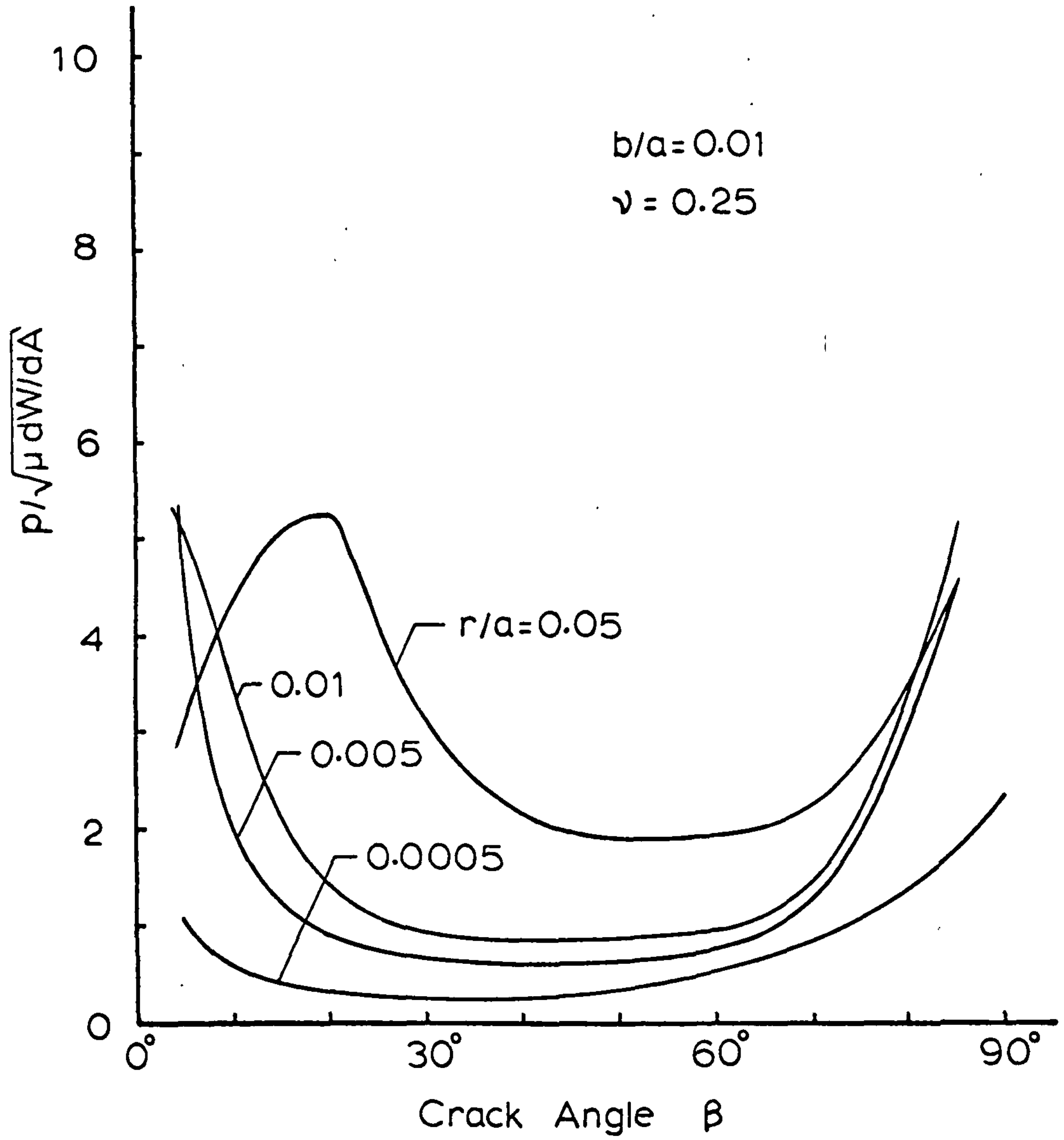


Figure 18 - Variation of Load with Crack Angle: b/a = 0.01; (Compression).

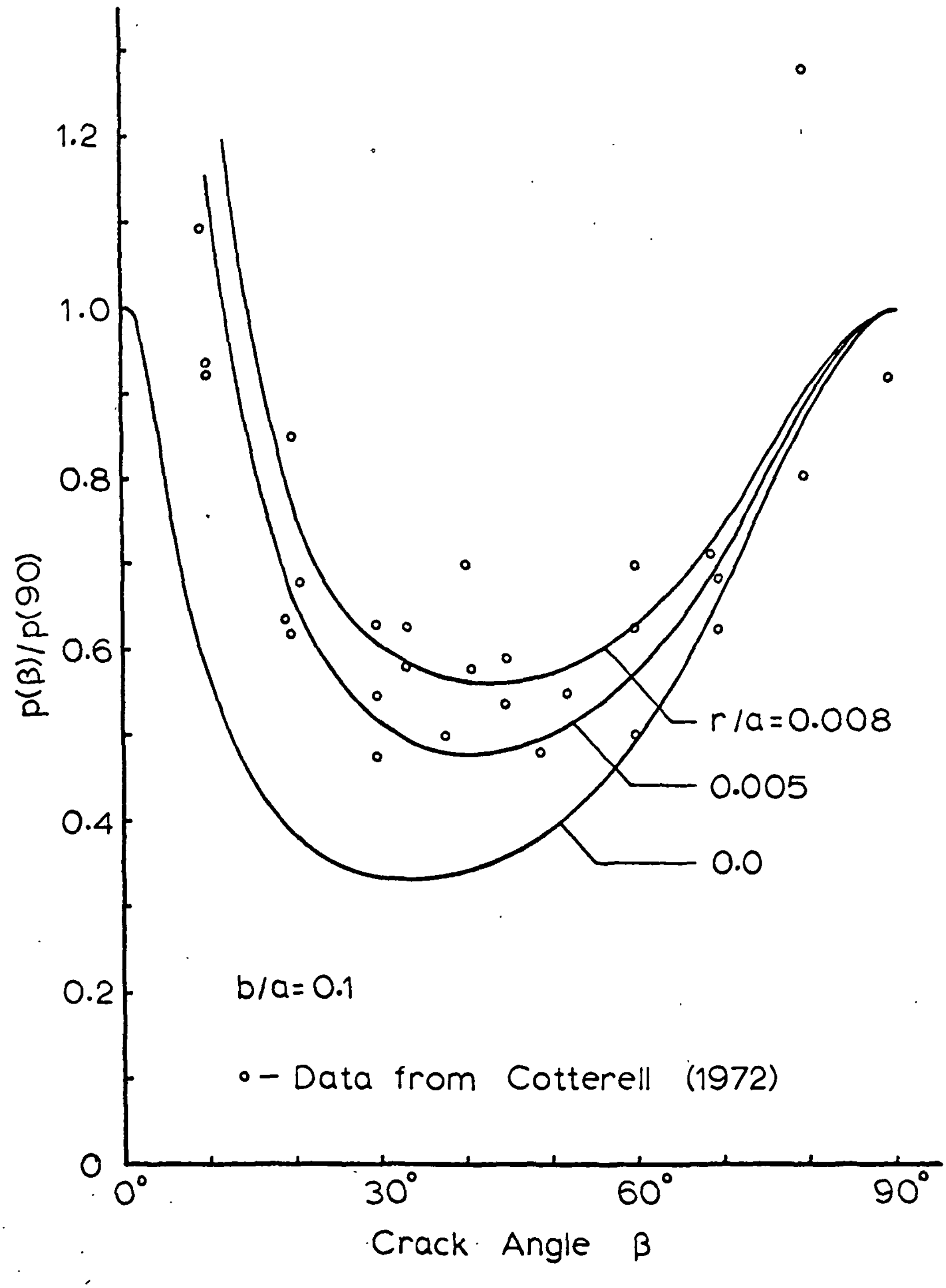


Figure 19 - Variation of Load with Crack Angle: b/a = 0.1; Data from Cotterell (1972); (Compression).

could choose a critical core dimension of, say, $r_o/a = 0.005$, and then use this dimension as set forth in previous sections for load predictions for other geometries in this material.

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