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ADAPTIVE SCHEME

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A "TYPE ONE" SERVO EXPLICIT MODEL FOLLOWING ADAPTIVE SCHEME

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Abstract

This paper describes "type 1" servo techniques applied to explicit model following systems. The advantages of using these are: (1) steady-state decoupling is provided for the controlled state variables as determined by a reference input (the pilot command for example), (2) overall stability of the decoupled adaptive scheme is guaranteed, (3) a zero steady-state error for those state variables incorporating integral feedback, to a constant input reference, and (4) improved transient performance. Example calculations have been performed using a mathematical model for a typical fighter aircraft and a desired performance reference model based on satisfactory aircraft flying qualities. Numerical data results are given.

Introduction

Model following control systems have received much attention over the past 10 to 15 years[1,2] with optimal control techniques receiving increased emphasis since Kalman's formulation of the problem. There are two categories of optimal control of model reference systems with a quadratic performance index (P.I.). In implicit systems, the model is contained in the P.I. but not in the control diagram. In explicit systems the model explicitly occurs in the system control diagram as illustrated in Figure 1.

The purpose of this paper is to discuss type 1 techniques as implemented in an explicit model following control system for possible application in a digital fly-by-wire flight control system. The method has been applied in an analog form to both the longitudinal and lateral dynamics of a representative fighter aircraft. Results are presented for the lateral case. Although the analysis and data given for the proposed control scheme are analog in nature, there is no essential difference between the analog scheme and an equivalent sampled data scheme. The gains computed for a sampled data system are a function of the sampling rate and will differ from the corresponding analog gains.

Problem Presentation

Problem presentation is made in three parts. The first part is a brief description of an explicit model following system in which the advantages and disadvantages of such a system are considered. In the second part a brief description and typical characteristics of type 1 servo are presented. In the third part a description of the proposed control scheme combining type 1 servo techniques and model following techniques to realize the advantages of both systems is presented.

Explicit Model Following System

Let the linear time invariant continuous model and plant be given by

$$\dot{x}_m = A_m x_m + B_m u_m \qquad \dot{x}_p = A_p x_p + B_p u_p \tag{1}$$

where the subscripts m and p denote model and plant, respectively; x denotes an nx1 state vector, A and B are the state and control matrices of order nxn and nxr, respectively; and u is an rx1 control input. The conventional explicit model following scheme is depicted in Figure 1.[1,2] The forward gain matrix K_{FF} is included to provide a driving force for the plant when the model and plant outputs are equal, that is, this system has a conditional feedback path. Thus the K_{FF} channel has an integral effect for the whole system as indicated in Figure 2. Figures 1 and 2 represent the same model following systems. The block diagram of Figure 2 has been rearranged to emphasize the conditional feedback characteristics of this type system. The dashed channel is drawn to show that this is an optional feature. This option is not used in the type 1 model following system to be described. The integral effect of this particular feed-forward channel is achieved by the dynamics of the system and model augmented state vectors. System gains K_{PF}, K_{PD}, K_{FF} may be calculated using linear regulator theory. It is worthwhile to point out that feedback gain K_{PD} is dependent on plant dynamics only and may be calculated independently of K_{PF} and K_{FF}. However, both K_{PF} and K_{FF} are functions of plant dynamics, plant feedback gains K_{PD} and model dynamics.

A Type 1 Servo

A type 1 system is depicted in Figure 3. This system is represented by the linear equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx = \dot{z} \end{aligned} \tag{2}$$

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where x, A, and B are as before. y is a q-dimensional output vector which is a subset of the state vector x. C is a qxn output matrix consisting of one unity element per row to select those elements of x which form the vector y. In a type 1 servo design, it is desired to control q states $(1 \leq q \leq n)$ of the plant such that those states have zero steady-state error to an external reference signal. This is possible provided that the system obtained by augmenting the original system by q integral states is controllable, that is, (A, B) is a controllable pair and the rank of $C A^{-1} B$ is equal to q.[3,4] The augmented equations for the system are

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{3}$$

where

$$z(t) = \int_0^t y \, d\tau \tag{4}$$

By applying linear regulator theory, optimal gains, with respect to a quadratic P.I., of Proportional Differential (K_{PD}) and Integral (K_I) type may be calculated. This converts the original system to a type 1 servosystem illustrated in Figure 3. As a benefit of this formulation, the number of integral feedbacks and the nature of the states to be fed back are determined by the desired performance, that is, either states to be controlled and/or decoupled. The input quantity, d_o, of Figure 3 represents a constant disturbance injected into the $\dot{x}$ summing junction. It does not effect the steady-state performance of this system because of the integral effect of the feedback. Y_R is the external reference signal. The closed-loop form of the system is now:

$$\dot{\hat{x}} = \hat{A} \hat{x} + \begin{bmatrix} d_0 \\ Y_R \end{bmatrix} \tag{5}$$

where $\hat{x} = \begin{bmatrix} x \\ Z \end{bmatrix}$ is the augmented state vector, and $\hat{A} = \left[\begin{array}{c|c} A + BK_{PD} & BK_I \\ \hline C & 0 \end{array}\right]$, is the augmented system closed-loop matrix.

A Type 1 Explicit Model Following System

Both conventional model following system and a type 1 servosystem[1,2,3] have some advantages that are highly desirable for an aircraft control system. The advantages are combined into a single control scheme, called a type 1 explicit model following system, in the manner described below.

Given the desired model

$$\dot{x}_m = A_m x_m + B_m u_m$$

$$Y_m = C_m x_m = \dot{z}_m$$

and the airplane dynamics

$$\dot{x}_p = A_p x_p + B_p u_p$$

$$Y_p = C_p x_p = \dot{z}_p$$

both model and plant are augmented separately to include Z_m and Z_p integral feedback, respectively, as described previously for a type 1 servomechanism. Optimal feedback gains for the model and for the plant are each computed separately. The P.I's. used in this state of calculating optimal gains are the standard quadratic ones.

$$P.I._m = \int_0^\infty \left\{ \langle \hat{x}_m, Q_1 \hat{x}_m \rangle + \langle u_m, R_1 u_m \rangle \right\} dt$$

$$P.I._p = \int_0^\infty \left\{ \langle \hat{x}_p, Q_2 \hat{x}_p \rangle + \langle u_p, R_2 u_p \rangle \right\} dt$$

where

$$\hat{x}_m^T \triangleq \left[x_m^T \,\vdots\, Z_m^T \right]; \hat{x}_p^T = \left[x_p^T \,\vdots\, Z_p^T \right]$$

While the choice of Q and R is subject to the usual problems of choosing weighting factors, there are other factors to consider. For the model, Q_1 and R_1 are chosen to achieve certain goals of the system designer, for example, "good" model dynamics to satisfy some aircraft handling qualities requirement, and/or to provide a decoupled system. For the augmented plant dynamics it is only necessary to choose Q_2 and R_2 such that the feedback gains are of reasonable magnitude since any choice of Q_2 and R_2 will provide a stable system and the model following requirement will insure good plant dynamic characteristics.

Model and plant closed-loop type 1 A matrices are

$$\hat{A}_m = \left[\begin{array}{c|c} A_m + B_m K_{PD_m} & B_m K_{I_m} \\ \hline C_m & 0 \end{array}\right] ; \quad \hat{A}_p = \left[\begin{array}{c|c} A_p + B_p K_{PD_p} & B_p K_{I_p} \\ \hline C_p & 0 \end{array}\right] \tag{6}$$

with the system equations for zero disturbance and no external reference signal given by

$$\dot{\hat{x}}_m = \hat{A}_m \hat{x}_m \qquad \dot{\hat{x}}_p = \hat{A}_p \hat{x}_p \tag{7}$$

These two independent type 1 systems may be formed into an explicit model following system through the addition of a control signal to the equations of the plant dynamics such that the plant equations become

$$\dot{\hat{x}}_p = \hat{A}_p \hat{x}_p + \hat{B}_p u_p \tag{8}$$

where

$$\hat{B}_p = \left[\begin{array}{c} B_p \\ \hline 0 \end{array}\right]$$

and combining the two systems. The following open-loop system is the result

$$\dot{\tilde{x}} = \tilde{A}\,\tilde{X} + \tilde{B}\,u_p \tag{9}$$

where

$$\tilde{x} = \left[\begin{array}{c} \hat{x}_m \\ \hline \hat{x}_p \end{array}\right] = \left[\begin{array}{c} x_m \\ Z_m \\ \hline x_p \\ Z_p \end{array}\right] ; \quad \tilde{B} = \left[\begin{array}{c} 0 \\ \hline \hat{B}_p \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline B_p \\ 0 \end{array}\right] ; \quad \tilde{A} = \left[\begin{array}{c|c} \hat{A}_m & 0 \\ \hline 0 & \hat{A}_p \end{array}\right] \tag{10}$$

The prefilter gains K_{PF} are calculated using linear regulator theory by minimizing $P.I._3$.

$$P.I._3 = \int_0^\infty \left(\langle H\tilde{x},\ QH\tilde{x}\rangle + \langle u_p,\ R\,u_p\rangle\right) dt \tag{11}$$

where $H = [I \vdots -I]$ of order $(n + q) \times 2(n + q)$. Hence the error term in the $P.I._3$ is $\hat{x}_m - \hat{x}_p$. The complete adaptive scheme is presented now as

$$\dot{\tilde{x}} = \tilde{A}_{c.l.}\ \tilde{x} + \tilde{y}_R \tag{12}$$

where

$$\tilde{A}_{c.l.} = \left[\begin{array}{c|c} \hat{A}_m & 0 \\ \hline \hat{B}_p \cdot K_{PF} & \tilde{A}_p \end{array}\right] ; \quad K_{PF} = \left[\begin{array}{c|c} K_1 & K_2 \\ \hline 0 & 0 \end{array}\right]$$

and

K_1 = Proportional Differential PreFilter gains

K_2 = Integral PF gains

$$\tilde{y}_R^T = \left[d_0' \vdots y_R \vdots d_0 \vdots y_R\right]^T$$

The passive adaptive scheme block diagram is shown in Figure 4, and the corresponding airplane control scheme is shown in Figure 5.

Results

Computer simulation runs were executed for both the lateral and longitudinal dynamics of a typical fighter aircraft. Data are presented herein for one flight condition in the lateral mode.

The desired characteristics for the lateral mode were chosen in accordance with appropriate military specifications for high-performance aircraft. Aircraft data used are representative of an existing fighter aircraft.

Lateral Mode Results

The objective of the control design is to decouple the roll angle and sideslip angle, ϕ and β. The selected model dynamics are given by

$$\begin{bmatrix} \dot{p}_m \\ \dot{r}_m \\ \dot{\beta}_m \\ \dot{\phi}_m \end{bmatrix} = \begin{bmatrix} -4. & 0.865 & -10. & 0 \\ -0.04 & -0.05 & 5.87 & 0 \\ 0 & -1.0 & -0.743 & 0.734 \\ 1.0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_m \\ r_m \\ \beta_m \\ \phi_m \end{bmatrix} + \begin{bmatrix} 20.0 & 3.3 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{a_m} \\ \delta_{r_m} \end{bmatrix} \quad \ldots \quad (13)$$

Choosing Q_1 and R_1 as previously described and solving for K_{PD_m} and K_{I_m} results in the following K_m matrix

$$K_m = \left[\underbrace{\begin{matrix} -3.025 & -0.466 & 0.634 & -5.475 \\ -0.426 & 3.19 & -2.102 & -0.791 \end{matrix}}_{K_{PD_m}} \;\vdots\; \underbrace{\begin{matrix} -3.143 & 0.347 \\ -3.347 & -3.143 \end{matrix}}_{K_{I_m}} \right]$$

The aircraft dynamics for a particular flight condition are

$$\begin{bmatrix} \dot{p}_p \\ \dot{r}_p \\ \dot{\beta}_p \\ \dot{\phi}_p \end{bmatrix} = \begin{bmatrix} -3.6 & 0.19 & -35.2 & 0 \\ -0.0377 & -0.3575 & 5.88 & 0 \\ 0.0688 & -0.996 & -0.216 & 0.073 \\ 0.9947 & 0.1027 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_p \\ r_p \\ \beta_p \\ \phi_p \end{bmatrix} + \begin{bmatrix} 14.65 & 6.54 \\ 0.217 & -3.086 \\ -0.0054 & 0.05 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{a_p} \\ \delta_{r_p} \end{bmatrix} \quad (14)$$

Now augmenting the aircraft in the same way as the model gives

$$K_p = \left[\underbrace{\begin{matrix} -4.063 & -1.638 & 2.97 & -6.249 \\ -1.989 & 4.331 & -4.298 & -1.791 \end{matrix}}_{K_{PD_p}} \;\vdots\; \underbrace{\begin{matrix} -3.218 & 0.7598 \\ -0.796 & -3.176 \end{matrix}}_{K_{I_m}} \right] \quad (15)$$

and

$$K_{PF} = \left[\underbrace{\begin{matrix} 1.07 & 0.309 & -0.5032 & 1.403 \\ 0.451 & -0.848 & 1.94 & 0.658 \end{matrix}}_{K_1} \;\vdots\; \underbrace{\begin{matrix} 0.36 & -0.313 \\ 0.0846 & 1.207 \end{matrix}}_{K_2} \right] \quad (16)$$

Typical time history results are shown in Figure 6 illustrating the decoupling property and zero steady-state error characteristics for this example.

Concluding Remarks

The use of type 1 servo techniques in an explicit model following passive adaptive scheme has been presented. This scheme provides the following advantages: (1) provide steady-state decoupling for arbitrarily selected state variables, (2) overall stability of the decoupled adaptive scheme is guaranteed, (3) zero steady-state error for the decoupled state variables, that is, those states incorporating integral feedbacks, is assured, and (4) better transient performance of the suggested type 1 explicit model following system than either a type 1 servocontrol system or a conventional explicit model following system.

Although the proposed control law was demonstrated using continuous systems, essentially there is no difference between the analog and sampled data system.

Preliminary work has been performed in closing an additional outer loop from the aircraft to the model to improve transient behavior. This work indicates that further improvements may be realized while retaining the previously cited advantages and will be the subject of further research.

References

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3. Porter, B.: Optimal Control of Multivariable Linear Systems Incorporating Integral Feedback. Elect. Letters, Vol. 7, No. 8, April 1971.

4. Young, P. C., and Willems, J. C.: An Approach to Linear Multivariable Servomechanism Problem. Int. J. of Control, Vol. 15, No. 5, 1972, pp. 961-979.

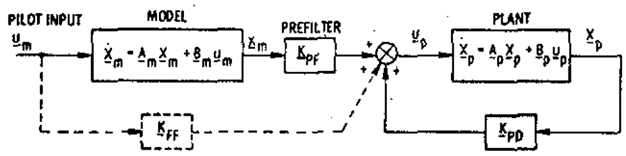


Figure 1. Explicit (model following) passive adaptive scheme.

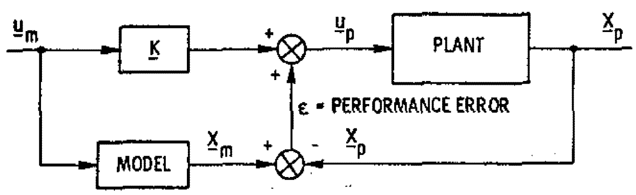


Figure 2. Conditional F.B.

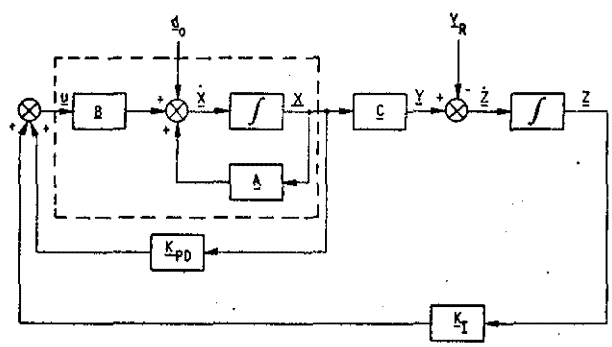


Figure 3. A "type one" servosystem.

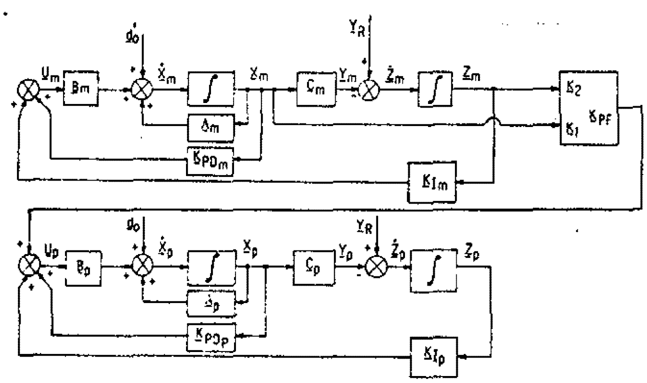


Figure 4. A "type one" servo explicit adaptive scheme.

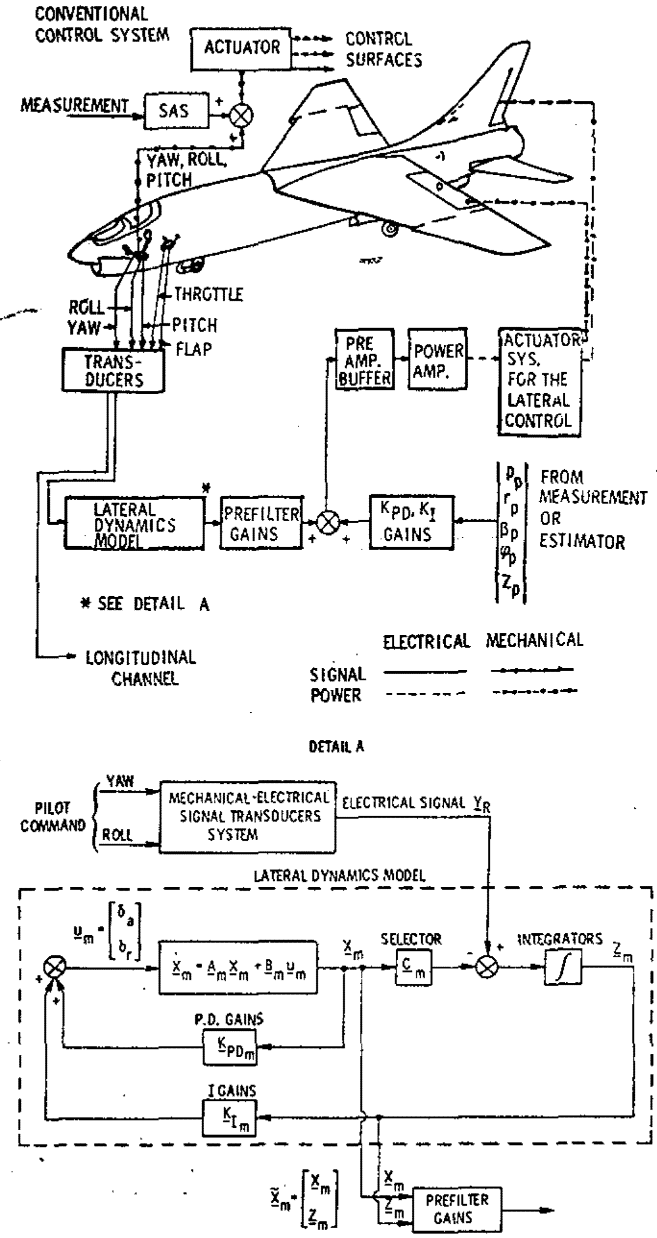
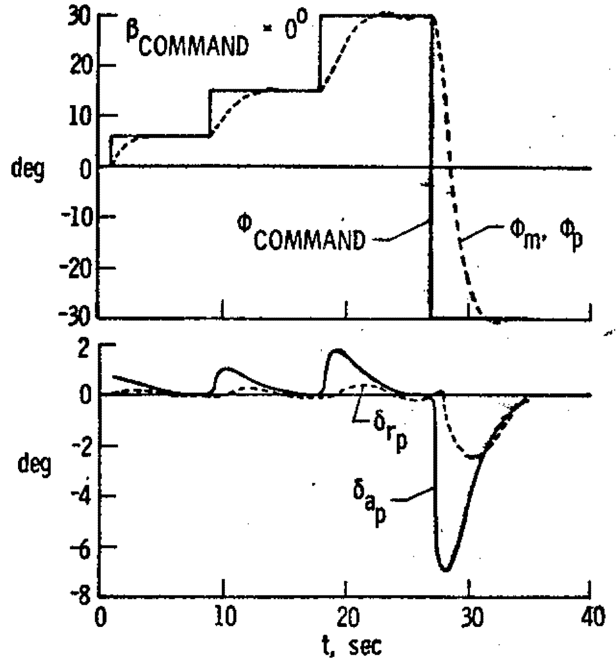
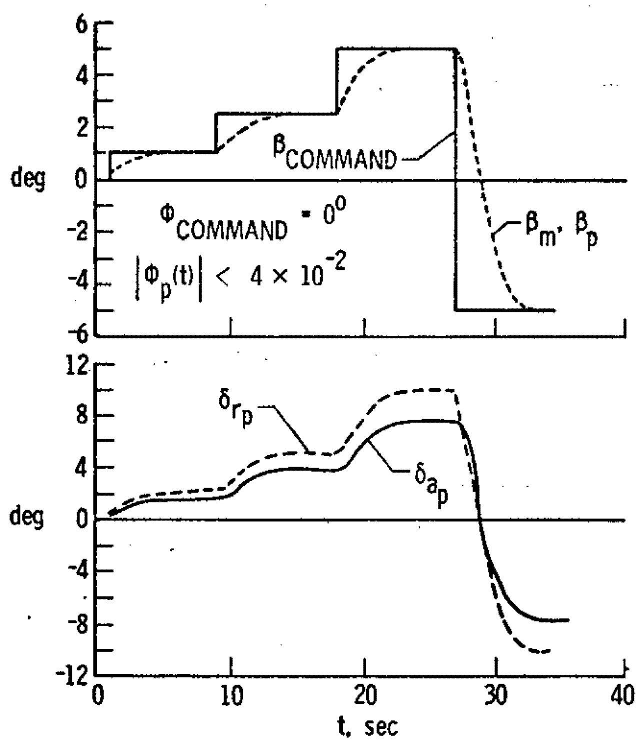


Figure 5. Schematic A/C FBW system.



(a) Roll command and zero sideslip.



(b) Sideslip command and zero roll.

Figure 6. Typical time history results.