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NASA TECH BRIEF

Lewis Research Center

Zeros of Certain Cross Products of Bessel Functions of Fractional Order

λ	s \ ν	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5
1.5	0	0.4027	1.2070	2.0080	2.8040	3.5930	4.3730	5.145	5.9040	6.6537	7.3934	8.1239
	1	6.3355	6.4450	6.6580	6.9660	7.3600	7.8300	8.365	8.9535	9.5884	10.2612	10.9649
2.0	0	0.3396	1.0115	1.6633	2.2870	2.8817	3.4523	4.0065	4.5500	5.0869	5.6193	6.1489
	1	3.2182	3.3877	3.7099	4.1575	4.6980	5.2992	5.9328	6.5771	7.2169	7.8439	8.4553
	2	6.3226	6.4040	6.5648	6.8011	7.1080	7.4810	7.9153	8.4058	8.9466	9.5293	10.1448
	3	9.4512	4.5048	9.6114	9.7698	9.9780	10.2340	10.5350	10.8800	11.2665	11.6926	12.1572
2.5	0	0.2945	0.8700	1.3950	1.8810	2.3389	2.7810	3.2150	3.6450	4.0719	4.4966	4.9196
	1	2.1840	2.3950	2.7760	3.2717	3.8150	4.3610	4.8850	5.3870	5.8706	6.3430	6.8032
	2	4.2360	4.3350	4.5320	4.8225	5.1991	5.6530	6.1650	6.7078	7.2537	7.7857	8.2990
	3	6.3150	6.3800	6.5090	6.7015	6.9532	7.2650	7.6320	8.0594	8.5379	9.0575	9.5995

The s in the table is the index of the zero.

Zeros of cross products of the derivatives of Bessel functions of fractional order of the form

$$J'_\nu(x)\,Y'_\nu(\lambda x) \;-\; J'_\nu(\lambda x)\,Y'_\nu(x)$$

are needed in problems related to mathematical physics and engineering acoustics. Higher zeros of such cross products, that is, higher values of x for which the cross product vanishes, may be calculated by McMahon's expansion, but lower zeros, which are of interest in engineering applications are not calculable by any known method.

Using Bessel functions of order $\nu = n + \frac{1}{2}$, n = 0,1,2 ..., which are characterized by closed form solutions, a set of zeros was obtained for a range of parameter λ.

Interpolation between the values given in the table is permitted provided that a curve is traced between at least three values from the table. The zeros have been obtained on a digital computer and the results were rounded off to the fourth decimal point. The table can also be used to give by interpolation zeros ν for any given x and λ, that is, values of ν for which the cross product vanishes.

Notes:

1. Zeros of the cross product of the derivatives of Bessel functions of fractional order have application in the area of acoustics.
2. Further information is available in the following report:
   NASA TM-X-2698 (N73-15705), Propagation of Waves of Acoustic Frequencies in Curved Ducts
   Copies may be obtained at cost from:
   Aerospace Research Applications Center
   Indiana University
   400 East Seventh Street
   Bloomington, Indiana 47401
   Telephone: 812-337-7833
   Reference: B74-10012
3. Specific technical questions may be directed to:
   Technology Utilization Officer
   Lewis Research Center
   21000 Brookpark Road
   Cleveland, Ohio 44135
   Reference: B74-10012

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