NASA TECH BRIEF

Langley Research Center

Generalized Current Distribution Rule

A method has been developed which helps to determine the branch current in a parallel-series network in relation to the total input current by inspection. The method is particularly useful for circuits with many elements when the branch elements are described as admittances. If the element values are variables, then these values may be expressed as admittances to find currents readily in desired branches.

A parallel-series network, N, with k branches b_i, $1 \leq i \leq k$, in parallel, with branch b_i comprising admittances $g_{i,1}$, $g_{i,2}$, ..., $g_{i,ni}$ in series, has current I entering its terminal node 1 and leaving its terminal node 1′ as shown in Figure 1. The objective is to obtain an expression for the current ratio I_i/I, where I_i is the current in the branch b_i.

By letting P_i be the product of all admittances in i-th branch, P_i becomes

$$P_i = \prod_{j=1}^{n_i} g_{i,j} \tag{1}$$

Further, Q_i is the sum of products of admittances in the i-th branch taken $(ni-1)$ at a time, if $(ni-1)>0$ and defined by $Q_i = 1$, if $ni = 1$. Next define Q as

$$Q = \prod_{i=1}^{k} Q_i \tag{2}$$

Finally, T_i is defined as the sum of admittance products of all trees that intersect in the branch b_i, which is the i-th tree admittance products sum given by

$$T_i = P_i Q Q_i^{-1} \tag{3}$$

To facilitate evaluation of T_i, a nontopological definition of T_i is given later after equation 5. The sum T of tree admittance products of N becomes

$$T = \sum_{i=1}^{k} T_i \tag{4}$$

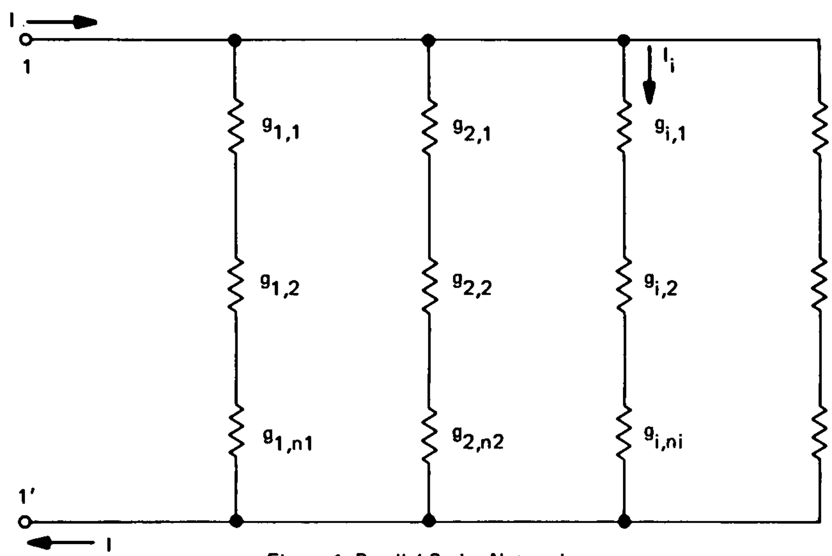

Figure 1. Parallel-Series Network

(continued overleaf)

Now the current ratio I_i/I is equal to the ratio of the admittance of branch b_i to the admittance of the total network. The admittance ratio can be shown to be:

$$\frac{\text{sum of tree admittance products of branch } b_i}{\text{sum of 2-tree admittance products of } b_i} \times \frac{\text{sum of 2-tree admittance products of N}}{\text{sum of tree admittance products of N}}$$

Hence, using the previously defined terms, the current ratio is described by

$$\begin{aligned} I_i/I &= (P_i/Q_i)(Q/T) = (P_iQQ_i^{-1})/T \\ &= T_i/T \end{aligned} \qquad (5)$$

Equation 5 gives the generalized current distribution rule in terms of topological concepts, which can be easily obtained by inspection. Familiarity with topological concepts, however, is not necessary in order to find the right-hand side of equation 5 in terms of $g_{i,j}$'s. The sum T_i can be looked upon as a product of k terms, each corresponding to a branch of N, with the i-th term consisting of the product of all admittances in the i-th branch and the

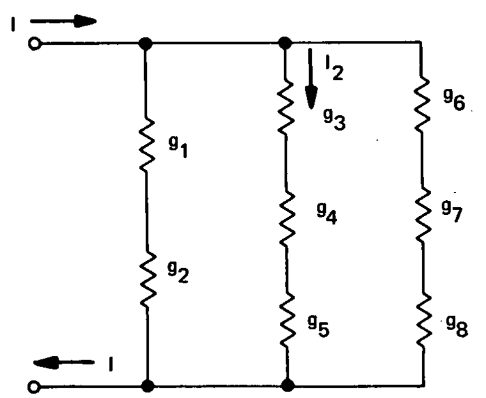

Figure 2. Parallel-Series Network With Eight Admittance Elements

j-th term, for all $j \neq i$, consisting of the sum of the products of all possible $(nj-1)$ admittances in the j-th branch, the j-th term being unity if and only if $nj=1$.

As an example, the developed method is applied to the network shown in Figure 2. Using equation 5, current ratio I_2/I is evaluated as follows:

$$\begin{aligned} I_2/I = {} & (g_1+g_2)(g_3g_4g_5)(g_6g_7+g_6g_8+g_7g_8) \\ & [(g_1g_2)(g_3g_4+g_3g_5+g_4g_5)(g_6g_7+g_6g_8+g_7g_8) \\ & +(g_1+g_2)(g_3g_4g_5)(g_6g_7+g_6g_8+g_7g_8) \\ & +(g_1+g_2)(g_4g_5+g_4g_3+g_5g_3)(g_6g_7g_8)]^{-1} \end{aligned}$$

Observe that:

$$I_2/I = T_2/T = \frac{T_2}{T_1+T_2+T_3}$$

where

$$\begin{aligned} T_1 &= (g_1g_2)(g_3g_4+g_3g_5+g_4g_5)(g_6g_7+g_6g_8+g_7g_8) \\ T_2 &= (g_1+g_2)(g_3g_4g_5)(g_6g_7g_6g_8+g_7g_8) \text{ and} \\ T_3 &= (g_1+g_2)(g_3g_4+g_3g_5+g_4g_5)(g_6g_7g_8) \end{aligned}$$

Thus, in a parallel-series network, the current that flows in a network branch is proportional to the sum of admittance products of trees of the network that include the branch.

Note:

No further documentation is available. Specific questions, however, may be directed to:

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NASA has decided not to apply for a patent.

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