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ON THE MINIMIZATION OF THE PRIME POWER CONSUMPTION OF A COUPLING-MODULATED GAS LASER TRANSMITTER

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ABSTRACT

The present document addresses itself to the prime power requirements of a coupling-modulated gas laser transmitter. The latter consists of a gas discharge tube and electro-optic modulator inside a laser resonator. In performing the calculations, the laser discharge length and the modulator voltage are simultaneously varied so that the transmitted power remains constant. In this way, tradeoffs can be made between the prime power supplied individually to the discharge tube and to the modulator driver to obtain a transmitter configuration which minimizes the total prime power consumption. An analytical expression is derived which describes the effects of information bandwidth and transmitter output power on the prime power requirements. Specific numerical results are obtained for a CO_2 laser transmitter based on presently available experimental data.

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1. INTRODUCTION

For many space-oriented applications, laser communication and tracking systems offer an attractive alternative to their more conventional microwave and millimeter wave counterparts. Highly accurate ground-based satellite ranging systems using lasers have already been implemented, and the applicability of lasers to point-to-point terrestrial communications is well established. In the next two decades, the large information-carrying capacity of laser communication systems may represent the optimum approach for transferring large volumes of data from earth observational satellites to the ground via an intermediate satellite as envisioned, for example, in NASA's planned Tracking and Data Relay Satellite System (TDRSS). The high gains available from relatively small optical antennas also allows substantial reductions in transmitter output power and in the prime power required to drive tracking and pointing servo systems.

The present document addresses itself to the prime power requirements of a coupling-modulated gas laser transmitter. The latter consists of a gas discharge tube and electro-optic modulator inside a laser resonator. The molecular linewidth is assumed to be homogeneously broadened; this is only true for gas pressures greater than about 20 torr, thus excluding the He-Ne laser from our analysis. In performing the prime power calculations, the laser discharge length and peak modulator voltage are simultaneously varied so that the output power of the transmitter remains constant. Increasing the discharge length increases the circulating power in the laser but also increases the prime power requirements of the discharge tube. An increase in the circulating power, however, allows a reduction in the modulator driver power consumption since the effective transmission provided by the modulator can be reduced. In this way, tradeoffs can be made between the prime power supplied individually to the discharge tube and to the modulator driver to obtain a transmitter configuration which minimizes the total prime power consumption. The manner in which the information bandwidth and transmitter output power influence the prime power consumption is also examined. Specific numerical results are obtained for a CO_2 laser transmitter based on presently available experimental data.

2. THEORY

2.1 COUPLING MODULATION

It has long been known that a birefringence can be induced in certain crystals by the application of a voltage across one dimension of a crystal rod. If a light beam propagating in the z-direction (see Figure 1) is polarized parallel to one of the orthogonal principal crystal axes (x and y in Figure 1), it sees an index of refraction given by $n_x = n_0 - \Delta n$ or $n_y = n_0 + \Delta n$ where n_0 is the index of refraction under zero field conditions and Δn is the change in index due to the slight restructuring of the crystal lattice by the electric field. If the resulting phase advance or delay $\Gamma/2$ is proportional to the applied voltage, the crystal is said to exhibit a "linear electro-optic effect", and this effect can be exploited to obtain amplitude, phase, or frequency modulation of the laser output.[1] While the crystal modulator can be placed inside or outside the laser cavity, it has been shown that the internal modulator requires significantly less driving voltage to achieve a given amount of modulated output.[2] This is due in part to the fact that an internal modulator acts on the circulating power within the laser while an external modulator acts only on the output power which is typically a small fraction of the circulating power. Thus, for a given modulated output, significantly less drive power is required for a modulator element within the optical cavity than for the same element externally situated provided the insertion loss is reasonably small. The insertion loss includes bulk absorption and

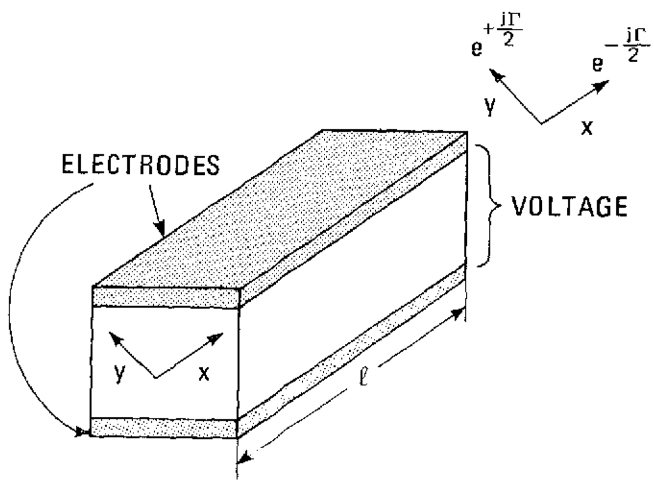


Figure 1. Electrically Induced Birefringence

scattering losses in optical materials, reflection and scatter at interfaces, and vignetting of the internal laser beam by the crystal aperture.

In coupling modulation, the polarization of the internal circulating radiation field is at a 45° angle with respect to the principal axes of the crystal modulator as in Figure 2. One end of the modulator rod can be coated for total reflection and the beam traverses the modulator length twice before encountering the Brewster window or other polarization selective device. The input field at the modulator surface (z = 0) is given by

$$\vec{E}_{in} = \frac{E_c}{\sqrt{2}} (\hat{e}_x + \hat{e}_y) \cos \omega_c t = \frac{E_c}{\sqrt{2}} (\hat{e}_x + \hat{e}_y) \operatorname{Re} e^{-j\omega_c t}$$

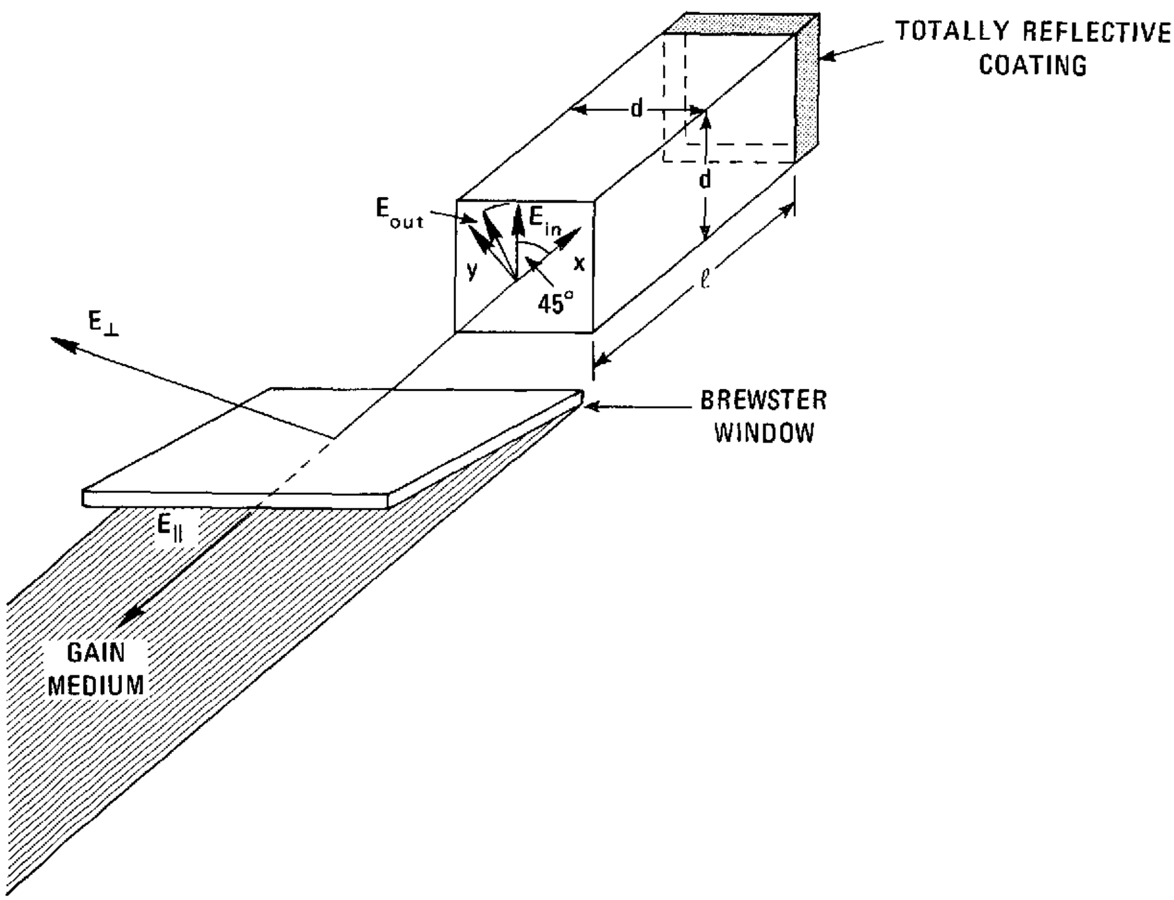


Figure 2. Coupling Modulation

where E_c is the amplitude of the circulating field, ω_c is the laser frequency, and $\hat{e}_x$ and $\hat{e}_y$ are unit vectors parallel to the principal crystal axes. The returning field after one round trip through the modulator is given by

$$\vec{E}_{out} = \frac{E_c}{\sqrt{2}} \,\mathrm{Re} \left\{ \left[e^{j\frac{2 n_x \omega_c \ell}{c}} \hat{e}_x + e^{j\frac{2 n_y \omega_c \ell}{c}} \hat{e}_y \right] e^{-j\omega_c t} \right\}$$

$$= \frac{E_c}{\sqrt{2}} \,\mathrm{Re} \left\{ \left[e^{-j\frac{2 \Delta n \omega_c \ell}{c}} \hat{e}_x + e^{j\frac{2 \Delta n \omega_c \ell}{c}} \hat{e}_y \right] e^{-j\left(\omega_c t - \frac{2 n_0 \omega_c \ell}{c}\right)} \right\}$$

$$= \frac{E_c}{\sqrt{2}} \,\mathrm{Re} \left\{ \left[e^{-j\Gamma(t)} \hat{e}_x + e^{j\Gamma(t)} \hat{e}_y \right] e^{-j\left(\omega_c t - \frac{2 n_0 \omega_c \ell}{c}\right)} \right\} \tag{1}$$

where

$$\Gamma(t) \equiv \frac{2 \Delta n(t) \omega_c \ell}{c} \tag{2}$$

Ignoring the constant phase shift $(2n_0 \omega_c \ell / c)$, the output field has a component perpendicular to the input polarization given by

$$E_\perp = E_c \sin \Gamma(t) \sin \omega_c t \tag{3}$$

whereas the parallel component is

$$E_\parallel = E_c \cos \Gamma(t) \cos \omega_c t \tag{4}$$

If we assume that $\Gamma(t)$ is of the form

$$\Gamma(t) = \Gamma_0 + \Gamma_m \cos \omega_m t \tag{5}$$

we obtain from Eq. (3)

$$E_\perp = E_c \sin \omega_c t \sin [\Gamma_0 + \Gamma_m \cos \omega_m t]$$

$$= E_c \sin \omega_c t \left[\sin \Gamma_0 \cos (\Gamma_m \cos \omega_m t) + \cos \Gamma_0 \sin (\Gamma_m \cos \omega_m t)\right]$$

$$= E_c \sin \omega_c t \left\{\sin \Gamma_0 \left[J_0 (\Gamma_m) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k} (\Gamma_m) \cos (2k \omega_m t)\right]\right.$$

$$\left. + 2 \cos \Gamma_0 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\Gamma_m) \cos \left[(2k + 1) \omega_m t\right]\right\}$$

$$= E_c \left\{\sin \Gamma_0 J_0 (\Gamma_m) \sin \omega_c t\right.$$

$$+ \sin \Gamma_0 \sum_{k=1}^{\infty} (-1)^k J_{2k} (\Gamma_m) \left[\sin \left[(\omega_c + 2k \omega_m) t\right] + \sin \left[(\omega_c - 2k \omega_m) t\right]\right.$$

$$+ \cos \Gamma_0 \sum_{k=0}^{\infty} (-1)^k J_{2k+1} (\Gamma_m) \left[\sin \left[\left(\omega_c + (2k + 1) \omega_m\right)\right] t\right]$$

$$\left. + \sin \left[\left(\omega_c - (2k + 1) \omega_m\right) t\right]\right]\Big\} \tag{6}$$

The time averaged power coupled from the laser at the various frequencies is thus given by

$$P (\omega_c + k \omega_m) = \begin{cases} \eta_C P_C \sin^2 \Gamma_0 J_k^2 (\Gamma_m) & k = \ldots, -4, -2, 0, +2, +4, \ldots \\ \\ \eta_C P_C \cos^2 \Gamma_0 J_k^2 (\Gamma_m) & k = \ldots, -3, -1, +1, +3, \ldots \end{cases} \tag{7}$$

where P_C is the time averaged circulating power in the cavity and η_C is the efficiency of the coupling element. Thus, the power in the carrier is equal to

$$P_{carrier} = \eta_C P_C \sin^2 \Gamma_0 J_0^2(\Gamma_m) \tag{8}$$

and the total power in the two sidebands at $\omega_c + \omega_m$ and $\omega_c - \omega_m$ is equal to

$$P_{sb} = 2\eta_C P_C \cos^2 \Gamma_0 J_1^2(\Gamma_m) \tag{9}$$

The remaining terms in Eq. (7) lead to harmonic distortion of the transmitted signal. It is shown in Appendix A that the total power transmitted is equal to

$$P_{total} = \sum_{k=-\infty}^{\infty} P(\omega_c + k\,\omega_m)$$

$$= \frac{\eta_C P_C}{2} \left[1 - J_0(2\Gamma_m) \cos 2\Gamma_0\right]$$

Thus the fractional power in all the harmonic distortion frequencies is equal to

$$F = \frac{1 - J_0(2\Gamma_m) \cos 2\Gamma_0 - 2 \sin^2 \Gamma_0 J_0^2(\Gamma_m) - 4 \cos^2 \Gamma_0 J_1^2(\Gamma_m)}{1 - J_0(2\Gamma_m) \cos 2\Gamma_0} \tag{10}$$

Equation (10) is plotted as a function of Γ_0 and Γ_m in Figure 3.

Similarly, Eqs. (4) and (5) yield

$$E_{\parallel} = E_c \cos \omega_c t \cos [\Gamma_0 + \Gamma_m \cos \omega_m t]$$

$$= E_c \cos \omega_c t \left[\cos \Gamma_0 \cos (\Gamma_m \cos \omega_m t) - \sin \Gamma_0 \sin (\Gamma_m \cos \omega_m t)\right]$$

$$= E_c \cos \omega_c t \left\{ \cos \Gamma_0 \left[J_0(\Gamma_m) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\Gamma_m) \cos (2k\,\omega_m t) \right. \right.$$

$$\left. \left. - 2 \sin \Gamma_0 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\Gamma_1) \cos \left[(2k + 1)\,\omega_m t \right] \right\} \right.$$

$$= E_c \left\{ \cos \Gamma_0 J_0(\Gamma_m) \cos \omega_c t \right.$$

$$+ \cos \Gamma_0 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\Gamma_m) \Big[\cos \left[(\omega_c + 2k\,\omega_m)\,t \right] + \cos \left[(\omega_c - 2k\,\omega_m)\,t \right]$$

$$- \sin \Gamma_0 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\Gamma_m) \Big[\cos \Big[\Big(\omega_c + (2k + 1)\,\omega_m \Big)\,t \Big]$$

$$\left. + \cos \Big[\Big(\omega_c - (2k + 1)\,\omega_m \Big)\,t \Big] \Big] \right\}$$

The time averaged power reentering the gain medium at the various frequencies is thus given by

$$P_R(\omega_c + k\,\omega_m) = \begin{cases} (1 - a_2)\,P_C \cos^2 \Gamma_0\, J_k^2(\Gamma_m) & k = \ldots, -4, -2, 0, +2, +4, \ldots \\ \\ (1 - a_2)\,P_C \sin^2 \Gamma_0\, J_k^2(\Gamma_m) & k = \ldots, -3, -1, +1, +3, \ldots \end{cases} \qquad \textbf{(11)}$$

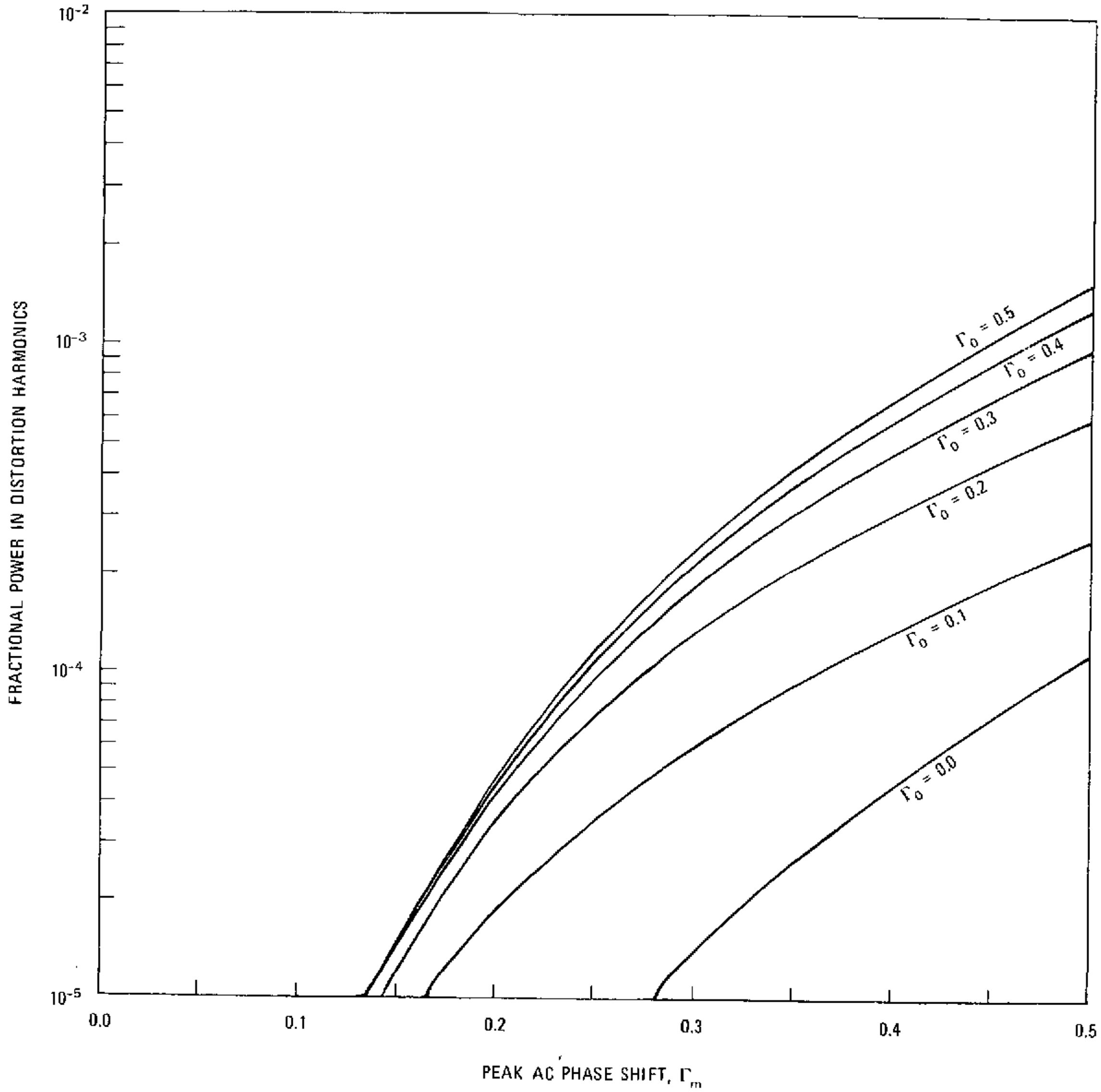


Figure 3. Fractional Power Contained in Distortion Harmonics as a Function of the DC and AC Phase Shifts, Γ_0 and Γ_m

where a_2 is a net dissipative loss at the modulator end of the cavity. The internal energy of the laser, however, cannot be varied (built up) at frequencies which are not resonant with the laser cavity.[4-5] Such buildups can occur only if (1) a sideband frequency is equal to the longitudinal mode spacing and (2) the response of the gain medium is sufficiently wide to support oscillation at the sideband frequencies. The latter condition is never satisfied for a low pressure CO_2 laser transmitter where the molecular linewidth is about 50 MHz as compared to a typical longitudinal mode spacing of 600 MHz (corresponding to a cavity length of 25 cm). Thus only the power returned to the gain medium at the carrier frequency can support the oscillation and the effective reflectivity at the modulator end of the cavity is given by

$$r_2 = (1 - a_2) \cos^2 \Gamma_0 J_0^2 (\Gamma_m) \tag{12}$$

from Eq. (11). The quantity $r_2/(1 - a_2)$ is plotted in Figure 4. It should be mentioned that certain bandwidth limitations do exist for coupling modulation. For modulation frequencies below 1 MHz, significant distortion results due to a resonant energy coupling between the molecular system and the oscillating optical mode.[6] Signal distortion also occurs at high modulation frequencies on the order of $\omega_m = 1/\tau$ where τ is the two-way transit time through the modulator.[1]

2.2 MODULATOR DRIVE POWER

In considering modulator drive power requirements, we will discuss two situations of practical interest. In the first system, it will be assumed that the modulator provides both the DC (Γ_0) and AC (Γ_m) phase retardation through the simultaneous application of DC and AC voltages to the modulator crystal. In the second system, it will be assumed that the DC bias is provided by a naturally birefringent plate or other device having a negligible electrical power consumption.

If a class A amplifier is designed to provide the required voltage across the capacitive modulator over a specified bandwidth, the equivalent circuit shown in Figure 5 can be used[7] where C_s is the driver and stray capacity, C_m is the modulator capacity and R is the equivalent shunt resistance (ideally the output impedance of the amplifier). The maximum modulation frequency is given by

$$f_{max} = \frac{1}{2\pi RC} \tag{13}$$

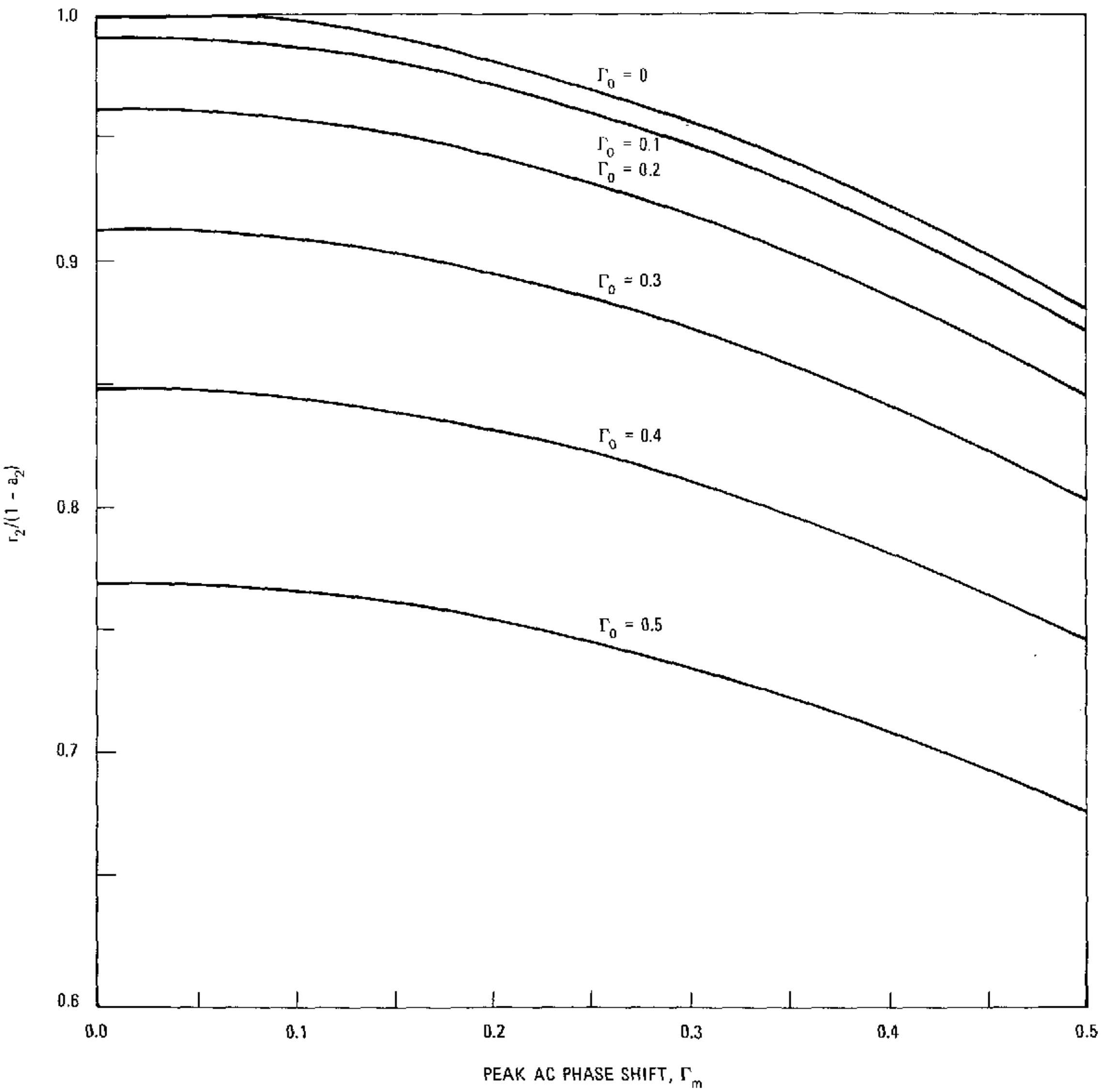


Figure 4. Effective Reflectivity of the Modulator as a Function of the DC and AC Phase Shifts, Γ_0 and Γ_m

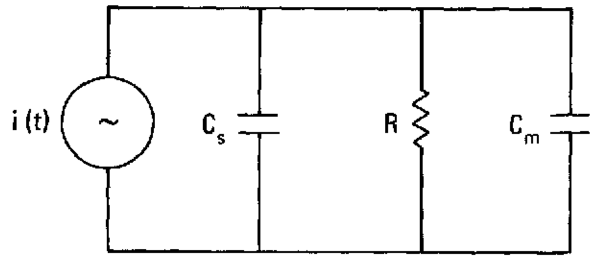


Figure 5. Equivalent Circuit of an Electro-Optic Modulator

where $C = C_s + C_m$. The power consumption by the modulator is equal to

$$P(t) = V(t)\,I(t) = \frac{V^2(t)}{R} = 2\pi f_{max} C V^2(t) \tag{14}$$

For the phase retardation assumed in Eq. (5), we can write

$$V(t) = V_0 + V_m \cos \omega_m t \tag{15}$$

when the modulator provides both the DC and AC phase shift. The time averaged power consumption of the modulator is then given by

$$P_M = 2\pi f_{max} C \left[V_0^2 + \frac{V_m^2}{2} \right] \tag{16}$$

For cubic crystals like CdTe (which has an extremely low absorption at 10.6 microns), the voltages V_0 and V_m are related to the phase shifts Γ_0 and Γ_m by the relations[1]

$$\Gamma_0 = \frac{2\pi}{\lambda_0} n_0^3 r_{41} V_0 \frac{2\ell}{d} \tag{17}$$

and

$$\Gamma_m = \frac{2\pi}{\lambda_0} n_0^3 r_{41} V_m \frac{2\ell}{d} \tag{18}$$

where n_0 is the zero-field index of refraction, λ_0 is the free space wavelength, r_{41} is the electro-optic coefficient, ℓ is the modulator crystal length, and d is the crystal thickness. In obtaining Eqs. (17) and (18), we have assumed the transverse mode of modulation — that is, the electric field is applied at right angles to a (110) plane with propagation at right angles to a $(\bar{1}10)$ plane. We can rewrite Eqs. (17) and (18) in terms of the "half-wave voltage" defined as the voltage required to impose a relative phase shift of π radians between the two polarization components for a one-way pass through the modulator and given by

$$V_\pi = \frac{d}{\ell} \frac{\lambda_0}{2 n_0^3 r_{41}} \tag{19}$$

We note from Eq. (19) that the half-wave voltage is a function of the crystal dimensions. A useful quantity is therefore,

$$V_\pi \frac{\ell}{d} = \frac{\lambda_0}{2 n_0^3 r_{41}} \tag{20}$$

which depends only on the material parameters and for CdTe is approximately 53 kilovolts at 10.6 micrometers.[3] Using Eqs. (17) through (20) we can rewrite Eq. (16) as

$$P_M = \frac{f_{max} C}{2\pi} \frac{\left(\frac{V_\pi \ell}{d}\right)^2}{\left(\frac{\ell}{d}\right)^2} \left[\Gamma_0^2 + \frac{1}{2} \Gamma_m^2\right] \tag{21}$$

where f_{max} is the maximum modulation frequency in cycles per second. If the modulator driver has an electrical efficiency equal to η_D, the total prime power consumed by the modulator driver is equal to

$$P_D = \frac{P_M}{\eta_D} = \frac{f_{max} C}{2\pi \eta_D} \frac{\left(\frac{V_\pi \ell}{d}\right)^2 \left[\Gamma_0^2 + \frac{1}{2} \Gamma_m^2\right]}{\left(\frac{\ell}{d}\right)^2} \tag{22}$$

If a birefringent plate is used to provide the DC output, Γ_0 is set equal to zero in Eq. (22).

2.3 LASER TUBE PRIME POWER

The CO_2 laser tubes which will be used in proposed laser communications and tracking experiments are powered by a DC voltage which produces and maintains the inverted population in the laser medium. Most of the voltage drop between the electrodes occurs over a small distance in the vicinity of the cathode referred to as the "cathode fall region." The electric field over the remainder of the cathode-anode gap ("the positive column") is fairly uniform. Even for short gap distances, a fair amount of electrical power is required to ionize the gas and produce the glow discharge. As a result, the power input requirements do not increase proportionately with discharge length but rather according to the relation

$$P_L = P_B + \left(\frac{dP}{dL}\right) L \tag{23}$$

where P_L is the total electrical power supplied to the discharge tube, P_B is the electrical power required to break down the gas in the limit of short discharge lengths, and (dP/dL) is the incremental electrical power per unit length required to sustain the discharge at the proper current for optimum laser output power.

To obtain the total prime power requirements of the laser tube, one must include the efficiency of the high voltage power supply, η_{HV}, yielding

$$P_{HV} = \frac{1}{\eta_{HV}} \left[P_B + \left(\frac{dP}{dL}\right) L \right] \tag{24}$$

for the total prime power consumed by the laser high voltage supply.

2.4 MINIMIZATION OF THE TOTAL PRIME POWER

Rigrod[8] has derived the following expression for the circulating power of a homogeneously broadened laser measured at mirror 2:

$$P_C = \frac{P_S \sqrt{r_1} \left[g_0 L + \ell n \sqrt{r_1 r_2} \right]}{\left[\sqrt{r_1} + \sqrt{r_2} \right] \left[1 - \sqrt{r_1 r_2} \right]} \tag{25}$$

where P_S is the saturation parameter, g_0 is the small signal gain, L is the discharge length, and r_1 and r_2 are the reflectivities of mirrors 1 and 2 respectively. At the modulator end, r_2 is given by Eq. (12). We now wish to simultaneously vary the three quantities L, Γ_0 and Γ_m under the two constraints that the power in both the carrier and in the sidebands at $\omega_c \pm \omega_m$ remain constant. We will find it convenient to choose Γ_m, as the independent variable. The two dependent variables, Γ_0 and L, can then be calculated using the appropriate equations imposed by the above-mentioned constraints on the transmitter output. From Eqs. (8) and (9), the ratio of sideband power to carrier power is given by

$$R \equiv \frac{P_{sb}}{P_{carrier}} = 2 \cot^2 \Gamma_0 \frac{J_1^2(\Gamma_m)}{J_0^2(\Gamma_m)} \tag{26}$$

As we change Γ_1, the ratio R is kept constant if we choose Γ_0 such that

$$\tan \Gamma_0 = \sqrt{\frac{2}{R}} \frac{J_1(\Gamma_m)}{J_0(\Gamma_m)} \tag{27}$$

Thus the dependent variable Γ_0 is uniquely determined by the assumed value for R and the value of the independent variable Γ_m. Having constrained the ratio of first sideband to carrier powers to the value R, it is now sufficient to fix the total power in the latter components. From Eqs. (8) and (9), this total power is given by

$$P_T = P_{carrier} + P_{sb} = \eta_C P_C \left[\sin^2 \Gamma_0 J_0^2(\Gamma_m) + 2 \cos^2 \Gamma_0 J_1^2(\Gamma_m)\right] \tag{28}$$

which can be solved for the circulating power P_C to yield

$$P_C = \frac{P_T}{\eta_C} \left[\sin^2 \Gamma_0 J_0^2(\Gamma_m) + 2 \cos^2 \Gamma_0 J_1^2(\Gamma_m)\right]^{-1} \tag{29}$$

Solving for the discharge length L in Eq. (15) gives

$$L = \frac{1}{g_0} \left[\frac{P_C}{P_S} \frac{\left[\sqrt{r_1} + \sqrt{r_2}\right]\left[1 - \sqrt{r_1 r_2}\right]}{\sqrt{r_1}} - \ln\sqrt{r_1 r_2} \right] \tag{30}$$

which is the constraining equation for the second dependent variable L. The latter equation is somewhat more complicated than the equation for Γ_0 since the quantities P_C and r_2 depend on Γ_0 and Γ_m through Eqs. (12) and (29). Nevertheless, if we specify values for g_0, P_S, and r_1, the second dependent variable, L, is uniquely determined by the value of Γ_m. Equation (29) simply reflects the fact that, if we increase the effective transmission at the modulator end, the circulating power in the laser can be reduced without sacrificing transmitted power. The reduced requirements on circulating power in turn allow a reduction of the discharge length and its associated prime power requirements. A higher effective transmission at the modulator end, however, implies an increase in the prime power consumption of the modulator driver. It is this trade off between modulator and discharge tube voltages that establishes a set of optimum operating conditions for the minimum consumption of prime power. The total prime power consumed is obtained by summing Eqs. (22) and (24), that is

$$P_P = \frac{f_{max} C}{2\pi \eta_D} \frac{\left(\frac{V_\pi \ell}{d}\right)^2}{\left(\frac{\ell}{d}\right)^2} \left[\Gamma_0^2 + \frac{1}{2}\Gamma_m^2\right] + \frac{1}{\eta_{HV}}\left[P_B + \left(\frac{dP}{dL}\right) L\right] \tag{31}$$

Since the modulator, gain medium (except for length), and component efficiency characteristics are fixed in a given system, Eq. (31) can be plotted versus the single parameter Γ_m provided we have decided on the output characteristics of the transmitter, i.e., P_T, R, and f_{max}. The first term in Eq. (31), corresponding to the prime power consumed by the modulator, clearly increases with increasing Γ_m while the second term, corresponding to the prime power consumed by the laser discharge tube, decreases through its dependence on the dependent variable L.

While one could in principle obtain the optimum value for Γ_m by setting the derivative of Eq. (31) with respect to Γ_m equal to zero, the dependence of P_P on Γ_m through the dependent variables Γ_0 and L is much too complicated to allow the extraction of an analytic solution. Such an analytic solution is obviously desirable, however, due to the large number of system parameters. As we shall see in the next section approximate analytic solutions can be obtained provided the optical losses at both ends of the resonator are assumed small compared to unity.

2.5 SMALL OPTICAL LOSS APPROXIMATION

2.5.1 DC Phase Delay Γ_0 Produced by Modulator

Under the assumption of small optical losses at both ends of the laser resonator, Eqs. (12) and (29) become

$$r_2 \approx 1 - \left(a_2 + \Gamma_0^2 + \frac{1}{2}\Gamma_m^2\right) \tag{32}$$

and

$$P_C \approx \frac{P_T}{\eta_C\left[\Gamma_0^2 + \frac{1}{2}\Gamma_m^2\right]} \tag{33}$$

Similarly Eq. (30) becomes

$$L = \frac{1}{g_0}\left[\frac{P_T}{\eta_C P_S\left[\Gamma_0^2 + \frac{1}{2}\Gamma_m^2\right]} + \frac{1}{2}\right]\left[a_1 + a_2 + \left(\Gamma_0^2 + \frac{1}{2}\Gamma_m^2\right)\right] \tag{34}$$

where $a_1 = 1 - r_1$ and we have used Eqs. (32) and (33). Substituting Eq. (33) into Eq. (31) allows us to write

$$P_P = \alpha_1 + \beta_1 x_1 + \frac{\gamma_1}{x_1} \tag{35}$$

where

$$\alpha_1 = \frac{P_B}{\eta_{HV}} + \frac{\left(\frac{dP}{dL}\right)}{\eta_{HV}\, g_0}\left(\frac{P_T}{\eta_C P_S} + \frac{a_1 + a_2}{2}\right) \tag{36}$$

$$\beta_1 \equiv \frac{\left(\frac{dP}{dL}\right)}{2\eta_{HV}\, g_0} + \frac{f_{max}\, C}{2\pi\eta_D} \frac{\left(\frac{V_\pi \ell}{d}\right)^2}{\left(\frac{\ell}{d}\right)^2} \tag{37}$$

$$\gamma_1 \equiv \frac{\left(\frac{dP}{dL}\right)}{\eta_{HV}\, g_0} \frac{P_T (a_1 + a_2)}{\eta_C\, P_S} \tag{38}$$

$$x_1 \equiv \Gamma_0^2 + \frac{1}{2}\Gamma_m^2 \tag{39}$$

Setting the derivative of Eq. (35) with respect to x_1 equal to zero gives

$$x_1 = \sqrt{\frac{\gamma_1}{\beta_1}} \tag{40}$$

as the minimum prime power condition. Substituting Eq. (40) into (35) yields a simple general expression for the minimum prime power requirements as a function of the various parameters, that is

$$P_{min} = \alpha_1 + 2\sqrt{\beta_1 \gamma_1} \tag{41}$$

For this minimum prime power configuration, the effective reflectivity at the modulator end is given by

$$r_2 = 1 - a_2 - \sqrt{\frac{\gamma_1}{\beta_1}} \tag{42}$$

while the circulating power is

$$P_C = \frac{P_T}{\eta_C}\sqrt{\frac{\beta_1}{\gamma_1}} \tag{43}$$

and the optimum discharge length is given by

$$L = \frac{1}{g_0}\left[\frac{P_T}{\eta_C\, P_S}\sqrt{\frac{\beta_1}{\gamma_1}} + \frac{1}{2}\right]\left[a_1 + a_2 + \sqrt{\frac{\gamma_1}{\beta_1}}\right] \tag{44}$$

In Section 3 we will see that, for typical system parameters, the approximate expressions derived above agree extremely well with numerical calculations based on the exact expressions derived in Section 2.4.

2.5.2 DC Phase Delay Γ_0 Produced by Birefringent Plate

If the DC phase delay is produced by a birefringent plate instead of a DC voltage applied to the modulator, the prime power requirements depend on the value of R corresponding to the ratio of information sideband power to carrier power. In this instance Γ_0 is set equal to zero in Eqs. (27) and (31) for the prime power supplied to the modulator driver. The equations corresponding to Eqs. (35) through (39) become

$$P_P = \alpha_2 + \beta_2\, x_2 + \frac{\gamma_2}{x_2} \tag{45}$$

where

$$\alpha_2 = \alpha_1 \equiv \frac{P_B}{\eta_{HV}} + \frac{\left(\frac{dP}{dL}\right)}{\eta_{HV}\, g_0}\left(\frac{P_T}{\eta_C\, P_S} + \frac{a_1 + a_2}{2}\right) \tag{46}$$

$$\beta_2 = \frac{\left(\frac{dP}{dL}\right)}{2\,\eta_{HV}\,g_0\left[1+\frac{1}{R}\right]} + \frac{f_{max}\,C}{2\pi\eta_D}\,\frac{\left(\frac{V_\pi \ell}{d}\right)^2}{\left(\frac{\ell}{d}\right)^2} \tag{47}$$

$$\gamma_2 = \frac{\left(\frac{dP}{dL}\right)}{\eta_{HV}\,g_0}\;\frac{P_T}{\eta_C\,P_S\left[1+\frac{1}{R}\right]} = \frac{\gamma_1}{\left[1+\frac{1}{R}\right]} \tag{48}$$

$$x_2 = \frac{\Gamma_m^2}{2} \tag{49}$$

The minimum prime power condition is now given by

$$x_2 = \sqrt{\frac{\gamma_2}{\beta_2}} \tag{50}$$

and the minimum prime power consumption is again given by an equation of the form

$$P_{min} = \alpha_2 + 2\sqrt{\beta_2\,\gamma_2} \tag{51}$$

The remaining equations are identical to Eqs. (42) through (44) with the subscript 1 replaced by the subscript 2. Examination of Eqs. (46) through (48) and Eq. (51) yields the expected result that the prime power consumption of a system utilizing a birefringent plate to produce the DC phase delay is less than that consumed by a system where the modulator provides the DC delay. For large values of R, however, the prime power consumption clearly approaches the consumption in Section 2.5.1 since β_2 and γ_2 approach the values β_1 and γ_1 as limiting values. Physically, large values of R correspond to situations where only a small fraction of the transmitted power is contained in the carrier component. For small values of R (low level modulation of the carrier), the parameter γ_2 tends to zero and the minimum prime power consumption tends to the value α_2 ($=\alpha_1$).

3. NUMERICAL RESULTS FOR CO_2

In Table 1 we list typical values for the parameters which appear in the equations of Section 2. They are listed in three columns corresponding to: (A) typical values available from "off-the-shelf" components; (B) "state-of-the-art" values measured in laboratory devices[9]; and (C) projected values based on industry "expectations". In Figure 6(a) we plot the total prime power requirements as a function of Γ_m where we have held the total output power P_T equal to 1 watt and assumed values of $R = 10$ and $f_{max} = 400$ MHz. The three curves are labelled to indicate the set of parameter values used in the calculation. The prime power supplied individually to the modulator driver and to the laser high voltage supply are plotted in Figures 6(b) and 6(c) respectively. As Γ_m increases, the power to the modulator must invariably increase as Figure (6b) clearly indicates. For very small values of Γ_m (corresponding to low transmission at the modulator end) the circulating power must be increased drastically to achieve the assumed one watt output level. The required circulating power is plotted in Figure 6(d). As the modulator transmission increases, the circulating power which maintains the one watt output falls. In Figure 6(e) we see that the required discharge length follows suit until Γ_m is so large that the laser is overcoupled by the modulator and the discharge length must be increased to preserve the one watt output. Prior to this overcoupled condition the prime power supplied to the laser high voltage power supply is falling [Figure 6(c)] while the prime power supplied to the modulator is steadily increasing [Figure 6(b)]. When the laser exceeds the critical coupling, more prime power must be supplied to <u>both</u> the modulator driver <u>and</u> the laser power supply to maintain the output power at the one watt level. Figures 6(a) and 6(e) indicate that the value of Γ_m that minimizes the prime power consumption lies well below the value that critically couples the laser. Furthermore, the prime power consumption depends critically on the value of Γ_m and the component parameters. The values of the various parameters (P_P, P_D, P_{HV}, P_C, L) at the minimum prime power configuration are indicated on Figures 6(a) through 6(e). These values are in excellent agreement (well within 1%) with values obtained using the approximate analytic expressions of Section 2.5. Thus, a rapid characterization of the minimum prime power configuration is possible using the equations in Section 2.5 for systems which are not well described by the parameter values listed in Table 2.

It is now worthwhile to consider the effect of changing the transmitter output characteristics (P_T, R and f_{max}) on the prime power requirements. Having established that the small-loss approximation is valid, we note from Eqs. (35) and (39) that the prime power consumption P_P is a function of the quantity $x_1 = \Gamma_0^2 + 1/2\,\Gamma_m^2$ rather than the individual quantities Γ_0 and Γ_m. This implies that the prime power consumption is independent of R, that is, if both the DC and AC phase delays are provided by the modulator, the power consumption depends

Table 1

Numerical Values for the Various Parameters

	Parameters	"A"	"B"	"C"
Molecular Parameters	g_0	.011/cm	.011/cm	.011/cm
	P_S	24.5 watts	24.5 watts	24.5 watts
	P_B	5.3 watts	5.3 watts	5.3 watts
	dP/dL	0.91 watts/cm	0.91 watts/cm	0.91 watts/cm
Modulator Crystal Parameters	$V_\pi \ell/d$	5.3×10^4 volts	5.3×10^4 volts	5.3×10^4 volts
	ℓ/d	20	30	40
	C	6pF	6pF	6pF
Component Efficiencies	η_D	0.20	0.33	0.40
	η_{HV}	0.70	0.70	0.80
	η_C	0.80	0.99	0.99
	a_1	0.020	0.010	0.005
	a_2	0.060	0.020	0.020
Transmitter Output	P_T	variable	variable	variable
	R	variable	variable	variable
	f_{max}	variable	variable	variable

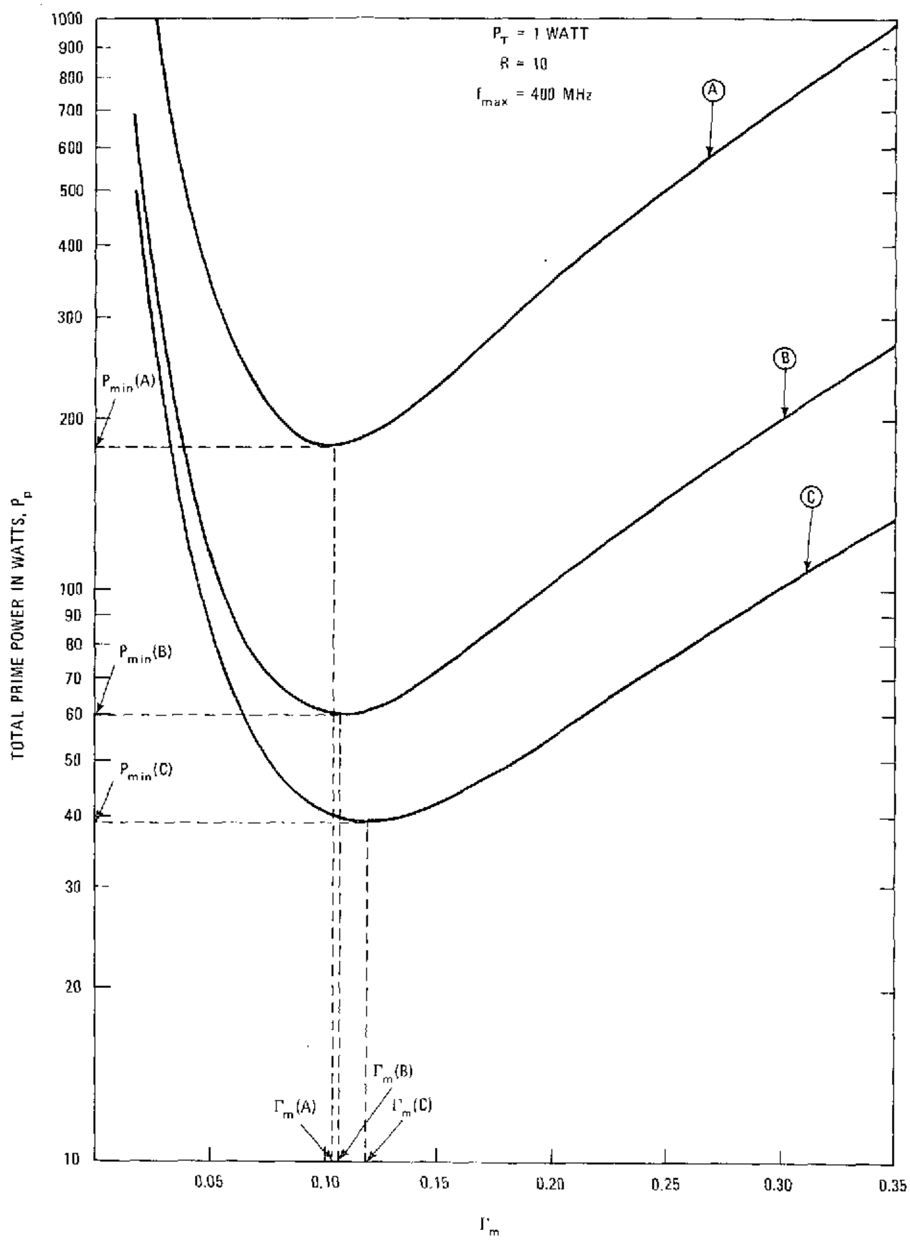
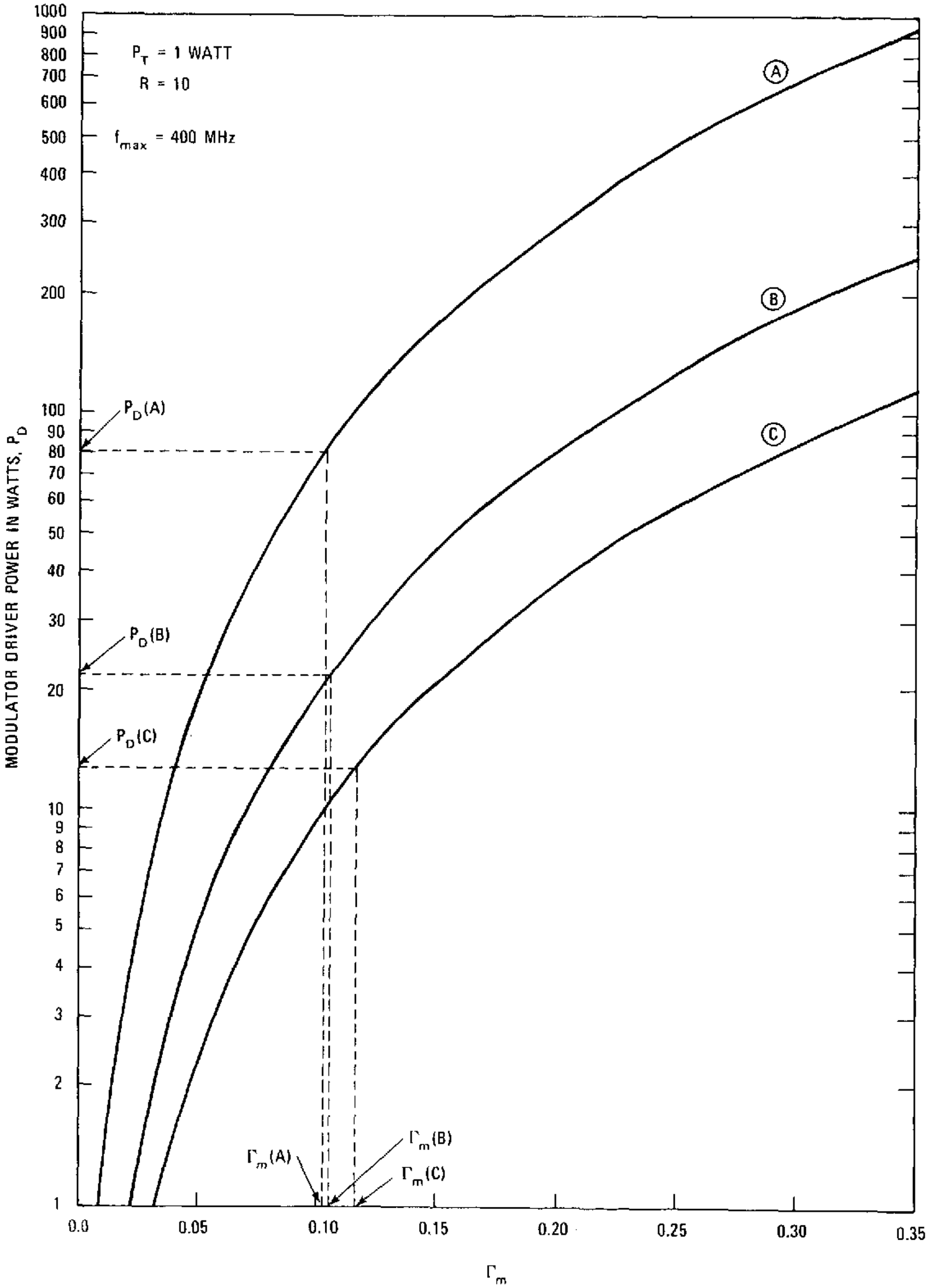


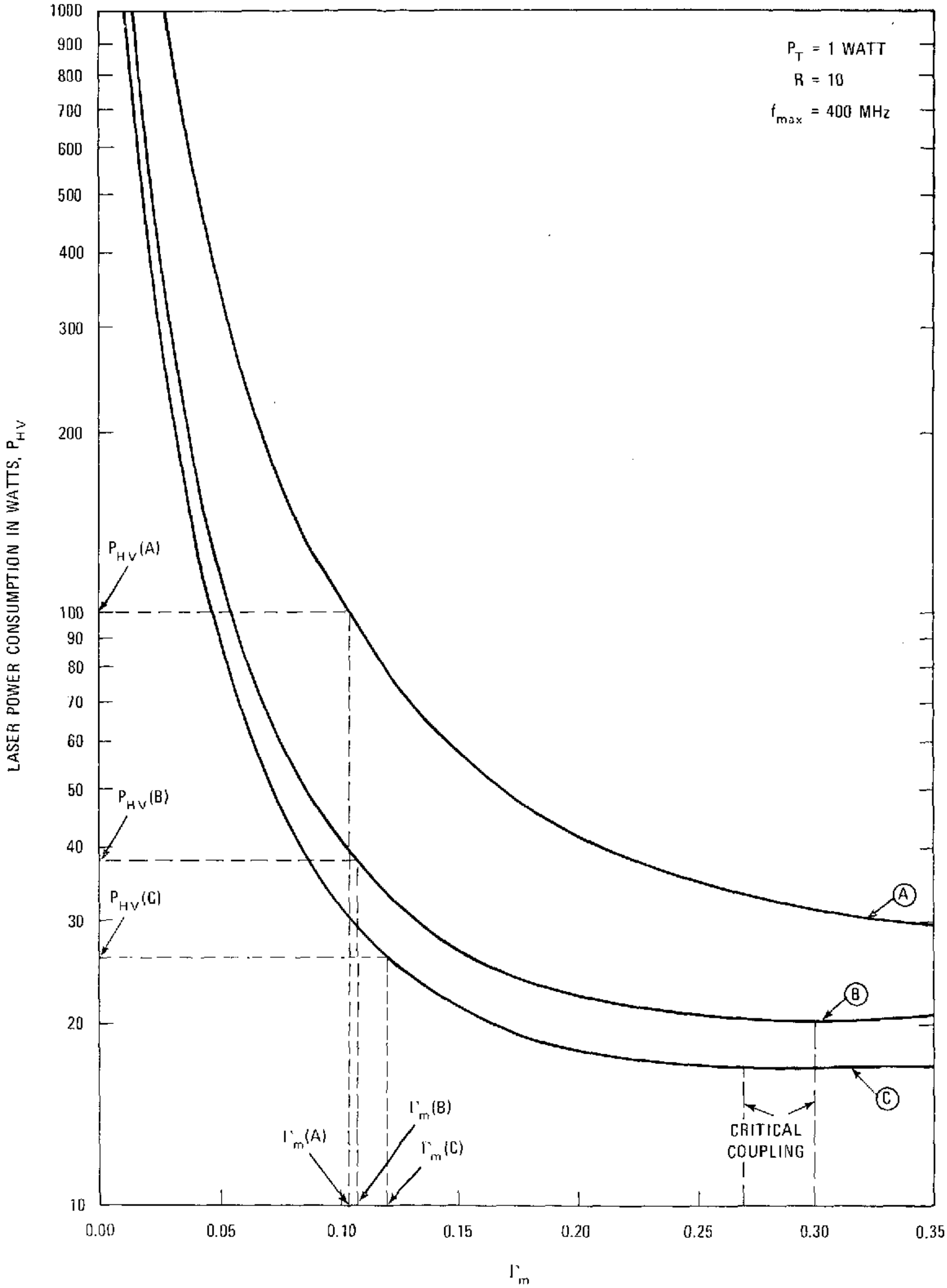
Figure 6(a)

Figure 6. The dependence of the (a) total prime power consumption P_P, (b) the modulator driver consumption P_D, (c) the laser high voltage supply consumption P_{HV}, (d) the circulating power P_C, and (e) the discharge length L on the peak AC phase shift Γ_m when the transmitter output power is held to a constant value of one watt, the ratio of sideband to carrier powers is equal to 10, and the maximum modulation frequency is 400 MHz. The curves are labelled by A, B, and C corresponding to the set of parameters in Table 1 used in the calculation. The values of the various quantities for the configuration which minimizes the total prime power consumption are indicated by the dashed lines.



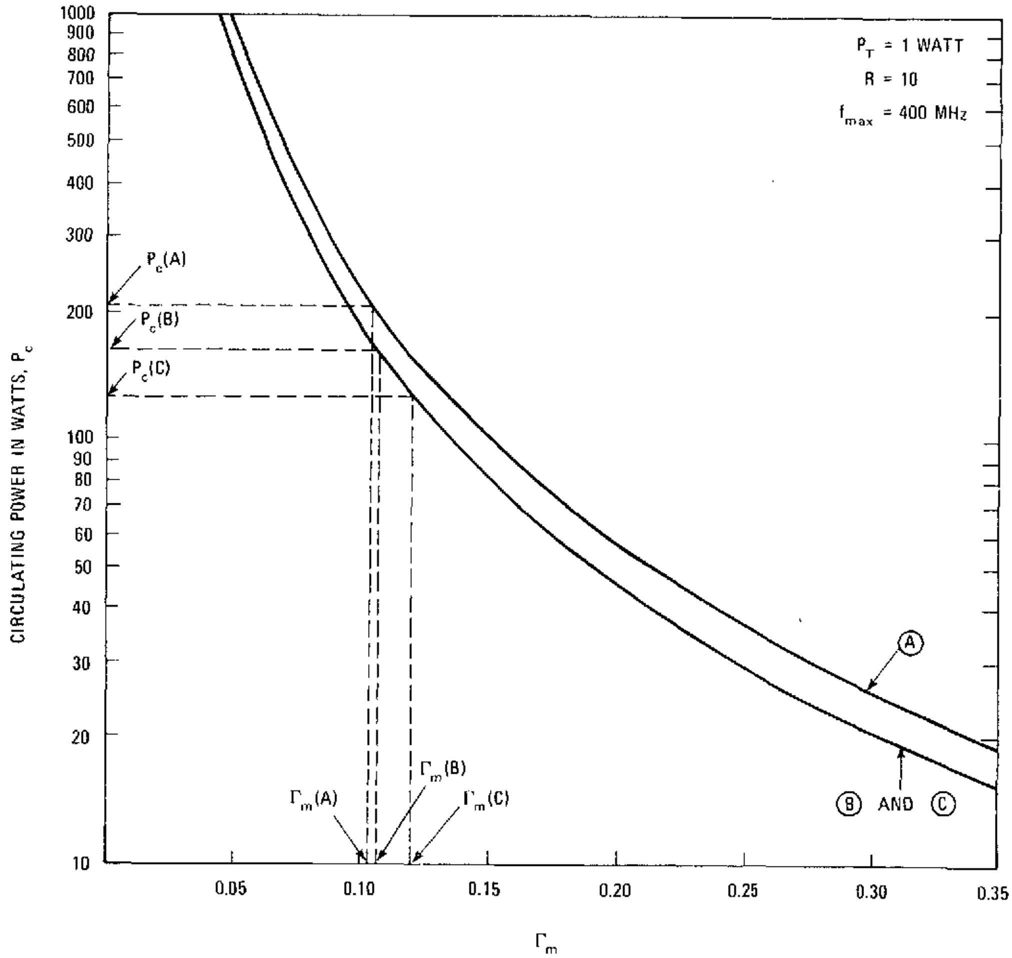
P_T = 1 WATT
R = 10
f_{max} = 400 MHz
MODULATOR DRIVER POWER IN WATTS, P_D
P_D(A)
P_D(B)
P_D(C)
A
B
C
Γ_m(A)
Γ_m(B)
Γ_m(C)
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0.0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Γ_m

Figure 6(b)



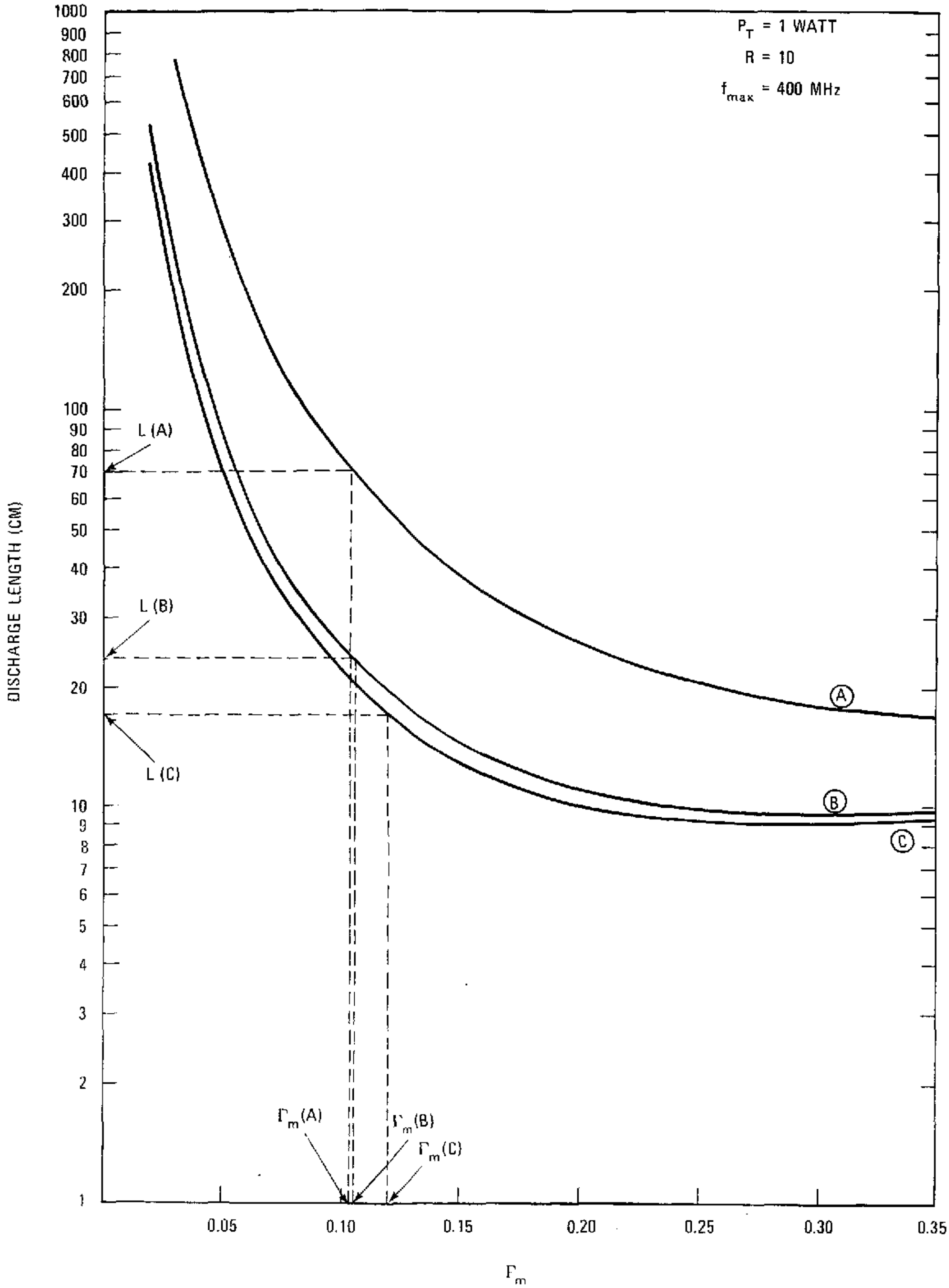
P_T = 1 WATT
R = 10
f_{max} = 400 MHz
LASER POWER CONSUMPTION IN WATTS, P_{HV}
$P_{HV}(A)$
$P_{HV}(B)$
$P_{HV}(C)$
$\Gamma_m(A)$
$\Gamma_m(B)$
$\Gamma_m(C)$
CRITICAL COUPLING
A
B
C
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Γ_m

Figure 6(c)



P_T = 1 WATT
R = 10
f_{max} = 400 MHz
CIRCULATING POWER IN WATTS, P_c
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
$P_c(A)$
$P_c(B)$
$P_c(C)$
$\Gamma_m(A)$
$\Gamma_m(B)$
$\Gamma_m(C)$
A
B AND C
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Γ_m

Figure 6(d)



P_T = 1 WATT
R = 10
f_{max} = 400 MHz
DISCHARGE LENGTH (CM)
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
L (A)
L (B)
L (C)
A
B
C
Γ_m(A)
Γ_m(B)
Γ_m(C)
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Γ_m

Figure 6(e)

only on the total power transmitted, P_T, and not on the relative distribution between carrier and information sidebands. We can therefore plot the minimum prime power requirements as a function of P_T and f_{max} only. In Figures 7(a) through 7(e) we plot the parameters P_{min}, P_D, P_{HV}, P_C, and L for the minimum prime power configuration against f_{max}. Again, for each value of P_T assumed (0.1 and 1 watt), we plot three curves corresponding to the three sets of parameters listed in Table 1.

For those situations where the DC phase delay Γ_0 is provided by a birefringent plate, the prime power consumption is less than that shown in Figure 7(a) and depends on the ratio of sideband-to-carrier powers. Clearly, for an unmodulated carrier (R = 0), no prime power is consumed by the modulator and the prime power used by the laser high voltage supply is equal to α_2 as given by Eq. (46). For values of R $\gtrsim$ 10, the prime power consumption is approximately given by Figure 7(a).

One important practical consideration which we have ignored up to now is whether or not a modulator driver can provide the RF power output we have just described as optimum for the information bandwidth and transmitted power we have chosen. In the commercial units presently available, tradeoffs must be made between the RF power output capability and the driver bandwidth. A value typical of wideband commercial drivers is 20 watts RF output. For the driver efficiencies listed in Table 1, a 20 watt output translates to a prime power consumption P_D equal to: (a) 100 watts; (b) 60 watts; and (c) 50 watts. A comparison of the latter values with Figure 7(b) indicates that an RF power output capability of 20 watts would be more than adequate except for the low efficiency case (A) for P_T = 1 watt and f_{max} > 700 MHz. Even in the latter situation, however, it can be shown that the RF output power limitation raises the total prime power consumption by only a few watts at gigahertz bandwidths. Present commercial units, however, only achieve the 20 watt level for bandwidths less than about ±210 MHz.

4. CONCLUSION

A method for minimizing the prime power consumption of a coupling-modulated gas laser transmitter has been presented. It has been demonstrated that the prime power consumption and the optimum length of the gain medium is extremely sensitive to component efficiencies. A set of general equations (Section 2.5) have been derived which give the optimum operating conditions for the transmitter provided the system parameters listed in Table 1 are known.

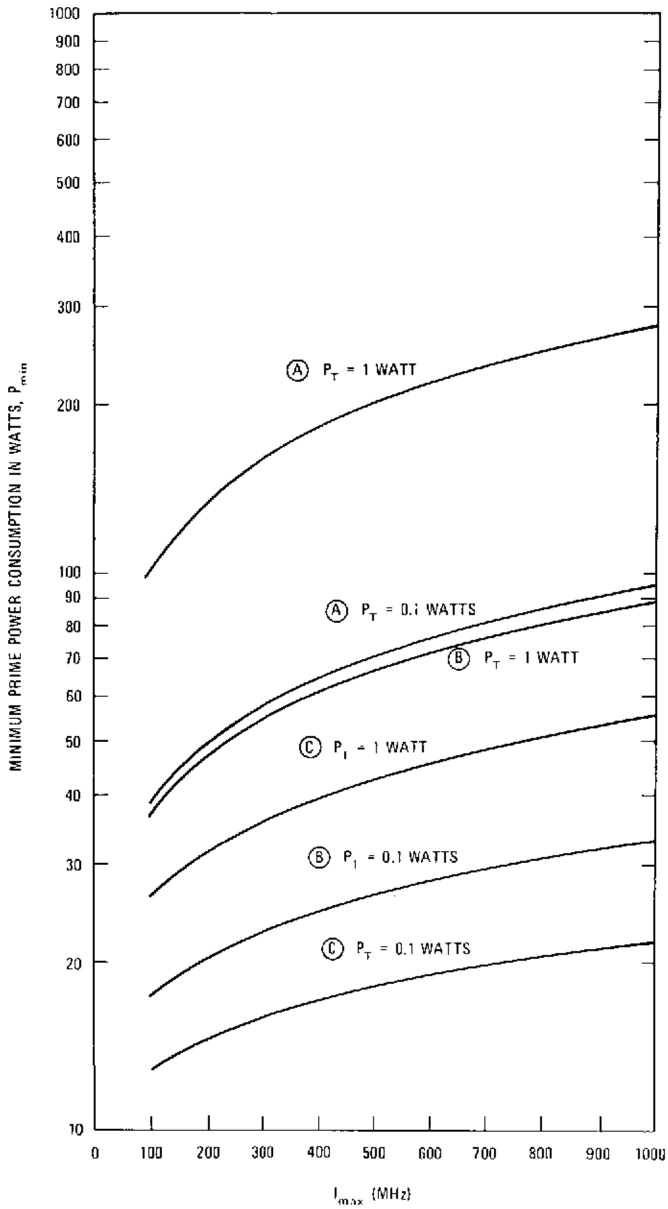
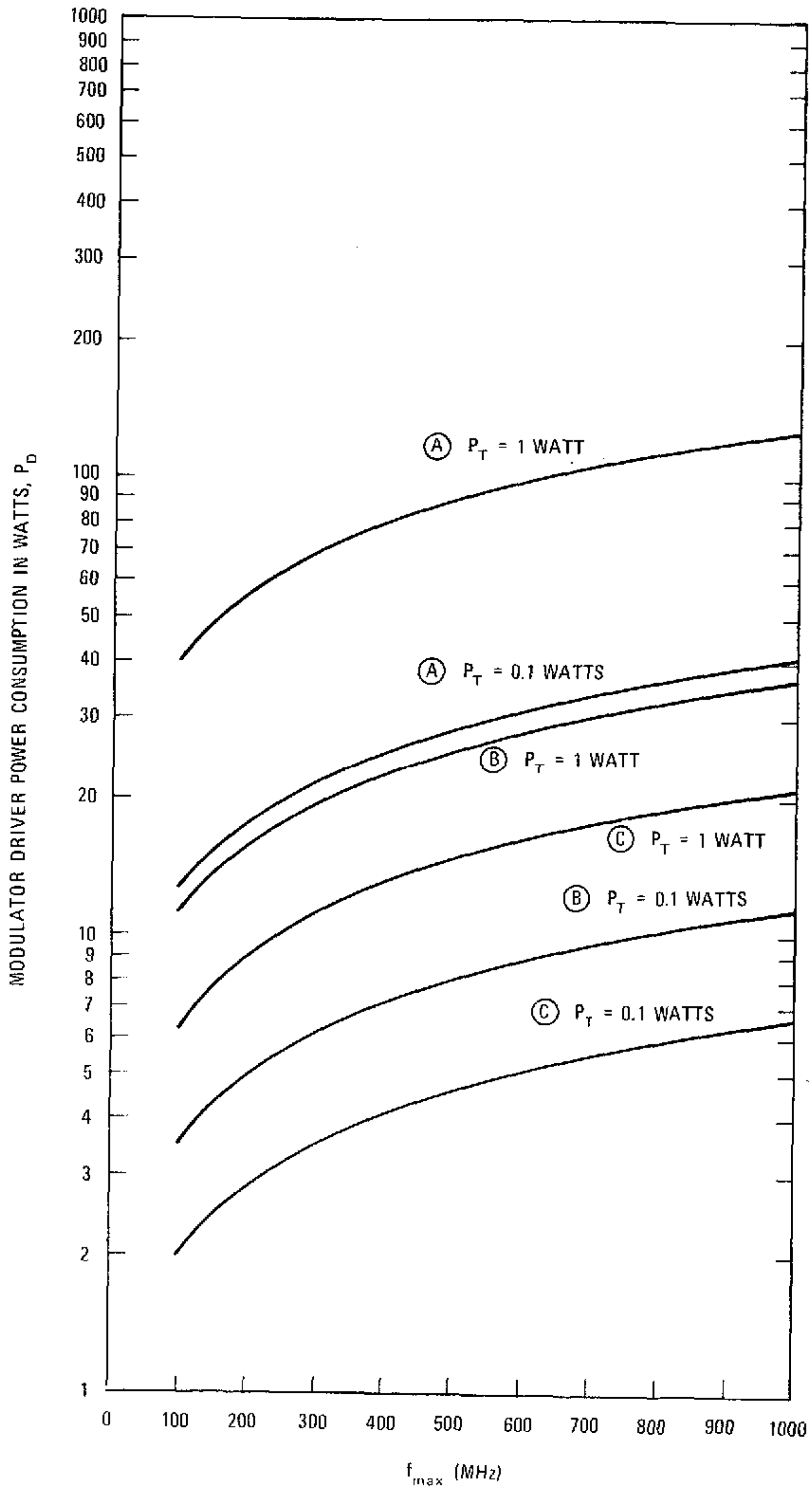


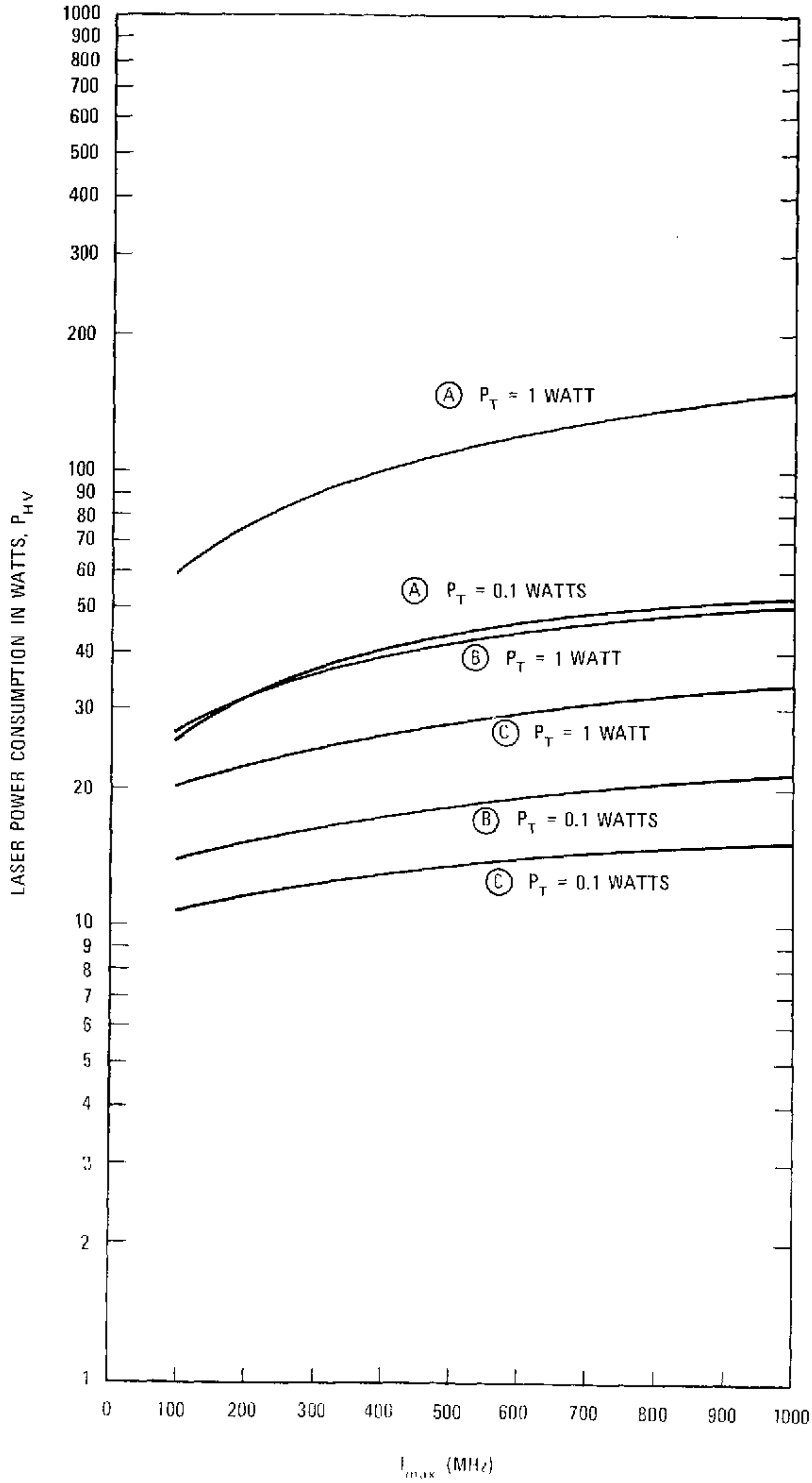
Figure 7(a)

Figure 7. The dependence of (a) the minimum prime power consumption P_{min}, (b) the modulator driver consumption P_D, (c) the laser high voltage supply consumption P_{HV}, (d) the circulating power P_C, and (e) the optimum discharge length L on the value of the transmitted power P_T and the maximum modulation frequency f_{max}. The curves are labelled by A, B, and C corresponding to the set of parameters in Table 1 used in the calculation. It is assumed that the modulator provides both the DC and AC phase shifts. If a birefringent plate provides the DC shift, the system utilizes less prime power than indicated in the plots (see Section 2.5). For values of R $\gtrsim$ 10, however, the deviation from the curves presented here is less than 10 percent.



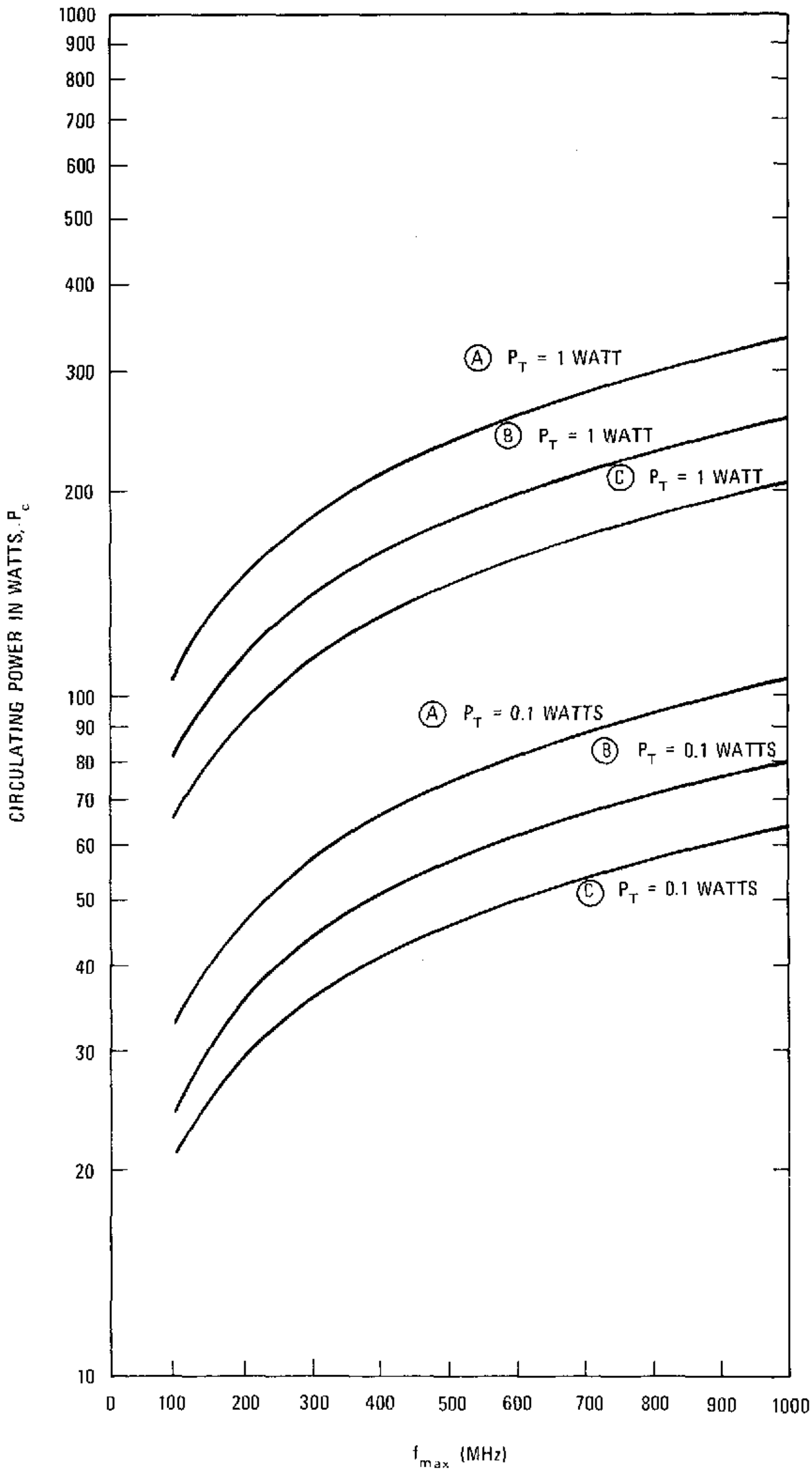
MODULATOR DRIVER POWER CONSUMPTION IN WATTS, P_D
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0
100
200
300
400
500
600
700
800
900
1000
f_{max} (MHz)
Ⓐ P_T = 1 WATT
Ⓐ P_T = 0.1 WATTS
Ⓑ P_T = 1 WATT
Ⓒ P_T = 1 WATT
Ⓑ P_T = 0.1 WATTS
Ⓒ P_T = 0.1 WATTS

Figure 7(b)



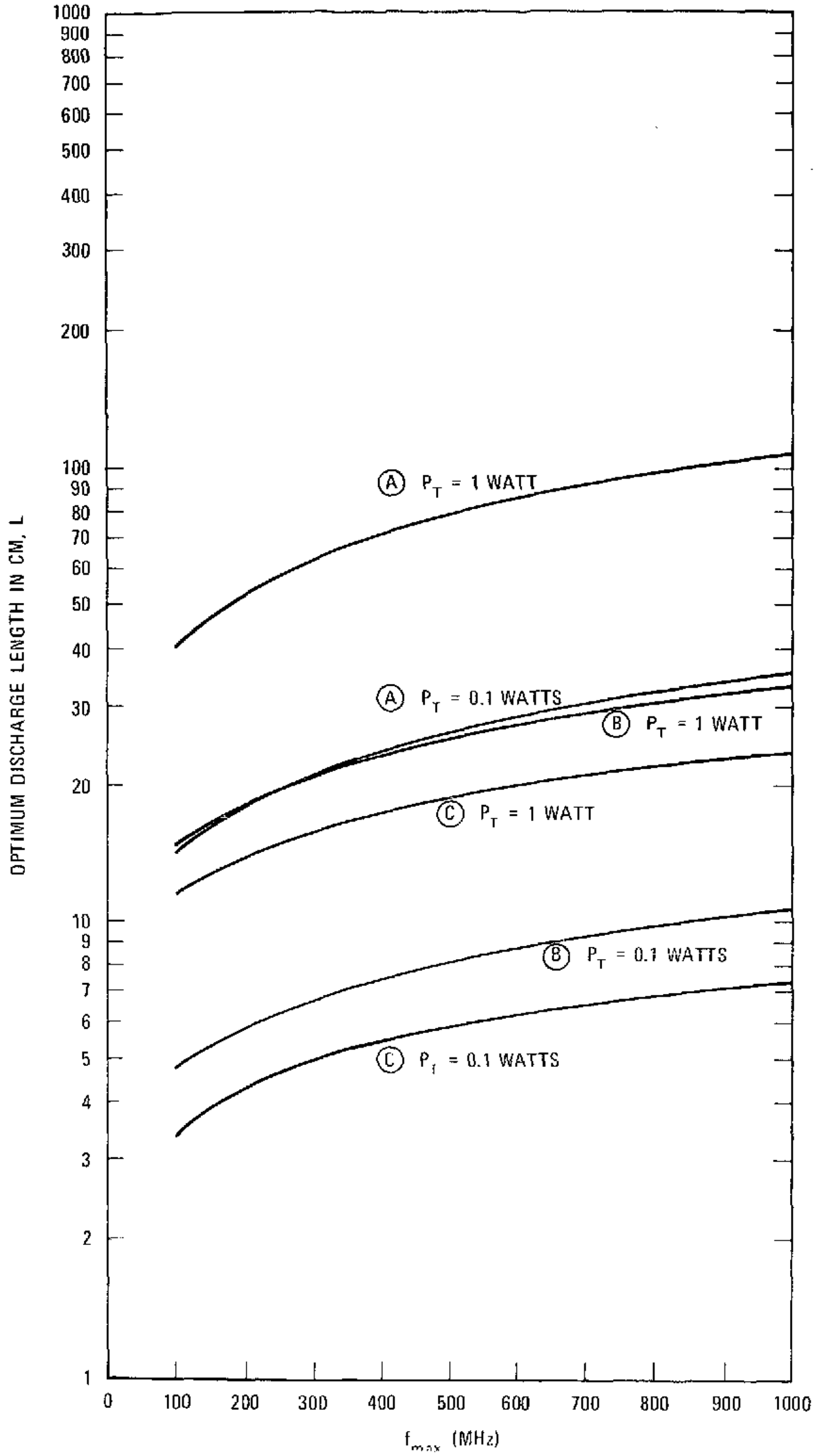
LASER POWER CONSUMPTION IN WATTS, P_{HV}
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0
100
200
300
400
500
600
700
800
900
1000
f_{max} (MHz)
Ⓐ P_T = 1 WATT
Ⓐ P_T = 0.1 WATTS
Ⓑ P_T = 1 WATT
Ⓒ P_T = 1 WATT
Ⓑ P_T = 0.1 WATTS
Ⓒ P_T = 0.1 WATTS

Figure 7(c)



1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
CIRCULATING POWER IN WATTS, P_c
Ⓐ P_T = 1 WATT
Ⓑ P_T = 1 WATT
Ⓒ P_T = 1 WATT
Ⓐ P_T = 0.1 WATTS
Ⓑ P_T = 0.1 WATTS
Ⓒ P_T = 0.1 WATTS
0
100
200
300
400
500
600
700
800
900
1000
f_{max} (MHz)

Figure 7(d)



1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
OPTIMUM DISCHARGE LENGTH IN CM, L
Ⓐ P_T = 1 WATT
Ⓐ P_T = 0.1 WATTS
Ⓑ P_T = 1 WATT
Ⓒ P_T = 1 WATT
Ⓑ P_T = 0.1 WATTS
Ⓒ P_f = 0.1 WATTS
0
100
200
300
400
500
600
700
800
900
1000
f_{max} (MHz)

Figure 7(e)

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9. Some of the values reported here have only been verified for modulator bandwidths of less than 300 MHz.

APPENDIX A

TOTAL TRANSMITTED POWER

In this appendix we calculate the total transmitted power contained in the carrier and all of the sidebands. From Eq. (7), the total power is equal to

$$P_{total} = \sum_{k=-\infty}^{\infty} P(\omega_c + k\,\omega_m)$$

$$= \eta_C P_C \left[\sin^2\Gamma_0 \sum_{k\ \mathrm{even}} J_k^2(\Gamma_m) + \cos^2\Gamma_0 \sum_{k\ \mathrm{odd}} J_k^2(\Gamma_m) \right] \qquad \text{(A-1)}$$

We can evaluate the sums appearing in Eq. (A-1) in the following manner:

$$\sum_{k\ \mathrm{even}} J_k^2(\Gamma_m) = \frac{1}{2}\left[\sum_{k=-\infty}^{\infty} J_k^2(\Gamma_m) + \sum_{k=-\infty}^{\infty} (-1)^k J_k^2(\Gamma_m) \right]$$

$$= \frac{1}{2}\left\{ \left[J_0^2(\Gamma_m) + 2\sum_{k=1}^{\infty} J_k^2(\Gamma_m) \right] + \left[J_0^2(\Gamma_m) + 2\sum_{k=1}^{\infty} (-1)^k J_k^2(\Gamma_m) \right] \right\} \qquad \text{(A-2)}$$

Using the Bessel relations

$$1 = J_0^2(z) + 2\sum_{k=1}^{\infty} J_k^2(z) \qquad \text{(A-3)}$$

and

$$J_0(2z) = J_0^2(z) + 2 \sum_{k=1}^{\infty} (-1)^k J_k^2(z) \tag{A-4}$$

Equation (A-2) becomes

$$\sum_{k\ \mathrm{even}} J_k^2(\Gamma_m) = \frac{1}{2}\left[1 + J_0(2\Gamma_m)\right] \tag{A-5}$$

Similarly

$$\sum_{k\ \mathrm{odd}} J_k^2(\Gamma_1) = \frac{1}{2}\left[\sum_{k=-\infty}^{\infty} J_k^2(\Gamma_m) - \sum_{k=-\infty}^{\infty} (-1)^k J_k^2(\Gamma_m)\right]$$

$$= \frac{1}{2}\left[1 - J_0(2\Gamma_m)\right] \tag{A-6}$$

Substituting (A-5) and (A-6) into (A-1) gives

$$P_{tot} = \frac{\eta_C P_C}{2} \left\{\sin^2 \Gamma_0 \left[1 + J_0(2\Gamma_m)\right] + \cos^2 \Gamma_0 \left[1 - J_0(2\Gamma_m)\right]\right\}$$

$$= \frac{\eta_C P_C}{2} \left[1 - J_0(2\Gamma_m) \cos 2\Gamma_0\right] \tag{A-7}$$