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A SIMPLE METHOD FOR THE DETERMINATION OF EFFECTIVE RAIN CELL DIMENSIONS AND ORIENTATION

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NOTICES

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ABSTRACT

A simple method is proposed for the determination of effective rain cell dimensions and orientation. Two rain cell models have been considered: the circular cell and the elliptical cell. In both models it is assumed that the rain rate is constant throughout the cell, that all cell locations are equally likely, and that the cell dimensions depend upon the rain rate. It is then shown that the effective cell dimensions and orientation may be deduced from rain rate statistics accumulated at two or three closely spaced sites, depending upon the model chosen. The results of this study along with previous estimates of rain cell dimensions indicate that the rain rate measurement points should be spaced on the order of one-half kilometer apart. The proposed method is simple and relatively inexpensive; thus, this approach is readily suited to the study of the dependence of rain cell size and orientation upon climatic region. The resulting rain cell characteristics are of direct value in the prediction of millimeter wavelength attenuation statistics on both single-terminal and diversity earth-space propagation paths as well as point-to-point terrestrial links.

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I. INTRODUCTION

The development of wide-bandwidth communication systems has led to an increased demand for utilization of the frequency spectrum above 10 GHz. Unfortunately, signals in this portion of the spectrum are attenuated by liquid water; and, in fact, severe attenuation may be encountered when the signal passes through regions of intense rain rates such as those associated with severe thunderstorms [1]. Thus, the overall communication system reliability may be ultimately determined by the spatial extent of those regions of intense rain rate through which the signal must pass and the probability of their occurrence.

A simple, cylindrical rain cell model was studied in an effort to determine its utility in the prediction of attenuation statistics. Although this model grossly oversimplifies the physical characteristics of the actual rain cells, it was found to be quite useful for the prediction of long term attenuation statistics within one or two decibels [2]. This model has been generalized to an ellipsoidal shape in order to more accurately reflect the effect of cell shape on space diversity communication systems where two spatially separated, parallel propagation paths are used to improve the overall system reliability. In the course of both of these modeling efforts it has been found that little data relating rain cell size, shape, orientation, and rain rate exists. Further, the dependence of such data upon climatic region is virtually nonexistant.

The use of radar for the determination of these characteristics has been proposed and is being pursued. However, this approach is expensive, and it is unlikely that the instrumentation will be widely implemented in the near future. Therefore, an alternative approach for the measurement of these cell characteristics is proposed herein. This approach would utilize either two or three rain rate gauges spaced relatively closely, on the order of one-half kilometer, and a means of simultaneously recording the measured rain rates. Thus, the instrumentation requirements would be minimal and the approach would be quite inexpensive to implement.

The radar approach yields size, shape, and orientation information concerning each rain cell observed. This information must then be processed to obtain statistical cell properties. In contrast, the rain gauge approach will not yield characteristics of individual cells but will considerably simplify the data processing required to obtain the desired effective cell characteristics. The radar approach can also yield information concerning the vertical distribution of rain rates; whereas, the rain gauge approach yields only surface rain rate statistics. Initially, both techniques should be compared at one site in order to determine the degree of agreement or disagreement.

II. THE RAIN CELL MODELS

The rain cell models to be considered were chosen because of their tractability. The intent was to represent rain cells in a phenomenal sense rather than to attempt to model individual rain cells. The first model considered was simply circular cylinder having a finite height above the earth's surface; the second model was an ellipsoid having two axes on the surface of a flat earth. The first model has two geometrical parameters, the diameter and the height, while the second model has four, the length, width, height, and orientation angle. Since the rain gauge method discussed herein deals only with surface rain rates, the vertical distributions of rain rate will be ignored. Hence, in the following, only the circular or elliptical surface cross sections of the cells will be considered. These geometries are shown in Fig. 1 where the x- and y-coordinate axes lie in the plane of the earth's surface. The orientation of this coordinate system is rather arbitrary; but, for the sake of

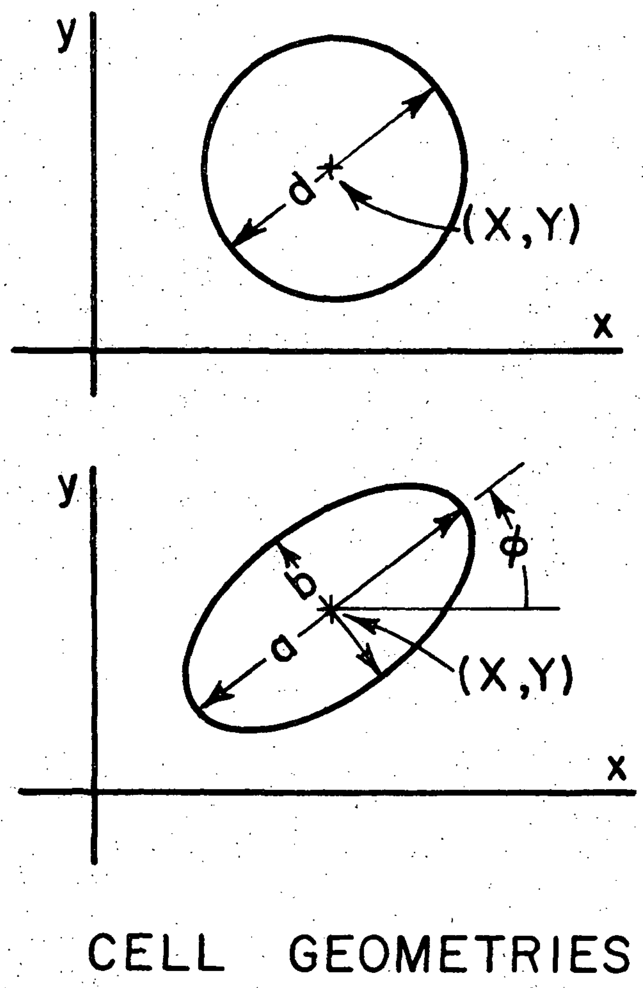


Fig. 1. The geometries of the circular and the elliptical rain cell models.

completeness, let us define the positive y-axis such that it is oriented in a northerly direction and the positive z-axis such that it is oriented upward. In both cases the coordinates X,Y denote the position of the center of the cell, and the remaining parameters are:

d = the diameter of the circular cell model,
a,b = the major and minor axes of the elliptical cell model, and
ϕ = the orientation angle of the elliptical cell model with respect to the x-axis.

The following basic assumptions are made in both cases:

(1) The rain rate is constant throughout each cell.
(2) In a statistical sense, there is a unique, single valued relationship between the cell rain rate and the cell geometrical parameters.
(3) All cell locations are equally likely.
(4) Only one cell influences the rain rate measurements at any instant of time.

Again, it must be stressed that the rain cells are to be modeled in a phenomenal, rather than physical, sense. Note, also, that assumption (2) implies that d, a, b, and ϕ are all dependent upon the rain rate, R. Finally, assumption (4) is in general too restrictive; however, in the cases to be examined here, the gauge spacings will be small compared to typical cell sizes. Therefore, since rain cells do not tend to occur immediately adjacent to one another, it is not likely that two different cells will simultaneously produce rain at two closely spaced gauges.

III. CIRCULAR CELL MODEL

If a rain gauge is placed at the origin of the coordinate system, a record of the rain rate, R, may be obtained as a function of time. The percentage of time during which the rain rate is less than or equal to a given value, R_0, may then be readily determined from such data as a function of R_0. If, then, the data gathering period is sufficiently long, it will be assumed that this percentage distribution is equivalent to the rain rate probability distribution. This distribution will be referred to as the point rain rate distribution, $P_{pt}(R)$, where the rain rate subscript has been dropped for simplicity. Since it has been assumed that all rain cell locations are equally likely, it follows that the point rain rate distribution does not depend upon the coordinate location of the measurement point. The point rain rate probability density function may then be defined in the usual manner,

$$(1) \qquad p_{pt}(R) = \frac{d\,P_{pt}(R)}{d\,R} \quad .$$

The preceding pragmatic line of reasoning applies to all of the probability density functions discussed in the following and, hence, will not be repeated.

A cell probability density function, $p_{cell}(R)$, will be associated with cells having rain rate R which are centered at X,Y. Again, $p_{cell}(R)$ does not depend upon X or Y since the cells were assumed to be uniformly distributed. It should be reiterated at this point that a unique effective cell diameter is assumed to be associated with each rain rate. Consequently, a cell of rain rate R centered within a circle of diameter d about the gauge will produce a rain rate R in the gauge, as shown in Fig. 2. This area about the gauge within which the occurrence of rain

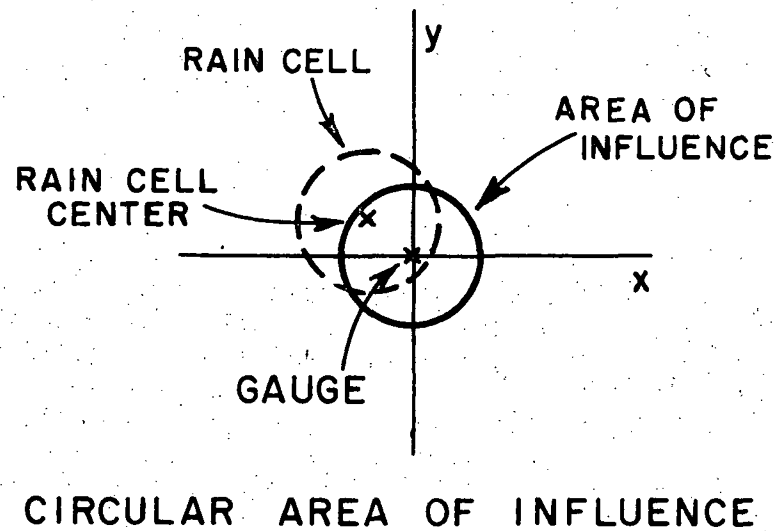


Fig. 2. Area of influence.

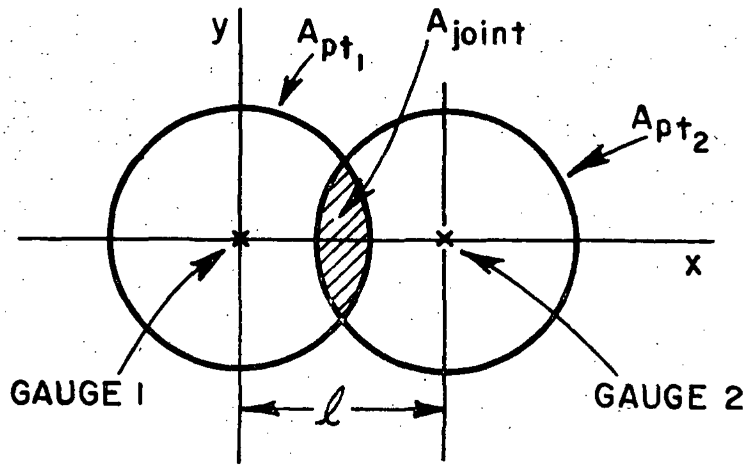
cell center will produce rain in the gauge will be referred to as the area of influence, $A_{pt}(d)$. This area of influence depends upon the cell diameter and, thus, implicitly upon the rain rate. In fact, it can be noted that the size and shape of the cell and the area of influence are identical for any given rain rate. Now, since the cells are uniformly distributed, it follows that the point rain rate and cell probability density functions are related by

(2) $$p_{pt}(R) = A_{pt}(d)\; p_{cell}(R)$$

or

(3) $$p_{pt}(R) = \frac{\pi}{4} d^2\; p_{cell}(R) \quad .$$

If, now, a second gauge is placed a distance ℓ along the x-axis from the first gauge, where ℓ is less than the cell diameter, d, the geometry shown in Fig. 3 results. For any given rain rate the areas of influence,



2 GAUGE GEOMETRY

Fig. 3. Two gauge geometry.

A_{pt_1} and A_{pt_2}, are identical. The joint area of influence, A_{joint}, is that area in which the occurrence of a rain cell center will produce rain at both gauges simultaneously. From geometrical considerations, the joint area of influence is

(4) $$A_{joint}(d,\ell) = \frac{d^2}{2} \sin^{-1} \sqrt{1 - \frac{\ell^2}{d^2}} - \frac{\ell}{2} \sqrt{d^2 - \ell^2} \quad .$$

The joint rain rate probability density function, $p_{joint}(R,\ell)$ associated with the two gauges may now be related to the cell probability density function by

(5) $$p_{joint}(R,\ell) = A_{joint}(d,\ell)\ p_{cell}(R) \quad .$$

This joint rain rate probability density function is dependent upon only one rain rate since the model considered permits the existence of only one rain cell at any instant of time. Thus, the simultaneous occurrence of dissimilar rain rates at the two gauges is not possible within the confines of this model.

Substituting (4) into (5) yields

(6) $$p_{joint}(R,\ell) = \tfrac{1}{2}\left[d^2 \sin^{-1} \sqrt{1 - \frac{\ell^2}{d^2}} - \ell\sqrt{d^2\ \ell^2}\ \right] p_{cell}(R)$$

Letting

$$(7) \qquad \gamma = \frac{\ell}{d}$$

and eliminating $p_{cell}(R)$ from Eq. (3) and (5) gives

$$(8) \qquad \frac{p_{joint}(R,\ell)}{p_{pt}(R)} = \frac{2}{\pi} \left[\sin^{-1}\sqrt{1-\gamma^2} - \gamma\sqrt{1-\gamma^2}\right] \quad .$$

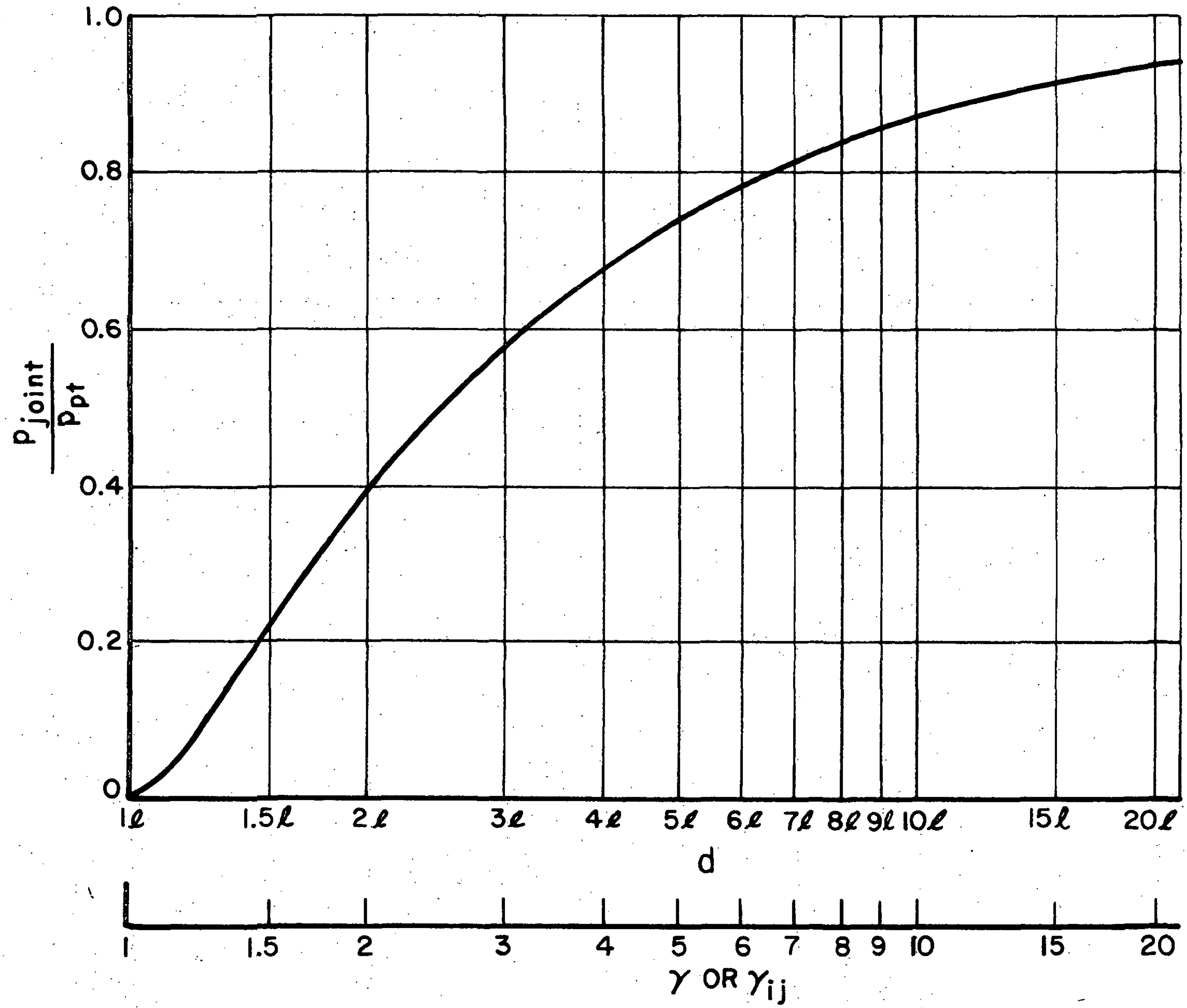
Here the ratio of the two rain rate probability density functions is a relatively simple function of the geometrical parameters ℓ and d. This functional relationship is shown in Fig. 4. Thus, if the two rain rate probability density functions are evaluated experimentally for a given value of rain rate and, of course, the gauge spacing is known, the effective rain cell diameter may be taken directly from Fig. 4. Therefore, the effective rain cell diameter, d(R), may be determined directly and simply from rain rate measurements at two sites.

It is evident from Fig. 4 that the useful range measurement of γ extends from roughly 1 to 20, i.e., d ranges from ℓ to 20ℓ. The implication here is that the gauge spacing should be small compared to the cell diameters of interest. Previous estimates of cell diameters indicate that they may range from 1 to 10 km with the smaller diameters being associated with the more intense rain rates [3]. Consequently, one may conclude that a gauge spacing of 0.5 km would be desirable for the implementation of this approach. Initially, this small spacing appears to be contradictory with rain gauge spacings commonly used in the past. However, these measurements have generally been concerned with distributions of point rain rates rather than the interpretation of the data in terms of areal rain rate distributions. This distinction between point and areal rain rate statistics has been pointed out by Epstein [4].

IV. ELLIPTICAL CELL MODEL

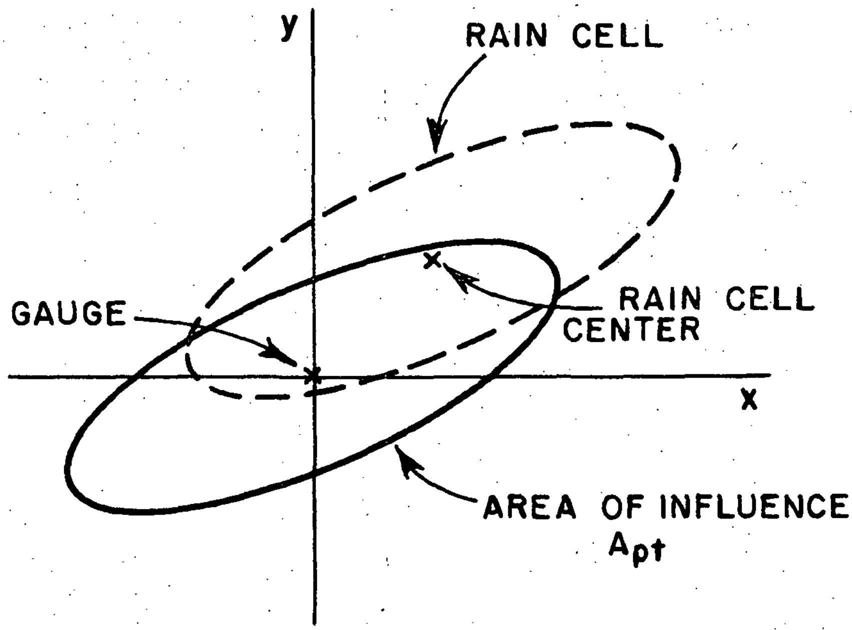
The treatment of the elliptical cell model follows that of the circular cell model with the exception of the increased number of degrees of freedom. The area of influence associated with a single gauge located at the origin of the coordinate system is shown in Fig. 5 and, again, is identical in size, shape, and orientation to the rain cell for any given rain rate. Locating an additional gauge at a separation distance of ℓ along both the positive x- and y-axes yields the geometry shown in Fig. 6. The areas of influence associated with the gauges are again equal for any given rain rate,

$$(9) \qquad A_{pt}(a,b,\phi) = A_1(a,b,\phi) = A_2(a,b,\phi) = A_3(a,b,\phi) \quad .$$



RATIO OF RAIN RATE PROBABILITY DISTRIBUTION FUNCTIONS vs. GEOMETRICAL PARAMETER

Fig. 4. Ratio of rain rate probability distribution functions versus geometrical parameter γ.



ELLIPTICAL AREA OF INFLUENCE

Fig. 5. Elliptical rain cell area of influence.

And, the point rain rate probability density function is given by

$$p_{pt}(R) = A_{pt}(a,b,\phi)\ p_{cell}(R) \tag{10}$$

$$p_{pt}(R) = \frac{\pi}{4}\ a\ b\ p_{cell}(R) \quad . \tag{11}$$

The joint areas of influence are denoted by $A_{ij}(a,b,\phi,\ell)$ and are given by

$$A_{12}(a,b,\phi,\ell) = \frac{ab}{2}\sin^{-1}\left[\sqrt{1-\ell^2\left(\frac{\cos^2\phi}{a^2}+\frac{\sin^2\phi}{b^2}\right)}\right] - \frac{\ell}{2}\sqrt{(b^2\cos^2\phi + a^2\sin^2\phi)\left[1-\ell^2\left(\frac{\cos^2\phi}{a^2}+\frac{\sin^2\phi}{b^2}\right)\right]} \tag{12}$$

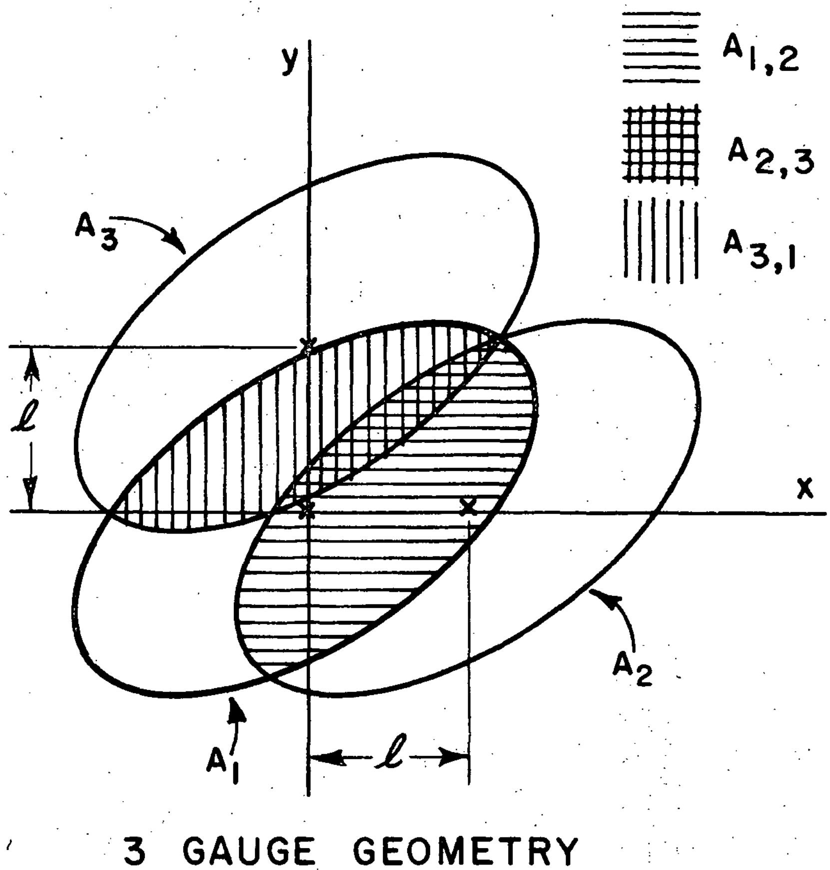


Fig. 6. 3 gauge geometry.

(13) $$A_{13}(a,b,\phi,\ell) = \frac{ab}{2}\sin^{-1}\left[\sqrt{1-\ell^2\left(\frac{\sin^2\phi}{a^2}+\frac{\cos^2\phi}{b^2}\right)}\right] - \frac{\ell}{2}\sqrt{(b^2\sin^2\phi + a^2\cos^2\phi)\left[1-\ell^2\left(\frac{\sin^2\phi}{a^2}+\frac{\cos^2\phi}{b^2}\right)\right]}$$

(14) $$A_{23}(a,b,\phi,\ell) = \frac{ab}{2}\sin^{-1}\left[\sqrt{1-\ell^2\left(\frac{(\sin\phi-\cos\phi)^2}{a^2}+\frac{(\sin\phi+\cos\phi)^2}{b^2}\right)}\right] - \frac{\ell}{2}\sqrt{\left[b^2(\sin\phi-\cos\phi)^2+a^2(\sin\phi+\cos\phi)^2\right]\left[1-\ell^2\left(\frac{(\sin\phi-\cos\phi)^2}{a^2}+\frac{(\sin\phi+\cos\phi)^2}{b^2}\right)\right]}$$

The ij subscript notation here and in the following obviously implies only the pairs: 1,2; 2,3; or 3,1. The three joint rain rate probability density functions may now be expressed as

$$(15) \qquad p_{ij}(R,\ell) = A_{ij}(a,b,\phi,\ell)\; p_{cell}(R) \quad .$$

Eliminating $p_{cell}(R)$ from Eqs. (11) and (15) yields

$$(16) \qquad \frac{p_{ij}(R,\ell)}{p_{pt}(R)} = \frac{2}{\pi}\left[\sin^{-1}\sqrt{1-\gamma_{ij}^2} - \gamma_{ij}\sqrt{1-\gamma_{ij}^2}\right]$$

where the geometrical parameters are given by

$$(17) \qquad \gamma_{12} = \ell\sqrt{\frac{\cos^2\phi}{a^2} + \frac{\sin^2\phi}{b^2}}$$

$$(18) \qquad \gamma_{23} = \ell\sqrt{\frac{(\sin\phi-\cos\phi)^2}{a^2} + \frac{(\sin\phi+\cos\phi)^2}{b^2}}$$

$$(19) \qquad \gamma_{31} = \ell\sqrt{\frac{\sin^2\phi}{a^2} + \frac{\cos^2\phi}{b^2}} \quad .$$

Comparing Eqs. (8) and (16), it is seen that the ratios of the probability density functions have the same functional dependence on the geometrical parameters for both models. Thus, given a set of joint and point rain rate probability density functions, the geometrical parameters, γ_{ij}, may be determined from Fig. 4. Eqs. (17), (18), and (19) may now be solved simultaneously to find a, b, and ϕ in terms of this set of geometrical parameters, γ_{ij}:

$$(20) \qquad \tan(2\phi) = \frac{\gamma_{12}^{\;2} + \gamma_{31}^{\;2} - \gamma_{23}^{\;2}}{\gamma_{12}^{\;2} - \gamma_{31}^{\;2}}$$

$$(21) \qquad a = \ell\sqrt{\frac{\cos^2\phi - \sin^2\phi}{\gamma_{12}^{\;2}\cos^2\phi - \gamma_{31}^{\;2}\sin^2\phi}}$$

$$(22) \qquad b = \ell \sqrt{\frac{\sin^2\phi - \cos^2\phi}{\gamma_{12}{}^2 \sin^2\phi - \gamma_{31}{}^2 \cos^2\phi}}$$

Note that the order of solution here is important since Eqs. (17)-(19) were not linear. The orientation angle, ϕ, must be determined first from Eq. (20) and then used in Eqs. (21) and (22) to find the ellipse axes. The orientation angle, ϕ, ranges from $-\pi/2$ to $\pi/2$; nevertheless, Eq. (20) still admits two possible solutions. This does not lead to an ambiguity since the value of a determined from these equations gives the length of the axis along the particular orientation angle chosen. Thus, a is not necessarily the major axis of the ellipse, it is the axis oriented in the direction of the orientation angle, ϕ, chosen from Eq. (20).

Thus, given a set of joint and point rain rate probability density functions and the gauge spacing, it is possible to deduce unique effective ellipse axes and the ellipse orientation angle for each rain rate. The conclusions regarding practical gauge spacings discussed in the context of the circular model apply in this case as well.

V. SUMMARY

A simple rain gauge method for the determination of effective rain cell dimensions and orientation has been described. Both circular and elliptical rain cell models have been treated. Either two or three rain gauges are required depending upon the model chosen. Further, the gauges should be spaced relatively closely, on the order of one-half kilometer. And, finally, the data processing required to extract the desired information from the rain rate statistics is quite simple and straightforward. Thus, a relatively simple, inexpensive technique is available for the statistical determination of effective rain cell dimensions and orientation.

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