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ANALYSIS AND DESIGN OF THREE DIMENSIONAL SUPERSONIC NOZZLES

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A DESIGN TECHNIQUE FOR MULTIPLE NOZZLE CONFIGURATIONS

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I. INTRODUCTION

In the design of hypersonic aircraft, the relatively high heating loads encountered make it difficult to provide an engine with a nozzle of sufficient length to achieve complete internal expansion. In this case, optimum performance of the system dictates cutting several walls of the nozzle short in order to achieve a savings in weight and making use of the vehicle undersurface as the final nozzle expansion surface. Two such hypersonic vehicles are represented schematically in Figure (1). In each case, the vehicle is propelled by a number of engines which all discharge through relatively short nozzles into a final single common nozzle formed to a large extent by the vehicle undersurface.

Since the upper wall of the common nozzle is formed by the lower surface of the vehicle its length is fixed by vehicle design considerations rather than by considerations of nozzle optimization. The optimum length for the nozzle cannot be defined without a series of calculations since its value is determined by a compromise between increased efficiency and increased structural weight. In order to perform the nozzle optimization, it is necessary to carry out a series of aerodynamic analyses which furnish a family of designs and their corresponding performance and to determine from these the relation between length, and

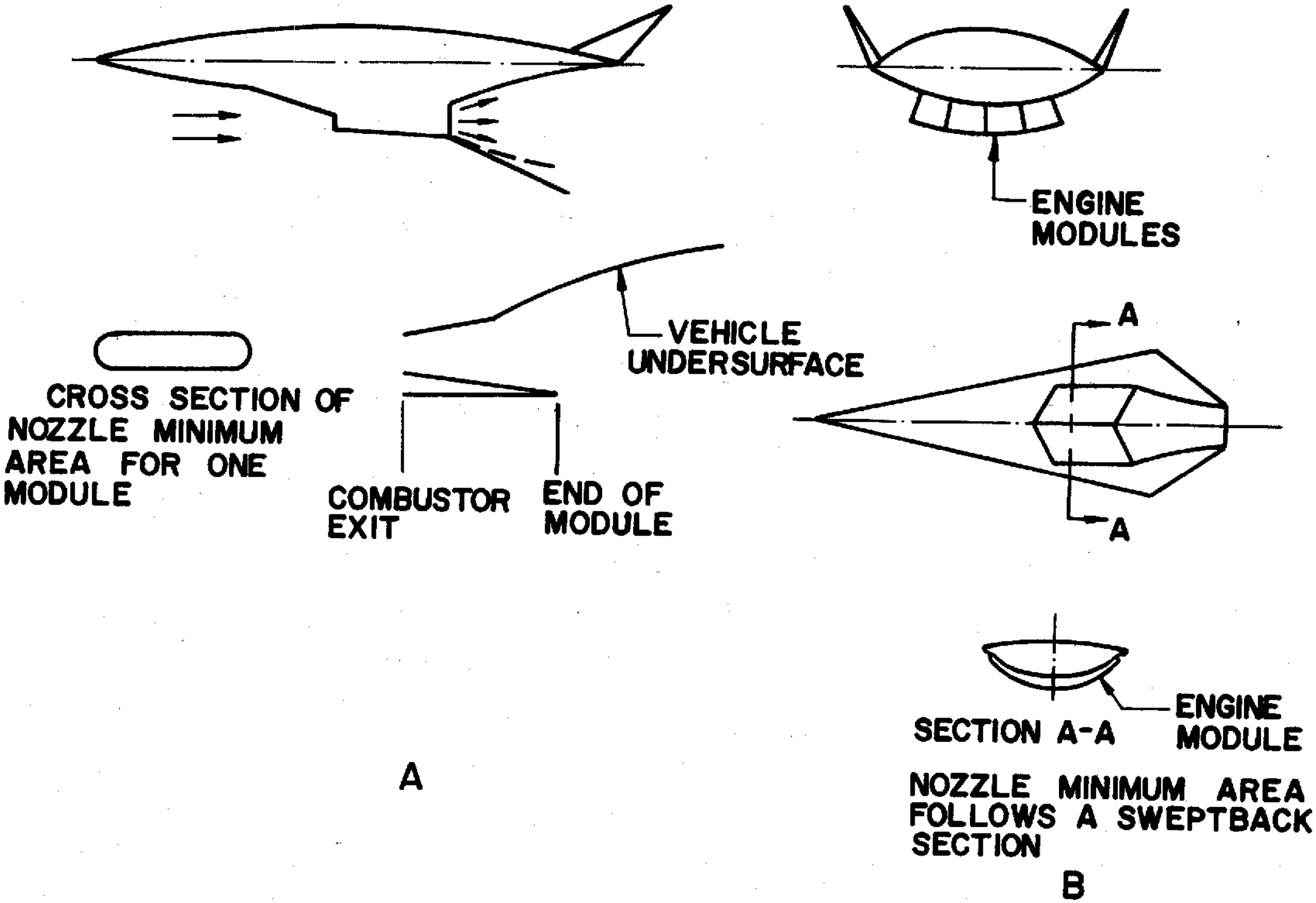

FIGURE 1. SKETCH OF TWO FAMILIES OF EXHAUST NOZZLES

therefore weight, and performance. It is the purpose of this volume to outline such a procedure.

In practical applications, the flow leaving the burners of ramjet and scramjet engines is far from uniform with large variations of stagnation conditions and flow properties existing across the burner exit plane. Moreover, the flow at the nozzle exit is also nonuniform, since the exit area is smaller than that which is required for full expansion and the nozzle length is necessarily shorter than that required for complete wave cancellation. As a result of this overall nonuniformity, it is not possible to develop exact criteria for the design of an optimum nozzle. Fortunately, the sensitivity of nozzle performance to minor perturbations of exit Mach number and flow inclination is quite small. Therefore, satisfactory nozzle designs may be obtained by means of an iteration procedure where initial values of several of the design parameters are assumed from simplified approximate considerations and are then varied systematically in order to determine the value of each parameter which optimizes nozzle performance.

This procedure can be carried out using the three dimensional nozzle analysis described in Volume I of this report. The geometric design parameters are initially selected from quasi-two dimensional considerations which will be described in the following sections and the performance of the selected configuration is determined using the computer programs described in Volume II of this report. Then, improvements in nozzle performance are obtained by perturbing the initially assumed values of the geometric parameters on the basis of the results of the computer analysis, this procedure being followed until no further performance improvements are obtained. The results of this study can then be used to evaluate the trade-off between increased nozzle efficiency and increased nozzle weight.

II. DESCRIPTION OF NOZZLE REQUIREMENTS

The geometry of a general multiple nozzle assembly is outlined schematically in Figure (2).

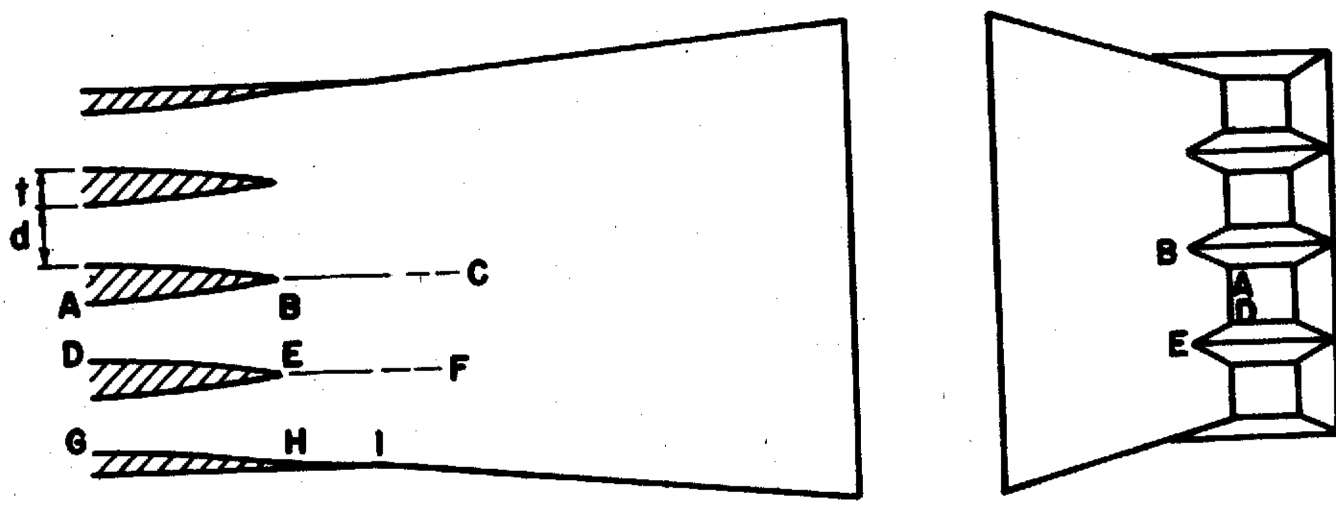

FIGURE 2. BOTTOM AND REAR VIEW OF GENERAL MULTIPLE NOZZLE CONFIGURATION

The walls of the nozzles expand simultaneously in two directions approximately normal to one another. Since the burner design fixes the thickness (t) of the walls which divide the individual nozzles, it therefore fixes the degree of lateral expansion which will be produced when these walls are removed.

Because the principal expansions occur on surfaces which are approximately normal to one another, the net change in Mach number produced can be accurately predicted by two dimensional theory using the sum of the turning produced by each individual wave as the total turning angle. The shapes of the waves, of course, cannot be predicted in this way and must be determined by the nozzle analysis program.

The surfaces BC and EF are planes whose orientation is known. The nozzle analysis does not require these planes to be parallel but their inclination must be fixed. As a first step, it is assumed that the wall HI is also given and planar until the station at which the solid surfaces AB, DE and GH are terminated. Other geometric boundary conditions could be imposed on the analysis, but these would only produce losses in thrust since lateral external expansion would take place before the flow was sufficiently expanded in the nozzle and would require that the end modules be calculated separately. Therefore, lateral expansions are considered only downstream of point H (surface HI is present, but I can coincide with H).

Since the nozzles are three dimensional, they expand in vertical as well as lateral planes. The general geometry of the nozzle in the vertical plane is outlined in Figure (3).

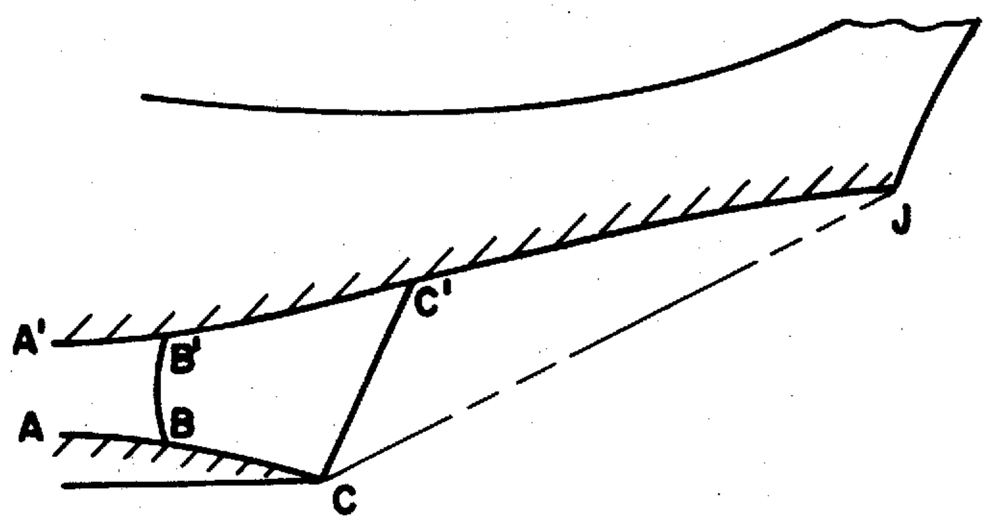

FIGURE 3. SIDE VIEW OF GENERAL MULTIPLE NOZZLE CONFIGURATION

The multiple primary nozzles terminate along a curved line BB'. The cowl surface, AC, is far shorter than the undersurface of the fuselage, A'J. The side walls of the nozzle GI, terminate along the line CC', which is inclined so as to form a supersonic trailing edge. The shape of the vehicle undersurface, C'J, is assumed to be given.

In practical aircraft designs, the length of the cowl surface AC is far shorter than that required to provide the possibility of cancelling the expansion waves eminating from A', while the vehicle length A'J is sufficiently large so that at least a portion of the waves produced at A can be cancelled. In general, the surfaces AB and A'B' are not sufficiently long to feel the effects of the expansion fans from the corners A and A'. Therefore, wall curvature upstream of the line BB' would only produce premature compression waves and reduce nozzle efficiency. As a result, these short surfaces are taken as straight walls and are completely characterized by their inclination to the vehicle centerline.

Summarizing, subject to the constraints of a given length and vertical rise for the vehicle undersurface A'J, and a given thickness, t, of the sidewalls of the multiple nozzles, the geometry of the system can be defined once the inclinations of the surfaces A'B' and AB and the sidewall shapes (AB, DE, etc.) have been specified. The cowl length, AC, can then be reduced and the resulting nozzle losses computed for a comparison with the weight savings effected.

It should be noted here, that since the waves from A and A' are not completely cancelled at the opposite walls, the flow properties along the line CJ are far from uniform. In this region, those waves produced along AA' are still present but are sufficiently weak to allow two dimensional criteria to be applied. This approximation will be used to select the inclinations of the surfaces AB and A'B'.

III. SELECTION CRITERIA FOR THE INITIAL NOZZLE GEOMETRY

Since the actual area available for the nozzle exit is usually insufficient to allow either complete expansion or complete wave cancellation, the nozzle exit flow will be nonuniform and at an average static pressure somewhat higher than ambient. The general wave pattern which will exist in a vertical reference plane through the nozzle is indicated in Figure (4) below.

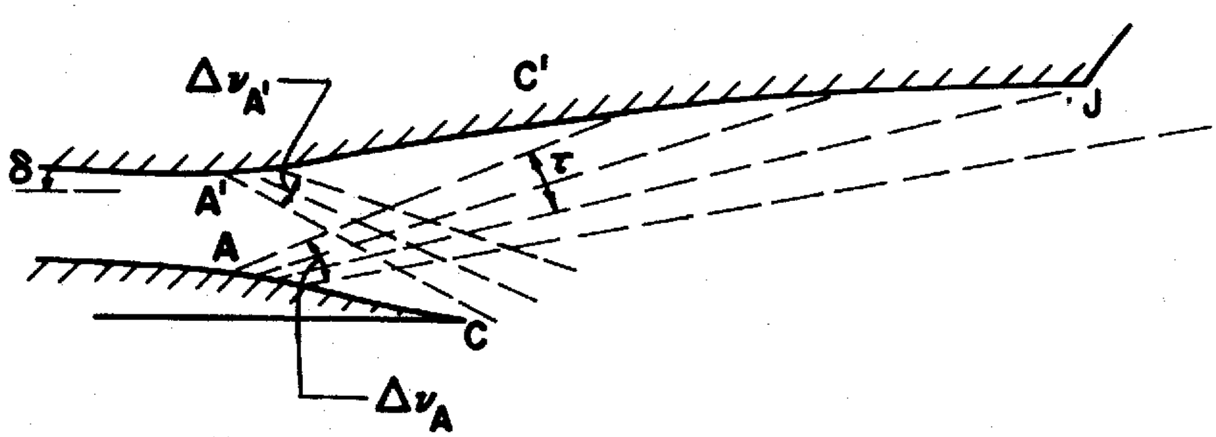

FIGURE 4. WAVE PATTERN IN VERTICAL NOZZLE REFERENCE PLANE

The difference in static pressure between the wall at J and the central streamtube is determined by the initial flow Mach number, the geometry and the degree of turning at A, the combination of which determines how much of the expansion fan generated at A reaches the wall A'J, and by the degree of wave cancellation along the wall. Similarly, the pressure difference between the cowl exit point C and the central streamtube is determined by the degree of turning at A' and the amount of wave cancellation along the cowl AC. In practical designs the length of the cowl is usually so short as to make the degree of cancellation along its surface small enough to be neglected.

Two important nozzle properties are determined by overall vehicle design considerations: the total nozzle exit area is fixed by vehicle geometric constraints and the angle of the thrust vector with respect to the free stream is determined by lift and pitching moment requirements. In addition, the initial flow inclination δ is a function of inlet and combustor design considerations and is assumed to be a prespecified parameter.

The first step in the nozzle design process is the definition of the entrance plane flow conditions. Since the nature of ramjet and scramjet engines is such that the combustor exit flow is usually highly nonuniform in both stagnation pressure and enthalpy, a simple treatment of the nozzle flow field requires that the entrance conditions be idealized somewhat. This is accomplished in the present case by dividing the nozzle entrance plane into regions as indicated in Figure (5). In each of these regions, an engineering approximation to the

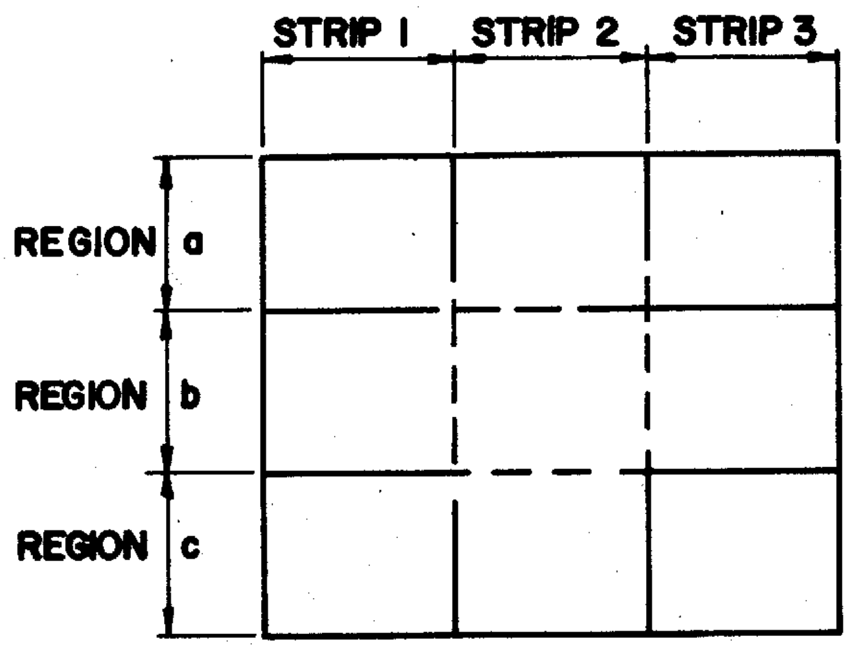

FIGURE 5. DIVISION OF THE NOZZLE ENTRANCE PLANE

total enthalpy may be obtained by dividing the area into a number of subregions of nearly constant properties and mass averaging the total enthalpies of each subregion. To obtain the average total pressure, the procedure is also straightforward but slightly more involved numerically. In each subregion, i, we must first determine the Mach number M_i, the mass flow m_{i_x} and the total temperature T_{T_i}. Then, the mass flow function N_i is calculated for each subregion using the definition

$$N_i = M_i\left(1 + \frac{\gamma-1}{2} M_i^2\right) \left(1 + \gamma M_i^2\right)^{-1} \tag{1}$$

The average mass flow function for the entire program is then given by Reference (1) to be

$$\overline{N} = \frac{\sum_{i=1}^{n} \sqrt{m_i} \sum_{i=1}^{n} \sqrt{m_i T_{T_i}}}{\sum_{i=1}^{n} \frac{m_i}{N_i} \sqrt{T_{T_i}}} \tag{2}$$

Knowing $\overline{N}$, the value of $\overline{M}$ is found by a solution of Equation (1). Assuming a constant value of static pressure, $\overline{p}$, across the nozzle entrance plane the average total pressure for the region is given by

$$\overline{p}_T = \overline{p}\left(1 + \frac{\gamma-1}{2} \overline{M}^2\right)^{-\frac{\gamma}{1-\gamma}} \tag{3}$$

Having defined the necessary average properties, each of the regions, is now expanded by means of a one dimensional analysis to a pressure no less than the local ambient value. The values of streamtube area, A, and gas velocity, V, are determined as functions of static pressure through the expansion as is the increase in two dimensional Prandtl-Meyer turning angle, ν, corresponding to the instantaneous value of Mach number.

Since the purpose of this analysis is simply to determine starting values for the nozzle geometric parameters used in the iteration procedure described in the previous section, a high degree of accuracy is not required. As a result, several assumptions have been and will be introduced in order to simplify the calculation. At this point, another simplification may be properly introduced: the value of γ may be assumed constant within prescribed ranges of temperature, jumping discontinuously at the boundary from its value in one range to its value in the next. By so doing, existing tables of one dimensional flow properties may be utilized for this portion of the calculation without a significant sacrifice in accuracy. The result of this computation is shown schematically in Figure (6).

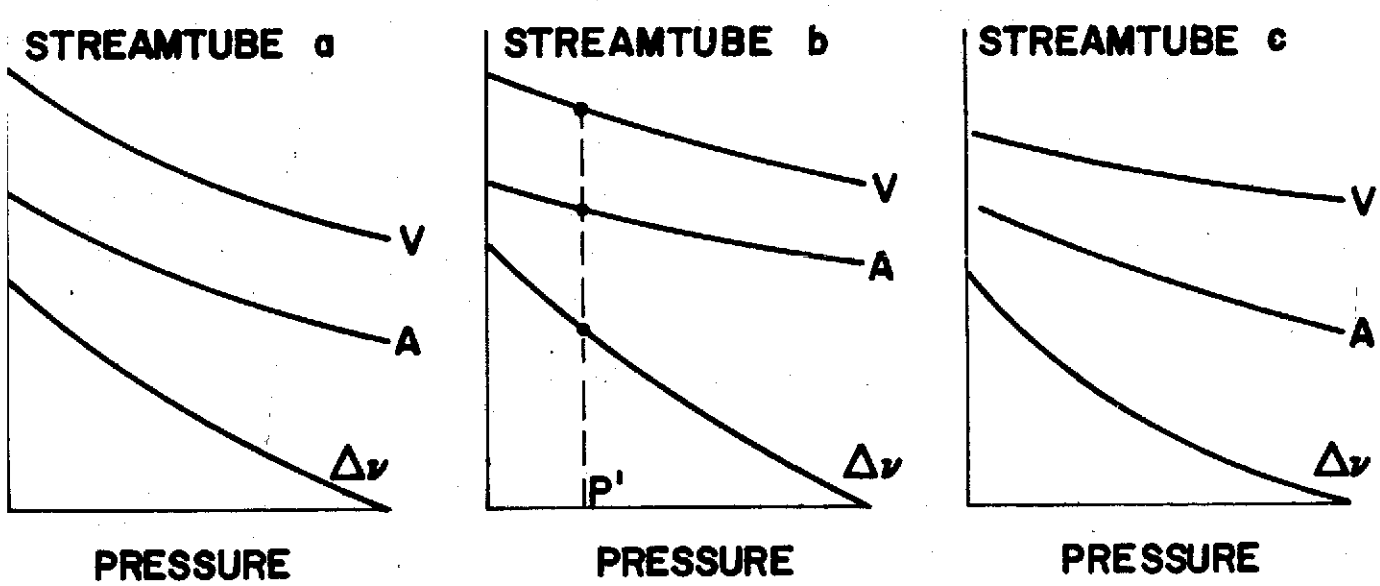

FIGURE 6. PROPERTIES THROUGH THE EXPANSION

The total increase in Prandtl-Meyer turning angle experienced by each of the three streams is simply

$$\Delta\nu_a = \Delta\nu_L + \Delta\nu_{A'} + \tau \tag{4}$$

$$\Delta\nu_b = \Delta\nu_L + \Delta\nu_A + \Delta\nu_{A'} \tag{5}$$

$$\Delta\nu_c = \Delta\nu_L + \Delta\nu_A \tag{6}$$

where $\Delta\nu_A$ and $\Delta\nu_{A'}$ are the degrees of turning produced by the corners A and A', $\Delta\nu_L$ is the degree of lateral expansion* and τ is the degree of turning produced by the waves from A' which reach the vehicle undersurface as indicated in Figure (4).

Denoting the inclination of the thrust vectors for the three regions with respect to the free stream direction 1 as σ_a, σ_b and σ_c, geometry dictates that

$$\sigma_a = \delta + \Delta\nu_{A'} - \tau \tag{7}$$

$$\sigma_b = \delta + \Delta\nu_{A'} - \Delta\nu_A \tag{8}$$

$$\sigma_c = \delta - \Delta\nu_A \tag{9}$$

*$\Delta\nu_L$ is calculated by taking the change in Prandtl-Meyer angle corresponding to the expansion from the initial entrance Mach number produced by the lateral area ratio (d+t)/d.

Since the inclination, σ_R of the resultant thrust vector is a given quantity, dictated by aircraft design considerations, we require that

$$(\sigma_a-\sigma_R)\ (m_aV_a+p_aA_a) + (\sigma_b-\sigma_R)\ (m_bV_b+p_bA_b) + \\ (\sigma_c-\sigma_R)\ (m_cV_c+p_cA_c) = 0 \tag{10}$$

Finally, since the total area, A, at the nozzle exit plane is fixed, we have the condition

$$A_a + A_b + A_c = A \tag{11}$$

Now, several values of the exit plane static pressure in streamtube b are selected, either equal to or greater than the local ambient static pressure. For each pressure p_b' selected, the total flow deviation $\Delta\nu_{b'}$, the velocity V_b, and the streamtube area A_b, are given by the one dimensional calculation represented in Figure (6). Using these values, Equations (4) through (11) are used to provide a numerical solution for the corresponding values of $\Delta\nu_A$, $\Delta\nu_{A'}$ and τ. The operation is repeated for several values of the pressure p_b' and the optimum set of values is selected for the initial guess as to nozzle geometry.

As stated earlier, since the length of the multiple nozzle section is relatively short very few, if any, of the waves generated at the corners A and A' will reach the cowl or vehicle surfaces upstream of the sidewall trailing edges CC'. As a result, these upper and lower surfaces should be kept linear within the confines of the multiple nozzles since premature curvature of these walls would produce envelope shocks and increase losses. However, the nozzle sidewalls must be reduced in thickness from their initial value of t to zero within the distance from A to C. Since, directional continuity at the trailing edge requires the formation of shock

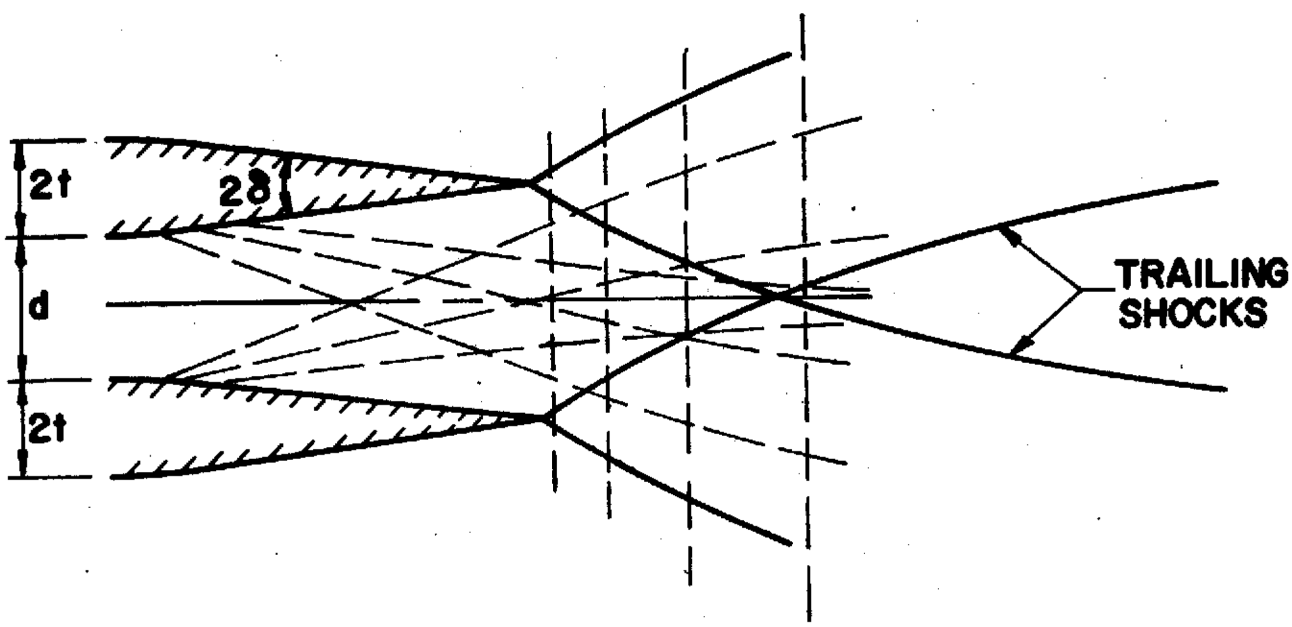

FIGURE 7. EFFECT OF STRAIGHT SIDEWALLS

waves of strength δ, restriction of the trailing shock losses to acceptable limits requires that the trailing edge angle be quite small. Thus, if the sidewalls were to be designed as straight surfaces and the shock losses were to be kept small, the sidewall length would become excessive. To avoid this problem, curvature of the sidewalls is indicated and the degree of this curvature becomes another design parameter.

Consider the lateral wave pattern depicted schematically in Figure (8). By turning the sidewalls AB and DE gradually

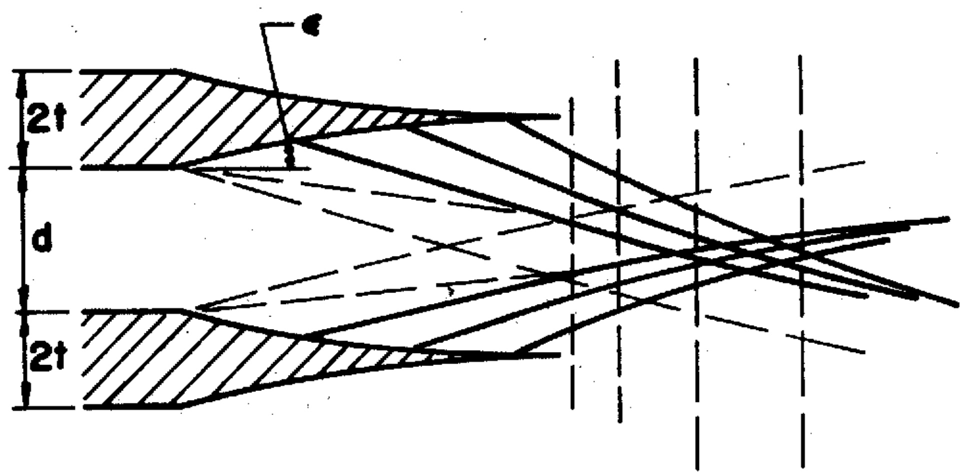

FIGURE 8. WAVE PATTERN WITH CURVED SIDEWALLS

back toward the axial direction, a series of compression waves are generated which coalesce downstream to form an envelope shock. Although the eventual strength of this wave, when fully formed, cannot exceed the strength δ of the wave formed by a straight wall design, we wish to design the wall to delay coalescence for the maximum possible distance. In this regard, the vertical expansion waves eminating from the corners A and A' are quite helpful, since they diffract the waves and tend to move the region of coalescence downstream.

Therefore, the design process should proceed along the following lines. For each of the three vertical regions, a, b and c defined earlier, the Mach numbers both upstream and downstream of the expansion fan from corners A and A' are known. In each of these regions, the sidewalls are designed using two dimensional characteristics techniques as described in Reference (2). For the two regions a and c bounded by the cowl and vehicle surfaces, the sidewalls are designed using the Mach number and flow deflection downstream of the corner expansion as the initial conditions: in region b, the central core, the combustor exit Mach number and direction are more appropriate. Once the three profiles are defined, the overall geometry may be defined by a linear variation with vertical rise from profile c to profile b to profile a. It should be noted that it is not necessary to utilize the averaged properties in each region for the characterisitcs computation since the methods presented in Reference (2) allow handling discontinuities in entropy and enthalpy across the flow field.

As a guide to the proper range of values for the initial sidewall expansion angle ε, consider the total amount of lateral expansion which must be produced as defined by the area ratio **(d+2t)/d**. Starting from the initial Mach number, this area ratio corresponds to a specific increase in the Prandtl-Meyer function $\Delta\nu_L$. This total change in ν is exactly twice the initial expansion angle required if uniform shock free flow were to be obtained in the lateral plane. This value will generally be fairly small and will lead to a rather long sidewall. Initial choices of larger lateral expansion angles, ε, will produce shorter walls and increasing wave coalescence. Summarizing

$$\varepsilon \geq \frac{1}{2} \Delta\nu_L \tag{12}$$

As noted earlier, the three dimensional effects in the nozzle diffract the sidewall compression waves and delay formation of the envelope shock. Therefore, the sidewall geometry designed using two dimensional techniques will be longer than actually required. As a consequence, once the flow field for the selected initial geometry is analyzed using the program described in Volume II of this report, the sidewall geometry may be perturbed in the direction of larger angles ε and correspondingly shorter overall lengths