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STUDY OF A CONTROL SYSTEM TO ALLEVIATE AIRCRAFT RESPONSE TO HORIZONTAL AND VERTICAL GUSTS

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16. Abstract

A study is made of the longitudinal response of a STOL airplane equipped with a vane-controlled gust-alleviation system. Effects of various combinations of design parameters on the responses to horizontal and vertical gusts and to elevator inputs are presented. Almost complete gust alleviation may be obtained with this system when the center of gravity of the STOL airplane is at rearward locations, but pitch stability augmentation is required for satisfactory control characteristics.

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STUDY OF A CONTROL SYSTEM TO ALLEVIATE AIRCRAFT RESPONSE TO HORIZONTAL AND VERTICAL GUSTS

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SUMMARY

An analysis is made of the longitudinal response of an airplane in three degrees of freedom, with provision for including the effects of a vane-controlled gust-alleviation system and a simple pitch stability augmentation system. The systems are designed to compensate for the effects of vertical and horizontal gusts. Examples are presented to show the effect of such systems with various combinations of parameters on the response of a STOL airplane with externally blown flaps in the landing-approach condition. The results include transient responses after penetration of step horizontal and vertical gusts, responses to step elevator deflections, roots of the characteristic equations, and magnitudes of the gust forcing terms.

The complete elimination of gust forcing terms in the longitudinal-response equations of an airplane, which is possible when horizontal gusts and speed changes are neglected, does not appear possible with systems of the type studied when these factors are taken into account. Nevertheless, the important forcing terms and the resulting response may be reduced to relatively small values by use of flaps with suitable aerodynamic characteristics, a rearward center-of-gravity location, and gearing of flaps to spoilers or other drag devices to reduce longitudinal force changes due to flap deflection. The stability characteristics of the alleviated airplane appear to be unsatisfactory without stability augmentation to provide pitch-attitude stabilization and improved damping of the phugoid mode.

INTRODUCTION

A previous analytical study of methods of reducing the vertical acceleration due to gusts (ref. 1) was limited to consideration of vertical gust inputs, and the airplane dynamics were simplified by assuming constant airspeed. These restrictions are justifiable for high-speed flight conditions. For STOL airplanes, particularly in the landing-approach condition, the relative sensitivity to horizontal gusts is increased, and effects of speed variations may be more critical because of the reduced period of the phugoid motion. For this reason, an extension of the theory to include speed variations and horizontal gusts is desirable.

The gust-alleviation system of reference 1 utilized a combination of wing flaps and elevators geared to move together so that, for analysis purposes, they could be considered as one control. These surfaces were moved in response to an angle-of-attack vane mounted ahead of the nose in such a way as to offset the forces and moments applied to the airplane by the gusts. In this sense, the system operates as an open-loop control. The vane position, however, is also affected by airplane motion. The resulting feedback of airplane motion to the control surfaces affects the longitudinal stability. The objective of the study of reference 1, therefore, was to reduce the gust inputs while retaining adequate stability and control characteristics. This objective was accomplished in the cruise flight regime without employing additional stability-augmentation devices. One objective of the present study is to investigate the possibility of offsetting the effects of both horizontal and vertical gusts, taking into account speed variations as well as vertical and pitch responses. The response of a STOL airplane in the landing-approach condition is used as an example to investigate this system.

The method of analysis used in reference 1 was based on the conventional linearized airplane stability theory, in which a first-order approximation to lag effects is employed. This method is advantageous because the addition of a gust-alleviation system does not increase the order of the equations above that of the basic airplane equations, and because most of the effects of the system may be visualized as modifications to the stability derivatives of the basic airplane. The modified derivatives provide physical insight into the effects of the alleviation system. Relations were derived in reference 1 which must be satisfied by the design parameters in order to eliminate completely the response to vertical gusts. The usefulness of these relations was later demonstrated in a research flight program on a gust-alleviated airplane (ref. 2). Extension of the theory to include horizontal gusts is desirable to determine whether similar relations can be found which will permit reduction of the response to horizontal gusts.

In the present investigation, therefore, longitudinal equations of motion for the airplane are set up in three degrees of freedom, considering both horizontal and vertical gust disturbances. A simple gust-alleviation system is assumed, consisting of a spring-loaded vane-type sensor operating wing flaps through a servomechanism. The spring-loaded vane-type sensor when used with a gust-alleviation system has the capability of providing gust alleviation over a range of speeds without the need for gain changes between the sensor motion and flap response. The vane-type sensor, although it is useful for research purposes, has practical disadvantages compared with internal instruments, such as accelerometers and gyros, because it is relatively fragile and exposed to damage. The relations derived for this type of system, however, are useful in providing a guide for the design characteristics of systems using internal sensors.

The analysis presented herein neglects flexibility effects and is based on the assumption of a conventional configuration in which forces and moments on the stabilizing surfaces can be calculated separately from those on the wing-fuselage combination. More detailed analysis would be required to investigate effects of structural flexibility and control-system dynamics in an actual configuration.

SYMBOLS

A aspect ratio

a_n normal acceleration, g units

C_L lift coefficient, $\dfrac{L}{\dfrac{\rho V^2}{2} S}$

$\hat{C}_L$ equilibrium lift coefficient

C_m pitching-moment coefficient, $\dfrac{M}{\dfrac{\rho V^2}{2} Sc}$

$C_{m,o,w}$ wing-body pitching-moment coefficient at zero lift with flaps deflected in approach condition

C_X longitudinal-force coefficient, $\dfrac{X}{\dfrac{\rho V^2}{2} S}$

C_Z vertical-force coefficient, $\dfrac{Z}{\dfrac{\rho V^2}{2} S}$

$\hat{C}_Z$ equilibrium vertical-force coefficient

c mean aerodynamic chord of wing

D differential operator, $\dfrac{d}{ds}$

F ratio of elevator deflection to pitch angle

G	ratio of elevator deflection to $D\theta$
K	ratio of flap deflection to vane deflection
K_y	ratio of radius of gyration to chord, k_y/c
k_y	radius of gyration in pitch
L	lift
ℓ	ratio of tail length to chord
ℓ_n	ratio of distance between vane and center of gravity to chord
M	pitching moment
M_s	spring moment on vane
m	mass of airplane
m'	ratio of flap deflection to elevator deflection with vane fixed
P	period, sec
q	nondimensional pitching velocity, $\dot{\theta}c/2V$
S	wing area
s	distance traveled in chord lengths, $\frac{V}{c}t$
$T_{1/2}$	time for oscillation to damp to one-half amplitude, sec
T_2	time for oscillation to double amplitude, sec
t	time
u'	increment of horizontal velocity

u	nondimensional horizontal velocity increment, u'/V
u_g'	horizontal gust velocity
u_g	nondimensional horizontal gust velocity, u_g'/V
V	true velocity with respect to undisturbed air mass
W	weight
w_g'	vertical gust velocity
w_o'	increment in vertical velocity with respect to undisturbed air mass
X	longitudinal force, positive forward
Z	vertical force, positive downward
α	increment in angle of attack
α_o	increment in angle of attack with respect to undisturbed air mass, $\frac{w_o'}{V}$ or $\theta - \gamma$
$\hat{\alpha}$	equilibrium angle of attack, $\hat{\theta} - \hat{\gamma}$
γ	increment in flight-path angle
$\hat{\gamma}$	equilibrium flight-path angle
δ_e	increment in elevator angle from equilibrium position
δ_f	increment in flap deflection from equilibrium position
δ_v	increment in vane angle from equilibrium position
δ_{v_u}	variation of vane angle with horizontal velocity, $\frac{\partial \delta_v}{\partial u}$
δ_{v_α}	variation of vane angle with angle of attack, $\frac{\partial \delta_v}{\partial \alpha}$

ϵ increment in downwash angle

ζ damping ratio

θ increment in pitch angle

$\hat{\theta}$ equilibrium pitch angle

μ relative density factor, $m/\rho Sc$

ρ air density

τ' time constant of servomechanism

τ nondimensional time constant of servomechanism, $\frac{V}{c}\tau'$, chords; or time constant of aperiodic mode, chords

τ_θ nondimensional time constant of servomechanism for pitch stability augmentation system, chords

ω frequency, rad/chord

Subscripts:

b value for basic airplane

g gust

o with respect to inertial axes fixed in undisturbed air mass

t tail

v vane

w wing-fuselage combination

Dot over quantity indicates differentiation with respect to time.

Stability derivatives are indicated by subscript notation; for example,

$$Z_{\alpha_W} = \frac{\partial Z}{\partial \alpha_W}$$

$$C_{Z_{\alpha_W}} = \frac{\partial C_Z}{\partial \alpha_W}$$

$$C_{Z_{Du}} = \frac{\partial C_Z}{\partial \frac{\dot{u}c}{2V}}$$

$$C_{Z_q} = \frac{\partial C_Z}{\partial \frac{\dot{\theta}c}{2V}}$$

Subscript after a stability derivative indicates component of airplane which contributes the derivative; for example, $\left(C_{Z_{\delta_e}}\right)_t$.

THEORETICAL ANALYSIS

The relative importance of vertical and horizontal gusts in producing changes in wing lift may be obtained from the relation

$$\Delta L = \left[\frac{w_g'}{V} C_{L_\alpha} + \frac{u_g'}{V}(2C_L)\right]\frac{\rho V^2}{2} S$$

Thus, the ratio of the incremental lift for horizontal and vertical gusts is

$$\frac{\Delta L_H}{\Delta L_V} = \frac{2C_L}{C_{L_\alpha}}$$

The mean-square values of the horizontal and vertical gust velocities are expected to be about equal. The ratio $\Delta L_H/\Delta L_V$ in cruising flight may therefore be in the range from

0.1 to 0.25. In the landing-approach condition, however, the value may exceed 1.0. Design of gust-alleviation systems for STOL airplanes should therefore include consideration of horizontal gusts.

Description of System

A sketch of the system under consideration is given in figure 1. The spring-loaded vane, which is discussed in detail subsequently, is sensitive to both horizontal and vertical gusts. The output of this vane operates the flaps through a servomechanism. The flaps are assumed to be capable of upward and downward motions from their initial deflected position. If necessary, gearing other controls to the flaps to modify their characteristics is considered within the scope of the analysis. Such controls might include the elevators, spoilers, drag brakes, or engine nozzles. In fact, one of these types of controls, such as spoilers, used alone might prove satisfactory. In this case, the same analysis can be applied by substituting the symbols and aerodynamic characteristics appropriate to spoiler control in place of those for flap control. The pilot's control input is also fed into the flap control system to provide direct lift control, inasmuch as the alleviation system prevents generation of lift for maneuvering through changes in angle of attack.

Equations of Motion

The longitudinal equations of motion are written first in dimensional form, considering separately the contributions of the wing and tail. The equations of motion are derived with respect to wind axes. The wing flaps are assumed to be available as controls. The axis system and angles used are given in figure 2. The equations are as follows:

$$m\dot{u}_o' = u_w' X_{u_w'} + u_t' X_{u_t'} + \alpha_w X_{\alpha_w} + \alpha_t X_{\alpha_t} + \delta_f X_{\delta_f} + \theta X_\theta$$

$$m(\dot{w}_o' - V\dot{\theta}) = u_w' Z_{u_w'} + u_t' Z_{u_t'} + \alpha_w Z_{\alpha_w} + \alpha_t Z_{\alpha_t} + \delta_f Z_{\delta_f} + \theta Z_\theta$$

$$mk_y^2 \ddot{\theta} = u_w' M_{u_w'} + u_t' M_{u_t'} + \alpha_w M_{\alpha_w} + \alpha_t M_{\alpha_t} + \delta_f M_{\delta_f} + \theta M_\theta$$

To nondimensionalize the equations, the following substitutions are made:

$$s = \frac{V}{c} t \qquad K_y = \frac{k_y}{c} \qquad u = \frac{u'}{V}$$

$$D = \frac{d}{ds} = \frac{c}{V}\frac{d}{dt} \qquad \mu = \frac{m}{\rho Sc}$$

$$C_X = \frac{X}{\frac{\rho V^2}{2} S} \qquad C_Z = \frac{Z}{\frac{\rho V^2}{2} S} \qquad C_m = \frac{M}{\frac{\rho V^2}{2} Sc}$$

The equations then become

$$2\mu Du_o = u_w C_{X_{u_w}} + u_t C_{X_{u_t}} + \alpha_w C_{X_{\alpha_w}} + \alpha_t C_{X_{\alpha_t}} + \delta_f C_{X_{\delta_f}} + \theta C_{X_\theta}$$

$$2\mu\left(D\frac{w'_o}{V} - D\theta\right) = u_w C_{Z_{u_w}} + u_t C_{Z_{u_t}} + \alpha_w C_{Z_{\alpha_w}} + \alpha_t C_{Z_{\alpha_t}} + \delta_f C_{Z_{\delta_f}} + \theta C_{Z_\theta}$$

$$2\mu K_y^2 D^2\theta = u_w C_{m_{u_w}} + u_t C_{m_{u_t}} + \alpha_w C_{m_{\alpha_w}} + \alpha_t C_{m_{\alpha_t}} + \delta_f C_{m_{\delta_f}} + \theta C_{m_\theta}$$

In the subsequent analysis, all variables are expressed in terms of u_o, u_g, α_o, α_g, and θ and their derivatives. Lag or lead in the occurrence of a velocity increment with respect to the motion at the wing is expressed by use of Laplace transform notation. The required relations are as follows:

$$u_w = u_o + u_g$$

$$\alpha_o = \frac{w'_o}{V} \qquad \alpha_g = \frac{w'_g}{V}$$

$$\alpha_w = \alpha_o + \alpha_g$$

$$u_t = u_o + u_g e^{-\ell D}$$

$$\alpha_t = \alpha_o - \alpha_o \frac{\partial\epsilon}{\partial\alpha} e^{-\ell D} + \alpha_g e^{-\ell D} - \alpha_g \frac{\partial\epsilon}{\partial\alpha} e^{-\ell D} - \delta_f \frac{\partial\epsilon}{\partial\delta_f} e^{-\ell D} + \ell D\theta$$

A vane sensor which is sensitive to the vertical and horizontal increments of velocity at the vane location is assumed to be mounted ahead of the nose. The velocities at the vane are given by the expressions

$$u_v = u_o + u_g e^{\ell_n D}$$

$$\alpha_v = \alpha_o + \alpha_g e^{\ell_n D} - D\theta\ell_n$$

The vane is assumed to respond to these velocity increments with no lag. A control law is now introduced which relates the motion of the wing flaps to the motion of the vane, with a constant time lag approximating the lag of the servomechanism which actuates the flaps:

$$\delta_f = K\delta_v e^{-\tau D}$$

where

$$\delta_v = \delta_{v_u} u_v + \delta_{v_\alpha} \alpha_v$$

It will be shown subsequently that for the type of vane considered, $\delta_{v_\alpha} = -1$. Hence

$$\delta_f = K(\delta_{v_u} u_v - \alpha_v)e^{-\tau D}$$

The lag or lead terms ℓ, ℓ_n, and τ are expressed in chords. These quantities may be converted to dimensional quantities in seconds by multiplying them by the ratio c/V.

In the subsequent examples, the elevators, spoilers, or drag devices may be assumed to be geared directly to the wing flaps. For purposes of analysis, the effect of these devices may be taken as a modification of the flap derivatives, inasmuch as their motion is in phase with that of the flaps.

The values for u_v and α_v may be substituted in the expression for δ_f, which is in turn substituted in the expression for α_t. All expressions for lag or lead are then represented by the linearized approximation:

$$e^{\tau D} = 1 + \tau D$$

or

$$e^{-\tau D} = 1 - \tau D$$

Discussion of the effect of this approximation is given in reference 1. The expressions for u_w, u_t, α_w, α_t, and δ_f are substituted in the equations of motion. After combining terms multiplying each variable or its derivatives, the equations of motion may be written in the following form:

$$2\mu Du - uC_{X_u} - \frac{1}{2} DuC_{X_{Du}} - \alpha_o C_{X_{\alpha_o}} - \frac{1}{2} D\alpha_o C_{X_{D\alpha_o}} - \theta C_{X_\theta} - \frac{1}{2} D\theta C_{X_q}$$

$$- \frac{1}{4} D^2\theta C_{X_{D^2\theta}} = \alpha_g C_{X_{\alpha_g}} + \frac{1}{2} D \alpha_g C_{X_{D\alpha_g}} + u_g C_{X_{u_g}} + \frac{1}{2} Du_g C_{X_{Du_g}} \quad (1a)$$

$$2\mu(D\alpha_o - D\theta) - uC_{Z_u} - \frac{1}{2} DuC_{Z_{Du}} - \alpha_o C_{Z_{\alpha_o}} - \frac{1}{2} D\alpha_o C_{Z_{D\alpha_o}} - \theta C_{Z_\theta} - \frac{1}{2} D\theta C_{Z_q}$$

$$- \frac{1}{4} D^2\theta C_{Z_{D^2\theta}} = \alpha_g C_{Z_{\alpha_g}} + \frac{1}{2} D\alpha_g C_{Z_{D\alpha_g}} + u_g C_{Z_{u_g}} + \frac{1}{2} Du_g C_{Z_{Du_g}} \quad (1b)$$

$$2\mu K_y^2 D^2\theta - uC_{m_u} - \frac{1}{2} DuC_{m_{Du}} - \alpha_o C_{m_{\alpha_o}} - \frac{1}{2} D\alpha_o C_{m_{D\alpha_o}} - \theta C_{m_\theta} - \frac{1}{2} D\theta C_{m_q}$$

$$- \frac{1}{4} D^2\theta C_{m_{D^2\theta}} = \alpha_g C_{m_{\alpha_g}} + \frac{1}{2} D\alpha_g C_{m_{D\alpha_g}} + u_g C_{m_{u_g}} + \frac{1}{2} Du_g C_{m_{Du_g}} \quad (1c)$$

The C_Z derivatives are given as follows:

$$C_{Z_u} = C_{Z_{u_g}} = C_{Z_{u_w}} + C_{Z_{u_t}} - K\delta_{v_u} \frac{\partial\epsilon}{\partial\delta_f} C_{Z_{\alpha_t}} + K\delta_{v_u} C_{Z_{\delta_f}} \quad (2a)$$

$$\frac{1}{2} C_{Z_{Du}} = K\delta_{v_u} \frac{\partial\epsilon}{\partial\delta_f} (\ell + \tau) C_{Z_{\alpha_t}} - K\delta_{v_u} \tau C_{Z_{\delta_f}} \quad (2b)$$

$$C_{Z_{\alpha_o}} = C_{Z_{\alpha_g}} = C_{Z_{\alpha_w}} + C_{Z_{\alpha_t}} \left(1 - \frac{\partial\epsilon}{\partial\alpha} + K \frac{\partial\epsilon}{\partial\delta_f}\right) - KC_{Z_{\delta_f}} \quad (2c)$$

$$\frac{1}{2} C_{Z_{D\alpha_o}} = \left[\ell \frac{\partial\epsilon}{\partial\alpha} - K \frac{\partial\epsilon}{\partial\delta_f} (\tau + \ell)\right] C_{Z_{\alpha_t}} + K\tau C_{Z_{\delta_f}} \quad (2d)$$

$$\frac{1}{2} C_{Z_q} = \left(\ell - K\ell_n \frac{\partial \epsilon}{\partial \delta_f}\right) C_{Z_{\alpha_t}} + K\ell_n C_{Z_{\delta_f}} \tag{2e}$$

$$\frac{1}{4} C_{Z_{D^2\theta}} = K\ell_n \frac{\partial \epsilon}{\partial \delta_f} (\tau + \ell) C_{Z_{\alpha_t}} - K\tau\ell_n C_{Z_{\delta_f}} \tag{2f}$$

$$\frac{1}{2} C_{Z_{D\alpha_g}} = \frac{1}{2} C_{Z_{D\alpha}} - \frac{1}{2} C_{Z_q} = \left[-\ell\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) + K \frac{\partial \epsilon}{\partial \delta_f} (\ell_n - \tau - \ell)\right] C_{Z_{\alpha_t}} - K(\ell_n - \tau) C_{Z_{\delta_f}} \tag{2g}$$

$$\frac{1}{2} C_{Z_{Du_g}} = -\ell C_{Z_{u_t}} - K\delta_{v_u} \frac{\partial \epsilon}{\partial \delta_f} (\ell_n - \tau - \ell) C_{Z_{\alpha_t}} + K\delta_{v_u} (\ell_n - \tau) C_{Z_{\delta_f}} \tag{2h}$$

The C_X and C_m derivatives can be obtained by substituting X and M for Z wherever it occurs.

The θ derivatives come from the change in the components of gravity forces acting on the airplane resolved along the wind axes resulting from an increment in θ. The expressions for these derivatives are as follows:

$$X_\theta = W \cos \hat{\gamma}$$

$$C_{X_\theta} = -\hat{C}_L \cos \hat{\gamma}$$

$$Z_\theta = W \sin \hat{\gamma}$$

$$C_{Z_\theta} = -\hat{C}_L \sin \hat{\gamma}$$

$$M_\theta = C_{m_\theta} = 0$$

Effect of Feedback of Pitch Angle and Pitch Rate to the Elevator

A simple stability augmentation system sensitive to pitch angle and pitch rate is considered in the subsequent analysis. The effect of this system may be analyzed, to the same degree of approximation as that used in the preceding derivation, by assuming the control law:

$$\delta_e = (F\,\theta + GD\theta)(1 - \tau_\theta D)$$

The nondimensional gain G relating δ_e to $D\theta$ may be converted to a dimensional gain G' relating δ_e to $\dot{\theta}$ by the formula

$$G' = G \frac{c}{V}$$

Likewise, the lag τ_θ expressed in chords may be converted to a lag τ'_θ in seconds by the formula

$$\tau'_\theta = \tau_\theta \frac{c}{V}$$

The increments to the C_Z stability derivatives provided by this stability augmentation system are as follows:

$$\Delta C_{Z_\theta} = F \left(C_{Z_{\delta_e}} \right)_t$$

$$\Delta C_{Z_{D\theta}} = 2\,(G - \tau_\theta F) \left(C_{Z_{\delta_e}} \right)_t$$

$$\Delta C_{Z_{D^2\theta}} = -4G\,\tau_\theta \left(C_{Z_{\delta_e}} \right)_t$$

Again, the increments to the C_X and C_m derivatives can be obtained by substituting X and m for Z wherever it occurs in these expressions.

Vane Characteristics and Control-System Parameters Required for Gust Alleviation

The forcing terms for the complete three degrees of freedom which are associated with gust effects may be collected from equations (1) as follows:

$$\alpha_g C_{X_\alpha} + \frac{1}{2} D\alpha_g \left(C_{X_{D\alpha}} - C_{X_q} \right) + u_g C_{X_u} + \frac{1}{2} Du_g C_{X_{Du_g}} \tag{3a}$$

$$\alpha_g C_{Z_\alpha} + \frac{1}{2} D\alpha_g \left(C_{Z_{D\alpha}} - C_{Z_q} \right) + u_g C_{Z_u} + \frac{1}{2} Du_g C_{Z_{Du_g}} \tag{3b}$$

$$\underline{\alpha_g C_{m_\alpha}} + \frac{1}{2}\, \underline{D\alpha_g \left(C_{m_{D\alpha}} - C_{m_q}\right)} + u_g C_{m_u} + \frac{1}{2}\, \underline{\underline{Du_g C_{m_{Du_g}}}} \tag{3c}$$

The response to vertical and horizontal gusts may be eliminated by reducing the coefficients of all these terms to zero. It is desirable, therefore, to determine how many of these terms may be reduced to zero by use of the system under consideration.

The response to vertical gusts under conditions of constant airspeed may be eliminated if the single-underlined terms are reduced to zero. This condition may be provided if the following relations, derived in reference 1, are satisfied:

$$K = \frac{C_{Z_{\alpha_w}}}{C_{Z_{\delta_f}}} \qquad \tau = \ell_n \tag{4a}$$

$$C_{m_{\delta_f}} = \frac{C_{m_{\alpha_w}}}{K} \tag{4b}$$

$$\frac{\partial \epsilon}{\partial \delta_f} = \frac{-\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)}{K} \tag{4c}$$

This result may be verified by substituting these relations in the α_g and $D\alpha_g$ terms of expressions (3) and using the values of the derivatives given in equations (2).

The required value of $C_{m_{\delta_f}}$ may be obtained with little performance penalty by gearing the elevator to move in phase with the flaps. The required value of $\frac{\partial \epsilon}{\partial \delta_f}$, however, is opposite in sign from that normally associated with wing flaps. In the research flight program described in reference 2, the reversed sign of $\frac{\partial \epsilon}{\partial \delta_f}$ was obtained by gearing an inboard segment of the flaps to move in the opposite direction from the flaps covering the remainder of the wing span. This method reduces the overall lift effectiveness of the flaps and might be undesirable for a STOL airplane in which flap deflection is required for improved maximum lift coefficient as well as for gust alleviation. Nevertheless, the ability to adjust $\frac{\partial \epsilon}{\partial \delta_f}$ to the desired value will be assumed in the present analysis.

The coefficient of the double-underlined term in expression (3b), C_{Z_u}, may be reduced to zero by use of a vane with the correct value of δ_{v_u} in conjunction with the values of K and $\frac{\partial \epsilon}{\partial \delta_f}$ given in equations (4a) and (4c). By setting the expression for this coefficient (eq. (2a)) equal to zero, the required value of δ_{v_u} is found to be

$$\delta_{v_u} = -\frac{\left(C_{Z_u}\right)_b}{\left(C_{Z_\alpha}\right)_b} \tag{5}$$

where the values of $\left(C_{Z_u}\right)_b$ and $\left(C_{Z_\alpha}\right)_b$ represent values for the basic airplane before the addition of the gust-alleviation system.

The required value of δ_{v_u} may be obtained by use of a suitable spring moment on the vane, tending to make it float down. The vane will then move up in response to either a rearward horizontal or an upward vertical gust. Consider the vane subject to velocity and angle-of-attack variations. The spring moment on the vane is equated to the aerodynamic moments by the relation

$$M_s = (\delta_v + \alpha)C_{m_{\delta_v}} \frac{\rho V^2}{2} S_v c_v$$

whence

$$\delta_v = \frac{M_s}{C_{m_{\delta_v}} \frac{\rho V^2}{2} S_v c_v} - \alpha$$

The spring moment is assumed not to vary with vane deflection.

The variations of vane angle with angle of attack and with velocity are then

$$\frac{\partial \delta_v}{\partial \alpha} = \delta_{v_\alpha} = -1$$

$$\frac{\partial \delta_v}{\partial u} = \delta_{v_u} = \frac{-2\ M_s}{C_{m_{\delta_v}} \frac{\rho V^2}{2} S_v c_v} = -\frac{\left(C_{Z_u}\right)_b}{\left(C_{Z_\alpha}\right)_b} \tag{6}$$

Equation (6) may be solved for M_s as follows:

$$M_s = \frac{\left(C_{Z_u}\right)_b C_{m_{\delta_v}} \frac{\rho V^2}{2} S_v c_v}{2 \left(C_{Z_\alpha}\right)_b}$$

This relation establishes the required strength of the spring moment on the vane, tending to make it float down.

The preceding formulas were derived on the assumption that the variation of lift with velocity is approximated by a linear relation for small changes in speed from the trim condition. This type of analysis might be required for a STOL airplane, for which the power effects on $\left(C_{Z_u}\right)_b$ are large. If the lift at a given angle of attack varies as V^2, as it does in the case of a rigid airplane with negligible slipstream effects, then the value of $\left(C_{Z_u}\right)_b$ equals $2\ \hat{C}_Z$, where $\hat{C}_Z$ is the equilibrium value of the vertical-force coefficient. The value of the spring moment is then

$$M_s = \frac{\hat{C}_Z C_{m_{\delta_v}} \frac{\rho V^2}{2} S_v c_v}{\left(C_{Z_\alpha}\right)_b}$$

Inasmuch as $\hat{C}_Z$ varies inversely as V^2, the spring moment required for correct alleviation of vertical and horizontal gusts is then constant throughout the speed range.

If one additional condition is specified, the triple-underlined terms of expressions (3b) and (3c) are also reduced to zero. This condition is

$$C_{Z_{u_t}} = \frac{\left(C_{Z_u}\right)_b}{\left(C_{Z_\alpha}\right)_b}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)C_{Z_{\alpha\,t}} = -\delta_{v_u}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)C_{Z_{\alpha\,t}} \tag{7}$$

The value of $C_{Z_{u_t}}$ depends on the tail load for trim, which the designer may control by varying the value of pitching moment of the wing-flap combination about the moment reference point or by varying the center-of-gravity location. Even if all these conditions are satisfied, however, the important terms $u_g C_{m_u}$ and all the forcing terms in the longitudinal force equations remain in the equations and are changed but not eliminated by the alleviation system.

NUMERICAL EXAMPLES

In order to determine the importance of the forcing terms remaining in the equations of motion, a number of examples have been studied utilizing the characteristics of a STOL airplane with externally blown flaps in the landing-approach condition. The parameters were estimated from reference 3.

Because the effects of some variables, such as flap deflection, were based on wind-tunnel measurements at only two deflections, these parameters are only approximate. The values are believed, nevertheless, to incorporate the main characteristics which show marked differences from those of a conventional airplane. These differences include, for example, large effects of power on the derivatives $\left(C_{Z_u}\right)_b$ and $\left(C_{m_u}\right)_b$ and large changes in drag due to flap deflection. The physical characteristics and derivatives for the basic airplane are given in table I. The stability and gust-response characteristics of the basic airplane and of the airplane with a gust-alleviation system of the type analyzed have been studied by calculating the roots of the characteristic equations, the coefficients of the forcing terms caused by gust inputs, and the transient responses to step vertical and horizontal gust inputs and to pilot control inputs. All calculations were made with a high-speed digital computer. These results are presented for six cases, as follows:

<table>
<tr><th></th><th>Response to gust inputs</th><th>Response to pilot control inputs</th></tr>
<tr><td>Case 1: Basic airplane; c.g. at 0.33c</td><td rowspan="2">Figure 3</td><td rowspan="2">Figure 6</td></tr>
<tr><td>Case 2: Alleviated airplane; c.g. at 0.33c</td></tr>
<tr><td>Case 3: Basic airplane; c.g. at 0.594c</td><td rowspan="2">Figure 4</td><td rowspan="2">Figure 7</td></tr>
<tr><td>Case 4: Alleviated airplane; c.g. at 0.594c</td></tr>
<tr><td>Case 5: Alleviated airplane; c.g. at 0.594c; $C_{X_{\delta_f}} = 0$</td><td rowspan="2">Figure 5</td><td rowspan="2">Figure 8</td></tr>
<tr><td>Case 6: Alleviated airplane with pitch autopilot; c.g. at 0.594c; $C_{X_{\delta_f}} = 0$</td></tr>
</table>

For the alleviated airplanes of cases 2, 4, and 5, the derivatives are the effective values as determined from equations (2), taking into account the contributions of the alleviation system. Values of the gain K, the vane floating tendency δ_{v_u}, and the lag τ are determined from equations (4a) and (5). The values of $C_{m_{\delta_f}}$ and $\frac{\partial \epsilon}{\partial \delta_f}$ are selected to satisfy equations (4b) and (4c). As a result, the single- and double-underlined terms in expressions (3b) and (3c) are eliminated, except for the effects of slight numerical errors. No attempt has been made to meet the condition of equation (7) which reduces the triple-underlined terms of expressions (3b) and (3c) to zero. An exception is made in case 6, in which the values of $C_{m_{\delta_f}}$ and $\frac{\partial \epsilon}{\partial \delta_f}$ are selected to give a small stable value of C_{m_α} of -0.45 rather than zero. The values of these parameters are listed in table II. Values of the roots of the characteristic equation for the six cases are given in table III. Values are given for the characteristic time in chords of real roots and for the damping ratio and frequency in radians per chord of oscillatory modes. In addition, the values are presented as time to one-half or double amplitude and period in seconds for the assumed flight speed of 32.61 meters per second (107 feet per second).

Values of the derivatives in the forcing terms on the right-hand side of the equations of motion (eqs. (1)) are given in table IV.

Transient responses for step inputs of vertical and horizontal gust velocity are given in figure 3 for the airplane with and without the gust-alleviation system for a center-of-gravity location at 0.33c and in figure 4 for the center-of-gravity location at 0.594c. Similar responses for the center-of-gravity location at 0.594c with two types of control systems incorporating a value of $C_{X_{\delta_f}}$ of 0 are shown in figure 5. The magnitude of the gust velocity for both vertical and horizontal gusts is 0.570 meter per second (1.87 feet per second). At the flight velocity of 32.61 meters per second, a vertical gust of this magnitude produces a change in flow angle of 1^{o}. The changes in angle of attack α_0 and velocity V are with respect to inertial axes fixed in the air mass and do not include the increment of gust velocity.

Transient response to step elevator deflections of 1^{o} are shown in figures 6, 7, and 8 for the same conditions as those for the gust inputs. In the cases with gust-alleviation systems, the flaps are operated as a function of the longitudinal control input to provide direct lift control in the manner shown in figure 1. The elevator is assumed to respond without lag to the pilot's control input, whereas the flap is driven through a servomechanism with a lag τ used in the gust-response calculations. With the vane fixed, the steady-state relation between the flap and elevator deflection is given by

$$\delta_f = m' \delta_e$$

The value of m' is taken to be -2.18 in all the six cases studied. The value of δ_e in this formula is the portion of the elevator deflection resulting from direct linkage to the control stick and does not include the portion resulting from linkage to the flap to adjust the flap pitching moments.

RESULTS AND DISCUSSION

An overall picture of the effects of the systems investigated can be obtained from the transient response calculations (figs. 3, 4, and 5). These responses were calculated for relatively small gust inputs; however, because of the linearity of the analysis, they may be multiplied by the appropriate factor to correspond to a larger input. The results for the basic airplane show that with the step gust inputs assumed, the changes in airplane acceleration are quite small and the need for an alleviation system for this STOL-type airplane in the landing-approach condition might be questioned. In landings made in high winds, however, particularly in the wake of buildings or other obstructions, severe turbulence may be encountered so that the resulting airplane accelerations may be objectionable. For this condition, the possibility of exceeding the range of validity of the linearized analysis must be kept in mind. The linearized analysis, however, is useful as a first step in showing the required characteristics of a gust-alleviation system.

For the forward center-of-gravity location (fig. 3), the alleviation system decreases the initial normal accelerations when the airplane encounters gusts but greatly increases the response of the poorly damped phugoid mode. The alleviation system also causes an initial response in the negative direction for horizontal gusts. For the rearward center-of-gravity location (fig. 4), the alleviation system again decreases the critical normal accelerations. The subsequent motion is slowly divergent. When the alleviation system is operating, both vertical and horizontal gusts cause an increase in forward inertial speed. This horizontal acceleration is a result of the decrease in drag when the flap is deflected upward. The initial horizontal acceleration due to the vertical gust is about 60 percent of the initial vertical acceleration of the unalleviated airplane. Such large horizontal accelerations due to gusts would probably be quite objectionable to the passengers.

In order to reduce the horizontal accelerations, a case was tried, again for the rearward center-of-gravity location, with the value of $C_{X_{\delta_f}}$ reduced to zero (case 5). This condition could possibly be obtained by gearing the flaps to drag devices or spoilers, so that the drag devices extend when the flaps move up. Alternatively, spoilers alone, trimmed up in the equilibrium approach condition, might provide the desired changes in lift with small changes in drag. The other flap derivatives $C_{Z_{\delta_f}}$ and $C_{m_{\delta_f}}$ were assumed to be unaffected when $C_{X_{\delta_f}}$ was reduced to zero. Although in practice these derivatives would be affected, an alleviation system equivalent to that assumed could always be attained, at least for some range of flap deflections, by modifying suitably the gearing between the flaps and elevators and by recalculating the flap gearing and downwash parameters. Because aerodynamic data for a suitable drag device were not available, the details of such an arrangement were not investigated. As shown in figure 5, the responses to the vertical and horizontal gusts for this case are almost eliminated.

An explanation of the behavior shown by the transient responses may be found by examining the roots of the stability equations and the forcing terms. Consider first the roots (table III). For the basic airplane with a forward center of gravity (case 1), for which the static margin is 0.277c, the normal short period and phugoid oscillation modes are encountered. The phugoid oscillation has poor damping and a period somewhat shorter than expected for this flight speed. The positive value of C_{m_u} tends to shorten the period. For the basic airplane with a rearward center of gravity (case 3), for which the static margin is 0.042c, the short period mode becomes a pair of rapid subsidences and the phugoid period increases considerably.

When the alleviation system is operating, the airplane has one real root with a very long time constant; this indicates a condition of near-neutral stability. This neutral stability is due to the choice of parameters to eliminate the gust forcing terms $C_{Z_{\alpha_g}}$ and $C_{m_{\alpha_g}}$, which are also equal to the values of C_{Z_α} and C_{m_α} associated with airplane motion. In addition, when the center of gravity of the airplane is at the rearward location, the phugoid mode is divergent and has a very long period. In practice, the pilot would probably interpret this motion as a straight divergence in airspeed.

The amount of disturbance of the airplane, even if it is neutrally stable or unstable, depends on the magnitude of the gust forcing terms given in table IV. The values of $C_{Z_{\alpha_g}}$, $C_{m_{\alpha_g}}$, $C_{Z_{D\alpha_g}}$, and $C_{m_{D\alpha_g}}$ are reduced to negligible values by the alleviation system. At the forward center of gravity (case 2), the value of $C_{m_{u_g}}$ is increased by the gust-alleviation system and is responsible for the large disturbance of the phugoid mode encountered in this case. At the rearward center of gravity (case 4), however, the value of $C_{m_{u_g}}$ is greatly reduced. The value of this derivative, as shown by equation (2a), depends on the value of C_{m_u} for the basic airplane and on the value of $C_{m_{\delta_f}}$. The positive value of C_{m_u} for the basic airplane with the forward center-of-gravity location may be explained as follows. The negative pitching moment of the wing with deflected flaps increases less rapidly than the square of the speed because a large part of the moment comes from the slipstream, whereas the balancing tail pitching moment varies as the square of the speed. As the center of gravity is moved rearward, the balancing tail load required is decreased; this results in a smaller positive value of C_{m_u}. The value of $C_{m_{\delta_f}}$ is also greatly reduced as the center of gravity moves back to a location near the center of lift of the flaps. In the configuration under consideration, both of these effects are reduced to zero at a center of gravity near 0.594c, the rearward center-of-gravity location assumed in this study.

Despite the reduction in the value of C_{m_u}, other forcing terms remain to produce a response to gusts of the alleviated airplane with the rearward center-of-gravity location. In view of the increase in forward speed noted previously in this case, it was thought that the large positive value of $C_{X_{u_g}}$ was primarily responsible for the remaining disturbance. This value may be reduced by reducing the value of $C_{X_{\delta_f}}$. For the case shown in figure 5, with the value of $C_{X_{\delta_f}}$ arbitrarily reduced to zero, the responses to vertical and horizontal gusts are almost eliminated. Even in this case, some forcing terms remain in the equations. In particular, the value of C_{X_α} remains close to that of the basic airplane. This derivative, however, apparently has only a small effect on the response.

Although the gust disturbances on a STOL airplane may be greatly reduced by the techniques studied, the stability characteristics without additional stability augmentation appear unsatisfactory. The very large time for subsidence of one of the real roots indicates that the airplane is indifferent as to pitch attitude. In other words, with C_{m_α} and C_{L_α} near zero, the airplane is in equilibrium at any angle of attack and the lift is independent of angle of attack. Any slight application of elevator control would cause a constantly changing pitch attitude.

The response to pilot control inputs is of importance in the overall evaluation of a gust-alleviation system. In the alleviation systems studied, a rapid response in normal acceleration to a longitudinal control input is provided by the direct-lift control feature in which the flap system moves in response to a pilot control input. The subsequent motion, however, is affected by the stability characteristics of the alleviated airplane.

As shown in figure 6, the response of the basic airplane with the forward center-of-gravity location (static margin, 0.277c) to an up-elevator control deflection is weak, with the normal acceleration reversing after a few seconds because of the decrease in airspeed. A somewhat greater initial response is obtained with the alleviation system and direct-lift control operating, but the poor damping of the phugoid mode may lead to control difficulties.

The response of the basic airplane with the rearward center-of-gravity location (static margin, 0.042c) appears unduly sensitive because of the low static stability (fig. 7). The airplane continues to diverge upward in pitch after the normal acceleration reverses. Normally, an airplane of this type would not be flown with such rearward center of gravity without stability augmentation.

The rearward center-of-gravity location was tried, in this case, to obtain improved gust alleviation. As shown previously, the airplane with a large drag increment due to flap deflection experienced an undesirable divergence in airspeed due to a gust disturbance. The longitudinal control characteristics appear undesirable because of the unusually large instability of the phugoid mode. Because of the instability and the long period, this motion would probably be interpreted by the pilot as a straight divergence.

The alleviated airplane with $C_{X_{\delta_f}} = 0$ (fig. 8) has somewhat less objectionable characteristics. The phugoid mode is again unstable, but the rate of divergence is reduced. Because of the effectively zero values of C_{m_α} and C_{m_u} for the alleviated airplane, the airplane continues to pitch up at a fairly constant rate as long as the elevator control is applied. Another case, for which data are not presented, was studied with a value of C_{m_α} of -0.45 which corresponds to the value of C_{m_α} of the basic airplane at the rearward center-of-gravity location. The results were not appreciably changed.

The foregoing results indicate that although the gust disturbances on a STOL airplane may be greatly reduced by the techniques studied, the stability characteristics without additional stability augmentation appear unsatisfactory. In order to investigate the effects of stability augmentation, a simple system sensitive to pitch angle and pitch rate was added to the alleviated airplane with $C_{X_{\delta_f}} = 0$. The sum of these feedback quantities was added to the motion of the elevator, as indicated in figure 1. The effects of this type of control on the gust response are shown in figure 5 and on the response to a control input in figure 8.

The response to gust inputs is still essentially eliminated by the alleviation system with the pitch stability augmentation system (fig. 5). The response to a control input appears to be more desirable for this case than for the other cases studied (fig. 8). The pitch angle responds in the correct direction but does not increase at a constant rate and the reduction of airspeed is more gradual. The roots (table III) are all aperiodic convergences except for the near-zero real root, which is a very slow divergence.

In all the examples presented, increased normal acceleration is accompanied by a fairly rapid decrease in airspeed. This characteristic is typical of airplanes operating at high values of lift coefficient. The pilot would be required to coordinate throttle action correctly with elevator control to make a correction to the flight path. The application of control decoupling techniques to the gust-alleviation system studied would appear to provide a means of further improving the longitudinal control characteristics.

As stated previously, an internal sensor, such as an accelerometer, has practical advantages over a vane. The flap motions on encountering a gust with such a sensor are similar to those with the vane. It therefore appears desirable to employ flaps having aerodynamic characteristics which reduce the gust forcing function just as in the case of the vane sensor. The use of flaps having these characteristics was shown in reference 1 to be desirable for a transport-type airplane in the cruise condition with an accelerometer sensor. Such flaps, in addition to producing a change in lift, should have pitching-moment and downwash characteristics similar to those shown to be required with the vane sensor. In addition, the use of a rearward center-of-gravity location to reduce C_{m_u} and $C_{m_{\delta_f}}$ and the gearing of spoilers or drag brakes to the flaps to reduce $C_{X_{\delta_f}}$ would be desirable.

A rearward center-of-gravity location requiring stability augmentation to provide adequate longitudinal stability also appears advantageous from the performance standpoint. The down load on the tail required for trim, which is very large for STOL airplanes with large trailing-edge flaps, is thereby reduced or reversed; this results in the requirement for less tail area as well as less wing area and thrust for the same landing-approach speed.

CONCLUDING REMARKS

This study considers the alleviation of response to horizontal and vertical gusts of a STOL airplane utilizing a vane-type sensor which operates flaps and other controls through an automatic control system. The response to horizontal gusts is shown to be important for a STOL airplane in the landing-approach condition.

The complete elimination of gust forcing terms in the longitudinal response equations of an airplane, which is possible when horizontal gusts and speed changes are neglected, does not appear possible when these factors are taken into account. Nevertheless, the important forcing terms may be reduced to relatively small values by use of suitable flap aerodynamic characteristics, a rearward center-of-gravity location, and gearing of flaps to spoilers or drag devices to reduce longitudinal-force changes due to flap deflection.

The stability characteristics of the alleviated airplane appear unsatisfactory without the use of stability augmentation to provide pitch-attitude stabilization and improved damping of the phugoid motion.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., June 21, 1973.

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TABLE I.- AIRPLANE CHARACTERISTICS AND DERIVATIVES USED IN ANALYSIS

(a) Physical characteristics

Dimensional data:

W	245 444 N (55 178 lbf)
m	25 028 kg (1 715 slugs)
c	3.203 m (10.51 ft)
S	74.45 m^2 (801.36 ft^2)
S_t	33.17 m^2 (357.0 ft^2)
k_y	3.667 m (12.03 ft)
V	32.61 m/sec (107 ft/sec)
ρ	1.226 kg/m^3 (0.002378 $slug/ft^3$)

Nondimensional parameters:

ℓ	3.50
ℓ_n	4.09
A	7.22
μ	85.63
K_y	1.144
C_L	5.058
$C_{m,o,w}$	-2.32
$\partial\epsilon/\partial\delta_f$	0.202
$\partial\epsilon/\partial\alpha$	0.430

Aerodynamic center of wing-fuselage combination 0.213c

TABLE I.- AIRPLANE CHARACTERISTICS AND DERIVATIVES

USED IN ANALYSIS – Continued

(b) Contributions to stability derivatives of wing-fuselage combination and of tail

$\left(C_{X_\alpha}\right)_w$	0.920
$\left(C_{Z_\alpha}\right)_w$	-8.02
$\left(C_{X_{\dot\alpha}}\right)_t$	-0.494
$\left(C_{Z_{\dot\alpha}}\right)_t$	-1.72
$\left(C_{X_u}\right)_w$	-0.120
$\left(C_{Z_u}\right)_w$	-5.84
$\left(C_{X_u}\right)_t$	-0.620
$\left(C_{Z_u}\right)_t$	0.330
$\left(C_{X_{\delta_f}}\right)_w$	-1.92
$\left(C_{Z_{\delta_f}}\right)_w$	-4.30
$\left(C_{m_{\delta_f}}\right)_w$ (about aerodynamic center of wing-fuselage combination)	-1.83

TABLE I.- AIRPLANE CHARACTERISTICS AND DERIVATIVES

USED IN ANALYSIS – Concluded

(c) Stability derivatives of basic airplane

	c.g. at 0.33c	c.g. at 0.594c
C_{X_α}	0.638	0.856
C_{Z_α}	-9.00	-9.00
C_{m_α}	-2.49	-0.376
$\frac{1}{2}C_{X_{D\alpha}}$	-0.743	-0.169
$\frac{1}{2}C_{Z_{D\alpha}}$	-2.59	-2.59
$\frac{1}{2}C_{m_{D\alpha}}$	-9.06	-9.06
C_{X_θ}	-5.06	-5.06
C_{Z_θ}	0	0
C_{m_θ}	0	0
$\frac{1}{2}C_{X_q}$	-1.73	-0.393
$\frac{1}{2}C_{Z_q}$	-6.02	-6.02
$\frac{1}{2}C_{m_q}$	-21.1	-21.1
C_{X_u}	-0.740	-0.023
C_{Z_u}	-5.51	-5.64
C_{m_u}	1.86	-0.0809
$\frac{1}{2}C_{X_{Du}}$	0	0
$\frac{1}{2}C_{Z_{Du}}$	0	0
$\frac{1}{2}C_{m_{Du}}$	0	0

TABLE II.- VALUES OF GAINS AND PARAMETERS FOR ALLEVIATED AIRPLANES

Case	c.g.	K	δ_{v_u}	τ	m'	$C_{m_{\delta_f}}$	$\frac{\partial \epsilon}{\partial \delta_f}$
2	0.33c	1.86	-0.612	4.09	-2.18	-1.34	-0.306
4	0.594c	1.86	-0.627	4.09	-2.18	-0.202	-0.306
5	0.594c	1.86	-0.627	4.09	-2.18	-0.202	-0.306
[a] 6	0.594c	1.89	-0.612	4.09	-2.18	0.0386	-0.266

[a] In addition, for case 6, the parameters of the pitch stability augmentation system are: F = 1.00; G = 3.41; τ_θ = 0.50.

TABLE III.- ROOTS OF CHARACTERISTIC EQUATION FOR CASES STUDIED

Case	ζ and ω, rad/chord, for oscillatory modes or τ, chords, for aperiodic modes		$T_{1/2}$, sec, and P, sec, for oscillatory modes or $T_{1/2}$ or T_2, sec, for aperiodic modes	
1: Basic airplane; c.g. at 0.33c (static margin, 0.277c)	ζ_1 = 0.774 ζ_2 = 0.034	ω_1 = 0.119 ω_2 = 0.040	$T_{1/2}$ = 0.733 $T_{1/2}$ = 48.6	P = 5.20 P = 15.5
2: Alleviated airplane; c.g. at 0.33c	τ_1 = 4270 ζ = -0.046	τ_2 = 3.41 ω = 0.0307	$T_{1/2}$ = 296 T_2 = 48.2	$T_{1/2}$ = 0.234 P = 20.2
3: Basic airplane; c.g. at 0.594c (static margin, 0.042c)	τ_1 = 22.9 ζ = 0.089	τ_2 = 7.24 ω = 0.013	$T_{1/2}$ = 1.56 $T_{1/2}$ = 58.2	$T_{1/2}$ = 0.493 P = 47.6
4: Alleviated airplane; c.g. at 0.594c	τ_1 = 4515 ζ = -0.76	τ_2 = 5.24 ω= 0.00776	$T_{1/2}$ = 308 T_2 = 11.5	$T_{1/2}$ = 0.358 P = 79.8
5: Alleviated airplane; c.g. at 0.594c; $C_{X_{\delta_f}}$ = 0	τ_1 = 8220 ζ = -0.200	τ_2 = 5.03 ω = 0.010	$T_{1/2}$ = 562 T_2 = 32.3	$T_{1/2}$ = 0.344 P = 61.8
6: Alleviated airplane with pitch autopilot; c.g. at 0.594c; $C_{X_{\delta_f}}$ = 0	τ_1 = -1497 τ_3 = 10.55	τ_2 = 231 τ_4 = 5.28	T_2 = 102 $T_{1/2}$ = 0.718	$T_{1/2}$ = 15.78 $T_{1/2}$ = 0.360

TABLE IV.- VALUES OF DERIVATIVES IN FORCING TERMS OF EQUATIONS (1)

(a) Longitudinal-force terms

Case	$C_{X_{\alpha_g}}$	$\frac{1}{2} C_{X_{D\alpha_g}}$	$C_{X_{u_g}}$	$\frac{1}{2} C_{X_{Du_g}}$
1: Basic airplane; c.g. at 0.33c	0.638	0.985	-0.74	2.17
2: Alleviated airplane; c.g. at 0.33c	4.48	0.00145	1.61	1.57
3: Basic airplane; c.g. at 0.594c	0.856	0.224	-0.023	-0.339
4: Alleviated airplane; c.g. at 0.594c	4.48	0.00033	2.25	-0.479
5: Alleviated airplane; c.g. at 0.594c; $C_{X_{\delta_f}} = 0$	0.919	0.00033	0.0170	-0.479

(b) Vertical-force terms

Case	$C_{Z_{\alpha_g}}$	$\frac{1}{2} C_{Z_{D\alpha_g}}$	$C_{Z_{u_g}}$	$\frac{1}{2} C_{Z_{Du_g}}$
1: Basic airplane; c.g. at 0.33c	-9.00	3.43	-5.51	-1.15
2: Alleviated airplane; c.g. at 0.33c	-0.023	0.0050	-0.014	-3.25
3: Basic airplane; c.g. at 0.594c	-9.00	3.43	-5.64	-0.07
4: Alleviated airplane; c.g. at 0.594c	-0.023	0.0050	-0.014	-2.84
5: Alleviated airplane; c.g. at 0.594c; $C_{X_{\delta_f}} = 0$	-0.023	0.0050	-0.014	-2.84

TABLE IV.- VALUES OF DERIVATIVES IN FORCING TERMS

OF EQUATIONS (1) – Concluded

(c) Pitching-moment terms

Case	$C_{m_{\alpha_g}}$	$\frac{1}{2} C_{m_{D\alpha_g}}$	$C_{m_{u_g}}$	$\frac{1}{2} C_{m_{Du_g}}$
1: Basic airplane; c.g. at 0.33c	-2.49	12.0	1.86	-11.6
2: Alleviated airplane; c.g. at 0.33c	-0.00415	0.017	3.38	-19.0
3: Basic airplane; c.g. at 0.594c	-0.375	12.0	-0.081	- 2.45
4: Alleviated airplane; c.g. at 0.594c	0.0001	0.017	0.154	- 9.96
5: Alleviated airplane; c.g. at 0.594c; $C_{X_{\delta_f}} = 0$	0.0001	0.017	0.154	- 9.96

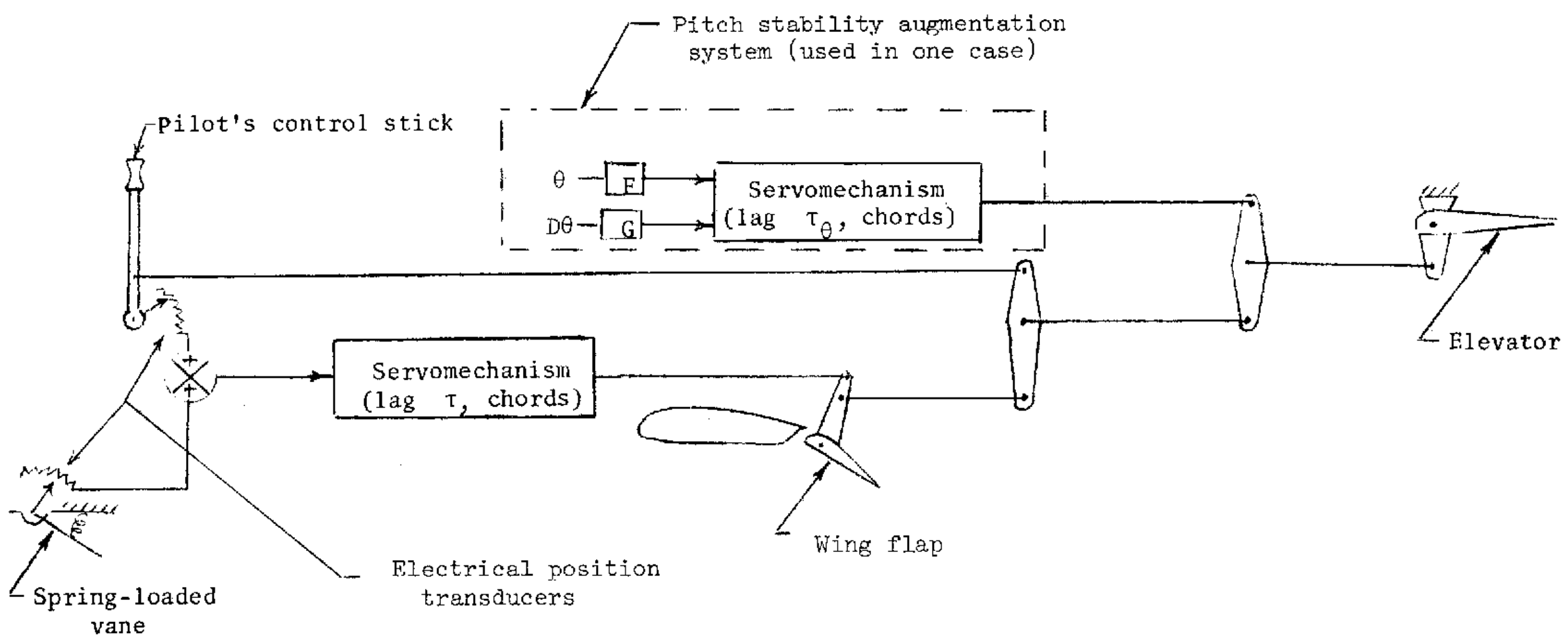

Figure 1.- Diagrammatic sketch of control system. (Linkages are not drawn to scale.)

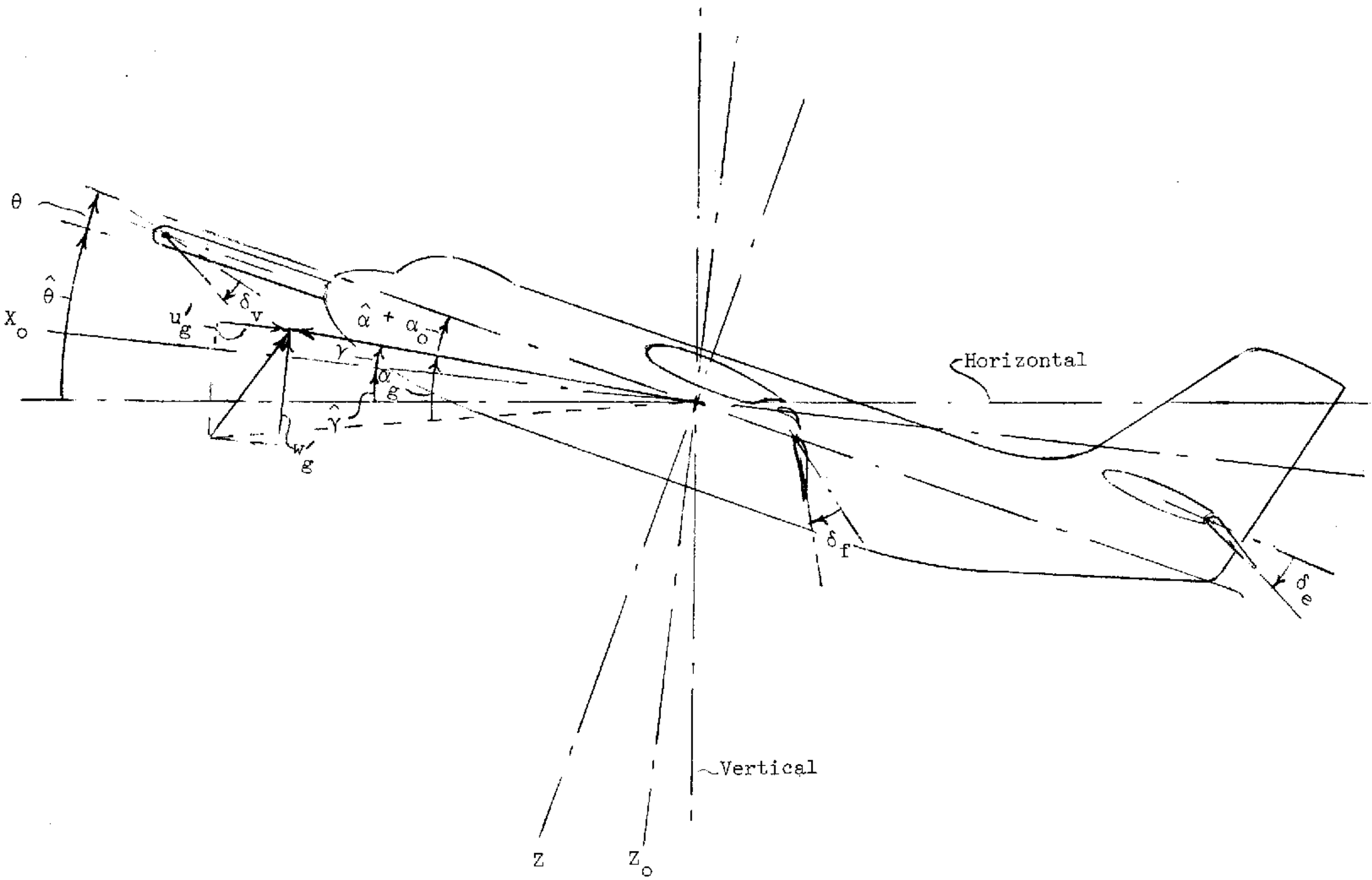

Figure 2.- Definition of axes and angles.

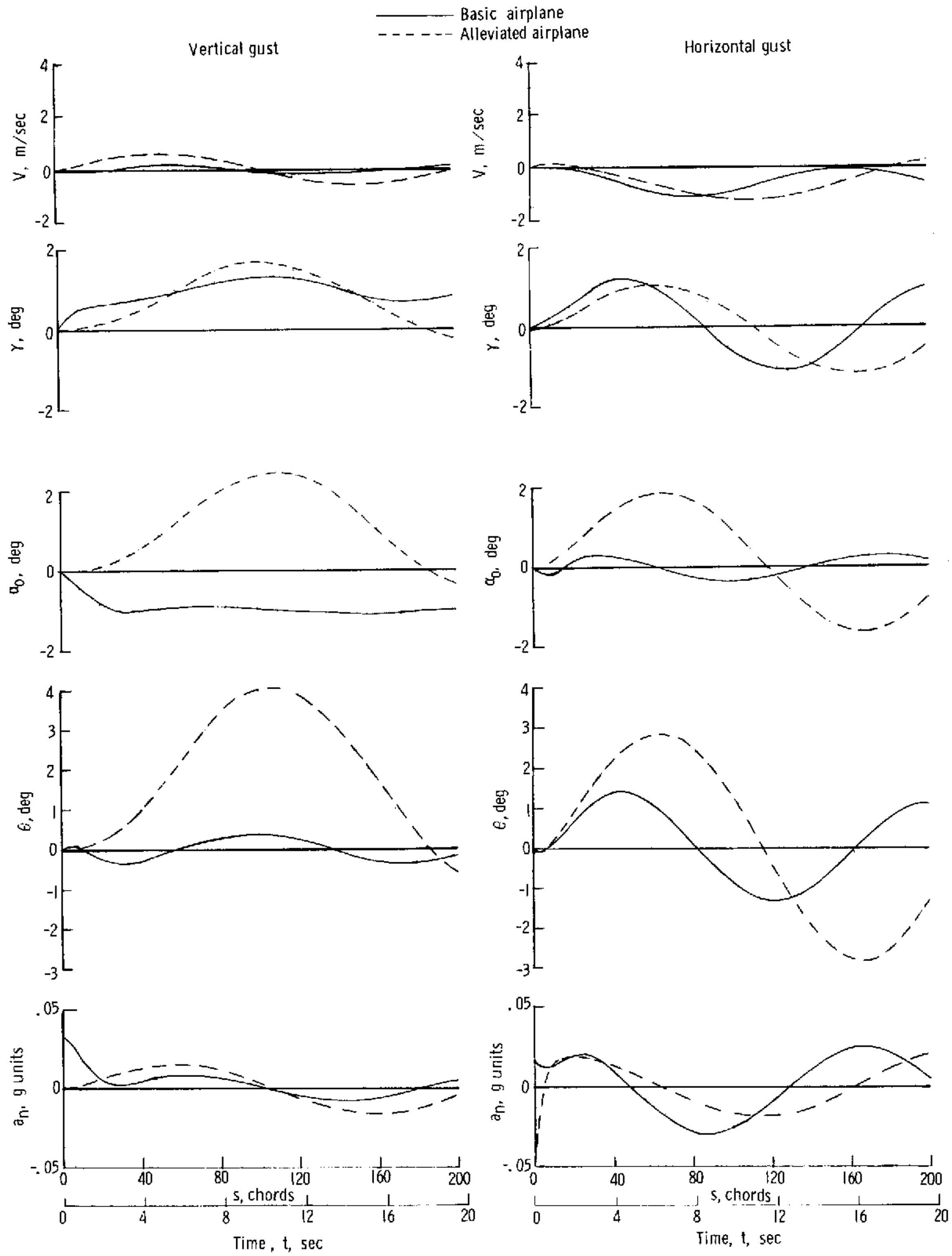

Figure 3.- Response to step vertical and horizontal gust inputs of 0.570 meter per second (1.87 feet per second) for a STOL airplane with and without gust-alleviation system and with c.g. at 0.33c.

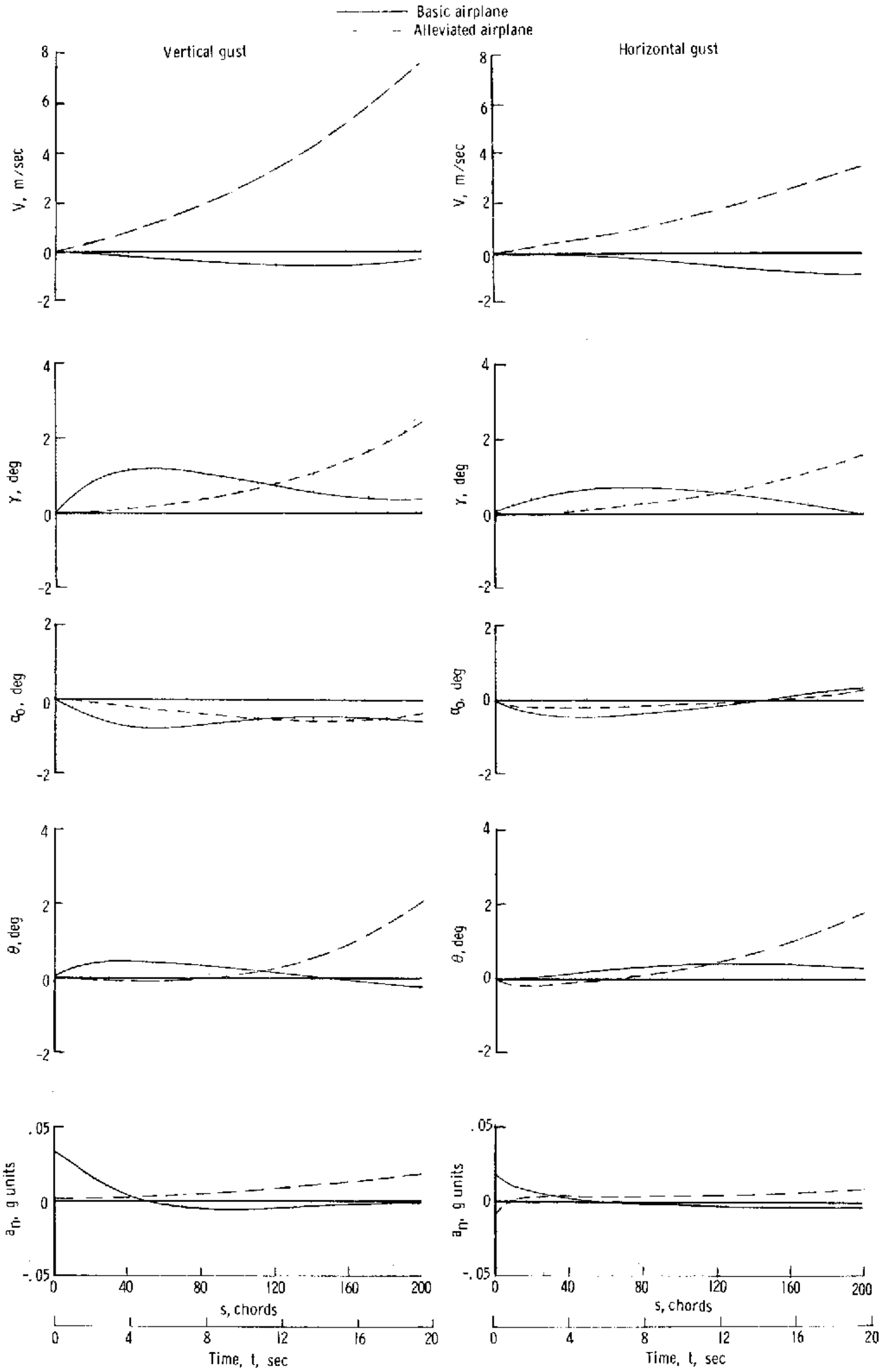

Figure 4.- Response to step vertical and horizontal gust inputs of 0.570 meter per second (1.87 feet per second) for a STOL airplane with and without gust-alleviation system and with c.g. at 0.594c.

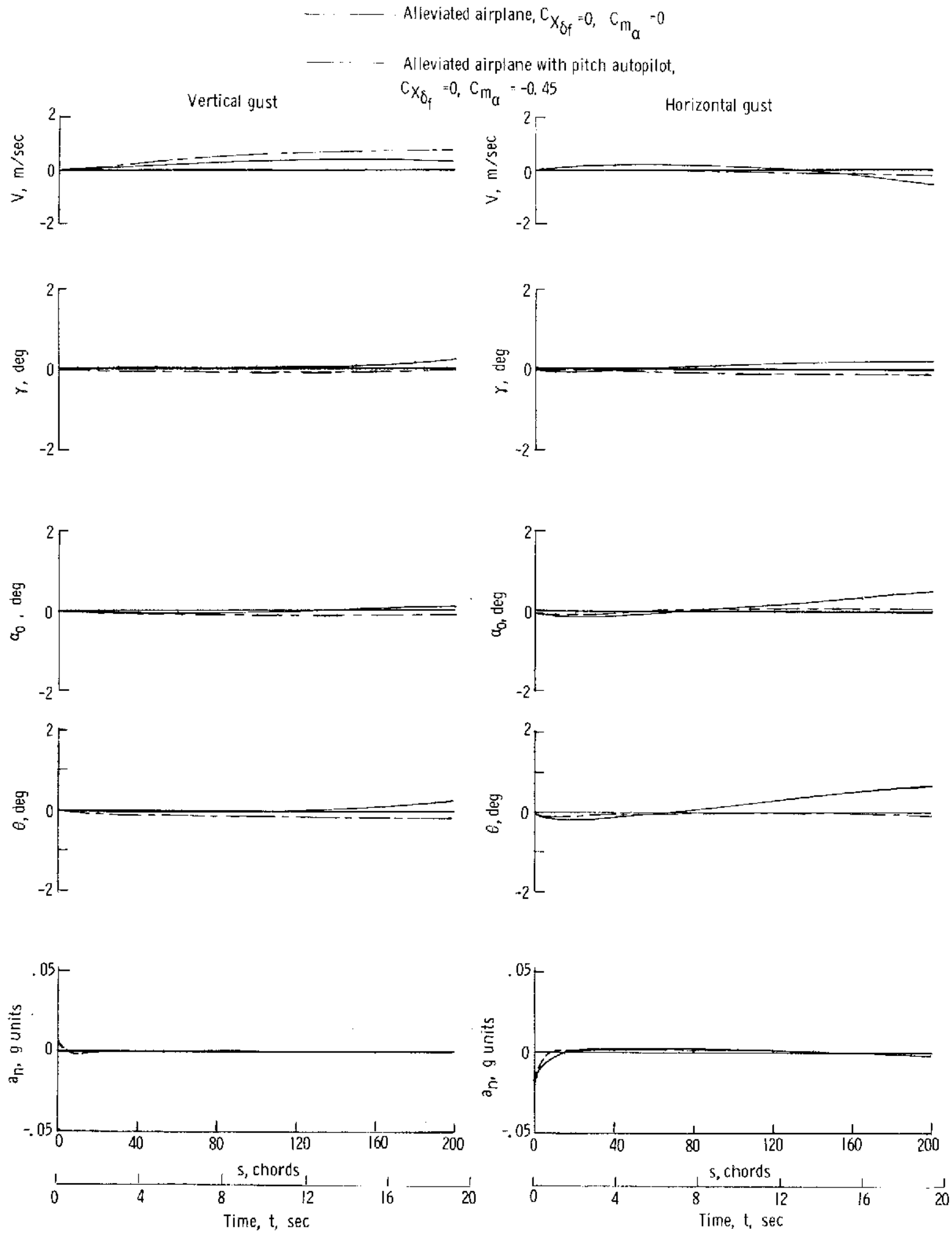

Figure 5.- Response to step vertical and horizontal gust inputs of 0.570 meter per second (1.87 feet per second) for a STOL airplane with two types of gust-alleviation systems, one of which includes a pitch autopilot. $C_{X_{\delta_f}} = 0$; c.g. at 0.594c.

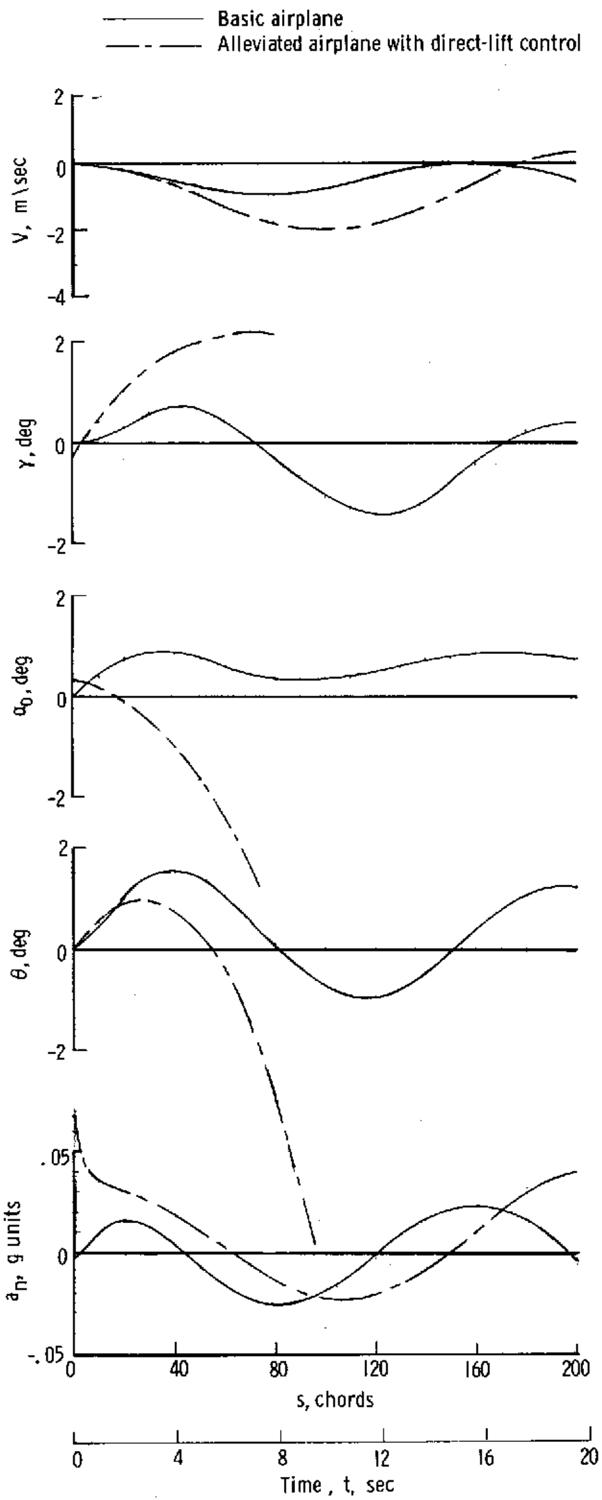

Figure 6.- Response to a step elevator deflection of 1° for a STOL airplane with and without a gust-alleviation system and with c.g. at 0.33c.

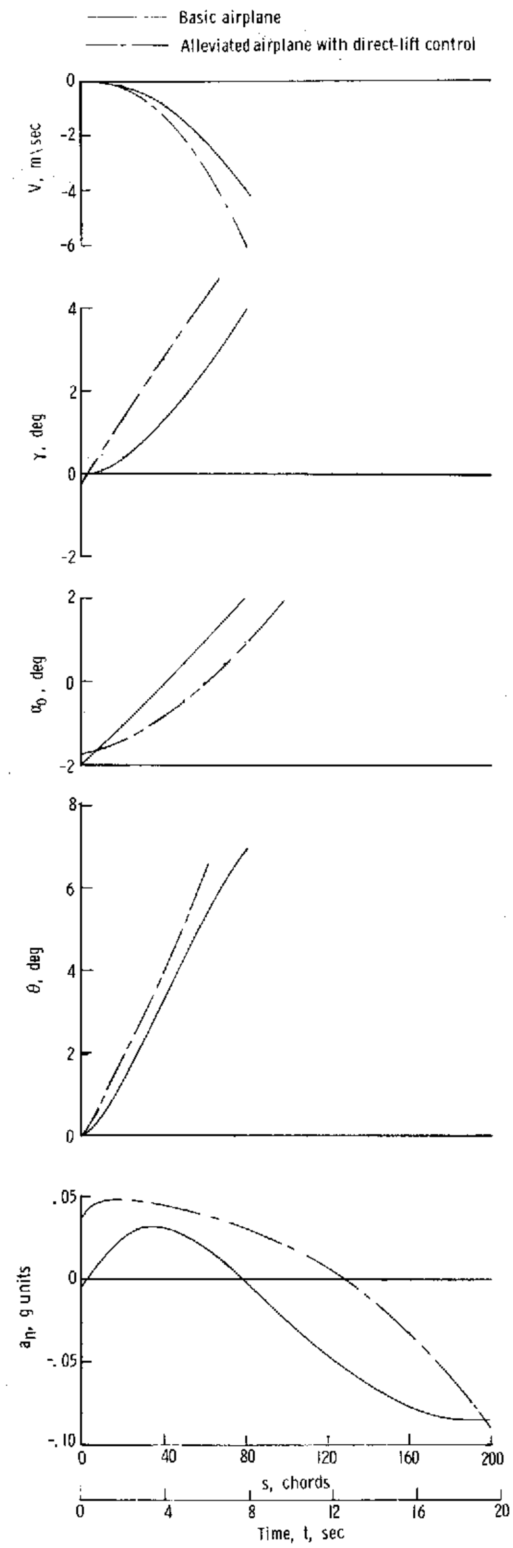

Figure 7.- Response to a step elevator deflection of 1° for a STOL airplane with and without a gust-alleviation system and with c.g. at 0.594c.

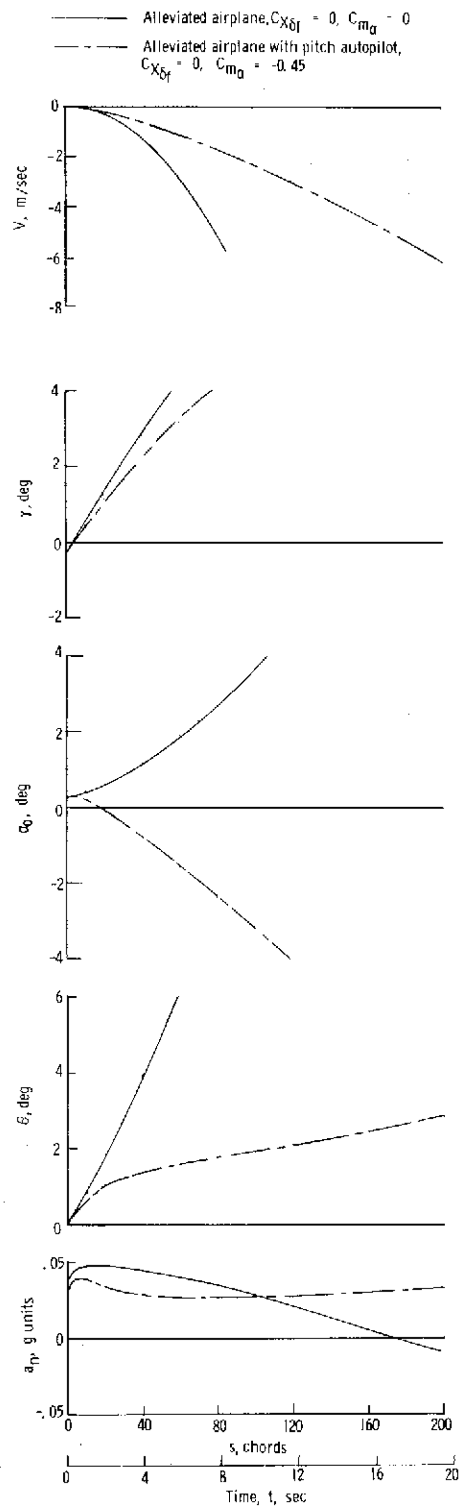

Figure 8.- Response to a step elevator deflection of 1° for a STOL airplane with two types of gust-alleviation systems incorporating direct lift control, one of which includes a pitch autopilot. $C_{X_{\delta_f}} = 0$; c.g. at 0.594c.

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