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# A NASTRAN DMAP ALTER FOR DETERMINING A LOCAL STIFFNESS MODIFICATION TO OBTAIN A SPECIFIED EIGENVALUE

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## SUMMARY

This paper describes a technique, which has been programmed as a DMAP Alter to Rigid Format 3, for determining a stiffness matrix modification to obtain a specified eigenvalue for a structure. The stiffness matrix modifications allowable are those that can be described as the product of a single scalar variable and a matrix of constant coefficients input by the user. The program solves for the scalar variable multiplier which will yield a specified eigenvalue for the complete structure (provided it exists), makes the modification to the stiffness matrix, and proceeds in Rigid Format 3 to obtain the eigenvalues and eigenvectors of the modified structure.

## INTRODUCTION

The motivation for devising a technique for determining a local stiffness modification to obtain a specified eigenvalue stemmed from several launch loads analyses performed at the Goddard Space Flight Center in which these analyses were updated using data from hardmount spacecraft vibration tests. Quite often, spacecraft are attached to their launch vehicle via a Marmon type clamp band. Generally, the clamp band attaches the spacecraft to an adapter section which in turn is bolted to the launch vehicle. However, the stiffness of the clamp band is often not known well enough to make an accurate analytical prediction of the fundamental mode of the spacecraft adapter structure when cantilevered from the base of the adapter, as it is in the spacecraft vibration tests. Thus, the original launch loads analyses are updated to reflect these discrepancies once the modes of the spacecraft-adapter structure have been measured in tests.



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Updating any finite element model to agree with modal data obtained from tests usually requires a trial and error process in which some local stiffness is adjusted until the fundamental mode of the model agrees with the test data. However, if a value for the local stiffness exists which will give the finite element model the measured eigenvalue, then this stiffness can be found analytically.

The DMAP Alter presented computes the value of the stiffness (or stiffness change) and adds this to the original stiffness matrix for the finite element model. The program then proceeds in Rigid Format 3 to compute the remaining eigenvalues and eigenvectors for the finite element model.

## THEORETICAL DESCRIPTION

In real eigenvalue analysis, NASTRAN solves for the eigenvalues and eigenvectors for the analysis (or $U_a$) degrees of freedom from

$$[K_{aa} - \lambda M_{aa}] \quad \{U_a\} = 0 \tag{1}$$

The stiffness matrix for the $U_a$ degrees of freedom is obtained from the original $U_g$ degrees of freedom through the application of constraints and Guyan reduction. The stiffness matrix $K_{gg}$ for the $U_g$ degrees of freedom can be considered to be the sum of two matrices

$$K_{gg} = K_{gg_o} + \Delta K_{gg} \tag{2}$$

where $K_{gg_o}$ contains the stiffnesses for the finite element model which will not be modified and $\Delta K_{gg}$ contains all of those stiffnesses that will be modified. The modification technique described in this paper is one in which the stiffnesses to be modified are <u>all</u> proportional to some scalar variable, which will be denoted as $\beta$. Thus, $\Delta K_{gg}$ can be written as

$$\Delta K_{gg} = \beta K'_{gg} \qquad (3)$$

where $K'_{gg}$ are the values of the $\Delta K_{gg}$ coefficients per unit value of the scalar variable $\beta$. The $K'_{gg}$ matrix could represent, for example, the portion of the finite element model represented by several beam elements of the same cross section whose moment of inertia we wanted to vary. In this case, $\beta$ would be the moment of inertia of those beams and $K'_{gg}$ would be the stiffness coefficients for these beams per unit moment of inertia.

In general, $\Delta K_{gg}$ can be any portion of the finite element model whose stiffness coefficients vary proportionally to some known variable. This variable could not, therefore, be the thickness of plate elements since the bending stiffness varies as the cube of the thickness while the transverse shear and membrane stiffnesses vary with the first power of the thickness. If, however, the plates were pure bending plates (no membrane or transverse shear), then all of the stiffness coefficients would vary with the cube of the thickness and we would be able to express the stiffness of those plate elements by an equation of the type in equation (3) where $\beta$ could be taken as the cube of the thickness or the bending rigidity D.

Thus, considering only those applications in which the stiffness matrix for a portion of the structure can be represented as in equation (3) where $\beta$ is a single scalar variable, the stiffness matrix for the complete structure (eq. (2)) becomes

$$K_{gg} = K_{gg_o} + \beta K'_{gg} \qquad (4)$$

The stiffness matrix in equation (4) can be reduced to the analysis set of degrees of freedom $U_a$ through the application of multi and single point constraints and through the Guyan reduction of the omitted points as mentioned above. The only restriction in the DMAP Alter presented herein is that the degrees of freedom that have stiffnesses that will be modified are not allowed to belong to the "O" set (omitted coordinates).

Following the normal procedures for reducing from the $U_g$ to the $U_a$ degrees of freedom (with the restrictions mentioned above), the eigenvalue problem as stated in equation (1) can be written as

$$[K_{aa_o} + \beta K'_{aa} - \lambda M_{aa}] \; \{U_a\} = 0 \qquad (5)$$

The problem is to find a value of $\beta$ that will result in one of the eigenvalues (usually the first nonzero eigenvalue) attaining a specified value, say $\lambda_1$. Setting $\lambda$ equal to the specified value $\lambda_1$ in equation (5) results in the equation

$$[(K_{aa_o} - \lambda_1 M_{aa}) + \beta K'_{aa}] \; \{U_a\} = 0 \qquad (6)$$

In order for there to be a nontrivial solution to equation (6), the determinant of the coefficient matrix must vanish. This will result in a polynomial in $\beta$ equal to zero, that is,

$$p(\beta) = 0$$

Thus, the solution for the value of $\beta$ that will provide a specified eigenvalue (provided such value of $\beta$ exists) may be obtained by solving an eigenvalue problem, using equation (6), for $\beta$. This can be readily accomplished in NASTRAN using the module READ by inputting to READ the matrix $(K_{aa_o} - \lambda_1 M_{aa})$ as the "stiffness" matrix and the matrix $K'_{aa}$ as the "mass" matrix. The resulting "eigenvalue" found by READ will be the value of $\beta$ that will provide the stiffness modification necessary for the structure to have the real eigenvalue $\lambda_1$.

It should be pointed out that there is no guarantee that the process will always work. There may be no modification of the portion of the structure we are attempting to modify that will result in the specified eigenvalue $\lambda_1$. However, the analyst can often tell, by comparison of his original finite element modes with those obtained from tests, what portion of the model appears to be too stiff or too flexible. In these instances, the procedure outlined in this paper for determining the stiffness modification should relieve the analyst of the burden of making arbitrary changes in the stiffnesses and solving repeated eigenvalue problems until the model agrees with the test. Since the technique outlined is one in which a stiffness change is determined which will provide one eigenvalue equal to a specified value, it appears that it will be most useful when there is disagreement between the original model and test results in a fundamental mode. It should also be mentioned that the stiffness change, while providing a specified fundamental mode, will obviously yield higher modes different from those obtained from the original or unmodified finite element model. There is no guarantee that these new higher modes will agree any better with the test modes than those from the original model.

## INPUT TO THE PROGRAM

The data deck required to make a run to modify part of the structure and obtain the resulting eigenvalues will be discussed in terms of changes to a normal deck for Rigid Format 3, real eigenvalue analysis.

### Case Control Deck

Two subcases are required. In the first subcase, a METHØD card selects an EIGB bulk data card which will be used for the eigenvalue extraction for $\beta$.

The second subcase contains the normal case control cards that the user would have in any Rigid Format 3 run including a METHØD card which selects the EIGR bulk data card for the real eigenvalues $\lambda$. The result of this subcase will be the normal real eigenvalue analysis output with one of the modes equal to the specified eigenvalue (to be specified in the Bulk Data Deck).

### Bulk Data Deck

1. Input of the normal finite element model of the structure which would be used in a real eigenvalue analysis. From this finite element model the stiffness matrix $K_{gg_o}$ will be built by NASTRAN. This could be the identical cards used to

describe the structure if an original modal analysis had been performed and the user were now rerunning it to modify part of the structure. In this case, the value of $\beta$ determined in the current run would be the change in stiffness of the modified part of the structure. Included in these cards, of course, is the EIGR card requested by subcase 2 which will find all desired modes subsequent to the modification.

2. DMIG input of $K'_{gg}$.

3. EIGB card requested by subcase 1 for finding the "eigenvalue" $\beta$. <u>The normalization for the eigenvector must be MASS</u>. If the scalar variable multiplier of $K'_{gg}$ is, for example, the moment of inertia of some of the beam elements, then the search range should be the range over which the user expects the change in this variable to lie (change with respect to the value that is in the finite element model in item 1).

4. A PARAM bulk data card with parameter name = FREQ and value equal to the frequency (in Hz) of the mode the user wishes to specify.

## DMAP ALTER DESCRIPTION

Appendix A lists the DMAP Alters to Rigid Format 3, Level 15.1.0, required to solve for the stiffness modification, to assemble the new stiffness matrix, and to proceed in Rigid Format 3 to obtain all of the desired eigenvalues and eigenvectors of the modified system. Several of the Alter statements are discussed in the appendix to clarify their function. In general, all the DMAP modules used but one are standard DMAP modules described in the NASTRAN User's or Programmer's Manuals. The module SCALAR, however, is a new module written and added to NASTRAN at the Goddard Space Flight Center and will be an available DMAP module in level 16 when it is released. Basically, this is a module that accepts matrices as input and will output one coefficient of the matrix as a NASTRAN complex, single or double precision parameter that can be used, for example, in the DMAP module ADD to multiply other matrices by. This was needed since the only way the scalar value of $\beta$ could be obtained as data that could be used in subsequent DMAP statements was in the matrix

KHHK output from module GKAM following the eigenvalue extraction for $\beta$. The module SCALAR was used to extract $\beta$ from KHHK. The matrix KHHK is the "modal stiffness" matrix found from the eigenvalue run to obtain $\beta$. If the normalization on the EIGB bulk data card requests normalization to unit modal mass, then the coefficient in KHHK will be $\beta$.

## SAMPLE PROBLEMS

Using the DMAP Alter program, two sample problems have been run. Figure 1 shows a beam finite element model of the UK-5 spacecraft and adapter to be flown on the Scout vehicle. The spacecraft and adapter are attached via a Marmon clamp, which in this finite element model is modeled as a scalar spring. In the original analysis, the model contained no scalar spring element for the clamp band and the adapter and spacecraft were assumed rigidly connected. The fundamental bending mode obtained from this finite element model was 43 Hz. Subsequent tests of the system indicated that the first mode was at 33 Hz and that the Marmon clamp did not appear "infinitely" stiff. Thus, the model was modified by including a spring between the adapter and spacecraft. The second run, made to determine the value that the spring should have to obtain a 33 Hz first bending mode contained the following changes:

1. removal of the MPC rigid constraint at the adapter/spacecraft interface that was used in the original analysis to simulate zero bending flexibility at that joint

2. addition of DMIG matrix input of a scalar spring stiffness matrix per unit value of stiffness:

$$K'_{gg} = \begin{bmatrix} 0 & 0 \ldots & 0 & 0 \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 \ldots & 1 & -1 \ldots & 0 \\ 0 & 0 \ldots & -1 & 1 \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where the nonzero values correspond to the rows and columns represented by the grid points and rotational degrees of freedom to which the scalar spring connects

3. EIGB bulk data card to find $k_s$ ($\beta$ is $k_s$ in this problem) with eigenvector normalization to MASS

4. PARAM FREQ bulk data card with value 33 Hz (complex single precision)

The data deck for this run is listed in Appendix B. The output from subcase 1 gave the value of $k_s$ needed to obtain a 33 Hz first bending mode, namely, $4.3 \times 10^9$ N/m ($24.5 \times 10^6$ lb/in). Subcase 2 then was executed to obtain the eigenvalues and eigenvectors for the system with this spring in the model. The resulting eigenvalues were a 33 Hz first mode with the second mode changing, in this case, by only a few percent from that obtained from the original model.

Figure 2 shows another problem run using the DMAP Alter. In this case, the structure is a stiffened plate simply supported on all four sides. The plate is stiffened with an I-beam whose area and offset distance are specified but whose moment of inertia (about the beam centroidal axis) may be varied. The problem is to determine the moment of inertia of the beam that will give a 40 Hz first symmetric bending mode of the structure. The structure was modeled with a 5x5 mesh of grid points equally spaced in one quadrant of the plate. The DMIG matrix $K'_{gg}$ in this problem consisted of the stiffness of the beams (due to the bending moment of inertia only) for all of the grid points to which the beams were attached. The Bulk Data input for the finite element model consisted of the normal input for such a structure but with zero bending inertia for the beams (the area and offset distance were input on the CBAR cards). The first subcase solved for the moment of inertia of the beams that would result in a 40 Hz first symmetric bending mode of the structure. This was determined as 855.8 $cm^4$ (20.56 $in^4$). Subcase 2 then proceeded to obtain the eigenvalues and eigenvectors of the modified system and it was determined that the first mode was at 40 Hz.

## ACKNOWLEDGEMENT

The help of Mr. Reginald Mitchell of the Goddard Space Flight Center is greatly appreciated for his programming efforts in writing the module SCALAR needed in the DMAP Alter.

## APPENDIX A

### DMAP ALTER FOR DETERMINING LOCAL STIFFNESS CHANGE TO OBTAIN A SPECIFIED EIGENVALUE (RIGID FORMAT 3)

```
 1  ALTER 45

 2  MTRXIN    ,MATPOOL,EQEXIN,SIL,/DKGGP,,/V,N,LUSET/V,N,NODKP/
              C,N,0/C,N,0  $

 3  SAVE      NODKP  $

 4  MATGPR    GPL,USET,SIL,DKGGP//C,N,G/C,N,G  $

 5  ALTER 48

 6  EQUIV     DKGGP,DKNNP/MPCF1  $

 7  ALTER 58

 8  MCE2      USET,GM,DKGGP,,,/DKNNP,,,  $

 9  ALTER 61

10  EQUIV     DKNNP,DKFFP/SINGLE  $

11  ALTER 64

12  UPARTN    USET,DKNNP/DKFFP,,,/C,N,N/C,N,F/C,N,S  $

13  ALTER 67

14  EQUIV     DKFFP,DKAAP/OMIT  $

15  ALTER 70

16  UPARTN    USET,DKFFP/DKAAP,,,/C,N,F/C,N,A/C,N,O  $

17  ALTER 75,76

18  ADD       MAA,/MAA1/C,Y,FREQ  $

19  ADD       MAA1,/MAA2/C,Y,FREQ  $

20  ADD       MAA2,KAA/DAA/C,N,(39.47842,0.0)/C,N,(-1.0,0.0)  $

21  DPD       DYNAMICS,GPL,SIL,USET/GPLD,SILD,USETD,,,,,,,EED,
              EQDYN/V,N,LUSET/V,N,LUSETD/V,N,NOTFL/V,N,NODLT/
              V,N,NOPSDL/V,N,NOFRL/V,N,NONLFT/V,N,NOTRL/
              V,N,NOEED/C,N,123/V,N,NOUE  $
```

```
22  SAVE      NOEED  $

23  COND      ERROR2,NOEED  $

24  CHKPNT    EED  $

25  READ      DAA,DKAAP,,,EED,USET,CASECC/LAMAK,PHIAK,MIK,OEIGSK/
              C,N,BUCKLING/V,N,NEIGVK/C,N,1  $

26  SAVE      NEIGV  $

27  OFP       LAMAK,OEIGSK,,,,//V,N,CARDNOK  $

28  SAVE      CARDNOK  $

29  GKAM      ,PHIAK,MIK,LAMAK,,,,,CASECC/MHHK,,KHHK,PHIDHK/
              C,N,-1/C,N,1/C,Y,LFREQ=0.0/C,Y,HFREQ=0.0/C,N,-1/
              C,N,-1/C,N,-1/V,N,NOCUP/V,N,FMODE  $

30  SCALAR    KHHK//C,N,1/C,N,1/V,N,BETA  $

31  SAVE      BETA  $

32  ADD       DKAAP,KAA/KAAT/V,N,BETA  $

33  COND      LBL6,REACT  $

34  RBMG1     USET,KAAT,MAA/KLL,KLR,KRR,MLL,MLR,MRR  $

35  ALTER 85,90

36  READ      KAAT,MAA,MR,DM,EED,USET,CASECC/LAMA,PHIA,MI,OEIGS/
              C,N,MODES/V,N,NEIGV/C,N,2  $

37  SAVE      NEIGV  $

38  CASE      CASECC,/CASEX2/C,N,TRAN/V,N,REPEATT=2/V,N,NOLOOP  $

39  ALTER 105,105

40  SDR2      CASEX2,CSTM,MPT,DIT,EQEXIN,SIL,,,BGPDP,LAMA,QG,
              PHIG,EST,,OQG1,OPHIG,OES1,OEF1,PPHIG/C,N,REIG  $

41  ALTER 109,109

42  PLOT      PLTPAR,GPSETS,ELSETS,CASEX2,BGPDT,EQEXIN,SIP,,PPHIG/
              PLOTX2/V,N,NSIL/V,N,LUSET/V,N,JUMPPLOT/V,N,PLTFLG/
              V,N,PFILE  $

43  ENDALTER
```

DESCRIPTION OF DMAP ALTER STATEMENTS

2. MTRXIN reads DMIG cards which contain the coefficients of the $K'_{gg}$ matrix input by the user. These are the stiffness coefficients (for the portion of the structure which will be modified) per unit value of the parameter that they vary with. These can easily be determined by running Rigid Format 1, up through GP4, with the bulk data containing all grid points, coordinate systems, and elements for the portion of the model to be modified.

5-16. These Alters perform the reduction on the $K'_{gg}$ matrix at the same location in Rigid Format 3 that the reductions are performed on the stiffness matrix for the remainder of the structure ($K_{gg_o}$).

18-20. Formulate the matrix $K_{aa_o} - \lambda_1 M_{aa}$ using the input parameter FREQ which is FREQ = $\frac{1}{2\pi}\sqrt{\lambda_1}$. That is, FREQ is the frequency in Hz of the mode we are specifying the eigenvalue for.

25. Solve an eigenvalue problem for $\beta$ using the buckling option in READ. The resulting "modal stiffness" matrix, KHHK, which will be output from module GKAM, will contain $\beta$ on the diagonal since the eigenvector normalization on the EIGB bulk data card is a normalization on unit modal mass.

29. GKAM outputs the matrix KHHK.

30. SCALAR (discussed above) extracts a value from KHHK and outputs it as a parameter (BETA).

32. ADD formulates the total stiffness $K_{aa_o} + \beta K'_{aa}$.

36. READ extracts the eigenvalues and eigenvector of the modified system, one of which will be the specified eigenvalue $\lambda_1$.

APPENDIX B

CASE CONTROL AND BULK DATA DECKS FOR UK-5 S/C - ADAPTER STIFFNESS MODIFICAT

```
TITLE = UK5 SPACECRAFT AND EH SECTION
SUBTITLE = CANTILEVERED MODE SHAPES (LATERAL)
LABEL = STIFFNESS CALCULATION FOR CLAMP BAND FOR 33 HZ FIRST BENDING
ECHO = UNSORT
MPC = 52
SUBCASE 1
  METHOD = 1
SUBCASE 2
  METHOD = 2
    OUTPUT
      VECTOR = ALL
      ELFORCE = ALL
      SPCF = ALL
BEGIN BULK
$
$ LATERAL MODES
$
GRDSET                                                  1345
BAROR                                     0.      1.      0.      1
EIGB    1       INV     5.+6    5.+7      1       1               1.-4    +EIG
+EIG1   MASS
EIGR    2       INV     25.     400.      3       3               1.-4    +EIG
+EIG2   MAX
PARAM   GRDPNT  0
PARAM   WTMASS  .002591
$
$ EH SECTION
$
GRID    501             47.77   0.        0.              123456
GRID    502             44.     0.        0.
GRID    503             40.     0.        0.
GRID    504             37.27   0.        0.
CBAR    5001    5001    502     501
CBAR    5002    5002    503     502
CBAR    5003    5003    504     503
PBAR    5001    5001     2.634    94.4    94.4    72.6    .989            +B50
PBAR    5002    5001     2.138    59.5    59.5    45.8    .989            +B50
PBAR    5003    5001     1.710    29.2    29.2    22.4    .989            +B50
+B5011                                                                    +B50
+B5021                                                                    +B50
+B5031                                                                    +B50
+B5012   .185    .185
+B5022   .185    .185
+B5032   .185    .185
MAT1    5001    1.+7    1.+7    .3
$
$ CONSTRAIN S/C - ADAPTER INTERFACE GRID POINTS TO BE THE
$ SAME EXCEPT IN ROTATIONAL DEGREE OF FREEDOM
$
MPC     51      504     1       1.0       601     1       -1.0
MPC     52      504     2       1.0       601     2       -1.0
```

```
$ ***************************************************
$ UK5 SPACECRAFT
$ ***************************************************
GRID    601             37.27   0.0     0.0
GRID    602             33.77   0.0
GRID    603             30.27   0.0
GRID    604             26.77   0.0
GRID    605             23.27   0.0
GRID    606             19.77   0.0
GRID    607             16.27   0.0
GRID    608             12.77   0.0
GRID    609             10.27   0.0
CBAR    6001    6001    602     601
CBAR    6002    6002    603     602
CBAR    6003    6003    604     603
CBAR    6004    6004    605     604
CBAR    6005    6005    606     605
CBAR    6006    6006    607     606
CBAR    6007    6007    608     607
CBAR    6008    6008    609     608
PBAR    6001    6001    3.062   37.05                   7.297           +B6011
PBAR    6002    6001    3.441   44.87                   4.072           +B6021
PBAR    6003    6001    3.525   64.                     8.861           +B6031
PBAR    6004    6001    3.225   89.1                    10.809          +B6041
PBAR    6005    6001    3.075   118.5                   13.492          +B6051
PBAR    6006    6001    3.165   156.5                   10.618          +B6061
PBAR    6007    6001    3.505   199.5                   7.328           +B6071
PBAR    6008    6001    3.965   238.6                   25.31           +B6081
+B6011                                                                  +B6012
+B6021                                                                  +B6022
+B6031                                                                  +B6032
+B6041                                                                  +B6042
+B6051                                                                  +B6052
+B6061                                                                  +B6062
+B6071                                                                  +B6072
+B6081                                                                  +B6082
+B6012  .1907
+B6022  .1697
+B6032  .1589
+B6042  .1591
+B6052  .1515
+B6062  .1349
+B6072  .1207
+B6082  .1067
MAT1    6001    1.+7    1.+7    .3
DMIG    DKGGP   0       6       1       2
DMIG    DKGGP   504     6               504     6        1.0            +DK1
DMIG    DKGGP   601     6               504     6       -1.0            +DK2
+DK1    601     6       -1.0
+DK2    601     6        1.0
PARAM   FREQ    33.     0.
ENDDATA
```

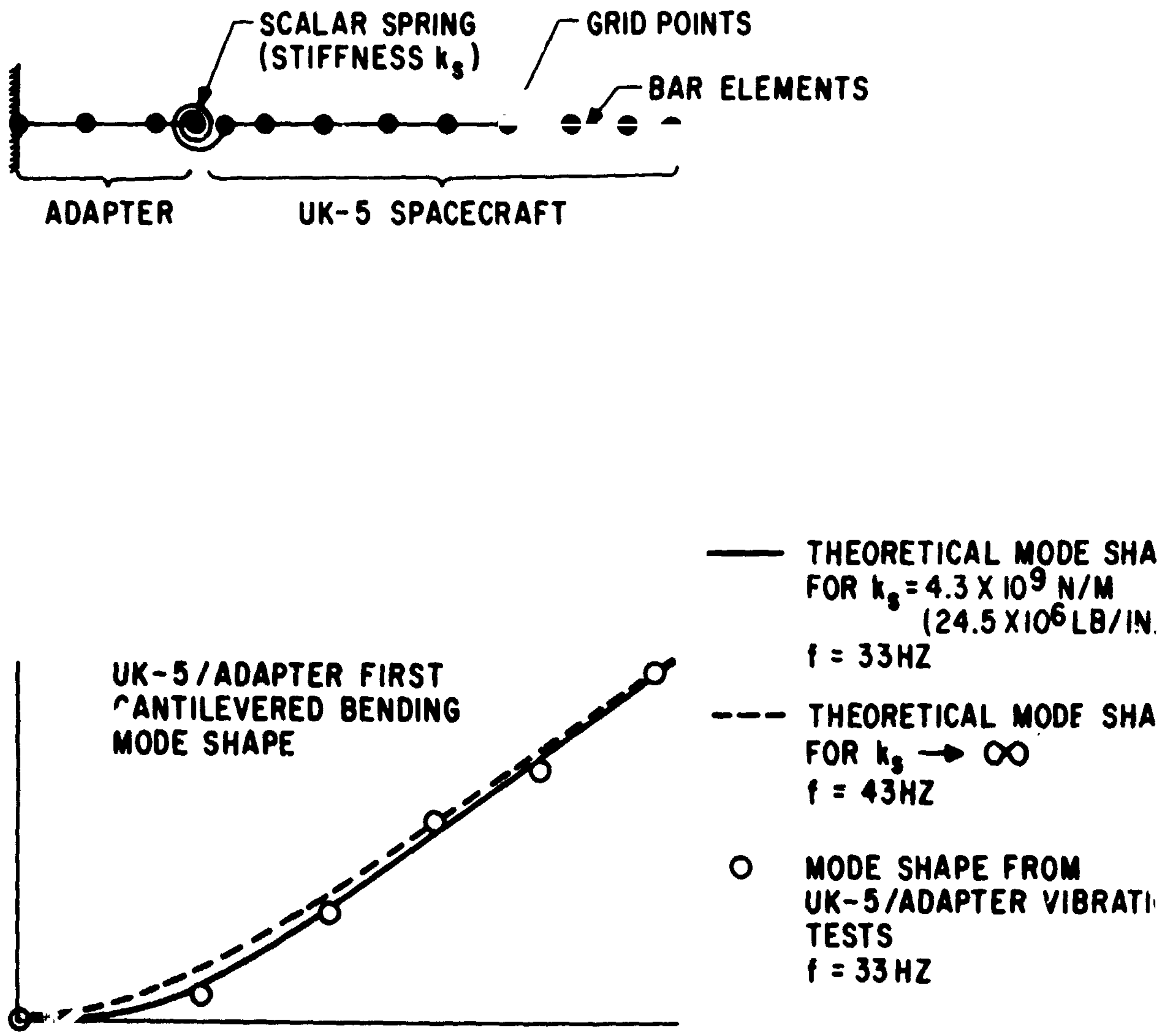


Figure 1.- Clamp-band stiffness modification to obtain 33 Hz first bending mode for the UK-5 spacecraft and adapter.

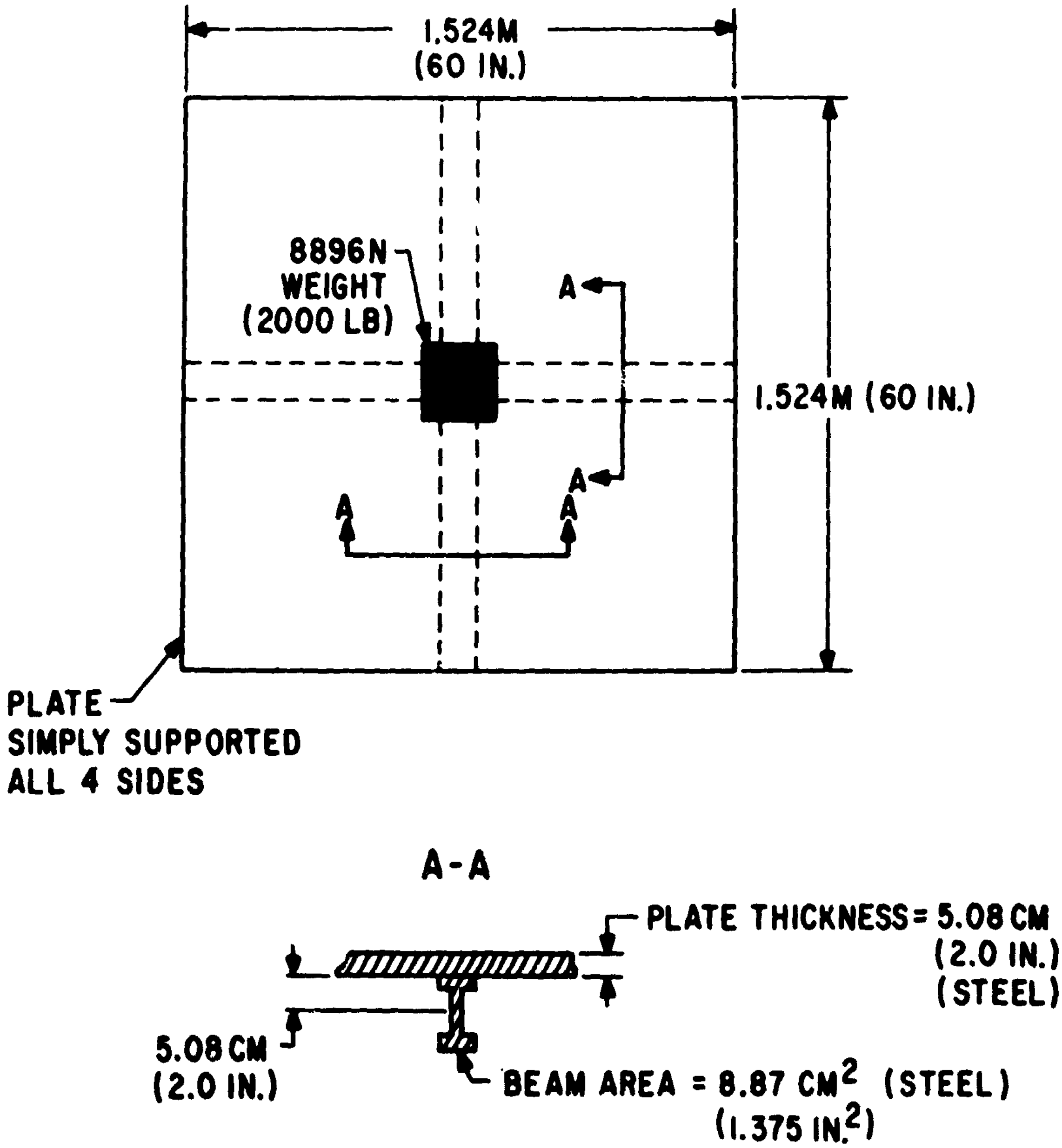


Figure 2.- Stiffener EI modification to obtain 40 Hz first mode for the simply supported stiffened plate.