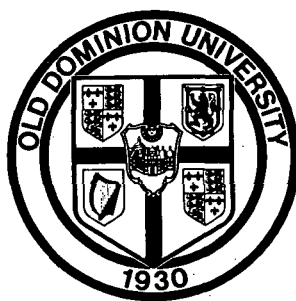


FINITE ELEMENT ANALYSIS OF A COMPOSITE MATERIAL INTERFACE

*A Technical Report
by
Kenneth H. Murray*

*Prepared for the
NATIONAL AERONAUTICS
AND
SPACE ADMINISTRATION
LANGLEY RESEARCH CENTER
HAMPTON, VIRGINIA* 23665

*Under
Master Contract Agreement NAS1-9434
Task Order No. 45*



*SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA*

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SUMMARY

A finite element model of a composite material interface is developed to study the influence of the interface on the thermal strain in the composite. A plane stress model is used with an axisymmetric model as a check. The interface thickness, thermal coefficient, modulus, Poisson's ratio and the percent of mineral in the composite are variables in the study. The results confirmed the usability of the finite element model in studying the polymer-mineral interface.

INTRODUCTION

Composite materials can be considered to be a combination of two separate phases or materials with an interphase or interface between them. The interface transmits the forces and displacements between the phases and thus makes possible the combined action of the phases[1]. These forces and displacements have been analyzed by means of finite elements to estimate the matrix stresses [2-4] as well as the effect of the interface [5-7]. In all of these analyses, perfect bonding and zero thickness interface between the phases was assumed, although such an assumption is physically unrealistic.

Consequently, a finite element analysis was made of a composite material

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consisting of a continuous polymer phase (the matrix) containing a discontinuous mineral phase (the filler). The phases were separated by a finite thickness interface to which a range of properties could be assigned. The effects of the interface properties on the thermal strain of the composite were determined and are reported in the next section.

MODELS AND ANALYSIS

Composite Model

The model polymer-mineral composite material had properties which approximated those of an epoxy-glass system in which the glass is in the form of small, uniformly sized, beads. The properties were:

	<u>Polymer</u>	<u>Mineral</u>
Elastic modulus, GN/m ²	3.047	68.95
Thermal expansion coef., K ⁻¹	88.6x10 ⁻⁶	8.5x10 ⁻⁶
Poisson's ratio	0.44	0.22
Specific gravity	1.15	2.45
Bead diameter, μ m	---	125

A composite with these properties was being investigated concurrently in an experimental program at NASA Langley Research Center.

Finite Element Model

The finite element model of the composite was basically a plane stress model of a unit cell of polymer surrounding a single glass bead, separated by a finite thickness interface (Figure 1). The bead was modeled as a hexagon because only straight line boundaries were permitted with the plane stress elements available in the computer program which was used in the analysis. The interface, which can vary in thickness and material properties, is essentially a geometrical extension of the hexagon. The bead diameter was taken as the distance between opposite sides of the hexagon. Since the diameter was fixed (125 μ m), different volume percentages of mineral were obtained by changing the size of the unit cell. Thus a large mineral

volume percentage (e.g. 60%) required a smaller unit cell than did a small percentage (e.g. 5%).

Using the basic unit cell, four finite element models were constructed. The first model (Figure 2) had a flexible link element as the interface. This link has been used successfully to model the bonding between concrete and reinforcing steel [8]. The material properties of this interface element were independent of the polymer and mineral because of the linking arrangement, so these properties could be varied over an extremely wide range. The second model used the same geometry except for the interface element (Figure 3). The interface element was a normal element geometrically but had its material properties bounded by the material properties of the polymer and mineral; e.g. it is unreasonable to expect the interface modulus to be larger than the mineral or smaller than the polymer modulus. The third model was made up of two of the second models stacked on top of each other (Figure 4). This was done to determine if the strain for two unit cells would differ from that for one cell. The fourth model was an axisymmetric idealization of the bead and polymer (Figure 5). This model used the same type of interface element as the second model. The idealization shown in Figure 5 was rotated 360° about the z-axis to develop a three-dimensional solid.

Analysis

The four models were analyzed using the finite element method by way of NASTRAN. NASTRAN, an acronym for NASA Structural Analysis, is a general purpose finite element computer program for structural analysis. A discussion of the finite element method and of NASTRAN is given in Appendix A.

The loading system for the models was a temperature change from 300 K (approximately room temperature) to 400 K. In the finite element method the thermal displacements are found by defining a set of nodal loads which approximate the thermal load on an element. The nodal loads from each element are summed to obtain the total load vector. This approximation along with the basic idealization of polymer-mineral composite are necessary to analyze the system using finite elements.

The boundary conditions for the first three models were the same

(Figures 3 and 4). The x-axis nodes (16, 17, 21, and 22) were restrained in the y-direction and the y-axis nodes (18, 22, and 23) were restrained in the x-direction. The boundary conditions approximated those of a piece of the composite two unit cells wide, resting on a plate, with the composite axis of symmetry coincident with the y-axis. As a result of the symmetry only half of the piece (one unit cell) had to be analyzed. The boundary conditions for the fourth (axisymmetric) model were zero displacement along the r-axis in the y-direction (see Figure 5).

The variables used in the analysis are listed in Table I. One composite variable, the volume percent of mineral, was included to confirm the results obtained with the other variables; if the mineral volume is increased, the thermal strain should decrease [9]. Four interface variables were used, the interface thickness plus the interface properties of elastic modulus E , coefficient of thermal expansion α , and Poisson's ratio μ . The interface properties were assumed to change little over the 300-400 K temperature range investigated, and indeed, the large drop in the epoxy modulus does not occur until a temperature of 440 K is reached [10]. The interface properties were assigned values equal to the polymer, the mineral, or the average of the two values. The particular value is indicated in the text and the tables by a lower case suffix p (polymer), m (mineral), or a (average). Thus the designation α_a , E_m means that the average value of the coefficient of thermal expansion ($48.55 \times 10^{-6}/K$) and the mineral value of the elastic modulus (68.95 GN/m^2) were assigned to the interface. Only the interface properties were varied; the polymer and mineral always were assigned their respective values listed in Table I.

The geometry of the models, the variable mineral volume, and the variable interface properties created a major data preparation problem. To reduce the effort and the possible errors associated with the input data, a computer program was written to punch the NASTRAN input deck. This program is given in Appendix B along with an outline of its operation.

RESULTS AND DISCUSSION

The results of the finite element program (NASTRAN) are displacements

at the nodes and stresses within the element. The displacements were divided by the length (height or width) of the unit cell to obtain strain values. These strain values are listed in Tables II through VII. An analysis of variance (ANOVA) of selected data is given in Tables VIII through X.

Preliminary Results

Model One: No results are presented in this report because the model never gave reasonable results. The material properties of the linking element were varied systematically from zero to 10^{20} and no combination of values gave reasonable results.

Poisson's ratio: Table II shows some of the results when the Poisson's ratio was varied in Model Two. It is obvious from the table that this quantity does not affect the thermal strain values, therefore it was eliminated from further study and an average value was used ($\mu = 0.33$) for the interface.

Interface thickness: Most researchers agree that the interface is a thin layer between the mineral and the polymer. Using this, the larger value of F did not appear valid. The remainder of the study uses only $F = 1$ and $10 \mu\text{m}$.

Model Three: The two cell model gave strain values identical to the one cell model (Model Two). The mineral content was 34% and the interface was $10 \mu\text{m}$ with all combinations of α_m , α_p , E_p , and E_m .

Variables examined: The final results are based on the following variables with their discrete values listed in parentheses. All combinations were studied for:

Volume Percent of Mineral - PER (5, 34, and 60%)

Interface thickness - F (1 and $10 \mu\text{m}$)

Interface thermal coefficient - α (α_p , α_m , and α_a)

Interface modulus - E (E_p , E_m , and E_a)

Models two and four

The reference model was Model Two with 34% mineral and an interface thickness of $1 \mu\text{m}$.

Final Results

Mineral content: Comparison of data in Tables III, IV, and V and the ANOVA in Table VIII indicate that the mineral content is a very significant variable. The thermal strain in the model decreases with increasing mineral content. This can be predicted because the mineral is less responsive to temperature change than the polymer, thus the more mineral the less thermal strain.

Interface thickness: Comparison of data in Tables IV and VI and the ANOVA in Table IX indicates that the interface thickness is a significant variable.

Model: Comparison of data in Tables IV and VII and the ANOVA in Table X for Models Two and Four indicates no apparent significance for the model. This, of course, is good because now only the plane stress model needs to be studied in detail.

Strain location: Comparison of Tables III through VII and the ANOVA in Tables VIII and IX indicates that the strain location is a significant variable. This is somewhat disturbing because it should not be significant. The variations in strain along the boundary is plotted in Figure 6. The low strain values in the middle region near the mineral bead are caused by the mineral's presence.

Table XI gives the results of the basic model with the boundary conditions varied at the corner node (22). Standard conditions fix the node in both the x- and y-directions. To help explain the nonuniform strain across the top and side of the model, this node (22) was released first in the x- and then in the y-direction. Only the two outside nodes and node 22 are presented, to observe the difference between them. It is quite obvious that the boundary conditions affect the final strains. The only way to eliminate this effect is to increase the number of elements near the boundaries to isolate the effect.

Interface material properties: Observation of Table IV for Model Two, $F = 1 \mu\text{m}$ and 34% mineral, indicates that as α and E increase the thermal strain increases. The thermal coefficient α has the most effect as can be seen from ANOVA Table IX. It should be noted that the total effect of both variables is less than 1% in general. This presents critical questions as to the real effect of the interface on the final thermal strains.

CONCLUDING REMARKS

The basic procedure used in this study was to model the polymer-mineral composite as a plane stress elasticity problem modeled with finite elements. A unit cell with one bead and a variable interface was idealized as plane stress triangles and quadrilaterals. The interface was varied in thickness and material properties through a realistic range to obtain data on its effect on the thermal strain of the unit cell during a temperature change.

For the standard model used in this study (34% mineral, interface thickness approximately 11% of the bead diameters, $1\text{ }\mu\text{m}$, and interface material properties equal to the polymer, mineral and their average) the following conclusions can be drawn:

1. The major factors influencing the thermal strain of the composite are:
 - a) Mineral content
 - b) Interface thickness
 - c) Interface thermal coefficient
2. The thermal strain decreases with increasing mineral content.
3. The thermal strain decreases with increasing interface stiffness.
4. More elements and nodes are needed to eliminate the variation in displacement if the plane stress elements are used.
5. Experimental results are needed to establish the correct interface properties.
6. The plane stress finite element idealization does a reasonable job representing the polymer-mineral composite; it but needs some more refinement.
7. A three-dimensional representation needs to be made to evaluate the usefulness of the plane stress model beyond what was done here.

ACKNOWLEDGEMENTS

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Appendix A

Finite Element Analysis - NASTRAN

I. Basic Procedure (Reference A.1)

A general continuous body can be idealized using the finite element method by dividing it into a discrete number of small elements. These elements are interconnected only at a finite number of points called nodes or joints. At each node a finite number of degrees of freedom, usually displacements and rotations, are prescribed. Thus the real system with displacements continuous throughout the body is idealized by a discrete system with displacements specified only at nodes. This approach to structural analysis has been used on framed structures for quite some time, but in the last twenty years it has been extended to include two- and three-dimensional continua. Because it is a systematic procedure it is very well suited for implementation on digital computers. It is this fact plus the versatility of the method that has led to its great popularity.

The first step in the analysis procedure is to idealize the continuum as a discrete number of finite elements. For each element a functional relationship between the internal displacements and the nodal displacements is written. Next the stress-strain-displacement relationships are developed and used to write the potential energy of the element. From the potential energy equation the stiffness matrix for the element is derived. The stiffness matrices of all elements are

then summed to determine the equations of equilibrium for the continuum. These equilibrium equations are then solved for the nodal displacements using modern numerical techniques. Once the nodal displacements have been computed, the forces, stresses and strains in each element are found.

II. Thermal Stress Analysis

The temperature load is input as a nodal force which is equivalent to the thermal strain associated with the temperature change. The thermal stress analysis by the finite element method can be best summarized by the following steps:

1) Idealization

The continuum is subdivided into a system of discrete or finite elements, interconnected at a discrete number of nodes. The unknown parameters of the problem are normally the displacements or rotations at each node.

2) Functional

A displacement function is selected which expresses the internal displacement of the element in terms of the unknown nodal displacements:

$$\{f\} = [N] \{\delta\}_e \quad (1)$$

3) Strain

The state of strain within an element is expressed in terms of the nodal displacements:

$$\{\epsilon\} = [B] \{\delta\}_e \quad (2)$$

4) Stress

The state of stress within an element is expressed in terms of the nodal displacements using the element strains, initial strains and constitutive properties of the material. In equation form:

$$\{\sigma\} = [D] (\{\epsilon\} - \{\epsilon_0\}) \quad , \quad (3a)$$

or

$$\{\sigma\} = [D] ([B] \{\delta\}_e - \{\epsilon_0\}) \quad . \quad (3b)$$

5) Total Potential Energy

The total potential energy is expressed in terms of the nodal displacements using the stress and strain functions:

$$\Pi = U + V \quad (4)$$

where U is the internal strain energy

$$U = 1/2 \int_{V_e} \{\epsilon\}^T \{\sigma\} dV_e$$

or after substituting equations (2) and (3b)

$$U = 1/2 \int_{V_e} \{\delta\}_e^T [B]^T [D] [B] \{\delta\}_e dV_e - 1/2 \int_{V_e} \{\delta\}_e^T [B]^T [D] \{\epsilon_0\} dV_e. \quad (5)$$

The potential energy of the applied forces can be written as

$$V = \int_{V_e} \{f\}^T \{p\} dV_e - \int_S \{f\}^T \{g\} dS - \{\delta\}_e^T F_n \quad (6)$$

where $\{p\}$ = body force per unit volume

$\{g\}$ = surface tractions per unit area

S = surface (area) over which $\{g\}$ acts

$\{F_n\}$ = applied nodal forces

For the case of thermal strain $V = 0$ and is omitted from the following. The total potential energy then is U ,

$$\Pi = 1/2 \int_{V_e} \{\delta\}_e^T [B]^T [D] [B] \{\delta\}_e dV_e - 1/2 \int_{V_e} \{\delta\}_e^T [B]^T [D] \{\epsilon_o\} dV_e \quad (7)$$

6) Minimize Total Potential Energy

The equilibrium of the system can be established by minimizing the total potential energy of the system. In equation form this can be stated:

$$\frac{\partial \Pi}{\partial \{\delta\}_e} = 0$$

Performing the minimization, after using equation (1), yields

$$\int_{V_e} [B]^T [D] [B] \{\delta\}_e dV_e - \int_{V_e} [B]^T [D] \{\epsilon_o\} dV_e = 0. \quad (8)$$

7) Nodal Forces

In the minimization above four load types are usually present, but ^{here} only one is present. Each must be converted into equivalent nodal forces. The second integral in equation (8) represents nodal loads resulting from initial strains or

$$\{F_o\} = \int_{V_e} [B]^T [D] \{\epsilon_o\} dV_e. \quad (9)$$

The equivalent nodal force then is

$$\{F\}_e = \{F_o\} \quad (10)$$

8) Stiffness Matrix

Another outgrowth of the minimization is the basic element relationship between load and displacement or the stiffness matrix $[k]$. The basic equilibrium equation in discrete coordinates can be written as

$$\{F\}_e = [k]\{\delta\}_e$$

and comparing it to equation (8) defines

$$[k] = \int_{V_e} [B]^T [D] [B] dV_e \quad (11)$$

9) Assemblage

Each element has now been reduced to a set of equilibrium equations. The continuum can now be represented by the sum of the equilibrium equations. If N denotes the total number of elements, then

$$\{F\} = [K]\{\delta\} \quad (12)$$

where

$$\{F\} = \sum_{e=1}^N \{F\}_e \quad (13a)$$

$$[K] = \sum_{e=1}^N [k]_e \quad (13b)$$

and

$$\{\delta\} = \sum_{e=1}^N \{\delta\}_e \quad (13c)$$

10) Boundary Conditions

The external constraints are applied using modern numerical technique procedures.

11) Solution

The unknown displacements of equation (12) with constraints are obtained using appropriate numerical procedures to solve the equations.

12) Stress Recovery

The displacements are used to determine the stress and strains in the continuum using equations (2) and (3a).

This entire procedure has been programed many times. The program used in this study is NASA's Structural Analysis Program - NASTRAN. The input for NASTRAN must contain the following information:

- 1) Type of analysis (static)
- 2) Type of element (plane stress)
- 3) Nodal point locations
- 4) Element connections
- 5) Material properties
- 6) Loads (temperature change)
- 7) Boundary conditions
- 8) Output requirements

Most of the input is quite easy, but the nodal locations and element connection cards can be difficult. For this reason a FORTRAN program was written to generate the geometry of the

polymer-mineral composite and then convert this into NASTRAN input format for punching. Appendix B gives a detailed description of this procedure.

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Appendix B

NASTRAN Input Generation Program

A FORTRAN program was written to generate the geometry of the mineral polymer composite and punch it in NASTRAN format. The Executive Control and Case Control decks are read into this program as data for the final NASTRAN deck. Also read into this program are the GRDSET, PROPERTY, MATERIAL and TEMP cards from the Bulk Data deck. All other cards are generated and punched. The mineral bead diameter, mineral content in percent volume, interface thickness and the number of basic blocks in the model are also read into this program. This allows great flexibility in defining the variables of this polymer-mineral composite study.

The program is listed on the following pages with a typical data deck.

```
DIMENSION DECK (30,18),GRIDX(35),GRIDY(35),IE(22),JE(22),KE(22)
DIMENSION LE(22)
```

```
B = BEAD RADIUS IN MICRONS
F = INTERFACE THICKNESS
PER = PERCENT BY VOLUME OF BEADS
A = INSCRIBED CIRCLE OF LARGE HEX
R = RADIUS TO NODE OF LARGE HEX
IN = LOGICAL UNIT FOR INPUT
IOO = LOGICAL UNIT FOR PUNCHED OUTPUT
IC = LOGICAL UNIT FOR PRINTED OUTPUT
NO = NUMBER OF HEXES
      MAXIMUM OF 2 FOR MODEL ONE AND 1 FOR MODEL TWO
NDEC = NUMBER OF LEADING CARDS
BHEX = RADIUS OF BEAD HEX
ITYPE = TYPE OF INTERFACE
      0 FOR MODEL ONE AND 1 FOR MODEL TWO
```

```
SET UP INPUT-OUTPUT UNITS
```

```
IN=2
IO=3
IOO=7
```

```
READ (IN,1) B,F,PER,NO,NDEC ,ITYPE
1 FORMAT (3F10.0,3I2)
NDEC1=NDEC+1
```

```
READ IN NASTRAN EXECUTIVE CONTROL DECK,
CASE CONTROL DECK AND
THE FOLLOWING BULK DATA CARDS
GRDSET
PROPERTY
MATERIAL
TEMP.
```

```
WRITE (IO,600)
600 FORMAT(1H1)
DO 15 I=1,NDEC1
READ (IN,11)(DECK(I,J),J=1,18)
WRITE(IO,11)(DECK(I,J),J=1,18)
15 CONTINUE
11 FORMAT (18A4)
BHEX = 1.15469*B
R = SQRT(75.0/PER)*B/.86603
GRIDX(1) = 0.
GRIDY(1) = 0.
IF (ITYPE) 300,19,300
```

```
LINKAGE ELEMENT MODEL ONE
```

```
19 DO 20 I=2,13
20 GRIDX(I)=BHEX
DO 21 I=14,25
21 GRIDX(I)=BHEX+F
DO 22 I=26,31
22 GRIDX(I)=R
JJ=1
```

```

      I=3
      GRIDY(2)=0
23  GRIDY(I)=GRIDY(I-1)+60.
      IF (I-31) 24,30,30
24  IF ( JJ=5      ) 25,26,27
25  I=I+1
      JJ=JJ+1
      GO TO 23
76  I=I+1
      JJ=1
      GRIDY(I)=0.
      I=I+1
      GO TO 23
27  WRITE(10,3)
      3 FORMAT('PROBLEMS')
      STOP
30  DO 40 I=1,31
      RR=GRIDX(I)
      TH=(-GRIDY(I)+60.)*0.01745329
      GRIDX(I)=RR*COS(TH)+R
40  GRIDY(I)=RR*SIN(TH)+0.86603*R
      GRIDX(32)=2.*R
      GRIDX(33)=2.*R
      GRIDX(34)=0.
      GRIDX(35)=0.
      GRIDY(32)=1.73206*R
      GRIDY(33)=0.
      GRIDY(34)=0.
      GRIDY(35)=1.73206*R
      DO 50 I=1,6
      IE(I)=1
      JE(I)=3+I-1
      LE(I)=0
50  KE(I)=2+I-1
      JE(6)=2
      DO 60 I=7,12
      IE(I)=2+I-7
      JE(I)=9+I-7
      KE(I)=21+I-7
60  LE(I)= 14+I-7
      JE(12)=8
      KE(12)=20
      DO 70 I=13,18
      IE(I)=20+I-13
      JE(I)=21+I-13
      KE(I)=27+I-13
70  LE(I)=26+I-13
      JE(18)=20
      KE(18)=26
      DO 80 I=19,22
      IE(I)=27+I-19
      JE(I)=32+I-19
      KE(I)=26+I-19
80  LE(I)=0
      JE(21)=30
      JE(22)=31
      KE(21)=34
      KE(22)=35
      DO 16 I=1,NDEC
16  WRITE(100,11) (DECK(I,J),J=1,18)
      DO 90 I=1,35

```

```

90 WRITE (100,4) I,GRIDX(I),GRIDY(I)
4 FORMAT ('GRID',4X,18,8X,2F8.3)
316 DO 100 I=1,6
100 WRITE (100,5) I,IE(I),JE(I),KE(I)
5 FORMAT ('CTRMEM',2X,18,7X,'1',318)
DO 110 I=19,22
110 WRITE (100,6) I,IE(I),JE(I),KE(I)
6 FORMAT ('CTRMEM',2X,18,7X,'2',318)
DO 120 I=7,12
120 WRITE (100,7) I, IE(I),JE(I),KE(I),LE(I)
7 FORMAT ('CQDMEM',2X,18,7X,'2',418)
DO 130 I=13,18
130 WRITE (100,8) I,IE(I),JE(I),KE(I),LE(I)
8 FORMAT ('CQDMEM',2X,18,7X,'1',418)
IF (1TYPE)380,139,380

```

C
C
C

SPC CARDS

```

139 I1=1
I2=34
I3=12
I4=30
I5=1
WRITE(100,12)I1,I2,I3,I4,I5
12 FORMAT('SPC'5X,318,8X,218)
I1=2
I2=35
I3=1
I4=29
I5=2
WRITE(100,12)I1,I2,I3,I4,I5
I1=3
I2=28
I3=2
I4=33
WRITE(100,12)I1,I2,I3,I4,I5
149 IF(NO-1)27,240,140
140 DC 141 I=1,35
141 GRIDY(I)=GRIDY(I)+1.73206*R
DO 150 I=1,27
II=I+35
150 WRITE(100,4)II,GRIDX(I),GRIDY(I)
II=63
WRITE(100,4)II,GRIDX(30),GRIDY(30)
II=64
WRITE(100,4)II,GRIDX(31),GRIDY(31)
II=65
WRITE(100,4)II,GRIDX(32),GRIDY(32)
II=66
WRITE(100,4)II,GRIDX(35),GRIDY(35)
DO 151 I=1,6
IE(I)=36
JE(I)=38+I-1
KE(I)=37+I-1
151 LE(I)=0
JE(6)=37
DO 161 I=7,12
IE(I)=37+I-7
JE(I)=44+I-7
KE(I)=56+I-7

```

```

160 LE(I)=49+I-7
    JE(12)=43
    KE(12)=55
    DO 170 I=13,18
    IE(I)=56+I-13
170 LE(I)=55+I-13
    IE(18)=55
    JE(13)=62
    JE(14)=26
    JE(15)=31
    JE(16)=63
    JE(17)=64
    JE(18)=61
    KE(13)=61
    KE(14)=62
    KE(15)=26
    KE(16)=31
    KE(17)=63
    KE(18)=64
    DO 180 I=19,22
180 LE(I)=0
    IE(19)=62
    IE(20)=26
    IE(21)=31
    IE(22)=63
    JE(19)=65
    JE(20)=32
    JE(21)=63
    JE(22)=64
    KE(19)=61
    KE(20)=62
    KE(21)=35
    KE(22)=66
    DO 200 I=1,6
    II=I+22
200 WRITE(100,5)II,IE(I),JE(I),KE(I)
    DO 210 I=19,22
    II=I+22
210 WRITE(100,6)II,IE(I),JE(I),KE(I)
    DO 220 I=7,12
    II=I+22
220 WRITE(100,7)II,IE(I),JE(I),KE(I),LE(I)
    DO 230 I=13,18
    II=I+22
230 WRITE(100,8)II,IE(I),JE(I),KE(I),LE(I)
    I1=4
    I2=63
    I3=1
    I4=66
    I5=1

C
C      SPC CARDS
C
    WRITE(100,12)I1,I2,I3,I4,I5
    IF(NO-1)27,240,241
240 WRITE(100,13)
    13 FORMAT('SPCADD'7X,'100'7X,'1'7X,'2'7X,'3')
    GO TO 131
241 WRITE(100,14)
    14 FORMAT('SPCADD'7X'100'7X'1'7X'2'7X'3'7X'4')
131 WRITE (100,9)

```


9 FORMAT ('ENDDATA')

STOP

C
C
C

REGULAR ELEMENT MODEL TWO

```
300 DO 310 I=2,7
310 GRIDX(I)=BHEX
    DO 311 I=8,13
311 GRIDX(I)=BHEX+F
    DO 312 I=14,19
312 GRIDX(I)=R
    JJ=1
    I=3
    GRIDY(2)=0
323 GRIDY(I)=GRIDY(I-1)+60.
    IF (I-1) 324,330,330
324 IF (JJ-5) 325,326,327
325 I=I+1
    JJ=JJ+1
    GO TO 323
326 I=I+1
    JJ=1
    GRIDY(I)=0.
    I=I+1
    GO TO 323
327 WRITE(10,3)
    STOP
```

C
C
C

DEVELOPE AND PUNCH GRID AND CONNECTION CARDS

```
330 DO 340 I=1,19
    RR=GRIDX(I)
    TH=(-GRIDY(I)+60.)*0.01745329
    GFIDX(I)=RR*COS(TH)+R
340 GRIDY(I)=RR*SIN(TH)+0.86603*R
    GFIDX(20)=2.*R
    GRIDX(21)=2.*R
    GRIDX(22)=0.
    GRIDX(23)=0.
    GRIDY(20)=1.73206*R
    GRIDY(21)=0.
    GRIDY(22)=0.
    GRIDY(23)=1.73206*R
    DO 350 I=1,6
    IE(I)=1
    JE(I)=3+I-1
    LE(I)=0
350 KE(I)=2+I-1
    JE(6)=2
    DO 360 I=7,18
    IE(I)=2+I-7
    JE(I)=3+I-7
    KE(I)=9+I-7
360 LE(I)=8+I-7
    JE(12)=2
    JE(18)=8
    KE(12)=8
    KE(18)=14
    DO 370 I=19,22
    KE(I)=20+I-19
370 LE(I)=0.
```

```

      IE(19)=14
      IE(20)=15
      IE(21)=17
      IE(22)=18
      JE(19)=15
      JE(20)=16
      JE(21)=18
      JE(22)=19
      DO416 I=1,NDEC
416  WRITE(100,11) (DECK(I,J),J=1,18)
      DO390 I=1,23
390  WRITE (100,4) I,GRIDX(I),GRIDY(I)
      GO TO 316
C
C      DEVELOPE AND PUNCH SPC CARDS
C
380  I1=1
      I2=22
      I3=12
      I4=18
      I5=1
      WRITE(100,12)I1,I2,I3,I4,I5
      I1=2
      I2=23
      I3=1
      I4=17
      I5=2
      WRITE(100,12)I1,I2,I3,I4,I5
      I1=3
      I2=16
      I3=2
      I4=21
      WRITE(100,12)I1,I2,I3,I4,I5
      GO TO 149
      END
C
C      EXAMPLE DATA DECK FOLLOWS
C
/*
125.      1.0      5.      126 1
ID POLYMER,MINERAL
APP DISP
SOL 1.0
TIME 10
CEND
TITLE = ONE HEX - 5 PERCENT MINERAL
SUBTITLE = INTERFACE = 1 , VARYING BOND REGULAR ELEMENT
ECHO = BOTH
SPCFORCES = ALL
OLOAD = ALL
ELSTRESS = ALL
DISPLACEMENT = ALL
SUBCASE 1
LABEL = ALPHA = MIN , E = MIN AND MU = 0.33
TEMPERATURE(LOAD)=1
SPC = 100
BEGIN BULK
GRDSET
PTRMEM      1      1      1.
PTRMEM      2      2      1.

```

PQDMEM	1	2	1.		
PQDMEM	2	3	1.		
MAT1	1	68.95		.22	1. 8.5E-6
MAT1	2	3.047		.44	1. 88.6E-6
MAT1	2	68.95		.33	1. 8.5E-6
TEMPD	1	100.			

TABLE I
Variables and their Values

Mineral Content by Volume	PER	5, 34 and 60%
Interface Elastic Modulus	E	E_p (3.047 GN/m ²), E_a (35.9985) and E_m (68.95)
Interface Thermal Coefficient	α	α_p ($88.6 \times 10^{-6}/^{\circ}\text{K}$) α_a (48.55×10^{-6}) and α_m (8.5×10^{-6})
Interface Thickness	F	1, 10, and 20 μm
Interface Poisson's Ratio	μ	μ_p (0.44), μ_a (0.33) and μ_m (0.22)
Unit Cell Length X-Direction	L_x	(1118.0, 428.7, 322.7) μm
Unit Cell Length Y-Direction	L_y	(968.2, 371.3, 279.5) μm

TABLE II
Effect of Poisson's Ratio

Average % Strain For $F = 10\mu\text{m}$

		μm	μa	μp
αm	$E\text{m}$	0.5136	0.5135	0.5134
αp	$E\text{m}$	0.5603	0.5621	0.5603
αm	$E\text{p}$	0.5249	0.5244	0.5234
αp	$E\text{p}$	0.5518	0.5538	0.5556

Average % Strain For $F = 1\mu\text{m}$

		μm	μa	μp
αm	$E\text{m}$	0.5491	0.5491	0.5491
αp	$E\text{m}$	0.5537	0.5540	0.5543
αm	$E\text{p}$	0.5502	0.5502	0.5501
αp	$E\text{p}$	0.5528	0.5531	0.5532

Notes:

All values are % strain averaged over all exterior nodes

Mineral content is 34% in model two

TABLE III Model Two
Interface Thickness = 1 μ m 5% Mineral

		X - Strain (%)			
		Node 21		Node 20	
		Disp.	Strain	Disp.	Strain
cm	Em	9.180648	.8211	8.849059	.7914
cm	Ea	9.181101	.8211	8.849708	.7915
cm	Ep	9.188110	.8218	8.859923	.7924
ca	Em	9.185943	.8216	8.856776	.7921
ca	Ea	9.185296	.8215	8.855821	.7920
ca	Ep	9.191171	.8220	8.864385	.7928
cp	Em	9.191225	.8220	8.864473	.7928
cp	Ea	9.189481	.8219	8.861919	.7926
cp	Ep	9.194225	.8223	8.868835	.7932
				9.074740	.8116
				9.075237	.8117
				9.083009	.8124
				9.080808	.8122
				9.080044	.8121
				9.086519	.8127
				9.086861	.8127
				9.084840	.8125
				9.090019	.8130

Lx = 1118.0 μ m

Ly = 968.2 μ m

		Y - Strain (%)			
		Node 23		Node 14	
		Disp.	Strain	Disp.	Strain
cm	Em	8.010276	.8272	7.727471	.7980
cm	Ea	8.010647	.8273	7.728028	.7981
cm	Ep	8.016359	.8279	7.736614	.7990
ca	Em	8.014427	.8277	7.733687	.7987
ca	Ea	8.013935	.8276	7.732952	.7986
ca	Ep	8.018758	.8281	7.740206	.7994
cp	Em	8.018567	.8281	7.739887	.7993
cp	Ea	8.017215	.8280	7.737863	.7991
cp	Ep	8.021150	.8284	7.743789	.7997
				7.733756	.7987
				7.734310	.7987
				7.742869	.7996
				7.739926	.7993
				7.739197	.7993
				7.746434	.8000
				7.746080	.8000
				7.744072	.7998
				7.749990	.8004
				7.922758	.8182
				7.923195	.8183
				7.929959	.8190
				7.927548	.8187
				7.92699	.8186
				7.932726	.8192
				7.932326	.8192
				7.930775	.8190
				7.935486	.8195

TABLE IV
Interface Thickness = 1 μ m
Model Two
34% Mineral

		X - Strain (%)			
		Node 15		Node 20	
		Disp.	Strain	Disp.	Strain
α m	Em	2.581950	.6022	2.084823	.4863
α m	Ea	2.582105	.6022	2.085121	.4863
α m	Ep	2.584319	.6028	2.090296	.4875
α a	Em	2.590835	.6043	2.097338	.4892
α a	Ea	2.589148	.6039	2.095041	.4886
α a	Ep	2.589499	.6040	2.097584	.4892
α p	Em	2.599698	.6064	2.109822	.4921
α p	Ea	2.596173	.6055	2.104936	.4910
α p	Ep	2.594666	.6052	2.104853	.4909

Lx = 428.7 μ m

Ly = 371.3 μ m

		Y - Strain (%)			
		Node 19		Node 14	
		Disp.	Strain	Disp.	Strain
α m	Em	2.322191	.6254	1.827982	.4923
α m	Ea	2.322403	.6255	1.828354	.4924
α m	Ep	2.325521	.6263	1.833783	.4939
α a	Em	2.329257	.6273	1.838657	.4925
α a	Ea	2.328002	.6270	1.836814	.4947
α a	Ep	2.329634	.6274	1.839994	.4955
α p	Em	2.336305	.6292	1.8349305	.4981
α p	Ea	2.333588	.6285	1.8452530	.4970
α p	Ep	2.333737	.6285	1.846189	.4972

		Node 20		Node 20	
		Disp.	Strain	Disp.	Strain
α m	Em	2.128779	.4878	2.128779	.5733
α m	Ea	2.129142	.4879	2.129142	.5734
α m	Ep	2.134732	.4895	2.134732	.5749
α a	Em	2.137257	.4907	2.137257	.5756
α a	Ea	2.135860	.4902	2.135860	.5752
α a	Ep	2.139659	.4912	2.139659	.5763
α p	Em	2.145714	.4936	2.145714	.5779
α p	Ea	2.142562	.4925	2.142562	.5770
α p	Ep	2.144574	.4929	2.144574	.5776

TABLE V Model Two

Interface Thickness = 1 μ m 60% Mineral

X - Strain (%)

	Node 21		Node 15		Node 20	
	Disp.	Strain	Disp.	Strain	Disp.	Strain
α m Em	1.455489	.4509	.7777187	.2408	1.342661	.4160
α m Ea	1.455686	.4510	.7780778	.2410	1.342902	.4160
α m Ep	1.458024	.4517	.7842753	.2430	1.345512	.4168
α a Em	1.465742	.4541	.7929214	.2456	1.353738	.4194
α a Ea	1.463813	.4535	.7901271	.2448	1.351682	.4188
α a Ep	1.464003	.4536	.7931280	.2457	1.351971	.4188
α p Em	1.475970	.4573	.8080860	.2503	1.364787	.4228
α p Ea	1.471919	.4560	.8021463	.2485	1.360439	.4215
α p Ep	1.469966	.4554	.8019585	.2484	1.358413	.4208

Lx = 322.7 μ mLy = 279.5 μ m

Y - Strain (%)

	Node 23		Node 19		Node 14		Node 20	
	Disp.	Strain	Disp.	Strain	Disp.	Strain	Disp.	Strain
α m Em	1.313377	.4698	.6586718	.2356	.6295804	.2252	.9786822	.3501
α m Ea	1.313512	.4699	.6590877	.2358	.6299332	.2253	.9790183	.3502
α m Ep	1.315411	.4706	.6646017	.2377	.6354205	.2273	.9846786	.3522
α a Em	1.321870	.4729	.6719461	.2404	.6430672	.2300	.9896197	.3540
α a Ea	1.320244	.4723	.6696079	.2395	.6406222	.2291	.9876866	.3533
α a Ep	1.320363	.4723	.6723304	.2405	.6432737	.2301	.9910420	.3545
α p Em	1.330343	.4759	.6851872	.2451	.6565203	.2348	1.0005300	.3579
α p Ea	1.326959	.4747	.6801019	.2433	.6512846	.2330	.9963333	.3564
α p Ep	1.325304	.4741	.6800399	.2432	.6511073	.2329	.9973896	.3568

Model Four

34% Mineral

	X - Strain (%)																			
	Node 4				Node 8				Node 15				Node 22				Node 26			
	Disp.		Strain		Disp.		Strain		Disp.		Strain		Disp.		Strain		Disp.		Strain	
om Em	1.866125	.4353	1.863651	.4347	1.865150	.4350	1.864674	.4349	1.881719	.4389										
om Ea	1.877305	.4379	1.873785	.4370	1.873817	.4370	1.875932	.4375	1.897558	.4426										
om Ep	1.891039	.4411	1.885982	.4399	1.882969	.4392	1.893998	.4418	1.919856	.4478										
oa Em	.961881	.2244	.953011	.2223	.854874	.1994	.949353	.2214	.935087	.2181										
oa Ea	.948323	.2212	.936498	.2184	.802615	.1872	.929207	.2167	.912071	.2127										
oa Ep	.940095	.2193	.922438	.2152	.700744	.1634	.901921	.2104	.884370	.2063										
op Em	1.737150	.4052	1.719656	.4011	1.523232	.3553	1.712238	.3994	1.682002	.3923										
op Ea	1.708915	.3986	1.685618	.3932	1.417848	.3307	1.670821	.3897	1.634386	.3812										
op Ep	1.691086	.3944	1.656277	.3863	1.213191	.2830	1.614441	.3766	1.576755	.3678										

$$L_x = 428.7 \mu\text{m}$$
$$L_y = 371.3 \mu\text{m}$$

	Node ₂₃			Node ₂₄			Y - Strain (%)			Node ₂₅			Node ₂₆		
	Disp.	Strain		Disp	Strain		Disp.	Strain		Disp.	Strain		Disp.	Strain	
αm Em	3.235837	.8715		3.269108	.8804		3.244813	.8739		3.215430	.8660				
αm Ea	3.263461	.8789		3.299375	.8886		3.268303	.8802		3.231418	.8703				
αm Ep	3.346422	.9013		3.360812	.9051		3.311613	.8919		3.269374	.8805				
αa Em	1.578804	.4252		1.529106	.4418		1.594495	.4294		1.633722	.4400				
αa Ea	1.492672	.4020		1.441762	.3883		1.530823	.4123		1.579264	.4253				
αa Ep	1.308284	.3523		1.259974	.3393		1.397209	.3763		1.457008	.3924				
αp Em	2.834025	.7633		2.731302	.7356		2.864509	.7715		2.945901	.7934				
αp Ea	2.658998	.7161		2.553586	.6877		2.734815	.7365		2.835386	.7636				
αp Ep	2.281927	.6146		2.183868	.5882		2.463257	.6634		2.587078	.6968				

TABLE VIII Analysis of Variance

Factor	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
Per cent Mineral (PER)	679.53556	2	339.76778	2472.8
Location of Strain	55.002587	6	9.1670972	66.7
Interface Thermal Coef. (α)	0.0109865	2	0.00549324	0.04
Interface Modulus (E)	0.0169027	2	0.0084513	0.06
Inter Error	24.182964	176	0.1374032	1.0
	all x 10^{-6}		all x 10^{-6}	

Notes:

Both Y- and X- Strains are included

Interface thickness is $1\mu\text{m}$

Model 2

Data from Tables III, IV, & V

TABLE IX Analysis of Variance

Factor	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
Interface Thickness (F)	0.59049875	1	0.59049875	27.4
Strain Location	37.240708	6	6.2067838	288.2
Interface Thermal Coef. (α)	0.9876678	2	0.49383391	22.9
Interface Modulus (E)	0.00863101	2	0.00431550	.2
Inter Error	2.455409	114	0.0215387	1.0
	all x 10^{-6}		all x 10^{-6}	

Notes:

Per cent mineral is 34%

Model 2

Data from Tables IV & VI

TABLE X Analysis of Variance

Factor	Sum of Squares	Degrees of Freedom	Mean Squares	Computed F
Model	2.3122947	1	2.31229	1.4
Location of Strain	89.936526	6	14.98942	8.8
Interface Thermal Coef. (α)	76.152995	2	38.62117	22.8
Interface Modulus (E)	0.7774556	2	0.388727	0.2
Inter Error	193.208262	114	1.694809	1.0
	all x 10^{-6}		all x 10^{-6}	

Notes:

Models two and four are compared

Interface thickness is $1\mu\text{m}$

Per cent mineral is 34%

Data from Tables IV & VII

TABLE XI Model Two

Effect of Boundary Conditions

Interface Thickness = 1 μ m 34% Mineral

		X - Strain (%)					
		Node 21			Node 20		
		Disp.	Strain	Disp.	Strain	Disp.	Strain
α p	Ep	2.612068	.6092	2.439823	.5691	0	0
α p	Ep	2.487895	.5803	2.466908	.5754	-.4721402	-.1101
α p	Ep	2.598213	.6053	2.484349	.5794	0	0
α m	Em	2.598778	.6061	2.425600	.5657	0	0
α m	Em	2.473490	.5769	2.453071	.5722	-.4798611	-.1119
α m	Em	2.582011	.6022	2.470542	.5762	0	0

Lx = 428.7 μ mLy = 371.3 μ m

		Y - Strain (%)					
		Node 23			Node 20		
		Disp.	Strain	Disp.	Strain	Disp.	Strain
α p	Ep	2.249300	.6058	2.157036	.5809	-.4625057	-.1246
α p	Ep	2.359073	.6353	2.057258	.5541	0	0
α p	Ep	2.334413	.6287	2.145183	.5777	0	0
α m	Em	2.236905	.6024	2.140730	.5765	-.4693512	-.1264
α m	Em	2.347228	.6322	2.039707	.5493	0	0
α m	Em	2.322228	.6254	2.128804	.5733	0	0

X - Node 22 x displacement released
Y - Node 22 y displacement released
O - Node 22 fixed

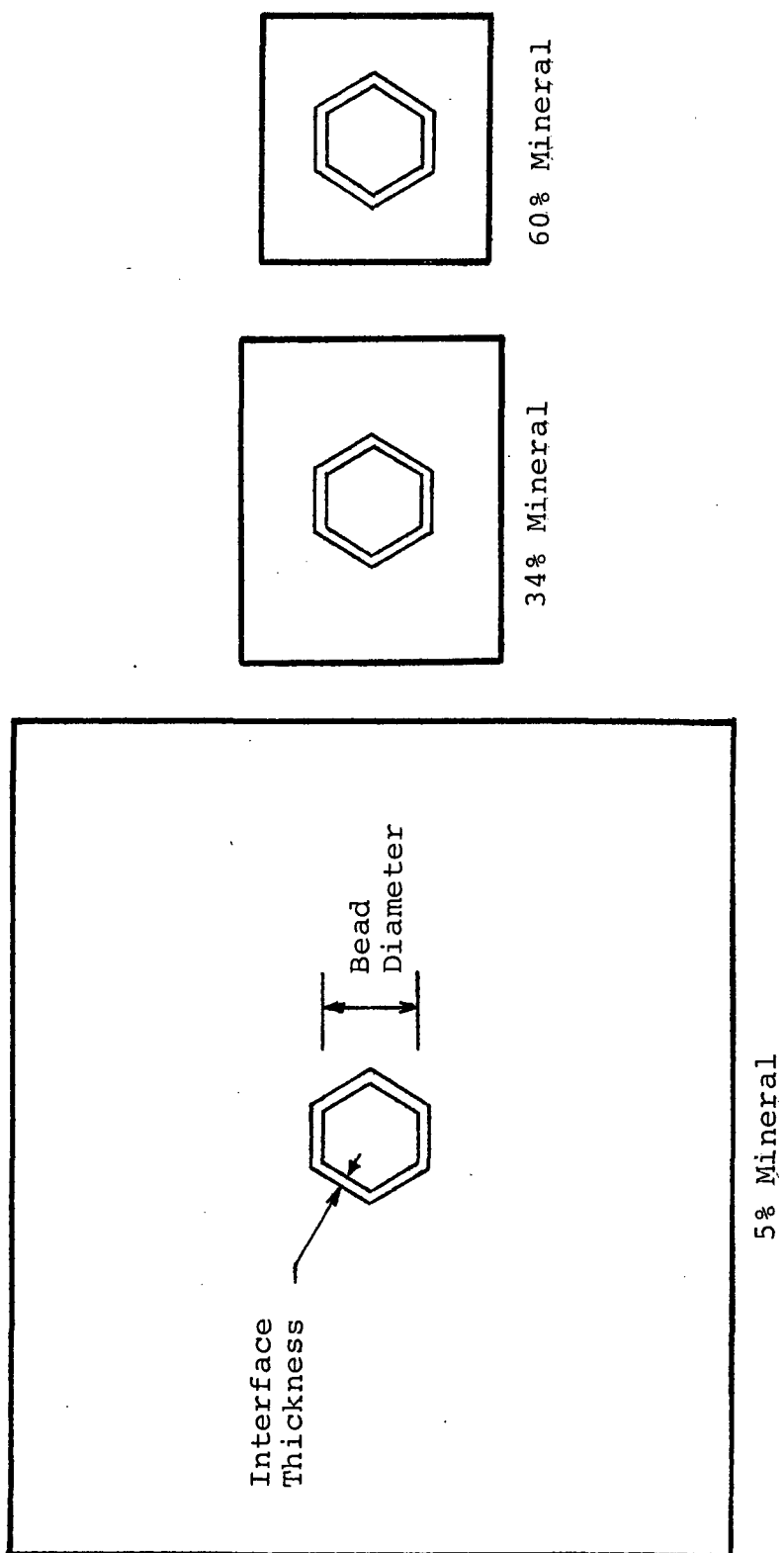


Figure 1 Basic Unit Cell Model of Polymer - Mineral Composite

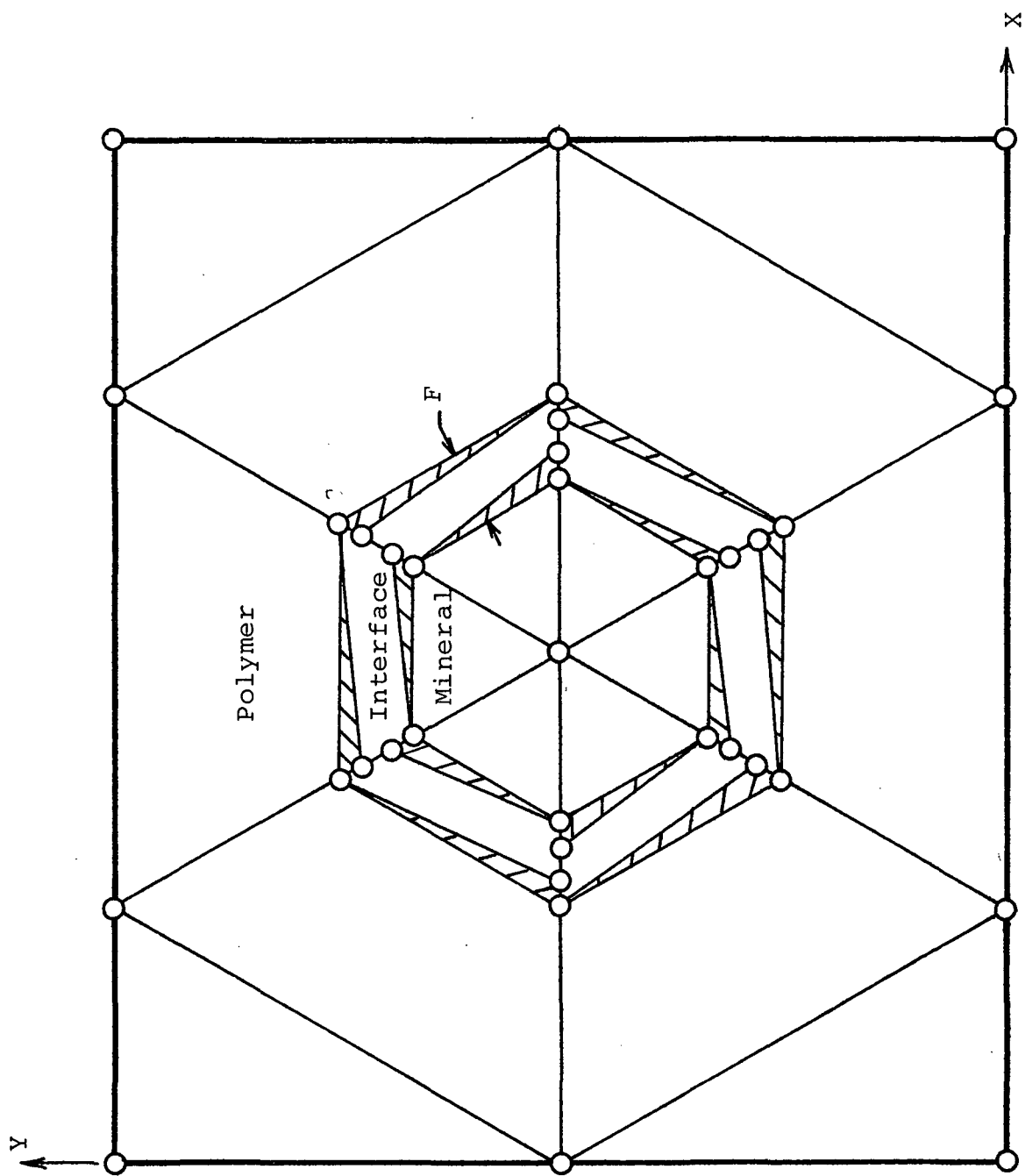


Figure 2 Model One

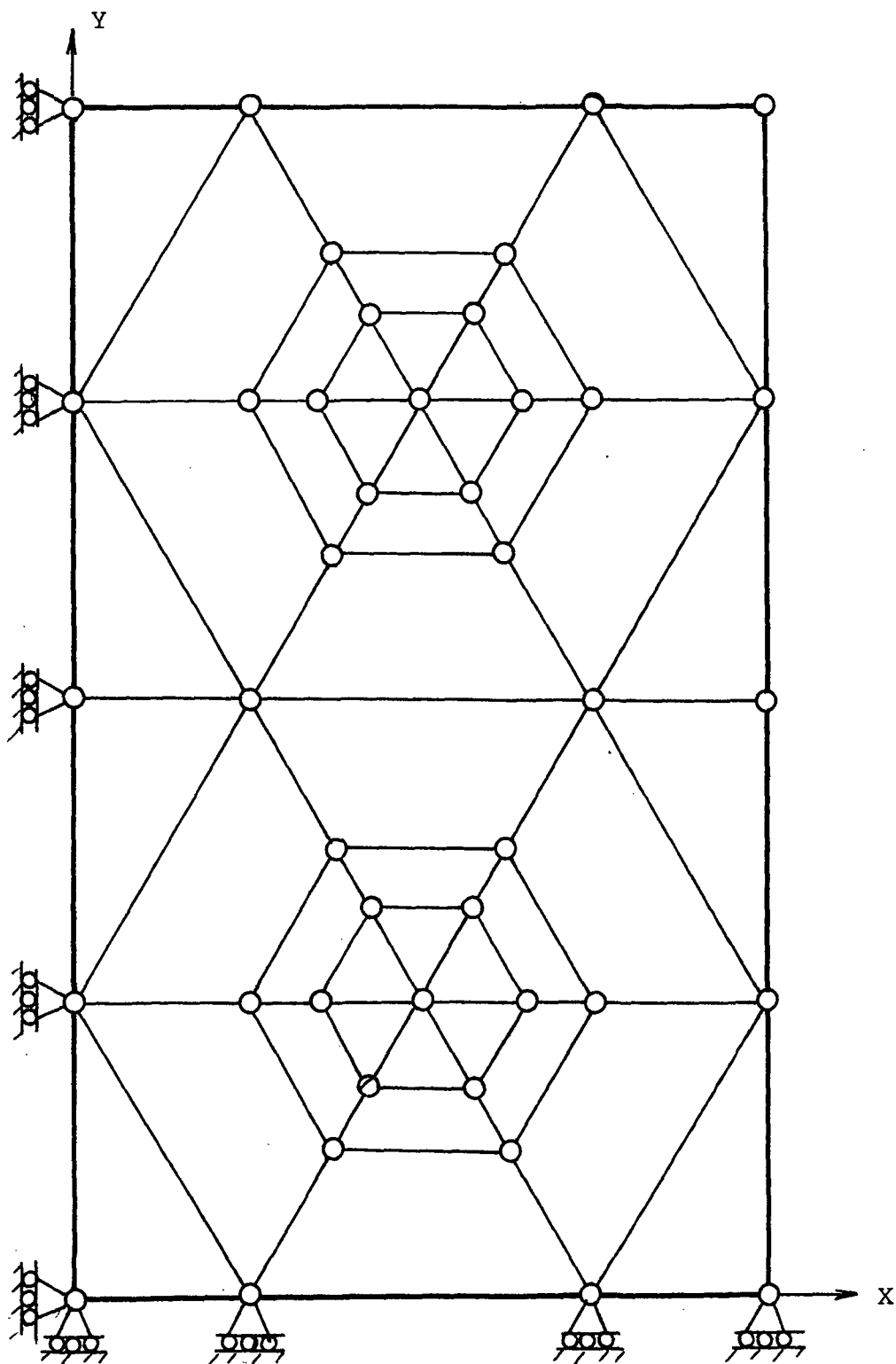


Figure 4 Model Three

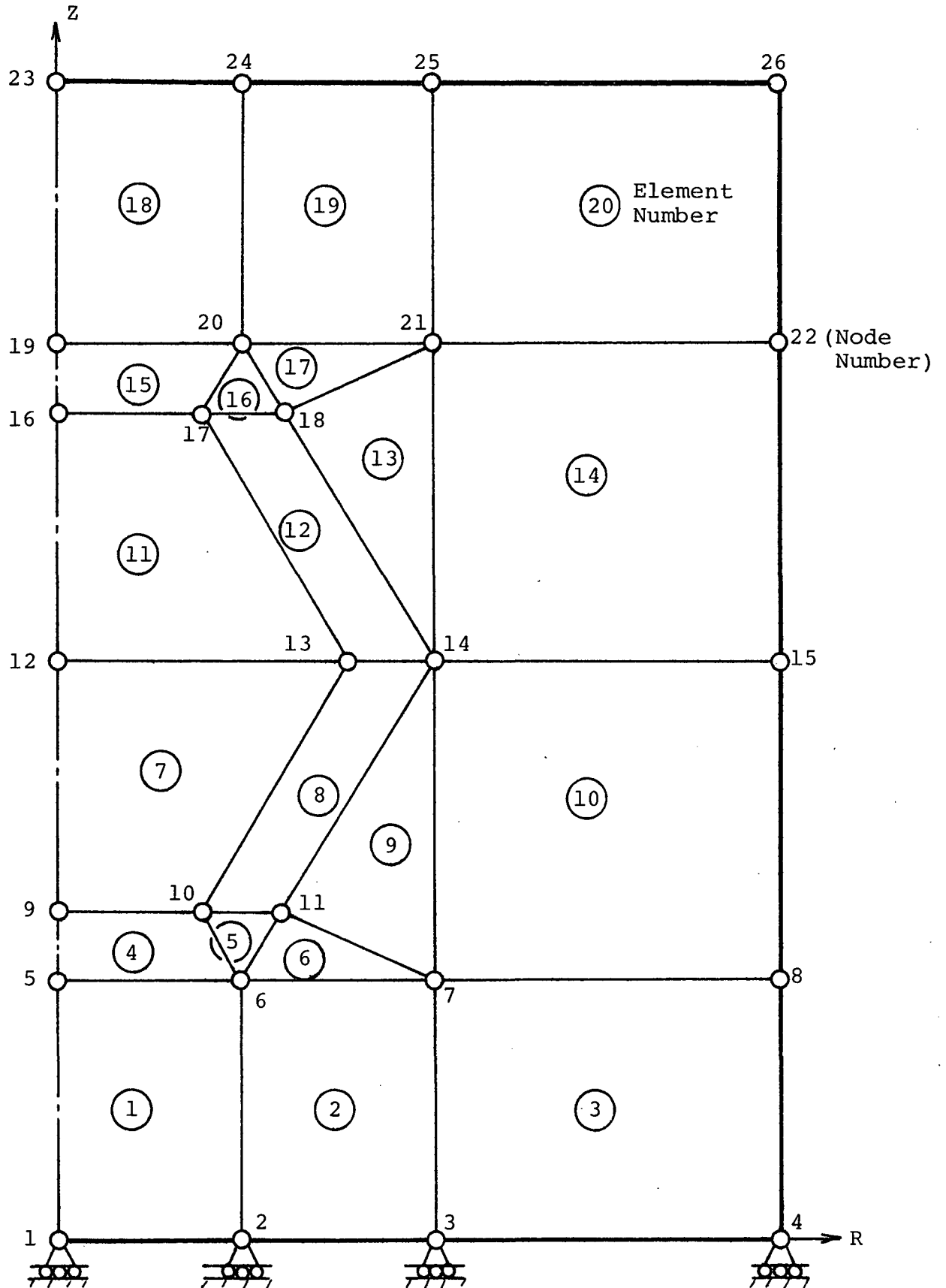


Figure 5 Model Four (Axisymmetric)

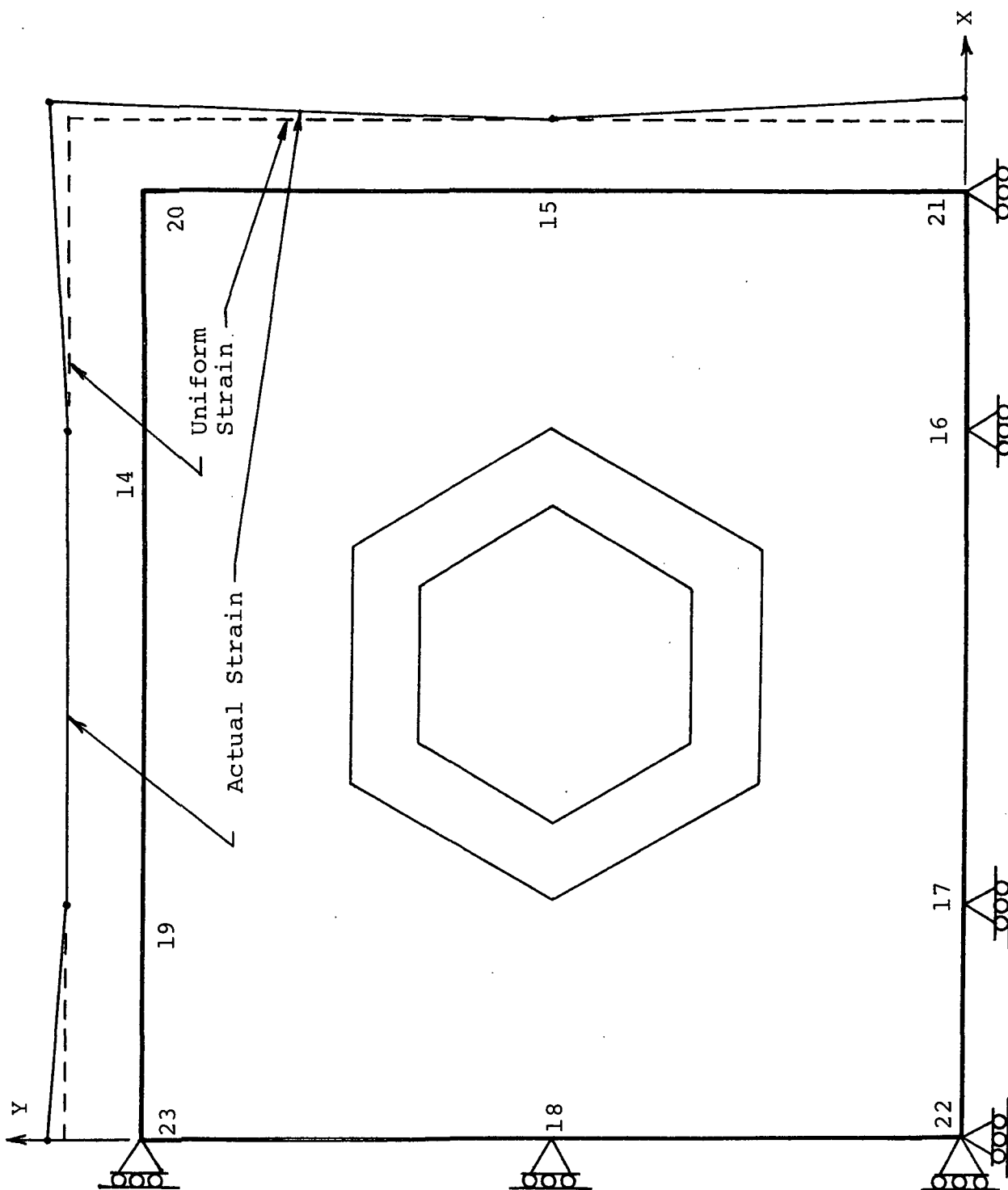


Figure 6 Non Uniform Strain