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APPLICATION OF CROSSED BEAM TECHNOLOGY TO DIRECT MEASUREMENTS OF SOUND SOURCES IN TURBULENT JETS (Part I)

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16. ABSTRACT

The mathematical bases for the direct measurement of sound source intensities in turbulent jets using the crossed-beam technique are discussed in detail. It is found that the problems associated with such measurements lie in three main areas: (1) measurement of the correct flow covariance, (2) accounting for retarded time effects in the measurements, and (3) transformation of measurements to a moving frame of reference. The determination of the particular conditions under which these problems can be circumvented is the main goal of the reported study.

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FOREWORD

This report was prepared by the IIT Research Institute for the National Aeronautics and Space Administration, Marshall Space Flight Center, on Contract NAS8-21035. The effort described here constitutes an analysis of applications of crossed-beam technology which is a part of the overall crossed-beam development program. This report includes an assessment of the ability to obtain direct measurements of sound source intensities in turbulent jets using the crossed-beam technique. The experimental activity and hardware development are described in separate reports.

Respectfully submitted,
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CONTENTS

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a_o	ambient sound speed
$B(\underline{X},t,\tau*)$	acoustic intensity autocovariance, equation (3.6)
e_{ij}	rate of strain tensor
$E(\underline{X},t,\omega)$	acoustic energy density, equation (3.6.1)
$G_E(\underline{Y},\underline{\Delta},T)$	Eulerian corssed-beam covariance
$G_L(\underline{Y},\underline{\lambda},T)$	Lagrangian crossed-beam covariance
$G_S(\underline{Y},\underline{\Delta},T)$	Schlieren covariance
$H_{ijk\ell}(\underline{Y},\kappa,\omega)$	spectral density transform of $P_{ijk\ell}$, equation (3.18)
i_A, i_B	detector signal fluctuations, beams A and B
$\overline{I}_A, \overline{I}_B$	mean detector signals, beams A and B
I_o	detector signal, no extinction
k	fluctuating extinction coefficient
K	mean extinction coefficient
ℓ	distance variable
L	turbulent scale length
$\underline{M}$	convection Mach number vector
M	$|\underline{M}|$
p	pressure
P_{ij}	stress tensor
P_r	Lagrangian scalar covariance
$P_{ijk\ell}(\underline{Y},\underline{\lambda},\tau)$	Lagrangian stress tensor covariance
$\underline{r}$	vector distance from disturbance to observer
r	$|\underline{r}|$
r_i	component of $\underline{r}$
$R_E(\underline{Y},\underline{\Delta},T)$	Eulerian scalar density covariance

$R_{ijk\ell}(\underline{Y},\underline{\Delta},\tau)$	Eulerian stress tensor covariance
$S(\underline{Y},t,\tau^*)$	sound source intensity
t	time
T_{ij}	Lighthill stress tensor
u_i	mean velocity
u_r	velocity component in observer's direction
U_c	mean convection speed
U_s	convection speed
$\underline{v}$	velocity vector
v_i	component of $\underline{v}$
v_r	component of $\underline{v}$ in direction $\underline{r}$
V	turbulence volume
x_i	component of observer's position
$\underline{X}$	vector locating observer
y_i	component of turbulence position
y_r	component of $\underline{Y}$ in observer's direction
y_s	component of $\underline{Y}$ normal to $\underline{r}$
$\underline{Y}$	vector locating turbulence source
$\underline{Z}$	vector locating turbulence source
α	frequency parameter
$\underline{\Delta}$	$(\underline{Y} - \underline{Z})$ separation between points in turbulence
$\underline{\Delta}_n$	component of $\underline{\Delta}$ normal to flow direction
$\underline{\Delta}_s$	component of $\underline{\Delta}$ normal to $\underline{r}$
θ	angle between $\underline{M}$ and $\underline{r}$
$\underline{\kappa}$	wave number vector
λ_r	Lagrangian distance variable in $\underline{r}$ direction

$\underline{\lambda}$ Lagrangian separation between points in turbulence

$\underline{\lambda}_n$ component of $\underline{\lambda}$ normal to $\underline{M}$

$\underline{\lambda}_s$ component of $\underline{\lambda}$ normal to $\underline{r}$

ρ density

ρ_o ambient density

σ Lagrangian time variable

τ retarded time difference

τ^* arbitrary time delay in sound source

T arbitrary time delay in crossed-beam signals

ω frequency

Operators

∇^2 Laplacian

$\square^2$ D'Alembertian

$\doteqdot$ indicates equality of two functions after integration

$\langle\ \rangle$ time average

APPLICATION OF CROSSED-BEAM TECHNOLOGY
TO DIRECT MEASUREMENTS OF SOUND SOURCES IN TURBULENT JETS

I. INTRODUCTION

The crossed-beam technique has been shown[1,2] to perform an area integration of correlations very similar to integrals appearing in the source term of the equations for turbulent noise production. This suggests then, that there is a possibility of applying crossed-beam techniques to the direct measurement of local sound source intensities in turbulent jets. The advantages of being able to do so are obvious: it becomes possible to study the basic nature of noise production and the effects of various noise reduction schemes upon the source itself. Indirect inference of sound source characteristics from far-field acoustic measurements can never rest on very firm ground.

This report represents a study of the mathematical concepts involved in applying crossed-beam technology to source measurements in order to indicate the conditions under which such an application is possible. For this purpose, much of this report is devoted to a redevelopment of basic concepts conceived by others. This is necessary to give the reader the required background and to present the pertinent equations in forms accessible to crossed-beam comparisons. The basic crossed-beam equations have been developed in several places, and are reviewed briefly in the following section. The approach to noise generation parallels quite closely to Ffowcs-Williams[3] four-dimensional Fourier transform approach since the equations in that form are manipulated readily for our purpose and point up the physical characteristics clearly.

II. CROSSED-BEAM PRINCIPLE

The operation of the crossed-beam technique has been described elsewhere,[4,5] and will therefore be discussed only briefly here. In principle, two narrow beams of radiation at selected wavelengths are positioned such that they intersect at a point in the region of interest in the jet. If the beam intensity of each beam is modulated by a process which is linearly related to the local gas density fluctuations, then the intensities can be expressed, apart from a calibration constant, by

$$i_A = \overline{I}_A \int \rho \, dy_2 \qquad (2.1)$$

$$i_B = \overline{I}_B \int \rho \, dy_3 \qquad (2.2)$$

where i_A, i_B, $\overline{I}_A$ and $\overline{I}_B$ are the fluctuating and mean signals, respectively, for each of the two beams A and B. If the two signals are cross-correlated over a sufficiently long integration time, the product mean value $\langle i_A \, i_B \rangle$ will give information concerning only the fluctuations common to each beam (i.e., in the intersection region). In this way, local information on turbulence has been obtained without the necessity of inserting physical probes into the flow field. From a more intuitive viewpoint, we might consider turbulence as consisting of discrete eddies that are slightly more or less transparent to the beams of radiation than the surroundings. As an eddy passes through a beam of radiation, the light intensity at the detector changes. When the two detector outputs are cross-correlated, only the information concerning eddies passing through the beam intersection point is retained. Signal fluctuations due to source instabilities, detector noise, etc., are normally uncorrelated, although these contributions do affect the overall signal-to-noise ratio of the system.

Cross-correlation of equations (2.1) and (2.2) gives

$$G_E(\underline{Y}, \Delta_1, T) = \frac{\langle i_A\, i_B \rangle}{\overline{I}_A\, \overline{I}_B} = \iint \langle \rho(y_1, y_2 + \Delta_2, y_3, t)\, \rho(y_1$$

$$+ \Delta_1, y_2, y_3 + \Delta_3, t + T) \rangle\; d\Delta_2\, d\Delta_3 \tag{2.3}$$

where i_A and i_B are correlated with an arbitrary time delay between signals, and Δ_1 is a displacement of beam B relative to beam A in the direction of their common normal. This equation expresses the correlation between two line integrals. If we can write equation (2.3) as

$$G_E(\underline{Y}, \Delta_1, T) = \iint \langle \rho(y_1, y_2, y_3, t)\, \rho(y_1 + \Delta_1, y_2 + \Delta_2, y_3$$

$$+ \Delta_3, t + T) \rangle\; d\Delta_2\, d\Delta_3$$

$$= \int R_E(\underline{Y}, \underline{\Delta}, T)\; d\underline{\Delta}_n \tag{2.4}$$

where $d\underline{\Delta}_n = d\Delta_2\, d\Delta_3$ or, equivalently as

$$G_E(\underline{Y}, \underline{\Delta}, T) = \left\langle \rho(\underline{Y}, t) \int \rho(\underline{Y} + \underline{\Delta}, t + T) \right\rangle\; d\underline{\Delta}_n \tag{2.5}$$

we are then dealing with the correlation between the area integral of points in a plane ($y_1 + \Delta_1$ = constant) with an outside point defined by (y_1, y_2, y_3). Such a cross-correlation has been assigned the term "point-area correlation," and is similar to the type occurring in turbulent sound source integrals.

If the turbulent flow is everywhere homogeneous, then the point-area formulation is obviously proper. If departures from homogeneity are pure odd functions of space coordinates about the point (y_1, y_2, y_3), then these departures tend to integrate out and the point-area concept is still valid.[5] In any event, the final proof of the point-area correlation assumption can come only from experiment. These experiments are the subject of a separate study.

III. THE EQUATIONS GOVERNING TURBULENT NOISE PRODUCTION

Lighthill[6,7] has developed the theory of aerodynamic noise by the use of an acoustical analogy, rearranging the exact equations of motion into the form of the inhomogeneous wave equation with a source term. If there are no sources of mass or external forces present, the source term becomes quadrupole in nature, existing in the form of the double divergence of momentum flux.

The wave equation becomes

$$\Box^2 \rho = \frac{\partial^2 \rho}{\partial t^2} - a_o^2 \, \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \tag{3.1}$$

where

$$T_{ij} = \rho v_i v_j + P_{ij} - a_o^2 \, \rho \delta_{ij} \tag{3.2}$$

from the exact equations of motion and represents the quadrupole strength. This equation can be written

$$(\rho - \rho_o)(\underline{X}, \, t) = \frac{1}{4\pi \, a_o^2} \int \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} (\underline{Y}, t - \frac{|\underline{X} - \underline{Y}|}{a_o}) \, \frac{d\underline{Y}}{|\underline{X} - \underline{Y}|} \tag{3.3}$$

Equation (3.3) is actually only the integral form of equation (3.1), not a true solution, since the stress tensor T_{ij} is itself a function of ρ. The only way a solution can be obtained is to approximate T_{ij} by terms independent of the density appearing in $\Box^2 \rho$. The foremost difficulty in using equation (3.3) for noise calculations, is in fact choosing the approximation for T_{ij} appropriate to the problem of interest.

The "solution" equation (3.3) is given with the integrand to be evaluated at the retarded time $|\underline{X} - \underline{Y}|/a_o$ which is the time difference between time of emission at $\underline{Y}$ and time the

sound wave reaches the observer at $\underline{X}$. This retarded time effect is the major stumbling block in casting the noise equation in a form identical with the crossed-beam equation. However, it must be pointed out that if retarded-time effects did not exist, equation (3.3) would integrate directly to zero for the far field (large $|\underline{X} - \underline{Y}|$) as will be shown later. Physically, this says that the quadrupole (or dipole) type of sound source creates a far-field disturbance only because of the phase mismatch which results from the varying distances to individual disturbances. This is the Stokes effect of classical acoustics.

Equation (3.3) can be reformulated in a more convenient way, and one which brings out the above points more clearly by separating out the retarded time variable using the definition of a delta function, i.e.,

$$(\rho - \rho_o)(\underline{X}, t) = \frac{1}{4\pi \, a_o^2} \int_V \int_{-\infty}^{+\infty} \frac{\partial^2 T_{ij}(\underline{Y}, t)}{\partial y_i \partial y_j} \frac{\delta(t - \tau_1)}{r} \, d\underline{Y} \, dt$$

(3.4)

where

$$\tau_1 = \frac{|\underline{X} - \underline{Y}|}{a_o} = \frac{r}{a_o} \quad \text{is a function of } \underline{Y}.$$

Rearranging terms,

$$(\rho - \rho_o)(\underline{X}, t) = \frac{1}{4\pi \, a_o^2} \iint \frac{\partial}{\partial y_i} \left(\frac{\delta}{r} \frac{\partial T_{ij}}{\partial y_j} \right) \, d\underline{Y} \, dt$$

$$- \iint \frac{\partial T_{ij}}{\partial y_j} \frac{\partial}{\partial y_i} \left(\frac{\delta}{r} \right) \, d\underline{Y} \, dt$$

Now the first term can be converted to a surface integral of

$$\frac{\delta}{r} \frac{\partial T_{ij}}{\partial y_j}$$

by the divergence theorem. If there are no surfaces present in the flow, this must vanish at infinity.

If we repeat this process we get

$$(\rho - \rho_o)(\underline{X}, t) = \frac{1}{4\pi \, a_o^2} \iint T_{ij} \frac{\partial^2}{\partial y_i \partial y_j} \left(\frac{\delta}{r}\right) d\underline{Y} \, dt$$

Differentiating by parts

$$\frac{\partial}{\partial y_i} \left(\frac{\delta}{r}\right) = \frac{\delta r_i}{r^3} - \frac{1}{a_o} \frac{\delta' r_i}{r^2}$$

where $\delta' = d\delta/dr$. Taking $\partial/\partial y_j$ of this and substituting into the above equation, we obtain

$$(\rho - \rho_o)(\underline{X}, t) = \frac{1}{4\pi a_o^4} \int \frac{\partial^2 T_{ij}(\underline{Y}, \, t + \frac{r}{a_o})}{\partial t^2} \frac{r_i r_j}{r^2} d\underline{Y}$$

where we have dropped terms of order $1/r^2$ and less (near field terms). This may be expressed as

$$(\rho - \rho_o)(\underline{X}, t) = \frac{1}{4\pi \, a_o^4} \int \frac{(x_i - y_i)(x_j - y_j)}{|\underline{X} - \underline{Y}|^3} \frac{\partial^2 T_{ij}(\underline{Y}, \, \tau_1)}{\partial t^2} d\underline{Y}$$

(3.5)

The autocovariance of the far field density fluctuations is then obtained by "squaring" equation (3.5) and averaging in time:

$$\left\langle (\rho - \rho_o)(\underline{X}, t)(\rho - \rho_o)(\underline{X}, t + \tau*) \right\rangle = B(\underline{X}, t, \tau*)$$

$$= \frac{1}{16\pi^2 a_o^8} \iint \frac{(x_i - y_i)(x_j - y_j)(x_k - z_k)(x_\ell - z_\ell)}{|\underline{X} - \underline{Y}|^3 |\underline{X} - \underline{Z}|^3} \times$$

$$\left\langle \frac{\partial^2 T_{k\ell}(\underline{Y}, \tau_1)}{\partial t^2} \frac{\partial^2 T_{k\ell}(\underline{Z}, \tau_2 + \tau*)}{\partial t^2} \right\rangle d\underline{Y} \, d\underline{Z} \qquad (3.6)$$

$\tau*$ is an arbitrary time delay, introduced only to be able to express B as an autocovariance. The acoustic energy density is then obtained from the Fourier transform of B

$$E(\underline{X}, t, \omega) = \int B(\underline{X}, t, \tau*) e^{-i\omega\tau*} \, d\tau* \qquad (3.6.1)$$

Substituting

$$\tau = \tau_2 + \tau* - \tau_1 = \tau* + \frac{|\underline{X} - \underline{Y}| - |\underline{X} - \underline{Z}|}{a_o}$$

$$= \tau* + \frac{\underline{\Delta} \cdot (\underline{X} - \underline{Y})}{a_o |\underline{X} - \underline{Y}|} \qquad (3.6.2)$$

and assuming $\underline{\Delta} \ll |\underline{X} - \underline{Y}|$ in equation (3.6),

$$B(\underline{X}, t, \tau*) = \frac{1}{16\pi^2 a_o^8} \iint \frac{(x_i - y_i)(x_j - y_j)(x_k - y_k)(x_\ell - y_\ell)}{r^6} \times$$

$$\frac{\partial^4 R_{ijk\ell}(\underline{Y}, \underline{\Delta}, \tau)}{\partial \tau^4} \, d\underline{\Delta} \, d\underline{Y} \qquad (3.7)$$

where

$$R_{ijk\ell}(\underline{Y}, \underline{\Delta}, \tau) = \left\langle T_{ij}(\underline{Y}, \tau_1) T_{k\ell}(\underline{Y} + \underline{\Delta}, \tau_1 + \tau) \right\rangle \qquad (3.7.1)$$

and is to be evaluated at the retarded time τ. τ is a function of $\underline{\Delta}$ and is, consequently a dependent variable. It must be treated as such in any manipulation of equation (3.7).

In a turbulent flow, the radiating quadrupoles are not at rest and move during the retarded time τ . This motion is correctly accounted for in equation (3.7) since the correlation tensor $R_{ijk\ell}$, measured in the frame of the observer includes the convection effects. If we consider the time derivative of such a correlation, we find there are two contributors, the true time change and the change due to convection of space derivatives. In most turbulent flows, it is the latter which dominates, i.e., apparent changes with time are due to the instantaneous convection of space derivatives. Even though these space derivatives may be very large they do not contribute to the integral of equation (3.7). This can be proven in the fashion discussed earlier by expressing volume integrals of divergence terms as area integrals which must vanish for large $\underline{\Delta}$ if

$$\lim_{\underline{\Delta} \to \infty} R_{ijk\ell}(\underline{Y}, \underline{\Delta}, \tau) = 0.$$

Thus, in order to present equation (3.7) in a form useful for noise prediction we must write it such that pure time effects are maximized. This can be done by introducing a moving frame of reference in which time scales are maximized. This of course is the frame of reference moving at the convection speed of the covariance. It must be stressed that this only minimizes the effects of convection of derivatives. They can be zero only in a frame which moves instantaneously with the fluid. The contribution of the velocity fluctuations poses another problem to us in the measurement of sound source strengths.

We note here that if a reference frame exists in which

$$\frac{\partial R_{ijk\ell}}{\partial t} = 0$$

then there can be no sound. This means that a convected field of "frozen" turbulence does not radiate sound. There are some very important exceptions to this and these will be discussed later.

Transforming to a moving axis using

$$\underline{\Delta} = \underline{\lambda} + a_o \underline{M} \tau \tag{3.8}$$

where $\underline{M}$ is the convection Mach number measured relative to $\underline{X}$, we define a moving-axis correlation tensor by

$$P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau) = R_{ijk\ell}(\underline{Y}, \underline{\Delta}, \tau) \tag{3.9}$$

The retarded time (see sketch of Figure 1) is then given by

$$a_o \tau = a_o \tau^* + (\underline{\lambda} + a_o \underline{M} \tau) \cdot \frac{(\underline{X} - \underline{Y})}{|\underline{X} - \underline{Y}|} \tag{3.10}$$

or

$$\tau = \frac{\underline{\lambda} \cdot (\underline{X} - \underline{Y}) + a_o \tau^* |\underline{X} - \underline{Y}|}{a_o \left\{ |\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y}) \right\}}$$

The time derivatives are related by

$$\frac{\partial R_{ijk\ell}}{\partial t} = \left\{ \frac{\partial}{\partial t} - a_o M_n \frac{\partial}{\partial \lambda_n} \right\} P_{ijk\ell} \tag{3.11}$$

Recall here, that τ is a dependent variable ($\tau = \tau(\underline{\lambda})$) and so $\partial/\partial\lambda_n$ includes some time effects; it is not entirely a space gradient. These effects are determined using equation (3.10) as follows:

$$\frac{\partial}{\partial \lambda_n} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau(\underline{\lambda})) = \left\{ \frac{\partial}{\partial \lambda_n} + \frac{\partial \tau}{\partial \lambda_n} \frac{\partial}{\partial \tau} \right\} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau)$$

$$= \left\{ \frac{\partial}{\partial \lambda_n} + \frac{x_n - y_n}{a_o \left[|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y}) \right]} \frac{\partial}{\partial \tau} \right\} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau)$$

$$\tag{3.12}$$

$$\underline{\Delta} \cdot \frac{(\underline{X} - \underline{Y})}{r} = (\lambda + a_o \underline{M} \tau) \cdot \frac{(\underline{X} - \underline{Y})}{r}$$
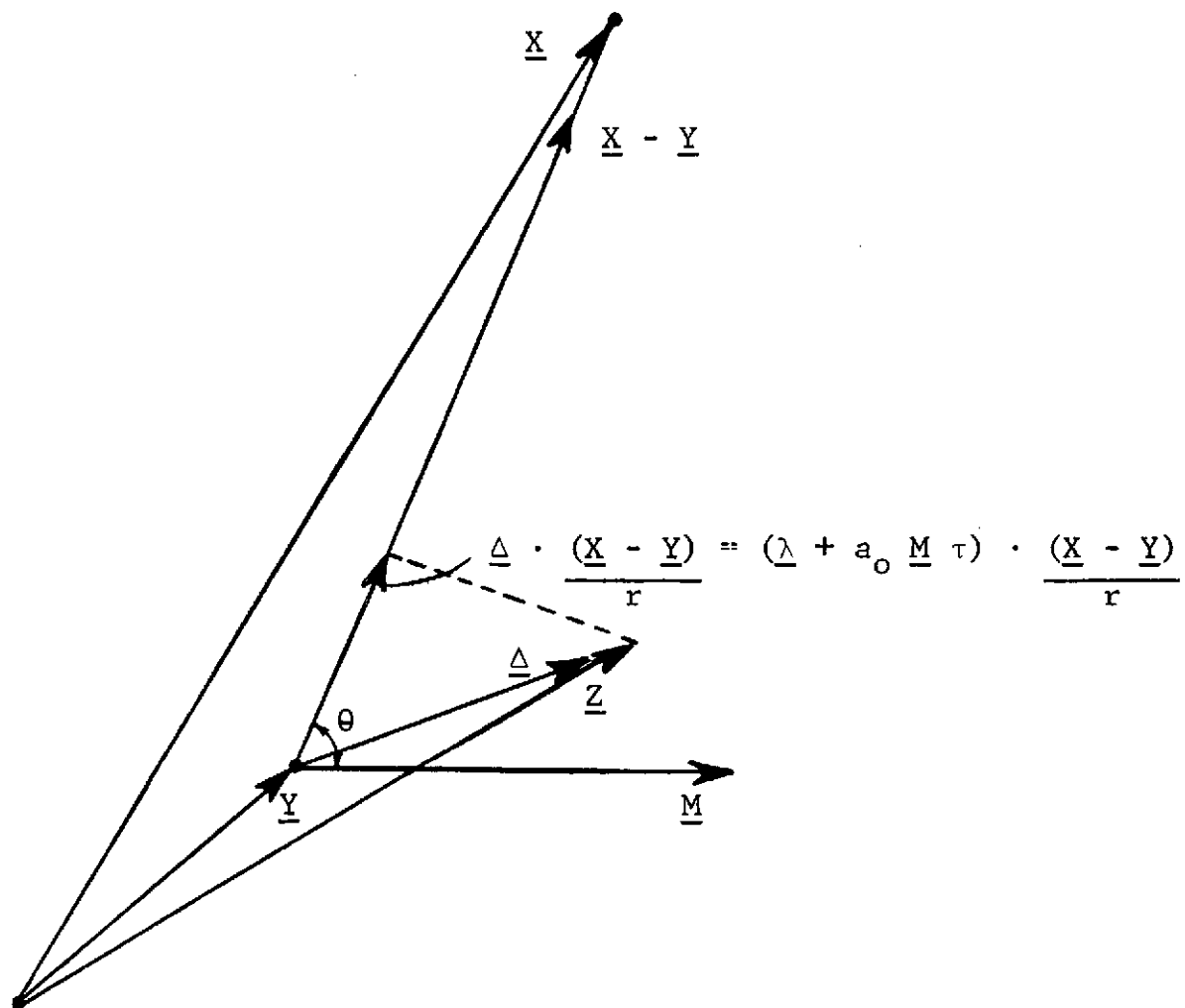
Figure 1   COORDINATE AXES SYSTEM

Here, $P_{ijk\ell}$ $(\underline{Y},\ \underline{\lambda},\ \tau(\lambda))$ signifies that τ is to be considered a function of $\underline{\lambda}$ prior to differentiation and $P_{ijk\ell}$ $(\underline{Y},\ \underline{\lambda},\ \tau)$ signifies that τ is to be considered an independent variable until after differentiation. Using equations (3-11) and (3-12) we get the final relation between time derivatives in the fixed and moving frames

$$\frac{\partial}{\partial \tau} R_{ijk\ell}(\underline{Y},\ \underline{\Delta},\ \tau) = \left\{ \frac{|\underline{X} - \underline{Y}|}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})} \frac{\partial}{\partial \tau} - a_o M_n \frac{\partial}{\partial \lambda_n} \right\}$$

$$\times P_{ijk\ell}(\underline{Y},\ \underline{\lambda},\ \tau) \qquad (3.13)$$

Again, the space derivative $\frac{\partial}{\partial \lambda_n} P_{ijk\ell}$ $(\underline{\lambda})$ will contribute nothing to the integral of equation (3.7) and we will drop that term. Differentiating three more times, equation (3.13) becomes

$$\frac{\partial^4}{\partial \tau^4} R_{ijk\ell}\ (\underline{Y},\ \underline{\Delta},\ \tau) \neq \frac{|\underline{X} = \underline{Y}|^4}{\left[|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})\right]^4} \frac{\partial^4}{\partial \tau^4} P_{ijk\ell}\ (\underline{Y},\ \underline{\lambda},\ \tau)$$

$$(3.14)$$

where $\neq$ signifies that the two sides of the equation are equivalent when integrated as in equation (3.7). This is, both sides of equation (3.14) produce the same far field effect although they are not strictly equal.

The Jacobian of the transformation gives

$$d\underline{\Delta} = \frac{|\underline{X} - \underline{Y}|}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})} d\underline{\lambda} \qquad (3.15)$$

Substituting equations (3.14) and 3.15) into (3.7) gives finally

$$B(\underline{X}, t, \tau^*) = \frac{1}{16\pi^2 a_o^8} \int \frac{(x_i - y_i)(x_j - y_j)(x_k - y_k)(x_\ell - y_\ell)}{|\underline{X} - \underline{Y}| \left[|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y})\right]^5} \times$$

$$\int \frac{\partial^4}{\partial \tau^4} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau) \, d\underline{\lambda} \, d\underline{Y} \qquad (3.16)$$

It is the inner integral which will concern us here since it contains all the local source characteristics. We will call this integral

$$S(\underline{Y}, t, \tau^*) = \int \frac{\partial^4}{\partial \tau^4} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau) \, d\underline{\lambda} \qquad (3.17)$$

The characteristics of this source integral have been discussed by Ffowcs-Williams by Fourier transforming $S(\underline{Y}, t, \tau^*)$ to obtain its sound power spectral density tensor. In this way we can see more clearly the effects of source frequencies and physical scales on the resulting sound frequencies.

The four-dimensional Fourier transform of the correlation tensor, defining a power spectral density, is given by

$$H_{ijk\ell}(\underline{Y}, \underline{\kappa}, \omega) = \frac{1}{(2\pi)^4} \iint P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \sigma) e^{-i\omega\sigma}$$

$$\times e^{-i\underline{\kappa} \cdot \underline{\lambda}} \, d\underline{\lambda} \, d\sigma \qquad (3.18)$$

from which

$$P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \sigma) = \iint H_{ijk\ell}(\underline{Y}, \underline{\kappa}, \omega) e^{i\omega\sigma} e^{i\underline{\kappa} \cdot \underline{\lambda}} \, d\underline{\kappa} \, d\omega$$

$$(3.19)$$

The four-dimensional transforms are used since we must determine the effects of both wave number (space) and frequency (time) effects within the source and the relation between them under the restraint of the retarded time relation, equation (3-13). In the far field a simple dispersion relation exists between $\underline{k}$ and ω and a one-dimensional transform will suffice.

Differentiation of equation (3-19) with respect to σ is equivalent to multiplication by $i\omega$ so that we can write, using equations (3.10), (3.17), 3.19)

$$
\begin{aligned}
S(\underline{Y}, t, \tau^*) &= (2\pi)^3 \iint \omega^4 H_{ijk\ell}(\underline{Y}, \underline{k}, \omega) \times \\
&\quad \int e^{i(\omega\tau + \underline{k} \cdot \underline{\lambda})} d\underline{\lambda}\, d\underline{k}\, d\omega \\[2mm]
&= (2\pi)^3 \iint \omega^4 H_{ijk\ell}(\underline{Y}, \underline{k}, \omega) \exp\left\{ \frac{i\omega|\underline{X} - \underline{Y}|\tau^*}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})} \right. \\[2mm]
&\quad \times \delta\left(\underline{k} \cdot \underline{\lambda} + \frac{\omega(\underline{X} - \underline{Y}) \cdot \underline{\lambda}}{a_o\left[|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})\right]} \right) d\underline{k}\, d\omega \\[2mm]
&= (2\pi)^3 \int \omega^4 H_{ijk\ell}\left(\underline{Y}, \frac{-\omega(\underline{X} - \underline{Y})}{a_o\left[|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})\right]}, \omega\right) \\[2mm]
&\quad \times \exp\left(\frac{i\omega|\underline{X} - \underline{Y}|\tau^*}{|\underline{X} - \underline{Y}| + \underline{M}\cdot(\underline{X} - \underline{Y})} \right) d\omega
\end{aligned}
\tag{3.20}
$$

As an interesting aside, the calculation procedure here is similar to that we do to reduce the crossed-beam power spectral density to a one-dimensional Fourier transform, the difference being that the crossed beam involves an area integral of a Fourier transform, whereas here we have a volume integral.

At very low speeds, $\underline{M} \longrightarrow 0$ at $a_o \rightarrow \infty$ and $\underline{\kappa} \rightarrow \frac{\omega}{a_o} = 0$. Thus at low speeds for fairly low frequencies $\underline{\kappa} \sim 0$ by equation (3.20.1). The frame of reference chosen ensures that time scales are maximized and hence frequencies will be limited.

If we take $\underline{\kappa} = 0$, then equation (3.20) gives

$$S(\underline{Y}, t, \tau^*) = (2\pi)^3 \int \omega^4 H_{ijk\ell} (\underline{Y}, 0, \omega) \exp\left\{\frac{i\omega|\underline{X} - \underline{Y}|\tau^*}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})}\right\} d\omega$$

(3.21)

$$= \int \frac{\partial^4}{\partial\tau^4} P_{ijk\ell} \left(\underline{Y}, \underline{\lambda}, \frac{\tau^*|\underline{X} - \underline{Y}|}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})}\right) d\underline{\lambda}$$

(3.22)

From these equations we can see several important characteristics of turbulent sources at low speeds.

The frequency is increased by the factor

$$\frac{|\underline{X} - \underline{Y}|}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})} = \frac{1}{1 - M\cos\theta}$$

a result of motion of the sources relative to the observer. This is just a doppler effect.

The acoustic energy at low speeds is independent of the retarded time τ, which was precisely Lighthill's low speed assumption. [6] The sound sources are compact in nature in the moving frame of reference so that retarded time differences are small; we have accounted for retarded time differences resulting from convection by the coordinate transformation. This is equivalent to assuming that all the acoustic power comes from zero wave number turbulence or, what is equivalent, the power comes only from true time varying signals (in the moving frame of reference). We must note here that $\underline{\kappa} \sim 0$ is

an approximation only. In actual fact if $\underline{\kappa} = 0$, there would be no sound since $\omega = 0$ then too.

As the convection speed increases, $\underline{\kappa}$ can no longer be neglected since $|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y})$ decreases and in face a singularity occurs in equations (3.20) and (3.20.1) when $|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y}) = 0$. The occurrence of the singularity introduces an interesting and important aspect of turbulent noise. Even though equation (3.16) appears to indicate that frozen turbulence creates no sound, we can now see an exception mentioned earlier. The singularity results in an indeterminate of the form 0/0 which can and does have a finite limiting value. The singularity condition is the case where the turbulence convection speed has a component toward the observer equal to the speed of sound. This is precisely the condition required for sustaining a Mach wave. Thus the eddies are analogous to solid bodies creating Mach waves which propagate toward the observer, and hence the name Mach wave emission for this type of (supersonic) turbulence-generated sound.

The retarded time difference becomes infinite at the singularity and each source component of the quadrupole then behaves as a separate source, no "Strokes effect" cancellations occurring. Thus Mach wave emission is very efficient (sourcelike). It will radiate for all $M = \dfrac{U_c}{a_o} > 1$ at the Mach angle θ relative to the flow direction defined by

$$\frac{\underline{M} \cdot (\underline{X} - \underline{Y})}{|\underline{X} - \underline{Y}|} = M \cos \theta \qquad (3.23)$$

The simplified expression (3.22) obviously does not hold for high speeds and we must go back to equation (3.20) which holds for all $\underline{M}$.

Introducing a new frequency variable α into equation (3.20), defined by

$$\alpha = \frac{\omega |\underline{X} - \underline{Y}|}{|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y})} \qquad (3.24)$$

we get

$$S(\underline{Y}, t, \tau^*) = (2\pi)^3 \int \alpha^4 \left(\frac{|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y})}{|\underline{X} - \underline{Y}|} \right)^5$$

$$\times H_{ijk} \left(\underline{Y}, \frac{\alpha(\underline{X} - \underline{Y})}{a_o |\underline{X} - \underline{Y}|}, \alpha \frac{|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y})}{|\underline{X} - \underline{Y}|} \right)$$

$$\times e^{-i\alpha\tau^*} d\alpha \qquad (3.25)$$

The singularity in equation (3.6) is cancelled by the corresponding term here and near the singularity the frequency term becomes zero so that

$$H_{ijk\ell} = H_{ijk\ell} \left(\underline{Y}, \frac{-\alpha(\underline{X} - \underline{Y})}{a_o |\underline{X} - \underline{Y}|}, 0 \right) \qquad (3.26)$$

This indicates that it is the wave number κ_r in the direction of propagation

$$\frac{(\underline{X} - \underline{Y})}{|\underline{X} - \underline{Y}|}$$

to the observer along the Mach acoustic ray at zero frequency, which produces the sound power. Note that $\omega = 0$ is required in order that α from equation (3.24) have a finite value. It is the physical scale rather than the time scale of the eddy that becomes important, the complete antithesis of the low speed case.

Fourier transforming equation (3.25),

$$H_{ijk\ell}\left(\underline{Y},\frac{-\alpha(\underline{X}-\underline{Y})}{a_o|\underline{X}-\underline{Y}|},\ 0\right) = \frac{a_o^4}{\alpha^4}\ \frac{1}{(2\pi)^4}\iint \frac{\partial^4}{\partial\lambda_r^4}\ P_{ijk\ell}(\underline{Y},\underline{\lambda}\sigma)$$

$$\times\ \exp\left\{\frac{i\alpha(\underline{X}-\underline{Y})\cdot\underline{\lambda}}{a_o|\underline{X}-\underline{Y}|}\right\}d\sigma\ d\lambda \qquad (3.27)$$

where λ_r is the component of $\underline{\lambda}$ in the direction of the observer corresponding to the wave number component $\kappa_r = \frac{\alpha}{a_o}$ in that direction.

Then by equation (3.25)

$$S(\underline{Y},\ t,\ \tau^*) = (1 - M\cos\theta)^5\ \frac{a_o^4}{2\pi}\iint\frac{\partial^4}{\partial\lambda_r^4}\ P_{ijk\ell}(\underline{Y},\ \underline{\lambda},\ \sigma)$$

$$\times\ \delta\left(\tau^* + \frac{\underline{\lambda}\cdot(\underline{X}-\underline{Y})}{a_o|\underline{X}-\underline{Y}|}\right)d\sigma\ d\underline{\lambda}$$

$$= (1 - M\cos\theta)^5\ a_o^5\iint\frac{\partial^4 P_{ijk\ell}}{\partial\lambda_r^4}\ (\underline{Y},\ \underline{\lambda},\ \sigma)\ d\sigma\ d\underline{\lambda}_s$$

$$(3.28)$$

where $\underline{\lambda}_s$ is in the area normal to λ_r defined by

$$(\underline{X}-\underline{Y})\cdot\underline{\lambda}_s = 0 \qquad (3.29)$$

and where

$$\lambda_r = -a_o\ \tau^* \qquad (3.29.1)$$

The important frame of reference then is one with two axes in the plane tangent to the Mach front and moving with it.

Thus as an eddy moves it emits continually as long as there exist a space-derivative fluctuation. It will continue to emit throughout its life-time which is finite, otherwise the time integral would give infinite power. The lifetime here is the timescale of $P_{ijk\ell}$. The smaller are the changes with time obviously the larger is the acoustic power radiated, in marked contrast with the low speed quadrupole case where changes with time are essential to the radiation.

Even at supersonic speeds there also exists quadrupole-type radiation at angles other than $\cos^{-1} \frac{1}{M}$. Since however, the quadrupole efficiency is so much smaller than that of the source-like Mach emission, the former may be ignored for the most part, except perhaps at far downstream observer positions.

In review, we have been able to write two integral expressions where retarded time differences can be accounted for readily. This ability is essential to us in applying a space-integrating device like the crossed beam to sound source measurements since different parts along each beam cannot contribute at different retarded times. The two expressions are applicable to

1. low subsonic speeds, where retarded times can be neglected in a frame of reference moving with the eddy and

2. high speed flows in which the ambient sound speed is somewhere exceeded by the eddies.

In this latter case the proper frame of reference is one in the plane of the Mach wave front and moving with it at the ambient sound speed toward the observer. Since points on this plane are equidistant from a far field observer, retarded time differences vary negligibly in the plane. Since the eddy (and the reference frame) travels at the speed of sound toward the observer, Stokes cancellation effects and retarded time changes in that direction do not enter the

formulation. In both the low speed and the high speed case, the correlation tensors must be measured in the moving reference frame specified.

The equations for source intensity are:

High Speed -- Mach Wave Emission

$$S(\underline{Y}, t, \tau^*) = (1 - M\cos\theta)^5 a_o^5 \iint \frac{\partial^4 P_{ijk\ell}(\underline{Y}, \underline{\lambda}_s, \sigma)}{\partial\lambda_r^4} \, d\underline{\lambda}_s \, d\sigma$$

(3.28)

Low Speed - M→0

$$S(\underline{Y}, t, \tau^*) = \int \frac{\partial^4}{\partial\tau^4} P_{ijk\ell}\left(\underline{Y}, \underline{\lambda}, \frac{\tau^*|\underline{X} - \underline{Y}|}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})}\right) d\underline{\lambda}$$

(3.22)

Under other conditions we must use the general expression

$$S(\underline{Y}, t, \tau^*) = \int \frac{\partial^4}{\partial\tau^4} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau) \, d\underline{\lambda}$$

(3.17)

$$= (2\pi)^3 \int \omega^4 H_{ijk\ell}\left(\underline{Y}, \frac{-\omega(\underline{X} - \underline{Y})}{a_o\left[|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})\right]}, \omega\right) \times$$

$$\exp\left\{\frac{i\omega|\underline{X} - \underline{Y}|\tau^*}{|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})}\right\} d\omega$$

(3.20)

where

$$\tau = \frac{\underline{\lambda}\cdot(\underline{X} - \underline{Y}) + a_o \tau^* |\underline{X} - \underline{Y}|}{a_o\left[|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})\right]}$$

(3.10)

$$\underline{\kappa} = \frac{-\omega(\underline{X} - \underline{Y})}{a_o\left[|\underline{X} - \underline{Y}| - \underline{M}\cdot(\underline{X} - \underline{Y})\right]}$$

(3.20.1)

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and

$$\frac{\partial^4}{\partial \tau^4} R_{ijk\ell}(\underline{Y}, \underline{\Delta}, \tau) \neq \frac{|\underline{X} - \underline{Y}|^4}{\left[|\underline{X} - \underline{Y}| - \underline{M} \cdot (\underline{X} - \underline{Y})\right]^4} \frac{\partial^4}{\partial \tau^4} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau)$$

$$(3.14)$$

The arguments given for cancellation of retarded time effects with reference axes in frames normal to the observer would also be true in the general case of equation (3.17). However, a volume integral is of importance here and retarded times are important in the observers direction. Whether or not these can ever be accounted for are left to the following sections for discussion, along with general discussions of the applications of all these equations to measurement by crossed-beam techniques.

IV. APPLICATION OF CROSSED BEAM TECHNOLOGY

The problems involved in applying crossed-beam technology to sound source measurements lie in three areas.

1. Covariance Measurements -- approximation of the covariance by parameters measurable by crossed-beam techniques.

2. Retarded Time Effects.

3. Moving Frame of Reference.

These are discussed in this section with subcategories for delineation of (1) low speed applications, (2) high speed applications and (3) general applications.

4.1 COVARIANCE MEASUREMENTS

The signal measured by a single beam of the crossed-beam system is determined by the extinction coefficient of the dominant light extinction process. The detector signal I is given by

$$I = I_0 \exp \left\{ -\int k d\ell \right\} \tag{4.1}$$

If the intensity is assumed to result from a mean $\bar{I}$ and a fluctuation i,

$$i + \bar{I} = I_0 \exp \left\{ -\int (K+k) d\ell \right\}$$

$$= \bar{I} \exp \left\{ -\int k d\ell \right\}$$

$$i = \bar{I} \left(\exp \left\{ -\int k d\ell \right\} - 1 \right)$$

and if the total integrated fluctuations are small

$$i(t) = \bar{I} \int k(\underline{Y}, t) d\ell \tag{4.2}$$

With proper calibration constant accounted for, the single or crossed-beam system then measures the statistics of the extinction coefficient whereas we wish to measure the statistics of

a fluid property. Within a gas, and for most extinction processes available to us, the extinction coefficient is strongly dependent upon the number of contributing particles or to the mass density if the particles causing the extinction are uniformly distributed on the average. The present generation of crossed-beam systems[8] uses absorption by naturally occurring CO_2 in the infrared region of the e.m. spectrum for the extinction process. Separate studies are underway to determine under what conditions of wave length, etc., k is proportional to ρ, and independent of pressure and temperature.

In any event it appears that density statistics are the most promising approach for crossed beams and we must therefore be able to express the sound source covariances in terms of density.

The source term[6] involves the quadrupole strength

$$T_{ij} = \rho v_i v_j + P_{ij} - a_o^2 \rho \delta_{ij}$$
$$\simeq \rho v_i v_j + (p - a_o^2 \rho) \delta_{ij}$$

(4.3)

if we neglect viscosity.

In isothermal flows

$$p \simeq a_o^2 \rho, \text{ and } T_{ij} \simeq \rho_o v_i v_j$$

(4.4)

If there are large entropy gradients in the flow, however, the pressure and density fluctuation may not cancel out and $p - a_o^2 \rho$ will be determined by entropy fluctuations.

Thus in a heated jet equation (4.3) might become

$$T_{ij} \simeq (p - a_o^2 \rho) \delta_{ij}$$

(4.5)

In the case of a supersonic jet, of course, all terms must be included in T_{ij}.

4.1.1   Low Speed Jets (M=0)

We will restrict our discussions to isothermal jets at low speeds. This is obviously not because of a lack of importance of heated-jet noise but is rather only to limit ourselves to the simpler case. A much more comprehensive study is required for the entropy fluctuation case and this is left to a future time.

Shear Flows

In most turbulence flows of interest to us here, there will be a large shear gradient in the flow, and turbulence will tend to be most intense in that region. If shear is present it has an amplifying effect upon the noise produced. From the momentum equation

$$\frac{\partial}{\partial t}(\rho v_i v_j) = P_{ij}\frac{\partial v_j}{\partial y_k} + P_{k\ell}\frac{\partial v_i}{\partial y_k} - \frac{\partial}{\partial y_k}(\rho v_i v_j v_k + P_{ik}v_j + P_{jk}v_i) \quad (4.6)$$

The last term will drop out when integrated over all space, (using the divergence theorem) and dropping viscous terms we get from equation (4.4)

$$\frac{\partial}{\partial t}(\rho v_i v_j) \int P\left(\frac{\partial v_i}{\partial y_j} + \frac{\partial v_j}{\partial y_i}\right) = p e_{ij} \quad (4.7)$$

The mean rate of strain tensor may be approximated by its mean value which we will assume to have a dominant direction y_1

$$e_{ij} \simeq e_{12} = \frac{du_1}{dy_2} \quad (4.8)$$

Then

$$\frac{\partial}{\partial t}(\rho v_i v_j) = p\frac{du_1}{dy_2} \quad (4.9)$$

and

$$\frac{\partial^2 T_{ij}}{\partial t^2} \int \frac{du_1}{dy_2}\frac{\partial p}{\partial t} = a_o{}^2 \frac{du_1}{dy_2}\frac{\partial p}{\partial t} \quad (4.10)$$

Taking the mean-square value of this,

$$\frac{\partial^4}{\partial\tau^4} R_{ijk\ell} \oint \left(\frac{du_1}{dy_2}\right)^2 a_o^4 \frac{\partial^2}{\partial\tau^2} R_E(\underline{Y},\underline{\Delta},\tau) \qquad (4.11)$$

where

$$R_E(\underline{Y},\underline{\Delta},\tau) = \langle \rho(\underline{Y},t) \; \rho(\underline{Y} + \underline{\Delta}, \; t+\tau) \rangle \qquad (4.11.1)$$

We are therefore concerned with the second derivative of the density covariance. It must be expressed in the moving reference frame for use in equation (3.28), but this will be left for later.

Homogeneous Turbulence

If there is no appreciable shear in a flow we can obtain $R_{ijk\ell}$ in terms of the density covariance using the approach of Ribner.[9] He considers the pressure field as composed of an ambient p_o, an incompressible part p^o and a compressible part p with corresponding densities related by $p = a_o^2 \rho$.

For incompressible flow $D\rho/Dt = 0$ and equation (3.1) becomes $-\nabla^2 p = (\partial^2 T_{ij}/\partial x_i \partial x_j)_{inc}$ where T_{ij} is evaluated at zero convection speed. Substituting into equation (3.1) we obtain

$$\frac{D^2\rho}{Dt^2} - a_o^2 \nabla^2\rho = -a_o^2 \nabla^2\rho^o \qquad (4.12)$$

where we have assumed $(T_{ij})_{comp} = (T_{ij})_{incomp}$ at zero convection speed and

$$\rho^o = \frac{1}{a_o^2} p^o$$

The source strength then becomes

$$\frac{\partial^4 R_{ijk\ell}}{\partial t^4} \neq -\frac{\partial^4}{\partial \tau^4} R(\underline{Y}, \underline{\Delta}, \tau) \qquad (4.13)$$

with

$$R(\underline{Y}, \underline{\Delta}, \tau) = \langle \rho(\underline{Y}, t)\rho(\underline{Y} + \underline{\Delta}, t + \tau) \rangle \qquad (4.11.1)$$

Thus at low speeds the source strength can be written in terms of the density covariance as required for crossed-beam measurements.

4.1.2 High Speed Jets (M=1)

From equation (3.28) the primary term in the sound source integral for Mach emission is

$$\frac{\partial^4}{\partial \lambda_r^4} P_{ijk\ell}(\underline{Y}, \underline{\lambda}_s, \tau) \qquad \text{with } \lambda_r = \text{constant}$$

Now by equations (3.4) and (3.16)

$$\frac{(x_i - y_i)(x_j - y_j)(x_k - y_k)(x_\ell - y_\ell)}{r^6(1 - M\cos\theta)^5} \frac{\partial^4 P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau)}{\partial \underline{\lambda}^4}$$

$$= \left\langle \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}(\underline{Y}, 0, t) \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}(Y, \underline{\lambda}, t + \tau) \right\rangle$$

and we are concerned with the component of this in the r direction.

Now,

$$\frac{1}{r}\left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\right]_{t = \frac{r}{a_o}} \neq \frac{r_j}{r^2 a_o} \frac{\partial^2 T_{ij}}{\partial t \partial y_i} = \frac{1}{r a_o} \frac{\partial^2 T_{ir}}{\partial t \partial y_i}$$

$$(4.14.1)$$

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$$\frac{\partial T_{ir}}{\partial y_i} = \frac{\partial}{\partial y_i}(\rho v_i v_j + P_{ri}) - a_o^2 \frac{\partial \rho}{\partial y_r}$$

$$= -a_o^2 \frac{\partial \rho}{\partial y_r} + \frac{\partial \rho v_r}{\partial t} \qquad (4.14.2)$$

We may write

$$\frac{1}{r} \frac{\partial \rho v_r}{\partial y_r} \neq \frac{-1}{a_o r} \frac{\partial \rho v_r}{\partial t} \qquad (4.14.3)$$

Substituting equations (4.14.2) and (4.14.3) into (4.14.1) we obtain

$$\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \neq \frac{-1}{a_o} \frac{\partial}{\partial t} \left(a_o \frac{\partial \rho}{\partial y_r} - \frac{\partial \rho v_r}{\partial y_r} \right)$$

$$\neq -\frac{\partial}{\partial t} \left((a_o - v_r) \frac{\partial \rho}{\partial y_r} - \rho \frac{\partial v_r}{\partial y_r} \right) \qquad (4.14.4)$$

If we let $v_r = u_r + v_r'$ in equation (4.14.4) we get[10]

$$\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} = (a_o - u_r) \frac{\partial^2 \rho}{\partial t \partial y_r} + \frac{\partial^2 \rho v_r'}{\partial t \partial y_r} + \frac{\partial \rho}{\partial t} \frac{\partial u_r}{\partial y_r} \qquad (4.15)$$

Shear Flows

If we are dealing with a high-shear region, only the third term is amplified by that shear. The other terms will tend to be small. Particularly, we can see that with $U_c = a_o$ that the factor $a_o - u_r$ will be small. Thus we will take

$$\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} = \frac{\partial u_r}{\partial y_r} \frac{\partial \rho}{\partial t} = \frac{(x_i - y_i)(x_j - y_j)}{r^2} \frac{\partial u_i}{\partial y_j} \frac{\partial \rho}{\partial t} \qquad (4.16)$$

where we have substituted

$$\frac{\partial u_r}{\partial y_r} = \frac{(x_i - y_i)(x_j - y_j)}{r^2} \frac{\partial u_i}{\partial y_i}$$

This gives

$$\frac{\partial^2 P_{ijk\ell}}{\partial \lambda^4} (\underline{Y}, \underline{\lambda}, \tau) = \left(\frac{\partial u_i}{\partial y_j}\right)^2 \left\langle \frac{\partial \rho (\underline{Y}, t)}{\partial t} \frac{\partial \rho (\underline{Y} + \underline{\lambda}, t + \tau)}{\partial t} \right\rangle \quad (4.17)$$

Since this is to be determined at $\lambda_r =$ const, and further, taking $\dfrac{\partial u_i}{\partial y_j} \cong \dfrac{du_1}{dy_2}$ equation (4.17) becomes

$$\frac{\partial^4}{\partial \lambda_r^4} P_{ijk\ell}(\underline{Y}, \lambda_s, \tau) \neq - \left(\frac{du_1}{dy_2}\right)^2 \left[\frac{\partial^2 R_E(\underline{Y}, \underline{\Delta}_s, \tau)}{\partial \tau^2}\right]_{\Delta_r = a_o(\tau - \tau^*)}$$

$$(4.17.1)$$

Thus

$$S(\underline{Y}, \tau^*) = -a_o^5 (1 - M\cos \theta)^5 \iint \left(\frac{du_1}{dy_2}\right)^2 \frac{\partial^2 R_E (\underline{Y}, \underline{\Delta}_s, \tau)}{\partial \tau^2} d\underline{\Delta}_s \, d\sigma \quad (4.18)$$

where

$$R_E(\underline{Y}, \underline{\Delta}_s, \tau) = \left\langle \rho(\underline{Y}, t) \rho(\underline{Y} + \underline{\Delta}, t + \tau)\right\rangle_{\Delta_r = a_o(\tau - \tau^*)} \quad (4.18.1)$$

Except for the sign, equation (4.18) is by coincidence identical with equation (4.11) for low-speed shear flow. The important observation here is that the source intensity is determined by the area integral of an Eulerian covariance. The gradient terms in the time derivative do not integrate out in an area integral and in fact dominate.

Homogeneous Turbulence

If the flow of a jet is supersonic there must be large shear gradients in the flow as the velocity goes from zero outside the jet to greater than speed of sound within it. There should be no necessity then of attempting a homogeneous solution for the Mach emission case.

4.1.3 General Case

In the general case, when we cannot assume either $M \cong 0$ or $M \cos\theta = 1$, it is not possible to approximate T_{ij} by a simple function of density. As stated earlier however, the Mach emission case probably covers all speeds of interest above M=1. Below M=1, the low-speed isothermal approximation is no doubt valid for isothermal jets over a broad range of speeds. More serious of course is the need for an expression in the case of heated flows. It appears that it should be possible, however, to express entropy fluctuations directly in terms of temperature, and consequently density fluctuations.

In an isothermal jet, it is not so much the approximation of the stress tensor covariance that presents the most severe problem. Rather, the retarded time differences become nonnegligible as convection speed increases, even in the moving frame of reference. Means of accounting for retarded time differences are discussed in the following paragraphs.

4.2 RETARDED TIME EFFECTS

In section II, it was shown that the crossed-beam system measures a covariance given by the expression

$$G_E(\underline{Y},\Delta_1,T) = \iint \left\langle \rho(y_1,y_2 + \Delta_2,y_3,t)\,\rho(y_1 + \Delta_1,y_2,y_3 + \Delta_3,t + T) \right\rangle \times d\Delta_2\, d\Delta_3$$

Under conditions of homogeneity or odd-function departures from it, this can be written as a point-plane correlation leading to

$$G_E(\underline{Y}, \Delta_1, T) = \int R_E(\underline{Y}, \underline{\Delta}, T) \, d\underline{\Delta}_n \qquad (2.4)$$

where $d\underline{\Delta}_n = d\Delta_2 d\Delta_3$ is a plane normal to Δ_1. The sound source integral on the other hand is generally of the form (from section III)

$$S(\underline{Y}, t, \tau^*) = \int \frac{\partial^4}{\partial \tau^4} P_{ijk\ell}(\underline{Y}, \underline{\lambda}, \tau) \, d\underline{\lambda}$$

$$\neq \quad (1-M\cos\theta)^5 \int \frac{\partial^4 R_{ijk\ell}(\underline{Y}, \underline{\Delta}, \tau)}{\partial \tau^4} \, d\underline{\Delta} \qquad (4.19)$$

Comparison of equations (2.4) and (4.19) suggests that we might obtain $S(\underline{Y}, \underline{\Delta}, \tau^*)$ from a crossed-beam measurement using a relation of the type

$$S(\underline{Y}, t, \tau^*) = (1 - M\cos\theta)^5 \int \frac{\partial^4 G_E(\underline{Y}, \Delta_1, T)}{\partial \tau^4} \, d\Delta_1 \qquad (4.20)$$

We can see now the need for obtaining the crossed-beam covariance in the form of the point-plane covariance is to approximate the correct covariance R under the area integral.

A difficulty involved in the use of equation (4.20) is immediately apparent; τ must be made identical with T, so that

$$T = \tau = \tau^* + \frac{\Delta \cos\theta}{a_o} \qquad (3.7.1)$$

In order that $G_E(\underline{Y},\Delta_1,T)$ be expressed in its point-plane form of equation (2.4), it was necessary to assume that we could make arbitrary coordinate translations normal to y_1 of the type $y_2 \rightarrow y_2 - \Delta_2$ without changing the statistical properties of the covariances. This is a good approximation probably, but when the translation is made we also vary τ^* through equation (3.7.1). It therefore becomes impossible in general to maintain the correct τ for arbitrary θ and Δ_2. There are, however some special cases where we do so and these are discussed in the following paragraphs.

Another problem is associated with taking the time delay derivative of G_E. In equation (4.20) we have assumed that

$$\int \frac{\partial^n R_E(\underline{Y},\underline{\Delta},T)}{\partial T^n} d\underline{\Delta}_n = \frac{\partial^n}{\partial T^n} \int R_E(\underline{Y},\underline{\Delta},T)\, d\underline{\Delta}_n$$

$$= \frac{\partial^n}{\partial T^n} G_E(\underline{Y},\Delta_1,T) \tag{4.21}$$

This is obviously not the case as long as $T = T(\underline{\Delta})$ which in general is the case as required by equation (3.7.1). Again there are special cases where this is done and these are discussed later.

4.2.1 Low Speed Jets

As shown in section III, if we transform to the moving frame of reference a large portion of the τ differences are accounted for by the convection equation. The time scales in the moving frame become very large in comparison with the retarded time differences $\sim L/a_o$, where L is a typical correlation length. If $a_o \gg U$, then retarded time differences may be neglected in comparison with fluctuation time scales. Flows which satisfy this condition have been said to contain "compact" sound sources.[11] In this case then ($U \ll a_o$) we can put $\tau = 0$ and equation (4.20) becomes valid.

Shear Flows

For high-shear flows,

$$\frac{\partial^4 R_{ijk\ell}}{\partial \tau^4} \neq \left(\frac{du_1}{dy_2}\right)^2 a_o^4 \frac{\partial^2 R_E(\underline{Y},\underline{\Delta},\tau)}{\partial \tau^2} \tag{4.11}$$

or in the moving reference frame

$$\frac{\partial^4 P_{ijk\ell}}{\partial \tau^4} \neq \frac{du_1}{dy_2} a_o^4 \frac{\partial^2 P(\underline{Y},\underline{\lambda},\tau)}{\partial \tau^2}$$

or

$$S(\underline{Y},t,\tau^*) = a_o^4 \left(\frac{du_1}{dy_2}\right)^2 \int \frac{\partial^2 P(\underline{Y},\underline{\lambda},\tau)}{\partial \tau^2} \, d\underline{\lambda} \tag{4.22}$$

If $G(\underline{Y},\Delta_1,T)$ is measured in a frame of reference moving with the convection speed of the flow and the flow direction is along y_1, then integral of equation (2.4) can be evaluated at $T = \tau = 0$. Then $T \neq T(\lambda)$ and we can use equation (4.24) to give

$$S(\underline{Y},t,\tau^*) = a_o^4 \left(\frac{du_1}{dy_2}\right)^2 \int \frac{\partial^2 G_L}{\partial T^2}(\underline{Y},\lambda_1,T) \, d\lambda_1 \tag{4.23}$$

The means of determining this moving axis covariance is treated in a later section.

Homogeneous Turbulence

For homogeneous turbulence we use equations (4.13) and (4.21) to give

$$S(\underline{Y},t,\tau^*) = a_o^4 \int \frac{\partial^4}{\partial T^4} G_L(\underline{Y},\lambda_1,T) \, d\lambda_1 \tag{4.24}$$

Again G_L must be measured in a moving axis frame.

4.2.2 High Speed Jets

In section III the source integral for Mach wave emission was developed to be

$$S(\underline{Y},t,\tau^*) = (1 - M \cos\theta)^5 \, a_o^5 \iint \frac{\partial^4 P_{ijk\ell}(\underline{Y},\lambda_s,\sigma)}{\partial \lambda_r^4} \, d\underline{\lambda}_s \, d\sigma \qquad (3.28)$$

where λ_s is tangent to the plane of the Mach front and λ_r is the distance variable in the direction of the ray. In this form the retarded time is approximately constant for the plane λ_r = const. and only enters into the problem for the time integration. Thus the surface integral itself has instantaneously no retarded time effects making it of suitable form for measurement by crossed-beam techniques.

In the case of shear flow, and the only one we shall consider here, equation (3.28) can be written using equation (4.18) as

$$S(\underline{Y},t,\tau^*) = -(1 - M \cos\theta)^5 \, a_o^5 \iint \left(\frac{du_1}{dy_2}\right)^2 \frac{\partial^2 R_E(\underline{Y},\underline{\Delta}_s,\tau)}{\partial \tau^2} \, d\underline{\Delta}_s \, d\sigma$$

Assuming du_1/dy_2 varies slowly with λ_r we can write

$$S(\underline{Y},t,\tau^*) = -(1 - M \cos\theta)^5 \, a_o^5 \left(\frac{du_1}{dy_2}\right)^2 \iint \frac{\partial^2 R_E(\underline{Y},\underline{\Delta}_s,\tau)}{\partial \tau^4} \, d\underline{\Delta}_s \, d\sigma \qquad (4.25)$$

From equation (3.8)

$$\Delta_r = \lambda_r + a_o\tau \qquad (4.26)$$

and from equation (3.29.1)

$$\lambda_r = -a_o\tau^*$$

Consequently

$$\frac{\Delta_r}{a_o} = \tau - \tau^*$$ (4.27)

The retarded time difference then is constant along planes of Δ_r = constant.

Now if we align the crossed beams such that they lie perpendicular to $\underline{r}$ and are spaced a distance Δ_r apart

$$G_E(\underline{Y},\Delta_r,T) = \int R_E(\underline{Y},\underline{\Delta}_s,T)\ d\underline{\Delta}_s$$ (4.28)

Apart from the differentiation required, it is necessary to measure G such that $T = \Delta_r/a_o + \tau^*$. This satisfies both the requirement for time retardation as well as following the fluid motion at the convection speed toward the observer, i.e., the measurement is then made in the moving frame of reference.

The time variable σ in equation (4.24) is the true time in the moving axis system. It also satisfies the retarded time relation. However since retarded time is independent of $\underline{\Delta}_s$, the differentiation may come outside of the area integral in equations (4.24), (4.27) and (4.28). Thus we can write

$$S(\underline{Y},t,\tau^*) = -(1 - M\cos\theta)^5\ a_o^5 \left(\frac{du_1}{dy_2}\right)^2 \int \left[\frac{\partial^2 G_E(\underline{Y},\Delta_r,T)}{\partial T^2}\right]_{T-\tau^* = \frac{\Delta_r}{a_o}} d\sigma$$

(4.29)

4.2.3 General Case

The success in choosing the coordinate system $(\Delta_r,\underline{\Delta}_s)$ to minimize retarded time effects in the supersonic, Mach emission case, leads us to consider the possibility of achieving like advantages in the general case.

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To this end consider the coordinate system (y_r, y_s) with y_r in the direction of the observer. Then $r_i = r_j = r$ and equation (3.16) becomes

$$B(\underline{X}, t, \tau^*) = \frac{1}{16\pi^2 a_o^8} \int \frac{r^2}{(1 - M\cos\theta)^5} S(\underline{Y}, t, \tau^*) \; d\underline{Y} \qquad (4.30)$$

with

$$S(\underline{Y}, t, \tau^*) = \int \frac{\partial^4}{\partial\tau^4} \; P_r(\underline{Y}, \underline{\lambda}, \tau) \; d\underline{\lambda} \qquad (4.30.1)$$

and

$$P_r = \left\langle T_{rr}(\underline{Y}, t) \; T_{rr}(\underline{Y} + \underline{\lambda}, t + \tau) \right\rangle$$

We can write equation (4.30.1) as

$$S(\underline{Y}, t, \tau^*) = \int \left[\int \frac{\partial^4}{\partial\tau^4} \; P_r(\underline{Y}, \underline{\lambda}, \tau) \; d\underline{\lambda}_s \right] d\lambda_r \qquad (4.31)$$

where the inner integral now becomes similar to the required cross-beam area integral. We note here that τ can be taken as constant within planes of $\lambda_r = $ constant and therefore write $\tau = \tau(\lambda_r)$ or $\tau = \tau(\Delta_r)$. By equations (3.6.2)

$$\tau = \tau^* + \frac{\Delta_r}{a_o}$$

Since P_r is to be measured in the moving frame of reference in order to minimize density gradient effects, we must restrict the time delay of the crossed beam to

$$T = \frac{\Delta_r}{a_o \; M\cos\theta}$$

Letting $\tau^* = 0$, since it has no relevance for our analysis,

$$\tau - T = \frac{\Delta_r}{a_o} \frac{(M \cos\theta - 1)}{M \cos\theta} \qquad (4.32)$$

This is zero, only when $M \cos\theta = 1$ or $a_o = \infty$. These are obviously the two limiting cases already discussed of Mach emission and very low speeds.

We are left then with two alternatives at this point, either (a) let the delay be $T = \Delta_r/a_o$ and check to make certain that gradient fluctuations do not dominate or (b) let $T = \Delta_r/a_o \, M \cos\theta$ and assume total error due to wrong retarded time is small. Alternative (a) is probably unacceptable if $M \cos\theta$ is much different from 1 but might suffice for $M \cos\theta$ close to 1, whereas alternative (b) may extend the range of applicability of the low speed case to higher speeds.

Retarded time errors which arise under alternative (b) above also prohibit taking the time delay derivatives out of the integral S. Thus we are not justified in taking the curvature of G_L. It would be necessary to differentiate signals before correlating the two beam signals.

In essence we are just concluding that some accuracy improvement in measuring sound sources in subsonic jets can be obtained by orienting the beams such that they are perpendicular to the direction of the observer and using equation (4.31).

4.3 MOVING FRAME OF REFERENCE

The covariances pertinent to the determination of sound source intensities must be measured in, or at least related to, a moving frame of reference given by the transformation

$$\underline{\lambda} = \underline{\Delta} \, a_o \, \underline{M} \, \tau$$

This is required in order to minimize convection effects on the measured covariances. We are faced with two problems in this regard. First, the inference of a Lagrangian covariance from a measured Eulerian covariance and secondly, assuring ourselves that the minimized gradients are sufficiently small to be negligible relative to the "true" time changes.

4.3.1 Minimizing Gradients Contribution

We are concerned primarily with two components of the vector $\underline{\lambda}$

1. $\lambda_1 = \Delta_1 - a_o M \tau$ (4.33)

Δ_1 is in the flow direction.

2. $\lambda_r = \Delta_r - a_o M \left[\cos\theta\right] \tau$ (4.34)

Δ_r is in the direction of the observer.

The first of these corresponds to the reference axis most suitable for low speed jets and the second for the Mach emission and "general" cases.

The covariances measured in the convection reference frame most certainly maximize the average time scales. However, it is believed that a measured convection speed is determined primarily by the energy-bearing eddies, other eddies being convected at different speeds. The time scale is therefore not necessarily maximized for all eddies, but only on the average. Those eddies which produce the sound (either $\kappa = 0$), or $\omega = 0$ in the two limiting cases) are not necessarily the energy-containing ones. If they are convected at a speed U_s, then

$$\left[\frac{d\rho}{dt}\right]_{true} = \frac{\partial\rho}{\partial t} + U_s \frac{\partial\rho}{\partial y_1} \qquad (4.35)$$

instead of

$$\left[\frac{d\rho}{dt}\right]_{\text{measured}} = \frac{\partial\rho}{\partial t} + U_c \frac{\partial\rho}{\partial y_1} \tag{4.36}$$

as is measured at the mean convection speed. Hence, we have a contribution of $(U_c - U_s) \partial\rho/\partial y_1$ which may exceed the true time variations and our measurement may be meaningless to sound source estimates. It is necessary therefore to determine the contribution of these fluctuating gradients experimentally. We can show that equation (4.36) can be used to give

$$\frac{\partial^2 P(\underline{Y},\lambda_1,\tau)}{\partial\tau^2} = \frac{\partial^2 R_E(\underline{Y},\Delta_1,\tau)}{\partial\tau^2} + U_c^2 \frac{\partial^2 R_E(\underline{Y},\Delta_1,\tau)}{\partial\Delta_1^2} \tag{4.37}$$

Using a similar equation obtained from equation (4.35), then the error in $\partial^2 P/\partial\tau^2$ is

$$\sigma^{(2)} P = (U_c^2 - U_s^2) \frac{\partial^2 R_E}{\partial\Delta_1^2} \tag{4.38}$$

$$\int \frac{\partial^2 R_E(\underline{Y},\Delta_1,\tau)}{\partial\Delta_1^2} \, d\underline{\Delta}_n$$

can be measured using crossed-Schlieren measurements[12] and the result then compared with the measurement of

$$\frac{\partial^2 G_L}{\partial\sigma^2} \; .$$

With proper data analysis the same assessment may be made for

$$\frac{\partial^4 G_L}{\partial\sigma^4} \; .$$

The curvature of the Lagrangian autocovariance $\partial^2 P/\partial\tau^2$ has been shown to be somewhat sensitive to signal filtering.

In particular, hot-wire measurements indicate a different lifetime at high frequencies than at low frequencies. There appeared to be very little sensitivity to frequency at low frequencies. However, crossed-beam spectra tend to peak at high frequencies in contrast to hot-wire spectra which show no peak, having a large low frequency content. It may be then that crossed-beam Lagrangian autocovariances are sensitive to band pass of the associated electronics. This must be assessed experimentally.

4.3.2 Measurement of Lagrangian Covariance Integrals

4.3.2.1 Low Speed Jets

Shear Flows

By equation (4.23)

$$S(\underline{Y},t,0) = a_o^4 \left(\frac{du_1}{dy_2}\right)^2 \int \frac{\partial^2 G_L(\underline{Y},\lambda_1,T)}{\partial T^2} \, d\lambda_1 \qquad (4.39)$$

With the crossed-beam system, we normally measure $G(\underline{Y},\Delta_1 = \text{const}, T)$ with Δ_1 as a parameter as illustrated in Figure 2.

The envelope of this curve is $G_L(\underline{Y},\lambda_1 = 0,T) = G_E(\underline{Y},\Delta_1 = U_c T,T)$ the Lagrangian autocovariance. The convection speed is then determined by $\lambda_1 = \Delta_1 = U_c T = 0$ evaluated at points on the envelope.

In low speed shear flows we are interested in $G_E(\underline{Y},\lambda_1, T = 0)$ or rather in its second derivative with respect to time delay. At $T = 0$, $\lambda_1 = \Delta_1$ and

$$S(\underline{Y},t,0) = a_o^4 \left(\frac{du_1}{dy_2}\right)^2 \int \frac{\partial^2 G(\underline{Y}\lambda_1, T = 0)}{\partial T^2} \, d\Delta_1 \qquad (4.40)$$
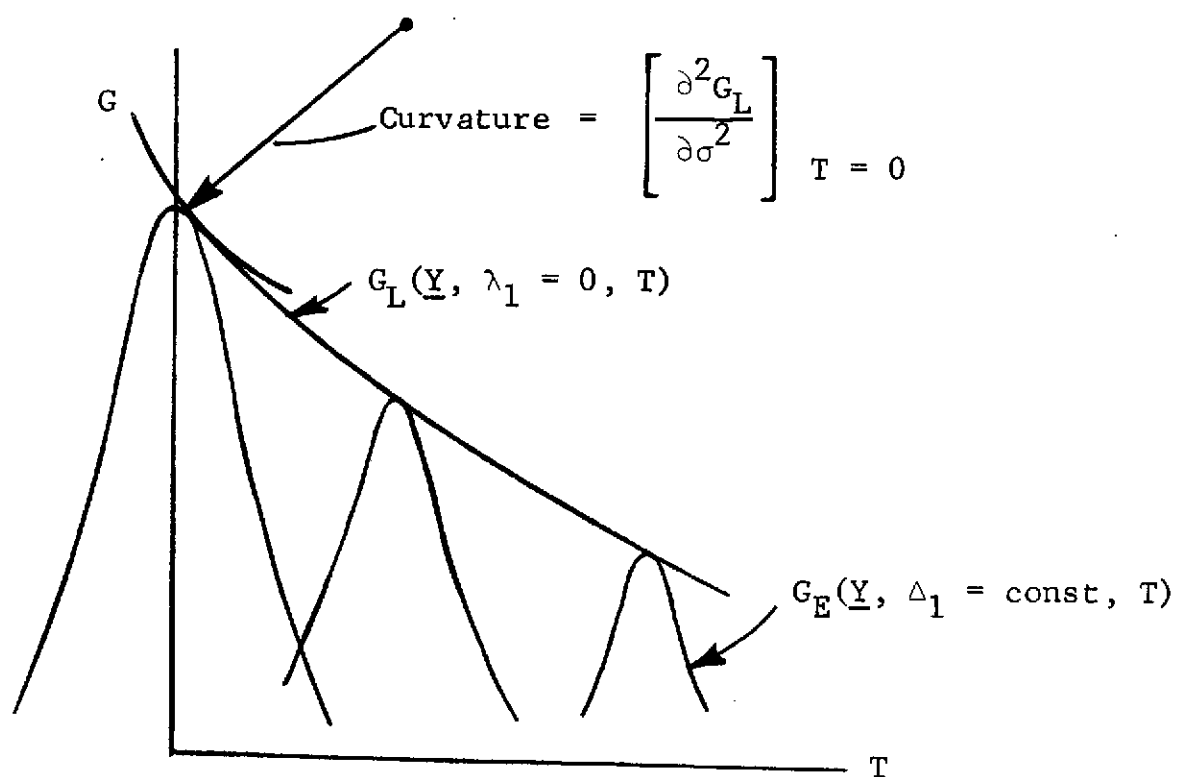
Figure 2   EULERIAN CROSS-COVARIANCES G_E, AND
LAGRANGIAN AUTOCOVARIANCE G_L

For purposes of evaluating this integral we can let $d\Delta_1 = U_c dT$ by Taylor's Hypothesis and equation (4.40) can be written

$$S(\underline{Y},t,0) = a_o^4 \left(\frac{du_1}{dy_2}\right)^2 U_c \int \frac{\partial^2 G_L(\underline{Y},0,0)}{\partial\sigma^2} \, dT \qquad (4.41)$$

We have used the time variable σ to indicate Lagrangian time where T refers to Eulerian. The integrand

$$\frac{\partial^2 G_L(\underline{Y},0_1,0)}{\partial\sigma^2}$$

is obtained from the curvature of $G_L(\underline{Y},\lambda_1,0)$ in the sketch. In general $G_L(\underline{Y},\lambda_1,T)$ varies slowly and its curvature is very small necessitating very accurate measurements at each value of λ_1. There are always experimental variations in the heights of the covariances at each value of Δ_1 and these may make derivative measurements impossible. Another approach is to differentiate the signals before correlating, to give

$$\frac{\partial^2 G_E(\underline{Y},\Delta_1,\tau)}{\partial T^2} \quad \text{directly.}$$

Here we are faced with the problem of accurately differentiating the signals. Only experience and a statistical error analysis can tell us which approach is the more accurate, although analysis by Krause in Reference 12, offers the possibility that statistical variation can actually be reduced by proper handling of the differentiated signals.

If $G_L(\underline{Y},\lambda_1,T)$ keeps a constant width as T increases, i.e., just decreases in amplitude, then

$$\frac{\partial^2 G_L}{\partial\sigma^2} = \frac{\partial G_L}{\partial\sigma} = 0 \quad \text{where } G_L = 0,$$

and we are probably not too wrong in taking

$$\frac{\partial^2 G_L(\underline{Y},\lambda_1,0)}{\partial\sigma^2} \bigg/ \frac{\partial^2 G_L(\underline{Y},0,0)}{\partial\sigma^2} = G_E(\underline{Y},\Delta_1,0) \bigg/ G_E(\underline{Y},0,0)$$

Then equation (4.41) becomes

$$S(\underline{Y},t,0) = M_c\ a_o^5 \left(\frac{du_1}{dy_2}\right)^2 \left[\frac{\partial^2 G_L(\underline{Y},\lambda_1 = 0,\sigma)}{\partial\sigma^2}\right]_{\sigma=0} \int \frac{G_E(\underline{Y},\Delta_1,T)}{G_E(\underline{Y},0,0)}\ dT$$

$$= M_c\ a_o^5 \left(\frac{du_1}{dy_2}\right)^2\ L_T\left[\frac{\partial^2 G_L(\underline{Y},\lambda_1, = 0,\sigma)}{\partial\sigma^2}\right]_{\sigma=0} \qquad (4.42)$$

where L_T is an Eulerian time scale. Whether approximations like these are valid are only determinable by actual experimental measurement.

Homogeneous Turbulence

Where there is no shear amplification the preceding analysis is still valid but the difficulty in taking the fourth derivative of the measured covariance becomes even more of a problem. It would seem, at this point, that the inaccuracies which would result would be difficult if not impossible to overcome.

4.3.2.2 High Speed Jets

In the Mach emission case

$$S(\underline{Y},t,0) = -\ (1 - M\ \cos\theta)^5 \left(\frac{du_1}{dy_2}\right)^2 \int\int \left[\frac{\partial^2 G_E(\underline{Y},\Delta_r,T)}{\partial T^2}\right]_{T=\Delta_r/a_o}\ d\sigma$$

$$(4.29)$$

Here, we must measure the curvature of the Eulerian covariance or the derivative covariance and integrate along the Lagrangian time variable. This can be done as sketched in Figure 3.

The lifetime $\sigma*$ in Lagrangian time must be sufficiently short that $y_2 = \Delta_1 \sin\theta = a_o \sigma* \sin\theta$ is small enough that du_1/dy_2 does not vary appreciably over that distance. If it does, then the mean gradient must be left under the integral in equation (4.29).

We have in this case somewhat of a lesser problem in taking the second derivative of G, since its curvature is much greater than that of G_L. Although $\partial^2 G_E/\partial T^2$ probably decreases as σ increases, as long as $\sigma*$ is not too long we can approximate equation (4.29) by

$$S(\underline{Y},t,0) = - (1 - M \cos\theta)^5 \, a_o^5 \left(\frac{du_1}{dy_2}\right)^2 \sigma* \left[\frac{\partial^2 G_E(\underline{Y},0,T)}{\partial T^2}\right]_{T=0} \quad (4.43)$$

Then we need only to measure the curvature of the crossed-beam autocovariance and the lifetime. Also, we can approximate the integrand by

$$\frac{\partial^2 G_E(\underline{Y},0,T)}{\partial T^2} = a_o^2 \, \frac{\partial^2 G_E(\underline{Y},\Delta_1,T)}{\partial \Delta_1^2}$$

$$= a_o^2 \, G_s(\underline{Y},\Delta_1,T) \quad (4.44)$$

where G_s is a gradient covariance,

$$G_s = \left\langle \frac{\partial \rho(\underline{Y},0,t)}{\partial y_r} \, \frac{\partial \rho(\underline{Y},0,t + \tau)}{\partial y_r} \right\rangle$$
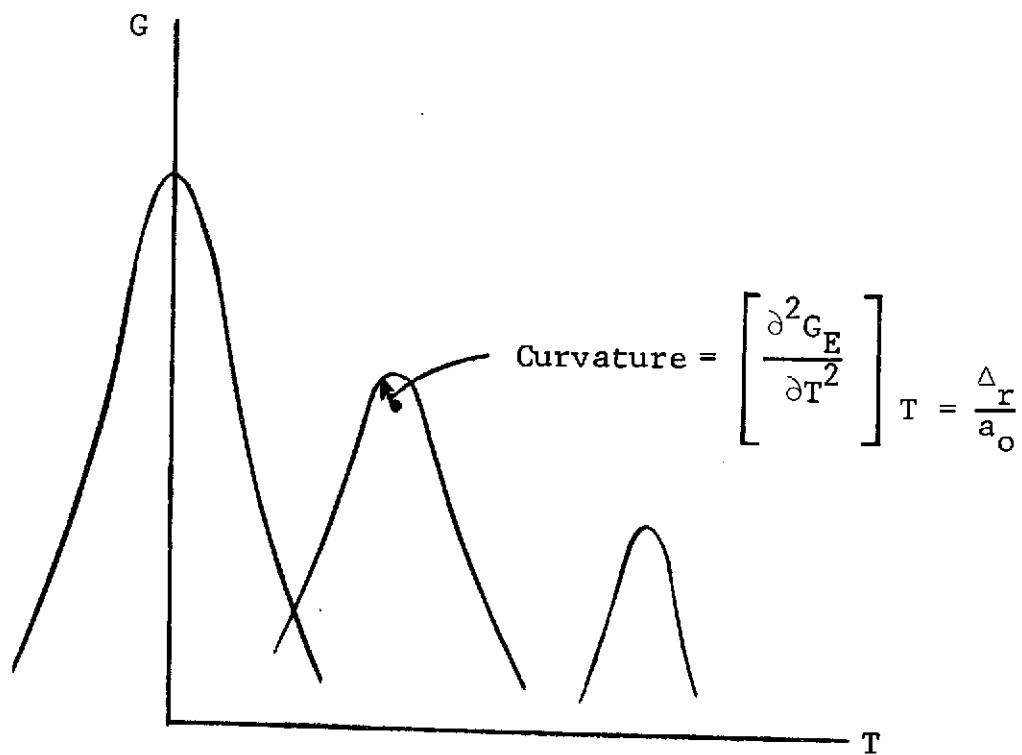
and is measurable by crossed-Schlieren techniques.[12]

Figure 3   CURVATURE OF EULERIAN CROSS-COVARIANCE
REQUIRED FOR MACH EMISSION CASE

Then equation (4.43) becomes

$$S(\underline{Y},t,0) = -(1 - M\cos\theta)^5 \, a_o^7 \left(\frac{du_1}{dy_2}\right) \sigma^* \, G_s(\underline{Y},0,0) \tag{4.45}$$

4.3.2.3 General Case

In the general case we have considered a frame of reference (y_r, y_2, y_3) with y_r in the direction of the observer and left undecided whether the reference frame should be altered to move at the speed of sound a_o or the convection speed component $a_o M \cos\theta$. The difference between these can be seen more clearly by reference to the sketch in Figure 4, showing contours of $G_E(\underline{Y}, \Delta_1, T)$.

Assuming that the stress tensor T_{rr} were relatable to ρ (which is not true in general but is at low speeds), and that we can take the time delay derivatives outside of the area integral, (which is not justified, section 4.2.3) we are then concerned with integrals of the form

$$S \sim \int \frac{\partial^n G(\underline{Y}, \lambda_r, T)}{\partial T^n} \, d\lambda_r$$

If λ_r is taken to move at sonic convection speed, $\lambda_r = \Delta_1 - a_o T$, and we must integrate along lines parallel to $\Delta_1 = -a_o MT$ in the sketch to satisfy retarded time requirements.

If λ_r is taken to move at the particle convection speed then we integrate along lines parallel to $\Delta_1 = a_o MT$, to minimize spatial gradients.

From the sketch we can see that integration along $\lambda_r = \Delta_r = a_o MT$ minimizes time scales whereas integration along $\lambda_r = \Delta_r - a_o T$ can give very short time scales resulting from convection effects. Only at $M\cos\theta = 1$, as in the sketch in
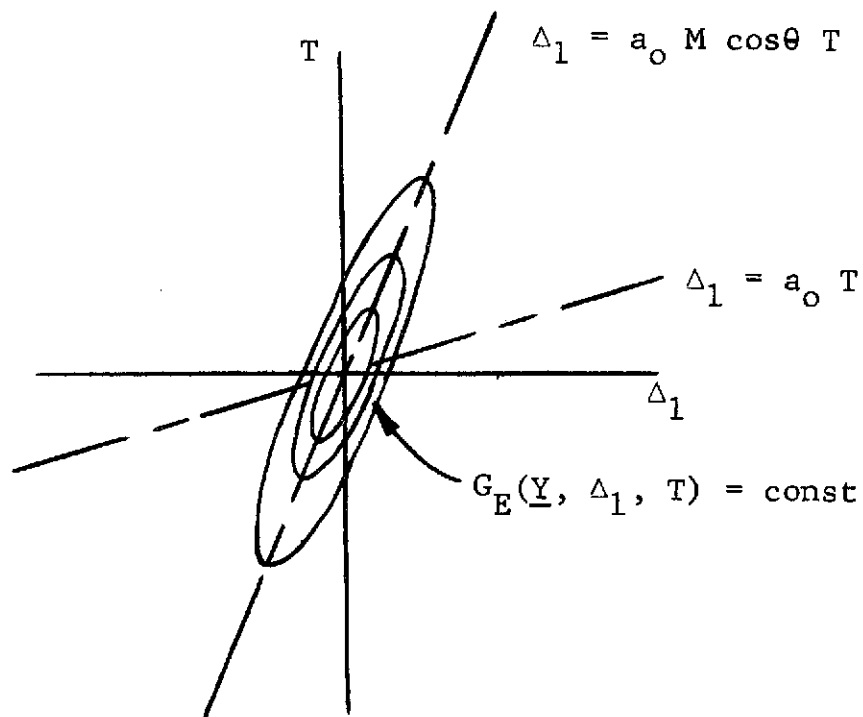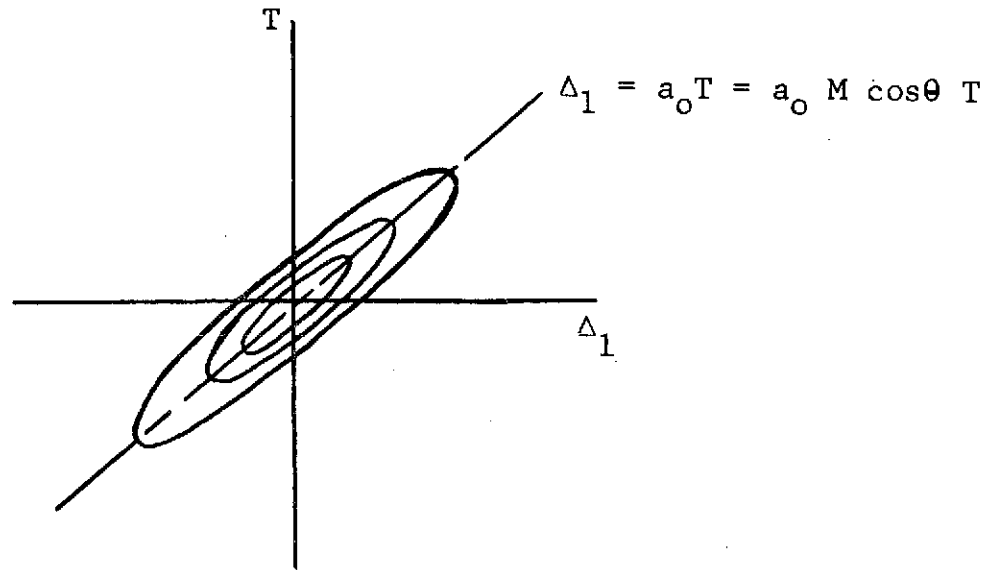
Figure 4   EFFECT OF MOTION OF OBSERVERS REFERENCE
FRAME ON EULERIAN COVARIANCE

Figure 5 are both requirements met. At $a = \infty$, the time delay becomes unimportant, (see Figure 5) since changes in T are very small. To assess which integration path should be used we need a complete knowledge of $G(\underline{Y}, \Delta_r, T)$ or at least an independent measurement of gradient fluctuations.

Integration Paths for Mach Emission ($M \cos\theta = 1$)
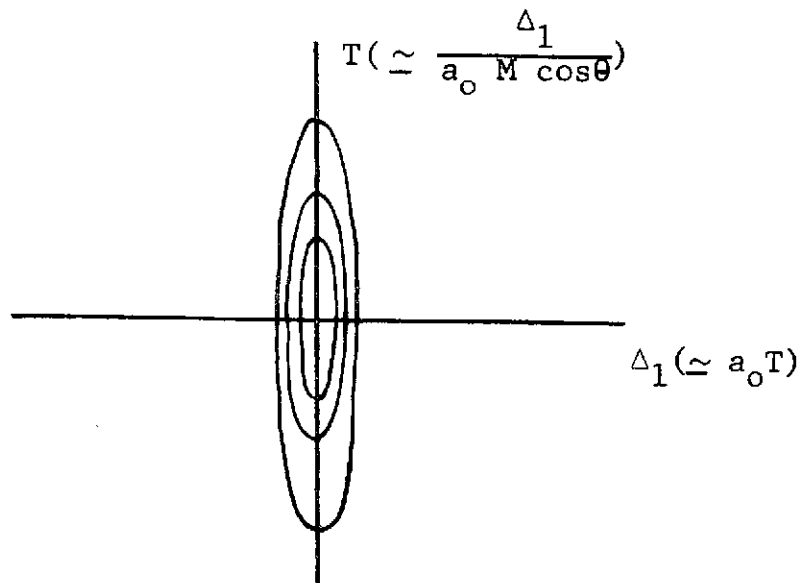
Integration Paths for Low Speed Case ($a_o = \infty$)

Figure 5   INTEGRATION PATHS FOR $M = 1$ AND $M = 0$ CASES

V. DISCUSSION

The problems associated with applying crossed-beam technology to the measurement of sound source intensities have been found to lie in the following general areas:

1. Measurement of correct covariance
2. Retarded time effects
3. Moving frame of reference

Another area, beyond the scope of this paper involves the applicability of point-area correlations and must be proven by experiments. Under certain conditions the main problems can be circumvented in principle at least.

Use of the infrared system holds promise of giving measurements directly relatable to density statistics. This does however limit us to sound source integral forms which are expressible in terms of density covariances. It has been suggested [11] that this representation may result in slowly converging integrals and therefore require good measurement accuracy.

Retarded time and moving axes requirements are made less stringent in two special limiting cases, viz., $M = 0$, and $M \cos\theta - 1$. In the first of these τ the retarded time variable can be considered everywhere constant and in the second we require only that τ = constant in the "plane" of the point-plane representation. This latter can be achieved by orienting the crossed beams both in the plane of the Mach front.

The moving frame of reference or Lagrangian covariances can be obtained from Eulerian covariances. The main difficulty lies in the requirement of obtaining a great many experimental measurements to map out fully $G_E(\Delta_1, T)$ and $G_L(\lambda_1, T)$. The time derivatives in the sound source integrals pose another possible

restriction in accuracy required of measurement. To obtain

$$\frac{\partial^4 G_L}{\partial T^4}$$

accurately may require a great many experimental points before differentiating. This is alleviated somewhat by the high-shear approximation, i.e., determining

$$\frac{\partial^2 G_L}{\partial T^2}$$

the second derivative rather than the fourth derivative.

The residual effects of density-gradient fluctuation in the convected reference frame may also pose a major problem if they are of the same order of magnitude as the "true" time derivatives. It is suggested that these be estimated from crossed-schlieren results.

An attempt to obtain a unified form of the sound source integral applicable to crossed-beam measurements at all speeds has been unsuccessful here. Opposing requirements on retarded time and space-derivative minimization are a major factor in this conclusion. Also, representation of the stress tensor in terms of a parameter (like density) which can be measured by crossed-beam methods appears to be impossible at other than $M = 0$ and $M \cos\theta = 1$. However it is believed that measurements using the $M = 0$ and $M \cos\theta = 1$ approximation should cover nearly the complete speed range of interest. Most serious of all assumptions made for low speed studies, is the assumption that the jet is isothermal. The range of validity of this approximation must be determined. Also further analysis on entropy fluctuation measurement must be made.

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